Chapter 03

MOTION IN A PLANE AND RELATIVE MOTION

1. INTRODUCTION TO MOTION IN A PLANE

1.1 Position Vector and displacement

The position vector $\vec{r}$ of a particle P located in a plane with reference to the origin of an x-y coordinate system is given by $\vec{r} = x\hat{i} + y\hat{j}$

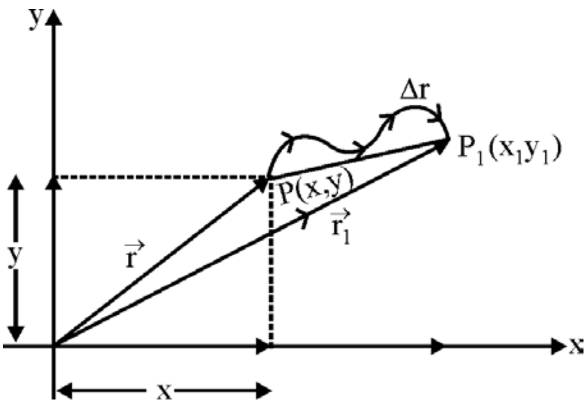


Fig. 3.1

Suppose the particle moves along the path as shown to a new position P_1 with the position vector $\vec{r}$

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$$

change in position = displacement

$$= \vec{r}_1 - \vec{r} = \left(x_1\hat{i} + y_1\hat{j}\right) - \left(x\hat{i} + y\hat{j}\right)$$

(By vector addition)

$$= (x_1 - x)\hat{i} + (y_1 - y)\hat{j}$$

$$= \Delta x\hat{i} + \Delta y\hat{j}$$

from above figure we can see that

$$\Delta\vec{r} = \vec{r}_1 - \vec{r}$$

1.2 Average Velocity

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t}$$

$$v_{avg} = \Delta v_x\hat{i} + \Delta v_y\hat{j}$$

NOTE:

Direction of the average velocity is same as that of $\Delta\vec{r}$

1.3 Instantaneous Velocity

$$\vec{v} = \lim_{\Delta t \to 0}\frac{\Delta v}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

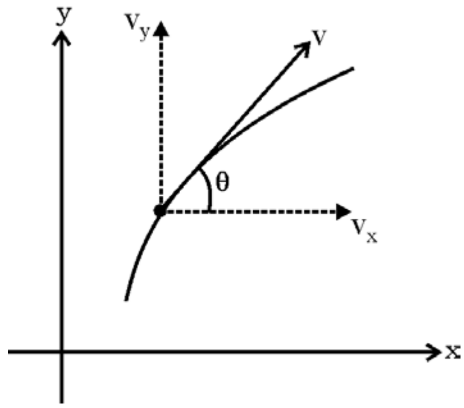


Fig. 3.2

where, $v_x = \frac{dx}{dt}$ and $v_y = \frac{dy}{dt}$

$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$, $|\vec{v}|$ represents speed/magnitude of velocity and

$$\tan\theta = \frac{v_y}{v_x}$$

or $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

NOTE:

The direction of instantaneous velocity at any point on the path of an object is tangent to the path at that point and is in the direction of motion.

1.4 Average Acceleration

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

$$\vec{a}_{avg} = a_x\hat{i} + a_y\hat{j}$$

1.5 Instantaneous Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

2. PROJECTILE MOTION

2.1 Introduction

When a particle is projected obliquely from the earth's surface, it moves simultaneously in horizontal and vertical directions in a curved trajectory as depicted in the diagram under. Motion of such a particle is called projectile motion.

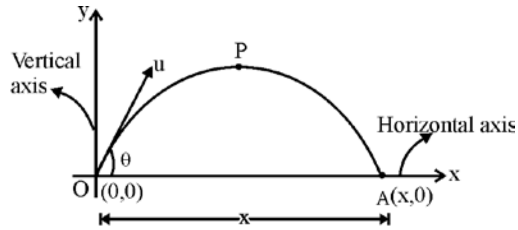


Fig. 3.3

2.2 Parameters in Projectile Motion

In this case a particle is projected at an angle θ with an initial velocity u. For this particular case we will calculate the following:

(a) time taken to reach A from O

(b) horizontal distance covered (OA)

(c) maximum height reached during the motion

(d) velocity at any time 't' during the motion

Horizontal axis	Vertical axis
$u_x = u\cos\theta$ $a_x = 0$ (In the absence of any external force a_x is assumed to be zero)	$u_y = u\sin\theta$ $a_y = -g$ $s_y = u_y t + \frac{1}{2}a_y t^2$ when the particle returns to same horizontal level, vertical displacement is 0 and time taken is called **time of flight (T)**, $0 = u\sin\theta T - \frac{1}{2}gT^2$ $T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$
$s_x = u_x t + 1/2a_x t^2$ $x - 0 = u\cos\theta\, t$ $x = u\cos\theta \times 2u_y/g$ $x = \frac{2u^2\cos\theta\sin\theta}{g}$ $(2\cos\theta\sin\theta = \sin 2\theta)$ $x = \frac{u^2\sin 2\theta}{g}$ horizontal distance covered is known as **Range**	$v_y = u_y + a_y t$ It depends on time 't' Its magnitude first decreases and then becomes zero and then increases.

Horizontal axis	Vertical axis
$v_x = u_x + a_x t$ $v_x = u\cos\theta$ It is independent of t and is constant	**Maximum height** attained by the particle Method 1: Using time of ascent time of ascent, $t = \frac{u\sin\theta}{g}$ $s_y = u_y t + \frac{1}{2}at^2$ $= u\sin\theta \times \frac{u\sin\theta}{g}$ $- \frac{1}{2}g\frac{u^2\sin^2\theta}{g^2}$ $H = \frac{u^2\sin^2\theta}{2g}$
Time of ascent = Time of descent At topmost point $y = 0$ $\Rightarrow 0 = u\sin\theta - gt$ $t_1 = \frac{u\sin\theta}{g}$ $t_2 = T - t_1 = \frac{u\sin\theta}{g}$ $t_1 = t_2 = \frac{T}{2} = \frac{u\sin\theta}{g}$	**Maximum height** attained by the particle Method 2: Using third equation of motion $v_y^2 - u_y^2 = 2a_y s_y$ $H = \frac{u^2\sin^2\theta}{2g}$

2.3 Maximum Range

$$R = \frac{u^2\sin 2\theta}{g} \text{ and } R_{max} = \frac{u^2}{g}$$

Range is maximum when sin 2θ is maximum

maximum value of sin 2θ = 1 **or**, θ = 45°

2.4 Analysis of Velocity in Case of a Projectile

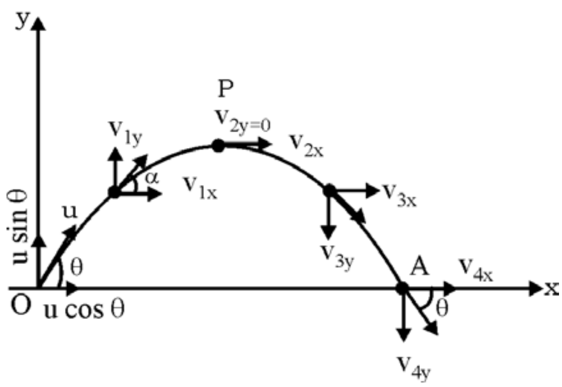


Fig. 3.4

From the above equations.

(i) $v_{1x} = v_{2x} = v_{3x} = v_{4x} = u_x = u\cos\theta$

which means that the velocity along x-axis remains constant

[as there is no external force acting along that direction]

(ii)

(a) magnitude of velocity along y-axis first decreases and then it increases after the topmost point

(b) at topmost point magnitude of velocity is zero.

(c) direction of velocity is in the upward direction while ascending and is in the downward direction while descending.

(d) magnitude of velocity at A is same as magnitude of velocity at O; but the direction is changed

(e) angle which the net velocity makes with the horizontal can be calculated by

$$\tan\alpha = \frac{v_y}{v_x} = \frac{\text{velocity along } y-\text{axis}}{\text{velocity along } x-\text{axis}}$$

net velocity is always along the tangent.

2.5 Equation of Trajectory

Trajectory is the path traced by the body. To find the trajectory we must find relation between y and x by eliminating time.

[Ref. to the earlier diag.]

Horizontal Motion	Vertical Motion
$u_x = u\cos\theta$ $a_x = 0$ $s_x = u\cos\theta t = x$ $t = \dfrac{x}{u\cos x}$	$u_y = u\sin\theta$ $a_y = -g$ $s_y = u_y t + a_y t^2$ $y = u\sin\theta\left(\dfrac{x}{u\cos\theta}\right)$ $-\dfrac{1}{2}g\dfrac{x^2}{u^2\cos^2\theta}$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \Rightarrow y = bx - ax^2$$

(i) This is an equation of a parabola

(ii) Because the coefficient of x^2 is negative, it is an inverted parabola.

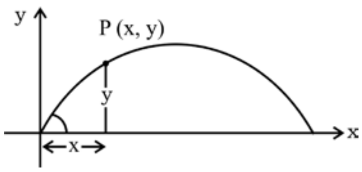


Fig. 3.5

Path of the projectile is a parabola

$$R = \frac{2u^2\sin\theta\cos\theta}{g} \text{ or } \frac{2u^2}{g} = \frac{R}{\sin\theta\cos\theta}$$

Substituting this value in the above equation we have,

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

3. PROJECTILE MOTION FROM A HEIGHT

3.1 Horizontal Direction:

(i) Initial velocity $u_x = u$

(ii) Acceleration $a_x = 0$

3.2 Vertical Direction:

(i) Initial velocity $u_y = 0$

(ii) Acceleration $a_y = -g$ (downward)

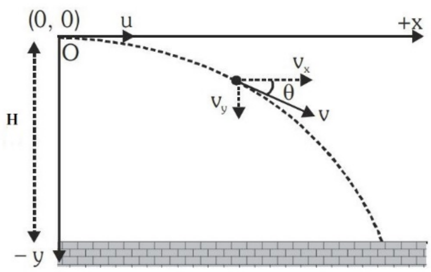


Fig. 3.6

The path traced by projectile is called its trajectory.

After time t,

Horizontal displacement x = ut

Vertical displacement $y = -\frac{1}{2}gt^2$

(Negative sign indicates that the direction of vertical displacement is downward.)

So $y = \frac{1}{2}g\frac{x^2}{u^2}\left(\because t = \frac{x}{u}\right)$ this is equation of a parabola

Above equation is called trajectory equation.

The equations for this type motion will be:

- Time of flight

$$T_f = \sqrt{\frac{2h}{g}}$$

- Horizontal Range

$$R = u_x t = u\sqrt{\frac{2h}{g}}$$

- Trajectory Equation

$$y = \frac{1}{2}g\frac{x^2}{u^2}\left(\because t = \frac{x}{u}\right)$$

This is equation of parabola

- Along vertical direction

$$v_y^2 = 0^2 + 2(h_1)(g)$$

$$v_y = \sqrt{2gh_1}$$

Along horizontal direction:

$$v_x = u_x = u$$

So, velocity

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh_1}$$

4. PROJECTILE ON AN INCLINE

- Motion of a particle along upward the inclined plane.

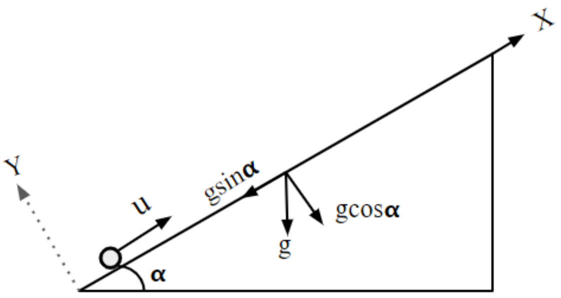


Fig. 3.7

$$u_x = u$$

$$a_x = -g\sin\alpha$$

$$v_x = u - (g\sin\alpha)t$$

$$x = ut - \frac{1}{2}(g\sin\alpha)t^2$$

$$u_y = 0$$

$$a_y = -g\cos\alpha$$

$$v_y = 0$$

$$y = 0$$

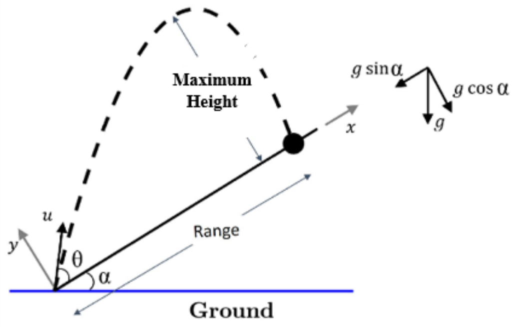


Fig. 3.8

Projectile up an inclined plane	
Motion along x-axis	**Motion along y-axis**
$u_x = u\cos\theta$ $a_x = -g\sin\alpha$ $v_x = u\cos\theta - g\sin\alpha t$ $x = u\cos\theta t - \frac{1}{2}g\sin\alpha t^2$	$u_y = u\sin\theta$ $a_y = -g\cos\alpha$ $v_y = u\sin\theta - g\cos\alpha t$ $y = u\sin\theta t - \frac{1}{2}g\cos\alpha t^2$

(i) Projectile up the plane

- **Time of Flight**

At point B displacement along y-direction is zero. So, substituting the proper values in $s_y = u_y t + \frac{1}{2}a_y t^2$, we get

$$0 = u\sin\theta t + \frac{1}{2}(-g\cos\alpha)t^2 \Rightarrow \therefore t = 0 \text{ and } \frac{2u\sin\theta}{g\cos\alpha}$$

t = 0, corresponds to initial point and $t = \frac{2u\sin\theta}{g\cos\alpha}$ corresponds to final point.

