20. Geometric Progressions

Exercise 20.1

1. Question

Show that each one of the following progressions is a G.P. Also, find the common ratio in each case :

i. 4, -2, 1, $-\frac{1}{2}$,....

ii. $-\frac{2}{3}, -6, -54, \ldots$

iii. $a, \frac{3a^2}{4}, \frac{9a^3}{16}, \ldots\ldots$

iv. $\frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \ldots.$

Answer

(i) Let a = 4, b = -2, c = 1.

In GP, b^2=ac

⇒ $(-2)^2$ = 4.1

⇒ 4 = 4

Common ratio = r $= \frac{-2}{4} = \frac{-1}{2}$

(ii) Let $a = \frac{-2}{3}$, b = -6, c = -54.

In GP, b^2=ac

⇒ $(-6)^2 = \frac{-2}{3} \times -54$

⇒ 36 = 36

Common ratio = r $= \frac{-6}{\frac{-2}{3}} = 9$

(iii) Let a = a, $b = \frac{3a^2}{4}, c = \frac{9a^3}{16}$

In GP, b^2=ac

⇒ $\left(\frac{3a^2}{4}\right)^2 = \frac{9a^3}{16} \times a$

⇒ $\frac{9a^4}{4} = \frac{9a^4}{16}$

Common ratio $= r = \frac{\frac{3a^2}{4}}{a} = \frac{3a}{4}$

(iv) Let $a = \frac{1}{2}, b = \frac{1}{3}, c = \frac{2}{9}$

In GP, b^2=ac

⇒ $\left(\frac{1}{3}\right)^2 = \frac{1}{2} \times \frac{2}{9}$

⇒ $\frac{1}{9} = \frac{1}{9}$

Common ratio $= r = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$

2. Question

Show that the sequence defined by $a_n = \frac{2}{3^n}$, n ∈ N is a G.P.

Answer

Put n = 1,2,3,4…

a_1, a_2, a_3, a_4

$\Rightarrow a_1 = \frac{2}{3}, a_2 = \frac{2}{9}, a_3 = \frac{2}{27}, a_4 = \frac{2}{81}, \ldots$

If $a_1, a_2, a_3 \ldots$

$\Rightarrow (a_2)^2 = a_1.a_3$

$\Rightarrow \left(\frac{2}{9}\right)^2 = \frac{2}{3} \times \frac{2}{27}$

$\Rightarrow \frac{4}{81} = \frac{4}{81}$

So, It is GP.

Common ratio $= r = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$

3 A. Question

Find:

the ninth term of the G.P. 1, 4, 16, 64, ….

Answer

$T_n = ar^{n-1}$

a = 4, $r = \frac{16}{4} = 4$

$\therefore T_9 = 4.(4^{9-1})$

$= 4.4^8$

$= 4^9$

∴ The 9 term is 4^9

3 B. Question

Find:

the 10^{th} term of the G.P. $-\frac{3}{4}, \frac{1}{2}, -\frac{1}{3}, \frac{2}{9}, \ldots$..

Answer

$T_n = ar^{n-1}$

$a = \frac{-3}{4}, r = \frac{\frac{1}{2}}{\frac{-3}{4}} = \frac{-2}{3}$

$\therefore T_9 = \frac{-3}{4}\left(\frac{-2}{3}\right)^{10-1}$

$= \frac{-3}{4} \times \frac{2^9}{3^9}$

$= \frac{2^7}{3^8}$

∴ The 10 term is $\frac{2^7}{3^8}$.

3 C. Question

Find:

the 8^{th} term of the G.P., 0.3, 0.06, 0.012, ….

Answer

$T_n = ar^{n-1}$

$a = \frac{3}{100}, r = \frac{\frac{6}{100}}{\frac{3}{10}} = \frac{2}{10}$

$\therefore T_8 = \frac{3}{100}\left(\frac{2}{10}\right)^{8-1}$

$= \frac{3}{100} \times \frac{2^7}{10^7}$

$= \frac{3.2^7}{10^9}$

∴ The 10 term is $\frac{3.2^7}{10^9}$.

3 D. Question

Find:

the 12th term of the G.P. $\frac{1}{a^3 x^3}$, ax, a^5, x^5, ….

Answer

$T_n = ar^{n-1}$

$a = \frac{1}{a^3x^3}, r = \frac{ax}{\frac{1}{a^3x^3}} = a^4x^4$

$\therefore T_{12} = \frac{1}{a^3x^3}(a^4x^4)^{12-1}$

$= \frac{1}{a^3x^3}(a^4x^4)^{11}$

$= a^{41}x^{41}$

∴ The 12 term is $a^{41}x^{41}$.

3 E. Question

Find:

nth term of the G.P. $\sqrt{3}, \frac{1}{\sqrt{3}}, \frac{1}{3\sqrt{3}}, \ldots.$

Answer

$T_n = ar^{n-1}$

$a = \sqrt{3}, r = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3}$

$\therefore T_n = \sqrt{3}\left(\frac{1}{3}\right)^{n-1}$

$\therefore$ The n term is $\sqrt{3}\left(\frac{1}{3}\right)^{n-1}$.

3 F. Question

Find:

the 10^{th} term of the G.P. $\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \ldots$

Answer

$T_n = ar^{n-1}$

$a = \sqrt{2}, r = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2}$

$\therefore T_{10} = \sqrt{2}\left(\frac{1}{2}\right)^{10-1}$

$= \frac{\sqrt{2}}{512}$

$= \frac{1}{256\sqrt{2}}$

$\therefore$ The 10 term is $\frac{1}{256\sqrt{2}}$.

4. Question

Find the 4^{th} term from the end of the G.P. $\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots, 162$.

Answer

Nth term from the end is given by,

$N = l\left(\frac{1}{r}\right)^{n-1}$

Where l = last term, n = nth term, and r = common ratio.

L = last term = 162

common ratio $= r = \frac{\frac{2}{9}}{\frac{2}{27}} = 3$

n = 4

$\therefore N = 162\left(\frac{1}{3}\right)^{4-1}$

$\Rightarrow N = 162 \times \frac{1}{27}$

$\Rightarrow 6$

∴ 4^{th} term from last is 6.

5. Question

which term of the progression 0.004, 0.02, 0.1, ... Is 12.5?

Answer

$T_n = ar^{n-1}$

$a = \frac{4}{1000}, r = \frac{\frac{2}{100}}{\frac{4}{1000}} = 5$, T_n = 12.5, n=?

$\therefore 12.5 = \frac{4}{1000} \times (5)^{n-1}$

$\Rightarrow \frac{125\times10^2}{4} = \frac{5^n}{5}$

$\Rightarrow \frac{625\times10^2}{4} = 5^n$

$\Rightarrow 5^n = 15625$

⇒ n = 6

6 A. Question

Which term of the G.P. :

$\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{1}{4\sqrt{2}}, \ldots$ is $\frac{1}{512\sqrt{2}}$?

Answer

$T_n = ar^{n-1}$

$a = \sqrt{2}, r = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2}, Tn = \frac{1}{512\sqrt{2}}$ n =?

$\therefore \frac{1}{512\sqrt{2}} = \sqrt{2} \times \left(\frac{1}{2}\right)^{n-1}$

$\Rightarrow \frac{1}{1024} = \left(\frac{1}{2}\right)^{n} \times 2$

$\Rightarrow \frac{1}{2048} = \left(\frac{1}{2}\right)^{n}$

$\Rightarrow 2^n = 2048$

⇒ n = 11

6 B. Question

Which term of the G.P. :

2, 2 √2,4, ... Is 128?

Answer

$T_n = ar^{n-1}$

$a = 2, r = \frac{2\sqrt{2}}{2} = \sqrt{2}, Tn = 128$ n =?

$\therefore 128 = 2 \times \left(\sqrt{2}\right)^{n-1}$

$\Rightarrow 64\sqrt{2} = (\sqrt{2})^n$

$\Rightarrow n = 13$

6 C. Question

Which term of the G.P. :

$\sqrt{3}, 3, 3\sqrt{3}, \ldots$ is 729?

Answer

$T_n = ar^{n-1}$

$a = \sqrt{3}, r = \frac{3}{\sqrt{3}} = \sqrt{3}, Tn = 729\ n = ?$

$\therefore 729 = \sqrt{3} \times (\sqrt{3})^{n-1}$

$729 = (\sqrt{3})^n$

$\Rightarrow n = 12$

6 D. Question

Which term of the G.P. :

$\frac{1}{3}, \frac{1}{9}, \frac{1}{27} \ldots$ is $\frac{1}{19683}$?

Answer

$T_n = ar^{n-1}$

$a = \frac{1}{3}, r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3}, T_n = \frac{1}{19683}, n = ?$

$\therefore \frac{1}{19683} = \frac{1}{3} \times \left(\frac{1}{3}\right)^{n-1}$

$\Rightarrow \frac{1}{19683} = \left(\frac{1}{3}\right)^n$

$\Rightarrow n = 9$

7. Question

Which term of the progression 18, -12, 8, ... is $\frac{512}{729}$?

Answer

$T_n = ar^{n-1}$

$a = 18, r = \frac{-12}{18} = \frac{-2}{3}, T_n = \frac{512}{729}, n = ?$

$\therefore \frac{512}{729} = 18 \times \left(\frac{-2}{3}\right)^{n-1}$

$\Rightarrow \frac{512}{729} = 18 \times \left(\frac{-2}{3}\right)^n \times \frac{-3}{2}$

$\Rightarrow \frac{512}{19683} = \left(\frac{2}{3}\right)^n$

$\Rightarrow n = 9$

8. Question

Find the 4th term from the end of the G.P. $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \ldots\ldots, \frac{1}{4374}$.

Answer

Nth term from the end is given by

N = $l\left(\frac{1}{r}\right)^{n-1}$

Where, l = last term, n = nth term, and r = common ratio.

L = last term $= \frac{1}{4374}$

common ratio $= r = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$

n = 4

∴ N = $\frac{1}{4374} \times (3)^{4-1}$

⇒ N = $27 \times \frac{1}{4374}$

⇒ $\frac{1}{162}$

∴ 4^{th} term from last is $\frac{1}{162}$.

9. Question

The fourth term of a G.P. is 27, and the 7th term is 729, find the G.P.

Answer

$T_n = ar^{n-1}$

a =a, r =?, T_n = 27 n=4

a =a, r =?, T_n = 729 n=7

∴ 27 = $a.r^{4-1}$

⇒ 27 =$a.r^3$...(1)

∴ 729 = $a.r^{7-1}$

⇒ 729 = $a.r^6$...(2)

Divide (2) by (1) we get

⇒ $\frac{729}{27} = \frac{ar^6}{ar^3}$

⇒ $r^3 = 27$

⇒ r = 3

Substituting r in 1 we get

a = 1

∴ GP = 1,3,9...

10. Question

The seventh term of a G.P. is 8 times the fourth term and 5th term is 48. Find the G.P.

Answer

$T_n = ar^{n-1}$

In the GP, the 7th term is 8 times the 4th term. So

$ar^6 = 8ar^3$, or

$r^3 = 8$ or $r = 2$.

$ar^4 = 48$,

$a.16 = 48$

$a = 3$

The first term is 3 and r = 2

∴ GP = 3, 6, 12, …

11. Question

If the G.P.'s 5, 10, 20, …. And 1280, 640, 320, … have their nth terms equal, find the value of n.

Answer

GP is given by

$a, ar, ar^2, \cdots, ar^{n-1}$

In the sequence 5, 10, 20, 40, …

First term, a = 5

Common ratio, r $= \frac{10}{5} = 2$

Equate the term to be found with the nnth term.

$ar^{n-1}=1280$

$5 \times 2^{n-1} =1280$

$2^{n-1} = 256$

$n - 1 = 8$

$n =9$

∴ 9^{th} term is equal.

12. Question

If 5^{th}, 8^{th} and 11^{th} terms of a G.P. are p, q and s respectively, prove that $a^2 = ps$.

Answer

$T_n = ar^{n-1}$

According to the question,

$\Rightarrow T_5 = a.r^4$

$\Rightarrow T_8 = a.r^7$

$\Rightarrow T_{11} = a.r^{10}$

To Prove: $q^2=p.s$

$\Rightarrow T_5 = a.r^4 = p$

⇒ $T_8 = a.r^7 = q$

⇒ $T_{11} = a.r^{10}=s$

$q^2= (a.r^7)^2 = a^2.r^{14}$...(1)

$p.s = (a.r^4)(a.r^{10}) = a^2.r^{14}$...(2)

∴ from (1) and (2) we get

⇒ $q^2=p.s$

Hence, Proved.

13. Question

The 4th term of a G.P. is square of its second term, and the first term is -3. Find its 7th term.

Answer

a = -1

$T_4 = (T_2)^2$

$T_n = ar^{n-1}$

∴ $a.r^3 = (ar)^2$

$r^3 = a.r^2$

⇒ r = a

∴ r =-1

$T_7 = ar^{7-1}$

$= (-1).r^6$

$= -1.(-1)^6$

= -1.

14. Question

In a GP the 3rd term is 24, and the 6th term is 192. Find the 10th term.

Answer

$T_n = ar^{n-1}$

a = a, r =?, T_n = 24 n = 3

a = a, r =?, T_n = 192 n = 6

∴ $24 = a.r^{3-1}$

⇒ $24 = a.r^2$...(1)

∴ $192 = a.r^{6-1}$

⇒ $192 = a.r^5$...(2)

Divide (2) by (1) we get

⇒ $\frac{192}{24} = \frac{ar^5}{ar^2}$

⇒ $r^3 = 8$

⇒ r = 2

Substituting r in 2 we get

a = 6

$T_{10} = 6.2^{10-1}$

$= 6.2^9$

= 3072.

15. Question

If a, b, c, d and p are different real numbers such that :

$(a^2 + b^2 + c^2)p^2 – 2(ab + bc + cd) p + (b^2 + c^2 + d^2) \le 0$, then show that a, b, c and d are in G.P.

Answer

We observe that the left side of the inequality could be written like:

$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \ge 0$ as each of these 3 terms is a

perfect square..........(1)

But by the given condition:

$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \le 0$ ....(2)

Therefore the conditions (1) and(2) can be satisfying iff the sum, $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$ which is possible iff each of the terms,

(ap - b) = (bp - c) = (cp - d) is equal to zero. So,

ap - b = 0, Or a/b = k.

bp - c = 0. Or b/c = k.

cp - d = o, Or a/d =k.

Therefore, a/ b = b/c = c/d = k the common ratio of the terms a, b, c, and d.

So a,b,c,and d are in geometric progression .

16. Question

If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx} \left(x \neq 0\right)$, then show that a, b, c, and d are in G.P.

Answer

Given: $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$

To Prove: a, b, c, and d are in G.P

Proof:

Applying componendo and dividend to the given expression, we get,

$$\frac{a+bx+a-bx}{a+bx-a+bx} = \frac{b+cx+b-cx}{b+cx-b+cx} = \frac{c+dx+c-dx}{c+dx-c+dx}$$

$$\frac{a}{bx} = \frac{b}{cx} = \frac{c}{dx}$$

$$\frac{a}{b} = \frac{b}{a} = \frac{c}{d}$$

Clearly, a, b, c, and d are in G.P.

Hence, Proved.

17. Question

If the p^{th} and q^{th} terms of a G.P. are q and p respectively, show that $(p + q)^{th}$ term is $\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$.

Answer

Given: p^{th} and q^{th} terms of a G.P. are q and p

Formula Used: $T_n = ar^{n-1}$

So, we get,

$q = ar^{p-1}$ ...(1)

$p = ar^{q-1}$...(2)

To Prove: $ar^{p+q-1} = \left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$

Proof:

Divide (1) by (2), we get

$\Rightarrow \frac{q}{p} = r^{p-q}$

$\Rightarrow r = \left(\frac{q}{p}\right)^{\frac{1}{p-q}}$

Substituting we get,

$\Rightarrow a = p\left(\frac{q}{p}\right)^{\frac{1-q}{p-q}}$

L.H.S = ar^{p+q-1}

$\text{L.H.S} = p\left(\frac{q}{p}\right)^{\frac{1-q}{p-q}} \times \left(\frac{q}{p}\right)^{\frac{p+q-1}{p-q}}$

$\text{L.H.S} = p\left(\frac{q}{p}\right)^{\frac{1-q+p+q-1}{p-q}}$

$\text{L.H.S} = p\left(\frac{q}{p}\right)^{\frac{p}{p-q}}$

$\text{L.H.S} = \left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$

L.H.S = R.H.S

Hence, Proved.

Exercise 20.2

1. Question

Find three numbers in G.P. whose sum is 65 and whose product is 3375.

Answer

Let the three numbers be $\frac{a}{r}, a, ar$.

∴ According to the question

⇒ $\frac{a}{r} + a + ar = 65$ ...(1)

⇒ $\frac{a}{r} \times a \times ar = 3375$ ...(2)

From 2 we get,

⇒ $a^3 = 3375$

∴ a = 15.

From 1 we get,

⇒ $\frac{a + ar + ar^2}{r} = 65$

⇒ $a + ar + ar^2 = 65r$ ...(3)

Substituting a = 15 in 3 we get

⇒ $15 + 15r + 15r^2 = 65r$

⇒ $15r^2 - 50r + 15 = 0$...(4)

Dividing (4) by 5 we get

⇒ $3r^2 - 10r + 3 = 0$

⇒ $3r^2 - 9r - r + 3 = 0$

⇒ 3r(r – 3) – 1(r – 3) = 0

∴ r = 3 or r = 1/3

∴ Now the equation will be

⇒ $\frac{15}{3}, 15, 15 \times 3$ or $\frac{15}{\frac{1}{3}}, 15, 15 \times \frac{1}{3}$

⇒ 5,15,45 or 45,15,5.

∴ The three numbers are 5,15,45.

2. Question

Find three number in G.P. whose sum is 38 and their product is 1728.

Answer

Let the three numbers be $\frac{a}{r}, a, ar$.

∴ According to the question

⇒ $\frac{a}{r} + a + ar = 38$ ...(1)

⇒ $\frac{a}{r} \times a \times ar = 1728$ ...(2)

From 2 we get

$\Rightarrow a^3 = 1728$

$\therefore a = 12.$

From 1 we get

$\Rightarrow \frac{a + ar + ar^2}{r} = 65$

$\Rightarrow a + ar + ar^2 = 65r$ ...(3)

Substituting a = 12 in 3 we get

$\Rightarrow 12 + 12r + 12r^2 = 38r$

$\Rightarrow 12r^2 – 26r + 12 = 0$...(4)

Dividing (4) by 2 we get

$\Rightarrow 6r^2 – 13r + 6 = 0$

$\Rightarrow 6r^2 – 12r – r + 6 = 0$

$\Rightarrow 6r(r – 2) – 1(r – 2) = 0$

$\therefore r = 2$ or $r = 1/2$

∴ Now the equation will be

$\Rightarrow \frac{12}{2}, 12, 12 \times 2 \text{ or } \frac{12}{\frac{1}{2}}, 12, 12 \times \frac{1}{2}$

⇒ 6,12,24 or 24,12,6.

∴ The three numbers are 6,12,24.

3. Question

The sum of first three terms of a G.P. is $\frac{13}{12}$, and their product is – 1. Find the G.P.

Answer

Let the three numbers be $\frac{a}{r}, a, ar$.

∴ According to the question

$\Rightarrow \frac{a}{r} + a + ar = \frac{13}{12}$ ...(1)

$\Rightarrow \frac{a}{r} \times a \times ar = -1$ ...(2)

From 2 we get

$\Rightarrow a^3 = – 1$

$\therefore a = – 1.$

From 1 we get

$\Rightarrow \frac{a + ar + ar^2}{r} = \frac{13}{12}$

$\Rightarrow 12a + 12ar + 12ar^2 = 13r$ ...(3)

Substituting a = – 1 in 3 we get

$\Rightarrow 12(– 1) + 12(– 1)r + 12(– 1)r^2 = 13r$

$\Rightarrow 12r^2 + 25r + 12 = 0$

$\Rightarrow 12r^2 + 16r + 9r + 12 = 0...(4)$

$\Rightarrow 4r(3r + 4) + 3(3r + 4) = 0$

$\therefore r = \frac{-3}{4}$ or $r = \frac{-4}{3}$

∴ Now the equation will be

$\Rightarrow \frac{-1}{\frac{-3}{4}}, -1, -1 \times \frac{-3}{4}$ or $\frac{-1}{\frac{-4}{3}}, -1, -1 \times \frac{-4}{3}$

$\Rightarrow \frac{4}{3}, -1, \frac{3}{4}$ or $\frac{3}{4}, -1, \frac{4}{3}$

∴ The three numbers are $\frac{4}{3}, -1, \frac{3}{4}$.

4. Question

The product of three numbers in G.P. is 125 and the sum of their products taken in pairs is $87\frac{1}{2}$. Find them.

Answer

Let the three numbers be $\frac{a}{r}, a, ar$.

∴ According to the question

$\Rightarrow \frac{a}{r} \times a \times ar = 125$ ...(1)

From 1 we get

$\Rightarrow a^3 = 125$

∴ a = 5.

