Kinematics

Study of motion of objects without taking into account the factors which cause the motion (i.e. nature of force.)

1. Frame of Reference

Motion of a body can be observed only if it changes its position with respect to some other body. Therefore, for motion to be observed there must be a body, which is changing its position with respect to another body and a person who is observing motion. The person observing motion is known as observer. The observer for the purpose of investigation must have its own clock to measure time and a point in the space attached with the other body as origin and a set of co-ordinate axes. These two things (the time measured by the clock and the co-ordinate system) are collectively known as reference frame.

1.1 Concept of Point Object

An object having its dimensions very small (negligible) as compared to the distances in consideration, will be considered as a point object.

1.2 Rest & Motion

If a body changes its position with time, it is said to be moving else it is at rest. Motion is always relative to the observer.

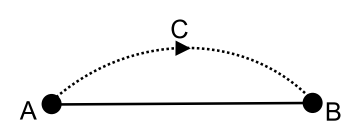
Motion is a combined property under study of object and the observer. There is no meaning of rest or motion without the viewer.

- To locate the position of a particle we need a reference frame. A commonly used reference frame is Cartesian coordinate system or x-y-z coordinate system.
- The coordinates (x, y, z) of the particle specify the position of the particle with respect to origin of that frame. If all the three coordinates of the particle remain unchanged as time passes it means the particle is at rest w.r.t. this frame.
- The reference frame is chosen according to problems.
- If frame is not mention, then ground is taken as reference frame.

2. Distance and Displacement

2.1 Distance

Total length of path covered by the particle, in definite time interval. Let a body moves from A to B via C. The length of path ACB is called the distance travelled by the body.



2.2 Displacement

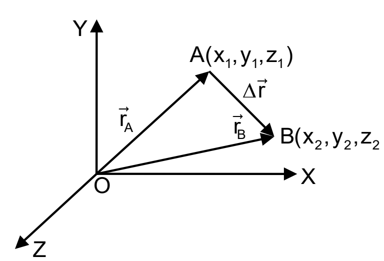
But overall, body is displaced from A to B. A vector from A to B, i.e. $\overrightarrow{AB}$ is its displacement vector or displacement that is the minimum distance and directed from initial position to final position.

2.3 Displacement in Terms of Position Vector

Let a body is displaced from

A(x_1, y_1, z_1) to B(x_2, y_2, z_2)

then its displacement is given by vector $\overrightarrow{AB}$.



$\because \quad \vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

and $\quad \vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\therefore \quad \Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

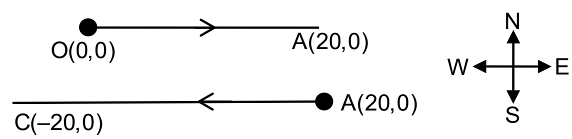
Key Points

- Distance is a scalar while displacement is a vector.
- Distance depends on path while displacement is independent of path but depends only on final and initial positions.
- For a moving body, distance cannot have zero or negative values but displacement may be positive, negative or zero.
- Infinite distances are possible between two fixed points because infinite paths are possible between two fixed points.
- Only single value of displacement is possible between two fixed points.
- If motion is in straight line without change in direction then
  Distance = |displacement| = magnitude of displacement.
- Magnitude of displacement may be equal or less than distance but never greater than distance.
  i.e., distance ≥ |displacement|

Example 1:

A particle starts from the origin, goes along the X-axis to the point (20 m, 0) and then returns along the same line to the point (–20 m, 0). Find the distance and displacement of the particle during the trip.

Solution:



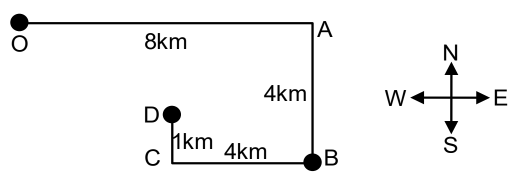
Distance = $\left|\overrightarrow{OA}\right| + \left|\overrightarrow{AC}\right|$ = 20 + 40 = 60 m

$\Delta\vec{r}$ = Displacement = $\overrightarrow{OA} + \overrightarrow{AC}$

= $20\hat{i} + (-40\hat{i}) = (-20\hat{i})$ m

Example 2:

A car moves from O to D along the path OABCD shown in fig. What is distance travelled and net displacement.



Solution:

Distance = $|\overrightarrow{OA}| + |\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{CD}|$

= 8 + 4 + 4 + 1 = 17 km

$\Delta\vec{r}$ = Displacement = $\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$

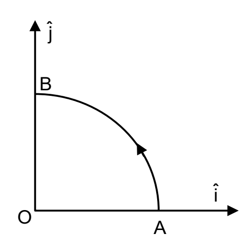
= $8\hat{i} + (-4\hat{j}) + (-4\hat{i}) + \hat{j} = 4\hat{i} - 3\hat{j}$

$\rightarrow$ $|\Delta\vec{r}|$ = |displacement| = $\sqrt{(4)^2 + (3)^2}$ = 5

So, Displacement = 5 km, 37^0 S of E

Example 3:

A particle goes along a quadrant from A to B of a circle radius 10 m as shown in fig. Find the magnitude of displacement and distance along path shown AB, and angle between displacement vector $\overrightarrow{AB}$ and x-axis?



Solution:

Displacement = $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 10\hat{j} - 10\hat{i}$

$\left|\overrightarrow{AB}\right| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ m

From ΔOBA $\quad \tan\theta = \dfrac{OA}{OB} = \dfrac{10}{10} = 1$

$\Rightarrow \theta = 45°$

Angle between displacement vector $\overrightarrow{OA}$ and x-axis = 90° + 45° = 135°

Distance of path AB = $\dfrac{1}{4}$(circumference)

= $\dfrac{1}{4}$ $(2\pi R)$ m = (5π) m

Example 4:

A body initially having position vector $r_A = 2\hat{i} + \hat{j} + 4\hat{k}$ moves to $r_B = 6\hat{i} + 9\hat{j} - 2\hat{k}$, then find the change in position vector of the body.

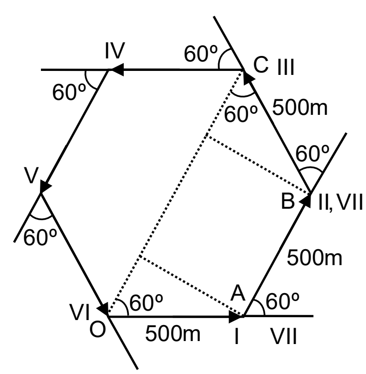
Solution:

Since $\Delta\vec{r} = \vec{r}_B - \vec{r}_A \quad = (6\hat{i} + 9\hat{j} - 2\hat{k}) - (2\hat{i} + \hat{j} + 4\hat{k}) = 4\hat{i} + 8\hat{j} - 6\hat{k}$

Example 5:

On an open ground a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of displacement with the total path length covered by the motorist in each case.

Solution:



At III turn

Displacement = $\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{OC}$

= 500 cos 60° + 500 + 500 cos 60°

= 1000 m from O to C

Distance = 500 + 500 + 500 = 1500 m

$\therefore \dfrac{\text{Displacement}}{\text{Distance}} = \dfrac{1000}{1500} = \dfrac{2}{3}$

At VI turn

$\because$ initial and final positions are same so

displacement = 0 and

distance = 500 × 6 = 3000 m

$\therefore \dfrac{\text{Displacement}}{\text{Distance}} = \dfrac{0}{3000} = 0$

At VIII turn

Displacement = 2(500) cos $\left(\dfrac{60°}{2}\right)$

= 1000 × cos 30° $\quad = 1000 \times \dfrac{\sqrt{3}}{2} = 500\sqrt{3}$m

Distance = 500 × 8 = 4000 m

$\therefore \dfrac{\text{Displacement}}{\text{Distance}} = \dfrac{500\sqrt{3}}{4000} = \dfrac{\sqrt{3}}{8}$

Example 6:

A particle moves on a circular path of radius ‘r’, It completes one revolution in 40 s. Calculate distance and displacement in 2 min 20 s.

Solution:

radius of path = r

Time period for one revolution = 40 sec

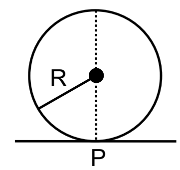
total number of revolution = $\frac{140}{40} = \frac{7}{2}$

Now distance covered = $2\pi r \times \frac{7}{2} = \frac{22}{7} \times r \times \frac{7}{2} = 22r$

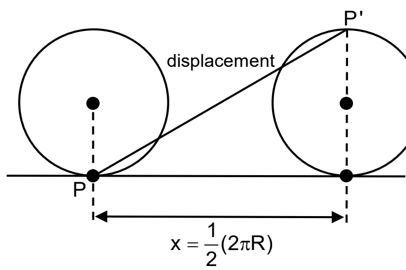
Displacement of particle in one revolution is zero so that displacement of particle = 2r.

Example 7:

A wheel of radius ‘R’ is placed on ground and its contact point is ‘P’. If wheel rolls down without slipping completing half a revolution, find the displacement of point P.



Solution:



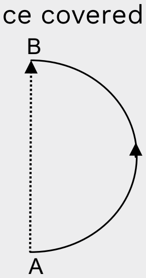
Displacement = $\sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4}$

Concept Builder-1

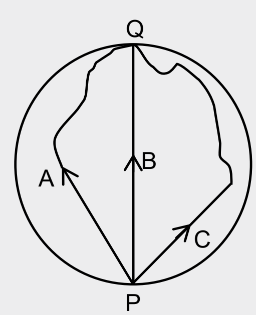
Q.1 A man moves 4 m along East direction, then 3 m along North direction, after that he climbs up a pole to a height 12 m. Find the distance covered by him and his displacement.

Q.2 A man has to go 50 m due North, 40 m due East and 20 m due South to reach a cafe from his home. (A) What distance he has to walk to reach the cafe? (B) What is his displacement from his home to the cafe?

Q.3 A person moves on a semicircular track of radius 40 m. If he starts at one end of the track and reaches the other end, find the distance covered and magnitude of displacement of the person.



Q.4 Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in figure. What is the magnitude of the displacement for each girl? For which girl is this equal to the actual length of path?



3. Speed and Velocity

3.1 Speed

The rate at which distance is covered with respect to time is called speed. It is a scalar quantity

Dimension $[M^{\circ}L^{1}T^{-1}]$

Unit: metre/second (S.I.), cm/s(C.G.S.)

Note For a moving particle speed can never be negative or zero it is always positive.

3.2 Uniform speed

When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed.

Uniform speed = $\dfrac{\text{Distance}}{\text{Time}}$

3.3 Non-uniform (variable) speed

In non-uniform speed particle covers unequal distances in equal intervals of time.

3.4 Average speed

The average speed of a particle for a given "interval of time" is defined as the ratio of total distance travelled to the time taken.

Average speed = $\dfrac{\text{Total Distance travelled}}{\text{Time taken}}$

i.e. $v_{av} = \dfrac{\Delta s}{\Delta t}$

Key Points

For average speed

- **When a particle moves with different uniform speeds** v_1, v_2, v_3 **........**v_n, in same time intervals t_1, t_2, t_3 **........**t_n, respectively, its average speed over the total time of journey is given as
- $v_{av} = \dfrac{v_1 + v_2 + v_3 + ...v_n}{n}$ (Arithmetic mean of speeds)
- When a particle describes same distances $s_1, s_2, s_3 ..... s_n$ with speeds $v_1, v_2, v_3 ....v_n$ respectively then the average speed of particle over the total distance will be given as
- $v_{av} = \dfrac{n}{\dfrac{1}{v_1} + \dfrac{1}{v_2} + \dfrac{1}{v_3} + ... + \dfrac{1}{v_n}}$ (Harmonic mean of speeds)

3.5 Instantaneous Speed

It is the speed of particle at a particular instant of time.

Instantaneous speed $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$

3.6 Velocity

The rate of change of position vector i.e. rate of displacement with time is called velocity.

It is a vector quantity

Dimension: $[M^{0}L^{1}T^{-1}]$

Unit: metre/second(S.I.), cm/s(C.G.S.)

3.7 Uniform Velocity

A particle is said to have uniform velocity, if magnitude as well as direction of its velocity remain same. This is possible only when it moves in a straight line without reversing its direction.

3.8 Non-uniform velocity

A particle is said to have non-uniform velocity, if both either magnitude or direction of velocity change.

3.9 Average Velocity

It is defined as the ratio of displacement to time taken by the body

Average velocity = $\dfrac{\text{Displacement}}{\text{Time taken}}$; $\vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$

3.10 Instantaneous velocity

It is the velocity of a particle at a particular instant of time.

Instantaneous velocity $\qquad \vec{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt}$

Key Points

- Velocity may be positive, negative or zero.
- Direction of velocity (Instantaneous) is always in the direction of change in position vector.
- Speedometer measures the instantaneous speed of a vehicle.
- Average speed ≥ I Average velocity I
- The direction of instantaneous velocity is always tangential to the path followed by the particle.
- When a particle is moving on any path, the magnitude of instantaneous velocity is equal to the instantaneous speed.
- A particle may have constant speed but variable velocity.
  Example: When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.
- When particle moves with uniform velocity then its instantaneous speed, magnitude of instantaneous velocity, average speed and magnitude of average velocity are all equal.
  $v = |\vec{v}| = v_{av} = |\vec{v}_{av}|$

Example 8:

If a body is moving with velocity $v = t^3 + 2t$. Find the average velocity between 1 and 2 second.

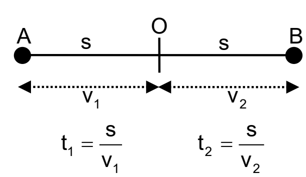
Solution:

As $< v_{av} >$ (average velocity) = $\dfrac{\int v\,dt}{\int dt} \qquad = \dfrac{\int v\,dt}{\int dt} = \dfrac{\int_1^2 (t^3 + 2t)dt}{\int_1^2 (dt)} = \dfrac{27}{4}$ m/s

Example 9:

If a particle travels the first half distance with speed v_1 and second half distance with speed v_2. Find its average speed during the journey.

Sol $\quad v_{avg.} = \dfrac{s+s}{t_1+t_2} = \dfrac{2s}{\dfrac{s}{v_1}+\dfrac{s}{v_2}} = \dfrac{2v_1v_2}{v_1+v_2}$

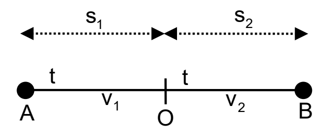


= Harmonic mean of v_1 and v_2 (For same distances covered with different speed)

Example 10:

If a particle travels with speed v_1 during first half time interval and with v_2 speed during second half time interval. Find its average speed during its journey.

Solution:



$v_{avg} = \frac{s_1 + s_2}{t+t} = \frac{(v_1+v_2)t}{2t} = \frac{v_1+v_2}{2}$ = Arithmetic mean of v_1, v_2 (For distances covered in equal time)

Example 11:

A particle of mass 2 kg moves on a circular path with constant speed 10 m/s. Find change in speed and magnitude of change in velocity. When particle completes half revolution.

Solution:

Change in speed = $v_2 - v_1 = 10 - 10 = 0$

change in velocity = $\vec{v}_2 - \vec{v}_1 = -10 - 10 = -20$ m/s

magnitude $\Delta v = 20$ m/sec

4. Acceleration

The rate of change of velocity of an object is called acceleration of the object.

It is vector quantity. It's direction is same as that of change in velocity (Not in the direction of the velocity)

Dimension: $[M^0L^1T^{-2}]$

Unit: metre/second^2(S.I.), cm/s^2(C.G.S.)

4.1 Uniform Acceleration

A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during motion of particle.

4.2 Non-Uniform Acceleration

A body is said to have non-uniform acceleration, if either magnitude or direction or both change during motion.

4.3 Average Acceleration

It is ratio of total change in velocity to the total time taken by the particle

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

4.4 Instantaneous Acceleration

It is the acceleration of a particle at a particular instant of time.

$$\vec{a} = \lim_{\Delta t \to o} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

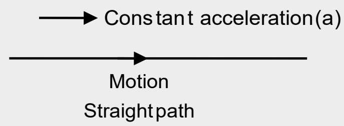
i.e. first derivative of velocity is called instantaneous acceleration.

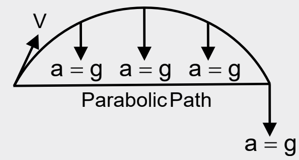
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \qquad \left[\text{As } \vec{v} = \frac{d\vec{r}}{dt}\right]$$

i.e. second derivative of position vector is called instantaneous acceleration.

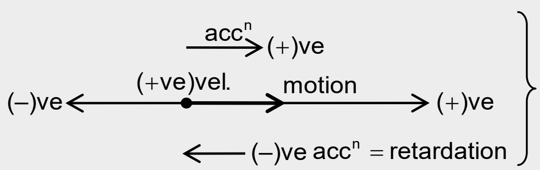
Key Points

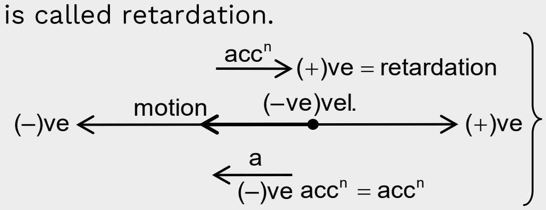
- When a particle moves with constant acceleration, then its path may be straight line or parabolic.





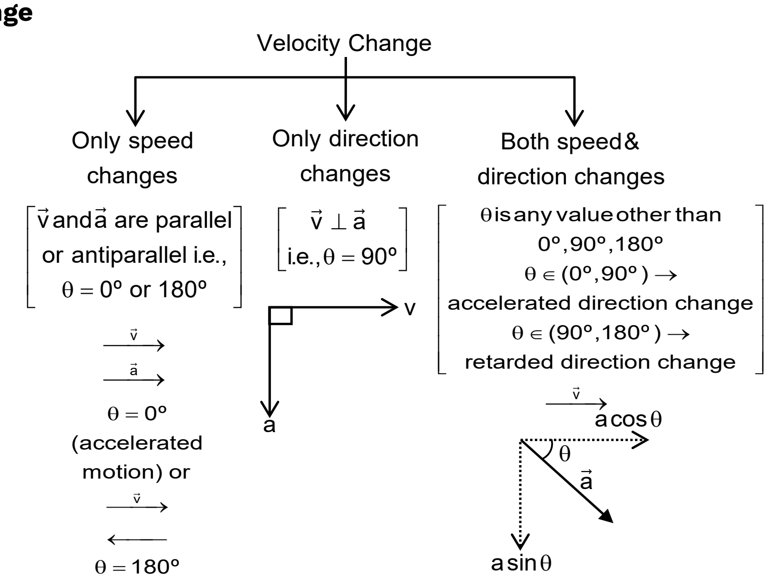
- When a particle starts from rest and moves with constant acceleration then its path must be a straight line.
- When a particle moves with variable velocity then acceleration must be present.
- When a particle moves continuously on a same straight line with uniform speed then acceleration of the particle is zero.
- When a particle moves continuously on a curved path with uniform speed then acceleration of the particle is non zero. For example uniform circular motion is an accelerated motion.
- For a particle moving with uniform velocity acceleration must be zero.
- When a particle moves with non-uniform speed then acceleration of the particle must be non zero.
- The direction of average acceleration vector is the direction of the change in velocity vector.
- Acceleration which opposes the motion of body is called retardation.





- Sign of velocity (+ve or -ve) represents the direction of motion but sign of acceleration indicates the direction of change in velocity.
- If velocity and acceleration both are having same sign, then magnitude of velocity (i.e. speed) is increasing and if both have opposite signs, then magnitude of velocity (i.e. speed) is decreasing.
- Momentarily rest is the condition when particle reverses it's direction and it's velocity is zero and acceleration is non-zero.

4.5 Velocity Change



Example 12:

The velocity of a particle is given by

$v = (2t^2 - 4t + 3)$ m/s where t is time in seconds. Find its acceleration at t = 2 second.

