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Magnetostatics

IN THIS CHAPTER ....

Magnetism is a phenomenon by the virtue of which there develops an attracting or repulsive force between two magnetic objects. The ability of an object or a material to produce invisible magnetic field lines in order to develop a force which attracts other magnetic materials such as iron, steel, etc., are known as *magnets*.

Bar Magnet and Magnetic Poles

Bar magnet consists of two equal and non-separate magnetic poles. One pole is designated as north pole (N) and the other as south pole (S).

Following are few characteristics of bar magnet

(i) A freely suspended magnet stays always in north-south direction. The end of the magnet which always rests in the geographical north direction is called the north pole and the end which always rests in the geographical south direction is called the south pole. These poles are represented by the letters N and S, respectively.

(ii) Like poles repel each other and unlike poles attract each other. This repulsion or attraction obeys the inverse-square law.

(iii) We cannot isolate the north or south pole of magnet. If a bar magnet is broken into two halves, we get two similar bar magnets with somewhat weaker properties.

Magnetic Field Lines

The magnetic field lines of a magnet are the imaginary lines which continuously represent the direction of that magnetic field or the direction of the magnetic force on a north monopole at any given position.

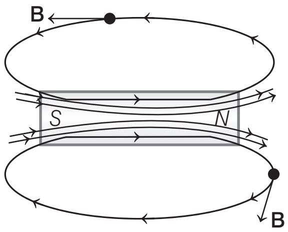
Important properties of magnetic field lines are given below

(i) These lines forms closed continuous curves.

(ii) The tangent drawn at any point of a field line represents the direction of net magnetic field.

(iii) These lines cannot intersect each other.

(iv) Outside a magnet, they are directed from north to south pole and inside a magnet, they are directed from south to north.



Magnetic Dipole

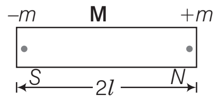
A magnetic dipole is an arrangement which consists two magnetic poles of equal and opposite strengths separated at a small distance. A bar magnet, a compass needle, etc., are the examples of magnetic dipoles.

Pole Strength (*m*)

It can be defined as the strength of magnetic pole to attract magnetic material towards itself. It is a scalar quantity and its SI unit is ampere-metre (A-m).

Magnetic Dipole Moment

The product of the distance ($2l$) between the two poles and the pole strength of either pole is called magnetic dipole moment.

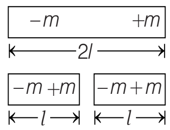
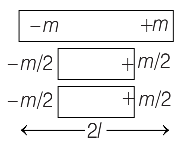
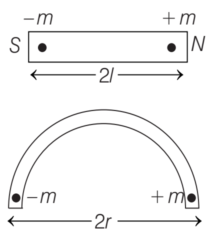
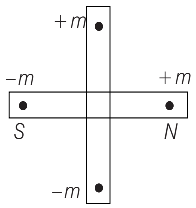
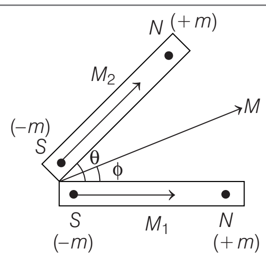


Magnetic dipole moment, $\mathbf{M} = m\,(2l)$

Its SI unit is joule/tesla (J/T) or ampere-metre^2 (A-m^2).

Its direction is from south pole towards north pole.

Pole Strength and Magnetic Dipole Moment in Special Cases

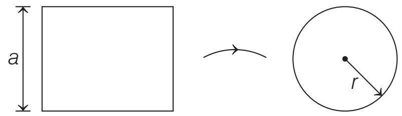
Special Cases	Figure	Effect on Pole Strength	New Magnetic Dipole Moment
If bar magnet is cut into two equal pieces such that the length of each piece becomes half		Remains unchanged	$M' = m \cdot \frac{2l}{2} = \frac{M}{2}$ (becomes half)
If bar magnet is cut into two equal pieces such that the width of each piece becomes half		Pole strength of each piece becomes half	$M' = \left(\frac{m}{2}\right)(2l) = \frac{M}{2}$ (becomes half)
If bar magnet is bent in the form of semi-circle		Remains unchanged	$M' = m(2r)\ (\because 2l = \pi r)$ $M' = m \times 2\left(\frac{2l}{\pi}\right) = \frac{2M}{\pi}$ $\left(\text{becomes } \frac{2}{\pi} \text{ times}\right)$
When two identical bar magnets are joined perpendicular to each other		Remains unchanged	$M = \sqrt{M_1^2 + M_2^2} = \sqrt{2}M$
When two bar magnets are inclined at an angle θ.		Remains unchanged	Resultant magnetic moment, $M' = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos\theta}$ Angle made by resultant magnetic moment (M) with M_1 is given by $\tan\phi = \frac{M_2\sin\theta}{M_1 + M_2\cos\theta}$

Example 1. *A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop (in A-m) will be* ***[JEE Main 2019]***

(a) $\frac{4m}{\pi}$ (b) $\frac{3m}{\pi}$ (c) $\frac{2m}{\pi}$ (d) $\frac{m}{\pi}$

Sol. (a) Let the given square loop has side a, then its magnetic dipole moment will be $m = Ia^2$.

When square is converted into a circular loop of radius r.



Then, wire length will be same in both area,

$$4a = 2\pi r \Rightarrow r = \frac{4a}{2\pi} = \frac{2a}{\pi}$$

Hence, area of circular loop formed is, $A' = \pi r^2 = \pi\left(\frac{2a}{\pi}\right)^2 = \frac{4a^2}{\pi}$

Magnitude of magnetic dipole moment of circular loop will be

$$m' = IA' = I\frac{4a^2}{\pi}$$

Ratio of magnetic dipole moments of both shapes is,

$$\frac{m'}{m} = \frac{I \cdot \frac{4a^2}{\pi}}{Ia^2} = \frac{4}{\pi} \Rightarrow m' = \frac{4m}{\pi}$$

Magnetic Field Strength at a Point due to Magnetic Dipole

The strength of a magnetic field at any point is defined as the force experienced by a hypothetical (unit magnetic) north pole placed at that point,

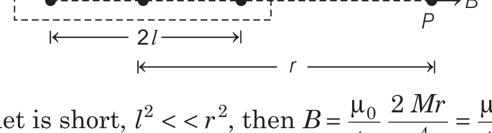
i.e. $\mathbf{B} = \frac{\mathbf{F}}{m}$, where m is the pole strength of hypothetical north pole. It is a vector quantity.

It's direction is the direction in which hypothetical north pole would tend to move, if free to do so.

There arises following cases

When Point lies on Axial or End-on Line of a Bar Magnet Magnetic field strength at point P due to bar magnet at a distance r from its centre is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2Mr}{(r^2 - l^2)^2}$$

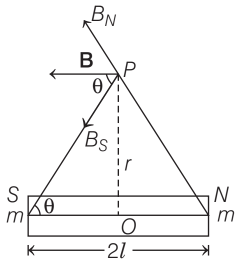


If magnet is short, $l^2 << r^2$, then $B = \frac{\mu_0}{4\pi} \frac{2Mr}{r^4} = \frac{\mu_0}{4\pi} \frac{2M}{r^3}$

The direction of magnetic field at point P is along NP.

When Point lies on Equatorial or Broad Side-on Line of a Bar Magnet Magnetic field strength at point P due to bar magnet at a distance r from its centre is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + l^2)^{3/2}}$$



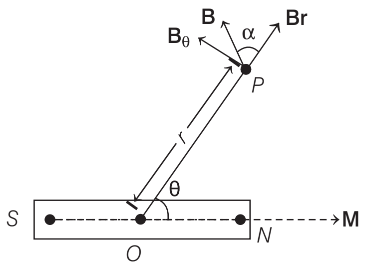
If $r^2 >> l^2$, then $B = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2)^{3/2}} = \frac{\mu_0}{4\pi} \frac{M}{r^3}$

The magnetic field at point P is directed parallel to the length of the magnet from its N to S-pole.

When Point makes Angle θ with Axis of a Bar Magnet At an angle θ with the axis of magnet, the magnetic field at point P at a distance r from centre of magnet in vacuum (or air) is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \sqrt{1 + 3\cos^2\theta}$$

and $\quad \tan\alpha = \frac{1}{2}\tan\theta$



For axial position of point $P, \theta = 0°$ and for equitorial position, $\theta = 90°$.

Example 2. *The pole strength of 12 cm long bar magnet is 20 A-m. The magnetic induction at a point 10 cm away from the centre of the magnet on its axial line is*

$\left(\text{Take, } \frac{\mu_0}{4\pi} = 10^{-7}\ Hm^{-1}\right)$

(a) 1.17×10^{-3} T (b) 2.20×10^{-2} T

(c) 1.17×10^{-2} T (d) 2.21×10^{-2} T

Sol. (a) Here, $2l = 12$ cm $= 0.12$ m,

$m = 20$ A-m, $d = 10$ cm $= 0.1$ m

On axial line, $B = \frac{\mu_0}{4\pi} \frac{2Md}{(d^2 - l^2)^2} = \frac{\mu_0}{4\pi} \frac{2m(2l)d}{(d^2 - l^2)^2}$

$$= 10^{-7} \times \frac{2(20)\,(0.12) \times 0.1}{[(0.1)^2 - (0.06)^2]^2}$$

$$= 1.17 \times 10^{-3}\ \text{T}$$

Example 3. *The earth's magnetic field at the equator is approximately* 0.4 G. *The earth's dipole moment is (Take,* R_e = 6.4 × 10^6 *m)*

(a) 4×10^2 A-m^2 (b) 1.05×10^{23} A-m^2
(c) 4×10^{-5} A-m^2 (d) 1.05×10^{-20} A-m^2

Sol. (*b*) The equitorial magnetic field is given by

$$B_e = \frac{\mu_0 m}{4\pi r^3}$$

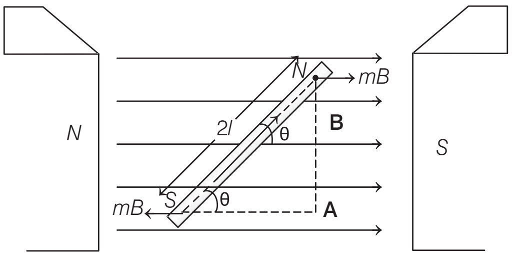
Given, $B_e \sim 0.4\ G = 4 \times 10^{-5}$ T, $R_e = 6.4 \times 10^6$ m

$$\therefore \quad m_e = \frac{4 \times 10^{-5} \times (6.4 \times 10^6)^3}{\frac{\mu_0}{4\pi}}$$

$$m_e = 4 \times 10^2 \times (6.4 \times 10^6)^3$$
$$= 1.05 \times 10^{23} \text{ A-m}^2$$

Torque on Bar Magnet in Uniform Magnetic Field

Consider a bar magnet of length $2l$ with strength of each pole m placed in a uniform magnetic field **B**. The magnet is held at angle θ with the direction of **B**.



Then, force on N-pole = mB, along **B**

Force on S-pole = mB, opposite to **B**

So, the torque that acts on bar magnet placed in magnetic field is given by, $\tau = MB \sin\theta$

In vector form, we can rewrite this equation as

$$\tau = \mathbf{M} \times \mathbf{B}$$

The direction of τ is perpendicular to the plane containing **M** and **B**.

Potential Energy of a Magnetic Dipole in a Uniform Magnetic Field

The torque acting on the dipole tends to align it in the direction of the field. Work has to be done in rotating the dipole against the action of the magnetic torque. This work done is stored in the form of potential energy of the dipole and is given as $U = W = -MB(\cos\theta - \cos\theta_0)$

In vector form, we may rewrite this equation as

$$U = -\mathbf{M} \cdot \mathbf{B}$$

Note When θ = 90°, U = 0, *i.e.* potential energy is zero, when the magnetic dipole is perpendicular to the field. Above equation shows that at θ = 0°, potential energy is minimum (= – *MB*), which is the most stable position. Further at θ = 180°, potential energy is maximum (= + *MB*), which is most unstable position.

Example 4. *A bar magnet when placed at an angle of* 30° *to the direction of magnetic field induction of* 5×10^{-2} *T, experiences a moment of couple* 2.5×10^{-6} *N-m. If the length of the magnet is* 5 *cm, its pole strength is*

(a) 2×10^2 A-m (b) 2×10^{-3} A-m
(c) 5 A-m (d) 5×10^{-2} A-m

Sol. (*b*) Here, θ = 30°, $B = 5 \times 10^{-2}$ T,

$\tau = 2.5 \times 10^{-6}$ N-m,

$2l = 5$ cm = 0.05 m, m = ?

∴ Torque, $\tau = MB \sin\theta = m(2l) B \sin\theta$

$$\Rightarrow \quad m = \frac{\tau}{B(2l)\sin\theta} = \frac{2.5 \times 10^{-6}}{5 \times 10^{-2} \times (0.05) \times \sin 30^\circ}$$

$$\Rightarrow \quad m = 2 \times 10^{-3} \text{ A-m}$$

Example 5. *A small bar magnet placed with its axis at* 30° *with an external field of* 0.06 *T experiences a torque of* 0.018 *N-m. The minimum work required to rotate it from its stable to unstable equilibrium position is* **[JEE Main 2020]**

(a) 9.2×10^{-3} J (b) 7.2×10^{-2} J
(c) 6.4×10^{-2} J (d) 11.7×10^{-3} J

Sol. (*b*) Torque on a bar magnet, $\tau = MB \sin\theta$

$$0.018 = M \times 0.06 \times \sin 30^\circ$$
$$\Rightarrow \quad M = 0.6 \text{ A-m}^2$$

Now, work done to rotate bar magnet,

$$W = MB[\cos\theta_1 - \cos\theta_2]$$

Here, $\theta_1 = 0°$ (for stable equilibrium position)

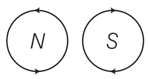
and $\theta_2 = 180°$ (for unstable equilibrium position).

So, $W = 0.6 \times 0.06 [\cos(0°) - \cos(180°)]$

$= 0.036 [(1) - (-1)] = 0.036 (2) = 0.072 = 7.2 \times 10^{-2}$ J

Current Carrying Loop as a Magnetic Dipole

A current carrying loop behaves as a magnetic dipole. If we look the upper face of the loop and current is flowing anti-clockwise, then it has a north polarity and if current is flowing clockwise, then it has a south polarity.



Magnetic dipole moment of a current carrying loop is given by $M = IA$

For N such turns, $M = NIA$

where, I = current and A = area of cross-section of the coil.

Magnetic Dipole Moment of a Revolving Electron in an Atom

The circular motion of an electron around the positively charged nucleus of an atom can be treated as a current carrying loop producing a magnetic field. Hence, it behaves like a magnetic dipole.

Consider m_e be the mass and $-e$ the charge of an electron revolving with speed v in a circular orbit of radius r.

The magnitude of the magnetic dipole moment **M** associated with the revolving electron is

$$\mathbf{M} = IA = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$$

The magnitude of the orbital angular momentum **L** of electron,

$$L = m_e vr \Rightarrow \frac{M}{L} = \frac{e}{2m_e}$$

Thus, **the ratio of the magnitude of the magnetic dipole moment to the magnitude of the angular momentum of the revolving electron is a constant. This ratio is called the gyromagnetic ratio.** Its value for an electron is 8.8×10^{10} C/kg.

The vector form of above equation can be written as

$$\mathbf{M} = -\left(\frac{e}{2m_e}\right)\mathbf{L}$$

$$\frac{M}{nh/2\pi} = \frac{e}{2m_e} \quad \Rightarrow \quad M = \frac{neh}{4\pi m_e}$$

When $n = 1$, $M = \mu$ (the elementary magnetic dipole moment), thus

$$\mu = \frac{eh}{4\pi m_e}$$

The elementary magnetic moment of a revolving electron is also known as Bohr magneton (μ).

Note Since, a current carrying loop acts as an magnetic dipole. So, the torque on the loop in a magnetic field is $\tau = NBiA \sin\theta = \mathbf{M} \times \mathbf{B}$.