$\Rightarrow \frac{a}{r} \times a + a \times ar + ar \times \frac{a}{r} = \frac{175}{2}$

$\Rightarrow a + ar + a^2 = \frac{175}{2}$

Substituting a = 5 in above equation we get

$\Rightarrow 5 + 5r + 25 = \frac{175}{2}$

$\Rightarrow 5r = \frac{175}{2} - 30$

$\Rightarrow r = \frac{175-60}{10}$

$\Rightarrow r = \frac{23}{2}$

$\Rightarrow \frac{5}{\frac{23}{2}}, 5, 5 \times \frac{23}{2}$

$\therefore \frac{10}{23}, 5, \frac{115}{2}$ are the three numbers.

5. Question

The sum of the first three terms of a G.P. is $\frac{39}{10}$, and their product is 1. Find the common ratio and the terms.

Answer

Let the three numbers be $\frac{a}{r}, a, ar$.

∴ According to the question

$\Rightarrow \frac{a}{r} + a + ar = \frac{39}{10}$ ...(1)

$\Rightarrow \frac{a}{r} \times a \times ar = 1$ ...(2)

From 2 we get

$\Rightarrow a^3 = 1$

$\therefore$ a = 1.

From 1 we get

$\Rightarrow \frac{a + ar + ar^2}{r} = \frac{39}{10}$

$\Rightarrow 10a + 10ar + 10ar^2 = 39r$ ...(3)

Substituting a = 1 in 3 we get

$\Rightarrow 10(1) + 10(1)r + 10(1)r^2 = 39r$

$\Rightarrow 10r^2 – 29r + 10 = 0$

$\Rightarrow 10r^2 – 25r – 4r + 10 = 0$...(4)

$\Rightarrow 5r(2r – 5) – 2(2r – 5) = 0$

$\therefore r = \frac{2}{5}$ or $r = \frac{5}{2}$

$\therefore$ Now the equation will be

$\Rightarrow \frac{1}{\frac{2}{5}}, 1, 1 \times \frac{2}{5}$ or $\frac{1}{\frac{5}{2}}, 1, 1 \times \frac{5}{2}$

$\Rightarrow \frac{5}{2}, 1, \frac{2}{5}$ or $\frac{2}{5}, 1, \frac{5}{2}$

$\therefore$ The three numbers are $\frac{5}{2}, 1, \frac{2}{5}$.

6. Question

The sum of three numbers in G.P. is 14. If the first two terms are each increased by 1 and the third term decreased by 1, the resulting numbers are in A.P. Find the numbers.

Answer

Let the three numbers be $\frac{a}{r}, a, ar$.

$\therefore$ According to the question

$\Rightarrow \frac{a}{r} + a + ar = 14$

$\Rightarrow a + ar + ar^2 = 14r$...(1)

First two terms are increased by 1, and third decreased by 1

$\therefore \frac{a}{r} + 1, a + 1, ar - 1$

The above sequence is in AP.

We know in AP.

2b = a + c

$\therefore 2(a + 1) = ar - 1 + \frac{a}{r} + 1$

$\Rightarrow 2a + 2 = \frac{ar^2 + a}{r}$

⇒ $2ar + 2r = ar^2 + a$

⇒ $ar^2 - 2ar + a = 2r$ ...(2)

Dividing 1 by 2 we get

⇒ $\frac{a+ar+ar^2}{ar^2-2ar+a} = \frac{14r}{2r}$

⇒ $\frac{1+r+r^2}{r^2-2r+1} = 7$

⇒ $1 + r + r^2 = 7r^2 - 14r + 7$

⇒ $6r^2 - 15r - 6 = 0$

⇒ $6r^2 - 12r - 3r - 6 = 0$

⇒ $6r(r - 2) - 3(r - 2) = 0$

⇒ $(6r - 3)(r - 2) = 0$

⇒ r = 2 or r = 1/2.

Substituting r = 2 in 2 we get

⇒ $a(2)^2 - 2a(2) + a = 2(2)$

⇒ 4a – 4a + a = 4

⇒ a = 4

Substituting r = 1/2 in 2 we get

⇒ $a(1/2)^2 - 2a(1/2) + a = 2(1/2)$

⇒ a = 4

∴ substituting a and r we get the numbers as 2,4,8.

7. Question

The product of three numbers in G.P. is 216. If 2, 8, 6 be added to them, the results are in A.P. Find the numbers.

Answer

Let the three numbers be $\frac{a}{r}, a, ar$.

∴ According to the question

⇒ $\frac{a}{r} \times a \times ar = 216$ ...(1)

⇒ $a^3 = 216$

⇒ a = 6

2,8,6 is added to them

∴ $\frac{a}{r} + 2, a + 8, ar + 6$

The above sequence is in AP.

We know in AP.

2b = a + c

⇒ $2(a + 8) = \frac{a}{r} + 2 + ar + 6$

Substituting a = 6 in above equation we get,

$\Rightarrow 2(6 + 8) = \frac{6}{r} + 2 + 6r + 6$

$\Rightarrow 28 = \frac{6 + 6r^2 + 8r}{r}$

⇒ $28r = 6 + 6r^2 + 8r$

⇒ $6r^2 – 20r + 6 = 0$

⇒ $6r^2 – 18r – 2r – 6 = 0$

⇒ 6r(r – 3) – 2(r – 3) = 0

⇒ (6r – 2) (r – 3) = 0

⇒ r = 3 or r = 1/3 .

∴ Now the equation will be

$\Rightarrow \frac{6}{3}, 6, 6 \times 3 \; or \; \frac{6}{\frac{1}{3}}, 6, 6 \times \frac{1}{3}$

⇒ 2,6,18 or 18,6,2

∴ The three numbers are 2,6,18.

8. Question

Find three numbers in G.P. whose product is 729 and the sum of their products in pairs is 819.

Answer

Let the three numbers be $\frac{a}{r}, a, ar$.

∴ According to the question

$\Rightarrow \frac{a}{r} \times a \times ar = 729$ ...(1)

From 1 we get

⇒ $a^3 = 729$

∴ a = 9.

$\Rightarrow \frac{a}{r} \times a + a \times ar + ar \times \frac{a}{r} = 819$

⇒ $a + ar + a^2 = 819$

Substituting a = 9 in above equation we get

⇒ 9 + 9r + 81 = 819

⇒ 9r = 729

⇒ r = 81

∴ $\frac{1}{9}, 9, 729$ are the three numbers.

9. Question

The sum of three numbers in G.P. is 21, and the sum of their squares is 189. Find the numbers.

Answer

Let the three numbers be a, ar, and ar^2

∴ According to the question

⇒ $a + ar + ar^2 = 21$

$a(1 + r + r^2) = 21$

Squaring both sides we get,

$a^2(1 + r + r^2)^2 = (21)^2$....(1)

And from the second condition,

$a^2 + a^2r^2 + a^2r^4 = 189$

$a^2(1 + r^2 + r^4) = 189$......(2)

Dividing both the equations we get,

$$\frac{a^2(1+r+r^2)^2}{a^2(1+r^2+r^4)} = \frac{21}{9}$$

$$\frac{1+r+r^2}{r^2-r+1} = \frac{7}{3}$$

Cross multiplying we get,

$3 + 3r + 3r^2 = 7r^2 – 7r + 7$

$4r^2 – 10r + 4 = 0$

$2r^2 – 5r + 2 = 0$

Factorizing the quadratic equation such that, on multiplication, we get 4 and on the addition, we get 5. So,

$2r^2 – (4r + r) + 2 = 0$

$2r(r – 2) –1(r – 2) = 0$

$(2r – 1)(r – 2) = 0$

$r = 1/2$, $r = 2$

Putting the value of r in equation 2 we get,

At r = 2,

$a^2(1 + r^2 + r^4) = 189$

$a^2(1 + 4 + 16) = 189$

$a^2 = \frac{189}{21}$

$a^2 = 9$

$a = \pm 3$

At r = 1/2

$$a^2\left(1+\frac{1}{4}+\frac{1}{16}\right) = 189$$

$$a^2\left(\frac{21}{16}\right) = 189$$

$$a^2 = \frac{189 \times 16}{21}$$

$a^2 = 9 \times 16$

$a = 3 \times 4 = 12$

The numbers are:

1, 9, 81 or 81, 9, 1

Exercise 20.3

1 A. Question

Find the sum of the following geometric progressions :

2, 6, 18, ... to 7 terms

Answer

Common Ratio = r = $\frac{6}{2} = 3$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r-1}$ ...(1)

⇒ a = 2, r = 3, n = 7

∴ Substituting the above values in (1) we get

⇒ $\frac{2(3^7 - 1)}{3-1}$

⇒ $3^7 - 1$

⇒ 2186

1 B. Question

Find the sum of the following geometric progressions :

1, 3, 9, 27, ... to 8 terms

Answer

Common Ratio = r = $\frac{3}{1} = 3$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r-1}$ ...(1)

⇒ a = 1, r = 3, n = 8

∴ Substituting the above values in (1) we get

⇒ $\frac{1(3^8 - 1)}{3-1}$

⇒ $(3^8 - 1)/2$

⇒ 3280

1 C. Question

Find the sum of the following geometric progressions :

$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$

Answer

Common Ratio = r = $\frac{\frac{-1}{2}}{1} = \frac{-1}{2}$

∴ Sum of GP till infinity = $\frac{a}{1-r}$ ...(1)

⇒ a = 1, $r = \frac{-1}{2}$

∴ Substituting the above values in (1), we get,

⇒ $\frac{1}{1-\left(\frac{-1}{2}\right)}$

$\Rightarrow \frac{1}{\frac{3}{2}}$

$\Rightarrow \frac{2}{3}$

1 D. Question

Find the sum of the following geometric progressions :

$\left(a^2 - b^2\right), \left(a - b\right), \left(\frac{a-b}{a+b}\right), \ldots$ to n terms

Answer

Common Ratio = r = $\frac{(a^2 - b^2)}{(a-b)} = \frac{(a+b)(a-b)}{(a-b)} = a + b.$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r-1}$ ...(1)

⇒ a = $(a^2 - b^2)$, r = (a + b), n = n

∴ Substituting the above values in (1) we get

$\Rightarrow \frac{(a^2 - b^2)((a+b)^n - 1)}{a + b - 1}$

1 E. Question

Find the sum of the following geometric progressions :

4, 2, 1, $\frac{1}{2}$..... to 10 terms.

Answer

Common Ratio = r = $\frac{2}{4} = \frac{1}{2}$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r-1}$ ...(1)

⇒ a = 4, $r = \frac{1}{2}$, n = 10

∴ Substituting the above values in (1) we get,

$\Rightarrow \frac{4\left(\left(\frac{1}{2}\right)^8 - 1\right)}{\frac{1}{2} - 1}$

$\Rightarrow \frac{-4 \times 255 \times 2}{-1 \times 256}$

$\Rightarrow \frac{255}{32}$

2 A. Question

Find the sum of the following geometric series :

0.15 + 0.015 + 0.0015 + ... to 8 terms;

Answer

Given expression can also be written as

$\Rightarrow \frac{15}{100}, \frac{15}{10^3}, \frac{15}{10^4}, \ldots.$

Common Ratio = r = $\frac{\frac{15}{100}}{\frac{15}{1000}} = 10$

∴ Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$ ...(1)

⇒ $a = \frac{15}{100}$, r = 10, n = 8

∴ Substituting the above values in (1) we get

⇒ $\frac{\frac{15}{100}(10^8-1)}{10-1}$

⇒ $\frac{15\times(10^8-1)}{100\times9}$

⇒ $\frac{(10^8-1)}{60}$

2 B. Question

Find the sum of the following geometric series :

$\sqrt{2}+\frac{1}{\sqrt{2}}+\frac{1}{2\sqrt{2}}+\ldots$ to 8 terms ;

Answer

Common Ratio = r = $\frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2}$

∴ Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$ ...(1)

⇒ a = √2 , $r = \frac{1}{2}$, n = 8

∴ Substituting the above values in (1) we get

⇒ $\frac{\sqrt{2}\left(\left(\frac{1}{2}\right)^8-1\right)}{\frac{1}{2}-1}$

⇒ $\frac{-\sqrt{2}\times255\times2}{-1\times256}$

⇒ $\frac{255\sqrt{2}}{128}$

2 C. Question

Find the sum of the following geometric series :

$\frac{2}{9}-\frac{1}{3}+\frac{1}{2}-\frac{3}{4}+\ldots$ to 5 terms ;

Answer

Common Ratio = r = $\frac{\frac{2}{9}}{\frac{-1}{3}} = \frac{-2}{3}$

∴ Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$ ...(1)

⇒ $a = \frac{2}{9}$, $r = \frac{-2}{3}$, n = 5

∴ Substituting the above values in (1) we get,

⇒ $\frac{\frac{2}{9}\left(\left(\frac{-2}{3}\right)^5-1\right)}{\frac{-2}{3}-1}$

⇒ $\frac{2}{9}$

$\Rightarrow \frac{255}{32}$

2 D. Question

Find the sum of the following geometric series :

$(x + y) + (x^2 + xy + y^2) + (x^3 + x^2 y + xy^2 + y^3) + ...$ to n terms ;

Answer

Let $S_n = (x + y) + (x^2 + xy + y^2) + (x^3 + x^2 y + xy^2 + y^3) + ...$ to n terms

Multiplying and dividing by (x – y) we get,

$$S_n = \frac{1}{x-y}[(x+y)(x-y) + (x^2+xy+y^2)(x-y) \dots \dots \text{upto n terms}$$

$(x - y) S_n = (x^2 - y^2) + x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$..upto n terms

$(x - y) S_n = (x^2 + x^3 + x^4 + \ldots$ n terms$) - (y^2 + y^3 + y^4 + \ldots$ n terms$)$

We know that,

Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$

We have two G.Ps in above sum, so,

$(x - y)S_n = x^2\left[\frac{x^n-1}{x-1}\right] - y^2\left[\frac{y^n-1}{y-1}\right]$

Hence, $S_n = \frac{1}{x-y}\left[x^2\left(\frac{x^n-1}{x-1}\right) - y^2\left(\frac{y^n-1}{y-1}\right)\right]$

2 E. Question

Find the sum of the following geometric series :

$\frac{3}{5} + \frac{4}{5^2} + \frac{3}{5^3} + \frac{4}{5^4} + \ldots$ to 2n terms;

Answer

Common Ratio = $r = \frac{\frac{3}{5}}{\frac{4}{5^2}} = \frac{15}{4}$

$\therefore$ Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$ ...(1)

$\Rightarrow a = \frac{3}{5}, r = \frac{15}{4}$, n = 2n

$\therefore$ Substituting the above values in (1) we get

$\Rightarrow \frac{\frac{3}{5}\left(\left(\frac{15}{4}\right)^{2n}-1\right)}{\frac{15}{4}-1}$

$\Rightarrow \frac{3\times\left(\left(\frac{15}{4}\right)^{2n}-1\right)\times 4}{5\times 11}$

$\Rightarrow \frac{12\times\left(\left(\frac{15}{4}\right)^{2n}-1\right)}{55}$

2 F. Question

Find the sum of the following geometric series :

$$\frac{a}{1+i}+\frac{a}{(1+i)^2}+\frac{a}{(1+i)^3}+\ldots+\frac{a}{(1+i)^n}.$$

Answer

Common Ratio = $r = \frac{\frac{a}{1+i}}{\frac{a}{(1+i)^2}} = 1 + i$

∴ Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$ ...(1)

⇒ $a = \frac{a}{1+i}$, r = 1 + i, n = n

∴ Substituting the above values in (1) we get

⇒ $\frac{\frac{a}{1+i}((1+i)^n-1)}{1+i-1}$

⇒ $\frac{a((1+i)^n-1)}{(1+i)\times i}$

⇒ $\frac{a((1+i)^n-1)}{i-1}$

2 G. Question

Find the sum of the following geometric series :

1, - a, a^2, - a^3, ... to n terms (a ≠1)

Answer

Common Ratio = $r = \frac{-a}{1} = -a$

∴ Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$ ...(1)

⇒ a = 1, r = - a, n = n

∴ Substituting the above values in (1) we get

⇒ $\frac{1((-a)^n-1)}{-a-1}$

⇒ $\frac{-1((-a)^n-1)}{a+1}$

2 H. Question

Find the sum of the following geometric series :

x^3, x^5, x^7, ... to n terms

Answer

Common Ratio = r = $\frac{x^5}{x^3} = x^2$

∴ Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$ ...(1)

⇒ a = x^3, r = x^2, n = n

∴ Substituting the above values in (1) we get

⇒ $\frac{x^3((x^2)^n-1)}{x^2-1}$

⇒ $\frac{x^{3+2n}-x^3}{x^2-1}$

2 I. Question

Find the sum of the following geometric series :

$\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \ldots$ to n terms

Answer

Common Ratio = r = $\frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r-1}$ ...(1)

⇒ a = √7 , r = √3 , n = n

∴ Substituting the above values in (1) we get

⇒ $\frac{\sqrt{7}((\sqrt{3})^n - 1)}{\sqrt{3}-1}$

⇒ $\frac{\sqrt{7}((\sqrt{3})^n - 1)}{\sqrt{3}-1}$

3 A. Question

Evaluate the following :

$$\sum_{n=1}^{11} (2 + 3^n)$$

Answer

The given expression can also be written as

⇒ $\sum_{n=1}^{11} 2 + \sum_{n=1}^{11} 3^n$...(1)

⇒ $\sum_{n=1}^{11} 2 = 22$ [∵ $\sum_{n=1}^{k} a = ka$]

Now this term is in GP.

3, 9, 27...to 11 terms

∴ Common Ratio = r = $\frac{9}{3} = 3$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r-1}$ ...(2)

⇒ a = 3, r = 3, n = 11

∴ Substituting the above values in (2) we get

⇒ $\frac{3(3^{11} - 1)}{3-1}$

⇒ 531438/2

⇒ 265719.

Now, Adding both these we will get the required solution.

∴ 22 + 265719

⇒ 265741

3 B. Question

Evaluate the following :

$$\sum_{k=1}^{n} (2^k + 3^{k-1})$$

Answer

The given expression can also be written as

$\Rightarrow \sum_{k=1}^{n} 2^k + \sum_{k=1}^{n} 3^{(k-1)}$...(1)

$\Rightarrow \sum_{k=1}^{n} 2^k$

Now this term is in GP.

2, 4, 8...to n terms

∴ Common Ratio = $r = \frac{4}{2} = 2$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r-1}$ ...(2)

⇒ a = 2, r = 2, n = n

∴ Substituting the above values in (2) we get,

$\Rightarrow \frac{2(2^n - 1)}{2-1}$

$\Rightarrow 2^{n+1} - 2.$

$\Rightarrow \sum_{k=1}^{n} 3^{(k-1)}$

Now this term is in GP.

1, 3, 9...to n terms

∴ Common Ratio = $r = \frac{3}{1} = 3$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r-1}$ ...(2)

⇒ a = 1, r = 3, n = n

∴ Substituting the above values in (2) we get,

$\Rightarrow \frac{1(3^n - 1)}{3-1}$

$\Rightarrow \frac{1}{2}(3^n - 1)$

Now, Adding both these we will get the required solution.

$\Rightarrow 2^{n+1} - 2 + \frac{1}{2}(3^n - 1)$

$\Rightarrow \frac{1}{2}(3^n - 1) + 2^{n+1} - 2$

3 C. Question

Evaluate the following :

$$\sum_{n=2}^{10} 4^n$$

Answer

$\Rightarrow \sum_{n=2}^{10} 4^n$

Now this term is in GP.

16, 64, 256...to 10 terms

∴ Common Ratio = $r = \frac{64}{16} = 4$

∴ Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$ ...(1)

⇒ a = 16, r = 4, n = 10

∴ Substituting the above values in (1) we get

⇒ $\frac{16(4^{10}-1)}{4-1}$

⇒ 5592400

4 A. Question

Find the sum of the following series :

5 + 55 + 555 + ... to n terms.

Answer

Taking 5 in common we get

5(1 + 11 + 111 + ...n)

Now Multiply and Divide by 9 we get

⇒ $\frac{5}{9} \times 9(1 + 11 + 111 + ...)$

⇒ $\frac{5}{9}(9 + 99 + 999 + ...)$

⇒ $\frac{5}{9}((10-1) + (100-1) + .. + (10^n - 1)$

⇒ $\frac{5}{9}(10 + 100 + 1000.. + 10^n) - (1 + 1 + 1 + ..n)$

Now First term is in GP.

10, 100, 1000...to n terms

∴ Common Ratio = r = $\frac{100}{10} = 10$

∴ Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$ ...(1)

⇒ a = 10, r = 10, n = n

∴ Substituting the above values in (1) we get

⇒ $\frac{10(10^n-1)}{10-1}$

⇒ $\frac{10}{9}(10^n - 1)$

For the second term the summation is n.

∴ $\frac{5}{9}\left(\frac{10}{9}(10^n - 1)\right) - n$

⇒ $\frac{50}{81}(10^n - 1) - \frac{5}{9}n.$

4 B. Question

Find the sum of the following series :

7 + 77 + 777 + ... to n terms.