Solution:

Acceleration (a) = $\frac{dv}{dt} = \frac{d}{dt}(2t^2 - 4t + 3) = 4t - 4$

Therefore, acceleration at t = 2s is equal to a = (4 x 2) - 4 =4 m/s^2

Example 13:

(a) If $s = 2t^3 + 3t^2 + 2t + 8$ then find time at which acceleration is zero.

(b) Velocity of a particle (starting at t = 0) varies with time as v = 4t. Calculate the displacement of particle between t = 2 to t = 4 s

Solution: (a) $V = \frac{ds}{dt} = 6t^2 + 6t + 2$

$\Rightarrow a = \frac{dv}{dt} = 12t + 6 = 0$

$\Rightarrow t = -\frac{1}{2}$ which is impossible.

Therefore, acceleration can never be zero.

(b) $\therefore \frac{ds}{dt} = v \qquad \therefore s = \int v\,dt = \int_2^4 4t\,dt = \left[2t^2\right]_2^4$

$= 2(4)^2 - 2(2)^2 = 32 - 8 = 24$ m

Example 14:

A body is moving with velocity $2\hat{i} + 3t^2\hat{j}$, find its average acceleration after 2 seconds.

Solution:

As $<\vec{a}>$ (average acceleration)

$= \frac{\Delta v(\text{Change in velocity})}{\Delta t(\text{Total time taken})}$

$\vec{v}_{initial} = 2\hat{i} + 0\hat{j}$, $\vec{v}_{final} = 2\hat{i} + 12\hat{j}$

therefore, $\Delta\vec{v} = \vec{v}_{final} - \vec{v}_{initial} = 0\hat{i} + 12\hat{j}$

$\Rightarrow <\vec{a}> = 0\hat{i} + 6\hat{j}$

Example 15:

A particle is moving along a straight line OX. At a time, t (in seconds) the distance x (in meters) of particle from point O is given by $x = 10 + 6t^2 - 2t^3$. How long would the particle travel before its acceleration becomes zero.

Solution:

Initial value of x at t = 0 $x_1 = 10$m

Velocity $v = \frac{dx}{dt} = 12t - 6t^2$

a = 12 – 12t

when a = 0, t = 1

x at t = 1, $x_2 = 10 + 6 - 2 = 14$

Distance travelled = $x_2 - x_1 = 14 - 10 = 4$m

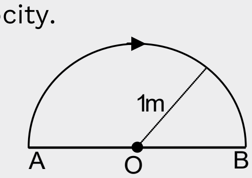
Concept Builder-2

Q.1 A particle moves on a straight line in such way that it covers 1st half distance with speed 3m/s and next half distance in 2 equal time intervals with speeds 4.5 m/s and 7.5 m/s respectively. Find average speed of the particle.

Q.2 Air distance between Kota to Jaipur is 260 km. and road distance is 320 km. A deluxe bus which moves from Jaipur to Kota takes 8 h while an aeroplane reaches in just 15 min. Find
(i) average speed of bus in km/h
(ii) average speed of aeroplane in km/h
(iii) average velocity of bus in km/h
(iv) average velocity of aeroplane in km/h

Q.3 The distance travelled by a particle in time t is given by $s = (2.5t^2)$ m. Find (a) the average speed of the particle during time 0 to 5.0 s and (b) the instantaneous speed at t = 5.0 s.

Q.4 A particle goes from point A to point B, moving in a semicircle of radius 1m in 1 second. Find the magnitude of its average velocity.



Q.5 The position of a particle moving on X-axis is given by $x = At^2 + Bt + C$. The material values of A, B and C are 7, -2 and 5 respectively and SI units are used. Find
(a) The acceleration of the particle at t = 5.
(b) The average acceleration during the interval t = 0 to t = 5.

Q.6 The acceleration of a particle moving in a straight line varies with its displacement as, a = 2s+1 velocity of the particle is zero at zero displacement. Find the corresponding velocity - displacement equation

5. Motion in Straight Line with Constant Acceleration

Equation of motion are valid when acceleration is constant.

- $v = u + at$
- $s = ut + \frac{1}{2}at^2$
- $v^2 - u^2 = 2as$
- $s = v_{av}\, t = \frac{(u+v)}{2} t$
- $s_n - s_{n-1} = s_{nth} = u + \frac{1}{2}a(2n-1)$
- All the equation of motion can be used in 2-D motion in vector form.

5.1 Concept of stopping distance and stopping time

- A body moving with a velocity u is stopped by application of brakes after covering a distance s. If the same body moves with velocity nu and same braking force is applied on it then it will come to rest after covering a distance of n^2s.

As $v^2 = u^2 - 2as$

$\Rightarrow\ 0 = u^2 - 2as$

$\Rightarrow\ s = \dfrac{u^2}{2a} \Rightarrow s \propto u^2$ [since a is constant]

So, we can say that if ‘u’ becomes n time then ‘s’ becomes n^2 times of previous value.

- Stopping time:

  $v = u - at$

  $\Rightarrow\ 0 = u - at$

  $\Rightarrow\ t = \dfrac{u}{a} \Rightarrow\ t \propto u$ [since a is constant]

  So we can say that if u becomes n times then t becomes n times that of previous value.

Example 16:

Two cars start off a race with velocities 2m/s and 4m/s travel in straight line with uniform accelerations 2 m/s^2 and 1 m/s^2 respectively. What is the length of the path if they reach the final point at the same time ?

Solution:

Since both particle reach at same position in same time t then from $s = ut + \dfrac{1}{2}at^2$

For 1st particle: $s = 4(t) + \dfrac{1}{2}(1)t^2 = 4t + \dfrac{t^2}{2}$,

For 2nd particle: $s = 2(t) + \dfrac{1}{2}(2)\,t^2 = 2t + t^2$

Equating above equation we get $4t + \dfrac{t^2}{2} = 2t + t^2$

$\Rightarrow\ t = 4$ s

Substituting value of t in above equation

$s = 4(4) + \dfrac{1}{2}(1)(4)^2 = 16 + 8 = 24$ m

Example 17:

A train, travelling at 20km/hr is approaching a platform. A bird is sitting on a pole on the platform. When the train is at a distance of 2 km from pole, brakes are applied which produce a uniform deceleration in it. At that instant the bird flies towards the train at 60 km/hr and after touching the nearest point on the train flies back to the pole and then flies towards the train and continues repeating itself. Calculate how much distance the bird covers before the train stops?

Solution:

For retardation of train $v^2 = u^2 + 2as$

$\Rightarrow 0 = (20)^2 + 2(a)(2) \Rightarrow a = -100\ km/hr^2$

Time required to stop the train $v = u + at$

$\Rightarrow 0 = 20 - 100t \Rightarrow\ t = \dfrac{1}{5}$ hr

For Bird, speed $= \dfrac{\text{Distance}}{\text{time}}$ $\quad \Rightarrow s_B = v_B \times t = 60 \times \dfrac{1}{5} = 12$ km.

5.2 Reaction Time

- It is the time gap between observation & execution
- During the reaction time the body travels with constant velocity.

Example 18:

A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the **reaction time** of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0 m/s^2. Find the distance travelled by the car after he sees the need to put the brakes on.

Solution:

Reaction time = 0.20 sec

$\Rightarrow$ velocity = 54 km/hr = 15 m/sec

Acceleration = 6 m/sec^2

Now distance travelled in 0.2 sec $= 15 \times 0.2 = 3$ m

distance travelled during deceleration $\Rightarrow 15^2 = 2 \times 6 \times x \Rightarrow x = 18.75$ m

Total distance = 21.75 m

Example 19:

A car start for rest and moving with constant acceleration 2 m/sec^2. It covers first 100-meter distance in time t_1 and second 100 meter in time t_2 then find ratio of time t_1: t_2.

Solution:

$a = 2$ m/sec^2

For 1st 100 m $\Rightarrow t_1 = \sqrt{\frac{2 \times 100}{2}} = 10$

$t_1 + t_2 = \sqrt{\frac{2 \times 100}{2}} = 10\sqrt{2}$

$t_2 = 10(\sqrt{2} - 1) \Rightarrow t_1 : t_2 = 1 : (\sqrt{2} - 1)$

Example 20:

A car start from rest and moving with constant acceleration 5 m/sec^2. The distance travelled in the first 5 sec is x_1 next 5 sec is x_2 and last 5 sec is x_3. Then $x_1 : x_2 : x_3$ is

Solution:

$a = 5$ m/sec^2

$x_1 = \frac{1}{2} \times 5 \times (5)^2 = \frac{125}{2}$

$x_2 = \frac{1}{2} \times 5 \times (10)^2 - x_1 = \frac{500}{2} - \frac{125}{2} = \frac{375}{2}$

$x_3 = \frac{1}{2} \times 5 \times (15)^2 - x_1 - x_2 = \frac{625}{2}$

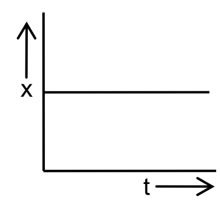
$x_1 : x_2 : x_3 = 1 : 3 : 5$

6. Graphical Section

6.1 Position Time Graph

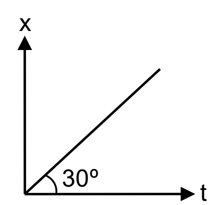
- Slope of x – t curve denotes instantaneous velocity at that point
- Area of x – t curve has no physical significance
- Slope of slope of x – t curve $\left(\frac{d^2x}{dt^2}\right)$ represents acceleration at any instant.
- To convert displacement–time curve into distance-time curve, we take the mirror image of section of negative slope about an imaginary line parallel to x-axis.

(A) → the slope of curve is zero:



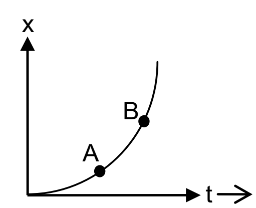
- v = 0
- so body is in rest condition

(B) → The slope of curve is constant so:



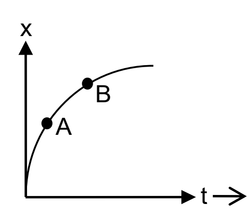
- $\vec{v}$ = constant = tan 30° = $\frac{1}{\sqrt{3}}$ unit, $\vec{a}$ = 0.
- So body moves with constant velocity & zero acceleration.

(C) → For the curve:



- Initial slope is zero i.e. u = 0
- Slope is positive throughout i.e. v = +ve
- $\left(\frac{dx}{dt}\right)_A < \left(\frac{dx}{dt}\right)_B \Rightarrow v_A < v_B$
- $\frac{d^2x}{dt^2}$ = +ve i.e. a = +ve
- So body starts from rest & moves with constant acceleration

(D) → For the curve:

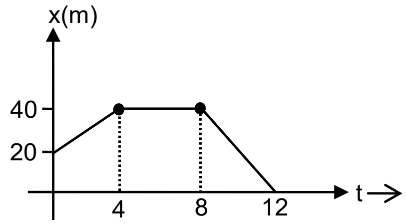


- Initial slope is +ve i.e. u = +ve
- Slope is +ve throughout i.e. v = +ve always
- $\left(\frac{dx}{dt}\right)_A > \left(\frac{dx}{dt}\right)_B$

  $\Rightarrow v_A > v_B$ so speed is decreasing
- $\frac{d^2x}{dt^2}$ = –ve i.e. a = –ve
- So body starts moving with a certain velocity is being retarded

Example 21:

For the following x – t curve calculates



(A) displacement (B) distance

(C) At what time average velocity is zero.

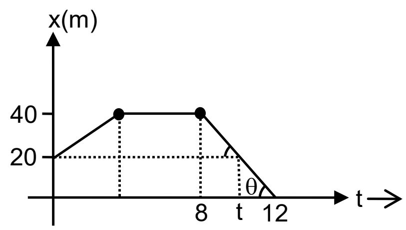
Solution:

(A) displacement = $x_f - x_i = 0 - 20 = -20$ m

(B) distance = |40 – 20| + |40 – 40| + |0 – 40| = 60 m

(C) Average velocity is zero where particle attains its initial position

$$\frac{40-20}{t-8} = \frac{20-0}{12-t} \Rightarrow t = 10 \text{ sec.}$$



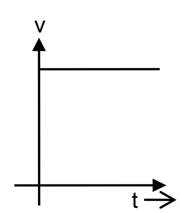
6.2 Velocity time graph

- Slope of v-t graph denotes instantaneous acceleration at a point
- Area of v-t graph provides distance/displacement
- Average velocity from v-t curve can be calculated as

$$\vec{v}_{av} = \frac{\text{Area enclosed with time axis}}{\text{Time} - \text{interval}}$$

- To convert v-t curve into speed-time curve, negative section of curve is made positive as speed cannot be negative.

Following Standard v-t curve represents: –



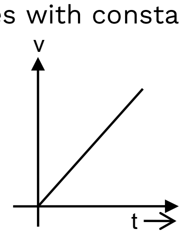
(A) The slope of curve is zero so

$\vec{a} = \vec{0}$ & $\vec{v}$ = constant

- So body moves with constant velocity.

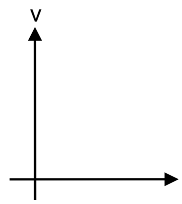
(B) The slope of curve is positive i.e. $\vec{a}$ = +ve also slope is constant which denotes $\vec{a}$ = constant

- So body starts from rest & moves with constant acceleration.



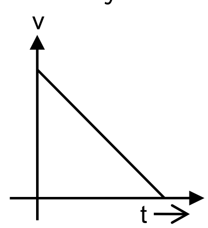
(C) v = 0, a = 0

- So the body is at rest



(D) The slope of curve is negative i.e. $\vec{a}$ = –ve

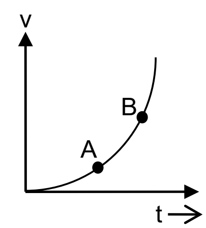
- So the body moving with certain velocity is retarding with constant retardation.



(E) Slope is changing continuously & is always positive i.e. $\vec{a}$ = variable

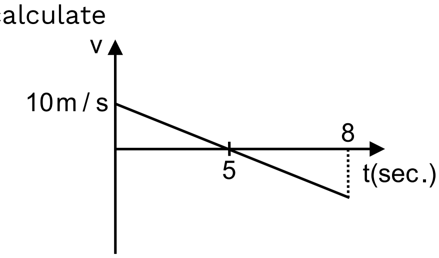
- Also $\left(\frac{dv}{dt}\right)_B > \left(\frac{dv}{dt}\right)_A$

  $\Rightarrow$ Acceleration is increasing
- So the body is moving with variable & increasing acceleration.



Example 22:

For the given v – t curve calculate



(A) Displacement (B) Average velocity

(C) Distance (D) Acceleration

Solution:

(A) Displacement = Area

Since $\frac{10}{5-0} = \frac{v}{8-5} \Rightarrow v = -6$

So Displacement

$= \left(\frac{1}{2} \times 10 \times 5\right) - \frac{1}{2}(3 \times 6) = 16$ m

(B) Average velocity = $\frac{16}{8} = 2$ m/s]

(C) Distance = $\left|\frac{1}{2} \times 10 \times 15\right| + \left|\frac{1}{2} \times 3 \times -6\right| = 34$ m

(D) acceleration = slope = v = –2 m/s^2

Example 23:

A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β, to come to rest. If the total time elapsed is t, evaluate (a) the maximum velocity attained and (b) the total distance travelled.

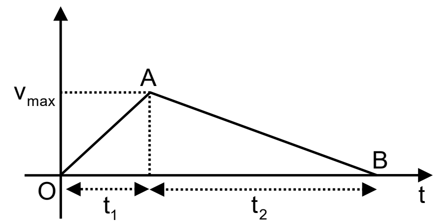
Solution:

(a) Let the car accelerates for time t_1 and decelerates for time t_2 then

$t = t_1 + t_2$ ......(i)

and corresponding velocity-time graph will be as shown in. fig.

From the graph α = slope of line OA = $\frac{v_{max}}{t_1}$



or $t_1 = \frac{v_{max}}{\alpha}$ ...(ii)

and $\beta = -$ slope of line OB = $\frac{v_{max}}{t_2}$

or $t_2 = \frac{v_{max}}{\beta}$ ...(iii)

From Eqs. (i), (ii) and (iii) $\frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} = t$

or $v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t$ or $v_{max} = \frac{\alpha\beta t}{\alpha+\beta}$

(b) Total distance = area under v-t graph $= \frac{1}{2} \times t \times v_{max} = \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha+\beta}$

Distance = $\frac{1}{2}\left(\frac{\alpha\beta t^2}{\alpha+\beta}\right)$

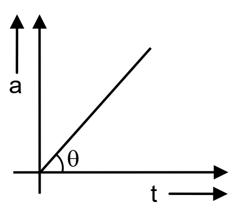
Note This problem can also be solved by using equations of motion (v = u + at, etc.).

6.3 Acceleration–Time Graph

- Slope of curve provides an idea about the rate by which acceleration changes but it does not represent any physical quantity.
- Area under the curve denotes change in velocity of any particle in motion
- Average acceleration from the curve can be calculated as

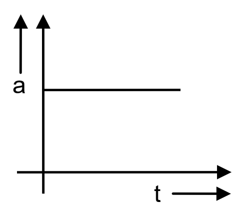
$$a_{Avg.} = \frac{\text{Area enclosed with time axis}}{\text{Time} - \text{interval}}$$

(i)



$a \propto t$, $\tan\theta$ = constant,
i.e. uniformly increasing acceleration

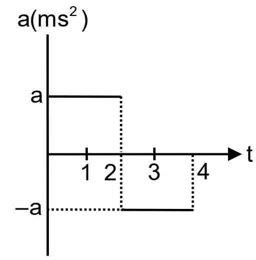
(ii)



$a \propto t^0$ i.e. uniform or constant acceleration

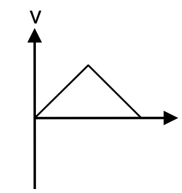
Example 24:

A particle starts from rest and undergoes an acceleration as shown in figure. The velocity-time graph from figure will have a shape.



Solution:

Till t = 2 s, there is constant acceleration a m/s^2 and after that there is constant deceleration –a m/s^2 (same magnitude). So velocity first increase from 0 to maximum and finally becomes zero.



Concept Builder-3

Q.1 The engine of a train passes an electric pole with a velocity u and the last compartment of the train crosses the same pole with a velocity v. Then find the velocity with which the mid-point of the train passes the pole. Assume acceleration to be uniform.

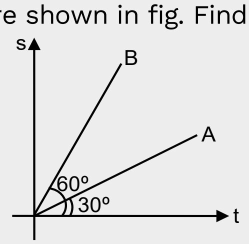
Q.2 A car moving along a straight highway with speed 126 kmh^{-1} is brought to a halt within a distance of 200 m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop ?

Q.3 If a body starts from rest and travels 120 cm in the 6^{th} second then what is the acceleration?

Q.4 A car is moving with speed u. Driver of the car sees red traffic light. His reaction time is t, then find out the distance travelled by the car after the instant when the driver decided to apply brakes. Assume uniform retardation 'a' after applying breaks

Q.5 A particle starts from rest, moves with constant acceleration for 15 s. If it covers s_1 distance in first 5 s then distance s_2 in next 10 s, then find the relation between s_1 & s_2.

Q.6 s-t graph of two particle A and B are shown in fig. Find the ratio of velocity of A to velocity of B.



