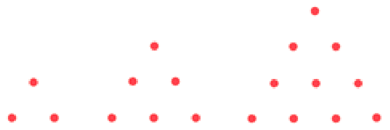
1. Arithmetic Sequences

Exercise Pg. 10

1. Question

Look at these triangles with dots. How many dots are there in each?



Compute the number of dots needed to make the next two triangles.

Answer

The number of dots in first 3 triangles are 3, 6 and 10 respectively.

The difference between the first two terms is 6-3 = 3

And 10-6 = 4

The series continues 3, 4, 5…

Hence, this forms an AP with first term as 3 and common difference as 1

Next two terms are 4 + 1 = 5

And 5 + 1 = 6

So 4^{th} pattern has 10 + 5 = 15 dots

5^{th} pattern has 15 + 6 = 21 dots

2. Question

Make the following number sequences from the sequence of equilateral triangles, squares, regular pentagons and so on, of regular polygons.

Number of sides – 3,4,5…

Sum of interior angles –

Sum of exterior angles –

One interior angle –

One exterior angle –

Answer

For the following sequences, let

a = First term of sequence

d = common difference if the sequence is an AP

a) Number of sides – 3,4,5,6,7…. And so on is an Arithmetic Progression.

Where a = 3 and d = 1

b) Sum of interior angles of a regular polygon with n sides = (n-2)×180

Putting n = 3, 4, 5, 6… and so on

The required sequence is 180,360,540,720… and so on.

An AP with a = 180 and d = 180

c) Sum of exterior angles of any closed polygon is 360

Hence the sequence is 360,360,360… and so on i.e. AP with a = 360 and d = 0

d) One Interior angle = (Sum of all interior angles)/(No. of sides)

$= \frac{(n-2)\times 180}{n}$

Putting n = 3, 4, 5… and so on…

The sequence is 60, 90, 108, 120… and so on. Here, the sequence is not an AP

e) One exterior angle = $\frac{\text{Sum of all exterior angles}}{\text{No.of sides}}$

(∵ no. of exterior angles = no. of sides)

$= \frac{360}{n}$

Putting n = 3, 4, 5… and so on

The sequence is 120, 90, 72, 60… and so on which is clearly not an AP.

3. Question

Write down the sequence of natural numbers leaving remainder 1 on division by 3 and the sequence of natural numbers leaving remainder 2 on division by 3.

Answer

Dividend = (Divisor × Quotient) + Remainder

Here, remainder = 1

Divisor = 3

Dividend = ?

On taking quotient as 1, 2, 3 and so on…

Dividend = 4, 7, 10… and so on which is an AP with

a = 4 and d = 3

Now, for 2nd case –

remainder = 2

Divisor = 3

Dividend = ?

On taking quotient as 1,2,3 and so on…

Dividend = 5,8,11… and so on which is an AP with

a = 5 and d = 3

4. Question

Write down the sequence of natural numbers ending in 1 or 6 and describe it in two other ways.

Answer

Sequence of natural numbers ending in 1 or 6 is,

1, 6, 11, 16, 21, 26, 31, 36, …

It can also be described as,

Arithmetic sequence with first term as 1 and common difference as 5

Or

Sequences of natural numbers, which leave 1 as the remainder when divided by 5.

5. Question

One cubic centimeter of iron weight 7.8 grams. Write as sequences, the volumes of weights of iron cubes of sides 1 centimetre, 2 centimetres and so on.

Answer

Volume of cube = side × side × side

For side = 1, Volume = 1 cc.

For side = 2, Volume = 8 cc.

For side = 3, Volume = 27 cc.

Hence, the sequence is 1, 8, 27… and so on which is clearly not an AP

Questions Pg-14

1. Question

Write the algebraic expression for each of the sequences below:

i) Sequence of odd numbers

ii) Sequence of natural numbers which leave remainder 1 on division by 3.

iii) The sequence of natural numbers ending in 1.

iv) The sequence of natural numbers ending in 1 or 6.

Answer

i) Sequence of odd numbers-

Odd numbers are those which are not divisible by 2

i.e. 1, 3, 5… and so on

i.e. (2-1),(4-1),(6-1)…and so on

Thus **Sn = 2n-1**, where n = 1, 2, 3… and so on

ii) Sequence of natural numbers which leave remainder 1 on division by 3. -

Dividend = (Divisor × Quotient) + (Remainder)

Here, Divisor = 3

Remainder = 1

For getting the required sequence, putting quotient as 1, 2, 3… and so on

Dividend = 4,7,10…

i.e. **Sn= 3n + 1**

iii) The sequence of natural numbers ending in 1.

Natural numbers ending in 1 are 1, 11, 21… and so on

These can be written as (10-9),(20-9),(30-9),… and so on

Hence, the sequence **Sn = 10n-9** where n = 1, 2, 3… and so on

iv) The sequence of natural numbers ending in 1 or 6.

The natural numbers ending in 1 or 6 are 1, 6, 11, 16, 21…and so on

Observing the pattern, these numbers can be written as (5-4), (10-4), (15-4),…and so on

Hence, the sequence **Sn = 5n-4**, where n = 1, 2, 3… and so on

2. Question

For the sequence of regular polygons starting with an equilateral triangle, write the algebraic expressions for the sequence of the sums of interior angles, the sums of the exterior angles, the measures of an interior angle, and the measures of an exterior angle.