Answer

Taking 7 in common we get

7(1 + 11 + 111 + ...n)

Now Multiply and Divide by 9 we get

$\Rightarrow \frac{7}{9} \times 9(1 + 11 + 111 + ...)$

$\Rightarrow \frac{7}{9}(9 + 99 + 999 + ...)$

$\Rightarrow \frac{7}{9}((10 - 1) + (100 - 1) + .. + (10^n - 1)$

$\Rightarrow \frac{7}{9}(10 + 100 + 1000.. + 10^n) - (1 + 1 + 1 + ..n)$

Now First term is in GP.

10, 100, 1000...to n terms

∴ Common Ratio = r = $\frac{100}{10} = 10$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r - 1}$ ...(1)

⇒ a = 10, r = 10, n = n

∴ Substituting the above values in (1) we get

$\Rightarrow \frac{10(10^n - 1)}{10 - 1}$

$\Rightarrow \frac{10}{9}(10^n - 1)$

For the second term the summation is n.

$\therefore \frac{7}{9}\left[\left(\frac{10}{9}(10^n - 1)\right) - n\right]$

$\Rightarrow \frac{70}{81}(10^n - 1) - \frac{7}{9}n.$

4 C. Question

Find the sum of the following series :

9 + 99 + 999 + … to n terms.

Answer

Taking 9 in common we get

9(1 + 11 + 111 + ...n)

Now Multiply and Divide by 9 we get,

$\Rightarrow \frac{9}{9} \times 9(1 + 11 + 111 + ...)$

$\Rightarrow \frac{9}{9}(9 + 99 + 999 + ...)$

$\Rightarrow \frac{9}{9}((10 - 1) + (100 - 1) + .. + (10^n - 1)$

$\Rightarrow \frac{9}{9}(10 + 100 + 1000.. + 10^n) - (1 + 1 + 1 + ..n)$

Now first term is in GP.

10, 100, 1000...to n terms

∴ Common Ratio = r = $\frac{100}{10} = 10$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r - 1}$ ...(1)

⇒ a = 10, r = 10, n = n

∴ Substituting the above values in (1) we get

$\Rightarrow \frac{10(10^n-1)}{10-1}$

$\Rightarrow \frac{10}{9}(10^n - 1)$

For the second term the summation is n.

$\therefore \left(\frac{10}{9}(10^n - 1)\right) - n$

$\Rightarrow \frac{10}{9}(10^n - 1) - n.$

4 D. Question

Find the sum of the following series :

0.5 + 0.55 + 0.555 + …. to n terms

Answer

Let

S = 0.5 + 0.55 + 0.555 + ...n terms

Taking 5 as common we get,

S = 5(0.1 + 0.11 + 0.111 + ...nterms)

Multiply and divide by 9

$\Rightarrow \frac{5}{9} \times 9(0.1 + 0.11 + 0.111 + ...)$

$\Rightarrow \frac{5}{9}(0.9 + 0.99 + 0.999 + ...)$

$\Rightarrow \frac{5}{9}((1-0.1) + (1-0.01) + .. + (1-0.001) + ..\text{nterms}$

$\Rightarrow \frac{5}{9}[(1 + 1 + 1 + ..n) - (0.1 + 0.01 + 0.001 + ..\text{nterms}$

Now 1 + 1 + 1 + ..n = n

For 0.1 + 0.01 + 0.001 + ..nterms

∴ Common Ratio = r = $\frac{0.01}{0.1} = \frac{1}{10}$

∴ Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$ ...(1)

⇒ a = 0.1, r = $\frac{1}{10}$, n = n

∴ Substituting the above values in (1) we get

$\Rightarrow \frac{0.1\left(\left(\frac{1}{10}\right)^n-1\right)}{\frac{1}{10}-1}$

$\Rightarrow \frac{-1}{9}\left(\left(\frac{1}{10}\right)^n - 1\right)$

For second term the summation is n.

$\therefore \frac{5}{9}\left[n + \left(\frac{-1}{9}\left(\left(\frac{1}{10}\right)^n - 1\right)\right)\right]$

$\Rightarrow \frac{5}{9}n - \frac{5}{81}\left[\left(\frac{1}{10}\right)^n - 1\right]$

4 E. Question

Find the sum of the following series :

0.6 + 0.66 + 0.666 + …. to n terms.

Answer

Let

S = 0.6 + 0.66 + 0.666 + …..n terms

Taking 6 as common we get

S = 6(0.1 + 0.11 + 0.111 + ...nterms)

Multiply and divide by 9

$\Rightarrow \frac{6}{9} \times 9(0.1 + 0.11 + 0.111 + ...)$

$\Rightarrow \frac{6}{9}(0.9 + 0.99 + 0.999 + ...)$

$\Rightarrow \frac{6}{9}((1 - 0.1) + (1 - 0.01) + .. + (1 - 0.001) + ..\text{nterms}$

$\Rightarrow \frac{6}{9}[(1 + 1 + 1 + ..n) - (0.1 + 0.01 + 0.001 + ..\text{nterms}$

Now 1 + 1 + 1 + ..n = n

For 0.1 + 0.01 + 0.001 + ..nterms

∴ Common Ratio = r = $\frac{0.01}{0.1} = \frac{1}{10}$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r - 1}$ ...(1)

⇒ a = 0.1, r = $\frac{1}{10}$, n = n

∴ Substituting the above values in (1) we get

$\Rightarrow \frac{0.1\left(\left(\frac{1}{10}\right)^n - 1\right)}{\frac{1}{10} - 1}$

$\Rightarrow \frac{-1}{9}\left(\left(\frac{1}{10}\right)^n - 1\right)$

For second term the summation is n.

$\therefore \frac{6}{9}\left[n + \left(\frac{-1}{9}\left(\left(\frac{1}{10}\right)^n - 1\right)\right)\right]$

$\Rightarrow \frac{6}{9}n - \frac{6}{81}\left[\left(\frac{1}{10}\right)^n - 1\right]$

5. Question

How many terms of the G.P. $3, \frac{3}{2}, \frac{3}{4}, \ldots$. Be taken together to make $\frac{3069}{512}$?

Answer

Given:

Sum of GP = $\frac{3069}{512}$

∴ Common Ratio = r = $\frac{\frac{3}{2}}{3} = \frac{1}{2}$

a = 3

To find: Number of terms = n.

Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$

$\Rightarrow \frac{3069}{512} = \frac{3\left(\left(\frac{1}{2}\right)^n-1\right)}{\frac{1}{2}-1}$

$\Rightarrow \frac{3069}{512\times3\times2} = 1-\left(\frac{1}{2}\right)^n$

$\Rightarrow \frac{3069}{3072} - 1 = -\left(\frac{1}{2}\right)^n$

$\Rightarrow \frac{-3}{3072} = -\left(\frac{1}{2}\right)^n$

$\Rightarrow \frac{1}{1024} = \left(\frac{1}{2}\right)^n$

$\Rightarrow \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^n$

∴ n = 10.

6. Question

How many terms of the series 2 + 6 + 18 + …. Must be taken to make the sum equal to 728?

Answer

Given:

Sum of GP = 728

∴ Common Ratio = r = $\frac{6}{2} = 3$

a = 2

To find: Number of terms = n.

Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$

$\Rightarrow 728 = \frac{2(3^n-1)}{3-1}$

⇒ 728 = 3^n - 1

⇒ 729 = 3^n

⇒ $3^6 = 3^n$

∴ n = 6.

7. Question

How many terms of the sequence $\sqrt{3}, 3, 3\sqrt{3}...$ must be taken to make the sum $39+13\sqrt{3}$?

Answer

Given:

Sum of GP = 39 + 13√3

∴ Common Ratio = r = $\frac{3}{\sqrt{3}} = \sqrt{3}$

a = √3

To find: Number of terms = n.

Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$

$\Rightarrow 39 + 13\sqrt{3} = \frac{\sqrt{3}(3^n - 1)}{\sqrt{3} - 1}$

$\Rightarrow (39 + 13\sqrt{3})(\sqrt{3} - 1) = \sqrt{3}(3^n - 1)$

$\Rightarrow 39\sqrt{3} - 39 + 39 - 13\sqrt{3} = \sqrt{3}(3^n - 1)$

$\Rightarrow 26\sqrt{3} = \sqrt{3}(3^n - 1)$

$\Rightarrow 27 = 3^n$

$\Rightarrow n = 3.$

8. Question

The sum of n terms of the G.P. 3, 6, 12, ... is 381. Find the value of n.

Answer

Given:

Sum of GP = 381

∴ Common Ratio = r = $\frac{6}{3} = 2$

a = 3

To find: Number of terms = n.

Sum of GP for n terms = $\frac{a(r^n - 1)}{r - 1}$

$\Rightarrow 381 = \frac{3(2^n - 1)}{2 - 1}$

$\Rightarrow 127 = 2^n - 1$

$\Rightarrow 128 = 2^n$

$\Rightarrow 2^7 = 2^n$

∴ n = 7.

9. Question

The common ratio of a G.P. is 3, and the last term is 486. If the sum of these terms be 728, find the first term.

Answer

Given: Common Ratio = 3

Sum of GP = 728

Sum of GP for n terms = $\frac{a(r^n - 1)}{r - 1}$

Last term say it be n

∴ $T_n = ar^{n-1}$

$\Rightarrow 486 = a3^{n-1}$

$\Rightarrow 486 = a.\frac{3^n}{3}$

$\Rightarrow 1458 = a.3^n$ ...(1)

$\Rightarrow 728 = \frac{a(r^n - 1)}{r - 1}$

$\Rightarrow 728 = \frac{a(3^n - 1)}{2}$

⇒ $1456 = a.3^n - a$...(2)

Subtracting 1 from 2 we get

⇒ $1458 - 1456 = a.3^n - a.3^n + a$

⇒ a = 2.

∴ The first term is 2.

10. Question

The ratio of the sum of the first three terms is to that of the first 6 terms of a G.P. is 125 : 152. Find the common ratio.

Answer

Sum of GP for n terms = $\frac{a(r^n - 1)}{r - 1}$

Sum of GP of 3 terms = 125

⇒ $125 = \frac{a(r^n - 1)}{r - 1}$

⇒ $125 = \frac{a(r^3 - 1)}{r - 1}$...(1)

Sum of GP of 6 terms = 152

⇒ $152 = \frac{a(r^n - 1)}{r - 1}$

⇒ $152 = \frac{a(r^6 - 1)}{r - 1}$...(2)

Dividing 1 by 2 we get

⇒ $\frac{125}{152} = \frac{\frac{a(r^3 - 1)}{r - 1}}{\frac{a(r^6 - 1)}{r - 1}}$

⇒ $\frac{125}{152} = \frac{r^3 - 1}{r^6 - 1}$

⇒ $\frac{125}{152} = \frac{r^3 - 1}{(r^3 - 1)(r^3 + 1)}$

⇒ $125r^3 + 125 = 152$

⇒ $r^3 = \frac{27}{125}$

⇒ $r = \frac{3}{5}$

11. Question

The 4th and 7th terms of a G.P. are $\frac{1}{27}$ and $\frac{1}{729}$ respectively. Find the sum of n terms of the G.P.

Answer

Nth term of GP is $T_n = ar^{n - 1}$

⇒ $\frac{1}{27} = a.r^3$...(1)

Nth term of GP is $T_n = ar^{n - 1}$

⇒ $\frac{1}{729} = a.r^6$...(2)

Divide (1) by (2)

$\Rightarrow \frac{\frac{1}{27}}{\frac{1}{729}} = \frac{ar^3}{ar^6}$

$\Rightarrow 27 = \frac{1}{r^3}$

$\Rightarrow r = \frac{1}{3}$

Substituting in 1 we get

a = 1

Sum of GP for n terms = $\frac{a(r^n - 1)}{r - 1}$

a = 1, r = $\frac{1}{3}$, n = n.

$\Rightarrow \frac{1\left(\left(\frac{1}{3}\right)^n - 1\right)}{\frac{1}{3} - 1}$

$\Rightarrow \frac{3\left(\frac{1}{3}\right)^n - 1}{-2}$

$\Rightarrow \frac{-3\left(\frac{1}{3}\right)^n - 1}{2}$

12. Question

Find the sum :

$$\sum_{n=1}^{10}\left\{\left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{5}\right)^{n+1}\right\}.$$

Answer

We can write the above expression as:

The given expression can also be written as

$\Rightarrow \sum_{n=1}^{10}\left(\frac{1}{2}\right)^{n-1} + \sum_{n=1}^{n}\left(\frac{1}{5}\right)^{(n+1)}$...(1)

Now for the first term is in GP.

$\Rightarrow 1, \frac{1}{2}, \frac{1}{4},$ ...upto 10 terms

∴ Common Ratio = r = $\frac{\frac{1}{2}}{1} = \frac{1}{2}$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r - 1}$ ...(2)

$\Rightarrow$ a = 1, $r = \frac{1}{2}$, n = 10

∴ Substituting the above values in (2) we get,

$\Rightarrow \frac{1(\left(\frac{1}{2}\right)^{10} - 1)}{\frac{1}{2} - 1}$

$\Rightarrow 2.\left(\frac{\frac{1}{1024} - 1}{-1}\right)$

$\Rightarrow \frac{1023}{512}$

Now for the second term is in GP.

$\Rightarrow \frac{1}{25}, \frac{1}{125}, \frac{1}{625}$, ...upto 10 terms

∴ Common Ratio = $r = \frac{\frac{1}{125}}{\frac{1}{25}} = \frac{1}{5}$

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r-1}$ ...(2)

$\Rightarrow = \frac{1}{25}, r = \frac{1}{5}$, n = 10

∴ Substituting the above values in (2) we get

$\Rightarrow \frac{\frac{1}{25}\left(\left(\frac{1}{5}\right)^{10} - 1\right)}{\frac{1}{5} - 1}$

$\Rightarrow \frac{-(\left(\frac{1}{5}\right)^{10} - 1)}{20}$

Total sum

$\Rightarrow \frac{1023}{512} + \frac{-(\left(\frac{1}{5}\right)^{10} - 1)}{20}$

13. Question

The fifth term of a G.P. is 81 whereas its second term is 24. Find the series and sum of its first eight terms.

Answer

Nth term of GP is $T_n = ar^{n-1}$

$T_5 = a.r^4$

$81 = a.r^4$...(1)

Nth term of GP is $T_n = ar^{n-1}$

$T_2 = a.r^1$

$24 = a.r^1$...(2)

Divide 1 by 2

$\Rightarrow \frac{81}{24} = \frac{a.r^4}{a.r^1}$

$\Rightarrow r^3 = \frac{27}{8}$

$\Rightarrow r = \frac{3}{2}$

Substituting r in 2 we get,

a = 16

∴ The series is 16, 24, 54, ...

∴ Sum of GP for n terms = $\frac{a(r^n - 1)}{r-1}$ ...(1)

$\Rightarrow a = 16, r = \frac{3}{2}, n = 8$

∴ Substituting the above values in (1) we get,

$\Rightarrow \frac{16\left(\left(\frac{3}{2}\right)^{8} - 1\right)}{\frac{3}{2} - 1}$

$\Rightarrow \frac{16\times2\times6305}{256}$

$\Rightarrow$ 788.125

14. Question

If S_1, S_2, S_3 be respectively the sums of n, 2n, 3n terms of a G.P., then prove that $S_1^2 + S_2^2 = S_1(S_2 + S_3)$

Question May be wrong.

Answer

Sum of GP for n terms $S_1 = \frac{a(r^n-1)}{r-1}$

$= \frac{a}{r-1}(r^n - 1)$ ...(1)

Sum of GP for 2n terms $S_2 = \frac{a(r^{2n}-1)}{r-1}$

$= \frac{a}{r-1}(r^{2n} - 1)$ ...(2)

Sum of GP for 3n terms $S_3 = \frac{a(r^{3n}-1)}{r-1}$ ...(3)

$= \frac{a}{r-1}(r^{3n} - 1)$

Let $\frac{a}{r-1} = k$

∴ 1, 2 and 3 becomes

$S_1 = K(r^n - 1)$

$S_2 = K(r^{2n} - 1)$

$S_3 = K(r^{3n} - 1)$

∴ $S_1^2 + S^2{}_2 = k^2(r^n - 1)^2 + k^2(r^{2n} - 1)^2$

$= k^2(r^{2n} + 1 - 2.r^n + r^{4n} + 1 - 2.r^{2n})$

$= k^2(r^{4n} - r^{2n} - 2r^n + 2)$

L.H.S $= k^2(r^{4n} - r^{2n} - 2r^n + 2)$

$S_1(S_2 + S_3) = K(r^n - 1)[\ (K(r^{2n} - 1) + K(r^{3n} - 1))]$

$= K^2(r^n - 1)[r^{2n} + r^{3n} - 2]$

$= k^2(r^{4n} - r^{2n} - 2r^n + 2)$

Hence, Proved.

15. Question

Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from (n + 1)th to (2n)th term is $\frac{1}{r^n}$.

Answer

First n terms of GP be a, ar, ar^2..., ar^{n-1}

From n + 1 term,

GP = ar^n, ar^{n+1}, ..., ar^{2n-1}

Sum of GP for n terms $S_1 = \frac{a(r^n-1)}{r-1}$

Sum of GP for next terms $S_2 = \frac{ar^n(r^n-1)}{r-1}$

$\therefore \frac{S_1}{S_2} = \frac{\frac{a(r^n-1)}{r-1}}{\frac{ar^n(r^n-1)}{r-1}}$

$\Rightarrow \frac{S_1}{S_2} = \frac{a}{ar^n}$

$\Rightarrow \frac{S_1}{S_2} = \frac{1}{r^n}$

Hence, Proved.

16. Question

If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are the roots $x^2 - 12x + q = 0$, where a, b, c, d form a G.P. Prove that (q + p) : (q – p) = 17 : 15.

Answer

Given that a and b are roots of $x^2 - 3x + p = 0$

⇒ a + b = 3 and ab = p ...(i)

It is given that c and d are roots of $x^2 - 12x + q = 0$

⇒ c + d = 12 and cd = q...(ii)

Also given that a, b, c, d are in G.P.

Let a, b, c, d be the first four terms of a G.P.

⇒ a = a, b = ar c = ar^2 d = ar^3

Now,

∴a + b = 3

⇒ a + ar = 3

⇒ a(1 + r) = 3...(iii)

c + d = 12

⇒ $ar^2 + ar^3 = 12$

⇒ $ar^2(1 + r) = 12$.....(iv)

From (iii) and (iv) we get

$3.r^2 = 12$

⇒ $r^2 = 4$

⇒ r = ±2

Substituting the value of r in (iii) we get a = 1

⇒ b = ar = 2∴

c = $ar^2 = 2^2 = 4$

d = $ar^3 = 2^3 = 8$

⇒ ab = p = 2and cd = 4×8 = 32

⇒ q + p = 32 + 2 = 34 and q−p = 32−2 = 30

$\Rightarrow$ q + p:q−p = 34:30 = 17:15

Hence, proved.

17. Question

How many terms of the G.P. $3, \frac{3}{2}, \frac{3}{4}$.....are needed to give the sum $\frac{3069}{512}$?

Answer

Given:

Sum of GP $= \frac{3069}{512}$

$\therefore$ Common Ratio = r $= \frac{\frac{3}{2}}{3} = \frac{1}{2}$

a = 3

To find: Number of terms = n.

Sum of GP for n terms = $\frac{a(r^n-1)}{r-1}$

$\Rightarrow \frac{3069}{512} = \frac{3\left(\left(\frac{1}{2}\right)^n-1\right)}{\frac{1}{2}-1}$

$\Rightarrow \frac{3069}{512\times3\times2} = 1-\left(\frac{1}{2}\right)^n$

$\Rightarrow \frac{3069}{3072} - 1 = -\left(\frac{1}{2}\right)^n$

$\Rightarrow \frac{-3}{3072} = -\left(\frac{1}{2}\right)^n$

$\Rightarrow \frac{1}{1024} = \left(\frac{1}{2}\right)^n$

$\Rightarrow \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^n$

$\therefore$ n = 10.

18. Question

A person has 2 parents, 4 grandparents, 8 great grand parents, and so on. Find the number of his ancestors during the ten generations preceding his own.

Answer

The number of ancestors are 2, 4, 8, 16....it is in GP common ratio = r $= \frac{4}{2} = 2$a = 2and n = 10Sum of GP for n terms $= \frac{a(r^n-1)}{r-1}$

$\Rightarrow$ a = 2, r = 2, n = 10

$\therefore$ Substituting the above values in (1) we get

$\Rightarrow \frac{2(2^{10}-1)}{2-1}$

$\Rightarrow$ 2(1024 - 1)

$\Rightarrow$ 2(1023)

$\Rightarrow$ 2046

19. Question

If S_1, S_2, ..., S_n are the sums of n terms of n G.P.'s whose first term is 1 in each and common ratios are 1, 2, 3, ..., n respectively, then prove that

$S_1 + S_2 + 2S_3 + 3S_4 + \ldots (n - 1) S_n = 1^n + 2^n + 3^n + \ldots + n^n$.