Thus, $T = \frac{2u\sin\theta}{g\cos\alpha}$

- **Range**

Horizontal component of initial velocity is $u_x = u\cos\theta$

Range $(R) = u_x T - \frac{1}{2}g\sin\alpha T^2$

$$= u\cos\theta\frac{2u\sin\theta}{g\cos\alpha} - \frac{1}{2}g\sin\alpha\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2$$

$$= \frac{2u^2\sin\theta\cos\theta}{g\cos\alpha} - \sin\alpha\frac{2u^2\sin^2\theta}{g\cos^2\alpha}$$

$$= 2u^2\sin\theta\left(\frac{\cos\alpha\cos\theta - \sin\alpha\sin\theta}{g\cos^2\alpha}\right)$$

$$= \frac{2u^2\sin\theta\cos(\alpha+\theta)}{g\cos^2\alpha}$$

Using, $\sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)$,

Range can also be written as,

$$R = \frac{u^2}{g\cos^2\alpha}\left[\sin(2\theta+\alpha) - \sin\alpha\right]$$

This range will be maximum when

$$2\theta + \alpha = \frac{\pi}{2} \text{ or } \theta = \frac{\pi}{4} - \frac{\alpha}{2} \text{ and}$$

$$R_{max} = \frac{u^2}{g\cos^2\alpha}[1 - \sin\alpha]$$

Here, also we can see that for $\alpha = 0$, range is maximum at $\theta = \frac{\pi}{4}$ or $\theta = 45°$

And $R_{max} = \frac{u^2}{g\cos^2 0°}(1 - \sin 0°) = \frac{u^2}{g}$

(ii) Projectile down the plane

- Motion of a particle along the downward inclined plane.

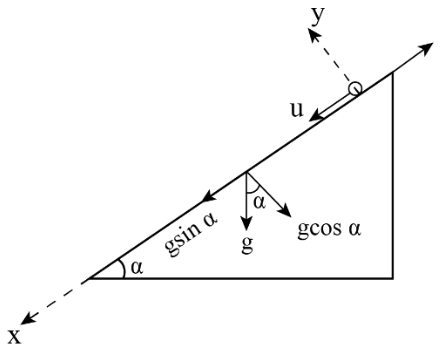


Fig. 3.9

$u_x = u$
$a_x = +g\sin\alpha$
$v = u + (g\sin\alpha)t$
$x = ut + \frac{1}{2}(g\sin\alpha)t$
$u_y = 0$
$a_y = -g\cos\alpha$
$v_y = 0$
$y = 0$

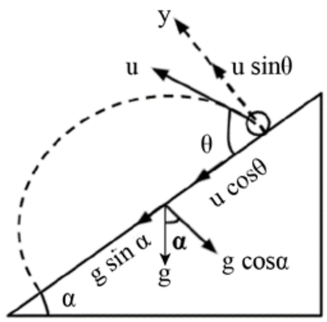


Fig. 3.10

Projectile down an inclined plane	
Motion along x-axis	**Motion along y-axis**
$u_x = u\cos\theta$ $a_x = g\sin\alpha$ $v_x = u\cos\theta + g\sin\alpha t$ $x = u\cos\theta t + \frac{1}{2}g\sin\alpha t^2$	$u_y = u\sin\theta$ $a_y = g\cos\alpha$ $v_y = u\sin\theta - g\cos\alpha t$ $y = u\sin\theta t - \frac{1}{2}g\cos\alpha t^2$

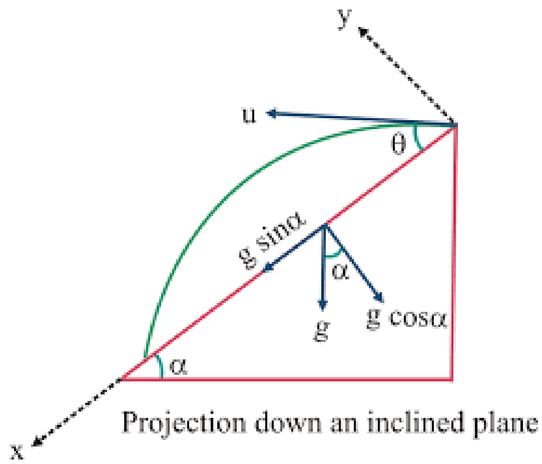


Fig. 3.11

Here, x and y-directions are down the plane and perpendicular to plane respectively as shown in figure.

Hence, $u_x = u\cos\theta, a_x = g\sin\alpha$

$u_y = u\sin\theta, a_y = -g\cos\alpha$

Proceeding in the similar manner, we get the following results:

- $T = \dfrac{2u\sin\theta}{g\cos\alpha}, R = \dfrac{u^2}{g\cos^2\alpha}\left[\sin(2\theta-\alpha)+\sin\alpha\right]$

5. RELATIVE MOTION

Relative is a very general term. In physics we use relative very often.
For e.g.

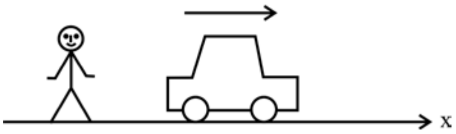


Fig. 3.12

Case I: If you are stationary and you observe a car moving on a straight road then you say velocity of car is 20 m/s which means velocity of car relative to you is 20 m/s or, velocity of car relative to the ground is 20 m/s.
(As you are stationary on the ground.)

Case II: If you go inside a car and observe you will find that the car is at rest while the road is moving backwards. you will say:
velocity of car relative to the you is 0 m/s
Mathematically, velocity of B relative to A is represented as

$\boxed{\vec{v}_{BA} = \vec{v}_B - \vec{v}_A}$

This being a vector quantity direction is very important

Velocity of Approach / Separation

- It is the component of relative velocity of one particle with respect to another, along the line joining them.

- If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation.

- In one dimension, since relative velocity is along the line joining A and B, hence velocity of approach /separation is simply equal to magnitude of relative velocity of A with respect to B.

Velocity of approach / separation in two dimensions

- It is the component of relative velocity of one particle with respect to another, along the line joining them.

- If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation.

6. RIVER–BOAT PROBLEMS

In river–boat problems we come across the following three terms:

$\vec{v}_r =$ absolute velocity of river.

$\vec{v}_{br} =$ velocity of boatman with respect to river and

$\vec{v}_b =$ absolute velocity of boatman.

Hence, it is important to note that $\vec{v}_{br}$ is the velocity of boatman with which he steers and $\vec{v}_b$ is the actual velocity of boatman relative to ground. Further $\vec{v}_b = \vec{v}_{br} + \vec{v}_r$

Now, let us derive some standard results and their special cases.

A boatman starts from point A on one bank of a river with velocity $\vec{v}_{br}$ in the direction shown in figure. River is flowing along positive x–direction with velocity $\vec{v}_r$. Width of the river is d. Then

$\vec{v}_b = \vec{v}_r + \vec{v}_{br}$

Therefore, $v_{bx} = v_{rx} + v_{brx} = v_r - v_{br}\sin\theta$

and $v_{by} = v_{ry} + v_{bry} = 0 + v_{br}\cos\theta = v_{br}\cos\theta$

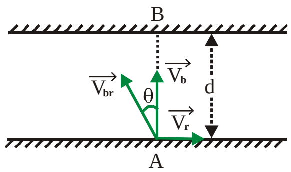


Fig. 3.13

Now, time taken by the boatman to cross the river is:

Further, displacement along x–axis when he reaches on the other bank (also called drift) is

$$\text{or } x = (v_r - v_{br}\sin\theta)\frac{d}{v_{br}\cos\theta} \qquad ...(ii)$$

Condition when the boatman crosses the river in shortest interval of time

From eq. (i) we can see that time (t) will be minimum when θ = 0° i.e., the boatman should steer his boat perpendicular to the river current.

Condition when the boatman wants to reach point B, i.e., at a point just opposite from where he started (shortest distance)

In this case, the drift (x) should be zero.

$\therefore x = 0$

$$\text{or} \quad (v_r - v_{br}\sin\theta)\frac{d}{v_{br}\cos\theta} = 0 \text{ or } v_r = v_{br}\sin\theta$$

$$\text{or} \quad \sin\theta = \frac{v_r}{v_{br}} \text{ or } \theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$$

Hence, to reach point B the boatman should row at an angle $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ upstream from AB.

$$\boxed{t = \frac{d}{v_b} = \frac{d}{\sqrt{v_{br}^2 - v_r^2}}}$$

Since $\sin\theta \not> 1$ So, if $v_r \geq v_{br}$, the boatman can never reach at point B. Because if $v_r = v_{br}$, $\sin\theta = 1$ or $\theta = 90°$ and it is just impossible to reach at B if $\theta = 90°$. Similarly, if $v_r > v_{br}$, $\sin\theta > 1$, i.e., no such angle exists. Practically it can be realized in this manner that it is not possible to reach at B if river velocity (v_r) is too high.

7. RELATIVE VELOCITY OF RAIN W.R.T THE MOVING MAN

Consider a man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$, represented by $\overrightarrow{OB}$ as shown in figure. To find the relative velocity of rain with respect to man (i.e., $\vec{v}_{rm}$.) bring the man at rest by imposing a velocity $-\vec{v}_m$ on man and apply this velocity on rain also. Now the relative velocity of rain with respect to man will be the resultant velocity of $\vec{v}_r\left(=\overrightarrow{OB}\right)$ and $-\vec{v}_m\left(=\overrightarrow{OC}\right)$, which will be represented by diagonal $\overrightarrow{OD}$ of rectangle OBDC.

$$\therefore \quad v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90°} = \sqrt{v_r^2 + v_m^2}$$

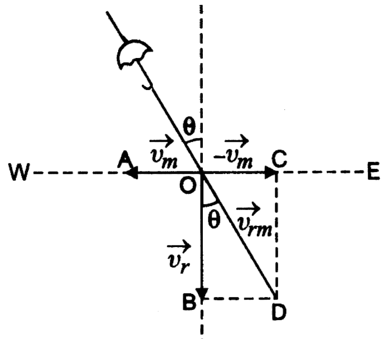


Fig. 3.14

If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction, then

$$\tan\theta = \frac{OD}{OB} = \frac{v_m}{v_r} \text{ or } \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

Here, angle θ is the angle that $\vec{v}_{rm}$ makes w.r.t vertical.

NOTE:

In the above problem if the man wants to protect himself from the rain, he should hold his umbrella in the direction of relative velocity of rain with respect to man i.e., the umbrella should be held making an angle θ (= $\tan^{-1}$ $(v_m/v)_r$) west of vertical.

Solved Examples

Example - 1

Velocity and acceleration of a particle at time t = 0 are $\vec{u} = (2\hat{i} + 3\hat{j})\,m/s$ and $\vec{a} = (4\hat{i} + 2\hat{j})\,m/s^2$ respectively. Find the velocity and displacement of particle at t = 2s.

Sol. Here, acceleration $\vec{a} = (4\hat{i} + 2\hat{j})\,m/s^2$ is constant.

So, we can apply

$$v = u + at \text{ and } s = ut + \frac{1}{2}at^2$$

Substituting the proper values, we get

$$\vec{v} = (2\hat{i} + 3\hat{j}) + (2)(4\hat{i} + 2\hat{j})$$

$$= (10\hat{i} + 7\hat{j})\,m/s$$

and $\vec{s} = (2)(2\hat{i} + 3\hat{j}) + \frac{1}{2}(2)^2(4\hat{i} + 2\hat{j})$

$$= (12\hat{i} + 10\hat{j})\,m$$

Therefore, velocity and displacement of particle at t = 2s are $(10\hat{i} + 7\hat{j})\,m/s$ and $(12\hat{i} + 10\hat{j})\,m$ respectively.

Example - 2

Velocity of a particle in x-y plane at any time t is $\vec{v} = (2t\hat{i} + 3t^2\hat{j})\,m/s$

At t = 0, particle starts from the co-ordinates (2m, 4m). Find acceleration of the particle at t = 1s.

