Motion in a Plane

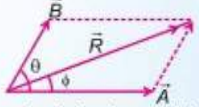
Chapter 4

1 SCALARS AND VECTORS

- **Scalar quantity:** It has only magnitude with proper unit. All base quantities are scalar. The rules combining scalars are rules of ordinary algebra.
- **Vector quantity:** It has both magnitude and direction and obeys the triangle law or parallelogram law of vector addition.
- **Equality of vector:** Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal, if and only if, they have same magnitude and direction.
- **Multiplication of vector by real numbers:** If a vector $\vec{A}$ is multiplied by real number λ, then $A' = \lambda|\vec{A}|$
  if $\lambda > 0$, magnitude will change and direction remains same
  if $\lambda < 0$, magnitude changes λ times and direction gets reverse.
- **Parallelogram law of vector addition:** For two co-initial vectors represented by two adjacent sides of a parallelogram, the diagonal of a parallelogram passing through same point will be resultant.

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\phi = \frac{B\sin\theta}{A + B\cos\theta}$$

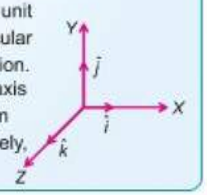


- **Subtraction of vector:** It can be defined as addition of a vector and negative of other vector.

$$\vec{S} = \vec{A} - \vec{B}$$

$$\vec{S} = \vec{A} + (-\vec{B}) \Rightarrow |\vec{S}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

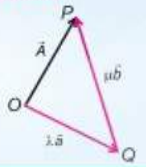
Unit Vectors: It is a vector of unit magnitude and points in a particular direction. It has no unit and dimension. Unit vectors along the x, y and z axis of a rectangular coordinate system represented by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively, called basic unit vectors.



2 RESOLUTION OF VECTORS

$$\vec{A} = \overrightarrow{OP} = \overrightarrow{OQ} + \overrightarrow{QP}$$

$$\vec{A} = \lambda\vec{a} + \mu\vec{b}$$



3 RECTANGULAR COMPONENTS

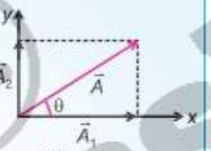
$$\vec{A} = \vec{A}_1 + \vec{A}_2$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$\vec{A} = A\cos\theta\hat{i} + A\sin\theta\hat{j}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A_y}{A_x},\ \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



- Resolution in three rectangular components

$$A_x = A\cos\alpha,\ A_y = A\sin\alpha$$

$$A_z = A\cos\gamma$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

4 MOTION IN A PLANE

$$\vec{r} = x\hat{i} + y\hat{j}$$

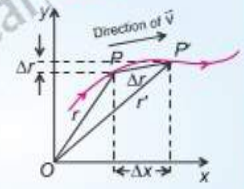
$$\vec{r}' = x'\hat{i} + y'\hat{j}$$

$$\overrightarrow{\Delta r} = \vec{r}' - \vec{r}$$

$$\overrightarrow{\Delta r} = (x' - x)\hat{i} + (y' - y)\hat{j}$$

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \vec{v}_x\hat{i} + \vec{v}_y\hat{j}$$

Instantaneous velocity, $\vec{v} = \frac{d\vec{r}}{dt}$



- The direction of velocity at any point on path is tangent to path and in direction of motion.

5 MOTION IN A PLANE WITH CONSTANT ACCELERATION

- $\vec{v} = \vec{v}_o + \vec{a}t$

$$v_x = v_{ox} + a_x t$$

$$v_y = v_{oy} + a_y t$$

- $\vec{r} = \vec{r}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2$, $x = x_o + v_{ox}t + \frac{1}{2}a_x t^2$

$$y = y_o + v_{oy}t + \frac{1}{2}a_y t^2$$

6 RELATIVE VELOCITY IN TWO DIMENSIONS

The velocity of object A relative to B

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

where $\vec{V}_A$ and $\vec{V}_B$ are velocities in the same frame.

Similarly, $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$\vec{V}_{AB} = -\vec{V}_{BA}$ and $|\vec{V}_{AB}| = |\vec{V}_{BA}|$

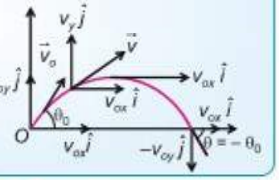
7 PROJECTILE MOTION

Equation of trajectory $y = x\tan\theta_0 - \frac{1}{2}\frac{gx^2}{v_o^2\cos^2\theta_0}$

This is equation of parabola.