Example 6. *A straight wire carrying current i is turned into a circular loop. If the magnitude of magnetic moment associated with it MKS unit is M, the length of wire will be*

(a) $\frac{4\pi}{M}$ (b) $\frac{M\pi}{4i}$

(c) $\sqrt{\frac{4\pi M}{i}}$ (d) $\sqrt{\frac{4\pi i}{M}}$

Sol. (c) Magnetic moment, $M = iA = i(\pi r^2)$, where $l = 2\pi r$

$$r = \sqrt{\frac{M}{\pi i}}$$

$$\therefore \quad l = 2\pi\sqrt{\frac{M}{\pi i}} = \sqrt{\frac{4\pi M}{i}}$$

Example 7. *The electron in hydrogen atom moves with a speed of 2.2×10^6 m/s in an orbit of radius 5.3×10^{-11}cm. The magnetic moment of the orbiting electron is*

(a) 3×10^{-20} *A-m*^{2} (b) 3.3×10^{-10} *A-m*^{2}

(c) 9.3×10^{-26} *A-m*^{2} (d) 9.6×10^{-10} *A-m*^{2}

Sol. (c) Given, $v = 2.2 \times 10^6$ ms^{-1},

$$r = 5.3 \times 10^{-11} \text{ cm} = 5.3 \times 10^{-13} \text{ m}$$

Frequency of revolution, $f = \frac{v}{2\pi r}$

The moving charge is equivalent to a current loop, given by

$$i = f \times e \quad \text{or} \quad i = \frac{ev}{2\pi r}$$

Let A be the area of the orbit, then the magnetic moment of the orbiting electron is,

$$M = iA = \left(\frac{ev}{2\pi r}\right)(\pi r^2) = \frac{evr}{2}$$

Putting the numerical values, we have

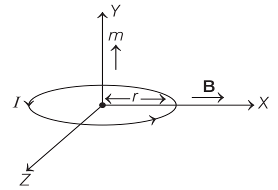
$$M = \frac{(1.6 \times 10^{-19})(2.2 \times 10^6)(5.3 \times 10^{-13})}{2}$$

$$\Rightarrow \quad M = 9.3 \times 10^{-26} \text{ A-m}^2$$

Example 8. *A circular coil having N turns and radius r carries a current I. It is held in the XZ-plane in a magnetic field $B\hat{\mathbf{i}}$. The torque on the coil due to the magnetic field (in N-m) is* **[JEE Main 2019]**

(a) $\frac{Br^2I}{\pi N}$ (b) $B\pi r^2 IN$ (c) $\frac{B\pi r^2 I}{N}$ (d) zero

Sol. (*b*) According to the question, the situation can be drawn as



Let the current I is flowing in anti-clockwise direction, then the magnetic moment of the coil is $m = NIA$.

where, N = number of turns in coil

and A = area of each coil = πr^2.

Its direction is perpendicular to the area of coil and is along Y-axis.

Then, torque on the current coil is

$$\tau = \mathrm{m} \times \mathrm{B} = mB \sin 90^\circ$$

$$= NIAB = NI\pi r^2 B \text{ (N-m)}$$

Bar Magnet as an Equivalent Solenoid

The resemblance of magnetic field lines of a bar magnet and a solenoid suggest that a bar magnet may be thought of as a large number of circulating currents in analogy with a solenoid.

- Cutting a bar magnet in half is like cutting a solenoid, we get two smaller solenoids with weaker magnetic properties.
- The field lines remain continuous, emerging from one face of the solenoid and entering into the other face.
- The magnitude of the magnetic moment of the solenoid is M = (total number of turns × current × cross-sectional area) = $n(2l)\,I(\pi a^2)$.
- A bar magnet and a solenoid produce similar magnetic fields. The magnetic moment of a bar magnet is thus equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field.

Example 9. *A closely wound solenoid of 2000 turns and area of cross-section* 1.6×10^{-4} m^2*, carrying a current of 4 A is suspended through its centre allowing it to turn in a horizontal plane. If a uniform horizontal magnetic field of* 7.5×10^{-2} *T is set up at angle of* $30°$ *with the axis of the solenoid, then the torque acting on the solenoid is*

(a) 0.048 J (b) 0.012 J (c) 48 J (d) 12 J

Sol. (a) Let M = magnetic moment of the solenoid, *i.e.*

$$M = NIA$$

Here, $N = 2000, I = 4\text{A}, A = 1.6 \times 10^{-4} \text{ m}^2$

$\therefore$ $M = 2000 \times 4 \times 1.6 \times 10^{-4}$

$M = 1.28 \text{ JT}^{-1}$

Torque, $\tau = MB \sin\theta$

Here, $\theta = 30°, B = 7.5 \times 10^{-2} \text{ T}, M = 1.28 \text{ JT}^{-1}$

$\tau = 1.28 \times 7.5 \times 10^{-2} \times \sin 30°$

$= 1.28 \times 7.5 \times 10^{-2} \times \frac{1}{2} = 0.048 \text{ J}$

Oscillations of a Freely Suspended Magnet

When a small bar magnet of magnetic moment **M** is placed in a uniform magnetic field **B** such as it is free to vibrate in a horizontal plane of magnetic field **B** about a vertical axis passing through its centre of mass. This bar magnet will oscillate. The restoring torque in this case will be

$$\tau = -MB\theta \quad (\because \text{for small oscillation, } \sin\theta \simeq \theta)$$

The deflecting torque on the magnet is

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

In equilibrium, deflecting torque = restoring torque

or $$\frac{d^2\theta}{dt^2} = \frac{-MB\theta}{I} = -\omega^2\theta, \text{ where } \omega = \sqrt{\frac{MB}{I}}$$

The period of vibration is given by, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MB}}$

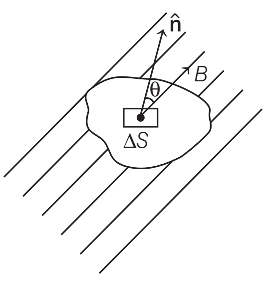
Magnetic field B can be calculated from above equation and is given as $B = \frac{4\pi^2 I}{MT^2}$

Magnetism and Gauss's Law

The net magnetic flux ϕ_B through any closed surface is always zero, *i.e.* the number of magnetic field lines leaving the surface is balanced by the number of lines entering it, *i.e.*

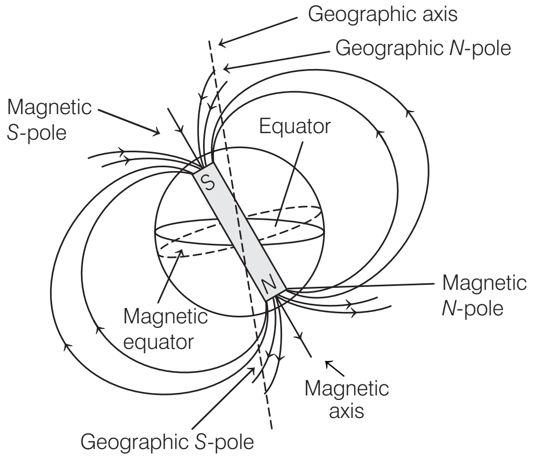
$\phi_B = \Sigma \mathbf{B} \cdot \Delta \mathbf{S} = 0$

where, B = magnetic field and ΔS = area element.



Earth's Magnetism

Our earth behaves as a huge powerful magnet. The value of magnetic field on the surface of earth is a few tenths of a gauss (1 G = 10^{-4} T) and its strength varies from place to place on the earth's surface. The earth's magnetic south pole is located near the geographic north pole and the earth's magnetic north pole is located near the geographic south pole.



Some definitions related to earth's magnetism are

Magnetic equator The huge circle whose plane is perpendicular to the earth's magnetic axis is called earth's magnetic equator.

Magnetic meridian The line joining the earth's magnetic poles is called the *magnetic axis* and a vertical plane passing through it is called the magnetic meridian.

Geographical meridian The line joining the geographical north and south poles is called the geographic axis and a vertical plane passing through it is called the geographical meridian.

Magnetic Elements of Earth

The earth's magnetic field at a place can be completely described by three parameters which are called elements of earth's magnetic field. These three elements are

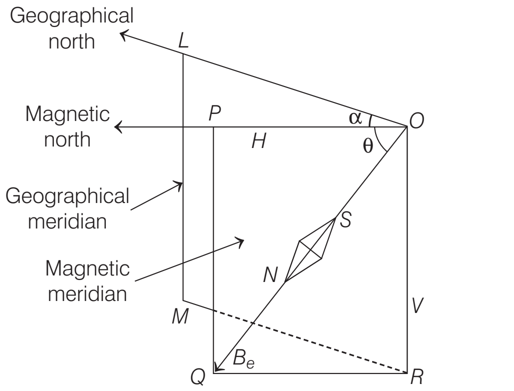
Angle of Declination or Magnetic Declination The angle between the magnetic meridian and geographical meridian at a place is called the angle of declination (or simply the declination) at that place.

Angle of Dip or Magnetic Inclination The angle of dip (θ) at a place is the angle between the direction of earth's magnetic field (B) and horizontal line in the magnetic meridian.

Horizontal-Vertical Component of the Earth's Magnetic Field Consider B_e be the net magnetic field at some point. H and V be the horizontal and vertical components of B_e. Let θ is the angle of dip at the some place, then we can see that,

$$H = B_e \cos\theta \quad \quad \ldots\text{(i)}$$

and $$V = B_e \sin\theta \qquad \ldots\text{(ii)}$$



Elements of earth's magnetic field

Squaring and adding Eqs. (i) and (ii), we get

$$B_e = \sqrt{H^2 + V^2}$$

Further, dividing Eqs. (ii) by Eq. (i), we get

$$\theta = \tan^{-1}\left(\frac{V}{H}\right)$$

Neutral Points

The point where resultant magnetic field due to magnet and due to earth is zero are called neutral points. Following are two cases related to neutral points

(i) When a bar magnet is placed along the magnetic meridian with its north pole pointing towards geographic north, then neutral points lie on the equatorial line of the magnet.

At each neutral point,

$$B_{eq} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + l^2)^{3/2}} = B_H$$

If $r >> l$, then $\frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} = B_H$

(ii) When a bar magnet is placed along the magnetic meridian with its north pole pointing towards geographic south, two neutral points are obtained on either side of the magnet along its axial line.

At each neutral point,

$$B_{axial} = \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - l^2)^2} = B_H$$

If $r >> l$, then $\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = B_H$

Example 10. *If a magnet is suspended at an angle 30° to the magnetic meridian, the dip needle makes an angle of 45° with the horizontal. The real dip is*

(a) $\tan^{-1}(\sqrt{3/2})$ (b) $\tan^{-1}(\sqrt{3})$

(c) $\tan^{-1}(\sqrt{3/2})$ (d) $\tan^{-1}(2/\sqrt{3})$

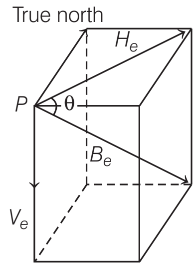
Sol. (*d*) As, $\tan\theta' = \frac{\tan\theta}{\cos\theta} = \frac{\tan 45°}{\cos 30°} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$

$\Rightarrow \qquad \theta' = \tan^{-1}(2/\sqrt{3})$

Example 11. *In the magnetic meridian of a certain place, the horizontal component of the earth's magnetic field is 0.26 G and the angle of dip is 60°. The magnetic field of the earth at this location is* **[NCERT]**

(a) 0.26 *G* (b) 0.42 *G* (c) 0.52 *G* (d) 0.80 *G*

Sol. (c) The earth's magnetic field is B_e and its horizontal and vertical components are H_e and V_e.



From the above figure, $\cos\theta = \frac{H_e}{B_e}$

Given, $H_e = 0.26$ G, $\theta = 60°$

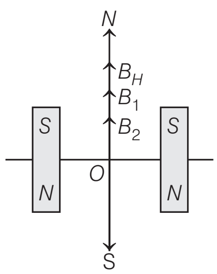
$\therefore \qquad B_e = \frac{H_e}{\cos 60°} = \frac{0.26}{\left(\frac{1}{2}\right)} = 0.52$ G

Example 12. *Two short bar magnets of length 1 cm each have magnetic moments 1.20 A-m² and 1.00 A-m², respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the south. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Take, horizontal component of the earth's magnetic induction = 3.6×10^{-5} Wb/m²)* **[JEE Main 2013]**

(a) 3.6×10^{-5} *Wb/m²* (b) 2.56×10^{-4} *Wb/m²*

(c) 3.50×10^{-4} *Wb/m²* (d) 5.80×10^{-4} *Wb/m²*

Sol. (*b*) Net magnetic field, $B_{net} = B_1 + B_2 + B_H$



$$B_{net} = \frac{\mu_0}{4\pi}\frac{(M_1 + M_2)}{r^3} + B_H = \frac{10^{-7}\ (1.2 + 1)}{(0.1)^3} + 3.6 \times 10^{-5}$$

$$= 2.56 \times 10^{-4}\ \text{Wb/m}^2$$

Example 13. *A magnetic compass needle oscillates 30 times per minute at a place, where the dip is 45° and 40 times per minute, where the dip is 30°. If B_1 and B_2 are respectively, the total magnetic field due to the earth at the two places, then the ratio $\frac{B_1}{B_2}$ is best given by* **[JEE Main 2019]**

(a) 1.8 T (b) 0.7 T (c) 3.6 T (d) 2.2 T

Sol. (b) Given, at first place, angle of dip, $\theta_1 = 45°$

Time period, $T_1 = \frac{60}{30} = 2$ s

At second place, angle of dip, $\theta_2 = 30°$

Time period, $T_2 = \frac{60}{40} = \frac{3}{2}$ s

Now, at first place,

$$B_{H_1} = B_1 \cos\theta_1 = B_1 \cos 45° = \frac{B_1}{\sqrt{2}} \quad \text{...(i)}$$

and at second place,

$$B_{H_2} = B_2 \cos\theta_2 = B_2 \cos 30° = \frac{\sqrt{3}}{2} B_2 \quad \text{...(ii)}$$

Also, time period of a magnetic needle is given by

$$T = 2\pi\sqrt{\frac{I}{MB_H}} \quad \text{...(iii)}$$

$$\therefore \quad T \propto \sqrt{\frac{1}{B_H}} \text{ or } \frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}} \quad \text{...(iv)}$$

By putting the values from Eqs. (i) and (ii) into Eq. (iv), we get

$$\frac{2}{\frac{3}{2}} = \sqrt{\frac{\frac{\sqrt{3}B_2}{2}}{\frac{B_1}{\sqrt{2}}}} \text{ or } \left(\frac{4}{3}\right)^2 = \frac{\sqrt{3}\times\sqrt{2}\,B_2}{2B_1}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{\sqrt{3}\times\sqrt{2}}{2} \times \frac{9}{16} = \frac{9\sqrt{3}}{16\sqrt{2}}$$

$$= \frac{9\times 1.732}{16\times 1.414} = \frac{15.588}{22.624} = 0.689 \approx 0.7 \text{ T}$$

Tangent Law

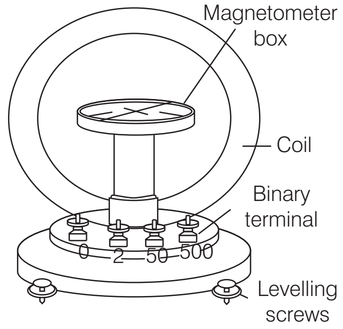
It states that, if a magnet is placed in two magnetic fields right angle to each other, then it will be acted upon by two couples tending to rotate it in opposite directions. It will be deflected through an angle θ, such that two couples balance each other.

Also, $$\tan\theta = \frac{B_1}{B_2}$$

where, θ is the angle between magnet and magnetic field B_2.

Tangent Galvanometer

It is a device used to measure very small current. It is a moving magnet type galvanometer and works on the principle of tangent law. In tangent galvanometer, a magnetic compass needle is placed horizontally at the centre of a vertical fixed current carrying coil whose plane is in the magnetic meridian. So, if the needle in equilibrium subtends an angle ϕ with the earth's magnetic field when current I flows through it, then



$$I = \frac{2R}{N\mu_0} H \tan\theta = \frac{H}{G}\tan\theta = K\tan\theta$$

where, $G = \frac{N\mu_0}{2R}$ is called *galvanometer constant* and $K = \frac{H}{G}$ is called *reduction factor* of tangent galvanometer.

Here, N is number of turns in the coil and R is radius of the coil.

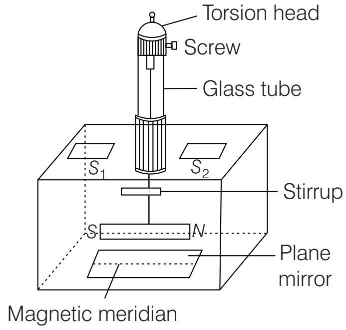
Tangent galvanometer is also called moving magnet type galvanometer.

Sensitivity It is the measure of change in the deflection produced by a unit current.

It is given by $$\frac{d\theta}{dI} = \frac{1}{k\sec^2\theta} = \frac{1}{k\left(1+\frac{I^2}{k^2}\right)}$$

Vibration Magnetometer

An instrument which is used to compare magnetic moments of two bar magnets or to determine the horizontal component of earth's magnetic field is known as vibration magnetometer. It is based on the principle that when a bar magnet suspended freely in a uniform magnetic field is displaced from its equilibrium position, it starts executing simple harmonic motion about the equilibrium position.



The time period of vibration of the magnet of moment of inertia I and magnetic moment M vibrating in uniform magnetic field of strength B_H is given by $T = 2\pi\sqrt{\frac{I}{MB_H}}$.

Uses of Vibration Magnetometer

(i) **To compare magnetic moments of two bar magnets** For two bar magnets A and B of same size and mass,

$$T_1 = 2\pi\sqrt{\frac{I}{M_1 B_H}}$$

and $$T_2 = 2\pi\sqrt{\frac{I}{M_2 B_H}}$$

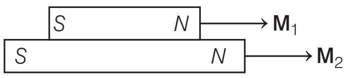
where, M_1 and M_2 be the magnetic moments of the magnets A and B respectively and I be the moment of inertia of each magnets.

$$\therefore \quad \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$$

(ii) **To compare the magnetic moments by sum and difference method**

(a) When the two magnets are placed with their poles in the same direction.