Answer

a) Sum of interior angles of any n sided regular polygon = (n-2) × 180 (for n>2)

Hence, Sn = (n-2) × 180

b) Sum of exterior angles of any regular polygon is 360

Hence, Sn= 360 (independent of n)

c) Measure of an interior angle = (Sum of all interior angles) ÷ (No. of sides) (∵ no. of angles = no. of sides)

Sn = ((n-2) × 180) ÷ n

d) Measure of an exterior angle = (Sum of all exterior angles) ÷ (no. of sides)

= 360÷n

S_n = 360/n; where n = 1, 2, 3… and so on

3. Question

Look at these pictures –

The second picture is obtained by removing the small triangle formed by joining the midpoints of the first triangle. The third picture is got by removing such a middle triangle from each of the red triangles of the second picture.

i) How many red triangles are there in each picture?

ii) Taking the area of the first triangles as 1, compute the area of a small triangle in each picture.

iii) What is the total area of all the red triangles in each picture?

iv) Write the algebraic expressions for these three sequences obtained by continuing this

process.

Answer

i) How many red triangles are there in each picture?

→ In 1^{st} picture, 1 red triangle

In 2^{nd} picture, 3 red triangles

In 3^{rd} picture, 9 red triangles

S_1 = 1, 3, 9…

ii) Taking the area of the first triangles as 1, compute the area of a small triangle in each

picture.

→ In 2^{nd} picture, each triangle is divided into 4 parts in which there are 3 red and 1`white triangle.

Hence, area of each small triangle = (1÷ 4) = 0.25 sq. units

In 3^{rd} picture, each smaller triangle from the 2^{nd} figure, except the middle triangle is further divided into further 4 equal parts.

Hence, area of each small triangle = (0.25/4) = 0.625 sq. units

S_2 = 1, 0.25, 0.625…

iii) What is the total area of all the red triangles in each picture?

→ In 1^{st} triangle, only red triangle is present.

In 2^{nd} triangle, 3 red and 1 white triangle is present each of equal areas = 0.25 sq. units

∴ Area of red triangles = 3 × 0.25 = 0.75 sq. units

In 3^{rd} picture, each smaller triangle from the 2^{nd} figure, except the middle triangle is further divided into further 4 equal parts.

Area of all red triangles = 9 × 0.0625 = 0.5625

S_3 = 1, 0.75, 0.5625…

iv) Write the algebraic expressions for these three sequences obtained by continuing this process.

For 1^{st} question, the sequence is 1, 3, 9…

Hence, the expression is $\mathbf{3^{n-1}}$, where n = 1,2,3…and so on

For 2^{nd} question, the sequence is 1, 0.25, 0.625…

Hence, the expression is $\mathbf{(1 \div 4)^{(n-1)} = (0.25)^{n-1}}$ **,**where n = 1, 2, 3… and so on

For 3^{rd} question, the sequence is 1,0.75,0.675…

Here, the expression is product of above 2 expressions

i.e. $3^{n-1} \times 0.25^{n-1} = \mathbf{0.75^{n-1}}$

Questions Pg-17

1. Question

Check whether each of the sequences given below is an arithmetic sequence. Give reasons.

For the arithmetic sequences, write the common difference also.

i) Sequence of odd numbers

ii) Sequence of even numbers

iii) Sequence of fractions got as half the odd numbers

iv) Sequence of powers of 2

v) Sequence of reciprocals of natural numbers

Answer

i) Sequence of odd numbers

→ Odd numbers are those which are not divisible by 2

i.e. 1,3,5,7…

As, can be seen from the above sequence, it is an Arithmetic Progression.

Common difference = 3-1 = 5-3 = 2

ii) Sequence of even numbers

→ Even numbers are those which are divisible by 2

I.e. 2,4,6…

As, can be seen from the above sequence, it is an Arithmetic Progression.

Common difference = 4-2 = 6-4 = 2

iii) Sequence of fractions got as half the odd numbers

→ Odd numbers are those which are not divisible by 2

i.e. 1,3,5,7…

The sequence mentioned is half the odd numbers…

I.e. 1/2, 3/2, 5/2, 7/2,…

As, can be seen from the above sequence, it is an Arithmetic Progression.

Common difference = $\left(\frac{3}{2}\right) - \left(\frac{1}{2}\right) = \left(\frac{5}{2}\right) - \left(\frac{3}{2}\right) = 1$

iv) Sequence of powers of 2

→ The sequence of powers of 2 is as follows

$2^1, 2^2, 2^3$,… and so on

i.e. 2,4,8,…

Difference of second and first term = 4-2 =2

Difference of third and second term = 8-4 =4

As the difference is not same, it is not an AP.

v) Sequence of reciprocals of natural numbers

→ Natural numbers are those starting from 1 with common difference as 1

Sequence of reciprocals of natural numbers is as follows –

1/1, 1/2, 1/3 , 1/4 , 1/5… and so on

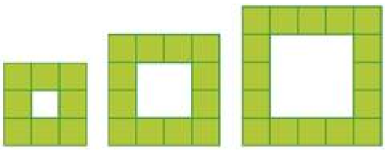
Difference of second and first term = (1/2)-(1/1) = -1/2

Difference of third and second term = (1/3) – (1/2) = -1/6

As the difference is not same, it is not an AP.

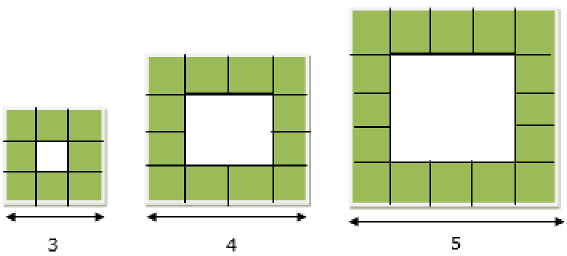
2. Question

Look at these pictures.



If the pattern is continued, do the numbers of coloured squares from an arithmetic sequence? Give reasons.