Sol. $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(2t\hat{i} + 3t^2\hat{j})$

$$= (2\hat{i} + 6t\hat{j})\,m/s^2$$

At t = 1s,

$$\vec{a} = (2\hat{i} + 6\hat{j})\,m/s^2$$

Example - 3

Anoop is moving due east with a velocity of 1 m/s and Dhyani is moving due west with a velocity of 2 m/s. What is the velocity of Anoop with respect to Dhyani?

Sol. It is a one dimensional motion. So, let us choose the east direction as positive and the west as negative.

Now, given that

v_A = velocity of Anoop = 1 m/s

and v_D = velocity of Dhyani $= -2\,m/s$

Thus, v_{AD} = velocity of Anoop with respect to Dhyani

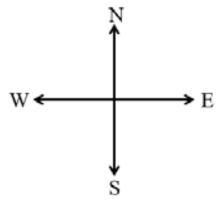
$$= v_A - v_D = 1 - (-2) = 3\,m/s$$

Hence, velocity of Anoop with respect to Dhyani is 3 m/s due east.

Example - 4

Car A has an acceleration of 2 m/s^2 due east and car B, 4 m/s^2 due north. What is the acceleration of car B with respect to car A?

Sol.

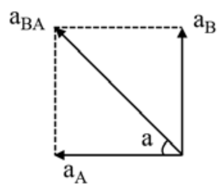


It is a two dimensional motion.

Therefore,

a_{BA} = acceleration of car B with respect to car A

$$= a_B - a_A$$



Here, a_B = acceleration of car B

$= 4\,m/s^2$ (Due north)

And a_A = acceleration of car A

$= 2\,m/s^2$ (Due east)

$$|a_{BA}| = \sqrt{(4)^2 + (2)^2} = 2\sqrt{5}\,m/s^2$$

and $\alpha = \tan^{-1}\left(\frac{4}{2}\right) = \tan^{-1}(2)$

Thus, a_{BA} is $2\sqrt{5}\,m/s^2$ at an angle of $\alpha = \tan^{-1}(2)$ from west towards north.

Example – 5

Car A and car B starts moving simultaneously in the same direction along the line joining them. Car A moves with a constant acceleration $a = 4m/s^2$, while car B moves with a constant velocity $v = 1m/s$. At time t = 0, car A is 10 m behind car B. Find the time when car A overtakes car B.

Sol.

Given, $u_A = 0, u_B = 1m/s, a_A = 4m/s^2$ and $a_B = 0$

Assuming car A to be at rest, we have

$u_{AB} = u_A - u_B = 0 - 1 = -1m/s$

$a_{AB} = a_A - a_B = 4 - 0 = 4m/s^2$

Now, the problem can be assumed in simplified form as shown below.

Substituting the proper values in equation $s = ut + \frac{1}{2}at^2$,

We get $10 = -t + \frac{1}{2}(4)(t^2)$

Or $2t^2 - t - 10 = 0$

Or $t = \frac{1 \pm \sqrt{1+80}}{4}$

$= \frac{1 \pm \sqrt{81}}{4}$

$= \frac{1 \pm 9}{4}$

Or t = 2.5 s and – 2s

Ignoring the negative value, the desired time is 2.5 s.

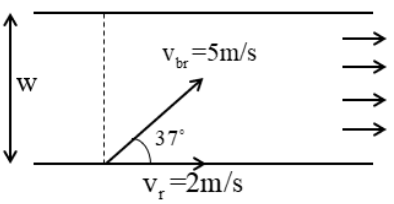
Example – 6

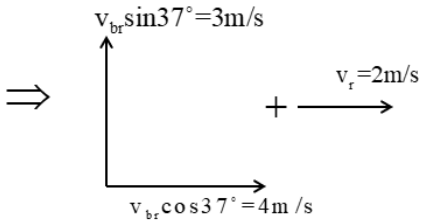
Width of a river is 30 m, river velocity is 2 m/s and rowing velocity is 5 m/s at 37° from the direction of river current

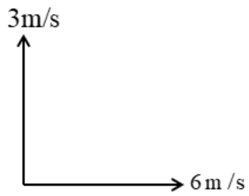
(a) Find the time taken to cross the river,

(b) Drift of the boatman while reaching the other shore.

Sol.







Net velocity of boatman

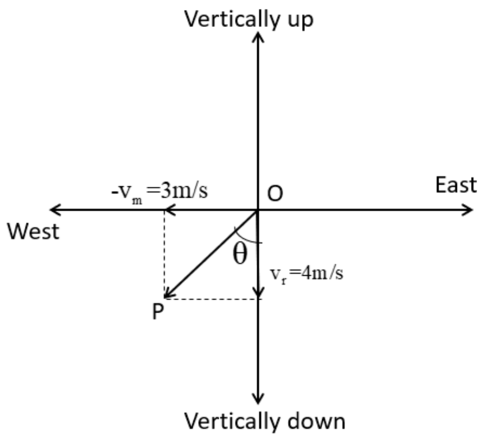
(a) Time taken to cross the river, $t = \frac{w}{3} = \frac{30}{3} = 10s$

(b) Drift along the river, $x = (6)(t) = 6 \times 10 = 60m$

Example – 7

A man is walking with 3 m/s, due east. Rain is falling vertically downwards with speed 4 m/s. Find the direction in which man should hold his umbrella, so that rain does not wet him.

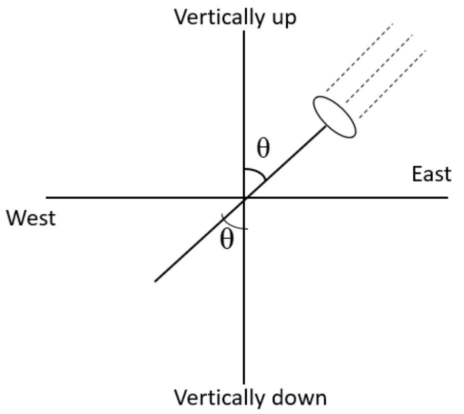
Sol. The man should hold his umbrella in the direction of v_{rm} or $v_r - v_m$



$OP = v_r + (-v_m) = v_r - v_m = v_{rm}$

$\Rightarrow \tan\theta = \frac{3}{4}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right) = 37°$



Therefore, man should hold his umbrella at an angle of 37º east of vertical (or 37º from vertical towards east)

Example – 8

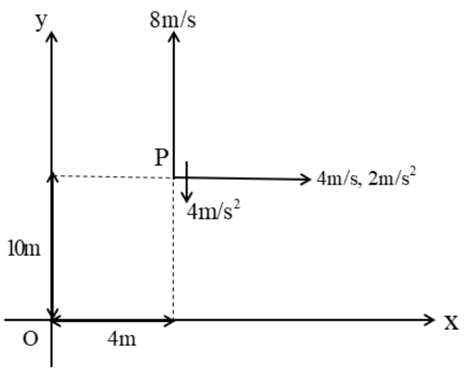
A particle is moving in x-y plane. Its initial velocity and acceleration are

$\vec{u} = (4\hat{i} + 8\hat{j})\, m/s$ and $\vec{a} = (2\hat{i} - 4\hat{j})\, m/s^2$.

Find the time when it crosses x-axis,

initial coordinates of particle are (4m, 10m).

Sol.



Particle starts from point P. Components of its initial velocity and acceleration are shown in figure.

At the time of crossing the x-axis, its y-coordinate should be zero or its y-displacement (w.r.t. initial point P) is – 10 m.

Using the equation, $s_y = u_y t + \frac{1}{2} a_y t^2$

$$-10 = 8t - \frac{1}{2} \times 4 \times t^2$$

Solving this equation, we get positive value of time,

$t = 5s$

Example – 9

A river is 20 m wide. River speed is 3 m/s. A boat starts with velocity perpendicular to river current. Velocity of boat is 5 m/s. How far from the point directly opposite to the starting point does the boat reach the opposite bank?

Sol. Let the horizontal drift of the boat at the opposite bank be L

Time taken by the boat to reach the opposite bank

$$t = \frac{d}{V_b}$$

$$\Rightarrow t = \frac{20}{5} = 4\,s$$

$$\therefore L = V_w t = 3 \times 4 = 12\,m$$

Example – 10

A river is flowing from west to east at a speed of 5 metre/minute. A man on the south bank of the river, capable of swimming at 10 metre/minute in still water, wants to swim across the river in the shortest time. He should swim in a direction,

Sol. Let $\vec{v}_m$ and $\vec{v}_r$ be velocities of the man and the river current w.r.t. the ground.

The velocity of the man in still water is equal to the relative velocity of the man w.r.t. water i.e.,

$$\vec{v}_{m/r} = \vec{v}_m - \vec{v}_r$$

Let $\vec{v}_{m/r}$ and $\vec{v}_m$ make angle α and θ with the north direction.

Let d be the width of the river. The time taken by the man to cross the river is given by

$$t = \frac{\text{width of the river}}{\text{component of man velocity along north}}$$

$$= \frac{d}{|\vec{v}_m| \cos\theta}$$

$$= \frac{d}{|\vec{v}_{m/r}| \cos\alpha}$$

$$\left(\because PQ = t|\vec{v}_m|\cos\theta = t|\vec{v}_{m/r}|\cos\alpha\right)$$

From above equation, t is minimum when denominator is maximum, Given, $|\vec{v}_{m/r}| = 10$ metre/min, a constant.

Thus, t become minimum when $\cos\alpha = 1$

i.e., $\alpha = 0$. Hence, the man takes the shortest time when he swims perpendicular to the river velocity i.e., towards north.

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

MOTION IN A PLANE

Introduction to Motion in a Plane & Projectile Motion

1. The co–ordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by

(a) $3t\sqrt{\alpha^2+\beta^2}$ (b) $3t^2\sqrt{\alpha^2+\beta^2}$

(c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $\sqrt{\alpha^2+\beta^2}$

2. A particle has an initial velocity $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is

(a) 10 unit (b) $7\sqrt{2}$ unit

(c) 7 unit (d) 8.5 unit

3. A particle is moving with velocity $\vec{v}=k\left(y\hat{i}+x\hat{j}\right)$, where k is a constant. The general equation for its path is

(a) $y = x^2$ + constant (b) $y^2 = x$ + constant

(c) xy = constant (d) $y^2 = x^2$ + constant

4. In a projectile motion, velocity at maximum height is

(a) $\frac{u\cos\theta}{2}$ (b) $u\cos\theta$

(c) $\frac{u\sin\theta}{2}$ (d) None of these

5. Two projectiles A and B are projected with angle of projection 30° for the projectile A and 45° for the projectile B. If R_A and R_B are the horizontal ranges for the two projectiles, then

(a) $R_A=R_B$

(b) $R_A>R_B$

(c) $R_A<R_B$

(d) the information is insufficient to decide the relation of R_A and R_B

6. Two bullets are fired with horizontal velocities of 50 m/s and 100 m/s from two guns placed at a height of 19.6 m. Which bullet will strike the ground first?

(a) First

(b) Second

(c) Simultaneously

(d) None of these

7. A body is thrown at angle 30° to the horizontal with the velocity of 30 m/s. After 1 sec, its velocity will be (in m/s) ($g = 10$ m/s^2)

(a) $10\sqrt{7}$ (b) $700\sqrt{10}$

(c) $100\sqrt{7}$ (d) $\sqrt{40}$

8. A projectile is fired at 30° to the horizontal. The vertical component of its velocity is 80 ms^{-1}. Its time of flight is T. What will be the velocity of the projectile at t = T/2

(a) 80 ms^{-1} (b) $80\sqrt{3}$ ms^{-1}

(c) $(80/\sqrt{3})$ ms^{-1} (d) 40 ms^{-1}

9. For a given velocity, a projectile has the same range *R* for two angles of projection. If t_1 and t_2 are the times of flight in the two cases then

(a) $t_1t_2 \propto R^2$ (b) $t_1t_2 \propto R$

(c) $t_1t_2 \propto \frac{1}{R}$ (d) $t_1t_2 \propto \frac{1}{R^2}$

10. If for a given angle of projection, the horizontal range is doubled, the time of flight becomes

(a) 4 times (b) 2 times

(c) $\sqrt{2}$ times (d) $1/\sqrt{2}$ times

11. A boy playing on the roof of a 10m high building throws a ball with a speed of 10 m/s at an angle of 30º with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground?