Answer

S_1 = n [First term is 1, common ratio 1; so sum to n terms = 1 + 1 + 1 + - - = n]ii) $S_2 = (2^n - 1)/(2 - 1) = (2^n - 1)$ iii) $S_3 = (3^n - 1)/2$ iv) $S_4 = (4^n - 1)/3$ .. v) So, $S_1 + S_2 + 2S_3 + 3S_4 + - - - - + (n - 1)S^n = = n + (2^n - 1) + (3^n - 1) + (4^n - 1) + - - - - - - - + (n^n - 1) = n + (- 1 - 1 - 1$ .... to n - 1 terms) $+ (2^n + 3^n + 4^n + ... + n^n) == n - (n - 1) + (2^n + 3^n + 4^n + ... + n^n) = 1 + (2^n + 3^n + 4^n + ... + n^n) = 1^n + 2^n + 3^n + 4^n + ... + n^n$ [Proved]

20. Question

A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying the odd places. Find the common ratio of the G.P.

Answer

Let there be n terms.

$a, ar, ar^2, ar^3, \ldots\ldots ar^{n-2}, ar^{n-1}$

The sum of a G.P. $= \frac{a(r^n-1)}{r-1}$

Odd terms of the sequence are:

$a, ar^2, ar^4 \ldots\ldots ar^{n-2}$

So sum of this series $= \frac{a(r^{(2)\frac{n}{2}}-1)}{r^2-1}$

$\Rightarrow \frac{a(r^n-1)}{r^2-1}$

According to the given problem:

$\Rightarrow \frac{a(r^n-1)}{r-1} = 5 \times \frac{a(r^n-1)}{r^2-1}$

$\Rightarrow \frac{1}{r-1} = 5 \times \frac{1}{(r+1)(r-1)}$

$\Rightarrow r + 1 = 5$

$\Rightarrow r = 4$.

21. Question

Let a_n be the nth term of the G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{an} = \alpha$ and $\sum_{n=1}^{100} a_{an-1} = \beta$, such that α ≠ β.

Prove that the common ratio of the G.P. is α /β

Answer

Let a be the first term and r be the common ratio of the G.P.

Given: $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$

Explanding the summation we get,

$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + a_6 + \ldots\ldots a_{200} = \alpha$

$ar + ar^3 + ar^5 + \ldots\ldots + ar^{199} = \alpha$

$ar\left(\frac{((1-(r^2)^{100})}{1-r^2}\right) = \alpha \ldots\ldots\ldots(1)$

Also,

$\sum_{n=1}^{100} a_{2n-1} = a_1 + a_3 + a_5 + \ldots\ldots + a_{199} = \beta$

$a + ar^2 + ar^4 + \ldots\ldots\ldots + ar^{198} = \beta$

$a\left(\frac{\left((1-(r^2)^{100}\right)}{1-r^2}\right) = \beta$ …………………(2)

From equation (1) and (2), Dividing them we get,

$r = \frac{\alpha}{\beta}$

Hence, Proved.

22. Question

Find the sum of 2n terms of the series whose every even term is ‘a’ times the term before it and every odd term is ‘c’ times the term before it, the first term being unity.

Answer

Let T indicate a term of the progression. $T_1, T_2, T_3, \ldots, T_n, \ldots T_{2n}$ $T_1 = 1$ $T_2 = a$ $T_3 = ca$ $T_4 = c.a^2$ $T_5 = c^2.a^2$ T_k **if k is even** $= a^{\frac{k}{2}}.c^{\frac{k}{2}-1}$

$T_{2n} = a^{\frac{2n}{2}}.c^{\frac{2n}{2}-1}$

$T_{2n} = a^n.c^{n-1}$

$S_{2n} = 1 + a + ca + c.a^2 + c^2.a^2 + c^2.a^3 \ldots\ldots a^n.c^{n-1} = 1 + [a + c.a^2 + c^2.a^3 \ldots + a^n.c^{n-1}] + [ca + c^2.a^2 + c^3.a^3 \ldots + c^{n-1}.a^{n-1}]$ The sum of a G.P. $= \frac{a(r^n-1)}{r-1}$

For $a + c.a^2 + c^2.a^3 \ldots + a^n.c^{n-1}$

a = a, r = ca, n = n

$\Rightarrow \frac{a(ca^n-1)}{ca-1}$

For $[ca + c^2.a^2 + c^3.a^3 \ldots + c^{n-1}.a^{n-1}]$

a = ca, r = ca, n = n

$\Rightarrow \frac{ca(ca^n-1)}{ca-1}$

∴ The required result $= 1 + \frac{a(ca^n-1)}{ca-1} + \frac{ca(ca^n-1)}{ca-1}$

$\Rightarrow \frac{a(ca^n-1) + ca(ca^n-1) + ca-1}{ca-1}$

Exercise 20.4

1 A. Question

Find the sum of the following series to infinity :

$$1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \frac{1}{3^4} + \ldots\infty$$

Answer

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = r = $\frac{\frac{-1}{3}}{1} = \frac{-1}{3}$

Sum of infinite GP = $\frac{a}{1-r}$,where a is the first term and r is the common ratio.

Note: We can only use the above formula if |r|<1

Clearly, a = 1 and r = $\frac{-1}{3}$

⇒ sum = $\frac{1}{1-\left(-\frac{1}{3}\right)} = \frac{3}{4}$

1 B. Question

Find the sum of the following series to infinity :

8 + $4\sqrt{2}$ + 4 + ... ∞

Answer

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = r = $\frac{4\sqrt{2}}{8} = \frac{1}{\sqrt{2}}$

Sum of infinite GP = $\frac{a}{1-r}$,where a is the first term and r is the common ratio.

Note: We can only use the above formula if |r|<1

Clearly, a = 8 and r = $\frac{1}{\sqrt{2}}$

⇒ sum = $\frac{8}{1-\frac{1}{\sqrt{2}}} = \frac{8\sqrt{2}}{\sqrt{2}-1}$

1 C. Question

Find the sum of the following series to infinity :

$2/5 + 3/5^2 + 2/5^3 + 3/5^4$ + ... ∞

Answer

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = r = $\frac{\frac{3}{5^2}}{\frac{2}{5}} = \frac{3}{10}$

Sum of infinite GP = $\frac{a}{1-r}$,where a is the first term and r is the common ratio.

Note: We can only use the above formula if |r|<1

Clearly, a = $\frac{2}{5}$ and r = $\frac{3}{10}$

⇒ sum = $\frac{\frac{2}{5}}{1-\frac{3}{10}} = \frac{4}{7}$

1 D. Question

Find the sum of the following series to infinity :

10 – 9 + 8.1 – 7.29 + ... ∞

Answer

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = r = $\frac{-9}{10}$

Sum of infinite GP = $\frac{a}{1-r}$,where a is the first term and r is the common ratio.

Note: We can only use the above formula if |r|<1

Clearly, a = 10 and r = $\frac{-9}{10}$

⇒ sum = $\frac{10}{1-(-\frac{9}{10})} = \frac{100}{19}$

1 E. Question

Find the sum of the following series to infinity :

$$\frac{1}{3}+\frac{1}{5^2}+\frac{1}{3^3}+\frac{1}{5^4}+\frac{1}{3^5}+\frac{1}{5^6}+.....\infty$$

Answer

We observe that above progression possess a common ratio, but alternatively , adjacent terms are not possessing a common ratio. So, it consists of 2 geometric progressions.

Let, S = $\frac{1}{3}+\frac{1}{5^2}+\frac{1}{3^3}+\frac{1}{5^4}+\frac{1}{3^5}+\frac{1}{5^6}+.....\infty$

⇒ S = $\left(\frac{1}{3}+\frac{1}{3^3}+\frac{1}{3^5}+\cdots\infty\right)+\left(\frac{1}{5^2}+\frac{1}{5^4}+\frac{1}{5^6}+.....\infty\right)$

Let us denote the two progressions with S_1 and S_2

∴ S = $S_1 + S_2$

$S_1 = \frac{1}{3}+\frac{1}{3^3}+\frac{1}{3^5}+\cdots\infty$

Common ratio = r = $\frac{\frac{1}{3^3}}{\frac{1}{3}} = \frac{1}{3^2} = \frac{1}{9}$

Sum of infinite GP = $\frac{a}{1-r}$,where a is the first term and r is the common ratio.

Note: We can only use the above formula if |r|<1

Clearly, a = $\frac{1}{3}$ and r = 1/9

⇒ $S_1 = \frac{\frac{1}{3}}{1-(\frac{1}{9})} = \frac{3}{8}$

$S_2 = \frac{1}{5^2}+\frac{1}{5^4}+\frac{1}{5^6}+.....\infty$

Common ratio = r = $\frac{\frac{1}{5^4}}{\frac{1}{5^2}} = \frac{1}{5^2} = \frac{1}{25}$

Sum of infinite GP = $\frac{a}{1-r}$,where a is the first term and r is the common ratio.

Note: We can only use the above formula if |r|<1

Clearly, a = $\frac{1}{25}$ and r = 1/25

⇒ $S_2 = \frac{\frac{1}{25}}{1-(\frac{1}{25})} = \frac{1}{24}$

Hence,

S = $\frac{1}{24}+\frac{3}{8} = \frac{10}{24} = \frac{5}{12}$

2. Question

Prove that :

$(9^{1/3} . 9^{1/9} . 9^{1/27} ....\infty) = 3.$

Answer

Using the properties of exponents:

The above term can be written as

Let S = $9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots\infty}$ ...(1)

We observe that above progression(in power of 9) possess a common ratio. So it is a geometric progression.

Let m = $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots\infty$

Common ratio = r = $\frac{\frac{1}{9}}{\frac{1}{3}}=\frac{1}{3}$

Sum of infinite GP = $\frac{a}{1-r}$,where a is the first term and r is the common ratio.

Note: We can only use the above formula if $|r|<1$

Clearly, a = $\frac{1}{3}$ and r = $\frac{1}{3}$

$\Rightarrow$ m = $\frac{\frac{1}{3}}{1-(\frac{1}{3})}=\frac{1}{2}$

From equation 1 we have,

$S = 9^m = 9^{1/2} = 3 = RHS$

Hence Proved

3. Question

Prove that :

$(2^{1/4} .4^{1/8} . 8^{1/16}. 16^{1/32}....\infty) = 2.$

Answer

Let, S = $2^{\frac{1}{4}}.2^{\frac{2}{8}}.2^{\frac{3}{16}}.2^{\frac{1}{8}}...$

Using the properties of exponents:

The above term can be written as:

$\Rightarrow S = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\cdots\infty}...$

Denoting the terms in power with x,

We have-

$S = 2^x$ where x = $\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\cdots\infty$

Clearly, we observe that x is neither possessing any common ratio or any common difference. But if you observe carefully you can see that numerator is possessing an AP and denominator of various terms are in GP

Many of similar problems are solved using the method of difference approach as solved below:

As x = $\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\cdots\infty$ .....Equation 1

Multiply both sides of the equation with 1/2,we have-

$$\frac{x}{2}=\frac{1}{2}\left(\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\cdots\infty\right)$$

$\Rightarrow \frac{x}{2}=\frac{1}{8}+\frac{2}{16}+\frac{3}{32}+\cdots\infty$ ....Equation 2

Subtract equation 2 from equation 1,we have:

$$x-\frac{x}{2}=\left(\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\cdots\infty\right)-\left(\frac{1}{8}+\frac{2}{16}+\frac{3}{32}+\cdots\infty\right)$$

TIP: Make groups get rid of difference in the numerator

$\Rightarrow \frac{x}{2}=\frac{1}{4}+\left(\frac{2}{8}-\frac{1}{8}\right)+\left(\frac{3}{16}-\frac{2}{16}\right)+\cdots\infty$

$\Rightarrow \frac{x}{2}=\frac{1}{4}+\left(\frac{1}{8}\right)+\left(\frac{1}{16}\right)+\cdots\infty$

$\Rightarrow x=\frac{1}{2}+\left(\frac{1}{4}\right)+\left(\frac{1}{8}\right)+\cdots\infty$

Clearly, we have a progression with common ratio = 1/2

∴ it is a Geometric progression

Sum of infinite GP = $\frac{a}{1-r}$,where a is the first term and r is the common ratio.

Note: We can only use the above formula if $|r|<1$

Clearly, $a=\frac{1}{2}$ and $r=\frac{1}{2}$

$\Rightarrow x=\frac{\frac{1}{2}}{1-\left(\frac{1}{2}\right)}=1$

From equation 1 we have,

$S = 2^x = 2^1 = 2$ = RHS

4. Question

If S_p denotes the sum of the series $1 + r^p + r^{2p} + \ldots$ to ∞ and s_p the sum of the series $1 - r^p + r^{2p} - \ldots$ to ∞, prove that $s_p + S_p = 2\,S_{2p}$.

Answer

Given,

$S_p = 1 + r^p + r^{2p} + \ldots$ to ∞

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = r^p and first term (a) = 1

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if $|k|<1$

As, $|r|<1 \Rightarrow |r^p|<1$ if (p>1)

∴ we can use the formula for the sum of infinite GP.

$\Rightarrow S_p=\frac{1}{1-r^p}$ ....equation 1

As, $s_p = 1 - r^p + r^{2p} - \ldots$ to ∞

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = $-r^p$ and first term (a) = 1

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if |k|<1

As, $|r|<1 \Rightarrow |r^p|<1$ if (p>1)

∴ we can use the formula for the sum of infinite GP.

$\Rightarrow s_p = \frac{1}{1-(-r^p)} = \frac{1}{1+r^p}$ ....equation 2

As we have to prove - $s_p + S_p = 2\ s_{2p}$

From equation 1 and 2, we get-

$\therefore S_p + s_p = \frac{1}{1-r^p} + \frac{1}{1+r^p}$

$\Rightarrow S_p + s_p = \frac{1+r^p+1-r^p}{(1-r^p)(1+r^p)} = \frac{2}{1-(r^p)^2}$ {using $(a+b)(a-b)=a^2-b^2$}

$\Rightarrow S_p + s_p = \frac{2}{1-r^{2p}}$

As $S_p = \frac{1}{1-r^p}$

∴ following the same analogy, we have-

$$\frac{1}{1-r^{2p}} = S_{2p}$$

$\therefore S_p + s_p = 2 \times \frac{1}{1-r^{2p}} = 2\ S_{2p}$

Hence,

$S_p + s_p = 2S_{2p}$

5. Question

Find the sum of the terms of an infinite decreasing G.P. in which all the terms are positive, the first term is 4, and the difference between the third and fifth term is equal to 32/81.

Answer

Let a denote the first term of GP and r be the common ratio.

We know that nth term of a GP is given by-

$a_n = ar^{n-1}$

As, a = 4 (given)

And $a_5 - a_3 = 32/81$ (given)

$\Rightarrow 4r^4 - 4r^2 = 32/81$

$\Rightarrow 4r^2(r^2 - 1) = 32/81$

$\Rightarrow r^2(r^2 - 1) = 8/81$

Let us denote r^2 with y

∴ 81y(y-1) = 8

$\Rightarrow 81y^2 - 81y - 8 = 0$

Using the formula of the quadratic equation to solve the equation, we have-

y = $\frac{-(-81)\pm\sqrt{81^2-4(-8)(81)}}{16}$

⇒ $y = \frac{81\pm\sqrt{6561-2592}}{162} = \frac{81\pm63}{162}$

∴ y = 18/162 = 1/9 or y = 144/162 = 8/9

⇒ r^2 = 1/9 or 8/9

$\therefore r = \pm\frac{1}{3} \text{ or } \pm\frac{2\sqrt{2}}{3}$

As GP is decreasing and all the terms are positive so we will consider only those values of r which are positive and |r|<1

∴ r = $\frac{1}{3}$ or $\frac{2\sqrt{2}}{3}$

∵ Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if |k|<1

∴ the sum of respective GPs are –

$S_1 = \frac{4}{1-\frac{1}{3}} = \frac{12}{2} = 6$ {sum of GP for r = 1/3}

$S_2 = \frac{4}{1-\frac{2\sqrt{2}}{3}} = \frac{12}{3-2\sqrt{2}}$ {sum of GP for r = (2√2)/3}

6. Question

Express the recurring decimal 0.125125125 … as a rational number.

Answer

Let,

x = 0.125125125 ….equation 1

As 125 is the repeating term, so in all such problems multiply both sides of the equation with a number such that complete repetitive part of number comes after the decimal.

∴ multiplying equation 1 with 1000 in both sides, we have –

1000x = 125.125125125… …equation 2

Subtracting equation 1 from equation 2,we get-

1000x – x = 125.125125125… - 0.125125125….

⇒ 999x = 125

∴ x = 125/999

7. Question

Find the rational number whose decimal expansion is $0.42\overline{3}$

Answer

Let,

x = 0.4233333333….. ….equation 1

As 3 is the repeating term, so in all such problems multiply both sides of the equation with a number such that complete repetitive part of number comes after the decimal.

∴ multiplying equation 1 with 100 in both sides, we have –

100x = 42.3333333333… …equation 2

Subtracting equation 1 from equation 2,we get-

100x – x = 42.3333333… - 0.423333333…

⇒ 99x = 41.91 {as letter terms gives zero only 42.33-0.42 gives result}

∴ x = 41.91/99

⇒ x = 4191/9900

Note: We can also solve these problems using geometric progression, but the above method is much simpler.

8 A. Question

Find the rational numbers having the following decimal expansions :

$0.\overline{3}$

Answer

Let,

x = 0.33333333…..

x = 0.3 + 0.03 + 0.003 + …∞

⇒ x = 3(0.1 + 0.01 + 0.001 + …∞)

⇒ x = $3\left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\cdots\infty\right)$

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = 1/10 and first term (a) = 1/10

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if |k|<1

∴ we can use the formula for the sum of infinite GP.

⇒ x = 3× $\frac{\frac{1}{10}}{1-(\frac{1}{10})} = 3\times\frac{1}{9}=\frac{1}{3}$

∴ x = 1/3

8 B. Question

Find the rational numbers having the following decimal expansions :

$0.\overline{231}$

Answer

Let,

x = 0.231231231231…..

x = 0.231 + 0.000231 + 0.000000231 + …∞

⇒ x = 231(0.001 + 0.00001 + 0.0000001 + …∞)

⇒ x = $231\left(\frac{1}{10^3}+\frac{1}{10^6}+\frac{1}{10^9}+\cdots\infty\right)$

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = 1/1000 and first term (a) = 1/1000

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if |k|<1

∴ we can use the formula for the sum of infinite GP.

⇒ x = 231× $\frac{\frac{1}{1000}}{1-(\frac{1}{1000})} = 231 \times \frac{1}{999} = \frac{231}{999}$

∴ x = 231/999

8 C. Question

Find the rational numbers having the following decimal expansions :

$3.5\overline{2}$

Answer

Let,

x = 3.522222222 .....

x = 3.5+0.02 + 0.002 + 0.0002 + ...∞

⇒ x = 3.5+2(0.01 + 0.001 + 0.0001 + ...∞)

⇒ x = $3.5 + 2\left(\frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \cdots \infty\right)$

⇒ x = 3.5 + 2S

Where S = $\frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \cdots \infty$

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = 1/10 and first term (a) = 1/100

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if |k|<1

∴ we can use the formula for the sum of infinite GP.

⇒ S = $\frac{\frac{1}{100}}{1-(\frac{1}{10})} = \frac{1}{90} = \frac{1}{90}$

∴ x = 3.5 + 2(1/90)

⇒ x = (35/10) + 1/45 = (315+2)/90 = 317/90

8 D. Question

Find the rational numbers having the following decimal expansions :

$0.6\overline{8}$

Answer

Let,

x = 0.688888888888.....

x = 0.6+0.08 + 0.008 + 0.0008 + ...∞

⇒ x = 0.6+8(0.01 + 0.001 + 0.0001 + ...∞)

$\Rightarrow$ x = $0.6 + 8\left(\frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \cdots\infty\right)$

$\Rightarrow$ x = 0.6 + 2S

Where S = $\frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \cdots\infty$

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = 1/10 and first term (a) = 1/100

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if |k|<1

∴ we can use the formula for sum of infinite GP.

$\Rightarrow$ S = $\frac{\frac{1}{100}}{1-(\frac{1}{10})} = \frac{1}{90} = \frac{1}{90}$

∴ x = 0.6 + 8(1/90)

$\Rightarrow$ x = (6/10) + 4/45 = (54+8)/90 = 62/90

9. Question

One side of an equilateral triangle is 18 cm. The mid-points of its sides are joined to form another triangle whose mid-points, in turn, are joined to form still another triangle. The process is continued indefinitely. Find the sum of the (i) perimeters of all the triangles. (ii) areas of all triangles.

Answer

As the midpoints of the triangles are joined successively to get another term and this is being a repeatedly infinite number of terms.

So we will be having an infinite number of side length for an infinite number of triangles.

Let ΔABC represents the equilateral triangle with side 18 cm.

D,E and F are the midpoints of side AB,BC and AC respectively

And thus ΔDEF represents another equilateral triangle.

We can find the length of DE using midpoint theorem of triangles.

If the midpoint of the 2 sides of a triangle are joined,it is parallel to the third side and is equal to 1/2 of it.

∴ DE = 1/2 × 18 = 9 cm

Similarly triangle inside DEF will have side = 9/2, and so on for other triangles.

We need to find sum of perimeters of all the triangles.