Net magnetic moment, $M_S = M_1 + M_2$

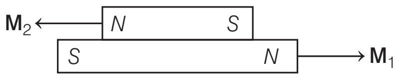


Net moment of inertia, $I_S = I_1 + I_2$

Time period of oscillation of this pair in Earth's magnetic field (B_H)

$$T_S = 2\pi\sqrt{\frac{I_S}{M_S B_H}} = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B_H}}$$

(b) When the two bar magnets are placed with their unlike poles in the same direction, the ratio of their magnetic moments can be given as



$$T_d = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)H}}$$

and $$\frac{M_1}{M_2} = \frac{T_d^2 + T_S^2}{T_d^2 - T_S^2}$$

(iii) **To compare horizontal components of Earth's magnetic field at two places**

$T = 2\pi\sqrt{\dfrac{I}{MB_H}}$, since I and M of the magnet are constant, so $T^2 \propto \dfrac{1}{B}$

$$\therefore \quad \frac{B_H}{B'_H} = \frac{T'^2}{T^2}$$

where, B_H and B'_H be the value of horizontal components of earth's magnetic field at two places A and B, respectively.

Example 14. *In a tangent galvanometer, when a current of 10 mA is passed, the deflection is 31°. By what percentage, the current has to be increased, so as to produce a deflection of 42°?*

(a) 10% (b) 50%
(c) 20% (d) 80%

Sol. (b) Here, $I_1 = 10$ mA, $\theta_1 = 31°$, $I_2 = ?$, $\theta_2 = 42°$

As, $$\frac{I_2}{I_1} = \frac{\tan\theta_2}{\tan\theta_1}$$

$$\Rightarrow \quad I_2 = I_1 \frac{\tan\theta_2}{\tan\theta_1} = 10 \times \frac{\tan 42°}{\tan 31°}$$

$$= \frac{10 \times 0.9}{0.6} = 15 \text{ mA}$$

Percentage increase in current,

$$\frac{(I_2 - I_1)}{I_1} \times 100 = \frac{(15 - 10)}{10} \times 100 = 50\%$$

Example 15. *The time period of vibration of two magnets in sum position is 3s. When polarity of weaker magnet is reversed, the combination makes 12 oscillations per minute. The ratio of magnetic moments of two magnets is*

(a) $\frac{16}{17}$ (b) $\frac{17}{8}$
(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

Sol. (b) Here, $T_1 = 3$ s, $T_2 = \frac{1}{12}$ min $= \frac{60 \text{ s}}{12} = 5$ s

$$\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$$

$$\therefore \quad \frac{M_1}{M_2} = \frac{5^2 + 3^2}{5^2 - 3^2} = \frac{34}{16} = \frac{17}{8}$$

Magnetisation of Materials

To describe the magnetic properties of materials, we have to understand the following terms

Magnetic Induction (B) The number of magnetic lines of force inside a magnetic substance crossing per unit area normal to their direction is called the magnetic induction or **magnetic flux density** inside the substance. It is denoted by **B**. The SI unit of **B** is tesla (T) or weber/metre^2 (Wb/m^2). The CGS unit is gauss (G).

$$1 \text{ Wb/m}^2 = 1 \text{ T} = 10^4 \text{ G}$$

Intensity of Magnetisation (I) The magnetic moment induced in unit volume of a magnetic substance placed in a magnetic field is called the intensity of magnetisation. It is denoted by I.

Thus, $$I = \frac{M}{V} = \frac{m}{A} = \frac{\text{Pole strength}}{\text{Area}}$$

i.e. The intensity of magnetisation may also be defined as the pole strength per unit cross-sectional area.

The unit of I is Am^{-1}.

Magnetic Intensity or Magnetic Field Strength (*H*) The capability of the magnetising field to magnetise the substance is expressed by means of a vector **H**, called the *magnetic intensity* of the field. It is defined through the vector relation,

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{I}$$

The SI unit of **H** is same as that of **I**, *i.e.* ampere/metre (A/m). The CGS unit is oersted.

Magnetic Permeability (μ) It is defined as the ratio of the magnetic induction **B** inside the magnetised substance to the magnetic intensity **H** of the magnetising field, *i.e.*

$$\mu = \frac{\mathbf{B}}{\mathbf{H}}$$

It is basically a measure of conduction of magnetic lines of force through a substance. The SI unit of magnetic permeability is weber/ampere-metre (Wb/A-m) or tesla metre-ampere^{-1} (TmA^{-1}).

Relative Magnetic Permeability (μ_r) It is the ratio of the magnetic permeability μ of the substance to the permeability of free space.

Thus, $\mu_r = \dfrac{\mu}{\mu_0}$

μ_r is a pure ratio, hence dimensionless.

For vacuum, its value is 1.

μ_r can also be defined as the ratio of the magnetic field B inside the substance when placed in magnetic field B_0.

Thus, $\mu_r = \dfrac{B}{B_0}$

Magnetic Susceptibility (χ_m) It is the property of the substance which shows how easily a substance can be magnetised when kept in a magnetising field. It can also be defined as the ratio of intensity of magnetisation (I) in a substance to the magnetic intensity (H) applied to the substance, *i.e.*

$$\chi_m = \frac{I}{H}$$

Relation between μ_r and χ_m We have, $B = \mu_0(I + H)$

or $B = B_0(\chi_m + 1)$

$\Rightarrow \mu_r = \chi_m + 1$

Example 16. *A paramagnetic substance in the form of a cube with sides* 1 cm *has a magnetic dipole moment of* 20×10^{-6} *J/T when a magnetic intensity of* 60×10^{3} *A/m is applied. Its magnetic susceptibility is* **[JEE Main 2019]**

(a) 3.3×10^{-4}

(b) 3.3×10^{-2}

(c) 4.3×10^{-2}

(d) 2.3×10^{-2}

Sol. (a) Given, side of cube = 1 cm = 10^{-2} m

$\therefore$ Volume, $V = 10^{-6}\text{m}^3$

Dipole moment, $M = 20 \times 10^{-6}$ J/T

Applied magnetic intensity, $H = 60 \times 10^{3}$ A/m

Intensity of magnetisation

$$I = \frac{M}{V} = \frac{20 \times 10^{-6}}{10^{-6}} = 20 \text{ A/m}$$

Now, magnetic susceptibility χ is,

$$\chi = \frac{\text{Intensity of magnetisation}}{\text{Applied magnetic intensity}} = \frac{I}{H} = \frac{20}{60 \times 10^3}$$

$$\Rightarrow \quad \chi = \frac{1}{3} \times 10^{-3} = 3.3 \times 10^{-4}$$

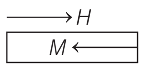
Classification of Magnetic Materials

According to behaviour of magnetic substances, they are classified into three categories as follows

Diamagnetic Substances

Those substances which when placed in an external magnetic field, acquire a very low magnetism in a direction opposite to the field, are called diamagnetic substances. These substances when brought near the end of a strong magnet, get repelled by it.

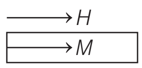
e.g. Copper (Cu), silver (Ag), bismuth (Bi), zinc (Zn), diamond (C), salt (NaCl), water (H_2O), mercury (Hg) , nitrogen (N_2), hydrogen (H_2), magnesium (Mg), gold (Au), etc.



Paramagnetic Substances

Those substances which when placed in an external field, acquire a feeble magnetism in the direction of field, are called paramagnetic substances. These substances when brought near the end of a strong magnet, get attracted towards it.

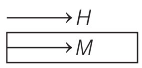
e.g. Aluminium (Al), sodium (Na), potassium (K), platinum (Pt), manganese (Mn), copper sulphate ($CuSO_4$), oxygen (O_2), etc.



Ferromagnetic Substances

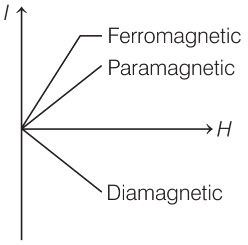
Those substances which when placed in a magnetic field, acquire a strong magnetism in the direction of field, are called ferromagnetic substances. These substances when brought near the end of a strong magnet get radially attracted towards it.

e.g. Iron (Fe), nickel (Ni), cobalt (Co), magnetite (Fe_3O_4) or natural magnet, etc.

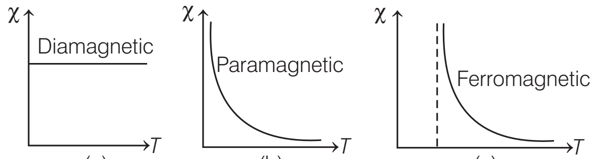


Following are few important graphs related to magnetic materials

- I-H curve



- χ-T curve



(a) (b) (c)

Curie's Law

According to this law, the magnetic susceptibility of paramagnetic substances is inversely proportional to absolute temperature, *i.e.*

$$\chi \propto \frac{1}{T} \Rightarrow \chi T = \text{constant}$$

Here, T = absolute temperature.

At Curie temperature, ferromagnetic substances changes into paramagnetic substances.

Curie-Wiess Law

Above Curie temperature or Curie point, the magnetic susceptibility of ferromagnetic substances is inversely proportional to $(T - T_C)$, *i.e.*

$$\chi \propto \frac{1}{T - T_C}$$

or

$$\chi = \frac{C}{T - T_C}$$

Here, T_C = Curie temperature.

Example 17. *A paramagnetic material has* 10^{28} *atoms/m*^{3}*. Its magnetic susceptibility at temperature 350 K is* 2.8×10^{-4}*. Its susceptibility at 300 K is* ***[JEE Main 2019]***

(a) 3.726×10^{-4} (b) 3.672×10^{-4}

(c) 2.672×10^{-4} (d) 3.267×10^{-4}

Sol. (*d*) From Curie's law for paramagnetic substance, we have

Magnetic susceptibility, $\chi \propto \frac{1}{T}$

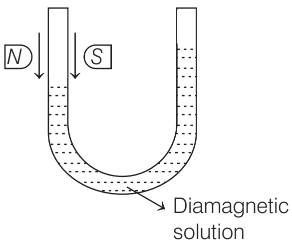
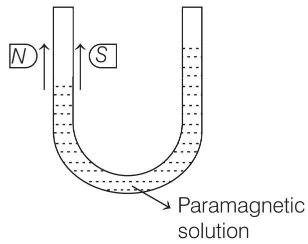
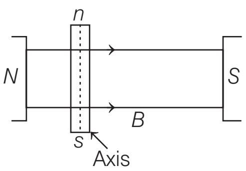
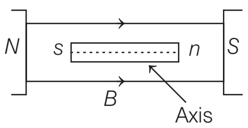
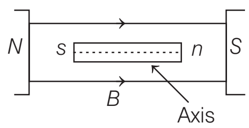
$$\therefore \quad \frac{\chi_2}{\chi_1} = \frac{T_1}{T_2}$$

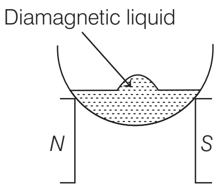
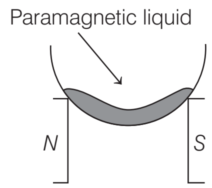
$$\Rightarrow \quad \chi_2 = \frac{\chi_1 \cdot T_1}{T_2}$$

$$\Rightarrow \quad \chi_2 = \frac{2.8 \times 10^{-4} \times 350}{300}$$

$$= 3.267 \times 10^{-4}$$

Comparative Study of Magnetic Materials

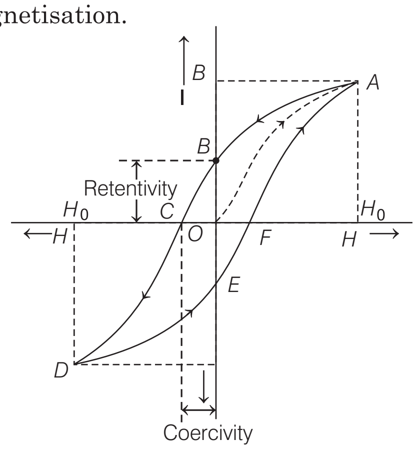
Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
When a diamagnetic solution is poured into a U-tube and one arm is placed between the poles of strong magnet, the level of solution in that arm is lowered. 	The level of the paramagnetic solution in that arm rises. 	No liquid is ferromagnetic.
If a rod of diamagnetic material is suspended freely between two magnetic poles, its axis becomes perpendicular to the magnetic field. 	Paramagnetic rod becomes parallel to the magnetic field. 	Ferromagnetic rod also becomes parallel to the magnetic field. 

Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
In non-uniform magnetic field, the diamagnetic substances are attracted towards the weaker fields, *i.e.* they move from stronger to weaker magnetic field.	In non-uniform magnetic field, they move from weaker to stronger part of the magnetic field slowly.	In non-uniform magnetic field, they move from weaker to stronger magnetic field rapidly.
Their permeability is less than one ($\mu < 1$).	Their permeability is slightly greater than one ($\mu > 1$).	Their permeability is much greater than one ($\mu >> 1$).
Their susceptibility is small and negative. Their susceptibility is independent of temperature.	Their susceptibility is small and positive. Their susceptibility is inversely proportional to absolute temperature which is Curie's law. *i.e.* $\chi \propto \frac{1}{T}$.	Their susceptibility is large and positive. They also follow Curie's law. *i.e.* $\chi \propto \frac{1}{T}$. At Curie temperature, ferromagnetic substances change into paramagnetic substances.
Shape of diamagnetic liquid in a glass crucible and kept over two magnetic poles. 	Shape of paramagnetic liquid in a glass crucible and kept over two magnetic poles. 	No liquid is ferromagnetic.
In these substances, the magnetic lines of force are farther than in air.	In these substances, the magnetic lines of force are closer than in air.	In these substances, magnetic lines of force are much closer than in air.
The resultant magnetic moment of these substances is zero.	These substances have a permanent magnetic moment.	These substances also have a permanent magnetic moment.

Hysteresis

The lagging of intensity of magnetisation (I) or magnetic induction (B) behind the magnetising field (H) during the process of magnetisation and demagnetisation of a ferromagnetic material is called hysteresis.

The following figure shows the magnetisation curve of a ferromagnetic material, when it is taken over a complete cycle of magnetisation.



The graph follows that

(i) Corresponding to point O, the magnetisation (H) is zero and likewise intensity of magnetisation (I) is also zero.

(ii) As magnetising field is increased, intensity of magnetisation I also increases along OA and becomes maximum at A. This maximum value is called *saturation value*.

(iii) If magnetic field is now decreased slowly, intensity of magnetisation decreases along the path AB. Corresponding to point B, magnetising field becomes zero, but some magnetisation equal to OB is still left in the specimen.

Here, OB gives the measure of *retentivity* of the material of the specimen.

(iv) If magnetic field H is reversed, the magnetisation decreases along BC, till it becomes zero corresponding to point C. Thus, to make I to be zero, magnetising field equal to OC has to be applied in reverse direction. Here, OC gives the measure of *coercivity* of the material of the specimen.

(v) When field H is further increased in reverse direction, *i.e.* along CD, the intensity of magnetisation attains saturation value corresponding to point D.

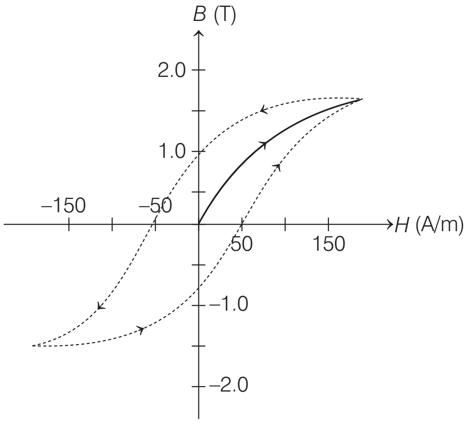
(vi) Finally, when magnetising field is increased in original direction, the point A is reached *via* EFA.

If the magnetising field is repeatedly changed between H_0 and $-H_0$, the curve $ABCDEFA$ is retraced. This curve is called the hysteresis loop.

The energy loss in magnetising and demagnetising a specimen is proportional to the area of hysteresis loop.

Note For steel, coercivity is large. However, retentivity is comparatively smaller in case of steel. Due to high value of coercivity and fairly large value of retentivity, steel is used to make permanent magnets. For soft iron, coercivity is very small and area of hysteresis loop is small. Because of these characteristics, soft iron is an ideal material for making electromagnets.

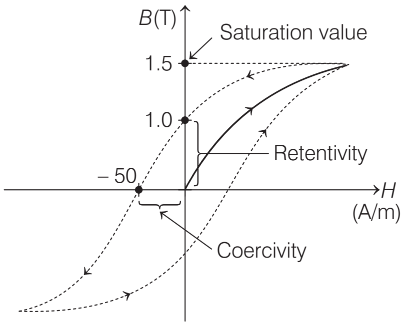
Example 18.



The figure gives experimentally measured B versus H variation in a ferromagnetic material. The retentivity, coercivity and saturation respectively of the material are ***[JEE Main 2020]***

(a) 1.0 *T*, 50 *A/m and* 1.5 *T*
(b) 150 *A/m*, 1.0 *T and* 1.5 *T*
(c) 1.5 *T*, 50 *A/m and* 1.0 *T*
(d) 1.5 *T*, 50 *A/m and* 1.0 *T*

Sol. *(a)* We can obtain the values of retentivity (the remaining value of magnetic field B when $H = 0$), coercivity (value of H when $B = 0$) and saturation value of induced magnetic field (B_{max}) as shown in the figure.