- Time of flight $T_f = \frac{2v_o\sin\theta_0}{g}$
- Maximum height $h_m = \frac{(v_o\sin\theta_0)^2}{2g}$
- Horizontal range $R = \frac{v_o^2\sin 2\theta_0}{g}$

for R_{max}, $\theta = 45°, R_{max} = \frac{v_o^2}{g}$

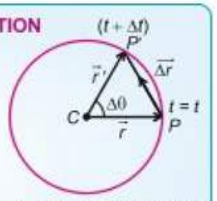


8 UNIFORM CIRCULAR MOTION

In uniform circular motion particle moves with constant speed.

- Angular displacement $\Delta\theta = \frac{\text{Arc } (PP')}{r}$
- Angular velocity $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi\nu$
- Linear speed $v = r\omega$
- Centripetal acceleration-Due to change in direction of velocity and is always directed towards centre.

$$a = \frac{v^2}{r} = r\omega^2 = 4\pi^2\nu^2 r = v\omega$$



Sharpen Your Understanding

1. Two vectors are said to be equal, if [NCERT Pg. 66]
   (1) They have equal magnitude only
   (2) Same direction only
   (3) They have equal magnitude and same direction
   (4) They have unequal magnitude and same direction
2. A null vector has [NCERT Pg. 68]
   (1) Zero magnitude, specified direction
   (2) Zero magnitude, arbitrary direction
   (3) Non-zero magnitude, no direction
   (4) Non-zero magnitude, arbitrary direction
3. To a person moving with a speed of 5 m/s towards east, rain appears to be falling vertically downward with speed $5\sqrt{3}$ m/s. The actual velocity of rain is [NCERT Pg. 69]
   (1) 10 m/s at 30° with vertical
   (2) 20 m/s at 30° with vertical
   (3) 10 m/s at 60° with vertical
   (4) 20 m/s at 60° with vertical
4. A vector can be resolved [NCERT Pg. 70]
   (1) Only in two components
   (2) Only in three components
   (3) In any number of components
   (4) Either two or three components

NCERT Based MCQs

5. The magnitude of component of a vector [NCERT Pg. 70]
   (1) Is always less than magnitude of vector
   (2) Is always equal to magnitude of vector
   (3) May be greater than magnitude of vector
   (4) Is always greater than magnitude of vector
6. A motor boat is racing towards north at 25 km/h and the water current in that region is 10 km/h in the direction of 60° east of south. The resultant velocity of the boat is nearly [NCERT Pg. 72]
   (1) 22 km/h
   (2) 12 km/h
   (3) 35 km/h
   (4) 26 km/h
7. In uniform circular motion, the centripetal acceleration is [NCERT Pg. 79]
   (1) Due to change in magnitude of velocity only
   (2) Due to change in direction of velocity only
   (3) Due to change in both magnitude and direction of velocity
   (4) Neither due to change in magnitude of velocity nor due to change in direction
8. In circular motion, the direction of angular velocity is [NCERT Pg. 80]
   (1) In the plane of circle
   (2) Perpendicular to plane of circle
   (3) In the direction of velocity
   (4) In the direction of acceleration
9. The shape of the trajectory of an object is determined by [NCERT Pg. 85]
   (1) Acceleration only
   (2) Velocity of projection only
   (3) Initial position and initial velocity only
   (4) Initial position, initial velocity and acceleration
10. Which of the following vector operation is meaningful? [NCERT Pg. 85]
    (1) Multiplication of any two vectors
    (2) Adding any two vectors
    (3) Adding a component of vector to the same vector
    (4) Both (2) and (3)
11. Which of the following quantities is/are vector? [NCERT Pg. 85]
    (1) Angular frequency
    (2) Angular velocity
    (3) Number of moles
    (4) Both (1) and (2)

12. Which of the following option is correct? [NCERT Pg. 86]

(1) Each component of a vector is always scalar

(2) Three vectors not lying in a plane can never add up to give null vector

(3) Two vectors of different magnitude can be add up to give null vector

(4) Minimum number of vectors to give null vector is five

13. A particle A is moving with velocity $\left(3\hat{i}+4\hat{j}\right)$ m/s and particle B is moving with velocity $\left(-3\hat{i}-4\hat{j}\right)$ m/s. The magnitude of velocity of B w.r.t A is [NCERT Pg. 76]

(1) 6 m/s

(2) 8 m/s

(3) 10 m/s

(4) 5 m/s

14. If two vectors $\vec{A}=a\hat{i}+6\hat{j}$ and $\vec{B}=b\hat{i}+c\hat{j}$ are equal then correct options for value of a, b and c is [NCERT Pg. 66]