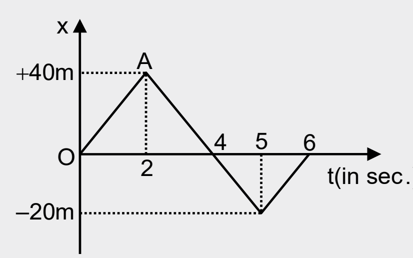
Q.7 s-t graph of a particle in motion is shown in fig. Calculate –

(i) Total distance covered

(ii) Displacement

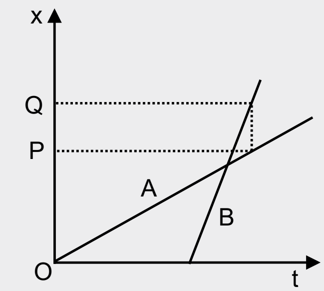
(iii) Average speed

(iv) Average velocity

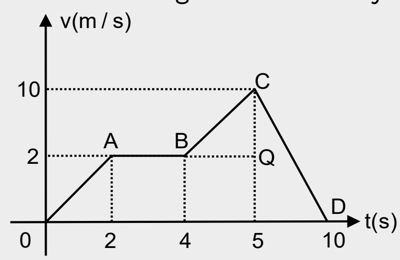


Q.8 The position -time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in fig. Choose correct entries in the brackets below:

(a) (A/B) lives closer to the school than (B/A)
(b) (A/B) starts from the school earlier than (B/A)
(c) (A/B) walks faster than (B/A)
(d) A and B reach home at the (same/different) time
(e) (A/B) overtake (B/A) on the road (once/twice)



Q.9 A particle moves on straight line according to the velocity – time graph shown in fig. Calculate-

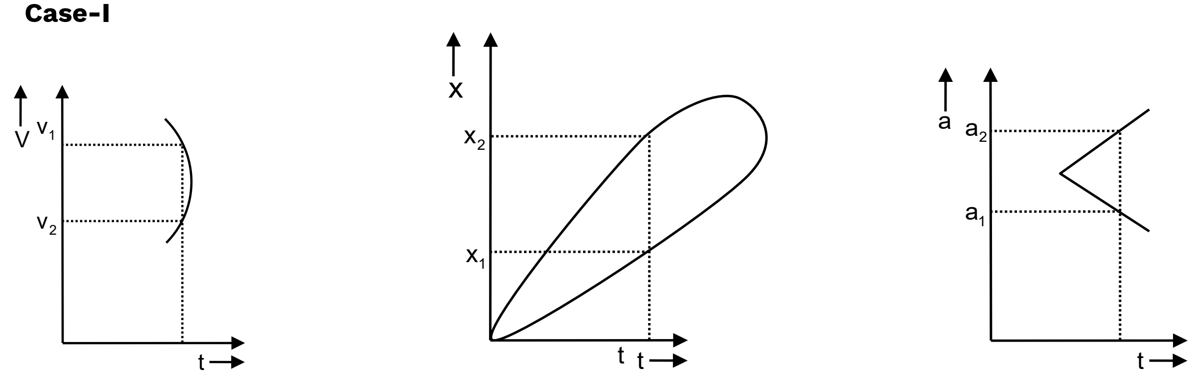


(i) Total distance covered
(ii) Average speed
(iii) In which part of the graph the acceleration is maximum and also find its value.
(iv) Retardation

Q.10 A body starts from rest and moves with a uniform acceleration of 10 ms^{-2} for 5 seconds. During the next 10 seconds it moves with uniform velocity. Find the total distance travelled by the body (using graphical analysis)

6.4 Following Graphs do not Exist in Practice

Case-I

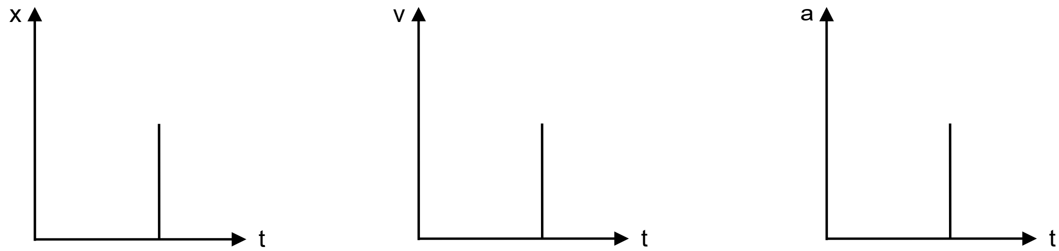


Explanation In practice, at any instant body cannot have two velocities or displacements or accelerations simultaneously.

Case-II

Explanation Speed or distance can never be negative

Case III



Explanation It is not possible to change any quantity without consuming time.

7. Motion Under Gravity (Free Fall)

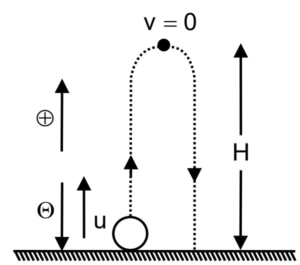
Acceleration produced in a body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

Value of g $=9.8\ m/s^2 = 980\ cm/s^2 = 32\ ft/s^2$

In the absence of air, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude (h<<earth's radius) is called motion under gravity. Free fall means acceleration of body is equal to acceleration due to gravity.

7.1 If a body is projected vertically upward

Positive / Negative directions are a matter of choice. You may take another choice



- Taking initial positions as origin and initial direction of motion (i.e. vertically up) as a positive, a = -g

  So, if a body is projected with velocity u and after time t it reaches a height h then
- For maximum height(H), v = 0

  So from above equation $u = g\,t_1$

  It is called time of ascent $(t_1) = u/g$

In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance.

Time of ascent (t_1) = Time of descent $(t_2) = u/g$

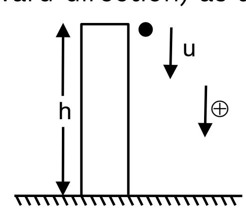
$\therefore$ Total time of flight $T = t_1 + t_2 = \dfrac{2u}{g}$

and $u^2 = 2gH \Rightarrow H = \dfrac{u^2}{2g}$

from $s = vt - \dfrac{1}{2}at^2 \Rightarrow$ t(Time of ascent) $= \sqrt{\dfrac{2H_{max}}{g}}$

7.2 If a body is projected vertically downward with some initial velocity from some height

Taking initial position as origin and direction of motion (i.e., downward direction) as a positive, we have



$v = u + gt,$ $\qquad h = ut + \dfrac{1}{2}gt^2$

$v^2 = u^2 + 2gh,$ $\qquad h_{n^{th}} = u + \dfrac{g}{2}(2n-1)$

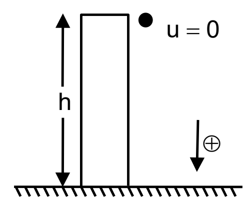
7.3 If a body is dropped from some height (initial velocity zero)

Taking initial position as origin and direction of motion (i.e. downward direction) as a positive, here we have

$u = 0$ [As body starts from rest]

$a = +g$

so $v = g\,t$, $h = \frac{1}{2}gt^2$, $v^2 = 2gh$



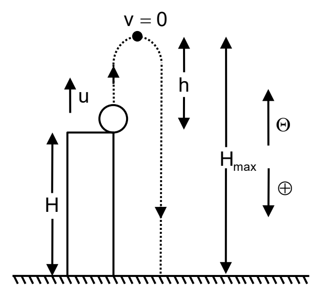
7.4 If a body is projected vertically upward with some initial velocity from a certain height

$v = -u + gt$

$H = -ut + \frac{1}{2}gt^2$

$v^2 = u^2 + 2gH$

$h_{n^{th}} = -u + \frac{g}{2}(2n - 1)$



Example 25:

A pebble is released from rest at a certain height and falls freely, reaching an impact speed of 4 m/s at the floor. Next, the pebble is thrown down with an initial speed of 3 m/s from the same height. What is its speed at the floor?

Solution:

1st case

$v^2 = u^2 + 2gh$

$16 = 0 + 2 \times 10 \times h \Rightarrow h = \frac{16}{20}$ m

IInd case

$v^2 = u^2 + 2gh$

$v^2 = 9 + 2 \times 10 \times \frac{16}{20}$ $\Rightarrow v^2 = 9 + 16 \Rightarrow v^2 = 25$ $\Rightarrow v = 5$ m/s

7.5 Application of Equation of motion in motion under gravity

Taking initial position as origin and upward direction as negative.

$v = -u + gt$; $H = -ut + \frac{1}{2}gt^2$

$v^2 = u^2 + 2gh$; $h_{n^{th}} = -u + \frac{g}{2}(2n-1)$

- Maximum height attained by the body

  $H_{max} = H + h = H + \frac{u^2}{2g}$

- Distance travelled by the body

  $H + 2h = H + \frac{u^2}{2g}$

  Time taken by the body to reach the ground

  $H = -ut + \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - ut - H = 0$

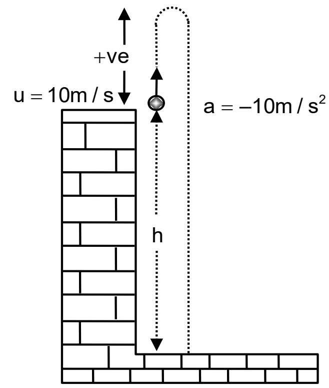
  $\Rightarrow gt^2 - 2ut - 2H = 0$

  After solving this equation, we get the result.

Example 26:

A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s, find the time when it strikes the ground ($g = 10\ m/s^2$)

Solution:



In the problem $u = +10$ m/s,

$a = -10\ m/s^2$ and $s = -40$ m

(at the point where ball strikes the ground)

Substituting in $s = ut + \frac{1}{2}at^2$

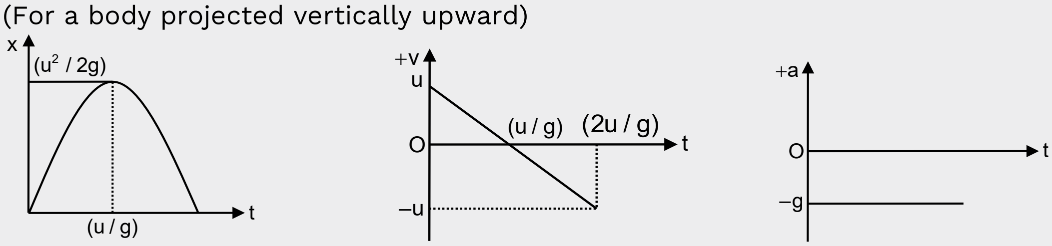
$-40 = 10t - 5t^2$ or $5t^2 - 10t - 40 = 0$

or $t^2 - 2t - 8 = 0$

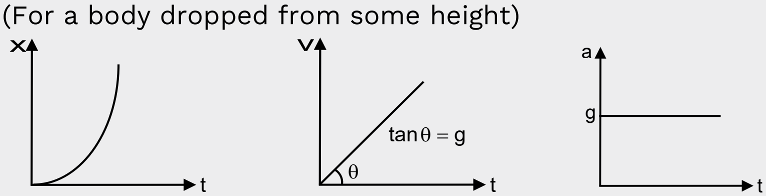
Solving this we have t = 4 s and – 2 s. Taking the positive value t = 4 s.

Key Points

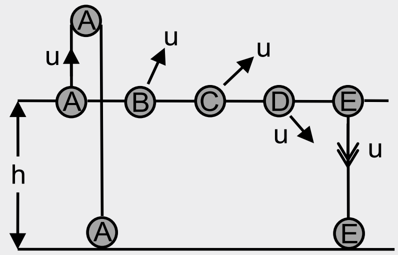
- In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.
- The magnitude of velocity at any point on the path is same whether the body is moving in upward or downward direction.
- Graph of displacement, velocity and acceleration with respect to time.
  (For a body projected vertically upward)



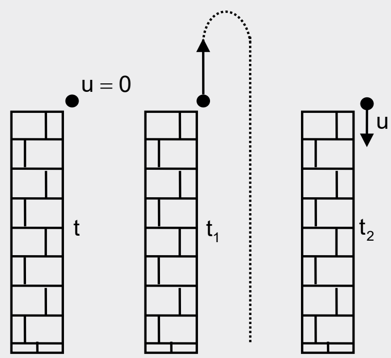
- As $h = (1/2)\,gt^2$, i.e. $h\ t^2$, distance covered in time t, 2t, 3t, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e. square of consecutive integers. (in case of free fall, from rest)
- A particle at rest, is dropped vertically from a height. The time taken by it to fall through successive distance of 1 m each will be in the ratio of the difference in the square roots of the integers i.e.
  $\sqrt{1},\ \left(\sqrt{2}-\sqrt{1}\right),\ \left(\sqrt{3}-\sqrt{2}\right),\ \left(\sqrt{4}-\sqrt{3}\right)$............
- The motion is independent of the mass of body, as mass is not involved in any equation of motion. It is due to this reason that a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity
  i.e., $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$
- The distance covered in the n^{th} second, (For a body dropped from some height)
  $h_n = \frac{1}{2}g(2n-1)$
- Graph of distance, velocity and acceleration with respect to time:
  (For a body dropped from some height)



- If various particles are thrown with same initial speed but in different directions, then.



- They strike the ground with same speed at different times irrespective of their initial direction of velocities.
- Time would be least for particle E which was thrown vertically downward.
- Time would be maximum for particle A which was thrown vertically upward.
- A ball is dropped from a building of height h and it reaches ground after time t.
  From the same building if two balls are thrown (one upwards and other downwards) with the same speed u and they reach the ground after t_1 and t_2 seconds respectively then t = $\sqrt{t_1 t_2}$



Example 27:

A body is dropped from a height h above the ground. Find the ratio of distances fallen in first one second, first two seconds, first three seconds, also find the ratio of distances fallen in 1^{st} second, in 2^{nd} second, in 3^{rd} second etc.

Solution:

From second equation of motion, i.e. $h = \frac{1}{2}gt^2$

$h_1 : h_2 : h_3 \ldots\ldots\ldots = \frac{1}{2}g(1)^2 : \frac{1}{2}g(2)^2 : \frac{1}{2}g(3)^2$

$= 1^2 : 2^2 : 3^2 \ldots\ldots\ldots = 1 : 4 : 9 : \ldots\ldots\ldots$

Now from the expression of distance travelled in n^{th} second $s_n = u + \frac{1}{2}a(2n-1)$

here u = 0, a = g

So, $S_n = \frac{1}{2}g(2n-1)$

therefore, $S_1 : S_2 : S_3 \ldots\ldots\ldots$

$= \frac{1}{2}g(2 \times 1 - 1) : \frac{1}{2}g(2 \times 2 - 1) : \frac{1}{2}g(2 \times 3 - 1)$

$= 1 : 3 : 5 \ldots\ldots\ldots$

Example 28:

A rocket is fired vertically up from the ground with a resultant vertical acceleration of 10 m/s^2. The fuel is finished in 1 minute and it continues to move up.

(a) What is the maximum height reached?

(b) After the fuel is finished, calculate the time for which it continues its upwards motion.

(Take g = 10 m/s^2)

Solution:

(a) The distance travelled by the rocket during burning interval (1minute = 60s) in which resultant acceleration is vertically upwards and 10 m/s^2 will be $h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2$ = 18000 m = 18 km and velocity acquired by it will bev = 0 +10 x 60 = 600 m/s

Now after 1 minute the rocket moves vertically up with initial velocity of 600 m/s and acceleration due to gravity opposes its motion. So, it will go to a height h_2 from this point, till its velocity becomes zero,

such that $0 = (600)^2 - 2gh_2$ or h_2 = 18000 m = 18 km [g = 10 ms^{-2}]

max height = 36 km

(b) As after burning of fuel the initial velocity 600 m/s and gravity opposes the motion of rocket, so from 1st equation of motion time taken by it till its velocity v = 0

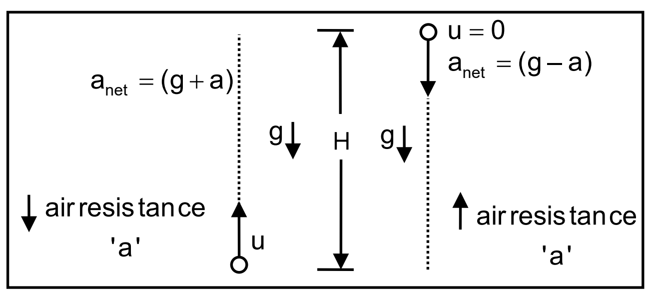
v = u + at

0 = 600 – 10t

t = 60 sec

7.6 Air Drag

- A body is thrown vertically upwards, if constant Air resistance is to be taken into account:



For upward motion:-

Net acceleration a_{net} = g + a (downwards)

If maximum height attained by the particle is 'H' then

$t_{ascent} = \sqrt{\dfrac{2H}{a_{Net}}} \Rightarrow t_{ascent} = \sqrt{\dfrac{2H}{g+a}}$

For downward motion: -

Net acceleration a_{Net} = g - a (downwards)

So $t_{descent} = \sqrt{\dfrac{2H}{g-a}}$

Also $t_{descent} > t_{ascent}$

- For downwards motion a and g will work in opposite directions because a always acts in direction opposite to motion and g always acts vertically downwards.

Concept Builder-4

Q.1 A ball thrown up vertically returns to the thrown after 8 seconds. Find:
(i) the speed with which it was thrown up.
(ii) the maximum height attained by the ball.
(iii) its position after 6 seconds ($g = 10\ m/s^2$)

Q.2 A ball is thrown upwards from the top of a tower 50 m high with a velocity of 20 m/s, find the velocity when it strikes the ground ($g = 10\ m/s^2$).

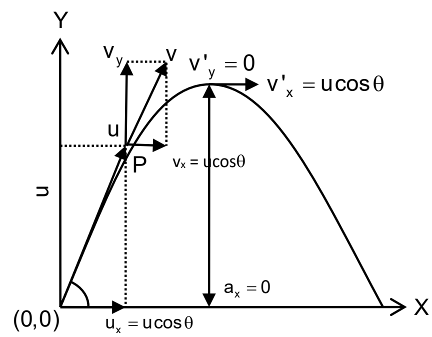
Q.3 A ball is thrown upwards from the top of a tower 30 m high with a velocity of 10 m/s, find the velocity when ball at 15 m high from the ground ($g = 10\ m/s^2$)

Q.4 A water droplet is dropped from a height 50m above the ground find the ratio of distance fallen in 1st second, in 2nd second, in 3rd second etc.

8. Projectile Motion

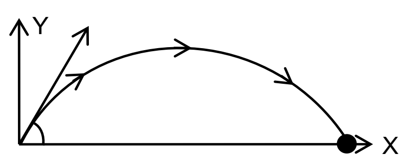
When a body is projected such that velocity of projection is not parallel to the force (θ other than 0° and 180°), then it moves along a curved path. This motion is called two dimensional motion. If force on the body is constant then curved path of the body is parabolic. This motion is studied under projectile motion.

(i) It is an example of motion with constant (or uniform) acceleration. Thus equations of motion can be used to analysis projectile motion.
(ii) A particle thrown in the space which moves under the effect of gravity only is called a "projectile".
(iii) If a particle posses a uniform acceleration in a directions oblique to its initial velocity, the resultant path will be parabolic. Let X-axis is along the ground and Y-axis is along the vertical, then path of projectile projected at an angle θ from the ground is as shown.

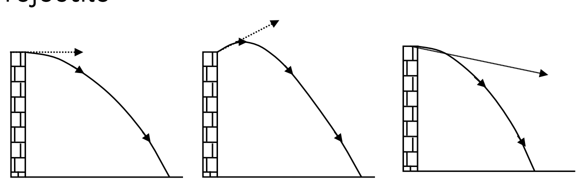


Types of Projectile

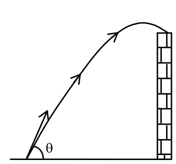
(i) Ground to Ground or Level to level Projectile



(ii) Hight to Ground Projectile



(iii) Ground to Hight Projectile



9. Ground to Ground Projection

Projectile motion can be considered as two mutually perpendicular motions, which are independent of each other i.e. Projectile motion = Horizontal motion + Vertical motion

9.1 Horizontal Motion

Initial velocity in horizontal direction

$= u\cos\theta = u_x$

Acceleration along horizontal direction

$= a_x = 0$. (Neglect air resistance)

Therefore, horizontal velocity remains unchanged.