Sum of Perimeter of all the triangles = P(say)

∴ P = 3×18 + 3×9 + 3×(9/2) + 3×(9/4) + ...∞

$\Rightarrow$ P = 54 + 27 (1 + 1/2 + 1/4 +...∞)

$\Rightarrow$ P = 54 + 27S

Where S = (1 + 1/2 + 1/4 +...∞)

We observe that above progression possess a common ratio. So it is a geometric progression.

Common ratio = 1/2 and first term (a) = 1

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if |k|<1

∴ we can use the formula for sum of infinite GP.

⇒ $S = \frac{1}{1-(\frac{1}{2})} = 2$

∴ P = 54 + 27×2 = 54+54 = 108

∴ Sum of the perimeters of all the triangles is 108 cm

We need to find sum of Area of all the triangles.

Sum of Perimeter of all the triangles = A(say)

As the area of an equilateral triangle is given by - $\sqrt{3}\frac{l^2}{4}$,where l represents the length of side of triangle.

∴ $A = \sqrt{3} \times \frac{18^2}{4} + \frac{\sqrt{3}}{4}\left(9^2 + \frac{9^2}{4} + \frac{9^2}{16} + \cdots \infty\right)$

⇒ $A = \sqrt{3} \times 81 + \frac{\sqrt{3} \times 9^2}{4}\left(1 + \frac{1}{4} + \frac{1}{16} + \cdots \infty\right)$

⇒ $A = \sqrt{3} \times 81 + 81\sqrt{3}\left(\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots \infty\right)$

⇒ P = 81√3 (1 + 1/4 + 1/16 +…∞)

⇒ P = 81√3 S'

Where S' = (1 + 1/4 + 1/16 +…∞)

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = 1/4 and first term (a) = 1

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if |k|<1

∴ we can use the formula for the sum of infinite GP.

⇒ $S' = \frac{1}{1-(\frac{1}{4})} = \frac{4}{3}$

∴ $A = 81\sqrt{3} \times \frac{4}{3}$ = 108√3

∴ Sum of the Area of all the triangles is 108√3 cm^2

10. Question

Find an infinite G.P. whose first term is 1 and each term is the sum of all the terms which follow it.

Answer

As we have the first term of GP. Let r be the common ratio.

∴ we can say that GP is 1 , r , r^2 , r^3 … ∞

As per the condition, each term is the sum of all terms which follow it.

If a_1,a_2 , … represents first, second, third term etc

∴ we can say that:

$a_1 = a_2 + a_3 + a_4 + \ldots\infty$

⇒ $1 = r + r^2 + r^3 + \ldots\infty$

Note: You can take any of the cases like $a_2 = a_3 + a_4 + ..$ all will give the same result.

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = r and first term (a) = r

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if |k|<1

∴ we can use the formula for the sum of infinite GP.

⇒ $S = \frac{r}{1-r}$

⇒ $\frac{r}{r-1} = 1$

⇒ r=1−r

∴ 2r=2 or r= 1/2

Hence the series is 1, 1/2, 1/4, 1/8, 1/16..............

11. Question

The sum of the first two terms of an infinite G.P. is 5, and each term is three times the sum of the succeeding terms. Find the G.P.

Answer

Suppose the 1st term is a and the common ratio is r.

∴ we can say that GP looks like: a ,ar ,ar^2 ,...

According to question:

a + ar = 5 ...equation 1

Also, $a_1 = 3(a_2+a_3+a_4+...\infty)$ {you can take any other combination}

⇒ $a = 3(ar+ar^2+ar^3 + ...\infty)$

⇒ $1 = 3(r + r^2 + r^3 +...\infty)$

We observe that above progression possess a common ratio. So it is a geometric progression.

Common ratio = r and first term (a) = r

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if |k|<1

∴ we can use the formula for the sum of infinite GP.

Therefore

⇒ $\frac{1}{3} = \frac{r}{1-r}$

1-r = 3r

$r = \frac{1}{4}$

From equation 1:

a+ar = a(1+r) = 5.

So,

⇒ $a = \frac{5}{1+\frac{1}{4}}$

$\Rightarrow$ a = 4

$\therefore$ GP is (4 , 1 , 1/4 , 1/16 , ….)

12. Question

Show that in an infinite G.P. with common ratio r (|r| < 1), each term bears a constant ratio to the sum of all terms that follow it.

Answer

Let a be the first term of GP.

Given common ratio = r

$\therefore$ we can write GP as : a ,ar ,ar^2 ,ar^3 …

We need to proof that: each term bears a constant ratio to the sum of all terms that follow it.

Means: $\frac{a}{ar+ar^2+\cdots} = \frac{ar}{ar^2+ar^3+\cdots} = \frac{ar^2}{ar^3+ar^4+\cdots\infty} = \cdots \text{constant}$

Proving for each and every individual term will be a tedious and foolish job.

So we will prove this for the n^{th} term, and it will validate the statement for each and every term.

N^{th} term is given by ar^{n-1}.

To prove: $\frac{ar^{n-1}}{ar^n+ar^{n+1}+\cdots\infty} = z(\text{say a constant})$

We know that sum of an infinite GP is given by:

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

$\therefore ar^n + ar^{n+1} + \ldots \infty = ar^n(1 + r + r^2 + \ldots\infty)$

$\therefore$ Sum = $ar^n \left(\frac{1}{1-r}\right) = \frac{ar^n}{1-r}$

Hence,

$$\frac{ar^{n-1}}{ar^n + ar^{n+1} + \cdots\infty} = \frac{ar^{n-1}}{\frac{ar^n}{1-r}} = \frac{1-r}{r} = \text{constant}$$

As the ratio is independent of the value of each and every term

And hence we say that it bears a constant ratio. Proved.

13. Question

If S denotes the sum of an infinite G.P. and S_1 denotes the sum of the squares of its terms, then prove that the first term and common ratio are respectively $\frac{2SS_1}{S^2+s_1}$ and $\frac{S^2-S_1}{S^2+S_1}$.

Answer

Let a be the first term, and r be the common ratio.

According to the question-

$a + ar + ar^2 + \ldots\infty = S$

$\Rightarrow S = a(1+r+r^2+\ldots\infty)$

We observe that the above progression possess a common ratio. So it is a geometric progression.

Common ratio = r and first term (a) = 1

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if |k|<1

$\therefore$ S = $\frac{a}{1-r}$ ...equation 1

Also, as per the question

$S_1 = a^2 + a^2r^2 + a^2r^4 + \ldots\infty$

$\Rightarrow S_1 = a^2 (1+r^2+r^4+\ldots\infty)$

We observe that above progression possess a common ratio. So it is a geometric progression.

Common ratio = r^2 and first term (a) = 1

Sum of infinite GP = $\frac{a}{1-k}$,where a is the first term and k is the common ratio.

Note: We can only use the above formula if |k|<1

$\therefore S_1 = \frac{a^2}{1-r^2}$

$\Rightarrow S_1 = \frac{a^2}{(1-r)(1+r)} = \frac{a}{1-r} \times \frac{a}{(1+r)}$

From equation 1,we have-

$\Rightarrow S_1 = \frac{aS}{1+r}$ ....equation 2

Dividing equation 1 by 2, we get-

$$\frac{S}{S_1} = \frac{\frac{a}{1-r}}{\frac{as}{1+r}}$$

$$\Rightarrow \frac{S^2}{S_1} = \frac{1+r}{1-r}$$

$\Rightarrow (1\text{-}r)S^2 = (1+r)S_1$

$\Rightarrow S^2 - S_1 = r(S^2 + S_1)$

$\therefore r = \frac{S^2-S_1}{S^2+S_1}$

Put the value of r in equation 1 to get a.

$a = \frac{2SS_1}{S^2+S_1}$

Exercise 20.5

1. Question

If a, b, c are in G.P., prove that log a, log b, log c are in A.P.

Answer

If a, b, c are in GP

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \text{common ratio}$ .....(i)

We know,

log a – log b = $\log\frac{a}{b}$ {property of logarithm}

and according to equation (i)

$\Rightarrow \log\frac{b}{a} = \log\frac{c}{b}$

⇒ log b – log a = log c – log b

⇒ 2 log b = log a + log c {property of arithmetic mean}

Hence they are in AP. ...proved

2. Question

If a, b, c are in G.P., prove that $\frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m}$ are in A.P.

Answer

Given:

a, b and c are in GP

∴ $b^2 = ac$ {property of geometric mean}

Taking log on both sides with base m –

$\log_m b^2 = \log_m ac$

⇒ $\log_m b^2 = \log_m a + \log_m c$ {using property of log}

⇒ $2\log_m b = \log_m a + \log_m c$ ...equation 1

Note: If three numbers a,b and c are in AP,we can say that –

2b = a + c

As equation 1 matches the form above, So

⇒ $\log_m a$, $\log_m b$ and $\log_m c$ are in AP. ...(1)

Now, applying base changing formula we get

⇒ $\log_a b = \frac{1}{\log_b a}$

∴ Applying base change on 1, we get

⇒ $\frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m}$ are in A.P

Hence, proved

3. Question

Find k such that k + 9, k – 6 and 4 form three consecutive terms of a G.P.

Answer

Let a = k + 9; b = k−6;

c = 4

Since, a, b and c are in GP, then

$b^2 = ac$ {using idea of geometric mean}

$\Rightarrow (k-6)^2 = 4(k + 9)$

$\Rightarrow k^2 – 12k + 36 = 4k + 36$

$\Rightarrow k^2 – 16k = 0$

$\Rightarrow k = 0$ or $k = 16$

4. Question

Three numbers are in A.P., and their sum is 15. If 1, 3, 9 be added to them respectively, they from a G.P. Find the numbers.

Answer

Let the original numbers be

a, a + d, and a + 2d

According to the question –

$a + a + d + a + 2d = 15$

$\Rightarrow 3a + 3d = 15$ or $a + d = 5$

$\Rightarrow d = 5 – a$

After the addition, the three numbers are:

a + 1, a + d + 3, and a + 2d + 9

they are now in GP, that is –

$\Rightarrow \frac{a+d+3}{a+1} = \frac{a+2d+9}{a+d+3}$

$\Rightarrow (a + d + 3)^2 = (a + 2d + 9)(a + 1)$

$\Rightarrow a^2 + d^2 + 9 + 2ad + 6d + 6a = a^2 + a + 2da + 2d + 9a + 9$

$\Rightarrow (5 – a)^2 – 4a + 4(5 – a) = 0$

$\Rightarrow 25 + a^2 – 10a – 4a + 20 – 4a = 0$

$\Rightarrow a^2 – 18a + 45 = 0$

$\Rightarrow a^2 – 15a – 3a + 45 = 0$

$\Rightarrow a(a – 15) – 3(a – 15) = 0$

$\Rightarrow a = 3$ or $a = 15$

$\therefore d = 5 – a$

$d = 5 – 3$ or $d = 5 – 15$

$d = 2$ or $– 10$

$\therefore$ The numbers are 3,5,7 or 15,5, – 5

5. Question

The sum of three numbers which are consecutive terms of an A.P. is 21. If the second number is reduced by 1 and the third is increased by 1, we obtain three consecutive terms of a G.P. Find the numbers.

Answer

Let the original numbers be

a, a + d, and a + 2d

According to the question –

3a + 3d = 21 or a + d = 7.

⇒ d = 7 – a

After the addition, the three numbers are:

a, a + d – 1, and a + 2d + 1

they are now in GP, that is –

⇒ $\frac{a+d-1}{a} = \frac{a+2d+1}{a+d-1}$

⇒ $(a + d - 1)^2 = a(a + 2d + 1)$

⇒ $a^2 + d^2 + 1 + 2ad - 2d - 2a = a^2 + a + 2da$

⇒ $(7 - a)^2 - 3a + 1 - 2(7 - a) = 0$

⇒ $49 + a^2 - 14a - 3a + 1 - 14 + 2a = 0$

⇒ $a^2 - 15a + 36 = 0$

⇒ $a^2 - 12a - 3a + 36 = 0$

⇒ a(a – 12) – 3(a – 12) = 0

⇒ a = 3 or a = 12

∴ d = 7 – a

d = 7 – 3 or d = 7 – 12

d = 4 or – 5

∴ The numbers are 3,7,11 or 12,7,2

6. Question

The sum of three numbers a, b, c in A.P. is 18. If a and b are each increased by 4 and c is increased by 36, the new numbers form a G.P. Find a, b, c.

Answer

Let d be the common difference of AP

∴ b = a + d ; c = a + 2d.

Given: a + b + c = 18

⇒ 3a + 3d = 18 or a + d = 6.

⇒ d = 6 – a

After the addition, the three numbers are:

a + 4, a + d + 4, and a + 2d + 36

they are now in GP, that is –

⇒ $\frac{a+d+4}{a+4} = \frac{a+2d+36}{a+d+4}$

$(a + d + 4)^2 = (a + 2d + 36)(a + 4)$

⇒ $a^2 + d^2 + 16 + 8a + 2ad + 8d = a^2 + 4a + 2da + 36a + 144 + 8d$

⇒ $d^2 - 32a - 128$

$\Rightarrow (6 - a)^2 - 32a - 128 = 0$

$\Rightarrow 36 + a^2 - 12a - 32a - 128 = 0$

$\Rightarrow a^2 - 44a - 92 = 0$

$\Rightarrow a^2 - 46a + 2a - 92 = 0$

$\Rightarrow a(a - 46) + 2(a - 46) = 0$

$\Rightarrow a = -2$ or $a = 46$

As,

$d = 6 - a$

$\therefore d = 6 - (-2)$ or $d = 6 - 46$

$d = 8$ or -40

$\therefore$ numbers are – 2, 6, 14 or 46, 6, – 34

7. Question

The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an A.P. Find the numbers.

Answer

Let the three numbers be $\frac{a}{r}, a, ar$

$\therefore$ According to the question

$\Rightarrow \frac{a}{r} + a + ar = 56$ ...(1)

$\Rightarrow a + ar + ar^2 = 56r$

$\Rightarrow a(1 + r + r^2) = 56r$

$\Rightarrow a = \frac{56r}{1 + r + r^2}$ ...(2)

Subtracting 1,7,21 we get,

$\Rightarrow \frac{a}{r} - 1, a - 7, ar - 21$

The above numbers are in AP

If three numbers are in AP, by the idea of the arithmetic mean, we can write 2b = a + c

$\therefore 2(a - 7) = \frac{a}{r} - 1 + ar - 21$

$\Rightarrow 2ar - 14r = a - r + ar^2 - 21r$

$\Rightarrow ar^2 - 8r + a - 2ar = 0$

$\Rightarrow a(r^2 - 2r + 1) = 8r$

From (2) we know the value of a

$\Rightarrow \frac{56r}{1 + r + r^2}(r^2 - 2r + 1) = 8r$

$\Rightarrow 56(r^2 - 2r + 1) = 8(1 + r + r^2)$

$\Rightarrow 7(r^2 - 2r + 1) = (1 + r + r^2)$

$\Rightarrow 7r^2 - 14r + 7 = 1 + r + r^2$

$\Rightarrow 6r^2 - 15r + 6 = 0$

$\Rightarrow 6r^2 - 12r - 3r + 6 = 0$

$\Rightarrow 6r(r - 2) - 3(r - 2) = 0$

$\Rightarrow r = 2$ or $r = 3/6 = 1/2$

When $r = 2 \Rightarrow a = 16$ {using equation 1)

$r = 1/2 \Rightarrow a = 16$

$\therefore$ the three numbers are (a/r, a, ar) = (8,16,32)

Or numbers are – (32,16,8)

8 A. Question

If a, b, c are in G.P., prove that :

$a(b^2 + c^2) = c(a^2 + b^2)$

Answer

Now, as a,b,c are in GP.

Using the idea of geometric mean we can write –

$\therefore b^2 = ac$ ...(1)

Put in the LHS of the given equation to be proved –

LHS = $a(ac + c^2)$ {putting $b^2 = ac$}

$\Rightarrow$ LHS = $a^2c + ac^2$

$\Rightarrow$ LHS = $c(a^2 + ac)$

Again put $ac = b^2$

$\Rightarrow$ LHS = $c(a^2 + b^2)$ = RHS

$\therefore$ L.H.S = R.H.S

Hence proved

8 B. Question

If a, b, c are in G.P., prove that :

$$a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$$

Answer

Now, as a,b,c are in GP.

$\therefore b^2 = ac$ ...(1)

Put in the LHS of the given equation to be proved –

$\Rightarrow$ LHS = $a^2(ac)c^2\left(\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}\right)$

$\Rightarrow$ LHS = $a^3.c^3\left(\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}\right)$

$\Rightarrow$ LHS = $c^3 + \frac{a^3c^3}{b^3} + a^3$

$\Rightarrow$ LHS = $c^3 + \frac{(ac)^3}{b^3} + a^3$

⇒ LHS = $c^3 + \frac{b^6}{b^3} + a^3$ {putting $b^2 = ac$ }

⇒ LHS = $a^3 + b^3 + c^3$ = RHS ...(Hence Proved)

8 C. Question

If a, b, c are in G.P., prove that :

$$\frac{(a+b+c)^2}{a^2+b^2+c^2} = \frac{a+b+c}{a-b+c}$$

Answer

As

a, b, c are in G.P, let r be the common ratio.

Therefore,

b = ar ... (1)

c = ar^2 ... (2)

To prove: $\frac{(a+b+c)^2}{a^2+b^2+c^2} = \frac{a+b+c}{a-b+c}$

As, LHS = $\frac{(a+b+c)^2}{a^2+b^2+c^2}$

⇒ LHS = $\frac{(a+ar+ar^2)^2}{a^2+a^2r^2+a^2r^4} = \frac{(1+r+r^2)^2}{1+r^2+r^4}$

⇒ LHS = $\frac{(1+r+r^2)^2}{1+2r^2+r^4-r^2} = \frac{(1+r+r^2)^2}{(1+r^2)^2-r^2}$

⇒ LHS = $\frac{(1+r+r^2)^2}{(1+r^2)^2-r^2} = \frac{(1+r+r^2)^2}{(1+r^2+r)(1+r^2-r)} = \frac{(1+r^2+r)}{(1+r^2-r)}$

As, RHS = $\frac{a+b+c}{a-b+c} = \frac{a(1+r^2+r)}{a(1+r^2-r)}$ = LHS

Clearly, LHS = RHS

Hence proved

8 D. Question

If a, b, c are in G.P., prove that :

$$\frac{1}{a^2-b^2} + \frac{1}{b^2} = \frac{1}{b^2-c^2}$$

Answer

Now, as a,b,c are in GP.

Using the idea of geometric mean we can write –

∴ b^2 = ac ...(1)

Put in the LHS of the given equation to be proved –

⇒ LHS = $\frac{b^2+a^2-b^2}{b^2(a^2-b^2)}$

⇒ LHS = $\frac{a^2}{b^2a^2-b^4}$

⇒ LHS = $\frac{a^2}{b^2a^2-a^2c^2}$

$\Rightarrow$ LHS = $\frac{1}{b^2 - c^2}$ = RHS

Hence Proved.

8 E. Question

If a, b, c are in G.P., prove that :

(a + 2b + 2c) (a – 2b + 2c) = a^2 + $4c^2$.

Answer

As,

a, b, c are in G.P, let r be the common ratio.

Therefore,

b = ar … (1)

c = ar^2 … (2)

To prove: $(ab + bc + cd)^2$ = (a + 2b + 2c) (a – 2b + 2c) = $a^2 + 4c^2$

As, LHS = (a + 2b + 2c) (a – 2b + 2c)

$\Rightarrow$ LHS = $(a + 2ar + 2ar^2)(a - 2ar + 2ar^2)$

$\Rightarrow$ LHS = $a^2(1 + 2r + 2r^2)(1 - 2r + 2r^2)$

$\Rightarrow$ LHS = a^2 $(1 + 4r^2 + 4r^4 - 4r^2)$

$\Rightarrow$ LHS = $a^2(1 + 4r^4)$

And RHS = $a^2 + 4a^2r^4 = a^2(1 + 4r^4)$

Clearly, LHS = RHS

Hence proved

9 A. Question

If a, b, c, d are in G.P, prove that :

$$\frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$$

Answer

a, b, c, d are in G.P.

Let r be the common ratio.

Therefore,

b = ar …(1)

c = ar^2 …(2)

and d = ar^3 …(3)

If somehow we use LHS and Make it equal to RHS, our job will be done.

we can manipulate the LHS of the given equation as –

$\Rightarrow$ LHS = $\frac{ab - cd}{b^2 - c^2}$

Put the values of a,b,c and d from equation 1,2 and 3

$\Rightarrow$ LHS = $\frac{a(ar)-(ar^2)(ar^3)}{(ar)^2-(ar^2)^2}$ = $\frac{a^2r-a^2r^5}{a^2r^2-a^2r^4}$

$\Rightarrow$ LHS = $\frac{a^2r(1-r^4)}{a^2r^2(1-r^2)}$ = $\frac{(1+r^2)(1-r^2)}{r(1-r^2)}$

$\Rightarrow$ LHS = $\frac{(1+r^2)}{r}$

Multiplying a in numerator and denominator –

$\Rightarrow$ LHS = $\frac{a(1+r^2)}{ar}$ = $\frac{a+ar^2}{ar}$

Again from equation 1, 2, and 3, we can see –

LHS = $\frac{a+ar^2}{ar}$ = $\frac{a+c}{b}$ = RHS …hence proved

9 B. Question

If a, b, c, d are in G.P, prove that :

$(a + b + c + d)^2 = (a + b)^2 + 2(b + c)^2 + (c + d)^2$

Answer

a, b, c, d are in G.P.