Clearly, retentivity = 1T, coercivity = 50 Am^{-1}

and saturation value = 1.5 T

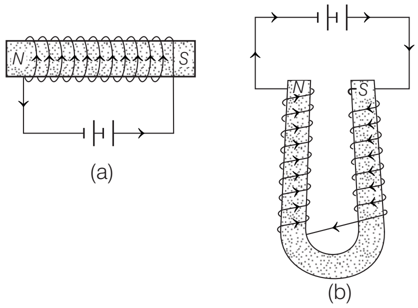
Permanent Magnets

Those substances which at room temperature retain their ferromagnetic property for a long period of time are called permanent magnets. An efficient way to make a permanent magnet is to place a ferromagnetic rod in a solenoid and pass a current. The magnetic field of the solenoid magnetises the rod.

The hysteresis curve allows us to select the suitable materials for permanent magnets. The material should have high retentivity, so that the magnet is strong and high coercivity, so that the magnetisation is not erased by stray magnetic fields, temperature fluctuations or minor mechanical damage.

Electromagnets

As we know that a current carrying solenoid behaves like a bar magnet. If we place a soft iron rod in the solenoid, then the magnetism of the solenoid increases hundreds of times and the solenoid is called an electromagnet. It is a temporary magnet.



The ferromagnetic property of the iron core causes the internal magnetic domains of the iron to line up with the smaller driving magnetic field produced by the current in the solenoid.

These are widely used in electric and electromechanical devices, including motors and generators. The main advantage of an electromagnet over a permanent magnet is that the magnetic field can be rapidly manipulated over a wide range by controlling the amount of electric current.

Example 19. *Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required?* ***[JEE Main 2020]***

(a) T: Large retentivity, small coercivity
(b) P : Large retentivity, large coercivity
(c) P : Small retentivity, large coercivity
(d) T : Large retentivity, large coercivity

Sol. (*b*) Permanent magnets (*P*) must be able to hold large magnetism for a long time. So, they must have large coercivity and large retentivity.

Electromagnet and transformer cores (*T*) must be able to loose magnetisation quickly when current in their coils are zero. So, they must have small retentivity.

Practice Exercise

ROUND I Topically Divided Problems

Bar Magnet and Magnetic Dipole

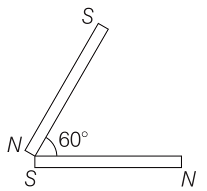
1. A magnet of magnetic moment M and pole strength m is divided in two equal parts, the magnetic moment of each part will be

(a) M (b) $M/2$
(c) $M/4$ (d) $2M$

2. Two magnets have the same length and the same pole strength. But one of the magnets has a small hole at its centre, then

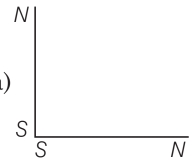
(a) both have equal magnetic moment
(b) one with hole has smaller magnetic moment
(c) one with hole has large magnetic moment
(d) one with hole loses magnetism through the hole

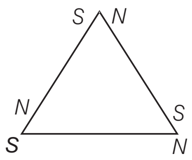
3. Two magnets of equal magnetic moments M each are placed as shown in figure. The resultant magnetic moment is

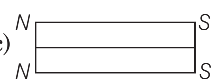


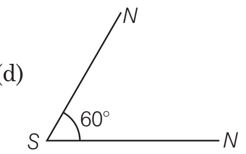
(a) M (b) $\sqrt{3}\,M$
(c) $\sqrt{2}\,M$ (d) $M/2$

4. In which orientation, the resultant magnetic moment of two magnets will be zero, if magnetic moment of each magnets is M in the following figures?

(a)


(b)


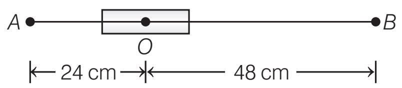
(c)


(d)


5. An insulating thin rod of length l has a linear charge density $\rho(x) = \rho_0 \dfrac{x}{l}$ on it. The rod is rotated about an axis passing through the origin ($x = 0$) and perpendicular to the rod. If the rod makes rotations per second, then the time averaged magnetic moment of the rod is **[JEE Main 2019]**

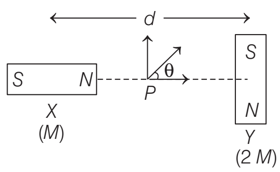
(a) $n\rho l^3$ (b) $\pi\, n\rho l^3$
(c) $\dfrac{\pi}{3} n\rho l^3$ (d) $\dfrac{\pi}{4} n\rho l^3$

6. A bar magnet of length 3 cm has a point A and B along axis at a distance of 24 cm and 48 cm on the opposite ends. Ratio of magnetic fields at these points will be



(a) 8 (b) 3
(c) 4 (d) $1/2\sqrt{2}$

7. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their mid-point P, at angle $\theta = 45°$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole) **[JEE Main 2019]**



(a) $\left(\dfrac{\mu_0}{4\pi}\right)\dfrac{M}{\left(\dfrac{d}{2}\right)^3} \times qv$ (b) 0

(c) $\sqrt{2}\left(\dfrac{\mu_0}{4\pi}\right)\dfrac{M}{\left(\dfrac{d}{2}\right)^3} \times qv$ (d) $\left(\dfrac{\mu_0}{4\pi}\right)\dfrac{2M}{\left(\dfrac{d}{2}\right)^3} \times qv$

8. The magnetic potential due to a magnetic dipole at a point on its axis distant 40 cm from its centre is found to be 2.4×10^{-5} JA^{-1}m^{-1}. The magnetic moment of the dipole will be

(a) 28.6 A-m^2 (b) 32.2 A-m^2

(c) 38.4 A-m^2 (d) None of these

9. A magnetic needle lying parallel to a magnetic field required W units of work to turn it through 60°. The torque required to maintain the needle in this position will be

(a) $\sqrt{3}\,W$ (b) W (c) $\sqrt{3}\,\dfrac{W}{2}$ (d) 2 W

10. Rate of change of torque τ with deflection q is maximum for a magnet suspended freely in a uniform magnetic field of induction B, when

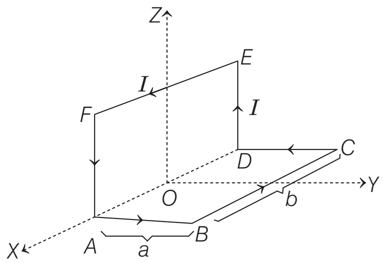
(a) $\theta = 0°$ (b) $\theta = 45°$ (c) $\theta = 60°$ (d) $\theta = 90°$

11. A rectangular coil (dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3A in the clockwise direction, is kept centred at the origin and in the XZ-plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is **[JEE Main 2019]**

(a) 0.27 N-m (b) 0.38 N-m

(c) 0.42 N-m (d) 0.55 N-m

12. A wire carrying current I is bent in the shape $ABCDEFA$ as shown, where rectangle $ABCDA$ and $ADEFA$ are perpendicular to each other. If the sides of the rectangles are of lengths a and b, then the magnitude and direction of magnetic moment of the loop $ABCDEFA$ is **[JEE Main 2020]**



(a) abI, along $\left(\dfrac{\hat{j}}{\sqrt{2}} + \dfrac{\hat{k}}{\sqrt{2}}\right)$

(b) $\sqrt{2}\,abI$, along $\left(\dfrac{\hat{j}}{\sqrt{2}} + \dfrac{\hat{k}}{\sqrt{2}}\right)$

(c) $\sqrt{2}\,abI$, along $\left(\dfrac{\hat{j}}{\sqrt{5}} + \dfrac{2\hat{k}}{\sqrt{5}}\right)$

(d) abI , along $\left(\dfrac{\hat{j}}{\sqrt{5}} + \dfrac{2\hat{k}}{\sqrt{5}}\right)$

13. A toroid of n turns, mean radius R and cross-sectional radius a carries current I. It is placed on a horizontal table taken as XY-plane. Its magnetic moment **m** **[NCERT Exemplar]**

(a) is non-zero and points in the z-direction by symmetry

(b) points along the axis of the toroid ($\mathbf{m} = m$)

(c) is zero, otherwise there would be a field falling as $\dfrac{1}{r^3}$ at large distances outside the toroid

(d) is pointing radially outwards

Earth's Magnetism

14. A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the centre of the magnet. If $B_H = 0.4$ G, the magnetic moment of the magnet is

(a) 2.880×10^3 JT^{-1} (b) 2.880×10^2JT^{-1}

(c) 2.880JT^{-1} (d) 28.80JT^{-1}

15. A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A rests with its plane normal to an external field of magnitude 5.0×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of 2.0/s. What is the moment of inertia of the coil about its axis of rotation?

(a) 1.2×10^{-4} g-cm^2 (b) 3×10^{-4} kg-m^2

(c) 0.3×10^{-4} kg-m^2 (d) 1.2×10^{-4} kg-m^2

16. Two bar magnets having same geometry with magnetic moments M and 2 M are firstly placed in such a way that their similar poles are at the same side. Time period of oscillations is T_1. Now, the polarity of one of the magnets is reversed and time period of oscillation is T_2

(a) $T_1 < T_2$ (b) $T_1 = T_2$

(c) $T_1 > T_2$ (d) $T_2 = \infty$

17. The period of oscillation of a freely suspended bar magnet is 4 s. If it is cut into two equal parts in length, then the time period of each part will be

(a) 4 s (b) 2 s

(c) 0.5 s (d) 0.25 s

18. A small circular loop of conducting wire has radius a and carries current i. It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and released, it starts performing simple harmonic motion of time period T. If the mass of the loop is m, then **[JEE Main 2020]**

(a) $T = \sqrt{\dfrac{\pi m}{iB}}$ (b) $T = \sqrt{\dfrac{2m}{iB}}$

(c) $T = \sqrt{\dfrac{2\pi m}{iB}}$ (d) $T = \sqrt{\dfrac{\pi m}{2iB}}$

19. Consider the two idealised systems: (i) a parallel plate capacitor with large plates and small separation and (ii) a long solenoid of length $L >> R$, radius of cross-section. In (i), E is ideally treated as a constant between plates and zero outside. In (ii), magnetic field is constant inside the solenoid and zero outside. These idealised assumptions, however contradict fundamental laws as below **[NCERT Exemplar]**

(a) case (i) contradicts Gauss's law for electrostatic fields.
(b) case (i) contradicts Gauss's law for magnetic fields.
(c) case (i) agrees with $\oint \mathbf{E} \cdot d\mathbf{l} = 0$.
(d) case (ii) contradicts $\oint \mathbf{H} \cdot d\mathbf{l} = I_{en}$.

20. The vertical component of earth's magnetic field is zero at or the earth's magnetic field always has a vertical component except at the **[NCERT Exemplar]**

(a) magnetic poles
(b) geographic poles
(c) every place
(d) magnetic equator

21. The magnetic field of the earth can be modelled by that of a point dipole placed at the centre of the earth. The dipole axis makes an angle of 11.3° with the axis of the earth. At Mumbai, declination is nearly zero, then **[NCERT Exemplar]**

(a) the declination varies between 11.3° W to 11.3° E
(b) the least declination is 0°
(c) the plane defined by dipole axis and earth axis passes through Greenwich
(d) declination averaged over earth must be always negative

22. At a certain place, the horizontal component of the earth's magnetic field is B_0 and the angle of dip is 45°. The total intensity of the field at that place will be

(a) B_0
(b) $\sqrt{2}\, B_0$
(c) $2\, B_0$
(d) B_0^2

23. The earth's magnetic field at a certain place has a horizontal component of 0.3 G and total strength of 0.5 G. Find angle of dip in $\tan^{-1}$.

(a) $\delta = \tan^{-1} \frac{4}{3}$
(b) $\delta = \tan^{-1} \frac{3}{4}$
(c) $\delta = \tan^{-1} \frac{5}{3}$
(d) $\delta = \tan^{-1} \frac{3}{5}$

Magnetic Materials and Hysteresis

24. The magnetic needle of a tangent galvanometer is deflected at an angle 30° due to a magnet. The horizontal component of earth's magnetic field 0.34×10^{-4} T is along the plane of the coil. The magnetic intensity is

(a) 1.96×10^{-4} T
(b) 1.96×10^{4} T
(c) 1.96×10^{-5} T
(d) 1.96×10^{5} T

25. Two tangent galvanometers having coils of the same radius are connected in series. A current flowing in them produces deflections of 60° and 45°, respectively.

The ratio of the number of turns in the coils is

(a) 4/3
(b) $(\sqrt{3}+1)/1$
(c) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$
(d) $\frac{\sqrt{3}}{1}$

26. A magnet freely suspended in a vibration magnetometer makes 40 oscillations per minute at a place A and 20 oscillations per minute at a place B. If the horizontal component of earth's magnetic field at A is 36×10^{-6} T, then its value at B is

(a) 36×10^{-6} T
(b) 9×10^{-6} T
(c) 144×10^{-6} T
(d) 228×10^{-6} T

27. Two magnets held together in earth's magnetic field with same polarity together make 12 vib min^{-1} and when opposite poles together make 4 vib min^{-1}. The ratio of magnetic moments is

(a) 9 : 1
(b) 1 : 3
(c) 1 : 9
(d) 5 : 4

28. A magnet performs 10 oscillations per minute in a horizontal plane at a place, where the angle of dip is 45° and the total intensity is 0.707 CGS units. The number of oscillations per minute at a place, where dip angle is 60° and total intensity is 0.5 CGS units will be

(a) 5
(b) 7
(c) 9
(d) 11

29. A vibration magnetometer consists of two identical bar magnets placed one over the other such that they are perpendicular and bisect each other. The time period of oscillation in a horizontal magnetic field is $2^{5/4}$ s. One of the magnets is removed and if the other magnet oscillates in the same field, then the time period in second is

(a) $2^{1/4}$
(b) $2^{1/2}$
(c) 2
(d) 4

30. The time of vibration of a dip needle vibrating in the vertical plane is 3 s. When magnetic needle is made to vibrate in the horizontal plane, the time of vibration is $3\sqrt{2}$ s. Then, the angle of dip is

(a) 30°
(b) 45°
(c) 60°
(d) 90°

31. A bar magnet is placed north-south with its north pole due north. The points of zero magnetic field will be in which direction from centre of magnet

(a) north and south
(b) east and west
(c) north-east and south-west
(d) north-east and south-east

32. A short bar magnet with the north pole facing north forms a neutral point P in the horizontal plane. If the magnet is rotated by 90° in the horizontal plane, the net magnetic induction at P is (Horizontal component of earth's magnetic field $= B_H$)

(a) zero (b) $2\,B_H$

(c) $\frac{\sqrt{5}}{2}B_H$ (d) $\sqrt{5}\,B_H$

33. A bar magnet 20 cm in length is placed with its south pole towards geographic north. The neutral points are situated at a distance of 40 cm from centre of the magnet. If horizontal component of earth's field 3.2×10^{-5} T, then pole strength of magnet is

(a) 5 A-m (b) 10 A-m

(c) 45 A-m (d) 20 A-m

34. The horizontal component of flux density of earth's magnetic field is 1.7×10^{-5} T. The value of horizontal component of intensity of earth's magnetic field will be

(a) 24.5 Am^{-1} (b) 13.5 Am^{-1}

(c) 1.53 Am^{-1} (d) 0.35 Am^{-1}

35. A bar magnet is cut into two equal halves by a plane parallel to the magnetic axis. Of the following physical quantities, the one which remains unchanged is

(a) pole strength

(b) magnetic moment

(c) intensity of magnetisation

(d) moment of inertia

36. A rod of ferromagnetic material with dimensions 10 cm × 0.5 cm × 0.2 cm is placed in a magnetic field of strength 0.5×10^4 Am^{-1} as a result of which a magnetic moment of 5 A-m^2 is produced in the rod. The value of magnetic induction will be

(a) 0.54 T (b) 6.28 T

(c) 0.358 T (d) 2.591 T

37. The space inside a toroid is filled with tungsten whose susceptibility is 6.8×10^{-5}. The percentage increase in the magnetic field will be

(a) 0.0068% (b) 0.068%

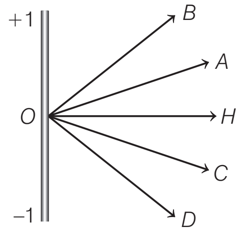
(c) 0.68% (d) None of these

38. A paramagnetic sample shows a net magnetisation of 8 Am^{-1} when placed in an external magnetic field of 0.6 T at a temperature of 4 K. When the same sample is placed in an external magnetic field of 0.2 T at a temperature of 16 K, the magnetisation will be **[NCERT Exemplar]**

(a) $\frac{32}{3}$ Am^{-1} (b) $\frac{2}{3}$ Am^{-1}

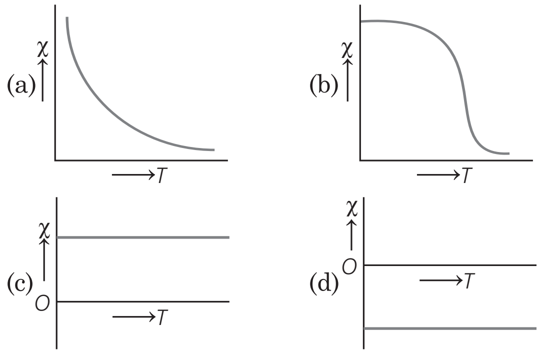
(c) 6 Am^{-1} (d) 2.4 Am^{-1}

39. The variation of the intensity of magnetisation (I) with respect to the magnetising field (H) in a diamagnetic substance is described by the graph in figure.



