QUADRATIC EQUATION

1. For $x \in \mathbb{R}$, then number of real roots of the equation $3x^2 - 4|x^2 - 1| + x - 1 = 0$ is ____.

[JEE(Advanced) 2022]

2. Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c, d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of $ac(a - c) + ad(a - d) + bc(b - c) + bd(b - d)$ is **[JEE(Advanced) 2020]**

(A) 0 (B) 8000 (C) 8080 (D) 16000

Paragraph for Question No. 3 and 4

Let p,q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$.

For n = 0,1,2,...., let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

3. If $a_4 = 28$, then $p + 2q =$ **[JEE(Advanced) 2017]**

(A) 14 (B) 7 (C) 12 (D) 21

4. $a_{12} =$ **[JEE(Advanced) 2017]**

(A) $2a_{11} + a_{10}$ (B) $a_{11} - a_{10}$ (C) $a_{11} + a_{10}$ (D) $a_{11} + 2a_{10}$

5. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

[JEE(Advanced) 2016]

(A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$ (C) $-2\tan\theta$ (D) 0

6. Let S be the set of all non-zero numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S ? **[JEE(Advanced) 2015]**

(A) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (B) $\left(-\frac{1}{\sqrt{5}}, 0\right)$ (C) $\left(0, \frac{1}{\sqrt{5}}\right)$ (D) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

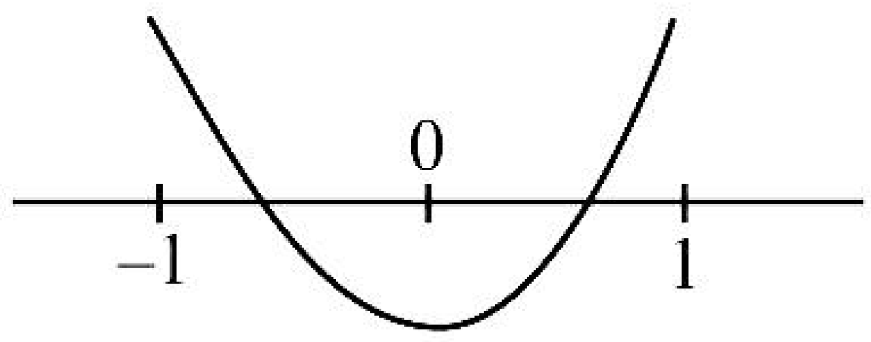
SOLUTIONS

1. Ans. (4)

Sol. $3x^2 + x - 1 = 4|x^2 - 1|$

If $x \in [-1, 1]$,

$3x^2 + x - 1 = -4x^2 + 4 \Rightarrow 7x^2 + x - 5 = 0$

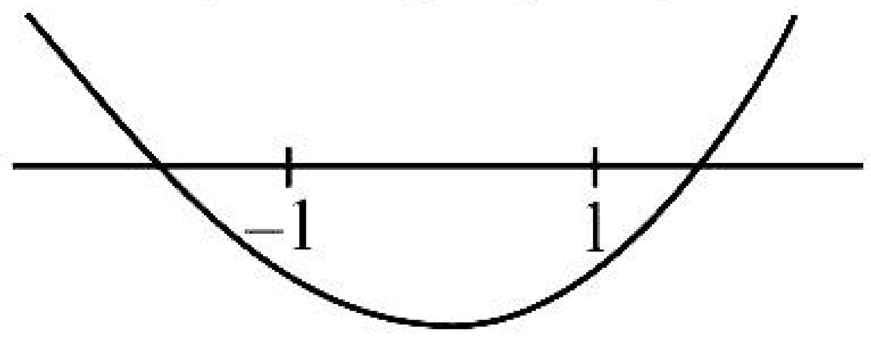


say $f(x) = 7x^2 + x - 5$

$f(1) = 3$; $f(-1) = 1$; $f(0) = -1$

[Two Roots]

If $x \in (-\infty, -1] \cup [1, \infty)$



$3x^2 + x - 1 = 4x^2 - 4 \Rightarrow x^2 - x - 3 = 0$

Say $g(x) = x^2 - x - 3$

$g(-1) = -1$; $g(1) = -3$

[Two Roots]

So total 4 roots.