($g = 10$ m/s^2, $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$)

(a) 8.66 m (b) 5.20 m

(c) 4.33 m (d) 2.60 m

12. Which of the following sets of factors will affect the horizontal distance covered by an athlete in a long–jump event

(a) Speed before he jumps and his weight

(b) The direction in which he leaps and the initial speed

(c) The force with which he pushes the ground and his speed

(d) The direction in which he leaps and the weight

13. For a projectile, the ratio of maximum height reached to the square of flight time is (g = 10 ms^{-2})

(a) 5 : 4 (b) 5 : 2

(c) 5 : 1 (d) 10 : 1

14. A cricketer can throw a ball to a maximum horizontal distance of 100 m. The speed with which he throws the ball is (to the nearest integer)

(a) 30 ms^{-1} (b) 42 ms^{-1}

(c) 32 ms^{-1} (d) 35 ms^{-1}

15. If two bodies are projected at 30° and 60° respectively, with the same speed, then

(a) Their ranges are same

(b) Their heights are same

(c) Their times of flight are same

(d) All of these

16. A particle covers 50 m distance when projected with an initial speed. On the same surface it will cover a distance, when projected with double the initial speed

(a) 100 m (b) 150 m

(c) 200 m (d) 250 m

17. The speed of a projectile at the highest point becomes $\frac{1}{\sqrt{2}}$ times its initial speed. The horizontal range of the projectile will be

(a) $\frac{u^2}{g}$ (b) $\frac{u^2}{2g}$

(c) $\frac{u^2}{3g}$ (d) $\frac{u^2}{4g}$

18. A projectile is projected with initial velocity $(6\hat{i}+8\hat{j})\text{m/sec}$. If g = 10 ms^{-2}, then horizontal range is

(a) 4.8 metre (b) 9.6 metre

(c) 19.2 metre (d) 14.0 metre

19. A projectile thrown with an initial speed u and angle of projection 15° to the horizontal has a range R. If the same projectile is thrown at an angle of 45° to the horizontal with speed 2u, its range will be

(a) 12 R (b) 3 R

(c) 8 R (d) 4 R

20. The velocity at the maximum height of a projectile is half of its initial velocity of projection u. Its range on the horizontal plane is

(a) $\sqrt{3}u^2/2g$ (b) $u^2/3g$

(c) $3u^2/2g$ (d) $3u^2/g$

21. A projectile is thrown from a point in a horizontal plane such that its horizontal and vertical velocity components are 9.8 m/s and 19.6 m/s respectively. Its horizontal range is

(a) 4.9 m (b) 9.8 m

(c) 19.6 m (d) 39.2 m

22. A ball thrown by one player reaches the other in 2 sec. the maximum height attained by the ball above the point of projection will be about

(a) 10 m (b) 7.5 m

(c) 5 m (d) 2.5 m

23. If the initial velocity of a projectile be doubled. Keeping the angle of projection same, the maximum height reached by it will

(a) Remain the same (b) Be doubled

(c) Be quadrupled (d) Be halved

24. The maximum horizontal range of a projectile is 400 m. The maximum height attained by it will be

(a) 100 m (b) 200 m

(c) 400 m (d) 800 m

25. Two bodies are projected with the same velocity. If one is projected at an angle of 30° and the other at an angle of 60° to the horizontal, the ratio of the maximum heights reached is

(a) 3 : 1 (b) 1 : 3

(c) 1 : 2 (d) 2 : 1

26. If time of flight of a projectile is 10 seconds. Range is 500 m. The maximum height attained by it will be

(a) 125 m (b) 50 m

(c) 100 m (d) 150 m

27. At the top of the trajectory of a projectile, the directions of its velocity and acceleration are

(a) Perpendicular to each other

(b) Parallel to each other

(c) Inclined to each other at an angle of 45°

(d) Antiparallel to each other

28. A man projects a coin upwards from the gate of a uniformly moving train. The path of coin for the man will be

(a) Parabolic

(b) Inclined straight line

(c) Vertical straight line

(d) Horizontal straight line

29. In a projectile motion, the velocity

(a) Is always perpendicular to the acceleration

(b) Is never perpendicular to the acceleration

(c) Is perpendicular to the acceleration for one instant only

(d) Is perpendicular to the acceleration for two instants

30. A particle is thrown upward with a speed u at an angle θ with the horizontal. When the particle makes an angle ϕ with the horizontal, its speed changes to v, then

(a) $v = u\cos\theta\cos\phi$ (b) $v = u\cos\theta\sec\phi$

(c) $v = u\cos\theta$ (d) $v = u \sec\theta \cos\phi$

31. A cricket ball is thrown with a velocity of 15 m/s at an angle of 30° with the horizontal. The time of flight of the ball will be ($g = 10\ m/s^2$)

(a) 1.5 s (b) 2.5 s

(c) 3.5 s (d) 4.5 s

32. A stone is thrown at an angle θ to the horizontal reaches a maximum height h. The time of flight of the stone is

(a) $\sqrt{(2h\sin\theta)/g}$ (b) $2\sqrt{(2h\sin\theta)/g}$

(c) $2\sqrt{(2h)/g}$ (d) $\sqrt{(2h)/g}$

33. Which of the following is largest, when the height attained by the projectile is the largest

(a) Range

(b) Time of flight

(c) Angle of projectile with vertical

(d) None of these

34. A ball whose kinetic energy is $\left(E = \frac{1}{2}mu^2\right)$, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be

(a) E (b) $\frac{E}{\sqrt{2}}$

(c) $\frac{E}{2}$ (d) zero

35. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?

(a) yes, 60° (b) yes, 30°

(c) no (d) yes, 45°

36. A particle of mass m is projected with velocity v making an angle of 45° with the horizontal. When the particle lands on the level ground the magnitude of the change in its momentum $(\vec{p} = m\vec{v})$ will be:

(a) 2 mv

(b) $\frac{mv}{\sqrt{2}}$

(c) $mv\sqrt{2}$

(d) zero

37. A particle is projected at 60° to the horizontal with a kinetic energy $\left(K = \frac{1}{2}mu^2\right)$. The kinetic energy at the highest point is

(a) K (b) zero

(c) $\frac{K}{4}$ (d) $\frac{K}{2}$

38. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the maximum area around the fountain that can get wet is:

(a) $\pi\frac{v^4}{g^2}$ (b) $\frac{\pi}{2}\frac{v^4}{g^2}$

(c) $\pi\frac{v^2}{g^2}$ (d) $\pi\frac{v^2}{g}$

39. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be

(a) $20\sqrt{2}$ m (b) 10 m

(c) $10\sqrt{2}$ m (d) 20 m

40. The maximum range of a gun on horizontal terrain is 1.0 km. If $g = 10ms^{-2}$, what must be the muzzle velocity of the shell?

(a) 400 m/s (b) 200 m/s

(c) 100 m/s (d) 50 m/s

Projectile Motion from a Height & Projectile on an Incline

41. An aeroplane is moving with a horizontal velocity u at a height h above the ground. If a packet is dropped from it the speed of the packet when it reaches the ground will be

(a) $(u^2+2gh)^{1/2}$ (b) $(2gh)^{1/2}$

(c) $(u^2-2gh)^{1/2}$ (d) 2 gh

42. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground? [g = 10 m/s², $\sin 30° = \frac{1}{2}$, $\cos 30° = 0.866$]

(a) 5.20 m (b) 4.33 m

(c) 2.60 m (d) 8.66 m

43. An inclined plane is making an angle β with horizontal. A projectile is projected from the bottom of the plane with a speed u at an angle α with horizontal then its maximum range R_{max} is

(a) $R_{max} = \frac{u^2}{g(1-\sin\beta)}$ (b) $R_{max} = \frac{u^2}{g(1+\sin\beta)}$

(c) $R_{max} = \frac{u}{g(1-\sin\beta)}$ (d) $R_{max} = \frac{u}{g(1+\sin\beta)}$

44. For a given velocity of projection from a point on the inclined plane, the maximum range down the plane is three times the maximum range up the incline. Then, the angle of inclination of the inclined plane is

(a) 30° (b) 45°

(c) 60° (d) 90°

45. A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is $\alpha = 30°$ and the angle of the barrel to the horizontal $\beta = 60°$. The initial velocity v of the shell is 21 m/sec. Then distance of point from the gun at which shell will fall

(a) 10 m (b) 20 m

(c) 30 m (d) 40 m

46. The maximum range of rifle bullet on the horizontal ground is 6 km its maximum range on an inclined of 30° will be

(a) 1 km (b) 2 km

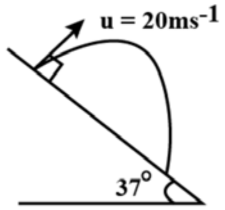
(c) 4 km (d) 6 km

47. A projectile is projected from the foot of an incline of angle 30° with a velocity 30 m/s. The angle of projection as measured from the horizontal is 60°. What would be its speed when the projectile is parallel to the incline?

(a) 10 m/s (b) $2\sqrt{3}$ m/s

(c) $5\sqrt{3}$ m/s (d) $10\sqrt{3}$ m/s

48. Find range of projectile on the inclined plane which is projected perpendicular to the inclined plane with velocity 20 m/s as shown in figure:



(a) 55 m (b) 75 m

(c) 84 m (d) 100 m

49. Find time of flight of a projectile thrown horizontally with speed $10ms^{-1}$ from a long-inclined plane which makes an angle of $\theta = 45°$ with the horizontal (Take $g = 10ms^{-2}$)

(a) $\sqrt{2}$ sec (b) $2\sqrt{2}$ sec

(c) 2 sec (d) none

50. The range of projectile on a downward inclined plane is ……… the range on upward inclined plane for the same velocity of projection and angle of projection.

(a) less than (b) more than

(c) equal to (d) none of these

RELATIVE MOTION

Relative Velocity in one & two Dimensions

51. A man projects a coin upwards from the gate of a uniformly moving train. The path of coin for the man will be

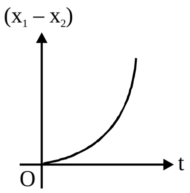
(a) Parabolic

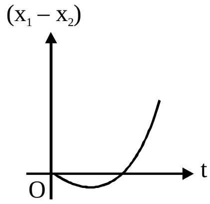
(b) Inclined straight line

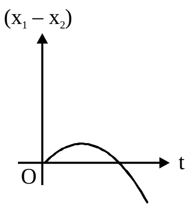
(c) Vertical straight line

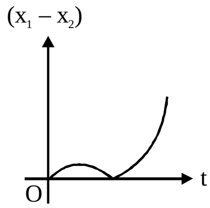
(d) Horizontal straight line

52. A body is at rest at x = 0. At t = 0, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through x = 0 moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time?

(a) 

(b) 

(c) 

(d) 

53. A bus starts from rest with an acceleration of 1 m/s^2. A man who is 48 m behind the bus starts with a uniform velocity of 10 m/s. Then the minimum time after which the man will catch the bus:

(a) 4 s (b) 10 s

(c) 12 s (d) 8 s

54. A 100 m long train at 15 m/s overtakes a man running on the platform in the same direction in 10 s. How long the train will take to cross the man if he was running in the opposite direction?

(a) 7 s (b) 5 s

(c) 3 s (d) 1 s

55. A particle is moving eastwards with a velocity of 5 m/s. In 10s the velocity changes to 5 m/s northwards. The average acceleration in this time is

(a) zero

(b) $\frac{1}{2}$ ms^{-2} towards north

(c) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north–east

(d) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north–west

56. A small body is dropped from a rising balloon. A person A stands on ground, while another person B is on the balloon. Choose the correct statement: Immediately, just after the body is released.

(a) A and B, both feel that the body is coming (going) down.

(b) A and B, both feel that the body is going up.

(c) A feel that the body is coming down, while B feels that the body is going up.

(d) A feel that the body is going up, while B feels that the body is going down.

57. A policeman moving on a highway with a speed of 30 kmh^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 kmh^{-1}. If the muzzle speed of the bullet is 150 ms^{-1} with what speed does the bullet hit the thief's car?

(a) 120 m/s (b) 90 m/s

(c) 125 m/s (d) 105 m/s

58. A bird is flying towards north with a velocity 40 km/h and a train is moving with a velocity 40 km/h towards east. What is the velocity of the bird noted by a man in the train?

(a) $40\sqrt{2}\,\text{km/h}\ N-E$ (b) $40\sqrt{2}\,\text{km/h}\ S-E$

(c) $40\sqrt{2}\,\text{km/h}\ N-W$ (d) $40\sqrt{2}\,\text{km/h}\ S-W$

River-boat & Rain-man Problems

59. A swimmer jumps from a bridge over a canal and swims 1 km upstream. After that first km, he passes a floating cork. He continues swimming for half an hour and then turns around and swims back to the bridge. The swimmer and the cork reach the bridge at the same time. The swimmer has been swimming at a constant speed. How fast does the water in the canal flow?