- At any instant horizontal velocity $u_x = u\cos\theta$
- At time t, x coordinate or displacement along X-direction is

$x = u_x t \Rightarrow x = (u\cos\theta)t$

9.2 Vertical Motion

It is motion under the effect of gravity

Initial velocity in vertical direction = $u\sin\theta = u_y$

Acceleration along vertical direction = $a_y = -g$

- At time t, vertical speed $v_y = u_y - gt = u\sin\theta - gt$
- In time t, displacement in vertical direction or "height" of the particle above the ground

$y = u_y t - \frac{1}{2}gt^2 = (u\sin\theta)t - \frac{1}{2}gt^2$

9.3 Net motion

Net initial velocity = $\vec{u} = u_x\,\hat{i} + u_y\,\hat{j}$

$= u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

Direction of u can be explained in terms of angle θ it makes with the ground

Net acceleration= $\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$ (direction of g is downwards)

9.4 Coordinates of particle at time t

Let (x, y) are coordinates of particle at general time t, then

$x = u_x t$ and $y = u_y t - \frac{1}{2}gt^2$

Net displacement in t time = $\sqrt{x^2 + y^2}$

9.5 Velocity of particle at time t

$\vec{v} = v_x\,\hat{i} + v_y\,\hat{j} = u_x\,\hat{i} + (u_y - gt)\,\hat{j}$

$= u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$

magnitude of velocity $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

If angle made by velocity with the ground is α, then

$\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x}$

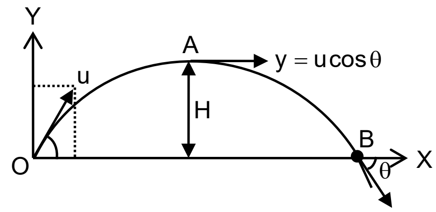
$\Rightarrow \tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$

9.6 Change in velocity and momentum of projectile

When particle returns to ground again at B point, its y coordinate is zero and the magnitude of its velocity is u at angle θ with ground. Total angular change = 2θ

Initial velocity $\vec{u}_i = u\cos\theta\hat{i} + u\sin\theta\hat{j}$

Final velocity $\vec{u}_f = u\cos\theta\hat{i} - u\sin\theta\hat{j}$



Total change in velocity $|\Delta\vec{v}| = 2mu\sin\theta$

Total change in momentum

$|\Delta\vec{p}| = m|\Delta\vec{v}| = 2mu\sin\theta$

- change in speed = 0

9.7 Time of flight (T)

At time T particle will be at ground again, i.e. displacement along Y-axis becomes zero.

$\therefore y = u_y t - \frac{1}{2}gt^2$ $\qquad \therefore 0 = u_y T - \frac{1}{2}gT^2$

or $\quad T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

Time of ascent = Time of descent

$= \frac{T}{2} = \frac{u_y}{g} = \frac{u\sin\theta}{g}$

at time $\frac{T}{2}$ particle attains maximum height of its trajectory.

9.8 Maximum height (H)

At maximum height, vertical component of velocity becomes zero. At this instant y coordinate is maximum

$\because v_y^2 = u_y^2 - 2gy$ $\qquad \because 0 = u_y^2 - 2gH_{max}$

$H_{max} = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}$

9.9 Horizontal range or Range (R)

It is the displacement of particle along X-direction during its complete flight.

$\because x = u_x t \quad \therefore R = u_x T = u_x \frac{2u_y}{g}$

$R = \frac{2u_x u_y}{g} = \frac{2(u\cos\theta)(u\sin\theta)}{g} = \frac{u^2\sin 2\theta}{g}$

9.10 Maximum horizontal range (R_{MAX})

If value of θ is increased from $\theta = 0°$ to 90°, then range increases from $\theta = 0°$ to 45° but it decreases beyond 45°. Thus range is maximum at θ = 45°

For maximum range, θ = 45° and

$R_{max} = \frac{u^2\sin 2(45°)}{g} = \frac{u^2\sin 90°}{g} \Rightarrow R_{max} = \frac{u^2}{g}$

9.11 Equation of Trajectory

Along horizontal direction

$x = u_x t$ or $x = (u\cos\theta)\, t$

Along vertical direction

$y = u_y t - \frac{1}{2}gt^2$ or $y = (u\sin\theta)\, t - \frac{1}{2}gt^2$

On eliminating from these two equations

$$y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2 \qquad \Rightarrow y = x\tan\theta - \frac{1}{2}g\,\frac{x^2}{u^2\cos^2\theta}$$

This is an equation of a parabola so it can be stated that projectile follows a parabolic path.

Again $y = x\tan\theta\left[1 - \frac{gx}{2u^2\sin\theta\cos\theta}\right] \qquad \Rightarrow \qquad y = x\tan\theta\left[1 - \frac{x}{R}\right]$

9.12 Kinetic Energy of Projectile

Kinetic energy = $\frac{1}{2}$ × Mass × (Speed)^2

Let a body is projected with velocity u at an angle θ.

Thus initial kinetic energy of projectile, $K_0 = \frac{1}{2}mu^2$

Since velocity of projectile at maximum height is $u\cos\theta$. So $K_h max = \frac{1}{2}mu^2\cos^2\theta = K_0\cos^2\theta$

which is the minimum kinetic energy during whole motion.

Key Points

- At maximum height, $v_y = 0$ and $v_x = u_x = u\cos\theta$ so at maximum height

  $v = \sqrt{v_x^2 + v_y^2} = u\cos\theta$
- At maximum height angle between velocity and acceleration is 90°.
- Magnitude of velocity at height 'h'.

  vertical component of velocity at height h

  $v_y^2 = u_y^2 - 2gh$

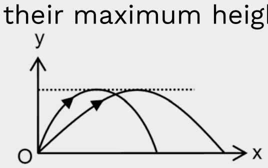
  $v_y^2 = (u\sin\theta)^2 - 2gh$

  horizontal component of velocity at height h

  $v_x = u\cos\theta$

  $|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2\cos^2\theta + (u^2\sin^2\theta - 2gh)} \qquad \Rightarrow |\vec{v}| = \sqrt{u^2 - 2gh}$
- $T = \frac{2u_y}{g}$; $\qquad H = \frac{u_y^2}{2g}$; $\qquad R = \frac{2u_x u_y}{g}$

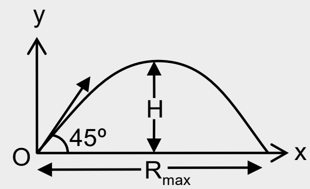
  T and H depend only upon initial vertical speed u_y
- If two projectiles thrown in different directions, have equal times of flight then their initial vertical speeds are same so that their maximum height are also same.



If $H_A = H_B$ then $(U_y)_A = (u_y)_B$ and $T_A = T_B$

- For situation shown in figure for $\theta = 45^\circ$

  Here $R_{max} = \dfrac{u^2}{g}$

  and $H = \dfrac{u^2 \sin^2 45º}{2g} = \dfrac{u^2}{4g}$

  $\therefore$ R_{max} = 4H = 4(maximum height attained)

- When R = H

  $R = \dfrac{u^2(2\sin\theta\cos\theta)}{g}$ and $H = \dfrac{u^2\sin^2\theta}{2g}$

  $\Rightarrow$ $\dfrac{R}{H} = 4\cot\theta = 1$ $\Rightarrow$ $4\cot\theta = 1$

  $\Rightarrow$ $\tan\theta = 4$ $\Rightarrow$ $\theta = \tan^{-1}(4)$

Example 29:

A projectile is thrown with speed u making angle θ with horizontal at t = 0. It just crosses two points of equal height at time t = 1s and t = 3s respectively. Calculate the maximum height attained by it? (g = 10 m/s^2)

Solution:

Displacement in y direction

$y = u_y \times 1 - \dfrac{1}{2} g \times (1)^2 = u_y \times 3 - \dfrac{1}{2} g(3)^2$ $\Rightarrow u_y = 2g = 20$ m/s

Maximum height attained $h_{max} = \dfrac{u_y^2}{2g} = 20$m.

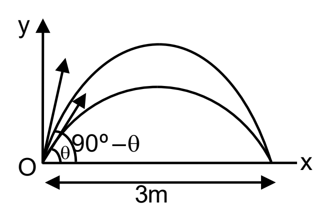
Example 30:

A stone is to be thrown so as to cover a horizontal distance of 3 m. If the velocity of the projectile is 7 m/s find:

(a) the angle at which it must be thrown

(b) the largest horizontal displacement that is possible with the projection speed of 7 m/s.

Solution:



(a) Range $R = \dfrac{u^2}{g}\sin 2\theta$ $\Rightarrow \sin 2\theta = \dfrac{gR}{u^2}$

$= \dfrac{9.8 \times 3}{(7)^2} = 0.6 = \sin 37°$ $\Rightarrow 2\theta = 37^\circ \Rightarrow \theta = 18.5^\circ$

angle of projection may also be $90^\circ - 18.5^\circ = 71.5^\circ$

(b) For largest horizontal displacement $\theta = 45^\circ$ maximum range

$R_{max} = \dfrac{u^2}{g} = \dfrac{(7)^2}{9.8} = \dfrac{49}{98} \times 10 = 5\text{m}.$

Example 31:

A ball is thrown from the ground to clear a wall 3 m high at a distance of 6 m and falls 18 m away from the wall, the angle of projection of ball is:-

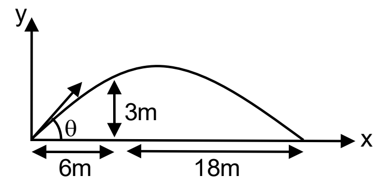
(A) $\tan^{-1}\left(\dfrac{3}{2}\right)$ (B) $\tan^{-1}\left(\dfrac{2}{3}\right)$ (C) $\tan^{-1}\left(\dfrac{1}{2}\right)$ (D) $\tan^{-1}\left(\dfrac{3}{4}\right)$

Ans. **(B)**

Solution:

From equation of trajectory, $y = x\tan\theta\left[1-\dfrac{x}{R}\right]$

$\Rightarrow 3 = 6\tan\theta\left(1-\dfrac{6}{24}\right) \Rightarrow \tan\theta = \dfrac{2}{3}$



Example 32:

A particle is projected with the velocity $v = a\hat{i} + b\hat{j}$ then calculate range, height and time of flight of particle.

Solution:

Horizontal velocity = $v_x = a$

Vertical velocity = $v_y = b$

Then time of flight = $\dfrac{2b}{g}$

Range of projectile = $\dfrac{2b}{g} \times a = \dfrac{2ab}{g}$

Maximum height of projectile = $b^2 = 2gh$

$\Rightarrow h = \dfrac{b^2}{2g}$

Concept Builder-5

Q.1 A football player kicks a ball at an angle of 30° to the horizontal with an initial speed of 20 m/s. Assuming that the ball travels in a vertical plane, calculate (a) the time at which the ball reaches the highest point (b) the maximum height reached (c) the horizontal range of the ball (d) the time for which the ball is in the air. ($g = 10\ m/s^2$)

Q.2 A cricketer can throw a ball to a maximum horizontal distance of 100 m. How high above the ground can the cricketer throw the ball, with the same speed?

Q.3 Two bodies are thrown with the same initial speed at angles α and $(90°-\alpha)$ with the horizontal. What will be the ratio of (a) maximum heights attained by them and (b) horizontal ranges ?

Q.4 The range of a particle when launched at an angle of 15° with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle 45° to the horizontal.

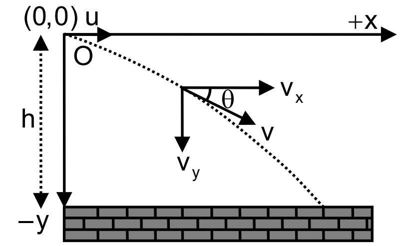
Q.5 A ball is thrown from the ground to clear a wall 5 m high at a distance of 10 m and falls 20 m away from the wall, the angle of projection of ball is:-

Q.6 A particle is projected with the velocity $v = 3\hat{i} + 4\hat{j}$ then calculate range, height and time of flight of particle.

10. Horizontal Projection from Height

Consider a projectile thrown from point O at some height h from the ground with a velocity u in horizontal direction.

Now we shall deal the characteristics of projectile motion separately along horizontal and vertical directions i.e.



Horizontal direction :	Vertical direction :
(i) Initial velocity $u_x = u$	Initial velocity $u_y = 0$
(ii) Acceleration $a_x = 0$	Acceleration $a_y = -g$

10.1 Trajectory Equation

The path traced by projectile is called its trajectory. After time t, x = ut and $y = -\frac{1}{2}gt^2$ negative sign indicates that the direction of vertical displacement is downward.

so $\quad y = -\frac{1}{2}g\frac{x^2}{u^2} \quad \left(\because t = \frac{x}{u}\right)$

This is equation of a parabola

Above equation is called trajectory equation

10.2 Velocity at a general point P (x,y)

$v = \sqrt{v_x^2 + v_y^2}$

Horizontal velocity of the projectile after time t is $v_x = u$ (remains constant)

Velocity of projectile in vertical direction after time t is

$v_y = 0 - (g)t = -gt$ (downward)

$\therefore v = \sqrt{v^2 + g^2t^2}$

and $\tan\theta = \frac{v_y}{v_x}$ or $\tan\theta = -\frac{gt}{u}$ (negative sign indicates clockwise direction)

10.3 Displacement

The displacement of the particle is expressed by $\vec{s} = x\hat{i} + y\hat{j}$

Where $|\vec{s}| = \sqrt{x^2 + y^2} = \left|(ut)\hat{i} - \left(\frac{1}{2}gt^2\right)\hat{j}\right|$

10.4 Time of flight

From equation of motion for vertical direction.

$h = u_yt + \frac{1}{2}gt^2$

At highest point $u_y = 0 \Rightarrow h = \frac{1}{2}gT^2$

$\Rightarrow T = \sqrt{\frac{2h}{g}}$

10.5 Velocity after falling a height h_1

Along vertical direction $v_y^2 = 0^2 + 2(h_1)(g)$

$v_y = \sqrt{2gh_1}$

Along horizontal direction $v_x = u_x = u$

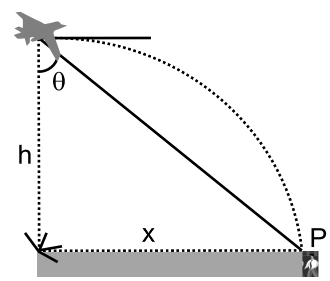
So velocity, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh_1}$

Example 33:

A relief aeroplane is flying at a constant height of 1960 m with 600 km/hr speed above the ground towards a point directly over a person struggling in flood water. At what angle of sight with the vertical should the pilot release a survival kit if it is to reach the person in water ?

($g = 9.8\ m/s^2$)

Solution:



Plane is flying at a speed = $600 \times \frac{5}{18} = \frac{500}{3}$ m/s horizontally (at a height 1960m)

time taken by the kit to reach the ground

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20\ s$

in this time the kit will move horizontally by

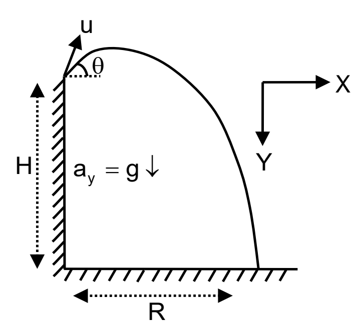
$x = ut = \frac{500}{3} \times 20 = \frac{10,000}{3}\ m$

So $\tan\theta = \frac{x}{h} = \frac{10,000}{3 \times 1960} = \frac{10}{5.88} = 1.7 = \sqrt{3}$

or $\theta = 60°$

11. Oblique Projection from a Certain Height

11.1 Projection from a Height at an Angle θ above horizontal



$u_x = u\cos\theta$ $\quad$ $u_y = -u\sin\theta$

$x = (u\cos\theta)\,t$ $\quad$ $a_y = g$

$H = (-u\sin\theta)\,t + \frac{1}{2}gt^2$

$gt^2 - (2u\sin\theta)\,t - 2H = 0$

After solving the above equation we get the result

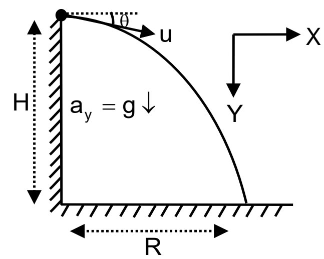
11.2 Velocity after falling height h

Along vertical direction; $v_y^2 = (-u\sin\theta)^2 + 2(h)(g)$

Along horizontal direction, $v_x = u_x = u\cos\theta$;

So, velocity, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$

11.3 Projection from a height at an angle θ below horizontal



$u_x = u\cos\theta$ $\qquad$ $u_y = u\sin\theta$

$c = (u\cos\theta)\,t$ $\qquad$ $a_y = g$

$H = (u\sin\theta)\,t + \frac{1}{2}gt^2$

$gt^2 + (2u\sin\theta)\,t - 2H = 0$

After solving the above equation we get the result.

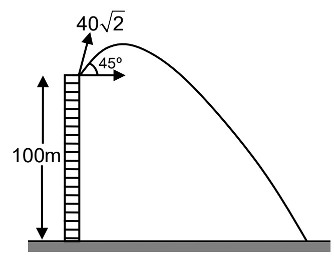
11.4 Velocity after falling height h

Along vertical direction, $v_y^2 = (u\sin\theta)^2 + 2hg$

Along horizontal direction, $v_x = u_x = u\cos\theta$;

So, velocity, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$

Example 34:



For shown projection from a tower, find the:

(i) time of flight,

(ii) horizontal range,

(iii) maximum height attained from ground,

(iv) striking speed. $\qquad$ ($g = 10\ m/s^2$)

Solution:

(i) By using $s = ut + \frac{1}{2}at^2$ in vertical direction

$-100 = 40(T) + \frac{1}{2}(-10)T^2$

$\Rightarrow T^2 - 8T - 20 = 0$

$\Rightarrow T = 10$ s

(ii) Horizontal range

$R = u_x T = (40)(10) = 400$ m

(iii) Maximum height $= 100 + \frac{u_y^2}{2g} = 100 + \frac{(40)^2}{2(10)} = 180$m

(iv) Striking speed

$v = \sqrt{u^2 + 2gh} = \sqrt{(40\sqrt{2})^2 + 2(10)(100)} = 20\sqrt{13}$ m/s

Concept Builder-6

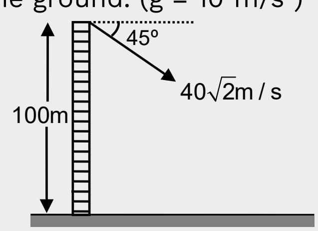
Q.1 A projectile is fired horizontally with a velocity of 98 ms^{-1} from the top of a hill 490 m high. Find (i) the time taken to reach the ground (ii) the distance of the target from the hill and (iii) the angle at which the projectile hit the ground. ($g = 9.8\ m/s^2$).

Q.2 Two tall buildings face each other and are at a distance of 180 m from each other. with what velocity must a ball be thrown horizontally from a window 55 m above the ground in one building, so that it enters a window 10 m above the ground in the second building? ($g = 10\ ms^{-2}$)

Q.3 Two paper screens A and B are separated by a distance of 100 m. A bullet pierces A and then B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity

Q.4 A ball is thrown up from the top of a tower with an initial velocity of 10 m/s at an angle of 30° with the horizontal. It hits the ground at a distance of 17.3 m from the base of tower. Calculate the height of the tower. ($g = 10\ m/s^2$)

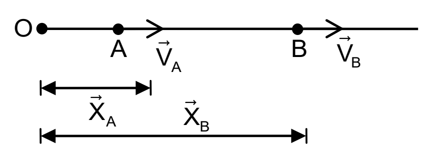
Q.5 For shown projection, find the:
(i) time of flight,
(ii) horizontal range,
(iii) speed just before striking the ground. ($g = 10\ m/s^2$)



of the bullet when it hits the screen B. ($g = 10\ m/s^2$)

12. Relative Velocity One Dimension (Relative Motion)

When we consider the motion of a particle, we assume a fixed point relative to which the given particle is in motion. For example, if we say that water is flowing or wind is blowing or a person is running with a speed u, we mean that these all are relative to the earth (which we have assumed to be fixed).