Answer



No. of coloured squares in

Figure 1 – 8

Figure 2 – 12

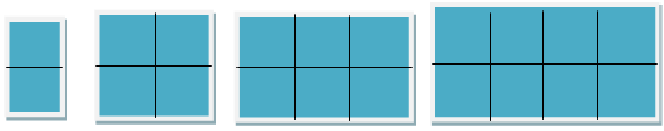
Figure 3 – 16

As can be seen from the figures, the side of each coloured square increases linearly from 3,4,5... and so on. Hence, it is an AP

With common difference = 12-8 = 16-12 = 4

3. Question

See this pictures below –



i) How many small squares are there in each rectangle?

ii) How many large squares?

iii) How many squares in all?

Continuing this pattern, is each such sequence of numbers, an arithmetic sequence?

Answer

i) How many small squares are there in each rectangle?

→ No. of small squares in

Rectangle 1 - 2

Rectangle 2 - 4

Rectangle 3 - 6

Rectangle 4 - 8

As is evident, it forms an AP with common difference = 4-2 =2

ii) How many large squares?

→ No. of large squares in

Rectangle 1 - 0

Rectangle 2 - 1 (intersection of all 4 small squares)

Rectangle 3 - 2 (2 same size overlapping squares as in rectangle 2)

Rectangle 4 - 3

As is evident, it forms an AP with common difference = 1-0 =1

iii) How many squares in all?

→ All squares = small squares + large squares

No. of all squares in

Rectangle 1 - 2 + 0 = 2

Rectangle 2 - 4 + 1 = 5

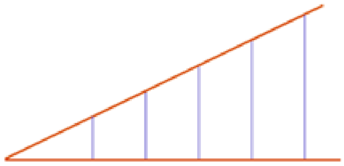
Rectangle 3 - 6 + 2 = 8

Rectangle 4 - 8 + 3 = 11

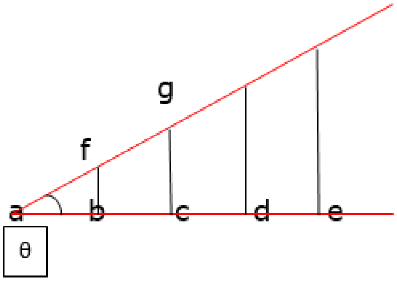
As is evident, it forms an AP with common difference = 2+1 = 3

4. Question

In this picture, the perpendiculars to the bottom line are equally spaced. Prove that, continuing like this, the lengths of perpendiculars form an arithmetic sequence.



Answer



In Δ fab , Δ gac , and so on…

The subtended angle θ is same

∴ tanθ = constant

∴ $\frac{l(fb)}{l(ab)} = \frac{l(gc)}{l(ac)}$

Now l(ab)=l(bc)=l(cd)… given

∴ l(ac)=2× l(ab)

l(ad)= 3× l(ab)

For, tanθ = constant

l(gc)=2× l(fb)

l(hd)=3× l(fb)

Hence, lengths fb,gc,hd…are in AP

i.e. the length of perpendiculars are in AP.

5. Question

The algebraic expression of a sequence is.

$X_n = n^3 – 6n^2 + 13n – 7$

Is it an arithmetic sequence?

Answer

For it to be an AP, $x_{n+1} – x_n$ = constant

$x_{n+1}-x_n = \{(n+1)^3-6(n+1)^2+13(n+1)-7\}-\{ n^3 – 6n^2+ 13n – 7\}$

$= (n+1)^3-n^3-6\{(n+1)^2-n^2\}+13(n+1-n)-7+7$

$= (n+1-n)(n^2+2n+1+n^2+n+n^2)-6(2n+1)+13$

$= 1× (3n^2+3n+1)-12n+7$

$= 3n^2-9n+8$

Which is not independent of n. Hence, the sequence is not an AP.

Questions Pg-21

1. Question

In each of the arithmetic sequences below, some terms are missing and their positions are marked with ∘. Find them.

i) 24, 42, ∘, ∘, … ii) ∘, 24, 42, ∘, …

iii) ∘, ∘, 24, 42, … iv) 24, ∘, 42, ∘, …

v) ∘, 24, ∘, 42, … vi) 24, ∘, ∘, 42, …

Answer

Let a = first term of arithmetic sequence

a_2 , a_3 , $a_{4...}$ are second, third, fourth terms and so on

d = common difference

i) Here, a = 24

$a_2 = 42$

d = 42-24 = 18

$a_3 = 42 + 18 = 60$

$a_4 = 60 + 18 = 78$

ii) Here, $a_2 = 24$

$a_3 = 42$

d = 42-24 = 18

$a = a_2 – d = 24-18 = 6$

$a_4 = 42 + 18 = 60$

iii) Here, $a_3 = 24$

$a_4 = 42$

$d = a_4 – a_3 = 42-24 = 18$

$a_2 = 24 – 18 = 6$

a = 6-18 = -12

iv) Here, a = 24

$a_3 = 42$

$a_3 = a + 2d$

Solving, d = 9

∴ $a_2 = 24 + 9 = 33$

$a_4 = 42 + 9 = 51$

v) Here, $a_2 = 24$

$a_4 = 42$

$a_4 = a + 3d$ …(eq)1

and,$a_2 = a + d$ …(eq)2

Eliminating a;

d = 9

$a = 24-9 = 15$

$a_3 = 24 + 9 = 33$

vi) Here, $a = 24$

$a_4 = 42$

$a_4 = a + 3d$

Solving, $d = 6$

$a_2 = 24 + 6 = 30$

$a_3 = 30 + 6 = 36$

Hence, all the answers are found.