Therefore,

bc = ad … (1)

b^2 = ac … (2)

c^2 = bd … (3)

If somehow we use RHS and Make it equal to LHS, our job will be done.

we can manipulate the RHS of the given equation as –

Note: Here we are manipulating RHS because working with a simpler algebraic equation is easier and this time RHS is looking simpler.

RHS = $(a + b)^2 + 2(b + c)^2 + (c + d)^2$

$\Rightarrow$ RHS = $a^2 + b^2 + 2ab + 2(c^2 + b^2 + 2cb) + c^2 + d^2 + 2cd$

$\Rightarrow$ RHS = $a^2 + b^2 + c^2 + d^2 + 2ab + 2(c^2 + b^2 + 2cb) + 2cd$

Put c^2 = bd and b^2 = ac, we get –

$\Rightarrow$ RHS = $a^2 + b^2 + c^2 + d^2 + 2(ab + ad + ac + cb + cd)$

You can visualize the above expression by making separate terms for $(a + b + c)^2 + d^2 + 2d(a + b + c) = \{(a + b + c) + d\}^2$

$\Rightarrow$ RHS = $(a + b + c + d)^2$ = LHS

Hence Proved.

9 C. Question

If a, b, c, d are in G.P, prove that :

(b + c) (b + d) = (c + a) (c + d)

Answer

a, b, c, d are in G.P.

Therefore,

$bc = ad$ … (1)

$b^2 = ac$ … (2)

$c^2 = bd$ … (3)

LHS = $b^2 + bd + bc + cd$

⇒ LHS = ac + bd + bc + cd {on substituting value of b^2 } …(1)

RHS = $c^2 + cd + ac + ad$

⇒ RHS = bd + cd + ac + bc {putting value of c^2} …(2)

From equation 1 and 2 we can say that –

LHS = RHS Hence proved

10 A. Question

If a, b, c are in G.P., prove that the following are also in G.P. :

a^2, b^2, c^2

Answer

As a, b, c are in G.P.

Therefore

$b^2 = ac$ … (1)

We have to prove a^2, b^2, c^2 are in GP or

we need to prove: $(b^2)^2 = (ac)^2$ {using idea of GM}

On squaring equation 1 we get,

⇒ $b^4 = a^2c^2$

⇒ $(b^2)^2 = (ac)^2$

Hence a^2,b^2,c^2 are in GP.

10 B. Question

If a, b, c are in G.P., prove that the following are also in G.P. :

a^3, b^3, c^3

Answer

As a, b, c are in G.P.

Therefore

$b^2 = ac$ … (1)

We have to prove a^3, b^3, c^3 are in GP or

we need to prove: $(b^3)^2 = (a^3c^3)$ {using idea of GM}

On cubing equation 1 we get,

⇒ $b^6 = a^3c^3$

⇒ $(b^3)^2 = (a^3c^3)$

Hence a^3,b^3,c^3 are in GP.

10 C. Question

If a, b, c are in G.P., prove that the following are also in G.P. :

$a^2 + b^2$, $ab + bc$, $b^2 + c^2$

Answer

a, b, c are in G.P

Therefore

$b^2 = ac$ … (1)

We have to prove $a^2 + b^2$, $ab + bc$, $b^2 + c^2$ are in GP or

we need to prove: $(ab + bc)^2 = (a^2 + b^2).(b^2 + c^2)$ {using GM}

Take LHS and proceed:

$\Rightarrow$ LHS $= (ab + bc)^2 = a^2b^2 + 2ab^2c + b^2c^2$

$\because b^2 = ac$

$\Rightarrow$ LHS $= a^2b^2 + 2b^2(b^2) + b^2c^2$

$\Rightarrow$ LHS $= a^2b^2 + 2b^4 + b^2c^2$

$\Rightarrow$ LHS $= a^2b^2 + b^4 + a^2c^2 + b^2c^2$ {again using $b^2 = ac$ }

$\Rightarrow$ LHS $= b^2(b^2 + a^2) + c^2(a^2 + b^2)$

$\Rightarrow$ LHS $= (a^2 + b^2)(b^2 + c^2)$ = RHS

Hence $a^2 + b^2$, $ab + bc$, $b^2 + c^2$ are in GP.

11 A. Question

If a, b, c are in G.P., prove that :

$(a^2 + b^2)$, $(b^2 + c^2)$, $(c^2 + d^2)$ are in G.P.

Answer

a, b, c, d are in G.P.

Therefore,

$bc = ad$ … (1)

$b^2 = ac$ … (2)

$c^2 = bd$ … (3)

To prove: $(a^2 + b^2)$, $(b^2 + c^2)$, $(c^2 + d^2)$ are in G.P, we need to prove that:

$(a^2 + b^2)(c^2 + d^2) = (b^2 + c^2)^2$ {deduced using GM relation}

$\therefore$ RHS $= (b^2 + c^2)^2 = b^4 + c^4 + 2b^2c^2$

$= a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2$ {using equation 2 and 3}

$= c^2(a^2 + b^2) + d^2(a^2 + b^2)$

$= (a^2 + b^2)(c^2 + d^2)$ = LHS

$\therefore (a^2 + b^2)$, $(b^2 + c^2)$, $(c^2 + d^2)$ are in G.P

Hence proved.

11 B. Question

If a, b, c are in G.P., prove that :

$(a^2 - b^2)$, $(b^2 - c^2)$, $(c^2 - d^2)$ are in G.P.

Answer

a, b, c, d are in G.P.

Therefore,

bc = ad … (1)

$b^2 = ac$ … (2)

$c^2 = bd$ … (3)

To prove: $(a^2 - b^2)$, $(b^2 - c^2)$, $(c^2 - d^2)$ are in G.P, we need to prove that:

$(a^2 - b^2)(c^2 - d^2) = (b^2 - c^2)^2$ {deduced using GM relation}

$\therefore$ RHS $= (b^2 - c^2)^2 = b^4 + c^4 - 2b^2c^2$

$= a^2c^2 + b^2d^2 - a^2d^2 - b^2c^2$ {using equation 2 and 3}

$= c^2(a^2 - b^2) - d^2(a^2 - b^2)$

$= (a^2 - b^2)(c^2 - d^2) =$ LHS

$\therefore$ $(a^2 - b^2)$, $(b^2 - c^2)$, $(c^2 - d^2)$ are in G.P

Hence proved.

11 C. Question

If a, b, c are in G.P., prove that :

$\frac{1}{a^2 + b^2}, \frac{1}{b^2 + c^2}, \frac{1}{c^2 + d^2}$ are in G.P

Answer

a, b, c, d are in G.P.

Therefore,

bc = ad … (1)

$b^2 = ac$ … (2)

$c^2 = bd$ … (3)

To prove: $\frac{1}{a^2 + b^2}, \frac{1}{b^2 + c^2}, \frac{1}{c^2 + d^2}$ are in G.P, we need to prove that:

$\frac{1}{(c^2 + d^2)(a^2 + b^2)} = \frac{1}{(b^2 + c^2)^2}$ {deduced using GM relation}

Or, $(b^2 + c^2)^2 = (a^2 + b^2)(c^2 + d^2)$

Take LHS and proceed to prove –

LHS $= (b^2 + c^2)^2 = b^4 + c^4 + 2b^2c^2$

$= a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2$ {using equation 2 and 3}

$= c^2(a^2 + b^2) + d^2(a^2 + b^2)$

$= (a^2 + b^2)(c^2 + d^2) =$ RHS

$\therefore \frac{1}{a^2 + b^2}, \frac{1}{b^2 + c^2}, \frac{1}{c^2 + d^2}$ are in GP

Hence Proved.

11 D. Question

If a, b, c are in G.P., prove that :

$(a^2 + b^2 + c^2)$, $(ab + bc + cd)$, $(b^2 + c^2 + d^2)$ are in G.P.

Answer

As,

a, b, c, d are in G.P, let r be the common ratio.

Therefore,

$b = ar$ … (1)

$c = ar^2$ … (2)

$d = ar^3$ … (3)

If we show that: $(ab + bc + cd)^2 = (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$

we can say that:

$(a^2 + b^2 + c^2)$, $(ab + bc + cd)$, $(b^2 + c^2 + d^2)$ are in G.P

As, $(ab + bc + cd)^2 = (a^2r + a^2r^3 + a^2r^5)^2$

$\Rightarrow (ab + bc + cd)^2 = a^4r^2(1 + r^2 + r^4)^2$ …(4)

As,

$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (a^2 + a^2r^2 + a^2r^4)(a^2r^2 + a^2r^4 + a^2r^6)$

$\Rightarrow (a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = a^4r^2(1 + r^2 + r^4)(1 + r^2 + r^4)$

$\Rightarrow (a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = a^4r^2(1 + r^2 + r^4)^2$ …(5)

From equation 4 and 5, we have:

$(ab + bc + cd)^2 = (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$

Hence,

We can say that $(a^2 + b^2 + c^2)$, $(ab + bc + cd)$, $(b^2 + c^2 + d^2)$ are in G.P.

12. Question

If (a – b), (b – c), (c – a) are in G.P., then prove that $(a + b + c)^2 = 3(ab + bc + ca)$

Answer

Given as (a – b), (b – c), (c – a) are in G.P

$\therefore \frac{b-c}{a-b} = \frac{c-a}{b-c} = \text{common ratio}$

$\Rightarrow (b - c)^2 = (a - b)(c - a)$

As we have to prove :$(a + b + c)^2 = 3(ab + bc + ca)$ so we proceed as follows:

$\Rightarrow b^2 + c^2 - 2bc = ac - a^2 - bc + ab$

$\Rightarrow a^2 + b^2 + c^2 = ac + ab + bc$

Add 2(ac + ab + bc) to both sides:

$\Rightarrow a^2 + b^2 + c^2 + 2(ac + ab + bc) = ac + ab + bc + 2(ac + ab + bc)$

$\Rightarrow (a + b + c)^2 = 3(ab + bc + ca)$

Hence Proved.

13. Question

If a, b, c are in G.P., then prove that :

$$\frac{a^2 + ab + b^2}{bc + ca + ab} = \frac{b + a}{c + b}$$

Answer

As a,b,c are in GP

Note:

1. In general, the GP series is like a,ar,ar^2..........

2. In this, b = ar and c = br = ar^2

So we proceed forward with the aim to equalize LHS and RHS of the equation to be proved using the above ideas.

L.H.S = $\frac{a^2 + ab + b^2}{bc + ca + ab}$

$\Rightarrow$ LHS = $\frac{a^2 + a(ar) + (ar)^2}{ar(ar^2) + ar^2(a) + a(ar)} = \frac{a^2(1 + r + r^2)}{a^2 r(r^2 + r + 1)}$

$\Rightarrow$ LHS = $\frac{1}{r}$

Now

R.H.S = $\frac{b + a}{c + b}$

$\Rightarrow$ RHS = $\frac{b + a}{c + b} = \frac{ar + a}{ar^2 + ar} = \frac{a(r + 1)}{ar(r + 1)} = \frac{1}{r}$

$\therefore$ RHS = 1/r

Clearly we observed that,

LHS = RHS = (1/r) ...Proved

14. Question

If the 4^{th}, 10^{th} and 16^{th}terms of a G.P. are x, y and z respectively. Prove that x, y, z are in G.P.

Answer

Let first term of GP be a and common ratio be r

As n^{th} term of GP is given as –

$T_n = ar^{n - 1}$

$\therefore T_4 = ar^{4 - 1} = ar^3$

Similarly $T_{10} = ar^9$

And $T_{16} = ar^{15}$

$\therefore x = ar^3$, $y = ar^9$ & $z = ar^{15}$

Clearly we observed that x, y, z have a common ratio.

$\therefore$ x,y,z are in GP with common ratio r^6.Hence proved.

15. Question

If a, b, c are in A.P. and a, b, d are in G.P., then prove that a, a – b, d – c are in G.P.

Answer

a, b, c are in AP

So, 2b = a + c ...(1)

b, c, d are in GP

So, b^2 = ad ...(2)

Multiply first equation with a and subtract it from 2nd.

$b^2 - 2ab = ad - ac - a^2$

$a^2 + b^2 - 2ab = a(d - c)$

$\Rightarrow (a - b)^2 = a(d - c)$

As a, (a – b), (d – c) satisfy the geometric mean relationship

Hence a, (a – b),(d – c) are in G.P.

16. Question

If pth, qth, rth and sth terms of an A.P., be in G.P., then prove that p – q, q – r, r – s are in G.P.

Answer

Given,

pth, qth rth and sth terms of an AP are in GP .

Firstly we should find out pth, qth, rth and sth terms

Let a is the first term and d is the common difference of an AP

so, pth term = a + (p – 1)d

qth term = a + (q – 1)d

rth term = a + (r – 1)d

sth term = a + (s – 1)d

$\therefore$ [a + (p – 1)d], [a + (q – 1)d], [a + (r – 1)d], [a + (s – 1)d] are in GP

so, Let first term of GP be α and common ratio is β

Then, [a + (p – 1)d] = α

[a + (q – 1)d] = αβ

[a + (r – 1)d] = $\alpha\beta^2$

[a + (s – 1)d] = $\alpha\beta^3$

now, here, it is clear that α, αβ, $\alpha\beta^2$, $\alpha\beta^3$ are in GP

NOTE: Using property of GP,we know that if a common term is multiplied with each number in a GP,series itself remains a GP

$\therefore$ α(1 – β), αβ(1 – β), $\alpha\beta^2(1 - \beta)$ are in GP

Where the first term is α(1 – β), and the common ratio is β

so, α(1 – β) = [a + (p – 1)d] – [a + (q – 1)d] = (p – q)

$\therefore$ α(1 – β) = (p – q) ...... (1)

Similarly, $\alpha\beta(1 - \beta) = \alpha\beta - \alpha\beta^2 = [a + (q - 1)d] - [a + (r - 1)d] = (q - r)$

$\therefore \alpha\beta(1 - \beta) = (q - r)$ ...... (2)

And $\alpha\beta^2(1 - \beta) = \alpha\beta^2 - \alpha\beta^3 = [\alpha + (r - 1)d] - [\alpha + (s - 1)d] = (r - s)$

$\therefore \alpha\beta^2(1 - \beta) = (r - s)$ ...... (3)

From the above explanation, we got $\alpha(1 - \beta)$, $\alpha\beta(1 - \beta)$, $\alpha\beta^2(1 - \beta)$ are in GP

$\therefore$ From equations (1), (2) and (3),

$(p - q), (q - r), (r - s)$ are in GP .

17. Question

If $\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are three consecutive terms of an A.P., prove that a, b, c are the three consecutive terms of a G.P.

Answer

Given $\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are in AP.

$\Rightarrow \frac{2}{2b} = \frac{1}{a+b} + \frac{1}{b+c}$ {taking arithmetic mean – to get the relationship}

$\Rightarrow \frac{1}{b} = \frac{b+c+a+b}{(a+b)(b+c)}$

$\Rightarrow ab + ac + b^2 + bc = b^2 + bc + ab + b^2$

$\Rightarrow b^2 = ac$

We know if a,b,c are consecutive terms of GP then $b^2 = ac$ holds.

$\therefore$ a,b,c are in GP.

18. Question

If $x^a = x^{b/2}z^{b/2} = z^c$, then prove that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Answer

Take logs of each expression, using $\ln(x^a) = a \ln(x)$ etc

$\ln(p*q) = \ln(p) + \ln(q)$:

Given,

$x^a = x^{b/2}z^{b/2} = z^c$

Taking log on each term –

$\Rightarrow a\ln x = \ln x^{\frac{b}{2}} + \ln z^{\frac{b}{2}} = c\ln z$ ...(1)

The equality of the first and third expressions tells us that

$\Rightarrow \ln z = \frac{a}{c}\ln x$ ...(2)

The second expression is equal to

$\Rightarrow \ln x^{\frac{b}{2}} + \ln z^{\frac{b}{2}} = \frac{b}{2}(\ln x + \ln z)$

$\Rightarrow a\ln x = \frac{b}{2}(\frac{a}{c}\ln x + \ln x)$ {using equation 1}

$\therefore a \ln x = \frac{b}{2}\ln x + \frac{ab}{2c}\ln x$

Divide through out by In x

$\therefore$ a = b/2 + ab/2c

$\Rightarrow$ 2ac = bc + ab

Dividing the equation by abc –

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

From this $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in GP.

19. Question

If a, b, c are in A.P. b, c, d are in G.P. and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P., prove that a, c, e are in G.P.

Answer

Given:

a,b,c are in AP

$\therefore$ 2b = a + c ...... (i)

b,c,d are in GP;

$\Rightarrow c^2 = bd$ ...... (ii)

1/c, 1/d, 1/e are in AP;

$\Rightarrow \frac{2}{d} = \frac{1}{c} + \frac{1}{e}$

$\Rightarrow d = \frac{2ce}{c+e}$...(iii)

From the above substituting for b & d in (ii) above,

$\Rightarrow c^2 = \frac{a+c}{2} \times \frac{2ce}{c+e}$

$\Rightarrow$ c(c + e) = (a + c) e

$\Rightarrow c^2 + ce = ae + ce$

$\Rightarrow c^2 = ae$

Thus a, c, e are in GP

20. Question

If a, b, c are in A.P. and a, x, b and b, y, c are in G.P., show that x^2, b^2, y^2 are in A.P.

Answer

If a,b,c are in AP it follows that

a + c = 2b........(1)

and a,x,b and b,y,c are in individual GPs which follows

$x^2 = ab$ ....(2)

$y^2 = bc$ ....(3)

Adding eqn 2 and 3 we get,

$x^2 + y^2 = ab + bc$

$= b(a + c)$

$= b.2b$ (from eqn 1)

$= 2b^2$

So we get $x^2 + y^2 = 2b^2$ which shows that they are in AP.

21. Question

If a, b, c are in A.P. and a, b, d are in G.P., show that a, (a – b), (d – c) are in G.P.

Answer

a, b, c are in AP

So, $2b = a + c$ …(1)

b, c, d are in GP

So, $b^2 = ad$ …(2)

Multiply first equation with a and subtract it from 2nd.

$b^2 – 2ab = ad – ac – a^2$

$\Rightarrow a^2 + b^2 – 2ab = a(d – c)$

Hence a, (a – b), (d – c) are in G.P.

22. Question

If a, b, c are three distinct real numbers in G.P. and $a + b + c = xb$, then prove that either $x < – 1$ or $x > 3$.

Answer

Let a be the first term of GP with r being the common ratio.

$\therefore b = ar$ …(1)

$c = ar^2$ …(2)

Given,

$(a + b + c) = xb$

$\Rightarrow (a + ar + ar^2) = x(ar)$

$\Rightarrow a(1 + r + r^2) = ar$

$\Rightarrow (1 + r + r^2) = xr$

$\Rightarrow r^2 + (1 – x)r + 1 = 0$

As r is a real number ⇒ Both solutions are real.

So discriminant of the given quadratic equation $D \geq 0$

As, $D \geq 0$

$\Rightarrow (1 – x)^2 – 4(1)(1) \geq 0$

$\Rightarrow x^2 – 2x – 3 \geq 0$

$\Rightarrow (x – 1)(x – 3) \geq 0$

$\therefore x < – 1$ or $x > 3$ …proved

23. Question

If p^{th}, q^{th} and r^{th} terms of an A.P. and G.P. are both a, b and c respectively, show that $a^{b - c} b^{c - a} c^{a - b} = 1$.

Answer

Let the A.P. be A, A + D, A + 2 D, ... and G.P be x, xR, xR^2, ... then

a = A + (p – 1)D, b = A + (q – 1)D, c = A + (r – 1)D

⇒ a – b = (p – q)D

Also, b – c = (q – r)D

And, c – a = (r – p)D

Also a = p^{th} term of GP

∴ a = xR^{p-1}

Similarly, b = xR^{q-1} & c = xR^{r-1}

Hence,

$(a^{b-c}).(b^{c-a}).(c^{a-b}) = [(xR^{p-1})^{(q-r)D}].[(xR^{q-1})^{(r-p)D}].[(xR^{r-1})^{(p-q)D}]$

$= x^{(q-r+r-p+p-q)D}.\ R^{[(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)]D}$

⇒ $(a^{b-c}).(b^{c-a}).(c^{a-b}) = x^0.\ R^0$

⇒ $(a^{b-c}).(b^{c-a}).(c^{a-b}) = 1$ ...proved

Exercise 20.6

1. Question

Insert 6 geometric means between 27 and $\frac{1}{81}$.

Answer

Let the six terms be a_1, a_2, a_3, a_4, a_5, a_6.

And,

A = 27, B $= \frac{1}{81}$

Now, these 6 terms are between A and B.

∴ A, a_1, a_2, a_3, a_4, a_5, a_6, B.

Now all of them are in GP

So we now have 8 terms in GP with the first term being 27 and eighth being 1/81.