(a) 1.5 km/h (b) 2 km/h

(c) 4 km/h (d) 1 km/h

60. A river is flowing from east to west at a speed of 5 m/min. A man on south bank of river, capable of swimming 10 m/min in still water, wants to swim across the river in the shortest time; he should swim:

(a) due north

(b) due north-east

(c) due north-east with double the speed of river

(d) none of the above

61. A steamer moves with velocity 3 km/h in and against the direction of river water whose velocity is 2 km/h. Calculate the total time for total journey if the boat travels 2 km in the direction of steam and then back to its place:

(a) 2 hrs (b) 2.5 hrs

(c) 2.4 hrs (d) 3 hrs

62. A boat crosses a river of width 1 km along the shortest path in 15 minutes. If the speed of boat in still water is 5 km/hr, then what is the speed of the river?

(a) 1 km/hr (b) 3 km/hr

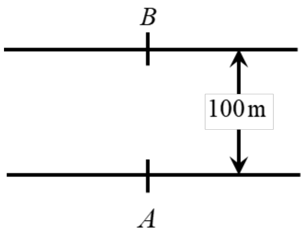
(c) 2 km/hr (d) 5 km/hr

63. Ship A is travelling with a velocity of 5 kmh^{-1} due east. The second ship is heading 30° east of north. What should be the speed of second ship if it is to remain always due north with respect to the first ship?

(a) 10 kmh^{-1} (b) 9 kmh^{-1}

(c) 8 kmh^{-1} (d) 7 kmh^{-1}

64. A man swims from a point A on one bank of a river of width 100 m. When he swims perpendicular to the water current, he reaches the other bank 50 m downstream. The angle to the bank at which he should swim, to reach the directly opposite point B on the other bank is



(a) 10° upstream (b) 20° upstream

(c) 30° upstream (d) 60° upstream

65. A boat is sent across (perpendicular) a river with a velocity of $8\,kmh^{-1}$. If the resultant velocity of the boat is $10\,kmh^{-1}$, the river is flowing with a velocity

(a) $6\,kmh^{-1}$ (b) $8\,kmh^{-1}$

(c) $10\,kmh^{-1}$ (d) $128\,kmh^{-1}$

66. A river is flowing from west to east at a speed of 5 m/min. A man on the south bank of the river, capable of swimming at 10 m/min in still water, wants to swim across the river in the shortest time. Finally, he will swim in a direction

(a) $\tan^{-1}(2)$ E of N (b) $\tan^{-1}(2)$ N of E

(c) 30°E of N (d) 60°E of N

67. A boat which has a speed of 5 km/h in still water crosses a river of width 1 km along the shortest possible path in 15 min. The velocity of the river water in km per hour is

(a) 1 (b) 3

(c) 4 (d) $\sqrt{41}$

68. Rain is falling vertically downwards with a speed of 4 kmh^{-1}. A girl moves on a straight, horizontal road with a velocity of 3 kmh^{-1}. The apparent velocity of rain with respect to the girl is

(a) 3 kmh^{-1} (b) 4 kmh^{-1}

(c) 5 kmh^{-1} (d) 7 kmh^{-1}

69. A man is walking on a road with a velocity $3\,kmh^{-1}$. Suddenly rain starts falling. The velocity of rain is $10\,kmh^{-1}$ in vertically downward direction. The relative velocity of the rain w.r.t. man is

(a) $\sqrt{7}\,kmh^{-1}$ (b) $\sqrt{13}\,kmh^{-1}$

(c) $13\,kmh^{-1}$ (d) $\sqrt{109}\,kmh^{-1}$

70. Rain is falling vertically with a velocity of 25 ms^{-1}. A woman rides a bicycle with a speed of 10 ms^{-1} in the north to south direction. What is the direction (angle with vertical) in which she should hold her umbrella to safe herself from the rain?

(a) $\tan^{-1}(0.4)$ (b) $\tan^{-1}(1)$

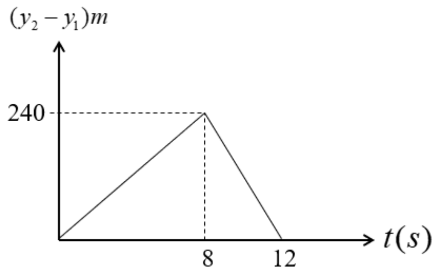
(c) $\tan^{-1}(\sqrt{3})$ (d) $\tan^{-1}(2.6)$

EXERCISE – 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

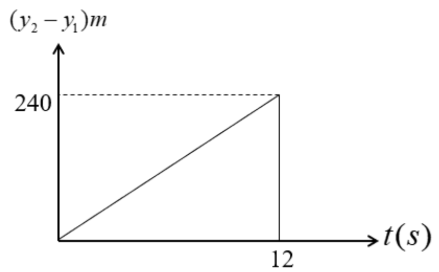
1. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? (assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2) (The figures are schematic and not drawn to scale.)

(2015)

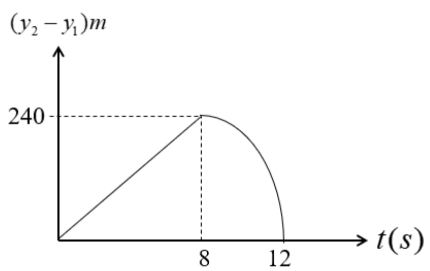
(a)



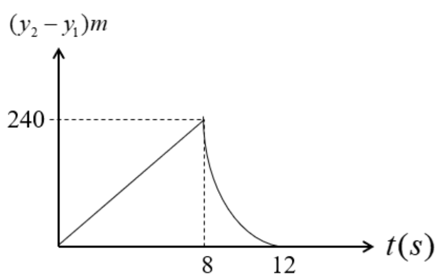
(b)



(c)



(d)



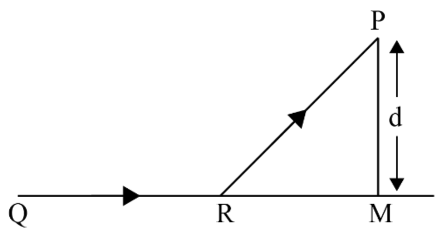
2. A car is standing 200 m behind a bus, which is also at rest. The two start moving at the same instant but with different forward accelerations. The bus has acceleration 2 m/s^2 and the car has acceleration 4 m/s^2. The car will catch up with the bus after a time of: **(2017)**

(a) $\sqrt{110}$s (b) $\sqrt{120}$s

(c) $10\sqrt{2}$s (d) 15 s

3. A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from the highway (from point M) as shown in the figure. Speed of the car in the field is half of that on the highway. What should be the distance RM, so that the time taken to reach P is minimum?

(2018)



(a) d (b) $\frac{d}{\sqrt{2}}$

(c) $\frac{d}{2}$ (d) $\frac{d}{\sqrt{3}}$

4. The position co-ordinates of a particle moving in a 3-D coordinate system is given by

x = a cos ωt

y = a sin ωt

and z = a ωt

The speed of the particle is: **(2019)**

(a) $\sqrt{2}a\omega$ (b) aω

(c) $\sqrt{3}a\omega$ (d) 2aω

5. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to: **(2019)**

(a) $\frac{2a_1a_2}{a_1+a_2}t$ (b) $\sqrt{2a_1a_2}\,t$

(c) $\sqrt{a_1a_2}\,t$ (d) $\frac{a_1+a_2}{2}t$

6. A particle moves from the point $\left(2.0\hat{i}+4.0\hat{j}\right)$m, at t = 0, with an initial velocity $\left(5.0\hat{i}+4.0\hat{j}\right)ms^{-1}$. It is acted upon by a constant force which produces a constant acceleration $\left(4.0\hat{i}+4.0\hat{j}\right)ms^{-2}$. What is the distance of the particle from the origin at time 2 s? **(2019)**

(a) 15 m (b) $20\sqrt{2}$m

(c) 5 m (d) $10\sqrt{2}$m

7. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight? **(2019)**

(a) 90° (b) 150°

(c) 120° (d) 60°

8. The position vector of a particle changes with time according to the relation $\vec{r}(t)=15t^2\hat{i}+\left(4-20t^2\right)\hat{j}$. What is the magnitude of the acceleration at t = 1 (in proper units)? **(2019)**

(a) 40 (b) 25

(c) 100 (d) 50

9. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is: **(2019)**

(a) $\frac{R}{4g}$ (b) $\frac{R}{g}$

(c) $\frac{R}{2g}$ (d) $\frac{2R}{g}$

10. Two particles are projected from the same point with same speed u such that they have the same range R, but different maximum height, h_1 and h_2. Which of the following is correct? **(2019)**

(a) $R^2 = 4h_1h_2$ (b) $R^2 = 16h_1h_2$

(c) $R^2 = 2h_1h_2$ (d) $R^2 = h_1h_2$

11. A particle is moving with a velocity $\vec{v}=K\left(y\hat{i}+x\hat{j}\right)$, where K is a constant. The general equation for its path is: **(2019)**

(a) $y = x^2$ + constant (b) $y^2 = x$ + constant

(c) $y^2 = x^2$ + constant (d) xy = constant

12. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they fire bullets in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is: **(2019)**

(a) 1:16 (b) 1:2

(c) 1:4 (d) 1:8

13. A body is projected at t = 0 with a velocity $10ms^{-1}$ at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1 s is R. Neglecting air resistance and taking acceleration due to gravity g = 10 ms^{-2}, the value of R is **(2019)**

(a) 10.3 m (b) 2.8 m

(c) 2.5 m (d) 5.1 m

14. A passenger train of length 60 m travels at a speed of 80 km/h. Another freight train of length 120 m travels at a speed of 30 km/h. The ratio of time taken by the passenger train to completely cross the freight train when they are moving in same direction to that when they are in the opposite directions is **(2019)**

(a) $\frac{11}{5}$ (b) $\frac{5}{2}$

(c) $\frac{3}{2}$ (d) $\frac{25}{11}$

15. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If 'v' is the speed of sound, speed of the plane is nv, where n is: **(2019)**

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$

(c) 1 (d) $\frac{1}{2}$

16. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr, where $\hat{i}$ points east and $\hat{j}$ north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in:

(2019)

(a) 4.2 hr (b) 2.6 hr

(c) 3.2 hr (d) 2.2 hr

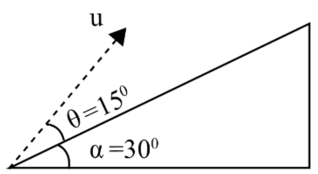
17. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal $m\gamma v^2$ (where m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is:

(2019)

(a) $\frac{1}{\sqrt{(\gamma g)}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$ (b) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(c) $\frac{1}{\sqrt{\gamma g}}\ln\left(\sqrt{\frac{\gamma}{g}}V_0\right)$ (d) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

18. A plane is inclined at an angle $\alpha = 30°$ with respect to the horizontal. A particle is projected with a speed $u = 2\text{ms}^{-1}$, from the base of the plane, as shown in figure. The distance from the base, at which the particle hits the plane is close to: (Take $g = 10\text{ms}^{-2}$)



(2019)

(a) 20 cm (b) 18 cm

(c) 26 cm (d) 14 cm

19. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ with speed v then (g = 10 ms^{-2}):

(2019)

(a) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v = \frac{5}{3}\text{ms}^{-1}$

(b) $\theta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v = \frac{3}{\sqrt{5}}\text{ms}^{-1}$

(c) $\theta = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v = \frac{3}{5}\text{ms}^{-1}$

(d) $\theta = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v = \frac{5}{3}\text{ms}^{-1}$

20. A particle moves such that its position vector $\vec{r}(t) = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$, where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle?