2. Question

The terms in two positions of some arithmetic sequences are given below. Write the first five terms of each:

i) 3rd term 34

6th term 67

ii) 3rd term 43

6th term 76

iii) 3rd term 2

5th term 3

iv) 4th term 2

7th term 3

v) 2nd term 5

5th term 2

Answer

i) $a_3 = 34$

$a_6 = 67$

$a_3 = a + 2d$ and $a_6 = a + 5d$

Solving, $a_6 = a_3 + 3d$

$\therefore d = 11$

$a_2 = 34-11 = 23$

$a = 23-11 = 12$

$a_4 = 34 + 11 = 45$

$a_5 = 45 + 11 = 56$

First 5 terms are 12,23,34,45 and 56

ii) $a_3 = 43$

$a_6 = 76$

$a_3 = a + 2d$ and $a_6 = a + 5d$

Solving, $a_6 = a_3 + 3d$

$\therefore$ d = 11

a_2 = 43-11 = 32

a = 32-11 = 21

a_4 = 43 + 11 = 54

a_5 = 54 + 11 = 65

First 5 terms are 21,32,43,54 and 65

iii) a_3 = 2

a_5 = 3

a_3 = a + 2d and a_5 = a + 4d

Solving, $a_5 = a_3 + 2d$

d = 0.5

a_2 = 2-0.5 = 1.5

a = 1.5-0.5 = 1

a_4 = 2 + 0.5 = 2.5

First 5 terms are 1,1.5,2,2.5 and 3

iv) a_4 = 2

a_7 = 3

a_4 = a + 3d

a_7 = a + 6d

Solving,$a_7 = a_4 + 3d$

$d = \frac{1}{3}$

$a_3 = 2 - \left(\frac{1}{3}\right) = \frac{5}{3}$

$a_2 = \left(\frac{5}{3}\right) - \left(\frac{1}{3}\right) = \frac{4}{3}$

$a = \left(\frac{4}{3}\right) - \left(\frac{1}{3}\right) = 1$

$a_5 = 2 + \left(\frac{1}{3}\right) = \frac{7}{3}$

First 5 terms are 1,4/3,5/3,2 and 7/3

v) a_2 = 5

a_5 = 2

a_2 = a + d

a_5 = a + 4d

Solving,$a_5 = a_2 + 3d$

d = -1

a = 5-(-1) = 6

$a_3 = 5-1 = 4$

$a_4 = 4-1 = 3$

$a_5 = 3-1 = 2$

First 5 terms are 6,5,4,3,2.

Hence solved.

3. Question

The 5th term of an arithmetic sequence is 38 and the 9th term is 66. What is its 25th term?

Answer

Given:

$a_5 = 38$ and,

$a_9 = 66$

We know, nth term is given as,

$[a_n = a + (n-1)\times d]$

$\Rightarrow a_5 = a + 4d$ ..(1)

And, $a_9 = a + 8d$ ...(2)

Solving eq(1) and eq(2), we get,

$a_9 = a_5 + 4d$

$\Rightarrow 66 = 38 + 4d$

$\Rightarrow d = \frac{66-38}{4} = 7.$

Now, from eq (1),

$a_5 = a + 4d$

$\Rightarrow 38 = a + 4(7)$

$\Rightarrow a = (38-(4\times7))$

$\Rightarrow a = 38 – 28$

$\Rightarrow a = 10$

And,

25^{th} term, $a_{25} = a + 24d$

$= 10 + (24\times7)$

$= 178$

Hence, 25^{th} term is 178

4. Question

Is 101 a term of the arithmetic sequence 13, 24,35,...? What about 1001?

Answer

$a = 13$

$a_2 = 24$

$a_3 = 35$

d = 35-24 = 11

Case i)

$a_n = 101$

$a_n = 13 + (n-1)d$

Solving, n = 9

∴ 101 is the term of this AP.

Case ii)

$a_n = 1001$

$a_n = 13 + (n-1)d$

Solving, n = 90.8

∴ 1001 is not the term of this AP.

5. Question

How many three-digit numbers are there, which leave a remainder 3 on division by 7?

Answer

Dividend = (divisor× quotient) + remainder

Here, divisor = 7

Remainder = 3

First 3 digit no. is 100

For, dividend = 100, quotient = 13.6…

Hence, first 3 digit no. divisible by 7 with remainder 3 comes with quotient>13.6… = 14

When quotient = 14, dividend = 101

Last 3 digit no. divisible by 7 with remainder 3 = 997

$a_n = 997$

a = 101

d = 7

$a_n = a + (n-1)\times d$

∴ n-1 = 128

∴ n = 129

Hence,129 numbers are present.

6. Question

Fill up the empty cells of the square below such that the numbers in each row and column from arithmetic sequences:

What if we use other numbers instead of 1, 4 28 and 7?

1			4
7			28

Answer

For 1st row,

$a = 1$ $a_4 = 4$

$a_4 = a + 3d$

$\therefore d = 1$

$a_2 = 1 + 1 = 2$

$\therefore$ **$A_{12} = 2$**

$a_3 = 2 + 1 = 3$

$\therefore$ **$A_{13} = 3$**

For 1st column,

$a = 1$ $a_4 = 7$

$a_4 = a + 3d$

$\therefore d = 2$

$a_2 = 1 + 2 = 3$

$\therefore$ **$A_{21} = 3$**

$a_3 = 3 + 2 = 5$

$\therefore$ **$A_{31} = 5$**

For 4th row,

$a = 7$ $a_4 = 28$

$a_4 = a + 3d$

$\therefore d = 7$

$a_2 = 7 + 7 = 14$

$\therefore$ **$A_{42} = 14$**

$a_3 = 14 + 7 = 21$

$\therefore$ **$A_{43} = 21$**

For 4th column,

$a = 4$ $a_4 = 28$

$a_4 = a + 3d$

$\therefore d = 8$

$a_2 = 4 + 8 = 12$ $\therefore$ **$A_{24} = 12$**

a_3 = 12 + 8 = 20 ∴ $\mathbf{A_{34} = 20}$

For 2^{nd} row,

a = 3 a_4 = 12

a_4 = a + 3d

∴ d = 3

a_2 = 3 + 3 = 6 ∴ $\mathbf{A_{42} = 6}$

a_3 = 6 + 3 = 9 ∴ $\mathbf{A_{23} = 9}$

For 3^{rd} row,

a = 5 a_4 = 20

a_4 = a + 3d

∴ d = 5

a_2 = 5 + 5 = 10

∴ $\mathbf{A_{32} = 10}$

a_3 = 10 + 5 = 15

∴ $\mathbf{A_{33} = 15}$

If we use numbers other than 1,4,7,and 28 it is not guaranteed that the grid will be completely filled such that each row and column is in AP.