Displacement of B with respect to A = Displacement of B as measured from A

$$\Rightarrow \quad \vec{x}_{BA} = \vec{x}_B - \vec{x}_A$$

$$\Rightarrow \quad \frac{d\vec{x}_{BA}}{dt} = \frac{d\vec{x}_B}{dt} - \frac{d\vec{x}_A}{dt}$$

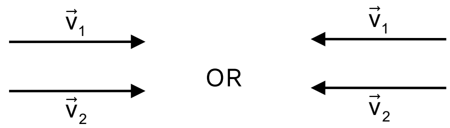
$$\Rightarrow \quad \vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

(Relative = Actual – Reference)

12.1 While moving in same frame of reference

- **For same direction:**

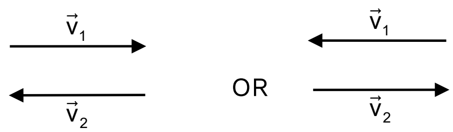
  When two particles are moving in the same direction, then magnitude of their relative velocity is equal to the difference between their individual speeds.



$|\vec{v}_{12}|$ or $|\vec{v}_{21}| = v_1 - v_2$

- **For opposite directions**

  When two particles are moving in the opposite directions, then magnitude of their relative velocity is always equal sum of their individual speeds.



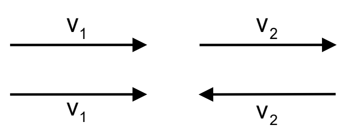
$|\vec{v}_{12}|$ or $|\vec{v}_{21}| = v_1 + v_2$

Note- When two particles move simultaneously then the concept of relative motion becomes applicable conveniently.

12.2 Numerical Applications

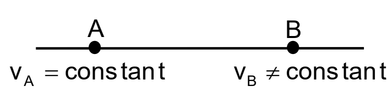
- When two particles are moving along a straight line with constant speeds then their relative acceleration must be zero and in this condition relative velocity is the ratio of relative displacement to time.

  v_1 = const. v_2 = const.



when $a_{rel.} = 0$ $\quad v_{rel} = \dfrac{s_{relative}}{time}$

- When two particles move in such a way that their relative acceleration is non zero but constant then we apply equation of motion in the relative form.



$a_A = 0$ $\quad a_B = a$

$a_{AB} = a_A - a_B$

$= 0 - a = -a \neq 0 = \text{constant}$

Equation of Motion (Relative)

(i) $v_{rel.} = u_{rel.} + a_{rel.}t$ (ii) $s_{rel.} = u_{rel.}t + \dfrac{1}{2}a_{rel.}t^2$

(iii) $v_{rel.}^2 = u_{rel.}^2 + 2.a_{rel.}s_{rel.}$ (iv) $s_{rel.} = \dfrac{1}{2}(u_{rel.} + v_{rel.})t$

Example 35:

Buses A and B are moving in the same direction with speeds 20 m/s and 15 m/s respectively. Find the relative velocity of A w.r.t. B and relative velocity of B w.r.t. A.

Solution:

Let their direction of motion be along + x-axis then

$\vec{v}_A = (20m/s)\hat{i}$ and $\vec{v}_B = (15m/s)\hat{i}$

(a) Relative velocity of A w.r.t. B is

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ = (actual velocity of A) – (velocity of B)

= (20m/s) $\hat{i}$ – (15 m/s) $\hat{i}$ = 5 m/s $\hat{i}$

i.e. A is moving with speed 5m/s w.r.t. B in the same direction.

(b) Relative velocity of B w.r.t. A is

$\vec{v}_{AB} = \vec{v}_B - \vec{v}_A$ (actual velocity of B) – (velocity of A)

= (15m/s) $\hat{i}$ – (20m/s) $\hat{i}$ = (– 5m/s) = (5m/s)(– $\hat{i}$)

i.e. B is moving in opposite direction w.r.t. A, at a speed 5 m/s.

Example 36:

A police van moving on a highway with a speed of 30 km/hr. fires a bullet at a thief's car which is speeding away in the same direction with a speed of 190 km/hr. If the muzzle speed of the bullet is 150 m/s, find the speed of the bullet with respect to the thief's car.

Solution:

$v_b \rightarrow$ velocity of bullet

$v_p \rightarrow$ velocity of police van

$v_t \rightarrow$ velocity of thief's car

$v_{bp} = v_b - v_p \Rightarrow \qquad v_{bp} = 150 \times \frac{18}{5} = 540$ km/hr

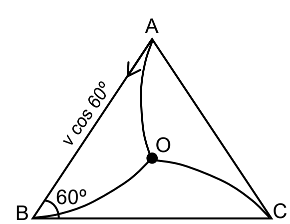
$v_b = v_{bp} + v_p = 540 + 30 = 570$ km/hr

$v_{bt} = v_b - v_t = 570 - 190 = 380$ km/hr.

Example 37:

Three boys A, B and C are situated at the vertices of an equilateral triangle of side d at t =0. Each of the boys move with constant speed v. A always moves towards B, B towards C and C towards A. When and where will they meet each other.

Solution:



By symmetry they will meet at the centroid of the triangle. Approaching velocity of A and B towards each other is v + v cos 60° and they cover distance d when they meet.

So that time taken, is given by

$$\therefore t = \frac{d}{v + v\cos 60^\circ} = \frac{d}{v + \frac{v}{2}} = \frac{2d}{3v}$$

Example 38:

Two trains A & B 100 km apart are travelling towards each other on different tracks with starting speed of 50 km/h for both. The train A accelerates at 20 km/h^2 and the train B retards at the rate 20 km/h^2. The distance covered by the train A when they cross each other is:

(1) 45 km (2) 55 km (3) 65 km (4) 60 km

Ans. **(4)**

Solution:

From relative motion equation

$s_r = u_r t + \frac{1}{2} a_r t^2 \Rightarrow u_r = 100$ km/hr

$a_r = 0$, $s_r = 100$

$100 = 100 \times t + 0 \Rightarrow t = 1$ hr

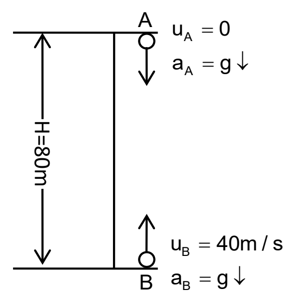
Distance travelled by A

$S = 50 \times 1 + \frac{1}{2} \times 20 \times (1)^2 = 60$ km

Example 39:

A stone is dropped from a tower of height 80 m. At the same instant another stone is thrown from the foot of the tower with a speed of 40 m/s. When & where the stones will cross each other.

Solution:



Here $S_{rel} = 80$ m,

$u_{rel} = 40$ m/s,

$a_{rel} = 0$ (Relative motion under gravity is a uniform motion)

So time $= \frac{S_{rel}}{U_{rel}} = 2$ s

Height from ground

$H = ut + \frac{1}{2} at^2$

$\Rightarrow H = 40(2) - \frac{1}{2} g(2)^2 = 60$ m

Concept Builder-7

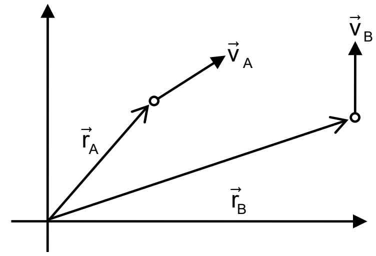
Q.1 Two trains A and B each of length 50 m, are moving with constant speeds. If one train A overtakes the other train B in 40s, when moving in same direction. While crosses the other in 20s. Find the speeds of each trains.

Q.2 Two trains A and B of length 400 m each of are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 m s^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between the guard of B & driver of A ?

Q.3 A bus is moving with a velocity 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what velocity should the scooterist chase the bus?

Q.4 A stone is thrown vertically upward from the foot of the tower of height 125 m with 50 m/sec. After 2 sec another stone is dropped from top of tower. when & where the stone will cross each other.

13. Relative Velocity in a Plane



For 2D motion

Relative velocity of A with respect to B can be calculated as

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$\Rightarrow |\vec{v}_{AB}| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$

13.1 For two particles to collide

- their combined relative displacement becomes zero.
- their combined vertical velocities will be same: if they are projected from same level (incase of projectiles)
- their combined motion can be converted into 1-D motions.

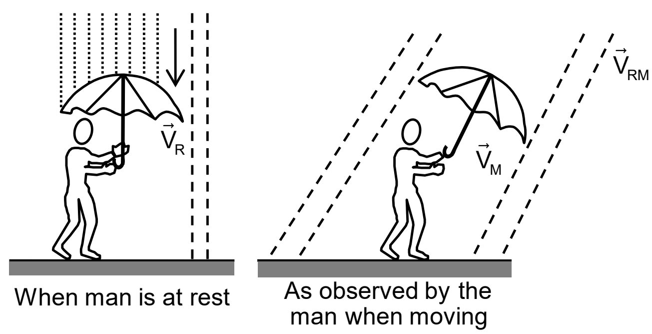
13.2 Relative path of a projectiles w.r.t. another projectile

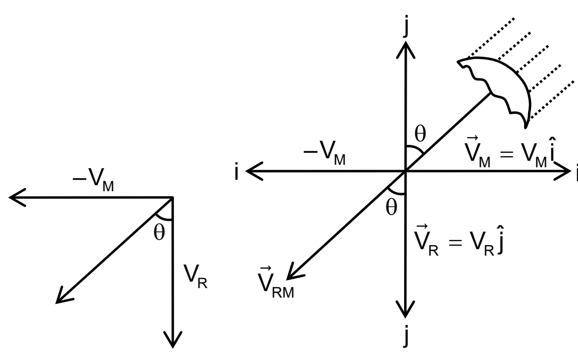
Two projectiles are thrown from ground with different velocities at different angles. Since both projectiles have equal accelerations so their relative acceleration is zero. Thus path of one projectile w.r.t. other is a straight line and motion of one projectile w.r.t. other is uniform.

- If $u_1 \cos\theta_1 = u_2 \cos\theta_2$ then relative path is a vertical line.
- If $u_1 \sin\theta_1 = u_2 \sin\theta_2$ then relative path is a horizontal line.

14. Rain – Man Problem

(i) If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with speed $\vec{v}_M$ the velocity of rain relative to observer will be





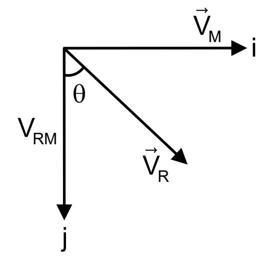
$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M \Rightarrow \vec{v}_{RM} = -v_R\hat{j} - v_M\hat{i}$

which by law of vector addition has magnitude

$v_{RM} = \sqrt{v_R^2 + v_M^2}$

The direction of $\vec{v}_{RM}$ is such that it makes an angle θ with the vertical given by $\theta = \tan^{-1}(v_m/v_R)$ as shown in figure.

(ii) If rain is already falling at an angle θ with the vertical with a velocity $\vec{v}_R$ and an observer is moving horizontally with speed $\vec{v}_M$ find that the rain drops are hitting on his head vertically downwards Here $\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$



$\vec{v}_{RM} = (v_R \sin\theta - v_M)\,\hat{i} \;\; - v_R \cos\theta\,\hat{j}$

Now for rain to appear falling vertically, the horizontal component of $\vec{v}_{RM}$ should be zero, i.e.

$$v_R \sin\theta - v_M = 0 \Rightarrow \sin\theta = \frac{v_M}{v_R}$$

and $|\vec{v}_{RM}| = v_R \cos\theta = v_R\sqrt{1-\sin^2\theta} \qquad = v_R\sqrt{1-\frac{v_M^2}{v_R^2}} \Rightarrow v_{RM} = \sqrt{v_R^2 - v_M^2}$

Example 40:

A person moves due east at a speed 6m/s and feels the wind is blowing towards south at a speed 6m/s.

(a) Find actual velocity of wind blow.

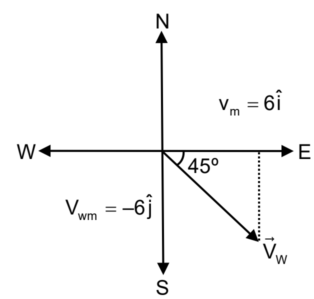
(b) If person doubles his velocity then find the relative velocity of wind blow w.r.t. man.

Solution:

(a) $\vec{v}_{act} = \vec{v}_{rel} + \vec{v}_{ref}$

$\vec{v}_w = \vec{v}_{wm} + \vec{v}_m \;\; = -6\hat{j} + 6\hat{i} \Rightarrow \vec{v}_w = 6\hat{i} - 6\hat{j}$

$|\vec{v}| = 6\sqrt{2}$ m/s and it is blowing along S-E



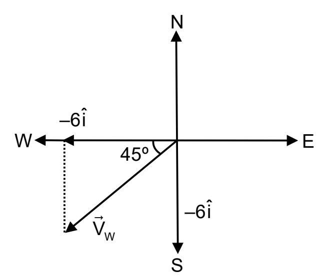
(b) Person doubles its velocity then $\vec{v}_m = 12\hat{i}$

but actual wind velocity remains unchanged.

$\vec{v}_{wm} = \vec{v}_w - \vec{v}_m = (6\hat{i} - 6\hat{j}) - 12\hat{i}$

$\Rightarrow \vec{v}_{wm} = -6\hat{i} - 6\hat{j}$

Now relative velocity of wind is $6\sqrt{2}$ m/s along S-W.



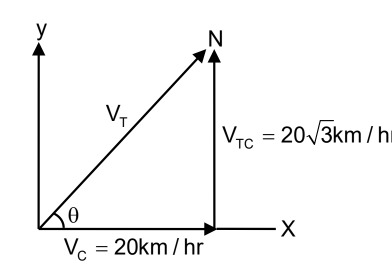
Example 41:

A man is going east in a car with velocity of 20 km/hr, a train appears to move towards north to him with a velocity of $20\sqrt{3}$ km/hr. What is the actual velocity and direction of motion of train ?

Solution:

$\vec{v}_{TC} = \vec{v}_T - \vec{v}_C \Rightarrow \vec{v}_{TC} + \vec{v}_C = 20\sqrt{3}\,\hat{j} + 20\hat{i}$

$|\vec{v}_T| = \sqrt{(20\sqrt{3})^2 + (20)^2} \;=\; \sqrt{1600} \;=$ 40 km/hr



$$\tan\theta = \frac{20\sqrt{3}}{20} \Rightarrow \theta = 60º$$

So direction of motion of train is 60° N of E or E - 60° – N

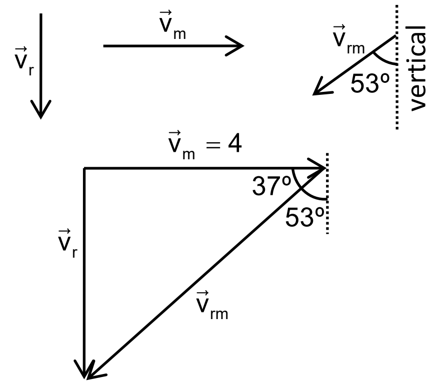
Example 42:

A man at rest observes the rain falling vertically. When he walks at 4 km/h, he has to hold his umbrella at an angle of 53° from the vertical. Find the velocity of raindrops.

Solution:

Assigning usual symbols $\vec{v}_m, \vec{v}_r$ and $\vec{v}_{r/m}$ to velocity of man, velocity of rain and velocity of rain relative to man, we can express their relationship by the following equation

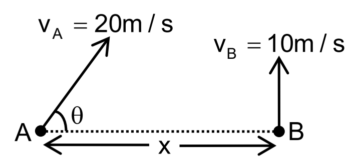
$\vec{v}_r = \vec{v}_m + \vec{v}_{r/m}$



The above equation suggests that a still man observes velocity $\vec{v}_r$ of rain, relative to the ground and while he is moving with velocity $\vec{v}_m$, he observes velocity of rain relative to himself $\vec{v}_{r/m}$. It is a common intuitive fact that umbrella must be held against $\vec{v}_{r/m}$ for optimum protection from rain. According to these facts, directions of the velocity vectors are shown in the adjoining figure.

Therefore $\quad v_r = v_m \tan 37° = 3$ km/h

Example 43:



Two particles A and B are projected from the ground simultaneously in the directions shown in the figure with initial velocities v_A = 20 m/s and v_B = 10 m/s respectively. They collide after 0.5 s. Find out the angle θ and the distance x.

Solution:

Both particle will collide if they are at same height in same time.

$$y_A = y_B \Rightarrow (u_y)_A t - \frac{1}{2}gt^2 = (u_y)_B t - \frac{1}{2}gt^2$$

$$\Rightarrow (u_y)_A = (u_y)_B \Rightarrow (v_A \sin\theta) = v_B$$

$$\Rightarrow 20 \sin\theta = 10 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

In 0.5s horizontal distance covered by A is $x = (u_x)_A\, t = (20 \cos 30°)0.5 = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$m

Concept Builder-8

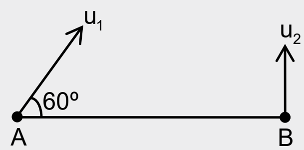
Q.1 A man 'A' moves in the north direction with a speed 10 m/s and another man B moves in E-30°-N with 10 m/s. Find the relative velocity of B w.r.t. A.

Q.2 Rain is falling vertically with a speed of 30 ms^{-1}. A woman rides a bicycle with speed of 10 ms^{-1} in the north to south direction. What is the direction in which she should hold her umbrella ?

Q.3 A man standing in the rain hold a umbrella at an angle 30° from vertical. He throws the umbrella and starts running with 10 km/hr. He find that the rain drops are hitting his head vertically. Find the velocity of rain w.r.t. running man ? Also calculate the actual speed of rain drops.

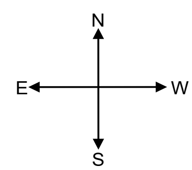
Q.4 A man is running up hill with velocity $(2\hat{i}+3\hat{j})$ m/s w.r.t. ground. He feels that the rain drops are falling vertically with velocity 4 m/s. If he runs down hill with same speed, find v_{rm}.

Q.5 A body is projected with velocity u_1 from point A as shown in figure. At the same time another body is projected vertically upwards with a velocity u_2 from point B. What should be value of $\frac{u_1}{u_2}$ for both bodies to collide.



Example 44:

Two bodies A and B are 10 km apart such that B is to the south of A. A and B start moving with the same speed 20 km/hr eastward and northward respectively then find.



(a) relative velocity of A w.r.t. B.

(b) minimum separation attained during motion.

(c) time elapsed from starting, to attain minimum separation.

Solution:

(a) $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 20\hat{i} - 20\hat{j}$

$\Rightarrow |\vec{v}_{AB}| = 20\sqrt{2}$

(b) minimum distance

$$= \frac{v_2 \times d}{\sqrt{v_1^2 + v_2^2}} = \frac{20 \times 10}{20\sqrt{2}} = 5\sqrt{2}$$

(c) time to attain minimum separation

$$= \frac{5\sqrt{2}}{20\sqrt{2}} = \frac{1}{4} \times 60 = 15 \text{ min}$$

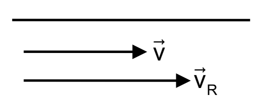
15. River-Boat (or man) Problem

A man can swim with velocity $\vec{v}$, i.e. it is the velocity of man w.r.t. still water.

If water is also flowing with the velocity $\vec{v}_R$ then velocity of man relative to ground $\vec{v}_m = \vec{v} + \vec{v}_R$

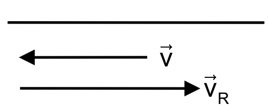
(i) If the swimming is in the direction of flow of water or along the downstream then

$v_m = v + v_R$



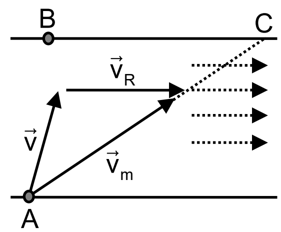
(ii) If the swimming is in the direction opposite to the flow of water or along the upstream then

$v_m = v - v_R$



(iii) If the man is crossing the river i.e. $\vec{v}$ and $\vec{v}_R$ are non colinear then use vector algebra.

$$\vec{v}_m = \vec{v} + \vec{v}_R$$



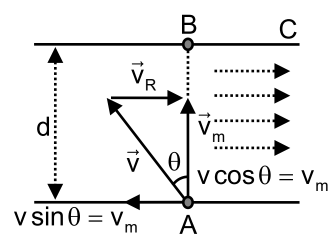
15.1 Crossing The River

To Cross a River

- Minimum distance of approach
  OR
  Cross the river along shortest possible path
  OR
  Cross the river and reach a point just opposite to the starting path
- Minimum time of approach
  OR
  Cross the river in shortest possible time

- **For shortest path**

  If a man wants to cross the river such that his "displacement should be minimum", it means he intends to reach just opposite point across the river. He should start swimming at an angle θ with the perpendicular to the flow of river towards upstream.