(a) OD (b) OC (c) OB (d) OA

40. The variation of magnetic susceptibility (χ) with temperature for a diamagnetic substance is best represented by figure



41. A copper rod is suspended in a non-homogeneous magnetic field region. The rod when in equilibrium will align itself

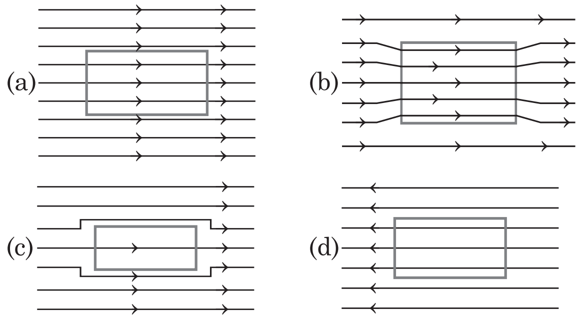
(a) in the region, where magnetic field is strongest

(b) in the region, where magnetic field is weakest and parallel to direction of magnetic field there

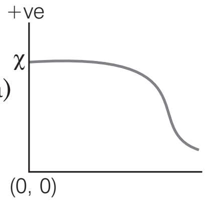
(c) in the direction in which it was originally suspended

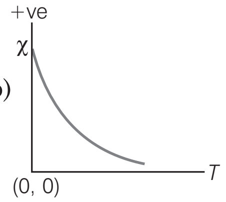
(d) in the region, where magnetic field is weakest and perpendicular to the direction of magnetic field there

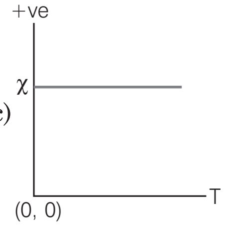
42. A uniform magnetic field parallel to the plane of paper, exsisted in space initially directed from left to right. When a bar of soft iron is placed in the field parallel to it, the lines of force passing through it will be represented by figure.

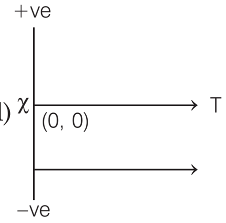


43. The variation of magnetic susceptibility (χ) with absolute temperature T for a ferromagnetic substance is given by

(a)


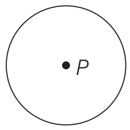
(b)


(c)


(d)


44. A perfectly diamagnetic sphere has a small spherical cavity at its centre,which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field **B.** Then, the field inside the paramagnetic substance is

[JEE Main 2020]



(a) much larger than $|\mathbf{B}|$ and parallel to **B**
(b) **B**
(c) zero
(d) much larger than $|\mathbf{B}|$ but opposite to **B**

45. The relative permeability of a substance X is slightly less than unity and that of substance Y is slightly more than unity, then

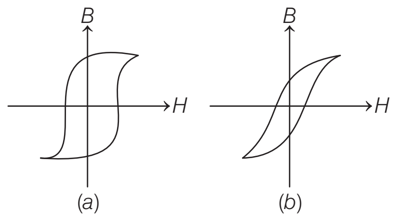
(a) X is paramagnetic and Y is ferromagnetic
(b) X is diamagnetic and Y is ferromagnetic
(c) X and Y both are paramagnetic
(d) X is diamagnetic and Y is paramagnetic

46. A bar magnet is demagnetised by inserting it inside a solenoid of length 0.2 m, 100 turns and carrying a current of 5.2 A. The coercivity of the bar magnet is

[JEE Main 2019]

(a) 1200 A/m (b) 285 A/m
(c) 2600A/m (d) 520A/m

47. Hysteresis loops for two magnetic materials a and b are as given below



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then, it is proper to use

[JEE Main 2016]

(a) a for electric generators and transformers
(b) a for electromagnets and b for electric generators
(c) a for transformers and b for electric generators
(d) b for electromagnets and transformers

ROUND II Mixed Bag

Only One Correct Option

1. The period of oscillation of a bar magnet in a vibration magnetometer is 2 s. The period of oscillation of another bar magnet whose magnetic moment is 4 times to that of Ist magnet is

(a) 4 s
(b) 1 s
(c) 2 s
(d) 0.5 s

2. A horizontal circular loop carries a current that looks anti-clockwise when viewed from above. It is replaced by an equivalent magnetic dipole N-S. Which of the following is true?

(a) The line N-S should be along a diameter of the loop
(b) The line N-S should be perpendicular to the plane of the loop
(c) North pole should be below the loop
(d) None of the above

3. The points A and B are situated perpendicular to the axis of 2 cm long bar magnet at large distances x and $3x$ from the centre on opposite sides. The ratio of magnetic fields at A and B will be approximately equal to

(a) 27 : 1 (b) 1 : 27
(c) 9 : 1 (d) 1 : 9

4. Which of the following statements are correct?

I. Electric monopoles do not exist whereas magnetic monopoles exist.
II. Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.
III. Magnetic field lines are completely confined within a toroid.
IV. Magnetic field lines inside a bar magnet are not parallel.
V. $\chi = -1$ is the condition for a perfect diamagnetic material, where χ is its magnetic susceptibility.

Choose the correct answer from the options given below. [JEE Main 2021]

(a) III and V only (b) II and IV only
(c) I and II only (d) II and III only

5. A magnetic needle of magnetic moment 6.7×10^{-2} A-m^2 and moment of inertia 7.5×10^{-6} kg -m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is [JEE Main 2017]

(a) 8.89 s (b) 6.98 s
(c) 8.76 s (d) 6.65 s

6. The coercivity of a small magnet, where the ferromagnet gets demagnetised is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetised when kept inside the solenoid is [JEE Main 2014]

(a) 30 mA (b) 60 mA
(c) 3 A (d) 6 A

7. The susceptibility of a paramagnetic material is K at 27 °C. At what temperature will its susceptibility be $K/2$?

(a) 600° C (b) 287° C (c) 54° C (d) 327° C

8. Force between two identical short bar magnets whose centres are r metre apart is 8.1 N, when their axes are along the same line. If separation is increased to 3 r and the axis are rearranged perpendicularly, the force between them would become

(a) 2.4 N (b) 1.2 N (c) 0.1 N (d) 1.15 N

9. A coil of 50 turns and area 1.25×10^{-3} m^2 is pivoted about a vertical diameter in a uniform horizontal magnetic field and carries a current of 2A. When the coil is held with its plane in N-S direction, it experience a couple of 0.04 N-m; and when its plane is E-W, the corresponding couple is 0.03 N-m. The magnetic induction is

(a) 0.2 T (b) 0.3 T (c) 0.4 T (d) 0.5 T

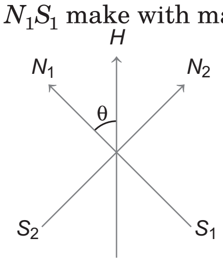
10. A long magnet is placed vertically with its S-pole resting on the table. A neutral point is obtained 10 cm from the pole due geographic north of it. If $H = 3.2 \times 10^{-5}$ T, then the pole strength of magnet is

(a) 8 ab-A-cm^{-1} (b) 16 ab-A-cm^{-1}
(c) 32 ab-A-cm^{-1} (d) 64 ab-A-cm^{-1}

11. A dip needle vibrates in the vertical plane perpendicular to magnetic meridian. The time period of vibration is found to be 2 s. The same needle is then allowed to vibrate in the horizontal plane and time period is again found to be 2 s. Then, the angle of dip is

(a) 0° (b) 30° (c) 45° (d) 90°

12. Two magnets of equal mass are joined at 90° to each other as shown in figure. Magnet N_1S_1 has a magnetic moment $\sqrt{3}$ times that of N_2S_2. The arrangement is pivoted, so that it is free to rotate in horizontal plane. When in equilibrium, what angle should N_1S_1 make with magnetic meridian?



(a) 75° (b) 60° (c) 30° (d) 45°

13. S is the surface of a lump of magnetic material. [NCERT Exemplar]

(a) Lines of B are necessarily continuous across S
(b) Some lines of B must be continuous across S
(c) Lines of H are necessarily continuous across S
(d) Both (a) and (c)

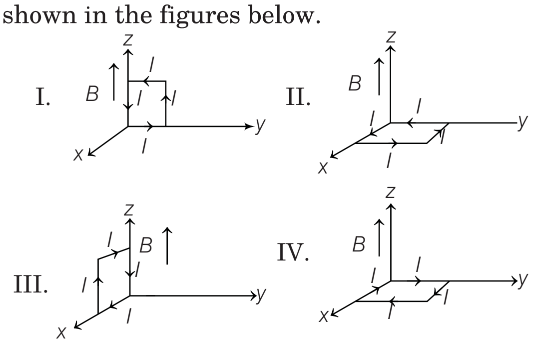
14. An iron rod of volume 10^{-3} m^3 and relative permeability 1000 is placed as core in a solenoid with 10 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod will be [JEE Main 2020]

(a) 50×10^2 A-m^2 (b) 5×10^2 A-m^2
(c) 500×10^2 A-m^2 (d) 0.5×10^2 A-m^2

15. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio of $\frac{B_1}{B_2}$ is [JEE Main 2018]

(a) 2 (b) $\sqrt{3}$
(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

16. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below.



If there is a uniform magnetic field of 0.3 T in the positive z-direction in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium? **[JEE Main 2015]**

(a) I and II respectively
(b) I and III respectively
(c) II and IV respectively
(d) II and III respectively

17. A steel wire of length l has a magnetic moment M. It is bent at its middle point at an angle of 60°, then the magnetic moment of new shape of wire will be

(a) $M/\sqrt{2}$ (b) $M/2$
(c) M (d) $\sqrt{2}\,M$

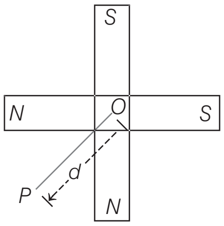
18. The mass of a specimen of a ferromagnetic material is 0.6 kg and the density is 7.8×10^3 kg m^{-3}. If the area of hysteresis loop of alternating magnetising field of frequency 50 Hz is 0.722 MKS units, then hysteresis loss per second will be

(a) 27.77×10^{-5} J (b) 2.777×10^{-5} J
(c) 27.77×10^{-4} J (d) 27.77×10^{-6} J

19. In an experiment with vibration magnetometer, the value of $4\pi^2 I / T^2$ for a short bar magnet is observed as 36×10^{-4}. In the experiment with deflection magnetometer with the same magnet, the value of $4\pi d^3 / 2\mu_0$ is observed as $10^8/36$. The magnetic moment of the magnet used is

(a) 50 A-m (b) 100 A-m
(c) 200 A-m (d) 1000 A-m

20. Two short bar magnets of equal dipole moment M are fastened perpendicularly at their centres as shown in the figure. The magnitude of resultant of two magnetic field at a distance d from the centre on the bisector of the right angle is



(a) $\frac{\mu_0}{4\pi}\frac{2\sqrt{2}\,M}{d^3}$ (b) $\frac{\mu_0}{4\pi}\frac{2M}{d^3}$
(c) $\frac{\mu_0}{4\pi}\frac{M}{d^3}$ (d) $\frac{\mu_0}{4\pi}\frac{\sqrt{2}\,M}{d^3}$

21. A bar magnet of length 10 cm and having pole strength equal to 10^{-3} Wb is kept in a magnetic field having magnetic induction B equal to $4\pi \times 10^{-3}$ T. It makes an angle of 30° with the direction of magnetic induction. The value of the torque acting on the magnet is

(a) 0.5 N-m (b) $2\pi \times 10^{-5}$ N-m
(c) $\pi \times 10^{-5}$ N-m (d) 0.5×10^{-5} N-m

22. A uniform magnetic needle is suspended from its centre by a thread. Its upper end is now loaded with a mass of 50 *mg* and the needle becomes horizontal. If the strength of each pole is 98.1 ab-amp-cm and $g = 981$ cms^{-2}, then the vertical component of earth's magnetic induction is

(a) 0.50 G (b) 0.25 G (c) 0.005 G (d) 0.05 G

23. In a vibration magnetometer, the time period of a bar magnet oscillating in horizontal component of earth's magnetic field is 2 s. When a magnet is brought near and parallel to it, the time period reduces to 1 s. The ratio F/H of the fields, F due to magnet and H, the horizontal component will be

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{3}$ (d) 3

24. The magnetic needle of an oscillation magnetometer makes 10 oscillations per min under the action of earth's magnetic field alone. When a bar magnet is placed at some distance along the axis of the needle, it makes 14 oscillations per min. If the bar magnet is turned, so that its poles interchange their positions, then the new frequency of oscillation of the needle is

(a) 10 vib min^{-1} (b) 2 vib min^{-1}
(c) 4 vib min^{-1} (d) 20 vib min^{-1}

25. A bar magnet has a magnetic moment equal to 5×10^{-5} Wb-m. It is suspended in a magnetic field which has a magnetic induction B equal to $8\pi \times 10^{-4}$ T. The magnet vibrates with a period of vibration equal to 15 s. The moment of inertia of magnet is

(a) 4.54×10^4 kg-m^2 (b) 4.54×10^{-5} kg-m^2
(c) 4.54×10^{-4} kg-m^2 (d) 4.54×10^5 kg-m^2

26. The plane of a dip circle is set in the geographic meridian and the apparent dip is δ_1. It is then set in a vertical plane perpendicular to the geographic meridian. The apparent dip angle is δ_2. The declination θ at the place is

(a) $\theta = \tan^{-1}(\tan\delta_1 \tan\delta_2)$
(b) $\theta = \tan^{-1}(\tan\delta_1 + \tan\delta_2)$
(c) $\theta = \tan^{-1}\left(\frac{\tan\delta_1}{\tan\delta_2}\right)$
(d) $\theta = \tan^{-1}(\tan\delta_1 - \tan\delta_2)$

27. A deflection magnetometer is adjusted in the usual way. When a magnet is introduced, the deflection observed is θ and the period of oscillation of the needle in the magnetometer is T. When the magnet is removed, the period of oscillation is T_0. The relation between T and T_0 is

(a) $T^2 = T_0^2 \cos\theta$ (b) $T = T_0 \cos\theta$
(c) $T = \frac{T_0}{\cos\theta}$ (d) $T^2 = \frac{T_0^2}{\cos\theta}$

28. Two short magnets with pole strengths of 900 ab-amp-cm and 100 ab-amp-cm are placed with their axes in the same vertical line, with similar poles facing each other. Each magnet has a length of 1 cm. When separation between the nearer poles is 1 cm, the weight of upper magnet is supported by the repulsive force between the magnets. If $g = 1000\,\text{cms}^{-2}$, then the mass of upper magnet is

(a) 100 g (b) 55 g (c) 45 g (d) 77.5 g

29. At a certain location in Africa, a compass points $12°$ west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points $60°$ above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at that location.

(a) $32°$ west of geographical meridian and 3.2×10^{-4} T
(b) $12°$ west of geographical meridian and 0.32×10^{-4} T
(c) $12°$ east of geographical meridian and 0.32×10^{-4} T
(d) $32°$ east of geographical meridian and 3.2×10^{-4} T

30. A circular coil has moment of inertia 0.8 kg-m^2 around any diameter and is carrying current to produce a magnetic moment of $20\,\text{A-m}^2$. The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4 T is applied along the vertical, it starts rotating around its horizontal diameter. The angular speed of the coil, acquired after rotating by $60°$, will be **[JEE Main 2020]**

(a) 10 rad s^{-1} (b) 20π rad s^{-1}
(c) 10π rad s^{-1} (d) 13.16 rad s^{-1}

31. A paramagnetic sample shows a net magnetisation of 6 A/m, when it is placed in an external magnetic field of 0.4 T at a temperature of 4 K. When the sample is placed in an external magnetic field of 0.3 T at a temperature of 24 K, then the magnetisation will be **[JEE Main 2020]**

(a) 4 A/m (b) 0.75 A/m
(c) 1 A/m (d) 2.25 A/m

32. A charged particle going around in a circle can be considered to be a current loop. A particle of mass m carrying charge q is moving in a plane with speed v under the influence of magnetic field $\mathbf{B}$. The magnetic moment of this moving particle is **[JEE Main 2020]**

(a) $\dfrac{mv^2\mathbf{B}}{2B^2}$ (b) $-\dfrac{mv^2\mathbf{B}}{2\pi B^2}$
(c) $-\dfrac{mv^2\mathbf{B}}{B^2}$ (d) $-\dfrac{mv^2\mathbf{B}}{2B^2}$

Numerical Value Questions

33. At a given place on the earth's surface, the horizontal component of earth's magnetic field is 3×10^{-5} T and resultant magnetic field is 6×10^{-6} T. So, the angle of dip (in degree) at this place is ………… .