7. Question

In the table below, some arithmetic sequences are given with two numbers against each. Check whether each belongs to the sequence or not.

Sequence	Numbers	Yes/No
11, 22, 33, ...	123	
	132	
12, 23, 34, ...	100	
	1000	
21, 32, 43, ...	100	
	1000	
$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$	3	
	4	
$\frac{3}{4}, 1\frac{1}{2}, 2\frac{1}{4}, \ldots$	3	
	4	

Answer

a) a = 11

d = 22-11 = 11

i) let a_n = 123

a_n = a + (n-1)×d

∴ n = 11.18

Since, n is not an integer, 123 **is not** a part of sequence

ii) let a_n = 132

a_n = a + (n-1)×d

∴ n = 12

Since, n is an integer, 132 **is a part** of sequence

b) a = 12

d = 23-12 = 11

i) let a_n = 100

$a_n = a + (n-1)\times d$

∴ n = 9

Since, n is an integer, 100 **is a part** of sequence

ii) let a_n = 1000

$a_n = a + (n-1)\times d$

∴ n = 90.81

Since, n is not an integer, 1000 **is not** a part of sequence

c) a = 21

d = 32-21 = 11

i) let a_n = 100

$a_n = a + (n-1)\times d$

∴ n = 8.18

Since, n is not an integer, 100 **is not** a part of sequence

ii) let a_n = 1000

$a_n = a + (n-1)\times d$

∴ n = 90

Since, n is an integer, 1000 **is a part** of sequence

d) a = 1/4

d = (1/2)-(1/4) = 1/4

i) let a_n = 3

$a_n = a + (n-1)\times d$

∴ n = 12

Since, n is an integer, 3 **is a part** of sequence

ii) let a_n = 4

$a_n = a + (n-1)\times d$

∴ n = 16

Since, n is an integer, 4 **is a part** of sequence

e) a = 3/4

d = (3/2)-(3/4) = 3/4

i) let a_n = 3

$a_n = a + (n-1)\times d$

∴ n = 4

Since, n is an integer, 3 **is a part** of sequence

ii) let $a_n = 4$

$a_n = a + (n-1)\times d$

∴ n = 5.33

Since, n is not an integer, 4 **is not** a part of sequence

Questions Pg-26

1. Question

Write three arithmetic sequences with 30 as the sum of the first five terms.

Answer

$S_n = 30$ where n = 5

For an AP with a = first term

d = common difference

$S_n = \frac{n\{2 \times a + (n-1) \times d\}}{2}$

Substituting in above equation…

∴ 2a + (5-1) × d = 12

∴ 2a + 4d = 12

∴ a + 2d = 6

i) Let a = 2, hence d = 2

Sequence is 2,4,6,8,10…

ii) Let a = 4, hence d = 1

Sequence is 4,5,6,7,8…

iii) Let a = 6, hence d = 0

Sequence is 6,6,6,6,6…

Hence, 3 sequences are found.

2. Question

The first term of an arithmetic sequence is 1 and the sum of the first four terms is 100. Find the first four terms.

Answer

a = 1 $S_4 = 100$ n = 4

$S_n = \frac{n\{2 \times a + (n-1) \times d\}}{2}$

Substituting everything in above equation,

2a + (4-1) × d = 50

∴ 3d = 48

∴ d = 16

First four terms are 1,17,33,49.

Hence, the answer is written above.

3. Question

Prove that for any four consecutive terms of an arithmetic sequence, the sum of the two terms on the two ends and the sum of the two terms in the middle are the same.

Answer

Let a,b,c and e are four consecutive terms of an AP with d = common difference.

b = a + d

c = a + 2d

e = a + 3d

First term + Last term = a + e = a + (a + 3d) = 2a + 3d ...(1)

Second term + Third term = b + c = (a + d) + (a + 2d) = 2a + 3d ...(2)

Hence,

First term + Last term = Second term + Third term

4. Question

Write four arithmetic sequences with 100 as the sum of the first four terms.

Answer

S_n = 100 where n = 4

For an AP with a = first term

d = common difference

$S_n = \frac{n\{2 \times a + (n-1) \times d\}}{2}$

∴ 2a + (4-1) × d = 50

∴ 2a + 3d = 50

Let a = 1, hence d = 16

Sequence is 1,17,33,49...

Let a = 4, hence d = 14

Sequence is 4,18,32,46...

Let a = 7 hence d = 12

Sequence is 7,19,31,43...

5. Question

The 8th term of an arithmetic sequence is 12 and its 12th term is 8. What is the algebraic expression for this sequence?

Answer

a_8 = 12,

a_{12} = 8

a_n = a + (n-1) × d

a_8 = a + 7d = 12 ...(eq)1

a_{12} = a + 11d = 8 ...(eq)2

(eq)2-(eq)1

4d = -4

d = -1

a = 19

So, sequence is 19,18,17,16…

6. Question

The Bird problem in Class 8 (The lesson, **Equations**) can be slightly changed as follows.