We know that, $T_n = ar^{n-1}$

Here $T_n = \frac{1}{81}$, a = 27 and

⇒ $\frac{1}{81} = 27r^{8-1}$

⇒ $\frac{1}{81\times27} = r^7$

⇒ $r = \frac{1}{3}$

∴ $a_1 = Ar = 27 \times \frac{1}{3} = 9$

$a_2 = Ar^2 = 27 \times \frac{1}{9} = 3$

$a_3 = Ar^3 = 27 \times \frac{1}{27} = 1$

$a_4 = Ar^4 = 27 \times \frac{1}{81} = \frac{1}{3}$

$a_5 = Ar^5 = 27 \times \frac{1}{243} = \frac{1}{9}$

$a_6 = Ar^6 = 27 \times \frac{1}{729} = \frac{1}{27}$

∴ The six GM between 27 and 1/81 are $9, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$.

2. Question

Insert 5 geometric means between 16 and $\frac{1}{4}$.

Answer

Let the five terms be a_1, a_2, a_3, a_4, a_5.

And,

$A = 16, B = \frac{1}{4}$

Now, these 5 terms are between A and B.

∴ A, a_1, a_2, a_3, a_4, a_5, B.

Now all of them are in GP

So we now have 7 terms in GP with the first term being 16 and seventh being 1/4.

∴ $T_n = ar^{n-1}$

Here $T_n = \frac{1}{4}$, a = 16 and

$\Rightarrow \frac{1}{4} = 16r^{7-1}$

$\Rightarrow \frac{1}{4\times16} = r^6$

$\Rightarrow r = \frac{1}{2}$

∴ $a_1 = Ar = 16 \times \frac{1}{2} = 8$

$a_2 = Ar^2 = 16 \times \frac{1}{4} = 4$

$a_3 = Ar^3 = 16 \times \frac{1}{8} = 2$

$a_4 = Ar^4 = 16 \times \frac{1}{16} = 1$

$a_5 = Ar^5 = 16 \times \frac{1}{32} = \frac{1}{2}$

∴ The six GM between 16 and 1/4 are $8, 4, 2, 1, \frac{1}{2}$.

3. Question

Insert 5 geometric means between $\frac{32}{9}$ and $\frac{81}{2}$.

Answer

Let the five terms be a_1, a_2, a_3, a_4, a_5.

And,

$A = \frac{32}{9}, B = \frac{81}{2}$

Now these 5 terms are between A and B.

∴ A, a_1, a_2, a_3, a_4, a_5, B.

Now all of them are in GP

So we now have 7 terms in GP with the first term being 32/9 and seventh being 81/2.

∴ $T_n = ar^{n-1}$

Here $T_n = \frac{81}{2}$, $a = \frac{32}{9}$ and

$\Rightarrow \frac{81}{2} = \frac{32}{9} r^{7-1}$

$\Rightarrow \frac{81 \times 9}{2 \times 32} = r^6$

$\Rightarrow r = \frac{3}{2}$

∴ $a_1 = Ar = \frac{32}{9} \times \frac{3}{2} = \frac{16}{3}$

$a_2 = Ar^2 = \frac{32}{9} \times \frac{9}{4} = 8$

$a_3 = Ar^3 = \frac{32}{9} \times \frac{27}{8} = 12$

$a_4 = Ar^4 = \frac{32}{9} \times \frac{81}{16} = 18$

$a_5 = Ar^5 = \frac{32}{9} \times \frac{243}{32} = 27$

∴ The six GM between 16 and 1/4 are $\frac{16}{3}, 8, 12, 18, 27$.

4. Question

Find the geometric means of the following pairs of numbers :

i. 2 and 8

ii. a^3b and ab^3

iii. –8 and –2

Answer

(i) GM = $\sqrt{ab}$

Let a = 2 and b =8

GM = $\sqrt{2 \times 8}$

= $\sqrt{16}$

= 4.

(ii) GM = $\sqrt{xy}$

Let $x = a^3b$ and $y = ab^3$

GM = $\sqrt{a^3b \times ab^3}$

= $\sqrt{a^4b^4}$

= a^2b^2.

(iii) GM = $\sqrt{ab}$

Let a = –2 and b = –8

GM = √–2×–8

= √–16

[we know that √–1 = i(iota)]

= 4i.

5. Question

If a is the G.M. of 2 and $\frac{1}{4}$, find a.

Answer

GM = √xy

Let X = 2 and Y = $\frac{1}{4}$

GM = $\sqrt{2 \times \frac{1}{4}}$

= $\sqrt{\frac{1}{2}}$

$\therefore a = \frac{1}{2}$

6. Question

Find the two numbers whose A.M. is 25 and GM is 20.

Answer

⇒ $A.M = \frac{a+b}{2}$

⇒ G.M = √ab

Given A.M=25, G.M = 20.

⇒ √ab = 20 …….(1)

⇒ $\frac{a+b}{2} = 25$…….(2)

⇒ a + b = 50

⇒ a = 50 – b

Putting the value of ‘a’ in equation (1), we get,

⇒ $\sqrt{(50-b)b} = 20$

⇒ $50b - b^2 = 400$

⇒ $b^2 - 50b + 400 = 0$

⇒ $b^2 - 40b - 10b + 400 = 0$

⇒ b(b – 40) – 10(b – 40) = 0

⇒ b = 40 or b = 10

⇒ If b = 40 then a = 10

⇒ If b = 10 then a = 40

7. Question

Construct a quadratic in x such that A.M. of its roots is A and G.M. is G.

Answer

Let the root of the quadratic equation be *a* and *b*.

According to the given condition,

$\Rightarrow \mathrm{AM} = \frac{a+b}{2} = \mathrm{A}$

$\Rightarrow$ a + b = 2A .....(1)

$\Rightarrow$ GM = √ab = G

= ab = G^2...(2)

The quadratic equation is given by,

$x^2 - x$ (Sum of roots) + (Product of roots) = 0

$x^2 - x(2A) + (G^2) = 0$

$x^2 - 2Ax + G^2 = 0$ [Using (1) and (2)]

Thus, the required quadratic equation is $x^2 - 2Ax + G^2 = 0$.

8. Question

The sum of two numbers is 6 times their geometric means, show that the numbers are in the ratio (3 + $2\sqrt{2}$) : (3 – $2\sqrt{2}$).

Answer

Let the two numbers be *a* and *b*.

GM = √ab

According to the given condition,

a + b = 6√ab ...(1)

$(a + b)^2 = 36ab$

Also,

$(a - b)^2 = (a + b)^2 - 4ab$

= 36ab – 4ab

= 32ab

$\Rightarrow$ a–b = √32ab

= 4√2ab .....(2)

Adding (1) and (2), we obtain

2a = (6 + 4√2)√ab

a = (3 + 2√2)√ab

substituting the value of *a* in (1), we obtain,

b =(3 – 2√2)√ab

$\therefore \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

Thus, the required ratio is (3+2√2) : 3–2√2.

9. Question

If AM and GM of roots of a quadratic equation are 8 and 5 respectively, then obtain the quadratic equation.

Answer

Let the root of the quadratic equation be *a* and *b*.

According to the given condition,

$\Rightarrow \mathrm{AM} = \frac{a+b}{2} = 8$

$\Rightarrow a + b = 16$ .....(1)

$\Rightarrow GM = \sqrt{ab} = 5$

$= ab = 25$ …(2)

The quadratic equation is given by,

$x^2 - x$ (Sum of roots) + (Product of roots) = 0

$x^2 - x(a + b) + (ab) = 0$

$x^2 - 16x + 25 = 0$ [Using (1) and (2)]

Thus, the required quadratic equation is $x^2 - 16x + 25 = 0$.

10. Question

If AM and GM of two positive numbers a and b are 10 and 8 respectively, find the numbers.

Answer

$\Rightarrow \mathrm{AM} = \frac{a+b}{2}$

$\Rightarrow GM = \sqrt{ab}$

Given AM = 10, GM = 8.

$\Rightarrow \frac{a+b}{2} = 10$

$\Rightarrow a + b = 20$

$\Rightarrow a = 20 - b$

$\Rightarrow \sqrt{(20-b)b} = 8$

$\Rightarrow 20b - b^2 = 64$

$\Rightarrow b^2 - 20b + 64 = 0$

$\Rightarrow b^2 - 16b - 4b + 64 = 0$

$\Rightarrow b(b - 16) - 4(b - 16) = 0$

$\Rightarrow b = 4$ or $b = 16$

$\Rightarrow$ If $b = 4$ then $a = 16$

$\Rightarrow$ If $b = 16$ then $a = 4$.

Hence, the numbers are 4 and 16.

11. Question

Prove that the product of n geometric means between two quantities is equal to the nth power of a geometric mean of those two quantities.

Answer

Let us suppose a and b are two numbers.

Let us say G is a number that is the Geometric mean of a and b

Therefore a, G and b must be in Geometric Progression or GP.

This means, common ratio = G/a = b/G

Or, $G^2 = ab$

Or, G = ?(ab)... (1)

Now, let us say G_1 , G_2 , G3 , ...Gn are n geometric means between a and b.

Which means that

a , G_1 , G_2 , G_3 ...... G_n , b form a G.P.

Note that the above GP has n+2 terms and the first term is a and the last term is b, which is also the $(n+2)^{th}$ term

Hence, $b = ar^{n+2-1}$

where a is the first term.

So,

$b = ar^{n+1}$

$r = (b/a)^{1/n+1}$ ....(2)

Now the product of GP becomes

Product = $G_1G_2G_3......G_n$

$= (ar)(ar^2)(ar^3).....(ar^n)$

$= a^n\ r^{(1+2+3+4+............+n)}$

$= a^n\ r^{n(1+n)/2}$

Putting the value of r from equation 2 , we get

$= a^n\ (b/a)^{n(}1+n^{)/2(}n+1)$

$= (ab)^{n/2}$

$= (?ab)^n$

Now, putting the value from equation 1, we get,

Product = G^n

Or, $G_1G_2G_3......G_n = G^n$

12. Question

If the A.M. of two positive numbers a and b (a > b) is twice their geometric mean. Prove that : a : b = (2 + $\sqrt{3}$) : (2 – $\sqrt{3}$).

Answer

Let the two numbers be *a* and *b*.

GM = √ab

According to the given condition,

$\Rightarrow \frac{a+b}{2} = 2\sqrt{ab}$

a + b = 4√ab ...(1)

$(a + b)^2 = 16ab$

Also,

$(a - b)^2 = (a + b)^2 - 4ab$

$= 16ab - 4ab$

$= 12ab$

$\Rightarrow a - b = 2\sqrt{3}ab$...(2)

Adding (1) and (2), we obtain

$2a = (4 + 2\sqrt{3})\sqrt{ab}$

$a = (2 + \sqrt{3})\sqrt{ab}$

substituting the value of *a* in (1), we obtain,

$b = (2 - \sqrt{3})\sqrt{ab}$

$\therefore \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

Thus, the required ratio is $(2+\sqrt{3}) : (2-\sqrt{3})$.

13. Question

If one A.M., A and two geometric means G_1 and G_2 inserted between any two positive numbers, show that $\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = 2A.$

Answer

Let the numbers be a and b.

Now $A = \frac{a+b}{2}$ or $2A = a+b$

Also, G_1 and G_2 are GM between a and b, then a, G_1, G_2, b are in G.P.

Let r be the common ratio.

Then, $b = ar^{4-1} = ar^3$

$\Rightarrow \frac{b}{a} = r^3$

$\Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$

$\therefore G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{3}}$

$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{3}}$

$\therefore \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = \frac{G_1^3+G_2^3}{G_1.G_2} = \frac{a^2b+ab^2}{ab} = a + b$

$a + b = 2A$

Very Short Answer

1. Question

If the fifth term of a G.P. is 2, then write the product of its 9 terms.

Answer

Given: Fifth term of GP is 2

⇒ Let the first term be a and the common ratio be r.

∴ According to the question,

$T_5 = 2$

We know,

$a_n = ar^{n-1}$

$a_5 = a.r^{5-1}$

$2 = ar^4$

GP = $a, ar, ar^2, \ldots, ar^8$

Product required = $a \times ar \times ar^2 \times \ldots \times ar^8$

$= a^9.r^{36}$

$= (ar^4)^9$

$= (2)^9$

= 512

2. Question

If $(p+q)^{th}$ and $(p-q)^{th}$ terms of a G.P. are m and n respectively, then write its pth term.

Answer

⇒ Let the first term be a and the common ratio be r.

∴ According to the question,

$a_{p+q} = m.$

$a_{p-q} = n.$

$a_n = ar^{n-1}$

$a_{p+q} = a.r^{p+q-1}$

$a_{p-q} = a.r^{p-q-1}$

∴ $a.r^{p+q-1} = m.$

$a.r^{p-q-1} = n.$

Multiplying above two equations we get

$a^2r^{(p+q-1+(p-q-1)} = a^2r^{(2p-2)}$

$a^2r^{(2p-2)} = m.n$

$(ar)^{2(p-1)} = m.n$

∴ $ar^{p-1} = \sqrt{m.n}$

⇒ P^{th} term is given by $a.r^{p-1}$

∴ $ar^{p-1} = \sqrt{m.n}$

3. Question

If $\log_x a, a^{x/2}$ and $\log_b x$ are in G.P., then write the value of x.

Answer

We know when three terms say a,b,c are in GP

We can write

$b^2 = a.c$

∴ According to the given data

We can write

$(a^{x/2})^2 = \log_x a \cdot \log_b x$

$a^x = \log_x a \cdot \log_b x$

$\Rightarrow a^x = \frac{\log_b a}{\log_b x} \times \log_b x$

$\Rightarrow a^x = \log_b a$

Multiplying by $\log_a$ to both sides we get

$\Rightarrow \log_a (a^x) = \log_a (\log_b a)$

$\Rightarrow x \log_a a = \log_a (\log_b a)$

$\Rightarrow x = \log_a (\log_b a)$

4. Question

If the sum of an infinite decreasing G.P. is 3 and the sum of the squares of its term is $\frac{9}{2}$, then write its first term and common difference.

Answer

Let the given GP be $a, ar, ar^2, \ldots$

Sum of infinite GP is given by $\frac{a}{1-r}$

∴ According to the question

$\Rightarrow \frac{a}{1-r} = 3$

$\Rightarrow a = 3(1-r)$

$\Rightarrow a = 3-3r$

$\Rightarrow a+3r = 3 \ldots(1)$

$\Rightarrow a^2, a^2r^2, a^3r^3, \ldots$

⇒ the first term is a^2 and the common ratio is r^2.

∴ According to the question

$\Rightarrow \frac{a^2}{1-r^2} = \frac{9}{2}$

$\Rightarrow \frac{a}{1-r} \times \frac{a}{1+r} = \frac{9}{2}$

$\Rightarrow 3 \times \frac{a}{1+r} = \frac{9}{2}$

$\Rightarrow \frac{a}{1+r} = \frac{3}{2}$

⇒ $2a = 3+3r$

⇒ $2a-3r = 3$…(2)

Equating 1 and 2 we get

$a = 2$ and $r = 1/3$

5. Question

If p^{th}, q^{th} and r^{th} terms of a G.P. are x, y, z respectively, then write the value of $x^{q-r}\ y^{r-p}\ z^{p-q}$.

Answer

Let the first term be a and the common ratio be R.

∴ According to the question,

$a_p = x$.

$a_q = t$

$a_r = z$.

We know that $a_n = aR^{n-1}$

∴ $a_p = aR^{p-1} = x$

$a_q = aR^{q-1} = y$

$a_r = aR^{r-1} = z$

⇒ $x^{q-r} = (aR^{p-1})^{q-r}$

⇒ $y^{r-p} = (aR^{q-1})^{r-p}$

⇒ $z^{p-q} = (aR^{r-1})^{p-q}$

Multiplying the above three equations we get

$x^{q-r}.y^{r-p}.z^{p-q} = (a^{q-r}.R^{pq-pr-q+r}).\ (a^{r-p}.R^{rq-pq-r+p}).\ (a^{p-q}.R^{pr-qr-p+q})$

$=(a^{q-r+r-p+p-q}.R^{pq-pr-q+r+rq-pq-r+p+pr-qr-p+q})$

$= (a^0.R^0)$

$= 1.$

6. Question

If A_1, A_2 be two AM's and G_1, G_2 be two GM's between a and b, then find the value of $\frac{A_1 + A_2}{G_1\ G_2}$.

Answer

As A_1 and A_2 are A.M between a and b

∴ we can write a, A_1, A_2, b

Let the first term of AP be a and the common difference be d.

∴ $A_1 = a+d$, $A_2 = a+2d$, $b = a+3d$

As G_1 and G_2 are G.M between a and b

∴ we can write a, G_1, G_2, b

Let the first term of AP be a and the common ratio be r.

∴ G_1 = aR, G_2 = ar^2 , b =ar^3

∴ $\frac{A1+A2}{G1G2} = \frac{a+d+a+2d}{ar.ar^2}$

⇒ $\frac{a+(a+3d)}{a.(ar^3)}$

⇒ $\frac{a+b}{a.b}$

7. Question

If second, third and sixth terms of an A.P. are consecutive terms of a G.P., write the common ratio of the G.P.

Answer

Given: Second, third and sixth terms of an A.P. are consecutive terms of a G.P.

Let the first term of AP be a and the common difference be d.

⇒ A_n = a+(n-1)d

⇒ A_2 = a+d

⇒ A_3 = a+2d

⇒ A_6 = a+5d

If a,b,c are consecutive terms of GP then we can write b^2 =a.c

∴ We can write $(a+2d)^2$ = (a+d).(a+5d)

⇒ $a^2+4d^2+4ad = a^2+6ad+5d^2$

⇒ $d^2+2ad = 0$

⇒ d(d+2a) =0

∴ d = 0 or d =-2a

When d = 0 then the GP becomes a,a,a.

∴ The common ration becomes 1.

When d = -2a then the GP becomes –a, -3a,-9a

∴ The common ratio becomes 3.

8. Question

Write the quadratic equation the arithmetic and geometric means of whose roots are A and G respectively.

Answer

Let two roots be a and b

∴ The arithmetic mean is given by $\frac{a+b}{2}$

⇒ $A = \frac{a+b}{2}$

⇒ Geometric mean is given by √a.b

⇒ $G = \sqrt{ab}$

Quadratic equation can be written as

⇒ $x^2 - (a+b)x + ab = 0$

Where a and b are roots of given equation

Substituting AM and GM

⇒ x^2-2Ax+G^2 = 0.

9. Question

Write the product of n geometric means between two number a and b

Answer

Let us suppose a and b are two numbers.

Let us say G is the Geometric mean of a and b.

∴ a, G and b must be in Geometric Progression or GP.

This means, common ratio = G/a = b/G

Or, G2 = ab

Or, G_n = n(ab) .... (1)

Now, let us say G1 , G2 , G3 ,.......Gn are n geomteric means between a and b.

Which means that

a , G1 , G2 , G3 ...... Gn , b form a G.P.

Note that the above GP has n+2 terms and the first term is a and last term is b, which is also the (n+2)th term

Hence, b = ar^{n+2-1}`

where a is the first term.

So,

b = ar^{n+1}

⇒ $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ ...(2)

Now the product of GP becomes

Product = G1G2G3......Gn

= $(ar)(ar^2)(ar^3)..(ar^n)$

= $a^n.r^{(1+2+3...+n)}$

=$a^n.r^{\frac{n(n+1)}{2}}$

Putting the value of r from equation 2 , we get

= $a^n.\left(\left(\frac{b}{a}\right)^{\frac{1}{n+1}}\right)^{\frac{n(n+1)}{2}}$

= $(a.b)^{\frac{n}{2}}$.

10. Question

If $a = 1 + b + b^2 + b^3 + \ldots \text{to } \infty$, then write b in terms of a given that $|b| < 1$.

Answer

Given:

a = 1+b+b^2+b^3+... to infinity

It is an infinite GP with first term as 1 and common ratio b.

Sum of infinite terms is given by

⇒ $s_{\infty} = \frac{a}{1-r}$

⇒ $a = \frac{b}{1-b}$

⇒ a-ab=b

⇒ b+ab =a

⇒ b(1+a) = a

⇒ $b = \frac{a}{1+a}$

MCQ

1. Question

Mark the correct alternative in each of the following:

If in an infinite G.P., first term is equal to 10 times the sum of all successive terms, then its common ratio is

A. 1/10

B. 1/11

C. 1/9

D. 1/20

Answer

Sum of infinite terms is given by

⇒ $s_{\infty} = \frac{a}{1-r}$

According to the question

⇒ $10(s_{\infty}-a)= a$

∴ $S_{\infty} = \frac{11a}{10}$

⇒ $\frac{11a}{10} = \frac{a}{1-r}$

⇒ 11-11r = 10

⇒ $r = \frac{1}{11}$.

2. Question

Mark the correct alternative in each of the following:

If the first term of a G.P. $a_1, a_2, a_3,\ldots$ is unity such that $4a_2 + 5a_3$ is least, then the common ratio of G.P. is

A. − 2/5

B. − 3/5

C. 2/5

D. none of these

Answer

Given GP has first term unity

∴ $a_1 = 1$.