(1) $a = b$

(2) $a = c$

(3) $c = 6$

(4) Both (1) and (3)

15. Equation of trajectory of projectile is $y=\sqrt{3}x-5x^2$. Then angle of projection with vertical is (Assume x-axis as horizontal and y-axis as vertical) [NCERT Pg. 78]

(1) 45°

(2) 30°

(3) 60°

(4) 53°

16. A projectile is projected with initial velocity $(10\hat{i}+20\hat{j})$ m/s from the ground. The velocity of the body just before hitting the ground is [NCERT Pg. 79]

(1) $10\hat{i}+20\hat{j}$

(2) $-10\hat{i}+20\hat{j}$

(3) $10\hat{i}-20\hat{j}$

(4) $-10\hat{i}-20\hat{j}$

17. The component of $\left(3\hat{i}+4\hat{j}\right)$ in the direction of $\left(\hat{i}-\hat{j}\right)$ is [NCERT Pg. 87]

(1) $\frac{\hat{j}-\hat{i}}{2}$

(2) $\frac{\hat{i}-\hat{j}}{2}$

(3) $\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$

(4) $\frac{1}{\sqrt{2}}(\hat{j}-\hat{i})$

18. The correct statement for a scalar quantity is [NCERT Pg. 87]

(1) It is conserved in a process

(2) It can never take negative values

(3) It does not vary from one point to another in space

(4) It has the same value for the observers with different orientations of axis

19. A man can swim with a speed of 5 km/h in still water. How long does he take to cross a river 1.0 km wide, if the river is flowing steadily at 3 km/h and he makes his strokes normal to the river current? [NCERT Pg. 86]

(1) 20 min

(2) 30 min

(3) 12 min

(4) 15 min

20. A particle starts from origin at t = 0 s with a velocity 4.0 $\hat{j}$ m/s and moves in x-y plane with a constant acceleration of $\left(6\hat{i}+4\hat{j}\right)$ m/s^2. The time after which y-coordinate of particle will be 48 m, will be [NCERT Pg. 87]

(1) 6 s

(2) 4 s

(3) 8 s

(4) 5 s

Thinking in Context

1. The _____ of a vector is called its absolute value. [NCERT Pg. 66]
2. Addition and subtraction of scalars make sense only for quantities with _____ units. However, you can multiply and divide scalars of _____ units. [NCERT Pg. 66]
3. Displacing a vector parallel to itself leaves the vector unchanged. Such vectors are called _____. [NCERT Pg. 66]
4. Multiplying a vector $\vec{A}$ by a negative number λ gives a vector $\lambda\vec{A}$ whose direction is _____ to the direction of $\vec{A}$. [NCERT Pg. 67]
5. Vector addition follows _____ law and _____ law [NCERT Pg. 68]
6. On adding two equal and opposite vectors, resultant will be a _____. [NCERT Pg. 68]
7. A unit vector is a vector of _____ magnitude. It has no _____. [NCERT Pg. 70]
8. The sum of the squares of direction cosines of a vector is _____. [NCERT Pg. 71]
9. The instantaneous acceleration is the limiting value of _____ as the time interval approaches zero. [NCERT Pg. 74]
10. In two or three dimensions, velocity and acceleration vectors may have any angle between _____. [NCERT Pg. 75]
11. In one dimension, the velocity and acceleration may have angle _____ between them. [NCERT Pg. 75]
12. Motion in a plane can be treated as superposition of two separate simultaneous _____ motions along two perpendicular directions. [NCERT Pg. 76]
13. The resultant velocity is the _____ sum of two velocities. [NCERT Pg. 77]
14. Particle A is moving with velocity $\vec{v}_A$ and particle B is moving with velocity $\vec{v}_B$ in same direction then their relative velocity is given by the _____ of two velocities. [NCERT Pg. 77]
15. In uniform circular motion, magnitude of velocity and acceleration remains _____ [NCERT Pg. 81]
16. In projectile motion x-component of velocity _____ while y-component of velocity undergoes a _____. [NCERT Pg. 79]
17. In projectile motion if air resistance is considered then both x and y component of velocities undergoes a _____. [NCERT Pg. 79]
18. When an object follows a circular path at a _____ the motion is said to be uniform circular motion. [NCERT Pg. 79]
19. The shape of the trajectory of motion is not determined by the _____ alone, but also depends on initial conditions of motion. [NCERT Pg. 85]
20. In uniform circular motion, acceleration is directed along the _____ of circular path [NCERT Pg. 81]

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