2. Ans. (D)

Sol. $x^2 + 20x - 2020 = 0$ has two roots $a,b \in R$

$x^2 - 20x + 2020 = 0$ has two roots $c,d \in$ complex

$ac(a - c) + ad(a - d) + bc(b - c) + bd(b - d)$

$= a^2c - ac^2 + a^2d - ad^2 + b^2c - bc^2 + b^2d - bd^2$

$= a^2(c + d) + b^2(c + d) - c^2(a + b) - d^2(a + b)$

$= (c + d)(a^2 + b^2) - (a + b)(c^2 + d^2)$

$= (c + d)((a + b)^2 - 2ab) - (a + b)((c + d)^2 - 2cd)$

$= 20[(20)^2 + 4040] + 20[(20)^2 - 4040]$

$= 20[(20)^2 + 4040 + (20)^2 - 4040]$

$= 20 \times 800 = 16000$

3. Ans. (C)

Sol. $\alpha^2 = \alpha + 1 \Rightarrow \alpha^4 = 3\alpha + 2$

$\therefore a_4 = 28 \Rightarrow p\alpha^4 + q\beta^4 = p(3\alpha + 2) + q(3\beta + 2) = 28$

$\Rightarrow \quad p(3\alpha + 2) + q(3 - 3\alpha + 2) = 28$

$\Rightarrow \quad \alpha(3p - 3q) + 2p + 5q = 28$

(as $\alpha \in Q^c$)

$\Rightarrow \quad p = q,\ 2p + 5q = 28 \Rightarrow p = q = 4$

$\therefore \quad p + 2q = 12$

4. Ans. (C)

Sol. $\alpha^2 = \alpha + 1 \Rightarrow \alpha^n = \alpha^{n-1} + \alpha^{n-2}$

$\Rightarrow p\alpha^n + q\beta^n = p(\alpha^{n-1} + \alpha^{n-2}) + q(\beta^{n-1} + \beta^{n-2})$

$a_n = a_{n-1} + a_{n-2}$

$\Rightarrow a_{12} = a_{11} + a_{10}$

5. Ans. (C)

Sol. $\alpha_1 = \dfrac{2\sec\theta + \sqrt{4\sec^2\theta - 4}}{2}$

$\beta_2 = \dfrac{-2\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2} \qquad \{\because \alpha_2 > \beta_2\}$

$\alpha_1 = \sec\theta + |\tan\theta| \qquad \{\because \alpha_1 > \beta_1\}$

$\beta_2 = -\tan\theta - \sec\theta$

$\alpha_1 = \sec\theta - \tan\theta \qquad \left(\because \theta \in \left(-\dfrac{\pi}{6}, -\dfrac{\pi}{12}\right)\right)$

$\alpha_1 + \beta_2 = -2\tan\theta$

6. Ans. (A, D)

Sol. $\alpha x^2 - x + \alpha = 0$

$D = 1 - 4\alpha^2$

distinct real roots $D > 0$

$\Rightarrow \alpha \in \left(-\dfrac{1}{2}, \dfrac{1}{2}\right) \qquad ...(i)$

given $|x_1 - x_2| < 1$

$\Rightarrow \dfrac{\sqrt{1 - 4\alpha^2}}{|\alpha|} < 1$

$\Rightarrow 1 - 4\alpha^2 < \alpha^2$

$\Rightarrow \alpha \in \left(-\infty, \dfrac{-1}{\sqrt{5}}\right) \cup \left(\dfrac{1}{\sqrt{5}}, \infty\right) \quad ...(ii)$

from (i) & (ii)

$\alpha \in \left(\dfrac{-1}{2}, \dfrac{-1}{\sqrt{5}}\right) \cup \left(\dfrac{1}{\sqrt{5}}, \dfrac{1}{2}\right)$