Hence the GP will become

⇒ $1, r, r^2, \ldots$

As given $4a_2+5a_3$ is least

∴ $4r + 5r^2$ is least

We can say $f(r) = 4r + 5r^2$

Now the given function will be least if df(r)/dr =0

$\therefore \frac{d(f(r))}{dr} = \frac{d(4r+5r^2)}{dr}$

⇒ 8+10r=0

⇒ 8=-10r

$\Rightarrow r = \frac{-2}{5}$.

3. Question

Mark the correct alternative in each of the following:

If a, b, c are in A.P. and x, y, z are in G.P., then the value of $x^{b-c}\ y^{c-a}\ z^{a-b}$ is

A. 0

B. 1

C. xyz

D. $x^a\ y^b\ z^c$

Answer

Given: a,b,c are in AP and x,y,z are in GP.

$\Rightarrow b = \frac{a+c}{2}, y^2 = xz$

⇒ 2b = a+c and y^2 = xz.

$\Rightarrow x = \frac{y^2}{z}$

L.H.S = $x^{b-c}.y^{c-a}.z^{a-b}$

$\Rightarrow \left(\frac{y^2}{z}\right)^{b-c}.y^{c-a}.z^{a-b}$

$\Rightarrow y^{2b-2c}.z^{-(b-c)}y^{c-a}z^{a-b}$

$\Rightarrow y^{2b-2c+c-a}.z^{-b+c+a-b}$

$\Rightarrow y^{a+c-c-a}.z^{-c-a+c+a}$

⇒1.1

⇒ 1.

4. Question

Mark the correct alternative in each of the following:

The first three of four given numbers are in G.P. and their last three are in A.P. with common difference 6. If first and fourth numbers are equal, then the first number is

A. 2

B. 4

C. 6

D. 8

Answer

Let, the last three numbers of the set which are in A.P be b,b+6,b+12 and the first number be a.

⇒ Thus, the four numbers are a,b,b+6,b+12

Given:

a=b+12...(1)

Also, given a,b,b+6 are in G.P

From equation (1)

⇒ b+12,b,b+6 are in G.P

⇒ $\frac{b}{b+12}=\frac{b+6}{b}$

⇒ b^2= (b+6)(b+12)

⇒ b^2= b^2+18b+72

⇒ 18b= -72

⇒ b= -4

⇒ a= -4+12

=8

Hence the four numbers are 8,-4,2,8

5. Question

Mark the correct alternative in each of the following:

If a, b, c are in G.P. and $a^{1/x}=b^{1/y}=c^{1/z}$, then xyz are in

A. AP

B. GP

C. HP

D. none of these

Answer

Given: a,b,c are in GP

Let us assume

⇒ $a^{\frac{1}{x}}=b^{\frac{1}{y}}=c^{\frac{1}{z}}=k$

⇒ $a=k^x, b=k^y, c=k^z$

As a,b,c, are in GP

b^2 =ac

⇒ k^{2y} =$k^x.k^z$

=k^{x+z}

⇒ 2y = x+z

∴ x,y,z are in AP

6. Question

Mark the correct alternative in each of the following:

If S be the sum, P the product and R be the sum of the reciprocals of n terms of a GP, then P^2 is equal to

A. S/R

B. R/S

C. $(R/S)^n$

D. $(S/R)^n$

Answer

Let the GP is a , ar , ar^2 , ar^3 , ... ar^{n-1}.

S = Sum of n terms

= a + ar + ar^2 + ar^3 + ...+ ar^{n-1}

$$= \frac{a(r^n - 1)}{r - 1}$$

and R = sum of reciprocal of n terms

$$= \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \cdots + \frac{1}{ar^{n-1}}$$

$$= \frac{1}{a}\left[\frac{1-\left(\frac{1}{r}\right)^n}{1-\frac{1}{r}}\right]$$

$$= \frac{r(r^n - 1)}{a(r-1)r^n}$$

P = product of n terms = a × ar × ar^2 × ar^3 × .....× ar^{n-1}

$=(a^{1+1+1+1+1+1+1+--1})(r^{1+2+3+-----(n-1)})$

$$= a^n . r^{\frac{n(n-1)}{2}}$$

take square both sides,

$$P^2 = a^{2n} . r^{n(n-1)}$$

$$\Rightarrow \frac{S}{R} = \frac{\frac{a(r^n-1)}{r-1}}{\frac{r(r^n-1)}{a(r-1)r^n}}$$

$$\Rightarrow \frac{S}{R} = a^2 . r^{n-1}$$

$$\Rightarrow \left(\frac{S}{R}\right)^n = a^{2n} . r^{n(n-1)}$$

Thus,

$$\Rightarrow \left(\frac{S}{R}\right)^n = P^2$$

7. Question

Mark the correct alternative in each of the following:

The fractional vale of $2.\dot{3}5\dot{7}$ is

A. 2355/1001

B. 2379/997

C. 2355/999

D. none of these

Answer

Let, x = $2.\dot{3}\dot{5}\dot{7}$ = 2.357357...... ...(1)

Multiply by 1000 both sides

1000x = 2357.357357...... ...(2)

Subtracting eq.(1) from eq.(2)

1000x – x = 2357.357357... - 2.357357...

999x = 2355

x = 2355/999

The fractional value of $2.\dot{3}\dot{5}\dot{7}$ is 2355/999

8. Question

Mark the correct alternative in each of the following:

If pth, qth and rth terms of an A.P. are in G.P., then the common ratio of this G.P. is

A. $\frac{p-q}{q-r}$

B. $\frac{q-r}{p-q}$

C. pqr

D. none of these

Answer

Let the first term of AP be a and the common difference be d.

⇒ A_n = a+(n-1)d

p^{th} term is a+(p-1)d.

q^{th} term is a+(q-1)d.

r^{th} term is a+(r-1)d.

As they are in GP.

Let the first term be A and common ratio be R.

⇒ a+(p-1)d =A...(1)

⇒ a+(q-1)d =AR...(2)

⇒ a+(r-1)d =AR^2...(3)

Subtracting 2 from 1 we get...

AR-A = a+(q-1)d-(a+(p-1)d)

= a+qd-d-a-pd+d

AR-A =qd-pd...

Subtracting 2 from 1 we get...

AR^2-AR = a+(r-1)d-(a+(q-1)d)

= a+rd-d-a-qd+d

R(AR-A) =rd-qd.

$\Rightarrow \frac{R(AR-A)}{AR-A} = \frac{d(q-p)}{d(r-q)}$

$\Rightarrow R = \frac{q-p}{r-q}$

9. Question

Mark the correct alternative in each of the following:

The value of $9^{1/3} \cdot 9^{1/9} \cdot 9^{1/27} \ldots$ to ∞, is

A. 1

B. 3

C. 9

D. none of these

Answer

Given: $9^{1/3} . 9^{1/9} . 9^{1/27} \ldots \infty$

$\Rightarrow 9^{1/3+1/9+1/27 \ldots \infty}$

$\Rightarrow \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots \infty$

We can see the above expression is GP

With first term as $\frac{1}{3}$ and the common ratio being $\frac{1}{3}$.

Sum of infinite terms is given by

$\Rightarrow S_{\infty} = \frac{a}{1-r}$

$\Rightarrow S_{\infty} = \frac{\frac{1}{3}}{1-\frac{1}{3}}$

$\Rightarrow S_{\infty} = \frac{1}{2}$

∴ The given equation turns out to be $9^{1/2}$

= 3.

10. Question

Mark the correct alternative in each of the following:

The sum of an infinite G.P. is 4 and the sum of the cubes of its terms is 192. The common ratio of the original G.P. is

A. 1/2

B. 2/3

C. 1/3

D. −1/2

Answer

Let, the first term of G.P. is a and common ratio is r.

We know that common ratio of infinite G.P. is belongs to

[0, 1)

G.P. ⇒ a, ar, ar^2, ……

Sum of infinite terms of G.P. $= \frac{a}{(1-r)} = 4$

⇒ a = 4(1 – r)

Cubic terms of a G.P. ⇒ a^3, a^3r^3, a^3r^6, ……

Sum of cubes of terms $= \frac{a^3}{1-r^3} = 192$

⇒ $a^3 = 192(1 - r^3)$

⇒ $4^3(1 - r)^3 = 92(1 - r^3)$

⇒ $(1 - r)^3 = 3(1 - r)(1 + r + r^2)$

Case I : 1 – r = 0

⇒ r = 1 (not possible)

Case II : $(1 - r)^2 = 3(1 + r + r^2)$

⇒ $2r^2 + 5r + 2 = 0$

⇒ (2r + 1)(r + 2) = 0

⇒ r = -2 (not possible) and r = -1/2

So, common ratio of original G.P. is -1/2

11. Question

Mark the correct alternative in each of the following:

If the sum of first two terms of an infinite GP is 1 and every term is twice the sum of all the successive terms, then its first term is

A. 1/3

B. 2/3

C. 1/4

D. 3/4

Answer

Let the first term be a and the common ratio be r.

Sum of infinite terms is given by

$\Rightarrow S_\infty = \frac{a}{1-r}$

The general term of any GP is given by ar^n.

The infinite sum of all successive terms is

$(ar^n)r + (ar^n)r^2 + (ar^n)r^3 + \ldots$

$= \frac{r.ar^n}{1-r}$

Therefore

$\Rightarrow ar^n = 2\frac{ar^n.r}{1-r}$

1-r = 2r

r = 1/3.

Also given that

a+ar = a(1+r) = 1.

∴ a = 1/(1+r) = 3/4.

12. Question

Mark the correct alternative in each of the following:

The nth term of a G.P. is 128 and the sum of its n terms is 225. If its common ratio is 2, then its first term is

A. 1

B. 3

C. 8

D. none of these

Answer

Let say GP is

a,ar,ar^2

where a is the first term and r is the common ratio

Also, given that r = 2

So, GP becomes

a,2a,4a

nth term = $ar^{(n-1)}$ = 128

a.$2^{(n-1)}$ = 128

$\Rightarrow \frac{a.2^n}{2} = 128$

⇒ a.2^n = 256

Sum of n terms of GP

$\Rightarrow S = \frac{a(1-r^n)}{1-r}$

$\Rightarrow S = \frac{a(1-2^n)}{1-2}$

$\Rightarrow -255 = a - a^{2n}$

Substituting a^{2n} in above equation we get

-255 = a-256

a =1.

13. Question

Mark the correct alternative in each of the following:

If second term of a G.P. is 2 and the sum of its infinite terms is 8, then its first term is

A. 1/4

B. 1/2

C. 2

D. 4

Answer

Let the GP be a,ar,ar^2

Sum of infinite terms is given by

$\Rightarrow s_\infty = \frac{a}{1-r}$

$\Rightarrow 8 = \frac{a}{1-r}$

Also according to the question ar =2

$\therefore r = \frac{2}{a}$

Substituting in above equation

$\Rightarrow 8 = \frac{a}{1-\frac{2}{a}}$

⇒ 8(a-2) =a^2

⇒ a^2-8a+16 = 0

⇒ $(a-4)^2$ = 0

⇒ a =4.

14. Question

Mark the correct alternative in each of the following:

If a, b, c are in G.P. and x, y are AM's between a, b and b, c respectively, then

A. $\frac{1}{x} + \frac{1}{y} = 2$

B. $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$

C. $\frac{1}{x} + \frac{1}{y} = \frac{2}{a}$

D. $\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$

Answer

a,b,c are in GP

∴ b^2 = a.c also c =ar^2

Where r is the common ratio

a,x,b,y,c are in AP

$\Rightarrow x = \frac{a+b}{2} = \frac{a+ar}{2} = \frac{a(1+r)}{2}$

$$\Rightarrow\ y=\frac{b+c}{2}=\frac{ar+ar^2}{2}=\frac{ar(1+r)}{2}$$

$$\Rightarrow\ \frac{1}{x}+\frac{1}{y}=\frac{2}{a(1+r)}+\frac{2}{ar(1+r)}$$

$$=\frac{2r+2}{ar(1+r)}$$

$$=\frac{2(1+r)}{ar(1+r)}$$

$$=\frac{2}{ar}$$

$$\Rightarrow\ \frac{1}{x}+\frac{1}{y}=\frac{2}{b}$$

15. Question

Mark the correct alternative in each of the following:

If A be one A.M. and p, q be two G.M.'s between two numbers, then 2 A is equal to

A. $\frac{p^3+q^3}{pq}$

B. $\frac{p^3-q^3}{pq}$

C. $\frac{p^2+q^2}{2}$

D. $\frac{pq}{2}$

Answer

Let the numbers be x,y.

x,A,y = 2A =x+y

x,p,q,y = p^2 = xq ; q^2 =py [a,b,c GM =b^2 =ac]

$\Rightarrow \frac{p^2}{q}=x;\frac{q^2}{p}=y$

$\Rightarrow x+y=2A=\frac{p^3+q^3}{pq}$

16. Question

Mark the correct alternative in each of the following:

If p, q be two A.M.'s and G be one G.M. between two numbers, then $G^2=$

A. $(2p-q)(p-2q)$

B. $(2p-q)(2q-p)$

C. $(2p-q)(p+2q)$

D. none of these

Answer

Let the numbers be a,b.

∴ a,p,q,b (two A.M.)

⇒ a,G,b(one GM)

G^2 = a.b(GM)

2p = a+q(AM)

2q = b+p(AM)

a = 2p-q

b = 2q-p

∴ a.b = (2p-q)(2q-p)

G^2 = (2p-q)(2q-p)

17. Question

Mark the correct alternative in each of the following:

If x is positive, the sum to infinity of the series $\frac{1}{1+x}-\frac{1-x}{(1-x)^2}+\frac{(1+x)^2}{(1+x)^3}-\frac{(1-x)^3}{(1+x)^4}+\ldots$ is

A. 1/2

B. 3/4

C. 1

D. none of these

Answer

It's first term is $\frac{1}{1+x}$ and common ratio is $-\frac{1-x}{1+x}$

Sum of infinite terms of G.P. = $\frac{a}{(1-r)}$

$$= \frac{\frac{1}{1+x}}{1-\left(-\frac{1-x}{1+x}\right)}$$

$$= \frac{1}{2}$$

18. Question

Mark the correct alternative in each of the following:

If $(4^3)(4^6)(4^9)(4^{12})\ldots(4^{3x})=(0.0625)^{-54}$, the value of x is

A. 7

B. 8

C. 9

D. 10

Answer

RHS = $(0.0625)^{-54}$

$\therefore \left(\frac{625}{10000}\right)^{-54} = \left(\frac{1}{16}\right)^{-54}$

$\Rightarrow (4^{-2})^{-54} = 4^{108}$

LHS = $4^{3(1+2+3...3x)}$

Sum of AP= $\frac{n}{2}(2a + (n-1)d$

$\Rightarrow \frac{3x}{2}(2.1 + (3x-1)1)$

$\Rightarrow \frac{3x}{2}(2 + 3x - 1)$

$\Rightarrow \frac{3x}{2}(3x + 1)$

LHS = $4^{\frac{9x}{2}(3x+1)} = 4^{108}$

$\Rightarrow 27x^2+9x-216 = 0$

$\Rightarrow 3x^2+x-24 = 0$

$\Rightarrow 3x^2+9x-8x-24 = 0$

$\Rightarrow 3x(x+3)-8(x-3) = 0$

$\Rightarrow \frac{8}{3} \times 3 = x$

$\therefore$ x =8.

19. Question

Mark the correct alternative in each of the following:

Given that x > 0, the sum $\sum_{n=1}^{\infty}\left(\frac{x}{x+1}\right)^{n-1}$ equals

A. x

B. x + 1

C. $\frac{x}{2x+1}$

D. $\frac{x+1}{2x+1}$

Answer

The Given sequence becomes an infinite GP where first term a = 1

And common ratio r $=\frac{x}{x+1}$

Sum of infinite terms is given by

$\Rightarrow s_{\infty} = \frac{a}{1-r}$

$\Rightarrow s_{\infty} = \frac{1}{1-\frac{x}{x+1}}$

$\therefore s_{\infty} = x + 1$

20. Question

Mark the correct alternative in each of the following:

In a G.P. of even number of terms, the sum of all terms is five times the sum of the odd terms. The common ratio of the G.P. is

A. $-\frac{4}{5}$

B. $\frac{1}{5}$

C. 4

D. none of these

Answer

Let,a be the first term and r be the common ratio. The

number of terms is 2n.

G.P. ⇒ a, ar, ar^2, ... (upto 2n terms)

Sum of all terms = $\frac{a(1-r^{2n})}{1-r}$

Odd terms G.P. ⇒ a, ar^2, ar^4, ... (upto n terms)

Sum of odd terms G.P. = $\frac{a(1-(r^2)^n)}{1-r^2} = \frac{a(1-r^{2n})}{1-r^2}$

Sum of all terms = 5×Sum of odd terms

$$\frac{a(1-r^{2n})}{1-r} = 5 \times \frac{a(1-r^{2n})}{1-r^2}$$

5(1 – r) = (1 – r^2)

r^2 – 5r + 4 = 0

(r – 1)(r – 4) = 0

r = 1(not possible) and r = 4

So, common ratio of the G.P. = 4

21. Question

Mark the correct alternative in each of the following:

Let x be the A.M. and y, z be two G.M.s between two positive numbers. Then, $\frac{y^3+z^3}{xyz}$ is equal to

A. 1

B. 2

C. $\frac{1}{2}$

D. none of these

Answer

Let the numbers be a,b.

a,x,b = 2x =a+b

a,y,z,b = y^2 = az ; z^2 =by [a,b,c GM =b^2 =ac]

$\Rightarrow \frac{y^2}{z} = a; \frac{z^2}{y} = b$

$\Rightarrow a + b = \frac{y^2}{z} + \frac{z^2}{y}$

$\Rightarrow 2x = \frac{y^3+z^3}{zy}$

$\Rightarrow \frac{y^3+z^3}{xyz} = 2.$

22. Question

Mark the correct alternative in each of the following:

The product $(32), (32)^{1/16} (32)^{1/36}$ ... to ∞ is equal to

A. 64

B. 16

C. 32

D. 0

Answer

Product of series = $32 \times 32^{1/6} \times 32^{1/32} \times \ldots$

$= 32^{(1 + 1/6 + 1/32 + \ldots)}$

$= 32^{\frac{a}{(1-r)}}$

$= 32^{\frac{1}{\left(1-\frac{1}{6}\right)}}$

$= (2^5)^{\frac{6}{5}}$

=64

23. Question

Mark the correct alternative in each of the following:

The two geometric means between the numbers 1 and 64 are

A. 1 and 64

B. 4 and 16

C. 2 and 16

D. 8 and 16

Answer

Let the GM be y,z

∴ 1,y,z,64

$y^2 = 1.z$; $z^2 = 64y$

$y^4 = z^2$

∴ $y^4 = 64y$

⇒ y = 4

∴ Z =16

∴ The two GM are 4,16.

24. Question

Mark the correct alternative in each of the following:

In a G.P. if the $(m+n)^{th}$ term is p and $(m-n)^{th}$ term is q, then its m^{th} term is

A. 0

B. pq

C. $\sqrt{pq}$

D. $\frac{1}{2}(p+q)$

Answer

⇒ Let the first term be a and the common ratio be r.

∴ According to the question,

$a_{m+n} = p.$

$a_{m-n} = q.$

$a_n = ar^{n-1}$

$a_{m+n} = a.r^{m+n-1}$

$a_{m-n} = a.r^{m-n-1}$

∴ $a.r^{m+n-1} = p.$

$a.r^{m-n-1} = n.$

Multiplying above two equations we get

$a^2r^{(m+n-1+(m-n-1)} = a^2r^{(2m-2)}$

$a^2r^{(2m-2)} = p.q$

$(ar)^{2(m-1)} = p.q$

∴ $ar^{m-1} = \sqrt{}p.q$

⇒ M^{th} term is given by $a.r^{m-1}$

∴ $ar^{m-1} = \sqrt{}p.q$

25. Question

Mark the correct alternative in each of the following:

Let S be the sum, P be the product and R be the sum of the reciprocals of 3 terms of a G.P. then $P^2R^3 : S^3$ is equal to

A. 1 : 1

B. $(\text{common ratio})^n : 1$

C. $(\text{First term})^2(\text{Common ratio})^2$

D. None of these

Answer

Let, 3 terms of the G.P. be a\r, a, ar

S = a\r + a + ar = $a\left(1 + r + \frac{1}{r}\right)$

$S^3 = a^3\left(1 + r + \frac{1}{r}\right)^3$

P = a\r × a × ar = a^3

$P^2 = a^6$

R = r\a + 1\a + 1\ar = $\frac{1}{a}\left(r + 1 + \frac{1}{r}\right)$

$R^3 = \frac{1}{a^3}\left(r + 1 + \frac{1}{r}\right)^3$

Then, $P^2R^3 : S^3$

$\Rightarrow a^6 \times \frac{1}{a^3}\left(r + 1 + \frac{1}{r}\right)^3 : a^3\left(1 + r + \frac{1}{r}\right)^3$

$\Rightarrow a^3\left(r + 1 + \frac{1}{r}\right)^3 : a^3\left(1 + r + \frac{1}{r}\right)^3$

⇒ 1 : 1