34. The earth's magnetic field may be considered due to a short magnet placed at the centre of earth and oriented along magnetic south-north direction. The ratio of magnitude of magnetic field on earth's surface at magnetic equator and that at magnetic poles will be $1 : x$, where the value of x is ……… .

35. If the magnetic susceptibility of a paramagnetic material at $-73\,°\text{C}$ is 0.0075, then its value at $-173°\,\text{C}$ will be ……… .

36. The intensity of magnetic field at a point X on the axis of a small magnet is equal to the field intensity at another point Y on its equatorial axis. The ratio of distances of X and Y from the centre of the magnet will be $2^{1/P}$, where the value of P is ……… .

37. A loop $ABCDEFA$ of straight edges has six corner points $A(0, 0, 0)$, $B(5, 0, 0)$, $C(5, 5, 0)$, $D(0, 5, 0)$, $E(0, 5, 5)$ and $F(0, 0, 5)$. The magnetic field in this region is $\mathbf{B} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{k}})$T. The quantity of flux through the loop $ABCDEFA$ (in Wb) is ………… . **[JEE Main 2020]**

Answers

Round I

1. (b)	2. (b)	3. (a)	4. (b)	5. (d)	6. (a)	7. (b)	8. (c)	9. (a)	10. (a)
11. (a)	12. (b)	13. (c)	14. (c)	15. (d)	16. (a)	17. (b)	18. (c)	19. (b)	20. (d)
21. (a)	22. (b)	23. (a)	24. (c)	25. (d)	26. (b)	27. (d)	28. (b)	29. (c)	30. (c)
31. (b)	32. (d)	33. (c)	34. (b)	35. (c)	36. (b)	37. (a)	38. (b)	39. (b)	40. (d)
41. (d)	42. (b)	43. (a)	44. (c)	45. (d)	46. (c)	47. (d)			

Round II

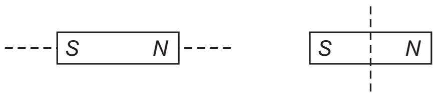
1. (b)	2. (b)	3. (a)	4. (a)	5. (d)	6. (c)	7. (d)	8. (c)	9. (c)	10. (c)
11. (c)	12. (c)	13. (a)	14. (b)	15. (c)	16. (c)	17. (b)	18. (c)	19. (b)	20. (a)
21. (a)	22. (b)	23. (d)	24. (b)	25. (d)	26. (c)	27. (a)	28. (b)	29. (b)	30. (d)
31. (b)	32. (d)	33. 60	34. 2	35. 0.015	36. 3	37. 175			

Solutions

Round I

1. If we cut along the axis of magnet of length l, then new pole strength, $m' = \frac{m}{2}$ and new length $l' = l$.

$\therefore$ New magnetic moment, $M' = \frac{m}{2} \times l = \frac{ml}{2} = \frac{M}{2}$



If we cut perpendicular to the axis of magnet, then new pole strength, $m' = m$ and new length, $l' = \frac{l}{2}$

$\therefore$ New magnetic moment,

$$M' = m \times \frac{l}{2} = \frac{ml}{2} = \frac{M}{2}$$

2. Hole reduces the effective length of the magnet and hence, magnetic moment reduces.

3. As, magnetic moments are directed along SN, angle between $\mathbf{M}$ and $\mathbf{M}$ is $\theta = 120°$.

$\therefore$ Resultant magnetic moment

$$= \sqrt{M^2 + M^2 + 2\,M \cdot M \cos 120°}$$

$$= \sqrt{M^2 + M^2 + 2\,M^2(-1/2)} = M$$

4. In Fig. (a), $M' = \sqrt{M^2 + M^2} = \sqrt{2}\,M$

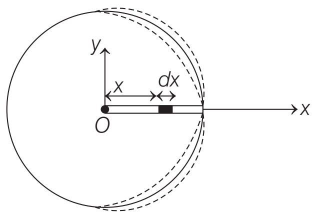
As magnetic moments are in a closed loop in Fig. (b),

$\Rightarrow \quad M' = 0$

In Fig. (c), $M' = M + M = 2\,M$

In Fig. (d), $M' = \sqrt{M^2 + M^2 + 2\,MM \cos 60°} = \sqrt{3}\,M$

5. Let dq be the charge on dx length of rod at a distance x from origin as shown in the figure below



The magnetic moment dm of this portion dx is given as

$$dm = (dI)\,A$$

$$dm = n\,dq\,A \qquad (\because I = qn,\ \therefore dI = n \cdot dq)$$

$$= n\,\rho\,dx\,A$$

where, ρ = charge density of rod $= \rho_0 \frac{x}{l}$.

So, $$dm = \frac{n\rho_0 x\,dx\,\pi x^2}{l}$$

$$= \frac{\pi\,n\,\rho_0}{l} \cdot x^3 \cdot dx$$

So, magnetic moment associated with complete rod is

$$M = \int_{x=0}^{x=l} dm = \int_0^l \frac{\pi\,n\,\rho_0}{l} \cdot x^3\ dx$$

$$= \frac{\pi\,n\,\rho_0}{l} \cdot \int_0^l x^3\ dx$$

$$= \frac{\pi\,n\,\rho_0}{l}\left[\frac{x^4}{4}\right]_0^l$$

$$= \frac{\pi\,n\,\rho_0\,l^3}{4}$$

At $\quad x = l, \rho = \rho_0$

$\therefore \quad M = \frac{\pi}{4} n\rho l^3$

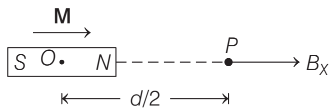
6. Here, $2l = 3$ cm, $d_1 = 24$ cm and $d_2 = 48$ cm

As, the magnet is short, $\frac{B_1}{B_2} = \frac{d_2^3}{d_1^3} = \left(\frac{48\text{ cm}}{24\text{ cm}}\right)^3 = 8$

7. Let $2l_1$ and $2l_2$ be the length of dipole X and Y respectively.

For dipole X, point P lies on its axial line. So, magnetic field strength at P due to X is

$$B_X = \frac{\mu_0}{4\pi} \cdot \frac{2Mr}{(r^2 - l_1^2)^2}, \text{ along } OP$$



Here, $\quad r = \frac{d}{2}$

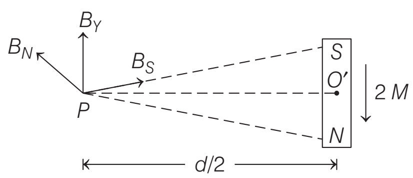
Also, $\quad d >> l_1$

$$\Rightarrow \quad |\mathbf{B}_X| = \frac{\mu_0}{4\pi} \cdot \frac{2M(d/2)}{(d/2)^4} = \frac{\mu_0}{4\pi} \frac{2M}{(d/2)^3}$$

Similarly, for dipole Y, point P lies on its equatorial line. So, magnetic field strength at P due to Y is

$$\mathbf{B}_Y = \frac{\mu_0}{4\pi} \cdot \frac{2M}{(r^2 + l_2^2)^{3/2}},$$

along a line perpendicular to $O'P$



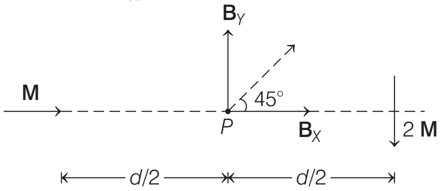
Here, $\quad r = \frac{d}{2}$

Also, $\quad d >> l_2$

$$\Rightarrow \quad |\mathbf{B}_Y| = \frac{\mu_0}{4\pi} \frac{2M}{(d/2)^3}$$

Thus, the resultant magnetic field due to X and Y at P is

$$\mathbf{B}_{net} = \mathbf{B}_X + \mathbf{B}_Y$$



Since, $|\mathbf{B}_Y| = |\mathbf{B}_X|$

Thus, the resultant magnetic field ($\mathbf{B}_{net}$) at P will be at 45° with the horizontal.

This means, direction of $\mathbf{B}_{net}$ and velocity of the charged particle is same.

$\therefore$ Force on the charged particle moving with velocity v in the presence of magnetic field which is

$$\mathbf{B} = q(\mathbf{v} \times \mathbf{B}) = q\,|\mathbf{v}||\mathbf{B}|\sin\theta$$

where, θ is the angle between $\mathbf{B}$ and $\mathbf{v}$.

According to the above analysis, we get

$$\theta = 0, \text{ therefore } F = 0$$

Thus, magnitude of force on the particle at that instant is zero.

8. Here, $r = 40$ cm $= 0.4$ m,

$\theta = 0°$ (an axial line)

$V = 2.4 \times 10^{-5}$ J/A-m, $M = ?$

As, $V = \dfrac{\mu_0}{4\pi} \dfrac{M\cos\theta}{r^2}$

$\Rightarrow \quad 2.4 \times 10^{-5} = 10^{-7} \dfrac{M \times 1}{(0.4)^2}$

or $\quad M = 38.4$ A-m^2

9. As, $W = BM\cos 60° = \dfrac{MB}{2}$ or $MB = 2W$

$\therefore \quad \tau = MB\sin\theta = (2W)\sin 60° = 2W\dfrac{\sqrt{3}}{2} = W\sqrt{3}$

10. As, $\tau = MB\sin\theta$ or $\dfrac{d\tau}{d\theta} = MB\cos\theta$. It will be maximum, when $\theta = 0°$

11. Given, area of the rectangular coil,

$A = 5 \text{ cm} \times 2.5 \text{ cm}$

$\Rightarrow \quad A = 12.5 \text{ cm}^2 = 12.5 \times 10^{-4} \text{ m}^2$

Number of turns, $N = 100$ turns

Current through the coil, $I = 3$ A

Magnetic field applied, $B = 1$ T

Angle between the magnetic field and area vector of the coil, $\theta = 45°$

As we know that, when a coil is tilted by an angle θ in the presence of some external magnetic field, then the net torque experienced by the coil is,

$$\tau = \mathbf{M} \times \mathbf{B} = NI(\mathbf{A} \times \mathbf{B})$$
$$= NIAB\sin\theta$$

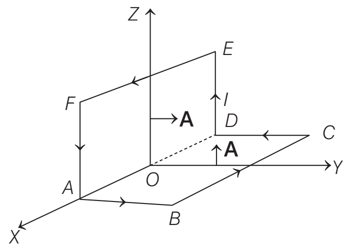
Substituting the given values, we get

$$\tau = 100 \times 3 \times 12.5 \times 10^{-4} \times 1 \times \sin 45°$$
$$\tau = 0.707 \times 100 \times 3 \times 12.5 \times 10^{-4} \text{ N-m}$$
$$= 2.651 \times 10^{-1} \text{ N-m} \approx 0.27 \text{ N-m}$$

12. Given rectangular loop can be viewed as a combination of two loops $ABCDA$ and $ADEFA$.

So, $\mathbf{m}$ (loop) $= \mathbf{m}$ ($ABCDA$) + $\mathbf{m}$ ($ADEFA$)

The area vector of $ABCDA$ is along Z-axis and that of $ADEFA$ is along Y-axis as shown in figure



Now, $\quad \mathbf{m}(ABCDA) = IA\hat{\mathbf{k}} = I(ab)\hat{\mathbf{k}}$

and $\quad \mathbf{m}(ADEFA) = IA\hat{\mathbf{j}} = I(ab)\hat{\mathbf{j}}$

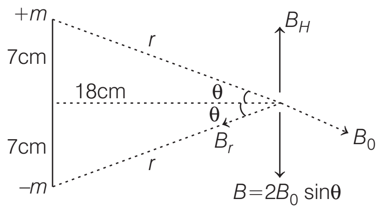
$\therefore \quad \mathbf{m}\text{ (loop)} = Iab\hat{\mathbf{k}} + Iab\hat{\mathbf{j}} = Iab(\hat{\mathbf{k}} + \hat{\mathbf{j}})$

$$= (Iab\sqrt{2})\left(\frac{\hat{\mathbf{k}}}{\sqrt{2}} + \frac{\hat{\mathbf{j}}}{\sqrt{2}}\right)$$

So, magnitude of $\mathbf{m} = \sqrt{2}Iab$ and direction of $\mathbf{m}$ is along $\dfrac{1}{\sqrt{2}}(\hat{\mathbf{j}} + \hat{\mathbf{k}})$.

13. In case of a toroid, the magnetic field is only confined inside the body of toroid in the form of concentric magnetic lines of force and there is no magnetic field outside the body of toroid. Thus, the magnetic moment of toroid is zero. Hence, option (c) is correct.

14.



i.e. $\dfrac{2\mu_0}{4\pi}\dfrac{m}{r^2} \times \dfrac{7}{r} = 0.4 \times 10^{-4}$

$\Rightarrow 2 \times 10^{-7} \times \dfrac{m \times 7}{(7^2 + 18^2)^{3/2}} \times 10^4 = 0.4 \times 10^{-4}$

$$m = \frac{4 \times 10^{-2} \times (373)^{3/2}}{14}$$

$$M = m \times 14 \text{ cm} = m \times \frac{14}{100}$$

$$= \frac{0.04 \times (373)^{32}}{14} \times \frac{14}{100}$$

$$= 4 \times 10^{-4} \times 7203.82 = 2.88 \text{ J/T}$$

15. Given, number of turns of circular coil, $n = 16$

Radius of circular coil, $r = 10$ cm $= 0.1$ m

Current, $I = 0.75$ A

Magnetic field, $B = 5.0 \times 10^{-2}$ T

Frequency, $f = 2 \text{ s}^{-1}$

Magnetic moment of the coil, $M = nIA$

$$= 16\times 0.75\times\pi\ (0.1)^2$$
$$= 16\times 0.75\times 3.14\times 0.1\times 0.1$$
$$= 0.377 \text{ J/T}$$

Frequency of oscillation of the coil, $f = \dfrac{1}{2\pi}\sqrt{\dfrac{M\times B}{I}}$

where, I = moment of inertia of the coil.

Squaring on both sides, we get

$$f^2 = \frac{1}{4\pi^2}\cdot\frac{MB}{I}$$

$$\Rightarrow \qquad I = \frac{MB}{4\pi^2 f^2} = \frac{0.377\times 5\times 10^{-2}}{4\times 3.14\times 3.14\times 2\times 2}$$
$$= 1.2\times 10^{-4} \text{ kg-m}^2$$

Thus, the moment of inertia of the coil is 1.2×10^{-4} kg-m^2.

16. When polarity is reversed, net magnetic moment $2M - M = M$, decreases. Therefore, time period of oscillation increases, *i.e.* $T_2 > T_1$.

17. From $T = 2\pi\sqrt{\dfrac{I}{MB}}$

When it is cut into two equal parts in length, mass of each part becomes 1/2, $I = \text{mass}\dfrac{(\text{length})^2}{12}$ becomes $\dfrac{1}{8}$th and M becomes $\dfrac{1}{2}$.

$$T' = 2\pi\sqrt{\frac{I/8}{(M/2)B}}$$
$$= \frac{1}{2}\left(2\pi\sqrt{\frac{I}{MB}}\right) = \frac{1}{2}T = 2\text{ s}$$

18. Time period of oscillation of a magnetic dipole in a uniform magnetic field, $T = 2\pi\sqrt{\dfrac{I}{\mu B}}$

where, μ = magnetic dipole.

Moment of dipole = Current × Area = $i\pi a^2$

I = moment of inertia of a coil = $\dfrac{ma^2}{2}$

So, $\quad T = 2\pi\sqrt{\dfrac{\dfrac{ma^2}{2}}{\pi a^2 iB}} = 2\pi\sqrt{\dfrac{m}{2\pi iB}} = \sqrt{\dfrac{2\pi m}{iB}}$

19. According to Gauss's law for electrostatic field,

$$\oint \mathbf{E}\cdot d\mathbf{S} = Q/\varepsilon_0$$

It does not contradict for electrostatic fields as the electric field lines do not form a continuous closed path.

According to Gauss's law in magnetic form, $\oint \mathbf{B}\cdot d\mathbf{S} = 0$

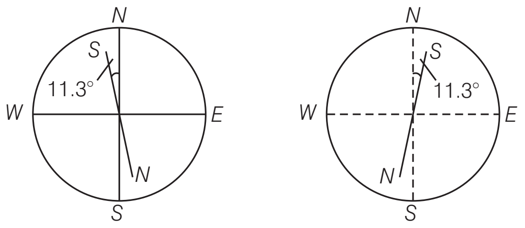
It contradicts for magnetic field because there is a magnetic field inside the solenoid and no field is present outside the solenoid carrying current but the magnetic field lines form the closed paths.

Hence, the correct option is (b).

20. At magnetic equator, the angle of dip is 0°. Then, the horizontal component,

$$B_H = B\cos\theta = 0$$

21. Since, the axis of the magnetic dipole placed at the centre of earth makes an angle 11.3° with the axis of earth, two possibilities arises as shown in Fig. (a) and (b). Hence, the declination varies between 11.3°*W* to 11.3°*E*.