  Such that its resultant velocity $\vec{v}_M = (\vec{v} + \vec{v}_R)$.

  It is in the direction of displacement AB.

  To reach at B $\quad v\sin\theta = v_R \Rightarrow \sin\theta = \dfrac{v_R}{v}$

  component of velocity of man along AB is $v\cos\theta$

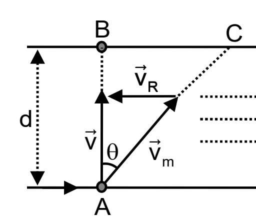
  so time taken $t = \dfrac{d}{v\cos\theta} = \dfrac{d}{\sqrt{v^2 - v_R^2}}$

- **For minimum time**

  To cross the river in minimum time, the velocity along AB ($v\cos\theta$) should be maximum.

  It is possible if $\theta = 0$, i.e. swimming should start perpendicular to water current.

  Due to effect of river velocity, man will reach at point C along resultant velocity, i.e. his displacement will not be minimum but time taken to cross the river will be minimum.



  $$t_{min} = \frac{d}{v}$$

  In time t_{min} swimmer travels distance BC along the river with speed of river v_R.

  distance travelled along river flow (drift of man)

  $= BC = t_{min}\, v_R = \dfrac{d}{v} v_R$

Example 45:

A boat moves along the flow of river between two fixed points A and B. It takes t_1 time when going downstream and takes t_2 time when going upstream between these two points. What time it will take in still water to cover the distance equal to AB.

Solution:

$$t_1 = \frac{AB}{v_b + v_R},\ t_2 = \frac{AB}{v_b - v_R} \quad \text{or} \quad v_b + v_R = \frac{AB}{t_1} \text{ and } v_b - v_R = \frac{AB}{t_2}$$

$$\Rightarrow \quad 2v_b = \frac{AB}{t_1} + \frac{AB}{t_2} = AB\left(\frac{t_1 + t_2}{t_1 t_2}\right) \quad \text{or} \quad \left(\frac{2t_1 t_2}{t_1 + t_2}\right) = \frac{AB}{v_b} \text{ time taken by the boat to cover AB}$$

Example 46:

A boat can be rowed at 5 m/s in still water. It is used to cross a 200 m wide river from south bank to the north bank. The river current has uniform velocity of 3 m/s due east.

(a) In which direction must it be steered to cross the river perpendicular to current ?

(b) How long will it take to cross the river in a direction perpendicular to the river flow ?

(c) In which direction must the boat be steered to cross the river in minimum time ? How far will it drift?

Solution:

(a) To cross the river perpendicular to current i.e. along shortest path

$$\sin\theta = \frac{u}{v} = \frac{3}{5} \Rightarrow \theta = 37º$$

(b) Time taken by boat,

$$t = \frac{d}{v\cos\theta} = \frac{200}{5 \times \frac{4}{5}} = 50s$$

(c) To cross the river in minimum time, $\theta = 0°$

Therefore $t_{min} = \frac{d}{v} = \frac{200}{5} s = 40s$

Drift = $u(t_{min})$ = 3(40)m = 120 m

Concept Builder-9

Q.1 A man can swim at a speed 2 ms^{-1} in still water. He starts swimming in a river at angle 150° to the direction of water flow and reaches the directly opposite point on the opposite bank.

(a) Find the speed of flowing water.

(b) If width of river is 1 km then calculate the time taken to cross the river.

Q.2 2 km wide river flowing at the rate of 5 km/hr. A man can swim in still water with 10 km/hr. He wants to cross the river along the shortest path. Find -

(a) in which direction should the person swim.

(b) crossing time.

Q.3 A person climbs up on a moving escalator in 60 sec & comes down on the same escalator in 150 seconds. How much time, he will take to walk up on stalled escalator.

ANSWER KEY FOR CONCEPT BUILDERS

CONCEPT BUILDER-1

1. 19 m, 13 m

2. (A) 110 m, (B) 50 m

3. 125.60 m, 80 m

4. Displacement = 2R
For 'B', Displacement = Actual length of path taken.

CONCEPT BUILDER-2

1. 4 m/s.

2. (i) 40 km/hr. (ii) 1040 km/hr
(iii) 32.5 km/hr (iv) 1040 km/hr

3. (A) 12.5 m/s, (B) 25 m/s

4. 2 m/s

5. (a) 14 m/s^2 (b) 14 m/s^2

6. $V^2 = 2s^2 + 2s$

CONCEPT BUILDER-3

1. $\sqrt{\frac{u^2+v^2}{2}}$ **2.** $\frac{49}{16}$ m/s^2, $\frac{80}{7}$ s

3. $a = \frac{240}{11}$ cm/s^2

4. $s = ut + \frac{u^2}{2a}$ **5.** $s_2 = 8\ s_1$

6. 1/3

7. (i) 120 m (ii) 0
(iii) 20 m/s (iv) 0 m/s

8. (a) A,B (b) A,B (c) B,A
(d) same (e) B over takes A once

9. (i) 37 m (ii) 3.7 m/s
(iii) 8 m/s^2 B to C (iv) 2 m/s^2

10. 625 m

CONCEPT BUILDER-4

1. (i) u = 40 m/s (ii) h_{max} = 80 m
(iii) s = 60 m from the hand

2. $v = 10\sqrt{14}$ m/s

3. v = 20 m/s

4. 1: 3: 5

CONCEPT BUILDER-5

1. (a) 1 sec (b) 5 m
(c) $20\sqrt{3}$m (d) 2 sec

2. 50 m

3. (a) $\tan^2\alpha$ (b) 1: 1

4. 3 km

5. $\theta = 37°$ **6.** $\frac{12}{5}$ m, $\frac{4}{5}$ m, $\frac{4}{5}$ s

CONCEPT BUILDER-6

1. (i) 10 sec (ii) 980 m.
(iii) $\theta = \pi/4$

2. v = 60 m/s **3.** $|\vec{v}| = 500\sqrt{2}$ m/s.

4. h = 10 m

5. (i) t = 2 sec = Time of flight
(ii) 80 m (iii) $|\vec{v}| = 20\sqrt{13}$ m/s

CONCEPT BUILDER-7

1. $v_A = \frac{15}{4}$ m/s $v_B = 5 - v_A = \frac{5}{4}$ m/s

2. 1250 m **3.** v = 20 m/s

4. Height from ground where both stones meet each other = 113.75 m
total time = 2 + 1.5 = 3.5 sec

CONCEPT BUILDER-8

1. $5\sqrt{3}\hat{i} - 5\hat{j}$, E – 30° – S

2. $\alpha = \tan^{-1}\left(\frac{1}{3}\right)$ = 18.4° towards south from vertical

3. $10\sqrt{3}$ km/h, 20 km/h]

4. $\sqrt{20}$ or $(-4\hat{i} - 2\hat{j})$ **5.** $\frac{2}{\sqrt{3}}$

CONCEPT BUILDER-9

1. (a) $v_R = \sqrt{3}$ m/s (b) t = 1000 sec

2. (i) Angle made by velocity of man to the river flow = $\frac{2\pi}{3}$
(ii) crossing time (t) = $\frac{2}{5\sqrt{3}}$ hrs.

3. $t = \frac{600}{7}$ seconds

Exercise - I

Distance and Displacement

1. An athlete completes one round of a circular track of radius R in 40 sec. What will be his displacement at the end of 2 min. 20 sec.

(1) Zero (2) 2R
(3) $2\pi R$ (4) $7\pi R$

2. Position of a particle moving along x–axis is given by $x = 2 + 8t - 4t^2$. The distance travelled by the particle from t = 0 to t = 2 is:–

(1) 0 (2) 8
(3) 12 (4) 16

3. A body moves 6 m north, 8 m east and 10 m vertically upwards, what is its resultant displacement from initial position:

(1) $10\sqrt{2}$ m (2) 10 m
(3) $10/\sqrt{2}$ m (4) 10 × 2 m

4. A person moves 30 m north and then 20 m towards east and finally $30\sqrt{2}$ m in south-west direction. The displacement of the person from the origin will be:

(1) 10 m along north (2) 10 m along south
(3) 10 m along west (4) Zero

Average Speed, Average Velocity and Acceleration

5. A person travels along a straight road for half the distance with velocity 10 m/s and the remaining half distance with velocity 20 m/s. The average velocity is given by:

(1) 15 m/s (2) $\frac{20}{3}$ m/s
(3) $\frac{80}{3}$ m/s (4) $\frac{40}{3}$ m/s

6. A car moves for half of its time at 80 km/h and for rest half of time at 40 km/h. Total distance covered is 60 km. What is the average speed of the car:

(1) 60 km / h (2) 80 km / h
(3) 120 km / h (4) 180 km / h

7. The ratio of the numerical values of the average velocity and average speed of a body is always:

(1) Unity
(2) Unity or less
(3) Unity or more
(4) Less than unity

8. If a car covers $2/5^{th}$ of the total distance with υ_1 speed and $3/5^{th}$ distance with υ_2 then average speed is:

(1) $\frac{1}{2}\sqrt{v_1v_2}$ (2) $\frac{v_1+v_2}{2}$
(3) $\frac{2v_1v_2}{v_1+v_2}$ (4) $\frac{5v_1v_2}{3v_1+2v_2}$

9. A car travels from A to B at a speed of 20 km/hr and returns at a speed of 30 km/hr. The average speed of the car for whole journey is:

(1) 25 km/hr (2) 24 km/hr
(3) 50 km/hr (4) 5 km/hr

10. A particle is constrained to move on a straight line path. It returns to the starting point after 10 sec. The total distance covered by the particle during this time is 30 m. Which of the following statements about the motion of the particle is false:

(1) Displacement of the particle is zero
(2) Average speed of the particle is 3 m/s
(3) Displacement of the particle is 30 m
(4) Both (1) and (2)

11. A particle moves along a semicircle of radius 10m in 5 seconds. The average velocity of the particle is:

(1) $2\pi\ ms^{-1}$ (2) $4\pi\ ms^{-1}$
(3) $2\ ms^{-1}$ (4)$4ms^{-1}$

Application of Calculus

12. An electron starting from rest has a velocity that increases linearly with the time that is $v = kt$, where $k = 2m / sec^2$. The distance travelled in the first 3 seconds will be:
(1) 9 m (2) 16 m
(3) 27 m (4) 36 m

13. The velocity of a body depends on time according to the equation $v = 20 + 0.1\ t^2$. The body is undergoing:
(1) Uniform acceleration
(2) Uniform retardation
(3) Non–uniform acceleration
(4) Zero acceleration

14. The acceleration 'a' in m/s^2 of a particle is given by $a = 3t^2 + 2t + 2$ where t is the time. If the particle starts out with a velocity $u = 2m /s$ at $t = 0$, then the velocity at the end of 2 second is:
(1) 12 m/s (2) 18 m/s
(3) 27 m/s (4)36 m/s

15. Starting from rest, acceleration of a particle is $a = 2(t - 1)$. The velocity of the particle at $t = 5s$ is:
(1) 15 m/sec (2) 25 m/sec
(3) 5 m/sec (4) None of these

16. A particle moves along a straight line such that its displacement at any time t is given by $S = t^3 - 6t^2 + 3t + 4$ metres. The velocity when the acceleration is zero is:
(1) $3\ ms^{-1}$ (2) $-12\ ms^{-1}$
(3) $42\ ms^{-1}$ (4) $-9\ ms^{-1}$

17. The displacement of a particle starting from rest (at $t = 0$) is given by $s = 6t^2 - t^3$. The time in seconds at which the particle will attain zero velocity again, is:
(1) 2 (2) 4
(3) 6 (4) 8

Equation of Motion

18. A particle experiences a constant acceleration for 20 sec after starting from rest. If it travels a distance S_1 in the first 10 sec and a distance S_2 in the next 10 sec, then:
(1) $S_1 = S_2$ (2) $S_1 = S_2/3$
(3) $S_1 = S_2/2$ (4) $S_1 = S_2/4$

19. The initial velocity of the particle is 10 m/sec and its retardation is $2m/sec^2$. The distance moved by the particle in 5th second of its motion is:
(1) 1 m (2) 19 m
(3) 50 m (4) 75 m

20. A body starts from rest. What is the ratio of the distance travelled by the body during the 4^{th} and 3^{rd} second:
(1) 7/5 (2) 5/7
(3) 7/3 (4) 3/7

21. A car moving with a speed of 50 km/hr, can be stopped by brakes after at least 6m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is:
(1) 6 m (2) 12 m
(3) 18 m (4) 24 m

22. An object accelerates from rest to a velocity 27.5 m/s in 10 sec then find distance covered by object in next 10 sec:
(1) 550 m (2) 137.5 m
(3) 412.5 m (4) 275 m

23. A motor car moving with a uniform speed of 20 m/sec comes to stop on the application of brakes after travelling a distance of 10 m. Its acceleration is:
(1) $20\ m / sec^2$ (2) $-20\ m / sec^2$
(3) $-40\ m / sec^2$ (4) $+ 2m / sec^2$

24. The velocity of a body moving with a uniform acceleration of $2m / sec^2$ is 10 m/sec. Its velocity after an interval of 4 sec is:
(1) 12 m / sec (2) 14 m / sec
(3) 16 m / sec (4) 18 m / sec

25. A body of mass 10 kg is moving with a constant velocity of 10 m/s. When a constant force acts for 4 seconds on it, it moves with a velocity 2 m/sec in the opposite direction. The acceleration produced in it is:

(1) 3 m/sec^2 (2) –3 m/sec^2
(3) 0.3 m/sec^2 (4) –0.3 m/sec^2

26. A car moving with a velocity of 10 m/s can be stopped by the application of a constant force F in a distance of 20 m. If the velocity of the car is 30 m/s, it can be stopped by this force in:

(1) $\frac{20}{3}$ m (2) 20 m
(3) 60 m (4)180m

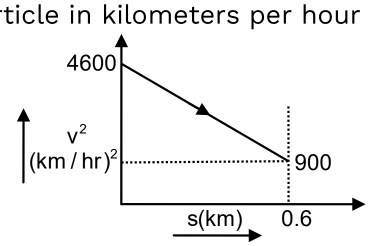
27. If a train travelling at 72 kmph is to be brought to rest in a distance of 200 metres, then its retardation should be:

(1) 20 ms^{-2} (2) 10 ms^{-2}
(3) 2 ms^{-2} (4)1ms^{-2}

28. Speed of two identical cars are u and 4u at a specific instant. The ratio of the respective distances in which the two cars are stopped from that instant is:

(1) 1: 1 (2) 1: 4
(3) 1: 8 (4)1: 16

29. A graph between the square of the velocity of a particle and the distance (s) moved is shown in figure. The acceleration of the particle in kilometers per hour square is:–



(1) 2250 (2) 3084
(3) – 2250 (4) – 3084

30. A car accelerates from rest at a constant rate of 2 m/s^2 for some time. Then, it retards at a constant rate of 4 m/s^2 and comes to rest. If it remains in motion for 3 second, then the maximum speed attained by the car is:–

(1) 2 m/s (2) 3 m/s
(3) 4 m/s (4) 6 m/s

Motion Under Gravity

31. A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of:

(1) 3 s (2) 5 s
(3) 7 s (4) 9 s

32. A stone thrown upward with a speed u from the top of the tower reaches the ground with a velocity 3u. The height of the tower is:

(1) $3u^2 / g$ (2) $4u^2 / g$
(3) $6u^2 / g$ (4) $9u^2/ g$

33. A man in a balloon rising vertically with an acceleration of 4.9 m/sec^2 releases a ball 2 sec after the balloon is let go from the ground. The greatest height above the ground reaches by the ball is (g = 9.8 m/sec^2)

(1) 14.7 m (2) 19.6 m
(3) 9.8 m (4)24.5 m

34. A rocket is fired upward from the earth's surface such that it creates an acceleration of 19.6 m/sec^2. If after 5 sec its engine is switched off, the maximum height of the rocket from earth's surface would be:

(1) 245 m (2) 490 m
(3) 980 m (4)735m

35. P, Q and R are three balloons ascending with velocities U, 4U and 8U respectively. If stones of the same mass be dropped from each, when they are at the same height, then:

(1) They reach the ground at the same time
(2) Stone from P reaches the ground first
(3) Stone from R reaches the ground first
(4) Stone from Q reaches the ground first

36. A body is projected up with a speed 'u' and the time taken by it is T to reach the maximum height H. Pick out the correct statement:

(1) It reaches H/2 in T/2 sec.
(2) It acquires velocity u/2 in T/2 sec
(3) Its velocity is u/2 at H/2
(4) Same velocity at 2T

37. Two balls A and B of same masses are thrown from the top of the building. A, thrown upward with velocity V and B, thrown downward with velocity V, then:
(1) Velocity of A is more than B at the ground
(2) Velocity of B is more than A at the ground
(3) Both A and B strike the ground with same velocity
(4) None of these

38. A body falling from a high Minaret travels 40 meters in the last 2 seconds of its fall to ground. Height of Minaret in meters is ($g = 10\ m/s^2$)
(1) 60 (2) 45
(3) 80 (4) 50

39. Two bodies of different masses m_a and m_b are dropped from two different heights a and b. The ratio of the time taken by the two to cover these distance are:
(1) a: b (2) b: a
(3) $\sqrt{a} : \sqrt{b}$ (4) $a^2 : b^2$

40. A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is:
(1) 4.9 m (2) 9.8 m
(3) 19.6 m (4) 24.5 m

41. A body is released from the top of a tower of height h. It takes t sec to reach the ground. Where will be the ball after time t/2 sec:
(1) At h/2 from the ground
(2) At h/4 from the ground
(3) Depends upon mass and volume of the body
(4) At 3h/4 from the ground

42. A body thrown with an initial speed of 96 ft/sec reaches the ground after (g = 32 ft/sec)
(1) 3 sec (2) 6 sec
(3) 12 sec (4) 8 sec

43. A stone is dropped from a certain height which can reach the ground in 5 second. If the stone is stopped after 3 second of its fall and then allowed to fall again, then the time taken by the stone to reach the ground for the remaining distance is:
(1) 2 sec (2) 3 sec
(3) 4 sec (4) None of these

44. A particle is dropped under gravity from rest from a height h ($g = 9.8\ m/s^2$) and it travels a distance 9h/25 in the last second, the height h is:
(1) 100 m (2) 122.5 m
(3) 145 m (4) 167.5 m

45. A balloon is at a height of 81 m and is ascending upwards with a velocity of 12 m/s. A body of 2 kg weight is dropped from it. If $g = 10\ m/s^2$, the body will reach the surface of the earth in:
(1) 1.5 s (2) 4.025 s
(3) 5.4 s (4) 6.75 s

46. A ball is thrown vertically upwards from the top of a tower at $4.9\ ms^{-1}$. It strikes the pond near the base of the tower after 3 seconds. The height of the tower is:
(1) 73.5 m (2) 44.1 m
(3) 29.4 m (4) None of these

47. A body starts to fall freely under gravity. The distance covered by it in first, second and third second are in ratio:
(1) 1: 3: 5 (2) 1: 2: 3
(3) 1: 4: 9 (4) 1: 5: 6

48. A stone is shot straight upward with a speed of 20 m/sec from a tower 200 m high. The speed with which it strikes the ground is approximately:
(1) 60 m/sec (2) 65 m/sec
(3) 70 m/sec (4) 75 m/sec

49. A body projected vertically upwards with a velocity u returns to the starting point in 4 seconds. If $g = 10\ m/sec^2$, the value of u is:
(1) 5 m/sec (2) 10 m/sec
(3) 15 m/sec (4) 20 m/sec

50. A body is thrown vertically up from the ground. It reaches a maximum height of 125m in 5 sec. After what time it will reach the ground from the maximum height position:

(1) 1.2 sec (2) 5 sec
(3) 10 sec (4)25 sec

51. A stone is dropped into a well in which the level of water is h below the top of the well. If v is velocity of sound, the time T after which the splash is heard is given by.