(2020)

(a) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$

(b) $\vec{v}$ and $\vec{r}$ both are parallel to $\vec{r}$

(c) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin

(d) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

21. A particle starts from the origin at t = 0 with an initial velocity of $\vec{u} = 3\hat{i}$ from origin and moves in the x-y plane with a constant acceleration $\vec{a} = (6\hat{i} + 4\hat{j})$ m/s^2. The x -coordinate of the particle at the instant when its y-coordinate is 32m is D meters. The value of D is: **(2020)**

(a) 60 (b) 50

(c) 32 (d) 40

22. Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hour. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible)

(2020)

(a) 29.5 ms^{-1} (b) 30.5 ms^{-1}

(c) 31.5 ms^{-1} (d) 28.5 ms^{-1}

23. Starting from the origin at time t = 0, with initial velocity $5\hat{j}\text{ms}^{-1}$, a particle moves in the x – y plane with a constant acceleration of $(10\hat{i} + 4\hat{j})\text{ ms}^{-2}$. At time t, its coordinates are $(20\text{ m}, y_0\text{ m})$. The values of t and y_0 are, respectively: **(2020)**

(a) 5 s and 25 m (b) 2 s and 18 m

(c) 2 s and 24 m (d) 4 s and 52 m

24. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops are coming at an angle 60º from the horizontal. On further increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45º. The value of β is close to: **(2020)**

(a) 0.50 (b) 0.73

(c) 0.37 (d) 0.41

25. A particle is moving along the x-axis with its coordinate with time t given by $x(t) = -3t^2 + 8t + 10$ m. Another particle is moving along the y-axis with its coordinate as a function of time given by $y = 5 - 8t^3$ m. At t = 1 s, the speed (in m/s) of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v(in m/s) is __________. **(2020)**

26. The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$ where α and β are constants and x & y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by

(2021)

(a) $\tan^{-1}\alpha, \dfrac{4\alpha^2}{\beta}$ (b) $\tan^{-1}\alpha, \dfrac{\alpha^2}{4\beta}$

(c) $\tan^{-1}\left(\dfrac{\beta}{\alpha}\right), \dfrac{\alpha^2}{\beta}$ (d) $\tan^{-1}\beta, \dfrac{\alpha^2}{2\beta}$

27. A mosquito is moving with a velocity $\vec{v} = 0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2s? **(2021)**

(a) $\tan^{-1}\left(\dfrac{2}{3}\right)$ from y-axis

(b) $\tan^{-1}\left(\dfrac{5}{2}\right)$ from x-axis

(c) $\tan^{-1}\left(\dfrac{2}{3}\right)$ from x-axis

(d) $\tan^{-1}\left(\dfrac{5}{2}\right)$ from y-axis

28. A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is. (Round off to the Nearest Integer) (Find the angle in degrees) **(2021)**

EXERCISE – 3 : ADVANCED OBJECTIVE QUESTIONS

Single Choice Questions

1. There are two values of time for which a projectile is at the same height. The sum of these two times is equal to

(a) 3T/2 (b) 4T/3

(c) 3T/4 (d) T

(T = time of flight of the projectile)

2. The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where a and b are constants and x and y are respectively horizontal and vertical distance of the projectile from the point of projection. The maximum height attained by the particle and the angle of projection from the horizontal are

(a) $\frac{b^2}{2a}, \tan^{-1}(b)$ (b) $\frac{a^2}{b}, \tan^{-1}(2a)$

(c) $\frac{a^2}{4b}, \tan^{-1}(a)$ (d) $\frac{2a^2}{b}, \tan^{-1}(a)$

3. A particle moves in the x-y plane according to the law x = kt and y = kt (1 – at), where k and a are positive constants and t is time. What is the equation of trajectory of the particle?

(a) y = kx (b) $y = x - \frac{\alpha x^2}{k}$

(c) $y = \frac{\alpha x^2}{k}$ (d) $y = \alpha x$

4. The equation of motion of a projectile is $y = 12x - \frac{3}{4}x^2$. Given that g = 10 ms^{-2}, what is the range of the projectile

(a) 12.4 m (b) 16 m

(c) 30.6 m (d) 36.0 m

5. A ball is dropped from the top of a tower in a high-speed wind. The wind exerts a steady force on the ball. The path followed by the ball will be

(a) Parabola (b) Circular arc

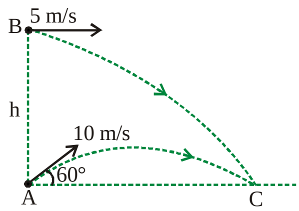
(c) Elliptical arc (d) Straight line

6. A particle is projected from the ground with an initial speed of u at an angle θ with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is

(a) $\frac{u}{2}\sqrt{1+2\cos^2\theta}$ (b) $\frac{u}{2}\sqrt{1+\cos^2\theta}$

(c) $\frac{u}{2}\sqrt{1+3\cos^2\theta}$ (d) u cos θ

7. A particle A is projected from the ground with an initial velocity of 10 m/s at an angle of 60° with horizontal. From what height h should another particle B be projected horizontally with velocity 5 m/s so that both the particles collide in ground at point C if both are projected simultaneously (g = 10 m/s²)



(a) 10 m (b) 30 m

(c) 15 m (d) 25 m

8. A particle is projected at an angle of 60° above the horizontal with a speed of 10 m/s. After some time the direction of its velocity makes an angle of 30° above the horizontal. The speed of the particle at this instant is

(a) $\frac{5}{\sqrt{3}}$ m/s (b) $5\sqrt{3}$ m/s

(c) 5 m/s (d) $\frac{10}{\sqrt{3}}$ m/s

9. In projectile motion, the modulus of rate of change of speed

(a) is constant

(b) first increases then decreases

(c) first decreases then increases

(d) none of these

10. Two particles A and B are projected simultaneously from a point situated on a horizontal plane. The particle A is projected vertically up with a velocity u_A while the particle B is projected up at an angle of 30° with horizontal with a velocity u_B. After 5 sec the particles were observed moving mutually perpendicular to each other. The velocity of projection of the particle u_A and u_B respectively are

(a) 50 ms^{-1}, 100 m/s

(b) 100 ms^{-1}, 50 ms^{-1}

(c) $u_A \geq 25$ m/s and $u_B < 50$ m/s

(d) none of these

11. A projectile is fired at an angle of 30° to the horizontal such that the vertical component of its initial velocity is 80 m/s. Its time of flight is T. Its velocity at t = T/4 has a magnitude of nearly

(a) 200 m/s (b) 300 m/s
(c) 140 m/s (d) 100 m/s

12. A particle A is projected vertically upwards. Another particle B of same mass is projected at an angle of 45°. Both reach the same height. The ratio of the initial kinetic energy of A to that of B is [given $KE = \frac{1}{2}mv^2$]

(a) 1:2 (b) 2:1
(c) $1:\sqrt{2}$ (d) $\sqrt{2}:1$

13. A body of mass m is thrown upwards at an angle θ with the horizontal with velocity v. While rising up the velocity of the mass after t seconds will be

(a) $\sqrt{(v\cos\theta)^2 + (v\sin\theta)^2}$

(b) $\sqrt{(v\cos\theta - v\sin\theta)^2 - gt}$

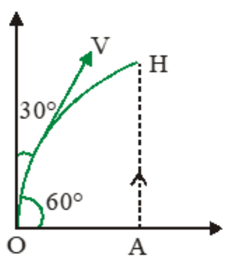
(c) $\sqrt{v^2 + g^2t^2 - (2v\sin\theta)gt}$

(d) $\sqrt{v^2 + g^2t^2 - (2v\cos\theta)gt}$

14. From the top of a tower 19.6 m high, a ball is thrown horizontally. If the line joining the point of projection to the point where it hits the ground makes an angle of 45° with the horizontal, then the initial velocity of the ball is

(a) 9.8 ms^{-1} (b) 4.9 ms^{-1}
(c) 14.7 ms^{-1} (d) 2.8 ms^{-1}

15. A particle is projected with a speed V from a point O making an angle of 30° with the vertical. At the same instant, a second particle is thrown vertically upwards with a velocity v from a point A. The two particles reach H, the highest point on the parabolic path of particle simultaneously. Then ratio $\frac{V}{v}$ is



(a) $3\sqrt{2}$ (b) $2\sqrt{3}$
(c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

16. A projectile is thrown in the upward direction making an angle of 60° with the horizontal direction with a velocity of 147 ms^{-1}. Then the time after which its inclination with the horizontal is 45° is

(a) 15 s (b) 10.98 s
(c) 5.49 s (d) 2.745 s

17. From the top of a tower of height 40 m a ball is projected upwards with a speed of 20 m/s at an angle of elevation of 30°. Then the ratio of the total time taken by the ball to hit the ground to its time of flight (time taken to come back to the same elevation) is (take g = 10 ms^2)

(a) 2:1 (b) 3:1
(c) 3:2 (d) 4:1

18. Three identical balls are thrown with same speed at angles of 15°, 45° and 75° with the horizontal respectively. The ratio of their distances from the point of projection to the point where they hit the ground will be

(a) $1:\sqrt{2}:1$ (b) 1 : 2 : 1
(c) 2 : 4 : 3 (d) $1:2:\sqrt{3}$

19. A projectile is thrown at an angle of 40° with the horizontal and its range is R_1. Another projectile is thrown at an angle 40° with the vertical and its range is R_2. What is the relation between R_1 and R_2?

(a) $R_1 = R_2$ (b) $R_1 = 2\,R_2$
(c) $R_2 = 2\,R_1$ (d) $R_1 = 4\,R_2/5$

20. A cricketer hits a ball with a velocity 25 m/s at 60° above the horizontal. How far (approximately) above the ground it passes over a fielder 50 m from the bat (assume the ball is struck very close to the ground)

(a) 8.2 m (b) 9.0 m
(c) 11.6 m (d) 12.7 m

21. From a point on the ground at a distance 2 metres from the foot of a vertical wall, a ball is thrown at an angle of 45° which just clears the top of the wall and afterward strikes the ground at a distance 4m on the other side. The height of the wall is

(a) $\frac{2}{3}$m (b) $\frac{3}{4}$m
(c) $\frac{1}{3}$m (d) $\frac{4}{3}$m

22. Two projectiles A and B are projected with same speed and angle of projection 30° for the projectile A and 45° for the projectile B. If R_A and R_B are the horizontal ranges for the two projectiles, then

(a) $R_A = R_B$

(b) $R_A > R_B$

(c) $R_A < R_B$

(d) the information is insufficient to decide the relation of R_A and R_B

23. A projectile is projected at an angle of 15° to the horizontal with some speed v. If another projectile is projected with the same speed, then it must be projected at what angle (other than 15°) with the horizontal so as to have the same range.

(a) It is never possible (b) 12.5°

(c) 75° (d) 65°

24. A fielder in a cricket match throws a ball from the boundary line to the wicket keeper. The ball describes a parabolic path. Which of the following quantities remains constant during the ball's motion in air? (neglect air resistance)

(a) its kinetic energy

(b) its speed

(c) the horizontal component of its velocity

(d) the vertical component of its velocity

25. The height y and the distance x along the horizontal plane of a projectile on a certain planet (with no surrounding atmosphere) are given by y = (8t – 5) metre and x = 6t metre where t is in seconds. The velocity of projection is

(a) 8 m/sec

(b) 6 m/sec

(c) 10 m/sec

(d) not obtained from the data

26. A body is projected horizontally with speed 20 m/s from top of a tower. What will be its speed nearly after 5 sec? Take $g = 10\ m/s^2$

(a) 54 m/s (b) 20 m/s

(c) 50 m/s (d) 70 m/s

27. A body is projected horizontally with speed 20 m/s from top of a tower, what will be the displacement of the body if it hits the ground after 5 sec and doesn't bounce (quote nearest integer)

(a) 100 m (b) 125 m

(c) 160 m (d) 225 m

28. A body is projected at an angle of 30° with the horizontal with speed 30 m/s. What is the angle with the horizontal after 1.5 seconds? Take $g = 10\ m/s^2$.

(a) 0° (b) 30°

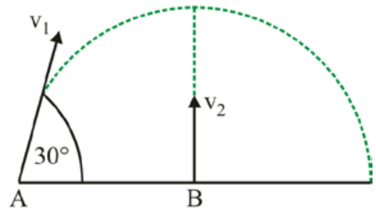
(c) 60° (d) 90°

29. From certain height, two bodies are projected horizontally with velocities 10 m/s and 20 m/s. They hit the ground in t_1 and t_2 seconds. Then

(a) $t_1 = t_2$ (b) $t_1 = 2\,t_2$

(c) $t_2 = 2\,t_1$ (d) $t_1 = \sqrt{2t_2}$

30. A body is projected with velocity v_1 from the point A as shown in figure. At the same time, another body is projected vertically upwards from B with velocity v_2. The point B lies vertically below the highest point. For both the bodies to collide, $\frac{v_2}{v_1}$ should be



(a) 2 (b) 0.5

(c) $\sqrt{3/2}$ (d) 1

31. An aeroplane is flying at a constant horizontal velocity of 600 km/h at an elevation of 6 km towards a point directly above the target on the earth's surface. At an appropriate time, the pilot released a ball so that it strikes the target on the earth. The ball will appear to be falling

(a) on a parabolic path as seen by pilot in the plane

(b) vertically along a straight path as seen by an observer on the ground near the target

(c) on a parabolic path as seen by an observer on the ground near the target

(d) on a zig-zag path as seen by pilot in the plane

32. Three particles A, B and C are thrown from the top of a tower 100 m in height with the same speed 10 m/s. A is thrown straight up, B is thrown straight down, and C is thrown horizontally. They hit the ground with the speeds v_A, v_B and v_C respectively. Then

(a) $v_A > v_B = v_C$ (b) $v_B > v_C > v_A$

(c) $v_A = v_B = v_C$ (d) $v_A = v_B > v_C$

33. A body is thrown horizontally with a velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level ground through the foot of the tower at a distance x from the tower. The value of x is

(a) h (b) h/2

(c) 2h (d) 2h/3

34. Consider a boy on a trolley who throws a ball with speed 20 m/s with respect to ground at an angle 37° with vertical and trolley is moving with a speed 10 m/s in horizontal direction then what will be maximum distance travelled by ball parallel to road :

(a) 20.2 m (b) 12 m

(c) 31.2 m (d) 62.4 m

35. Two men A and B, A standing on the extended floor nearby a building and B is standing on the roof of the building. Both throw a stone towards each other. Then which of the following will be correct.