One bird said:

“We and we again, together with half of us and half of that, and one more is a natural number”

Write the possible number of birds in order. For each of these, write the sum told by the bird also.

Find the algebraic expression for these two sequences.

Answer

Let the possible no of birds be x,

Therefore, according to given condition

$x + x + \frac{x}{2} + \frac{x}{4} + 1 = N$

Where N is any natural number.

$\Rightarrow \frac{4x+4x+2x+x}{4} = N$

⇒ 11x = 4N

$\Rightarrow x = \frac{4N}{11}$

Also, no of birds should also be a natural number, therefore N should be a multiple of 11, i.e. N = 11n for any natural no 'n'

Hence possible values of N = 11, 22, 33…

And possible values of x = 4, 8, 12, …

Algebraic expression for N is 11n

And

For x is 4n.

7. Question

Prove that the arithmetic sequence with first term 1/3 and common difference 1/6 contains all natural numbers.

Answer

For the sequence,

a = $\frac{1}{3}$

d = $\frac{1}{6}$

a_n = a + (n-1) × d

Let p be any natural number

p = $\frac{1}{3}$ + (n-1) × $\frac{1}{6}$

$= \frac{n}{6} + \frac{1}{6}$

$= \frac{n+1}{6}$

6p = n + 1

Thus, for every natural number p, a value of n exists.

Hence, this arithmetic sequence contains all natural numbers.

8. Question

Prove that the arithmetic sequence with first term 1/3 and common difference 2/3 contains all odd numbers, but no even number.

Answer

For the sequence,

$a = \frac{1}{3}$

$d = \frac{2}{3}$

$a_n = a + (n-1) \times d$

$= \frac{1}{3} + (n-1) \times \frac{2}{3}$

$= \frac{2n-1}{3}$

= k(2n-1) (where k = constant)

As a_n is a multiple of (2n-1) and not (2n) it contains all odd numbers but no even number..

9. Question

Prove that the squares of all the terms of the arithmetic sequence 4, 7, 10, … belong to the sequence.

Answer

a = first term = 4

d = common difference = 7-4 = 3

$a_n = a + (n-1) \times d$

$a_n = 4 + 3n-3 = 3n + 1$

$(a_n)^2 = (3n + 1)^2 = 9n^2 + 6n + 1 = 3n(3n + 2) + 1 = 3\{n(3n + 2)\} + 1$

As, $(a_n)^2$ is also of the form 3n + 1, the squares of all the terms of the arithmetic sequence 4, 7, 10, … belong to the sequence.

10. Question

Prove that the arithmetic sequence 5, 8, 11, … contains no perfect squares.

Answer

a = 5

d = 8-5 = 3

$a_n = a + (n-1) \times d$

$a_n = 5 + (n-1) \times 3$

$\therefore a_n = 3n + 2$

Let p be a natural number

$p^2 = 3n + 2$

$n = \frac{(p^2-2)}{3}$

Now, for all integers from 0 to 9(i.e. p from 0 to 9), n does not come out to be an integer.

Hence, the arithmetic sequence 5, 8, 11, ... contains no perfect squares.

11. Question

The angles of a pentagon are in arithmetic sequence. Prove that its smallest angle is greater than 36°.

Answer

Sum of all angles of a regular polygon with n sides = (n-2) × 180

Let a,a + d,a + 2d,a + 3d,a + 4d are 5 terms of an AP.

Sum of all angles = 5a + 10d = (5-2) × 180 = 540

∴ a + 2d = 108

The minimum angle will be obtained when a = d.

a + 2a = 3a = 108

∴ a = 36 is the minimum angle.

12. Question

Write the whole numbers in the arithmetic sequence $\frac{11}{8}, \frac{14}{8}, \frac{17}{8} \ldots$. Do they form an arithmetic sequence?

Answer

Let $a = \frac{11}{8}$

$a_2 = \frac{14}{8}$

$a_3 = \frac{17}{8}$

$a_2 - a = \frac{14}{8} - \frac{11}{8} = \frac{3}{8}$

$a_3 - a_2 = \frac{17}{8} - \frac{14}{8} = \frac{3}{8}$

As $a_3 - a_2 = a_2 - a$

Hence, the common difference is same, i.e. the sequence forms an AP.

Questions Pg-32

1 A. Question

Find the sum of the first 25 terms of each of the arithmetic sequences below.

11, 22, 33, ...

Answer

a = 11 = first term

d = 22-11 = 11 = common difference

$S_n = \frac{n\{2 \times a + (n-1) \times d\}}{2}$

Here, n = 25

$\therefore S_{25} = \frac{25\{2\times11 + (25-1)\times11\}}{2}$

= 25× 11× 13

= 3575

1 B. Question

Find the sum of the first 25 terms of each of the arithmetic sequences below.

12, 23, 34, ….

Answer

a = 12

d = 23-12 = 11

$S_n = \frac{n\{2\times a + (n-1)\times d\}}{2}$

Here, n = 25

$\therefore S_{25} = \frac{25\{2\times12 + (25-1)\times11\}}{2}$

= 25× 24× 6

= 3600

1 C. Question

Find the sum of the first 25 terms of each of the arithmetic sequences below.

21, 32, 43, …

Answer

a = 21

d = 32-21 = 11

$S_n = \frac{25\{2\times21 + (25-1)\times11\}}{2}$

Here, n = 25

$\therefore S_{25} = 3825$

1 D. Question

Find the sum of the first 25 terms of each of the arithmetic sequences below.