(1) $T = \frac{2h}{v}$ (2) $T = \sqrt{\frac{2h}{g}} + \frac{h}{v}$

(3) $T = \sqrt{\frac{2h}{v}} + \frac{h}{g}$ (4) $T = \sqrt{\frac{h}{2g}} + \frac{2h}{v}$

52. A body is falling from height 'h' it takes t_1 time to reach the ground. The time taken to cover the first half of height is:–

(1) $t_2 = \frac{t_1}{\sqrt{2}}$ (2) $t_1 = \frac{t_2}{\sqrt{2}}$

(3) $t_2 = \sqrt{3}\, t_1$ (4) None of these

53. Drops of water falls from the roof of a building 9 m. high at regular intervals of time. When the first drop reaches the ground, at the same instant fourth drop starts to fall. What are the distances of the second and third drops from the roof:–

(1) 6 m and 2 m (2) 6 m and 3 m
(3) 4 m and 1 m (4) 4 m and 2 m

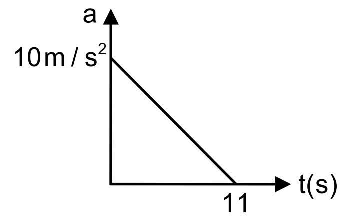
Graph Related Questions

54. The displacement–time graph for two particles A and B are straight lines inclined at angles of 30º and 60º with the time axis. The ratio of velocities of $V_A : V_B$ is:

(1) 1: 2 (2) 1: $\sqrt{3}$

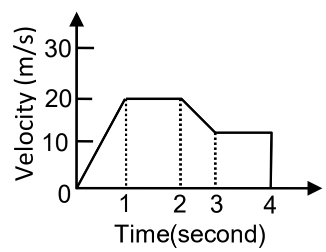
(3) $\sqrt{3}$: 1 (4) 1: 3

55. A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be:



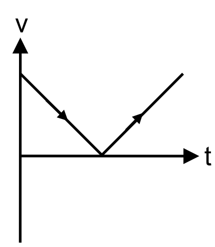
(1) 110 m/s (2) 55 m/s
(3) 550 m/s (4) 660 m/s

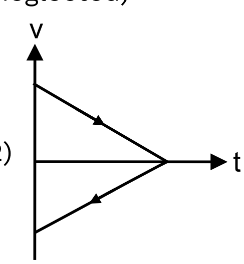
56. The variation of velocity of a particle with time moving along a straight line is illustrated in the following figure. The distance travelled by the particle in four seconds is:

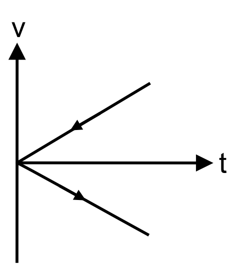


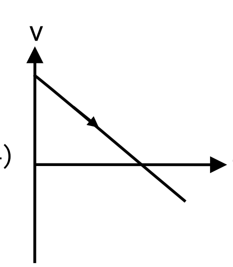
(1) 60 m (2) 55 m
(3) 25 m (4) 30 m

57. A ball is thrown vertically upwards. Which of the following graph(s) represent velocity–time graph of the ball during its flight (air resistance is neglected)

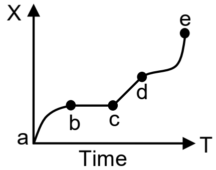
(1)


(2)


(3)


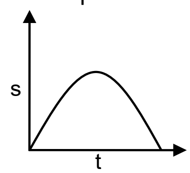
(4)


58. The displacement versus time graph for a body moving in a straight line is shown in figure. Which of the following regions represents the motion when no force is acting on the body:

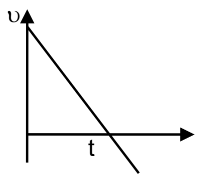


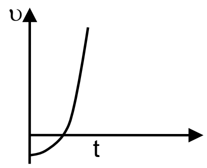
(1) ab (2) bc
(3) cd (4) de

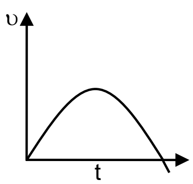
59. The graph of displacement υ/s time is,

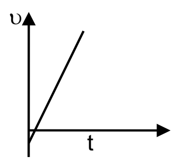


Its corresponding velocity–time graph will be:

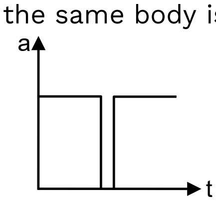
(1) 

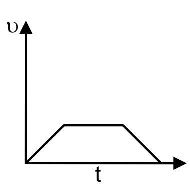
(2) 

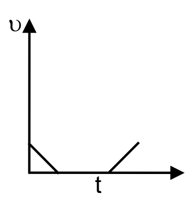
(3) 

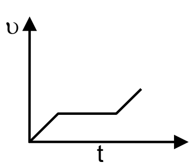
(4) 

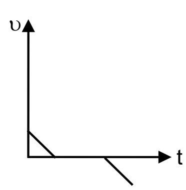
60. Acceleration–time graph of a body is shown. The corresponding velocity–time graph of the same body is:



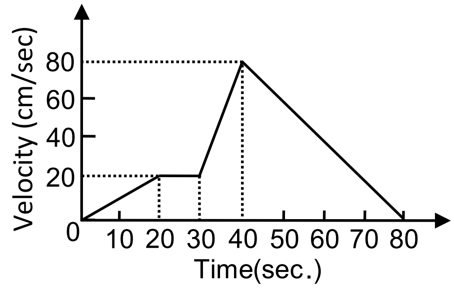
(1) 

(2) 

(3) 

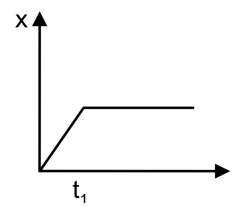
(4) 

61. The υ – t graph of a moving object is given in figure. The maximum acceleration is



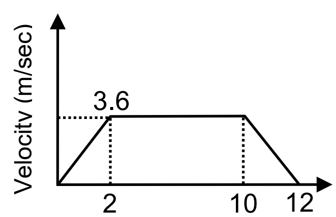
(1) 1cm/sec^2 (2) 2cm/sec^2
(3) 3cm/sec^2 (4) 6cm/sec^2

62. The x–t graph shown in figure represents



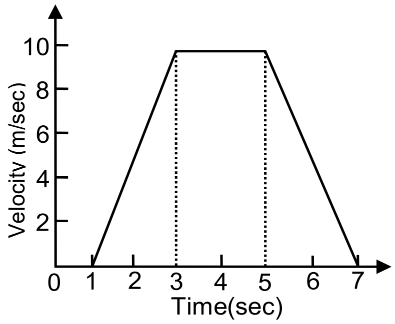
(1) Constant velocity
(2) Velocity of the body is continuously changing
(3) Instantaneous velocity
(4) The body travels with constant speed upto time t_1 and then stops

63. A lift is going up. The variation in the speed of the lift is as given in the graph. What is the height to which the lift takes the passengers:



(1) 3.6 m
(2) 28.8 m
(3) 36.0 m
(4) Cannot be calculated from the above graph

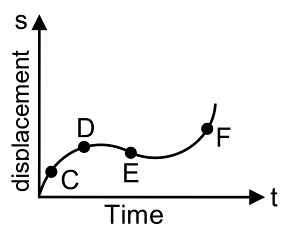
64. For the velocity–time graph shown in figure below the distance covered by the body in last two seconds of its motion is what fraction of the total distance covered by it in all the seven seconds.



(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $\frac{1}{3}$ (4) $\frac{2}{3}$

65. The displacement–time graph of moving particle is shown below,

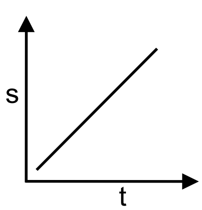


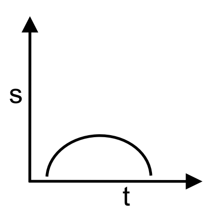
The instantaneous velocity of the particle is negative at the point:

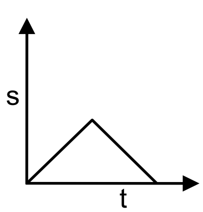
(1) D (2) F

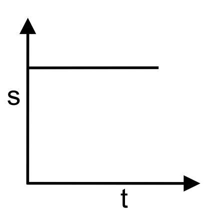
(3) C (4) E

66. Which of the following graph represents uniform motion:

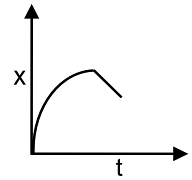
(1) 

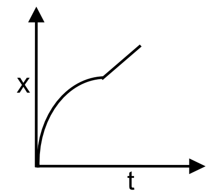
(2) 

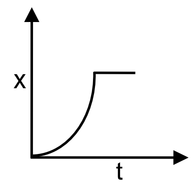
(3) 

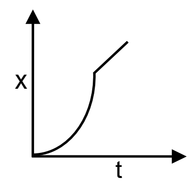
(4) 

67. A car starts from rest accelerates uniform by for 4 second and then moves with uniform velocity which of the x-t graph represent the motion of the car:–

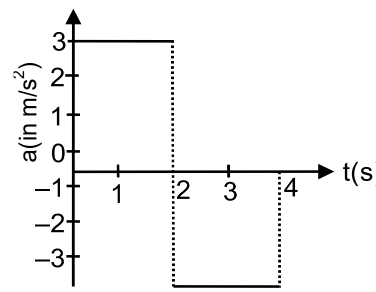
(1) 

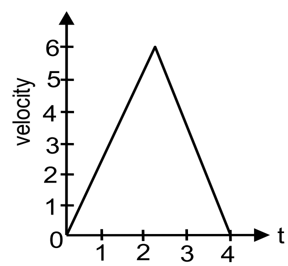
(2) 

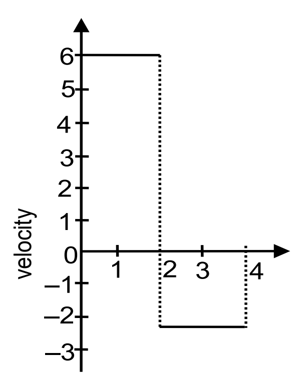
(3) 

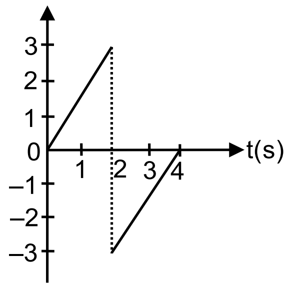
(4) 

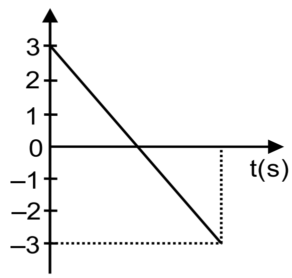
68. For motion of a particle acceleration time graph is shown in figure then the velocity time curve for the duration of 0-4 sec is:–



(1) 

(2) 

(3) 

(4) 

Paragraph (Question No. 69to 71)

69. The height y and the distance x along the horizontal plane of a projectile on a certain planet (with on surrounding atmosphere) are given by $y = (8t - 5t^2)$ meter and $x = 6t$ meter, where t is in second. The velocity with which the projectile is projected is:

(1) 8 m/s
(2) 6 m/s
(3) 10 m/s
(4) Not obtainable from the data

70. Referring to above question, the angle with the horizontal at which the projectile was projected is:

(1) $\tan^{-1}(3/4)$
(2) $\tan^{-1}(4/3)$
(3) $\sin^{-1}(3/4)$
(4) Not obtainable from the data

71. Referring to the above questions, the acceleration due to gravity is given by:

(1) 10 m/s^2 (2) 5 m/s^2
(3) 20 m/s^2 (4) 2.5 m/s^2

72. When a body is thrown with a velocity u making an angle θ with the horizontal plane, the maximum distance covered by it in horizontal direction is:

(1) $\frac{u^2 \sin\theta}{g}$ (2) $\frac{u^2 \sin\theta}{2g}$
(3) $\frac{u^2 \sin 2\theta}{g}$ (4) $\frac{u^2 \cos 2\theta}{g}$

73. If a projectile is fired at an angle θ with the vertical with velocity u, then maximum height attained is given by:–

(1) $\frac{u^2 \cos\theta}{2g}$ (2) $\frac{u^2 \sin^2\theta}{2g}$
(3) $\frac{u^2 \sin^2\theta}{g}$ (4) $\frac{u^2 \cos^2\theta}{2g}$

74. A ball is thrown at an angle θ with the horizontal and the range is maximum. The value of $\tan\theta$ is:

(1) 1 (2) $\sqrt{3}$
(3) $\frac{1}{\sqrt{3}}$ (4) 2

75. The range of a particle when launched at an angle of 15º with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle of 45º to the horizontal:

(1) 1.5 km (2) 3.0 km
(3) 6.0 km (4) 0.75 km

76. A cricketer can throw a ball to a maximum horizontal distance of 100 m. The speed with which he throws the ball is (to the nearest integer):

(1) 30 ms^{-1} (2) 42 ms^{-1}
(3) 32 ms^{-1} (4) 35 ms^{-1}

77. The horizontal range of a projectile is $4\sqrt{3}$ times its maximum height. Its angle of projectile will be:

(1) 45º (2) 60º
(3) 90º (4) 30º

78. Two bodies are thrown up at angles of 45º and 60º respectively, with the horizontal. If both bodies attain same vertical height, then the ratio of velocities with which these are thrown is:

(1) $\sqrt{\frac{2}{3}}$ (2) $\frac{2}{\sqrt{3}}$
(3) $\sqrt{\frac{3}{2}}$ (4) $\frac{\sqrt{3}}{2}$

79. A bullet is fired with a speed of 1000 m/sec in order to hit a target 100 m away. If $g = 10\ m/s^2$, the gun should be aimed:
(1) Directly towards the target
(2) 5 cm above the target
(3) 10 cm above the target
(4) 15 cm above the target

80. The range of a projectile when fired at 75° with the horizontal is 0.5 km. what will be its range when fired at 45° with same speed:–
(1) 0.5 km. (2) 1.0 km.
(3) 1.5 km. (4) 2.0 km.

81. The speed at the maximum height of a projectile is $\frac{\sqrt{3}}{2}$ times of its initial speed 'u' of projection. Its range on the horizontal plane:–
(1) $\frac{\sqrt{3}u^2}{2g}$ (2) $\frac{u^2}{2g}$
(3) $\frac{3u^2}{2g}$ (4) $\frac{3u^2}{g}$

82. If R is the maximum horizontal range of a particle, then the greatest height attained by it is:–
(1) R (2) 2R
(3) $\frac{R}{2}$ (4) $\frac{R}{4}$

83. A projectile is thrown into space so as to have the maximum possible horizontal range equal to 400m. Taking the point of projection as the origin, the coordinates of the point where the velocity of the projectile is minimum are:–
(1) (400, 100) (2) (200, 100)
(3) (400, 200) (4) (200, 200)

84. Two stones are projected with the same speed but making different angles with the horizontal. Their ranges are equal. If the angle of projection of one is $\frac{\pi}{3}$ and its maximum height is y_1 then the maximum height of the other will be:–
(1) $3y_1$ (2) $2y_1$
(3) $\frac{y_1}{2}$ (4) $\frac{y_1}{3}$

85. An arrow is shot into the air. Its range is 200 metres and its time of flight is 5 s. If the value of g is assumed to be $10\ ms^{-2}$, then the horizontal component of the velocity of arrow is:–
(1) 25 m/s (2) 40 m/s
(3) 31.25 m/s (4) 12.5 m/s

Projectile from a Tower

86. An aeroplane moving horizontally with a speed of 180 km/hr. drops a food packet while flying at a height of 490 m. The horizontal range of the packet is:–
(1) 180 m (2) 980 m
(3) 500 m (4) 670 m

87. A plane is flying horizontally at $98\ ms^{-1}$ and releases an object which reaches the ground in 10 s. The angle made by it while hitting the ground is:–
(1) 55° (2) 45°
(3) 60° (4) 75°

88. An aeroplane is moving with a velocity u. It drops a packet from a height h. The time t taken by the packet in reaching the ground will be:
(1) $\sqrt{\left(\frac{2g}{h}\right)}$ (2) $\sqrt{\left(\frac{2u}{h}\right)}$
(3) $\sqrt{\left(\frac{h}{2g}\right)}$ (4) $\sqrt{\left(\frac{2h}{g}\right)}$

89. An aeroplane is moving with horizontal velocity u at height h. The velocity of a packet dropped from it on the earth's surface will be (g is acceleration due to gravity)
(1) $\sqrt{u^2 + 2gh}$ (2) $\sqrt{2gh}$
(3) 2gh (4) $\sqrt{u^2 - 2gh}$

90. Particle is dropped from the height of 20 m on horizontal ground. There is wind blowing due to which horizontal acceleration of the particle becomes 6 ms^{-2}. Find the horizontal displacement of the particle till it reaches ground.

(1) 6m (2) 10 m
(3) 12m (4) 24 m

Equation of Trajectory

91. The equation of a projectile is $y = \sqrt{3}x - \frac{gx^2}{2}$ the angle of projection is:–

(1) 30° (2) 60°
(3) 45° (4) none

92. The equation of projectile is $y = 16x - \frac{x^2}{4}$ the horizontal range is:–

(1) 16 m (2) 8 m
(3) 64 m (4)12.8 m

Relative Motion in one Dimension

93. A jet air plane travelling at a speed of 500 km/h ejects its products of combustion at a speed of 1500km/h relative to the jet plane. The speed of the latter with respect to an observer on the ground is:

(1) 1500 km/h (2) 2000 km/h
(3) 1000 km/h (4) 500 km/h

94. A train of 150m length is going towards north direction at a speed of 10 ms^{-1}. A parrot flies at a speed of 5ms^{-1} towards south direction parallel to the railway track. The time taken by the parrot to cross the train is equal to:–

(1) 12 s (2) 8 s
(3) 15 s (4) 10 s

95. Two trains, each 50m long, are travelling in opposite directions with velocity 10 m/s and 15 m/s. The time of crossing is:–

(1) 2 s (2) 4 s
(3) $2\sqrt{3}$s (4) $4\sqrt{3}$s

96. A thief is running away on a straight road in a jeep moving with a speed of 9 m s^{-1}. A police man chases him on a motor cycle moving at a speed of 10 m s^{-1}. If the instantaneous separation of the jeep from the motorcycle is 100m, how long will it take for the police man to catch the thief ?

(1) 1s (2) 19s
(3) 90s (4) 100s

97. A man is 45 m behind the bus when the bus start accelerating from rest with acceleration 2.5 m/s^2. With what minimum velocity should the man start running to catch the bus?