(a) (b)

22. For $H = R\cos\delta$,

$$\Rightarrow \qquad R = \frac{H}{\cos\delta} = \frac{B_0}{\cos 45^\circ} \qquad (\because H = B_0)$$
$$= \sqrt{2}\,B_0$$

23. Horizontal component of earth's magnetic field is given by

$$B_H = B\cos\delta$$

or $\qquad \cos\delta = \dfrac{B_H}{B} = \dfrac{0.3}{0.5} = \dfrac{3}{5}$

$\therefore \qquad \dfrac{1}{\cos\delta} = \sec\delta = \dfrac{5}{3}$

Now, $\qquad \tan\delta = \sqrt{\sec^2\delta - 1} = \sqrt{\left(\dfrac{5}{3}\right)^2 - 1}$

$$= \sqrt{\frac{25}{9} - 1} = \frac{4}{3}$$

$\therefore \qquad \delta = \tan^{-1}\dfrac{4}{3}$

24. In a tangent galvanometer, applying tangent law,

$$B = H\tan\theta$$

$\Rightarrow \qquad B = 0.34\times 10^{-4}\tan 30^\circ = 0.34\times 10^{-4}\times\dfrac{1}{\sqrt{3}}$

$$B = 1.96\times 10^{-5}\text{ T}$$

25. In series, same current flows through two tangent galvanometers.

i.e. $\qquad i = \dfrac{2rH}{\mu_0 n_1}\tan\theta_1 = \dfrac{2rH}{\mu_0 n_2}\tan\theta_2$

$\therefore \qquad \dfrac{n_1}{n_2} = \dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\tan 60^\circ}{\tan 45^\circ} = \sqrt{3} : 1$

26. Here, $n_1 = 40, n_2 = 20, H_1 = 36\times 10^{-6}$ T, $H_2 = ?$

$\therefore \qquad \dfrac{H_2}{H_1} = \dfrac{n_2^2}{n_1^2} = \dfrac{(20)^2}{(40)^2} = \dfrac{1}{4}$

or $\qquad H_2 = \dfrac{36\times 10^{-6}}{4} = 9\times 10^{-6}$ T

27. As, $\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2} = \frac{n_2^2 + n_1^2}{n_1^2 - n_2^2}$

$\Rightarrow \quad \frac{M_1}{M_2} = \frac{4^2 + 12^2}{12^2 - 4^2} = \frac{160}{128} = 5:4$

28. Here, $n_1 = 10$ oscillations per min,

$\delta_1 = 45°$, $R_1 = 0.707$ CGS units,

$n_2 = ?$, $\delta_2 = 60°$, $R_2 = 0.5$ CGS units

$\therefore \quad \frac{n_2}{n_1} = \sqrt{\frac{H_2}{H_1}} = \sqrt{\frac{R_2 \cos \delta_2}{R_1 \cos \delta_1}}$

$\Rightarrow \quad \frac{n_2}{10} = \sqrt{\frac{0.5 \cos 60°}{0.707 \cos 45°}} = \sqrt{\frac{0.5 \times 1/2}{0.5 \times \sqrt{2} \times 1/\sqrt{2}}} = \frac{1}{\sqrt{2}}$

or, $\quad n_2 = \frac{10}{\sqrt{2}} = 7.07 \approx 7$

29. When two identical bar magnets are held perpendicular to each other.

$\therefore \quad M_1 = \sqrt{M^2 + M^2} = M\sqrt{2}, I_1 = I$

and $\quad T_1 = 2^{5/4}$ s, $T_2 = ?$

and $M_2 = M$ (as one magnet is removed)

$I_2 = I_1 / 2$

$T_2 = T_1 \times \frac{1}{2^{1/4}} = 2^{5/4} \times \frac{1}{2^{1/4}} = 2$ s

30. As $t_1 = 3 = 2\pi\sqrt{\frac{I}{MR}}$, where R resultant intensity of earth's field.

$\therefore \quad t_2 = 3\sqrt{2} = 2\pi\sqrt{\frac{I}{MH}}$

Dividing $\frac{t_1}{t_2}$, we get

$\frac{1}{\sqrt{2}} = \sqrt{\frac{H}{R}} = \sqrt{\frac{R\cos\delta}{R}} = \sqrt{\cos\delta}$

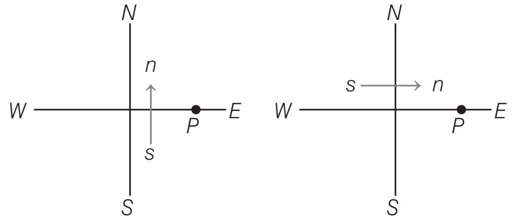
$\Rightarrow \quad \cos\delta = \frac{1}{2}$

$\Rightarrow \quad \delta = 60°$

31. Points of zero magnetic field, *i.e.* neutral points lie on equatorial line of magnet, *i.e.* along east and west.

32. In Fig. (a), at neutral point P,

$$B_H = \frac{\mu_0}{4\pi}\left(\frac{M}{d^3}\right)$$



(a) (b)

In Fig. (b),

Net magnetic induction at P = resultant of

$\frac{\mu_0}{4\pi}\frac{2M}{d^3} = 2B_H$ along horizontal and B_H along vertical

$\Rightarrow$ Net magnetic induction at P

$= \sqrt{(2B_H)^2 + (B_H)^2}$

$= \sqrt{5}\, B_H$

33. Here, $\quad 2l = 20$ cm

$\Rightarrow \quad l = 10$ cm, $d = 40$ cm

As, neutral point, $H = B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2 - l^2)^2}$

$\Rightarrow \quad 3.2 \times 10^{-5} = \frac{10^{-7} \times 2M\,(0.4)}{15 \times 15 \times 10^{-4}}$

$\therefore \quad M = \frac{3.2 \times 15 \times 15 \times 10^{-4} \times 10^{-5}}{0.8 \times 10^{-7}} = 9$

$\therefore \quad m = \frac{M}{2l} = \frac{9}{0.2} = 45$ A-m

34. Here, $B = 1.7 \times 10^{-5}$ T, $H = ?$

Now, $H = \frac{B}{\mu_0} = \frac{1.7 \times 10^{-5}}{4\pi \times 10^{-7}} = 13.5$ Am^{-1}

35. For each half, pole strength m becomes half, *i.e.* $M = m \times 2l$ and volume $V = a \times 2l$ also becomes half. Therefore, $I = M/V$, remains constant.

36. Here, $\quad V = (10 \times 0.5 \times 0.2)$ cm^3 = 1 cm^3 = 10^{-6} m^3,

$H = 0.5 \times 10^4$ Am^{-1}, $M = 5$ A-m^2, $B = ?$

$\therefore \quad I = \frac{M}{V} = \frac{5}{10^{-6}} = 5 \times 10^6$ Am^{-1}

From $\quad B = \mu_0(I + H)$

$B = 4\pi \times 10^{-7}(5 \times 10^6 + 0.5 + 10^4) = 6.28$ T

37. When space inside the toroid is filled with air,

$$B_0 = \mu_0 H$$

When filled with tungsten,

$B = \mu H = \mu_0\mu_r H$

$= \mu_0(1 + \chi_m)\, H$

Percentage increase in magnetic field/induction

$= \frac{(B - B_0) \times 100}{B_0}$

$= \frac{\mu_0\chi_m H \times 100}{\mu_0 H} = \chi_m \times 100$

$= 6.8 \times 10^{-5} \times 100$

$= 0.0068\%$

38. Here $I_1 = 8$ Am^{-1}, $B_1 = 0.6$ T, $t_1 = 4$ K,

$I_2 = ?$, $B_2 = 0.2$ T, $t_2 = 16$ K

According to Curie law,

$$I \propto \frac{B\,(\text{magnetic field induction})}{t\,(\text{in kelvin})}$$

$\therefore \quad \frac{I_2}{I_1} = \frac{B_2}{B_1} \times \frac{t_1}{t_2} = \frac{0.2}{0.6} \times \frac{4}{16} = \frac{1}{12}$

or $\quad I_2 = I_1 \times \frac{1}{12} = \frac{8}{12} = \frac{2}{3}\ \text{Am}^{-1}$

39. For a diamagnetic substance, I is negative and $-I \propto H$. Therefore, the variation is represented by OC or OD. As magnetisation is small, so OC is the better choice.

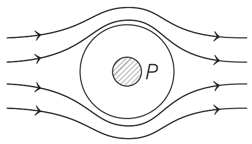
40. For diamagnetic substances, the magnetic susceptibility is negative and it is independent of temperature.

41. Copper is a diamagnetic material, therefore its rod align itself, where magnetic field is weakest and perpendicular to the direction of magnetic field at that position.

42. Because of large permeability of soft iron, magnetic lines of force prefer to pass through it. Concentration of lines in soft iron bar increases as shown in Fig. (b).

43. As temperature of a ferromagnetic material is raised, its susceptibility χ remains constant first and then decreases as shown in Fig. (a).

44. As, the sphere is perfectly diamagnetic, it completely expels field as shown below



Therefore, there will be no magnetic field inside the cavity of sphere.

So, the paramagnetic sample at the spherical cavity at the centre of sphere remains uneffected by external field and field inside paramagnetic sample is zero.

45. As, $\mu_r < 1$ for substance X, it must be diamagnetic and $\mu_r > 1$ for substance Y, so it is must be paramagnetic.

46. Coercivity of a bar magnet is the value of magnetic field intensity (H) that is needed to reduce magnetisation to zero.

Since, for a solenoid, magnetic induction is given as,

$$B = \mu_0 nI \quad \text{...(i)}$$

where, n is the number of turns (N) per unit length (l) and I is the current.

Also, $\quad B = \mu_0 H \quad \text{...(ii)}$

$\therefore$ From Eqs. (i) and (ii), we get

$$\mu_0 nI = \mu_0 H$$

or $\quad H = nI = \frac{N}{l} I$

Substituting the given values, we get

$$H = \frac{100}{0.2} \times 5.2 = 2600\ \text{A/m}$$

Thus, the value of coercivity of the bar magnet is 2600 A/m.

47. Area of hysteresis loop is proportional to the net energy absorbed per unit volume by the material, as it is taken over a complete cycle of magnetisation.

For electromagnets and transformers, energy loss should be low.

i.e. Thin hysteresis curves.

Also, $|B| \to 0$ when $H = 0$ and $|H|$ should be small when $B \to 0$.

Hence, option (d) is correct.

Round II

1. As, $T \propto \frac{1}{\sqrt{M}}$; when M becomes 4 times, then T becomes its half. Therefore, new $T = 1$ s.

2. Field due to circular loop carrying current is perpendicular to the plane of the loop. As, current is anti-clockwise. So, N pole lies above the loop and south pole lies below the loop.

3. On equatorial line, magnetic field due to magnet varies inversely as cube of the distance, therefore

$$\frac{B_1}{B_2} = \left(\frac{3x}{x}\right)^3 = 27 : 1$$

4. Statement III is correct, because the magnetic field outside the toroid is zero and they form closed loops inside the toroid itself.

Statement V is correct because we know that superconductors are materials inside which the net magnetic field is always zero and they are perfect diamagnetic.

$$\mu_r = 1 + \chi$$
$$\chi = -1$$
$$\mu_r = 0$$

5. Time period of oscillation is

$$T = 2\pi \sqrt{\frac{I}{MB}}$$

$\Rightarrow \quad T = 2\pi \sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = 0.665\ \text{s}$

Hence, time for 10 oscillations is $t = 6.65$ s.

6. For solenoid, the magnetic field needed to magnetise, the magnet is given by

$$B = \mu_0 nI$$

where, n is number of turns per unit length $= N/l$

and $\quad N = 100, l = 10\ \text{cm} = \frac{10}{100}\ \text{m} = 0.1\ \text{m}$

$\Rightarrow \quad 3 \times 10^3 = \frac{100}{0.1} \times I \Rightarrow I = 3\ \text{A}$

7. For a paramagnetic material, $K \propto \frac{1}{T}$

$\therefore \quad \frac{K_2}{K_1} = \frac{T_1}{T_2}$ or $\frac{K/2}{K} = \frac{27 + 273}{T_2}$

$\Rightarrow \quad T_2 = 600\ \text{K or } 327°\text{C}$

8. When axes are in the same line,

$$F = \frac{\mu_0}{4\pi}\frac{6M_1M_2}{r^4}$$

i.e. $F \propto \frac{1}{r^4}$

When r becomes thrice, F becomes $\frac{1}{(3)^4}$ times, *i.e.* $\frac{1}{81}$ times. Therefore, $F' = \frac{8.1}{81} = 0.1$ N.

9. As, $M = niA = 50 \times 2 \times 1.25 \times 10^{-3} = 0.125$ A-m^2

If normal to the face of the coil makes an angle θ with the magnetic induction B, then in first case,

$$\text{torque} = MB\cos\theta = 0.04$$

and in second case,

$$\text{torque} = MB\sin\theta = 0.03$$

$\therefore$ $MB = \sqrt{(0.04)^2 + (0.03)^2} = 0.05$

$\Rightarrow$ $B = \frac{0.05}{M} = \frac{0.05}{0.125} = 0.4$ T

10. As, the magnet is long, we assume that the upper north pole produces no effect. But due to south pole strength of the magnet which is equal and opposite to horizontal component of earth's magnetic field, neutral point is obtained. *i.e.*

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^2} = H$$

In CGS system, $\frac{\mu_0}{4\pi} = 1$

$\therefore$ $1 \times \frac{m}{10^2} = 3.2 \times 10^{-5} \times 10^4$ (gauss)

$$m = 32 \text{ ab-A-cm}^{-1}$$

11. As, $T_1 = 2\pi\sqrt{\frac{I}{MV}}$

and $T_2 = 2\pi\sqrt{\frac{I}{MH}}$

$\therefore$ $\frac{T_2}{T_1} = \sqrt{\frac{V}{H}} = \sqrt{\tan\theta}$

or $\tan\theta = \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$

$\Rightarrow$ $\theta = 45°$

12. In equilibrium, the resultant magnetic moment will be along magnetic meridian. Let N_1S_1 makes an angle θ with this resultant,

$\therefore$ $\tan\theta = \frac{M_2}{M_1} = \frac{M}{\sqrt{3}M} = \frac{1}{\sqrt{3}}$

$\therefore$ $\theta = 30°$

13. The lines of magnetic field induction **B** are necessarily continuous across the surface S of a lump of magnetic material. Outside the lump of magnetic material, $H = B/\mu_0$ and inside the lump of magnetic material, $H = B/\mu_0\mu_r$, where μ_r is the relative permeability of material. Thus, the lines of H cannot all be continuous across surface S.

14. Magnetic moment of an iron core solenoid,

$$M = \mu_r NiA$$

where, μ_r = relative permeability = 1000,

N = number of turns = 10 turns/cm,

i = current = 0.5 A

and A = area of cross-section.

Now, $M = \mu_r\, Ni\frac{V}{l}$ $\left(\because A = \frac{V}{l}\right)$

Substituting all the given values in above equation, we get

$$M = 1000 \times \frac{10}{10^{-2}} \times 0.5 \times 10^{-3} \quad (\because V = 10^{-3}\text{m}^3)$$

$$= 500 \text{ A-m}^2 = 5 \times 10^2 \text{ A-m}^2$$

15. Initially, $m = I\pi R^2$ and $B_1 = \frac{\mu_0 I}{2R_1}$

Finally, $m' = 2m = I\pi R_2^2$

$\Rightarrow$ $2I\pi R_1^2 = I\pi R_2^2$ or $R_2 = \sqrt{2}R_1$

So, $B_2 = \frac{\mu_0 I}{2(R_2)} = \frac{\mu_0 I}{2\sqrt{2}R_1}$

Hence, $\frac{B_1}{B_2} = \frac{\left(\frac{\mu_0 I}{2R_1}\right)}{\left(\frac{\mu_0 I}{2\sqrt{2}R_1}\right)} = \sqrt{2}$

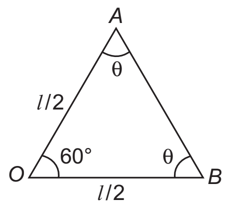
16. Since, **B** is uniform only torque acts on a current carrying loop.

As, $\tau = \mathbf{M} \times \mathbf{B}$

$\Rightarrow$ $|\tau| = |M||B|\sin\theta$

For orientation shown in (II) $\theta = 0^0$, $\tau = 0$ (stable equilibrium) and for (IV) $\theta = \pi, \tau = 0$ (unstable equilibrium)

17. When the wire is bent at its middle point O at 60°, then



$$60° + \theta + \theta = 180°$$

or $2\theta = 180° - 60° = 120°$

$\Rightarrow$ $\theta = 60°$

$\because \Delta OAB$ is an equilateral triangle.