(a) stone will hit A, but not B

(b) stone will hit B, but not A

(c) stone will not hit either of them, but will collide with each other

(d) none of these

36. A particle is projected from a point (0, 1) on Y–axis (assume + Y direction vertically upwards) aiming towards a point (4, 9). It fell on ground along x axis in 1 sec.

Taking g = 10 m/s^2 and all coordinate in metres. Find the x–coordinate of the point where it fell.

(a) 3 (b) 4

(c) 2 (d) $2\sqrt{5}$

37. The position vector of a particle is given as $\vec{r} = (t^2 - 4t + 6)\hat{i} + (t^2)\hat{j}$. The time after which the velocity vector and acceleration vector becomes perpendicular to each other is equal to

(a) 1 sec (b) 2 sec

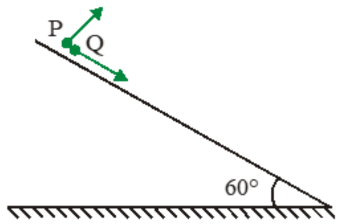
(c) 1.5 sec (d) not possible

38. A particle is projected up an inclined plane with initial speed v = 20 m/s at an angle $\theta = 30^\circ$ with plane. The component of its velocity perpendicular to plane when it strikes the plane is

(a) $10\sqrt{3}\,m/s$ (b) 10 m/s

(c) $5\sqrt{3}\,m/s$ (d) data is insufficient

39. A particle P is projected from a point on the surface of smooth inclined plane (see figure). Simultaneously another particle Q is released on the smooth inclined plane from the same position. P and Q collide on the inclined plane after t = 4 second. The speed of projection of P is nearly:



(a) 5 m/s (b) 10 m/s

(c) 15 m/s (d) 20 m/s

40. A ball is projected horizontally with a speed v from the top of a plane inclined at an angle 45° with the horizontal. How far from the point of projection will the ball strike the plane?

(a) $\frac{v^2}{g}$ (b) $\sqrt{2}\frac{v^2}{g}$

(c) $\frac{2v^2}{g}$ (d) $\sqrt{2}\left[\frac{2v^2}{g}\right]$

41. Position vector of a particle moving in x-y plane at time t is: $\vec{r} = a(1 - \cos\omega t)\hat{i} + a\sin\omega t\,\hat{j}$. The path of the particle is

(a) a circle of radius a and centre at (a, 0)

(b) a circle of radius a and centre at (0, 0)

(c) an ellipse

(d) neither a circle nor an ellipse

42. A particle moves in x-y plane. The position vector of particle at any time t is $\vec{r} = \{(2t)\hat{i} + (2t^2)\hat{j}\}$ m. The rate of change of θ at time t = 2 second. (where θ is the angle which its velocity vector makes with positive x-axis) is

(a) $\frac{2}{17}$ rad/s (b) $\frac{1}{14}$ rad/s

(c) $\frac{4}{7}$ rad/s (d) $\frac{6}{5}$ rad/s

43. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is:

(a) 10 unit (b) 7 unit

(c) $7\sqrt{2}$ unit (d) 8.5 unit

44. Velocity and acceleration of a particle initially are $\vec{v}=\left(3\hat{i}+4\hat{j}\right)$ m/s and $\vec{a}=-\left(6\hat{i}+8\hat{j}\right)$ m/s² respectively. Initially particle is at origin. maximum x–coordinate of particle will be:

(a) 1.5 m (b) 0.75 m

(c) 2.25 m (d) 4.0 m

45. Let $\vec{v}$ and $\vec{a}$ denote the velocity and acceleration respectively of a particle moving in a circular path then,

(a) $\vec{v}\,.\,\vec{a}<0$ all the time

(b) $\vec{v}\,.\,\vec{a}>0$ all the time

(c) $\vec{v}\,.\,\vec{a}=0$ all the time

(d) (a),(b) & (c) all are possible depending upon the direction of net acceleration.

46. A person walks up a stationary escalator in time t_1. If he remains stationary on the moving escalator, then it can take him up in time t_2. How much time would it take him to walk up the moving escalator.

(a) $\dfrac{t_1+t_2}{2}$ (b) $\sqrt{t_1+t_2}$

(c) $\dfrac{t_1\,t_2}{t_1+t_2}$ (d) t_1+t_2

47. A horizontal wind is blowing with a velocity v towards north-east. A man starts running towards north with acceleration a. The time after which man will feel the wind blowing towards east is :

(a) $\dfrac{v}{a}$ (b) $\dfrac{\sqrt{2}\,v}{a}$

(c) $\dfrac{v}{\sqrt{2}\,a}$ (d) $\dfrac{2v}{a}$

48. Two trains are each 50 m long starts moving parallel towards each other at speeds 10 m/s and 15 m/s respectively, after how much time will they pass each other?

(a) 8s (b) 4s

(c) 2s (d) 6s

49. On a calm day a boat can go across a lake and return in time T_0 at a speed v. On a rough day there is uniform current at speed u to help the onward journey and impede the return journey. If the time taken to go across and return on the rough day be T, then T/T_0 is:

(a) $1-\dfrac{u^2}{v^2}$ (b) $\dfrac{1}{1-\dfrac{u^2}{v^2}}$

(c) $1+\dfrac{u^2}{v^2}$ (d) $\dfrac{1}{1+\dfrac{u^2}{v^2}}$

50. A river is flowing from West to East at a speed of 5 metres per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still water, wants to swim across the river in shortest time. He should swim in a direction

(a) due North (b) 30° East of North

(c) 30° West of North (d) 60° East of North

51. A river is flowing from west to east at a speed of 20 m/min. A man on the south bank of the river, capable of swimming at 10 m/min in still water, wants to swim across the river without any drift. He should swim in a direction:

(a) due north

(b) 30° east of north

(c) 30° west of north

(d) zero drift is not possible

52. The rowing speed of a man relative to water is 5 km/h and the speed of water flow is 3 km/h. At what angle to the river flow should he head if he wants to reach a point on the other bank, directly opposite to starting point:

(a) 127° (b) 143°

(c) 120° (d) 150°

53. Two cars are moving in the same direction with the same speed of 30 km/h. They are separated by 5 km. What is the speed of the car moving in the opposite direction if it meets the two cars at an interval of 4 minutes?

(a) 15 km/h (b) 30 km/h

(c) 45 km/h (d) 60 km/h

Multiple Choice Questions

54. An observer moves with a constant speed along the line joining two stationary objects. He will observe that the two objects.

(a) have the same speed

(b) have the same velocity

(c) move in the same direction

(d) move in opposite direction

55. A particle is projected at an angle θ from ground with speed u (g = 10 m/s^2)

(a) if u = 10 m/s and θ = 30°, then time of flight will be 1 sec.

(b) if u = $10\sqrt{3}$ m/s and θ = 60°, then time of flight will be 3 sec.

(c) if u = $10\sqrt{3}$ m/s and θ = 60°, then after 2 sec velocity becomes perpendicular to initial velocity.

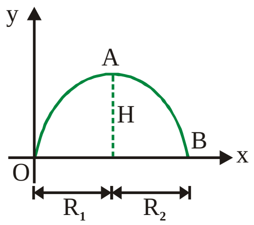
(d) if u = 10 m/s and θ = 30°, then velocity never becomes perpendicular to initial velocity during its flight.

56. A particle leaves the origin with an initial velocity $\vec{u} = (3\hat{i})$ m/s and a constant acceleration $\vec{a} = (-1.0\,\hat{i} - 0.5\,\hat{j})$ m/s^2. its velocity $\vec{v}$ and position vector $\vec{r}$ when it reaches its maximum x-co-ordinate are:

(a) $\vec{v} = -2\hat{j}$ (b) $\vec{v} = (-1.5\hat{j})\,m/s$

(c) $\vec{r} = (4.5\hat{i} - 2.25\hat{j})\,m$ (d) $\vec{r} = (3\hat{i} - 2\hat{j})\,m$

57. In a projectile motion let $t_{OA} = t_1$ and $t_{AB} = t_2$. the horizontal displacement from O to A is R_1 and from A to B is R_2. Maximum height is H and time of flight is T. If air drag is to be considered, then choose the correct alternative (s)



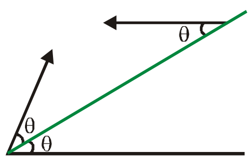
(a) t_1 will decrease while t_2 will increase

(b) H will increase

(c) R_1 will decrease while R_2 will increase

(d) T may increase or decrease

58. From an inclined plane two particles are projected with same speed at same angle θ, one up and other down the plane as shown in figure. Which of the following statement(s) is/are correct?



(a) the particles will collide the plane with same speed

(b) the times of flight of each particle are same

(c) both particles strike the plane perpendicularly

(d) the particles will collide in mid-air if projected simultaneously and time of flight of each particle is less than the time of collision.

59. Choose the correct alternative(s)

(a) If the greatest height to which a man can throw a stone is h, then the greatest horizontal distance up to which he can throw the stone is 2h

(b) The angle of projection for a projectile motion whose range R is n times the maximum height H is $\tan^{-1}$ (4/n)

(c) The time-of-flight T and the horizontal range R of a projectile are connected by the equation $gT^2 = 2R \tan\theta$ where θ is the angle of projection

(d) A ball is thrown vertically up. Another ball is thrown at an angle θ with the vertical. Both of them remain in air for the same period of time. Then the ratio of heights attained by the two balls is 1 : 1.

60. Two particles A and B are located in x-y plane at points (0, 0) and (0, 4 m). They simultaneously start moving with velocities.

$\vec{v}_A = 2\hat{j}$ m/s and $\vec{v}_B = 2\hat{i}$ m/s. Select the correct alternative(s)

(a) the distance between them is constant

(b) the distance between them first decreases and then increases

(c) the shortest distance between them is $2\sqrt{2}$ m

(d) time after which they are at minimum distance is 1 s

61. The co-ordinate of the particle in x-y plane are given as $x = 2 + 2t + 4t^2$ and $y = 4t + 8t^2$ the motion of the particle is

(a) along a straight line

(b) uniformly accelerated

(c) along a parabolic path

(d) nonuniformly accelerated

62. River is flowing with a velocity $\vec{v}_R = 4\hat{i}$ m/s. A boat is moving with a velocity $\vec{v}_{BR} = (-2\hat{i} + 4\hat{j})$ of m/s relative to river. The width of the river is 100 m along y-direction. Choose the correct alternative(s)

(a) the boatman will cross the river in 25 s

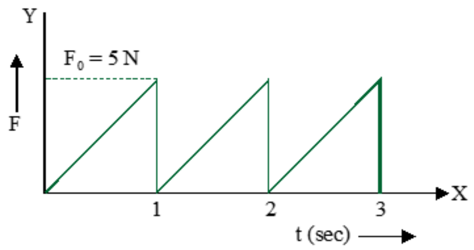
(b) absolute velocity of boatman is $2\sqrt{5}$ m/s

(c) drift of the boatman along the river current is 50 m

(d) the boatman can never cross the river

Numerical Value Type Questions

63. A particle of mass m = 2 kg is projected along X–axis with velocity V_0 = 5 ms^{-1}. It is acted on by a variable force acting along Y–axis as shown in figure. What is the magnitude of its velocity at 2 seconds? (in ms^{-1})



64. A man standing on a road has to hold his umbrella at 37° with the vertical to keep the rain away. He throws the umbrella and starts running at 12 km/h. He finds that raindrops are hitting his head vertically. Find the speed (in km/hr) of raindrops with respect to the moving man.

65. $\vec{V}_A = (x\hat{i} + 2\hat{j})$ m/s and $\vec{V}_B = (3\hat{i} + 2\hat{j})$ m/s find x such that, the relative speed of A with respect to B becomes 5 m/s.

66. A particle is projected up an inclined plane of inclination β at an elevation α to the horizontal. Find the ratio between $\tan\alpha$ and $\tan\beta$, if the particle strikes the plane horizontally.

67. A train takes 2 minutes to acquire its full speed 60 kmph from rest and 1 minute to come to rest from the full speed. If somewhere in between two stations 1 km of the track be under repair and the limited speed on this part be fixed to 20 kmph, find the late running of the train (in sec) on account of this repair work, assuming otherwise normal at running of the train between the stations.

Assertion & Reason

(A) If both Assertion and reason are true and reason is the correct explanation of the assertion.

(B) If both assertion and reason are true but reason is not the correct explanation of the assertion.

(C) If assertion is true but reason is false.

(D) If assertion is false but reason is true.

(E) If both assertion and reason are false.

68. **Assertion:** For a particle moving along a straight line or in a plane, the average velocity vector over a time interval can be equal to instantaneous velocity at the end of the interval, even if velocity of particle is not constant.