(1) 12 m/s (2) 14 m/s
(3) 15 m/s (4) 16 m/s

Relative Motion in Two Dimension

98. A bird is flying with a speed of 40 km/hr in the north direction. A train is moving with a speed of 40 km/hr in the west direction. A passenger sitting in the train will see the bird moving with velocity:–

(1) 40 $\sqrt{2}$ km/hr in NE direction
(2) 40 $\sqrt{2}$ km/hr in NE direction
(3) 40 km/hr in NW direction
(4) 40 $\sqrt{2}$ km/hr in NW direction

99. A ship is travelling due east at 10 km/h. A ship heading 30° east of north is always due north from the first ship. The speed of the second ship in km/h is -

(1) 20 $\sqrt{2}$ (2) 20 $\sqrt{3/2}$
(3) 20 (4) 10

100. Two balls are thrown simultaneously, (A) vertically upwards with a speed of 20 m/s from the ground and (B) vertically downwards from a height of 40 m with the same speed and along the same line of motion. At which point the balls will collide:–
(take g = 10 m/sec^2)

(1) 15 m above from the ground
(2) 15 m below from the top of the tower
(3) 20 m above from the ground
(4) 20 m below from the top of the tower

Relative Motion in River Flow

101. A river is flowing from east to west at a speed of 5m/min. A man on south bank of river, capable of swimming 10 m/min in still water, wants to swim across the river in shortest time; he should swim:–

(1) due north
(2) due north-east
(3) due north-east with double the speed of river
(4) none of the above

102. A boat is sailing at a velocity $(3\hat{i}+4\hat{j})$ with respect to ground and water in river is flowing with a velocity $(-3\hat{i}-4\hat{j})$. Relative velocity of the boat with respect to water is:–

(1) $8\hat{j}$ (2) $5\sqrt{2}$
(3) $6\hat{i}+8\hat{j}$ (4) $-6\hat{i}-8\hat{j}$

103. A boat takes 2 hours to go 8 km and come back in still water lake. With water velocity of 4km/hr,the time taken for going upstream of 8 km and coming back is:–

(1) 140 min (2) 150 min
(3) 160 min (4) 170 min

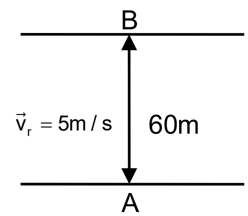
104. A boat, which has a speed of 5 km/h in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/h is -

(1) 1 (2) 3
(3) 4 (4) $\sqrt{41}$]

105. A man wishes to swim across a river 0.5 km wide. If he can swim at the rate of 2 km/h in still water and the river flows at the rate of 1 km/h. The angle (w.r.t. the flow of the river) along which he should swim so as to reach a point exactly opposite his starting point, should be:–

(1) 60° (2) 120°
(3) 145° (4) 90°

106. A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across at distance of 60 m in 5s. His velocity in still water should be:



(1) 12 m/s (2) 13 m/s
(3) 5 m/s (4) 0 m/s

107. A man is walking on a road with a velocity 3 km/hr. Suddenly rain starts falling. The velocity of rain is 10 km/hr in vertically downward direction. the relative velocity of the rain with respect to man is:–

(1) $\sqrt{13}$ km/hr (2) $\sqrt{7}$ km/hr
(3) $\sqrt{109}$ km/hr (4) 13 km/hr

108. It is raining vertically downwards with a velocity of 3 km h^{-1} . A man walks in the rain with a velocity of 4 kmh^{-1}. The rain drops will fall on the man with a relative velocity of ;

(1) 1 kmh^{-1} (2) 3 kmh^{-1}
(3) 4 kmh^{-1} (4) 5 kmh^{-1}

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	2	2	1	3	4	1	2	4	2	3	4	1	3	2	1	4	2	2	1	1	4	3	2	4	2
Que.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
Ans.	4	4	4	4	3	2	2	1	4	2	2	3	2	3	4	4	2	3	2	3	3	1	2	4	2
Que.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	2	1	3	4	2	2	4	3	1	3	4	4	3	2	4	1	4	1	3	2	1	3	4	1	2
Que.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	3	4	3	2	2	1	4	2	4	2	3	2	4	1	3	2	3	3	4	2	4	3	2	3	1
Que.	101	102	103	104	105	106	107	108																	
Ans.	1	3	3	2	2	2	3	4																	

Exercise - II

1. The engine of a train passes an electric pole with a velocity 'u' and the velocity with which the mid-point of the train passes the pole is '2u' then the last compartment of the train crosses the same pole with a velocity:-

(1) 3 u (2) $\frac{3u}{2}$

(3) $\sqrt{7}u$ (4) $\sqrt{\frac{3}{2}}\,u$

2. A person walks up a stalled escalator in 90 sec. When standing on the same escalator now moving, he is carried in 60s. The time he would take to walk up the moving escalator will be:–

(1) 27 s (2) 72 s (3) 18 s (4) 36 s

3. A body is projected vertically up at t = 0 with a velocity of 98 m/s. Another body is projected from the same point with same velocity after time 4 seconds. Both bodies will meet after:–

(1) 6 s (2) 8 s

(3) 10 s (4) 12 s

4. A boy is running on the plane road with velocity (v) with a long hollow tube in his hand. The water is falling vertically downwards with velocity (u). At what angle to the vertical, he must incline the tube so that the water drops enters in it without touching its side:–

(1) $\tan^{-1}\left(\frac{v}{u}\right)$ (2) $\sin^{-1}\left(\frac{v}{u}\right)$

(3) $\tan^{-1}\left(\frac{u}{v}\right)$ (4) $\cos^{-1}\left(\frac{v}{u}\right)$

5. Virat and Rohit are running on a straight road. If at an instant Rohit has uniform speed 28 m/s and Virat starts from rest with acceleration of 4 m/s^2 from a point 80 m ahead of Rohit, if both are running in same direction then find out time when Virat overtakes Rohit again:-

(1) 4 sec

(2) 10 sec

(3) Both (1) and (2) are correct

(4) None of these

6. A particle moves along a straight line OX. At a time t (in seconds) the distance x (in metres) of the particle from O is given by $x = 40 + 12t - t^3$. How long would the particle travel before coming to rest:-

(1) 24 m (2) 16 m

(3) 56 m (4) 40 m

7. Two bodies, A (of mass 1 kg) and B (of mass 3 kg), are dropped from heights of 16 m and 25 m respectively. The ratio of the time taken by them to reach the ground is:-

(1) 5 / 4 (2) 12 / 5

(3) 5 / 12 (4) 4 / 5

8. For angles of projection of a projectile at angles (45° – θ) and (45° + θ), the horizontal ranges described by the projectile are in the ratio of:-

(1) 1: 1 (2) 2: 3

(3) 1: 2 (4) 2: 1

9. A car runs at a constant speed on a circular track of radius 100 m, taking 62.8 s for every circular lap. The average velocity and average speed for each circular lap respectively is:

(1) 0, 0 (2) 0, 10 m/s

(3) 10 m/s, 20 m/s (4) 20 m/s, 0

10. The position x of a particle with respect to time t along x-axis is given by $x= 9t^2-t^3$ where x is in metres and t in seconds. What will be the position of this particle when it achieves maximum speed along the + x direction ?

(1) 24 m (2) 32 m

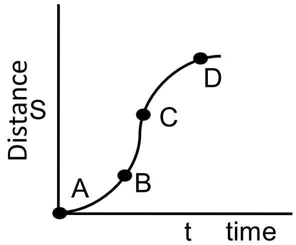
(3) 54 m (4) 81 m

11. A particle moves in a straight line with a constant acceleration. It changes its velocity from $10 ms^{-1}$ to $20\ ms^{-1}$ while passing through a distance 135 m in t second. The value of t is:-

(1) 12 (2) 9

(3) 10 (4) 1.8

12. A particle shows distance-time curve as given in this figure. The maximum instantaneous velocity of the particle is around the point:-



(1) D (2) A
(3) B (4) C

13. A bus is moving with a speed of 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus:-

(1) 10 ms^{-1} (2) 20 ms^{-1}
(3) 40 ms^{-1} (4) 25 ms^{-1}

14. The speed of a projectile at its maximum height is half of its initial speed. The angle of projection is:-

(1) 15° (2) 30°
(3) 45° (4) 60°

15. A ball is dropped from a high-rise platform at t = 0 starting from rest. After 6 seconds another ball is thrown downwards from the same platform with a speed v. The two balls meet at t =18s. What is the value of v ?
(take g = 10 m/s^2)

(1) 60 m/s (2) 75 m/s
(3) 55 m/s (4) 40 m/s

16. A particle covers half of its total distance with speed v_1 and the rest half distance with speed v_2. Its average speed during the complete journey is:

(1) $\frac{v_1v_2}{v_1+v_2}$ (2) $\frac{2v_1v_2}{v_1+v_2}$

(3) $\frac{v_1^2v_2^2}{v_1^2+v_2^2}$ (4) $\frac{v_1v_2}{2}$

17. A car moves from X to Y with a uniform speed v_u and returns to X with a uniform speed v_d. The average speed for this round trip is:

(1) $\sqrt{v_uv_d}$ (2) $\frac{v_uv_d}{v_d+v_u}$

(3) $\frac{v_u+v_d}{2}$ (4) $\frac{2v_uv_d}{v_d+v_u}$

18. The distance travelled by a particle starting from rest and moving with an acceleration 4/3 ms^{-2}, in the third second is:-

(1) $\frac{10}{3}$m (2) $\frac{19}{3}$m
(3) 6 m (4) 4 m

ANSWER KEY																		
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
Ans.	3	4	4	1	2	2	4	1	2	3	2	4	2	4	2	2	4	1

Exercise – III (Previous Year Question)

1. A body is moving with velocity 30 m/s towards east. After 10 seconds its velocity becomes 40 m/s towards north. The average acceleration of the body is:- **[AIPMT-2011]**

(1) 1 m/s^2 (2) 7 m/s^2
(3) $\sqrt{7}$ m/s^2 (4) 5 m/s^2

2. A boy standing at the top of a tower of 20 m height drops a stone. Assuming g = 10 ms^{-2}, the velocity with which it hits the ground is:- **[AIPMT-2011]**

(1) 10.0 m/s (2) 20.0 m/s
(3) 40.0 m/s (4) 5.0 m/s

3. A missile is fired for maximum range with an initial velocity of 20 m/s. If g = 10 m/s^2, the range of the missile is:- **[AIPMT 2011]**

(1) 40 m (2) 50 m
(3) 60 m (4) 20 m

4. A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection, is: **[AIPMT 2011]**

(1) 60° (2) $\tan^{-1}\frac{1}{2}$
(3) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (4) 45°

5. The motion of a particle along a straight line is described by equation x = 8 + 12t − t^3 where x is in metre and t in second. The retardation of the particle when its velocity becomes zero is:- **[AIPMT PRE.-2012]**

(1) 6 ms^{-2} (2) 12 ms^{-2}
(3) 24 ms^{-2} (4) zero

6. A particle has initial velocity $\left(2\hat{i}+3\hat{j}\right)$ and acceleration $\left(0.3\hat{i}+0.2\hat{j}\right)$. The magnitude of velocity after 10 seconds will be: **[AIPMT -2012]**

(1) 5 units (2) 9 units
(3) 9$\sqrt{2}$ units (4) 5$\sqrt{2}$ units

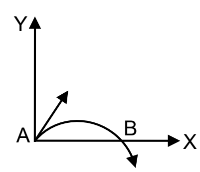
7. The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is:- **[AIPMT -2012]**

(1) $\theta = \tan^{-1}(2)$ (2) θ = 45°
(3) $\theta = \tan^{-1}\left(\frac{1}{4}\right)$ (4) $\theta = \tan^{-1}(4)$

8. A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 seconds, the next 5 seconds and the next 5 seconds respectively. the relation between h_1, h_2 and h_3 is: **[AIPMT 2013]**

(1) $h_1 = h_2 = h_3$ (2) $h_1 = 2h_2 = 3h_3$
(3) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$ (4) $h_2 = 3h_1 = h_3 = 3h_2$

9. The velocity of a projectile at the initial point A is $\left(2\hat{i}+3\hat{j}\right)$m/s. It's velocity (in m/s) at point B is: **[AIPMT 2013]**



(1) $2\hat{i}+3\hat{j}$ (2) $-2\hat{i}-3\hat{j}$
(3) $-2\hat{i}+3\hat{j}$ (4) $2\hat{i}-3\hat{j}$

10. A particle is moving such that its position coordinate (x, y) are
(2m, 3m) at time t = 0
(6m, 7m) at time t = 2 s and
(13m, 14m) at time t = 5s.
Average velocity vector ($\vec{v}$) from t = 0 to t =5s is: **[AIPMT 2014]**

(1) $\frac{1}{5}\left(13\hat{i}+14\hat{j}\right)$ (2) $\frac{7}{3}\left(\hat{i}+\hat{j}\right)$
(3) $2\left(\hat{i}+\hat{j}\right)$ (4) $\frac{11}{5}\left(\hat{i}+\hat{j}\right)$

11. A projectile is fired from the surface of the earth with a velocity of $5 ms^{-1}$ and angle θ with the horizontal. Another projectile fired from another planet with a velocity of $3 ms^{-1}$ at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) is:
(given g = 9.8 m/s^2) **[AIPMT 2014]**

(1) 3.5 (2) 5.9
(3) 16.3 (4) 110.8

12. A ship A is moving Westwards with a speed of 10 km h^{-1} and a ship B, 100 km South of A, is moving Northwards with a speed of 10 km h^{-1}. The time after which the distance between them becomes shortest, is: **[AIPMT 2015]**

(1) 5 h (2) $5\sqrt{2}$ h
(3) $10\sqrt{2}$ h (4) 0 h

13. A particle of unit mass undergoes one dimensional motion such that its velocity varies according to $v(x) = \beta x^{-2n}$ where β and n are constants and x is the position of the particle. The acceleration of the particle as a function of x, is given by: **[AIPMT-2015]**

(1) $-2n\beta^2 x^{-4n-1}$ (2) $-2\beta^2 x^{-2n+1}$
(3) $-2n\beta^2 e^{-4n+1}$ (4) $-2n\beta^2 x^{-2n-1}$

14. Two particles A and B, move with constant velocities $\vec{v}_1$ and $\vec{v}_2$. At the initial moment their position vectors are $\vec{r}_1$ and $\vec{r}_2$ respectively. The condition for particle A and B for their collision is:- **[ReAIPMT-2015]**

(1) $\vec{r}_1 - \vec{r}_2 = \vec{v}_1 - \vec{v}_2$ (2) $\dfrac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \dfrac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$

(3) $\vec{r}_1 . \vec{v}_1 = \vec{r}_2 . \vec{v}_2$ (4) $\vec{r}_1 \times \vec{v}_1 = \vec{r}_2 \times \vec{v}_2$

15. If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it between 1s and 2s is: **[NEET-I-2016]**

(1) $\frac{3}{2}A + 4B$ (2) 3A + 7B
(3) $\frac{3}{2}A + \frac{7}{3}B$ (4) $\frac{A}{2} + \frac{B}{3}$

16. Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $x_p(t) = at + bt^2$ and $x_Q(t) = ft - t^2$. At what time do the cars have the same velocity? **[NEET-II-2016]**

(1) $\dfrac{a+f}{2(1+b)}$ (2) $\dfrac{f-a}{2(1+b)}$
(3) $\dfrac{a-f}{1+b}$ (4) $\dfrac{a+f}{2(b-1)}$

17. Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be **[NEET-2017]**

(1) $\dfrac{t_1 t_2}{t_2 - t_1}$ (2) $\dfrac{t_1 t_2}{t_2 + t_1}$
(3) $t_1 - t_2$ (4) $\dfrac{t_1 + t_2}{2}$

18. The x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$ respectively, where x and y are in meters and t in seconds. The acceleration of the particle at t = 2s is:- **[NEET-2017]**

(1) 5 m/s^2 (2) – 4 m/s^2
(3) – 8 m/s^2 (4) 0

19. The speed of a swimmer in still water is 20 m/s. The speed of river water is 10 m/s and is flowing due east. If he is standing on the south bank and wishes to cross the river along the shortest path, the angle at which he should make his strokes w.r.t. north is given by: **[NEET-2019]**

(1) 30° west (2) 0°
(3) 60° west (4) 45° west

20. When an object is shot from the bottom of a long smooth inclined plane kept at an angle 60° with horizontal, it can travel a distance x_1 along the plane. But when the inclination is decreased to 30° and the same object the shot with the same velocity, it can travel x_2 distance. Then x_1: x_2 will be **[NEET-2019]**

(1) $1:\sqrt{2}$ (2) $\sqrt{2}:1$

(3) $1:\sqrt{3}$ (4) $1:2\sqrt{3}$

21. A person standing on the floor of an elevator drops a coin. The coin reaches the floor in time t_1 if the elevator is at rest and in time t_2 if the elevator is moving uniformly. Then:- **[NEET-2019(Odisha)]**

(1) $t_1 < t_2$ or $t_1 > t_2$ depending upon whether the lift is going up or down

(2) $t_1 < t_2$

(3) $t_1 > t_2$

(4) $t_1 = t_2$

22. Two bullets are fired horizontally and simultaneously towards each other from roof tops of two buildings 100 m apart and of same height of 200m with the same velocity of 25 m/s. When and where will the two bullets collide. (g =10 m/s^2)

[NEET-2019(Odisha)]

(1) after 2s at a height 180 m

(2) after 2s at a height of 20 m

(3) after 4s at a height of 120 m

(4) they will not collide

23. A person travelling in a straight line moves with a constant velocity v_1 for certain distance 'x' and with a constant velocity v_2 for next equal distance. The average velocity v is given by the relation

[NEET-2019(Odisha)]

(1) $\frac{1}{v}=\frac{1}{v_1}+\frac{1}{v_2}$ (2) $\frac{2}{v}=\frac{1}{v_1}+\frac{1}{v_2}$

(3) $\frac{v}{2}=\frac{v_1+v_2}{2}$ (4) $v=\sqrt{v_1v_2}$

24. A ball is thrown vertically downward with a velocity of 20m/s from the top of a tower, It hits the ground after some time with a velocity of 80m/s. The height of the tower is:

(g= 10m/s^2) **[NEET - 2020]**

(1) 320 m (2) 300 m

(3) 360 m (4) 340 m

25. A person sitting in the ground floor of a building notices through the window, of height 1.5 m, a ball dropped from the roof of the building crosses the window in 0.1 s. What is the velocity of the ball when it is at the topmost point of the window ?

(g = 10 m/s^2) **[NEET_Covid_2020]**

(1) 15.5 m/s (2) 14.5 m/s

(3) 4.5 m/s (4) 20 m/s

26. A small block slides down on a smooth inclined plane, starting from rest at time t = 0. Let S_n be the distance travelled by the block in the interval t = n – 1 to t = n.

Then, the ratio $\frac{S_n}{S_{n+1}}$ is: **[NEET - 2021]**

(1) $\frac{2n-1}{2n}$ (2) $\frac{2n-1}{2n+1}$

(3) $\frac{2n+1}{2n-1}$ (4) $\frac{2n}{2n-1}$

27. A car starts from rest and accelerates at 5 m/s^2. At t = 4 s, a ball is dropped out of a window by a person sitting in the car. What is the velocity and acceleration of the ball at t = 6 s ?(Take g = 10 m/s^2)

[NEET - 2021]

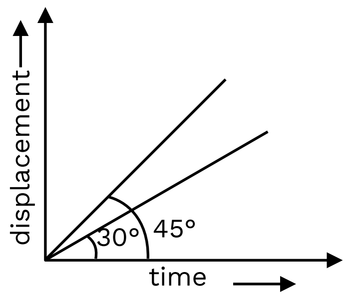
(1) 20 m/s, 5 m/s^2

(2) 20 m/s, 0

(3) $20\sqrt{2}$ m/s, 0

(4) $20\sqrt{2}$ m/s, 10m/s^2

28. The displacement-time graphs of two moving particles make angles of 30° and 45° with the x-axis as show int the figure. The ratio of their respective velocity is: **[NEET - 2022]**



(1) $\sqrt{3}:1$ (2) 1: 1

(3) 1: 2 (4) $1:\sqrt{3}$

29. The ratio of the distance travelled by a freely falling body in the 1^{st}, 2^{nd}, 3^{rd} and 4^{th} second: **[NEET - 2022]**

(1) 1: 2: 3: 4 (2) 1: 4: 9: 16

(3) 1: 3: 5: 7 (4) 1: 1: 1: 1

30. A ball is projected with a velocity, 10 ms^{-1}, at an angle of 60° with the vertical direction. Its speed at the highest point of its trajectory will be: **[NEET - 2022]**

(1) Zero (2) $5\sqrt{3}\ ms^{-1}$

(3) 5 ms^{-1} (4) 10 ms^{-1}

ANSWER KEY

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
Ans.	4	2	1	2	2	4	4	3	4	4	1	1	1	2	3	2	2	2	1	3	4	1	2	2	2
Que.	**26**	**27**	**28**	**29**	**30**																				
Ans.	2	4	4	3	2																				