19, 28, 37, …

Answer

a = 19

d = 28-19 = 9

$S_n = \frac{n\{2\times a + (n-1)\times d\}}{2}$

Here, n = 25

$\therefore S_{25} = \frac{25\{2\times19 + (25-1)\times9\}}{2}$

= 3175

1 E. Question

Find the sum of the first 25 terms of each of the arithmetic sequences below.

1, 6, 11, …

Answer

a = 1

d = 6-1 = 5

$S_n = \frac{n\{2\times a + (n-1)\times d\}}{2}$

Here, n = 25

$\therefore S_{25} = \frac{25\{2\times 1 + (25-1)\times 5\}}{2}$

= 1525

Hence, the sum of first 5 terms for each sequence is found out.

2. Question

What is the difference between the sum of the first 20 terms and the next 20 terms of the arithmetic sequence 6, 10, 14, …?

Answer

For the arithmetic sequence,

a = 6

d = 10-6 = 4

$S_n = \frac{n\{2\times a + (n-1)\times d\}}{2}$ …(eq)1

Sum of first 20 terms = S_{20}

Sum of next 20 terms = S_{40}-s_{20}

Substituting values in eq1

$S_{20} = \frac{20\{2\times 6 + (20-1)\times 4\}}{2}$

= 10× 88

= 880

$S_{40} = \frac{40\{2\times 6 + (40-1)\times 4\}}{2}$

= 20× 168

= 3360

Sum of first 20 terms = S_{20} = 880

Sum of next 20 terms = S_{40}-s_{20} = 3360-880 = 2480

∴ Difference between sum of first 20 terms = 2480-880 = 1600

And next 20 terms

Hence, answer is found out.

3. Question

Calculate the difference between the sum of the first 20 terms of the arithmetic sequences 6, 10, 14, … and 15, 19, 23, …

Answer

Case i)

a = 6

d = 10-6 = 4

n = 20

$S_{n1} = \frac{20\{2\times6+(20-1)\times4\}}{2}$

= 10×88

= 880

Case ii)

a = 15

d = 19-15 = 4

n = 20

$S_{n2} = \frac{20\{2\times15+(20-1)\times4\}}{2}$

= 10×106

= 1060

The difference between the sum of the first 20 terms of the arithmetic sequences 6, 10, 14, … and 15, 19, 23, … = $S_{n1} - S_{n2}$

= 1060- 880

= 180

Hence, answer is found out.

4. Question

Find the sum of all three digit numbers, which are multiples of 9.

Answer

Three digit numbers which are multiples of 9 are in AP

The sequence is 108,117..999

a = 108

a_n = 999

d = 9

a_n = a + (n-1)×d

999 = 108 + (n-1)× 9

∴ n = 100

$S_n = \frac{n(a+an)}{2}$

$= \frac{100(108+999)}{2}$

= 50× 1107

= 55350

5. Question

Find n is the equation $5^2 \times 5^4 \times 5^6 \times \ldots \times 5^{2n} = (0.008)^{-30}$.

Answer

$a^m \times a^n = a^{m+n}$

LHS = $5^2 \times 5^4 \times 5^6 \times \ldots \times 5^{2n}$

$= 5^{2 + 4 + 6 + \ldots 2n}$

$= 5^{2(1 + 2 + 3 + \ldots n)}$

$\{1 + 2 + 3 + \ldots n = \frac{n(n+1)}{2}\}$

$= 5^{2 \times \frac{n(n+1)}{2}}$

$= 5^{n(n + 1)}$

RHS = $(0.008)^{-30} = (8 \times 10^{-3})^{-30}$

$= (2^3 \times (2 \times 5)^{-3})^{-30}$

$= (2^3 \times 2^{-3} \times 5^{-3})^{-30}$

$= 5^{90}$

$\because (a^m)^n = a^{mn}$

LHS = RHS

$\therefore 5^{n(n + 1)} = 5^{90}$

$\therefore n(n + 1) = 90$

$\therefore n^2 + n - 90 = 0$

Solving for n,

n = 9 and n = -10

But, $n<0$ is not possible

Hence, n = 9

6 A. Question

The expressions for the sum to n terms of some arithmetic sequences are given below. Find the expression for the nth term of each:

$n^2 + 2n$

Answer

$S_n = n^2 + 2n$

$a_{n+1} = S_{n+1} - S_n$

$= \{(n + 1)^2 + 2(n + 1)\} - \{n^2 + 2n\}$

$= n^2 + 4n + 4 - n^2 - 2n$

$= 2n + 4$

$= 2(n + 1) + 2$

$\therefore a_n = 2n + 2$

6 B. Question

The expressions for the sum to n terms of some arithmetic sequences are given below. Find the expression for the nth term of each:

$2n^2 + n$

Answer

$S_n = 2n^2 + n$

$a_{n+1} = S_{n+1} - S_n$

$= \{2(n+1)^2 + (n+1)\} - \{2n^2 + n\}$

$= (2n^2 + 4n + 4 + n + 1) - (2n^2 + n)$

$= 4n + 5$

$= 4(n+1) + 1$

$\therefore a_n = 4n + 1$

6 C. Question

The expressions for the sum to n terms of some arithmetic sequences are given below. Find the expression for the nth term of each:

$n^2 – 2n$

Answer

$S_n = n^2 – 2n$

$a_{n+1} = S_{n+1} - S_n$

$= \{(n+1)^2 - 2(n+1)\} - (n^2 – 2n)$

$= n^2 + 2n + 1 - 2n - 2 - n^2 + 2n$

$= 2n - 1$

$= 2(n+1) - 3$

$a_n = 2n - 3$

6 D. Question

The expressions for the sum to n terms of some arithmetic sequences are given below. Find the expression for the nth term of each:

$2n^2 – n$

Answer

$S_n = 2n^2 - n$

$a_{n+1} = S_{n+1} - S_n$

$= \{2(n+1)^2 - (n+1)\} - \{2n^2 - n\}$

$= (2n^2 + 4n + 2 - n - 1) - (2n^2 - n)$

$= 4n + 1$

$= 4(n+1) - 3$

$a_n = 4n - 3$

6 E. Question

The expressions for the sum to n terms of some arithmetic sequences are given below. Find the expression for the nth term of each:

$n^2 – n$

Answer

$S_n = n^2 - n$

$a_{n+1} = S_{n+1} - S_n$

$= \{(n+1)^2 - (n+1)\} - (n^2 - n)$

$= n^2 + 2n + 1 - n - 1 - n^2 + n$

$= 2n$

$= 2(n+1) - 2$

$a_n = 2n - 2$

7. Question

Calculate in head, the sums of the following arithmetic sequences.

i) 51 + 52 + 53 + ... + 70

ii) $1\frac{1}{2} + 2\frac{1}{2} + \ldots + 12\frac{1}{2}$

iii) $\frac{1}{2} + 1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} + \ldots + 12\frac{1}{2}$

Answer

(i) Here, we will use formula in which we can calculate sum of AP by first term(a) and last term(a_n) i.e.

$$S_n = \frac{n}{2}(a + a_n)$$

Where n is no of terms

Clearly first term, a = 51

Last term, $a_n = 70$

And no of terms, n = 70 - 51 + 1 = 20

Hence, sum is $= \frac{20}{2}(51 + 70)$

= 10(121)

= 1210

(ii)

Here, we will use formula in which we can calculate sum of AP by first term(a) and last term(a_n) i.e.

$$S_n = \frac{n}{2}(a + a_n)$$

Where n is no of terms

Clearly first term, $a = 1\frac{1}{2}$

Last term, $a_n = 12\frac{1}{2}$

Common difference, $d = 2\frac{1}{2} - 1\frac{1}{2} = 1$

Therefore, we can calculate no of terms usinh,

$a_n = a + (n-1)d$

$\Rightarrow 12\frac{1}{2} = 1\frac{1}{2} + (n-1)1$

⇒ 11 = n - 1

⇒ n = 12

Hence, we have sum as

$S_n = \frac{12}{2}(1\frac{1}{2} + 12\frac{1}{2})$

⇒ S_n = 6(14) = 84

(iii)

Here, we will use formula in which we can calculate sum of AP by first term(a) and last term(a_n) i.e.

$S_n = \frac{n}{2}(a + a_n)$

Where n is no of terms

Clearly first term, $a = \frac{1}{2}$

Last term, $a_n = 12\frac{1}{2}$

Common difference, $d = 1 - \frac{1}{2} = \frac{1}{2}$

Therefore, we can calculate no of terms usinh,

$a_n = a + (n-1)d$

$\Rightarrow 12\frac{1}{2} = \frac{1}{2} + \frac{(n-1)1}{2}$

$\Rightarrow 12 = \frac{n-1}{2}$

⇒ n = 25

Hence, we have sum as

$S_n = \frac{25}{2}(\frac{1}{2} + 12\frac{1}{2})$

$\Rightarrow S_n = \frac{25}{2}(13) = \frac{325}{2}$

$\Rightarrow S_n = 162\frac{1}{2}$

8. Question

The sum of the first 10 terms of an arithmetic sequence is 350 and the sum of the first 5 terms is 100. Write the algebraic expression for the sequence.

Answer

S_{10} = 350

S_5 = 100

$S_n = \frac{n\{2\times a + (n-1)\times d\}}{2}$

When n = 10,

$S_{10} = 350 = \frac{10\{2\times a + (10-1)\times d\}}{2}$

∴ 2a + 9d = 70 ...(eq)1

$S_5 = 100 = \frac{5\{2\times a + (5-1)\times d\}}{2}$

∴ a + 2d = 20 …(eq)2

Solving equations 1 and 2…

d = 6

a = 8

Hence, the sequence is 8,14,20…

$\mathbf{a_n}$ = a + (n-1)×d = 8 + (n-1)×6 = **6n + 2**

9. Question

Prove that the sum of any number of terms of the arithmetic sequence 16, 24, 32, … starting from the first, added to 9 gives a perfect square.

Answer

a = 16

d = 24-16 = 8

Let number of terms = n

$S_n = \frac{n\{2\times a + (n-1)\times d\}}{2}$

$= \frac{n\{2\times 16 + (n-1)\times 8\}}{2}$

= n(12 + 4n)

$S_n + 9 = 4n^2 + 12n + 9 = (2n + 3)^2$

Hence, the sum of any number of terms of the arithmetic sequence 16, 24, 32, …. starting from the first, added to 9 gives a perfect square.

10. Question

4			
7	10		
13	16	19	
22	25	28	31

………………………………………

………………………………………

………………………………………

Write the next two lines of the pattern above. Calculate the first and last terms of the 20th line.

Answer

The first terms of each line are 4,7,13,22…

The difference between each consecutive term is

7-4 = 3

13-7 = 6

22-13 = 9

i.e. 3,6,9… in AP

For 4th line,

First term = 22 + 12 = 34 , d = 3

For 5th line

First term = 34 + 15 = 49 , d = 3

Also from observation, nth line has n terms

Hence, 4th line

34,37,40,43

And 5th line

49,52,55,58,61

The sequence of common difference is

The difference between the first terms of each line forms an AP which is

3,6,9…

For first term of 20th line, sum of 19 terms of above sequence to be added to first term of first line.

Sum of 19 terms = $\frac{19\{2\times3+(19-1)\times3\}}{2}$

= 570

For 20th line,

First term = 4 + 570 = **574**

It has 20 terms

Last term = 574 + 19×3 = 574 + 57 = **631**