Reason: $\frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{d\vec{r}}{dt}$

(a) A (b) B
(c) C (d) D
(e) E

69. **Assertion:** Two stones are simultaneously projected from level ground from same point with same speeds but different angles with horizontal. Both stones move in same vertical plane. Then the two stones may collide in mid-air.

Reason: For two stones projected simultaneously from same point with same speed at different angles with horizontal, their trajectories may intersect at some point.

(a) A (b) B
(c) C (d) D
(e) E

70. **Assertion:** In a plane to plane projectile motion, the angle between instantaneous velocity vector and acceleration vector can be anything between 0 to π (excluding the limiting case).

Reason: In plane to plane projectile motion, acceleration vector is always pointing vertical downwards. (neglect air friction).

(a) A (b) B
(c) C (d) D
(e) E

71. **Assertion:** Two particles of different mass, projected with same velocity and angle of projection, the maximum height attained by both the particle will be same.

Reason: The maximum height of projectile is independent of particle mass.

(a) A (b) B
(c) C (d) D
(e) E

72. **Assertion:** When a body is dropped or thrown horizontally from the same height, it would reach the ground at the same time.

Reason: Horizontal velocity has no effect on the vertical direction.

(a) A (b) B
(c) C (d) D
(e) E

73. **Assertion:** In order to hit a target, a man should point his rifle in the same direction as target.

Reason: The horizontal range of the bullet is independent of the angle of projection with horizontal.

(a) A (b) B
(c) C (d) D
(e) E

Match the Column

74. A ball is projected from the ground with velocity v such that its range is maximum.

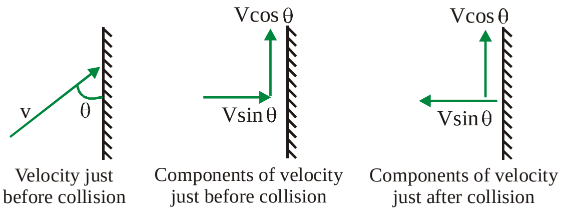
Column–I	**Column–II**
(A) Velocity at half of the maximum height	(P) $\sqrt{3}\ v/2$
(B) Velocity at the maximum height	(Q) $\frac{v}{\sqrt{2}}$
(C) Change in its velocity when it returns to the ground	(R) $v\sqrt{2}$
(D) Average velocity when it reaches the maximum height	(S) $\frac{v}{2}\sqrt{\frac{5}{2}}$

Paragraph Type Questions

Using the following Comprehension, Solve Q. 75 to Q. 78

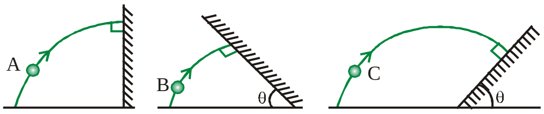
Passage

We know how by neglecting the air resistance, the problems of projectile motion can be easily solved and analysed. Now we consider the case of the collision of a ball with a wall. In this case the problem of collision can be simplified by considering the case of elastic collision only. When a ball collides with a wall, we can divide its velocity into two components, one perpendicular to the wall and other parallel to the wall. If the collision is elastic, then the perpendicular component of velocity of the ball gets reversed with the same magnitude.



The other parallel component of velocity will remain constant if wall is given smooth.

Now let us take a problem. Three balls ‘A’ and ‘B’ & ‘C’ are projected from ground with same speed at same angle with the horizontal. The balls A, B and C collide with the wall during their flight in air and all three collide perpendicularly with the wall as shown in figure.



75. Which of the following relation about the maximum height H of the three balls from the ground during their motion in air is correct:

(a) $H_A = H_C > H_B$ (b) $H_A > H_B = H_C$
(c) $H_A > H_C > H_B$ (d) $H_A = H_B = H_C$

76. If the time taken by the ball A to fall back on ground is 4 seconds and that by ball B is 2 seconds. Then the time taken by the ball C to reach the inclined plane after projection will be:

(a) 6 sec (b) 4 sec
(c) 3 sec (d) 5 sec

77. The maximum height attained by ball 'A' from the ground is

(a) 10 m

(b) 15 m

(c) 20 m

(d) insufficient information

78. The maximum height attained by ball B from ground is:

(a) 20 m (b) 5 m

(c) 15 m (d) none of these

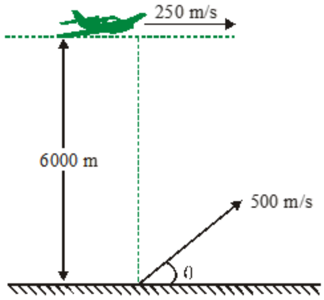
Paragraph Type Questions

Using the following comprehension, solve Q. 79 & 80

Passage

An aircraft moving with a speed of 250 m/s is at a height of 6000 m, just overhead of an anti-aircraft gun.

79. If the muzzle velocity of the shell is 500 m/s, the firing angle θ should be



(a) 30º (b) 45º

(c) 60º (d) None of these

80. The time after which the aircraft is hit is:

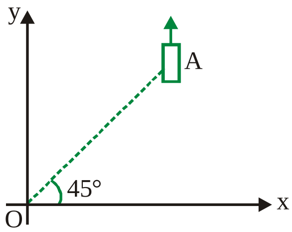
(a) $20\sqrt{3}$ s (b) $15\sqrt{3}$ s

(c) 20 s (d) $10\sqrt{3}$ s

EXERCISE – 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Single Choice Questions

1. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $\left(\sqrt{3}-1\right)$ m/s. At a particular instant when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley. **(2002)**



(a) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

(b) Find the speed of the ball with respect to the surface, if $\phi = 4\theta/3$.

Assertion & Reason

(a) If Statement I is true. Statement II is true; Statement II is the correct explanation for Statement I.

(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.

(c) If Statement I is true; Statement II is false.

(d) If Statement I is false; Statement II is true.

2. **Statement–I:** For an observer looking out through the window of a fast-moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

Statement–II: If the observer and the object are moving at velocities $\vec{v}_1$ and $\vec{v}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{v}_2 - \vec{v}_1$.

(2008)

(a) A (b) B

(c) C (d) D

3. A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is. **(2011)**

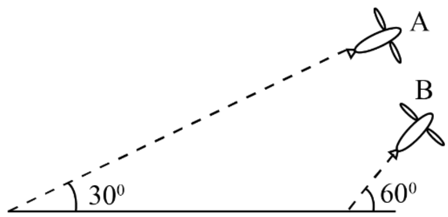
4. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s, where $\hat{i}$ is a along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is **(2013)**

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

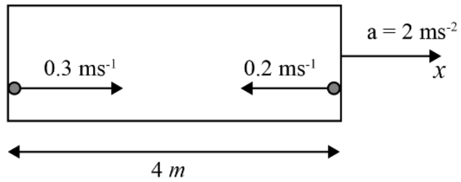
5. Airplanes A and B are flying with constant velocity in the same vertical plane of angle 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is:

(2014)



6. A rocket is moving in a gravity free space with a constant acceleration of 2ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in -x direction with a speed of 0.2ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is:

(2014)



7. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, it metres, is........... **(2018)**

8. Starting at time t = 0 from the origin with speed $1ms^{-1}$, a particle follows a two-dimensional trajectory in the x-y plane so that its coordinates are related by the equation $y = \frac{x^2}{2}$. The x and y components of its acceleration are denoted by a_x and a_y, respectively. Then **(2020)**

(a) $a_x = 1ms^{-2}$ implies that when the particle is at the origin, $a_y = 1ms^{-2}$

(b) $a_x = 0$ implies $a_y = 1ms^{-2}$ at all times

(c) at t = 0, the particle's velocity points in the x-direction

(d) $a_x = 0$ implies that at t = 1 s, the angle between the particle's velocity and the x axis is 45°

9. A projectile is thrown from a point O on the ground at an angle 45° from the vertical and with a speed $5\sqrt{2}m/s$. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x meters from the point O. The acceleration due to gravity. $g = 10m/s^2$

The value of t is ___ **(2021)**

10. A projectile is thrown from a point O on the ground at an angle 45° from the vertical and with a speed $5\sqrt{2}m/s$. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x meters from the point O. The acceleration due to gravity. $g = 10m/s^2$

The value of x(in m) is ___ **(2021)**

Answer Key

CHAPTER -3 | MOTION IN A PLANE AND RELATIVE MOTION

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1.	(b)	**2.**	(b)	**3.**	(d)	**4.**	(b)
5.	(d)	**6.**	(c)	**7.**	(a)	**8.**	(b)
9.	(b)	**10.**	(c)	**11.**	(a)	**12.**	(b)
13.	(a)	**14.**	(c)	**15.**	(a)	**16.**	(c)
17.	(a)	**18.**	(b)	**19.**	(c)	**20.**	(a)
21.	(d)	**22.**	(c)	**23.**	(c)	**24.**	(a)
25.	(b)	**26.**	(a)	**27.**	(a)	**28.**	(c)
29.	(c)	**30.**	(b)	**31.**	(a)	**32.**	(c)
33.	(b)	**34.**	(c)	**35.**	(a)	**36.**	(c)
37.	(c)	**38.**	(a)	**39.**	(d)	**40.**	(c)
41.	(a)	**42.**	(d)	**43.**	(b)	**44.**	(a)
45.	(c)	**46.**	(c)	**47.**	(d)	**48.**	(b)
49.	(c)	**50.**	(b)	**51.**	(c)	**52.**	(b)
53.	(d)	**54.**	(b)	**55.**	(d)	**56.**	(d)
57.	(d)	**58.**	(c)	**59.**	(d)	**60.**	(a)
61.	(c)	**62.**	(b)	**63.**	(a)	**64.**	(d)
65.	(a)	**66.**	(b)	**67.**	(b)	**68.**	(c)
69.	(d)	**70.**	(a)				

EXERCISE - 2 : PREVIOUS YEARS JEE MAIN QUESTIONS

1.	(c)	**2.**	(c)	**3.**	(d)	**4.**	(a)
5.	(c)	**6.**	(b)	**7.**	(c)	**8.**	(d)
9.	(d)	**10.**	(b)	**11.**	(c)	**12.**	(a)
13.	(b)	**14.**	(a)	**15.**	(d)	**16.**	(b)
17.	(a)	**18.**	(a)	**19.**	(a)	**20.**	(d)
21.	(a)	**22.**	(a)	**23.**	(b)	**24.**	(b)
25.	(580.00)	**26.**	(b)	**27.**	(a)	**28.**	(120.00)

EXERCISE – 3 :
ADVANCED OBJECTIVE QUESTION

1.	(d)	**2.**	(c)	**3.**	(b)	**4.**	(b)
5.	(d)	**6.**	(c)	**7.**	(c)	**8.**	(d)
9.	(a)	**10.**	(a)	**11.**	(c)	**12.**	(a)
13.	(c)	**14.**	(a)	**15.**	(c)	**16.**	(c)
17.	(a)	**18.**	(b)	**19.**	(a)	**20.**	(a)
21.	(d)	**22.**	(c)	**23.**	(c)	**24.**	(c)
25.	(c)	**26.**	(a)	**27.**	(c)	**28.**	(a)
29.	(a)	**30.**	(b)	**31.**	(c)	**32.**	(c)
33.	(c)	**34.**	(d)	**35.**	(d)	**36.**	(c)
37.	(a)	**38.**	(b)	**39.**	(b)	**40.**	(d)
41.	(a)	**42.**	(a)	**43.**	(c)	**44.**	(b)
45.	(d)	**46.**	(c)	**47.**	(c)	**48.**	(b)
49.	(b)	**50.**	(a)	**51.**	(d)	**52.**	(a)
53.	(c)	**54.**	(a,b,c)	**55.**	(a,b,c,d)	**56.**	(b,c)
57.	(a,d)	**58.**	(b)	**59.**	(a,b,c,d)	**60.**	(b,c,d)
61.	(a,b)	**62.**	(a,b,c)	**63.**	(7.5)	**64.**	(16)
65.	(8)	**66.**	(2)	**67.**	(160)	**68.**	(c)
69.	(c)	**70.**	(b)	**71.**	(a)	**72.**	(a)
73.	(e)	**74.**	$(A \rightarrow P; B \rightarrow Q; C \rightarrow R; D \rightarrow S)$				
75.	(a)	**76.**	(c)	**77.**	(c)	**78.**	(c)
79.	(c)	**80.**	(d)				

EXERCISE – 4 :
PREVIOUS YEARS JEE ADVANCED QUESTIONS

1.	((a) 45°, (b) 2 m/s)	**2.**	(b)
3.	(5 m/s^2)	**4.**	(b)
5.	(5)	**6.**	(8)
7.	(30.00)	**8.**	(a,b,c,d)
9.	(0.5)	**10.**	(7.5)