$\therefore$ $AB = 2l' = l/2$

New magnetic moment,

$$M' = m(2l') = \frac{ml}{2} = \frac{M}{2}$$

18. If V = volume of the material = mass/density,
A = area of hysteresis loop,
ν = frequency of alternate magnetic field applied
and t = time for which field is applied,
then energy loss in the material in t second is

$$E = VA\nu t = \left(\frac{m}{d}\right) A\nu t$$

So, energy loss in 1 s $= \left(\frac{m}{d}\right) A\nu$

$$= \frac{0.6}{7.8\times10^3}\times0.722\times50$$

$$= 2.777\times10^3 = 27.77\times10^{-4} \text{ J}$$

19. In a vibration magnetometer,

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

$$\therefore \quad 4\pi^2\frac{1}{T^2} = MH = 36\times10^{-4} \quad \ldots\text{(i)}$$

In a deflection magnetometer,

$$H = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

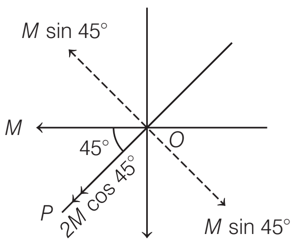
$$\frac{4\pi d^3}{2\mu_0} = \frac{M}{H} = \frac{10^8}{36} \quad \ldots\text{(ii)}$$

Multiplying Eq. (i) and Eq. (ii), we get

$$M^2 = 36\times10^{-4}\times\frac{10^8}{36} = 10^4$$

or $\quad M = 10^2 = 100$ A-m

20. Resolving the magnetic moments along OP and perpendicular to OP. From figure, we find that component of OP perpendicular to it cancel out. Resultant magnetic moment along OP is
$= M\cos45° + M\cos45°$.



$$= 2M\cos45° = \frac{2M}{\sqrt{2}} = \sqrt{2}\,M$$

The point P lies on axial line of magnetic of moment
$= \sqrt{2}\,M$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{2(\sqrt{2}\,M)}{d^3}$$

21. In SI, the unit of pole strength is A-m. Here, the pole strength is given in weber, which is the unit of $(\mu_0 m)$ and

$$\mu_0 m = 10^{-3} \text{ Wb}$$

or $\quad m = \dfrac{10^{-3}}{\mu_0}$

Magnetic moment of magnet,

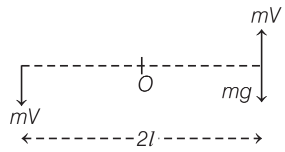
$$M = m\times2l = \frac{10^{-3}}{\mu}(0.1) = \frac{10^{-4}}{\mu_0}$$

$\therefore$ Torque, $\quad \tau = MB\sin\theta$

$$= \left(\frac{10^{-4}}{\mu_0}\right)(4\pi\times10^{-3})\sin30°$$

$$\Rightarrow \quad = \frac{10^{-4}}{4\pi\times10^{-7}}(4\pi\times10^{-3})\times\frac{1}{2} = 0.5 \text{ N-m}$$

22. As shown in figure,



Mass, $M = 50$, $mg = 50\times10^{-3}\ g$
Strength of each pole, $m = 98.1$ ab-amp-cm,
$g = 981$ cms^{-2}, $V = ?$

At equilibrium, $mV\times2l = Mg\times l$

or $\quad V = \dfrac{Mg}{2m}$

or $\quad V = \dfrac{50\times10^{-3}\times981}{2\times98.1} = 0.25$ G

23. As, $\quad T = 2\pi\sqrt{\dfrac{I}{MH}}$

$$\therefore \quad 2 = 2\pi\sqrt{\frac{I}{MH}} \quad \ldots\text{(i)}$$

When an external magnet is brought near and parallel to H and the time period reduces to 1 s, net field must be $(F + H)$.

$$\therefore \quad 1 = 2\pi\sqrt{\frac{1}{M(F+H)}} \quad \ldots\text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{2}{1} = \sqrt{\frac{F+H}{H}} = \sqrt{\frac{F}{H}+1}$$

$$\Rightarrow \quad \frac{F}{H}+1 = 4$$

$$\Rightarrow \quad \frac{F}{H} = 4-1 = 3$$

24. In the first case, $T_1 = \dfrac{60}{10} = 6$ s

$$\Rightarrow \quad T_1 = 2\pi\sqrt{\frac{I}{MH}} = 6 \quad \ldots\text{(i)}$$

In the second case, $T_2 = \dfrac{60}{14} = \dfrac{30}{7}$ s

If B magnetic induction due to external magnet, then

$$T_2 = 2\pi\sqrt{\frac{I}{M(H+B)}} = \frac{30}{7} \quad \ldots\text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{6}{30/7} = \sqrt{\frac{H+B}{H}} = \sqrt{1+\frac{B}{H}}$$

or $$\left(\frac{7}{5}\right)^2 = 1+\frac{B}{H}$$

or $$\frac{B}{H} = \frac{49}{25} - 1 = \frac{24}{25}$$

If n is number of vibrations/min in the third case when polarity of external magnet is reversed, then

$$T_3 = \frac{60}{n} = 2\pi\sqrt{\frac{I}{M(H-B)}} \quad \ldots\text{(iii)}$$

Dividing Eq. (i) by Eq. (iii), we get

$$\frac{6}{60/n} = \sqrt{\frac{H-B}{H}} = \sqrt{1-\frac{B}{H}} = \sqrt{1-\frac{24}{25}}$$

$\Rightarrow$ $$\frac{n}{10} = \frac{1}{5} \text{ or } n = 2 \text{ vib per min}$$

25. Here, magnetic moment is given in weber-metre, which is the unit of $\mu_0 M$.

$\therefore$ $$\mu_0 M = 5\times 10^{-5} \text{ Wb-m}$$

or $$M = \frac{5\times 10^{-5}}{\mu_0} \text{ A-m}^2$$

Also, $$B = 8\pi\times 10^{-4} = \mu_0 H$$

$\therefore$ $$H = \frac{8\pi\times 10^{-4}}{\mu_0}$$

Now, $$T = 2\pi\sqrt{\frac{I}{MH}}$$

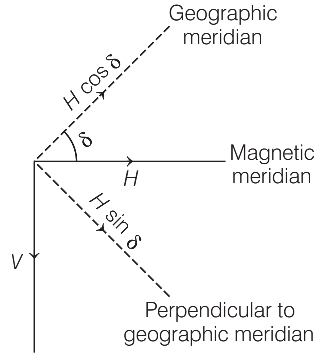
$\therefore$ $$I = \frac{MHT^2}{4\pi^2} = \frac{5\times 10^{-5}\times 8\pi\times 10^{-4}\times 15^2}{4\pi^2\mu_0^2}$$

$$= \frac{5\times 10^{-5}\times 8\pi\times 10^{-4}\times 225}{4\pi^2(4\pi\times 10^{-7})^2}$$

or $$I = \frac{2250\times 10^{-9}}{\pi(16\pi^2)\times 10^{-14}}$$

$$= 4.54\times 10^5 \text{ kg-m}^2$$

26. Let θ be the declination at the place. As it is clear from figure.



$$\tan\delta_1 = \frac{V}{H\cos\theta}$$

and $$\tan\delta_2 = \frac{V}{H\cos(90^\circ-\theta)} = \frac{V}{H\sin\theta}$$

$\because$ $$\frac{\tan\delta_1}{\tan\delta_2} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$\Rightarrow$ $$\theta = \tan^{-1}\left(\frac{\tan\delta_1}{\tan\delta_2}\right)$$

27. In the usual setting of deflecting magnetometer, field due to magnet (F) and horizontal component (H) of earth's field are perpendicular to each other. Therefore, the net field on the magnetic needle is $\sqrt{F^2+H^2}$.

$\therefore$ $$T = 2\pi\sqrt{\frac{I}{M\sqrt{F^2+H^2}}} \quad \ldots\text{(i)}$$

When magnet is removed,

$$T_0 = 2\pi\sqrt{\frac{I}{MH}} \quad \ldots\text{(ii)}$$

Also, $$\frac{F}{H} = \tan\theta$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{T}{T_0} = \sqrt{\frac{H}{\sqrt{F^2+H^2}}}$$

$$= \sqrt{\frac{H}{\sqrt{H^2\tan^2\theta + H^2}}}$$

$$= \sqrt{\frac{H}{H\sqrt{\sec^2\theta}}}$$

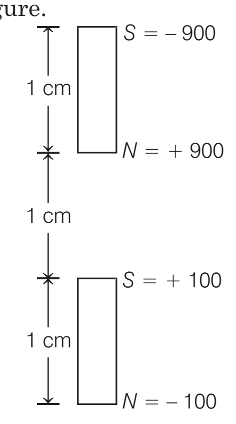
$$= \sqrt{\cos\theta}$$

or $$\frac{T_2^2}{T_0^2} = \cos\theta$$

$\Rightarrow$ $$T^2 = T_0^2\cos\theta$$

28. In CGS system, $\frac{\mu_0}{4\pi} = 1$

In equilibrium, net repulsion due to magnetic interaction = weight of upper magnet. Therefore, it is clear from figure.



$$\frac{900\,(100)}{1^2} + \frac{900\,(-100)}{2^2} - \frac{900\,(100)}{2^2} - \frac{900\,(-100)}{3^2} = m \times g$$

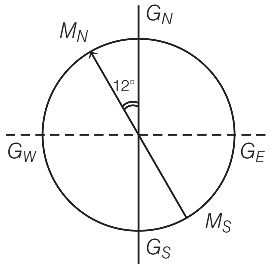
$$\Rightarrow \quad 900 \times 100 \left(\frac{1}{1^2} - \frac{1}{2^2} - \frac{1}{2^2} + \frac{1}{3^2} \right) = m \times 1000$$

$$\Rightarrow \quad 90 \left(\frac{11}{18} \right) = m$$

$$\Rightarrow \quad m = 55 \text{ g}$$

29. Given, angle of declination,

$$\theta = 12° \text{ west}$$



Angle of dip, $\delta = 60°$

Horizontal component of earth's magnetic field,

$$H = 0.16 \text{ G}$$

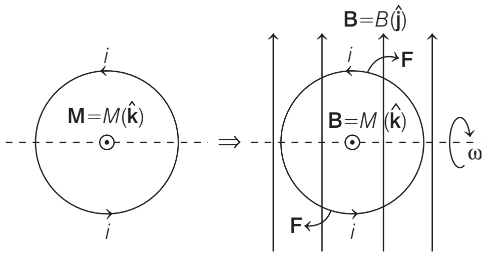
Let the magnitude of earth's magnetic field at that place be R.

Using the formula, $H = R \cos \delta$

or
$$R = \frac{H}{\cos \delta}$$
$$= \frac{0.16}{\cos 60°}$$
$$= \frac{0.16 \times 2}{1}$$
$$= 0.32 \text{ G}$$
$$= 0.32 \times 10^{-4} \text{ T}$$

The earth's magnetic field lies in a vertical plane 12° west of geographical meridian at an angle 60° above the horizontal.

30.



Before applying the magnetic field

After applying the magnetic field

Given that, $I_{dia} = 0.8 \text{ kg-m}^2$,

$$M = 20 \text{ A-m}^2,\ B = 4 \text{ T}$$

$\theta =$ angle between **M** and **B** after rotation of 60° by coil $= 90° - 60° = 30°$ and $\omega = ?$

Applying the law of conservation of energy,

$$(\text{TE})_{\text{initial}} = (\text{TE})_{\text{final}}$$
$$U_i + K_i = U_f + K_f$$
$$0 + 0 = -MB\cos\theta + \frac{1}{2} I_{dia}\omega^2$$
$$0 = -20 \times 4 \cos 30° + \frac{1}{2} \times 0.8 \times \omega^2$$
$$0 = -20 \times 4 \times \frac{\sqrt{3}}{2} + 0.4\,\omega^2$$
$$0 = -40\sqrt{3} + 0.4\,\omega^2$$
$$\Rightarrow \quad 40\sqrt{3} = 0.4\,\omega^2$$
$$\Rightarrow \quad \omega^2 = \frac{40\sqrt{3}}{0.4} = 100\sqrt{3} = 100 \times 1.732 = 173.2$$
$$\omega = \sqrt{173.2} = 13.16 \text{ rad/s}$$

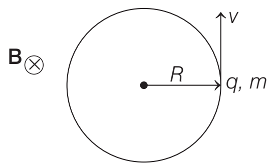
31. From Curie's law for paramagnetic materials,

$$\chi_m \propto \frac{1}{T}$$
$$\Rightarrow \quad \frac{I}{H} \propto \frac{1}{T}$$
$$\Rightarrow \quad \frac{I}{(B_0/\mu_0)} \propto \frac{1}{T} \Rightarrow \frac{\mu_0 I}{B_0} \propto \frac{1}{T}$$
$$\Rightarrow \quad I \propto \frac{B_0}{\mu_0 T} \Rightarrow I \propto \frac{B_0}{T} \qquad (\because \mu_0 = \text{constant})$$
$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{(B_0)_1}{(B_0)_2} \times \frac{T_2}{T_1}$$

On putting the given values in above equation, we get

$$\frac{6}{I_2} = \frac{0.4}{0.3} \times \frac{24}{4}$$
$$\Rightarrow \quad \frac{6}{I_2} = \frac{4}{3} \times \frac{6}{1}$$
$$\Rightarrow \quad \frac{6}{I_2} = 8 \Rightarrow I_2 = \frac{6}{8}$$
$$\Rightarrow \quad I_2 = \frac{3}{4} \text{ A/m}$$
$$\Rightarrow \quad I_2 = 0.75 \text{ A/m}$$

32. Let us consider the direction of magnetic field be perpendicular to the plane in inward direction as shown below



Current produced by the motion of charge particle,

$$i = \frac{q}{T} = \frac{qv}{2\pi R}$$

If motion of a charged particle is considered as a current loop, then magnetic moment of charged particle,

$$M = iA = \frac{qv}{2\pi R} \times \pi R^2$$

$$\Rightarrow \quad M = \frac{1}{2} qvR \quad \ldots\text{(i)}$$

As we know that, while moving in a circle, necessary centripetal force is provided by magnetic force.

i.e. $\frac{mv^2}{R} = qvB$ or $R = \frac{mv}{qB}$

Putting this value in Eq. (i), we get

$$M = \frac{1}{2} qv\left(\frac{mv}{qB}\right)$$

or $$M = \frac{mv^2}{2B}$$

Here, the direction of **M** is determined by right hand thumb rule which is perpendicular to plane but in outward direction, *i.e.* opposite to that of **B**.

$$\therefore \quad \mathbf{M} = \frac{mv^2}{2B}(-\hat{\mathbf{B}})$$

or $$\mathbf{M} = -\frac{mv^2\mathbf{B}}{2B^2}$$

33. Horizontal component of earth's magnetic field,

$$B_H = B_E \cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{B_H}{B_E} = \frac{3 \times 10^{-6}}{6 \times 10^{-5}} = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

34. Any point on magnetic equator is on equatorial line of magnet and at poles, it is on axial line.

$$\therefore \quad \frac{B_{\text{equatorial}}}{B_{\text{axial}}} = \frac{1/2}{1} = \frac{1}{2}$$

$$\therefore \quad x = 2$$

35. Magnetic susceptibility,

$$\chi_{m_1} = 0.0075, T_1 = -73^\circ\text{ C}$$
$$= (-73 + 273)\text{ K} = 200\text{ K}$$
$$\chi_{m_2} = ?, T_2 = -173^\circ\text{ C} = (-173 + 273)\text{ K} = 100\text{ K}$$

As, $\chi_m \propto \frac{1}{T}$

$$\Rightarrow \quad \frac{\chi_{m_2}}{\chi_{m_1}} = \frac{T_1}{T_2} = \frac{200}{100} = 2$$

$$\therefore \quad \chi_{m_2} = 2\chi_{m_1} = 2 \times 0.0075 = 0.015$$

36. If d_1 is distance of point X on axial line and d_2 is distance of point Y on equatorial line,

then $$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d_1^3}, B_2 = \frac{\mu_0}{4\pi}\frac{M}{d_2^3}$$

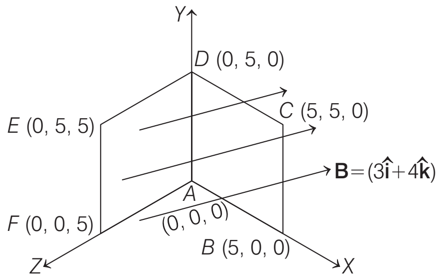
As, $$B_1 = B_2$$

$$\therefore \quad \frac{\mu_0}{4\pi}\frac{2M}{d_1^3} = \frac{\mu_0}{4\pi}\frac{M}{d_2^3}$$

$$d_1^3 = d_2^3$$

$$\Rightarrow \quad \frac{d_1}{d_2} = 2^{1/3}$$

37. If given situation, loop and magnetic field are given as shown in the figure



Flux through loop $ABCDEFA$ = Flux through part $ABCDA$ + Flux through part $ADEFA$

$$= \mathbf{B}\cdot\mathbf{A}_1 + \mathbf{B}\cdot\mathbf{A}_2$$

Here, from figure, $\mathbf{A}_1 = 25\hat{\mathbf{k}}$ and $\mathbf{A}_2 = 25\hat{\mathbf{i}}$

So, flux associated with complete loop

$$= (3\hat{\mathbf{i}} + 4\hat{\mathbf{k}})\cdot 25\hat{\mathbf{k}} + (3\hat{\mathbf{i}} + 4\hat{\mathbf{k}})\cdot 25\hat{\mathbf{i}}$$
$$= 100 + 75$$
$$= 175\text{ Wb}$$