Wave Optics

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. In a double slit experiment using light of wavelength 600 nm, the angular width of a fringe on a distant screen is 0.1°. The spacing between the two slits is

(a) 3.44×10^{-4} m (b) 1.54×10^{-4} m

(c) 1.54×10^{-3} m (d) 1.44×10^{-3} m

2. A diffraction pattern is obtained by using beam of red light what will happen, if red light is replaced by the blue light?

(a) Bands disappear.

(b) Bands become broader and farther apart.

(c) No change will take place.

(d) Diffraction bands become narrow and crowded together.

3. A screen is placed 50 cm from a single slit which is illuminated with light of wavelength 6000 Å. If the distance between the first and third minima in the diffraction pattern is 3.0 mm. The width of the slit is

(a) 1×10^{-4} m (b) 2×10^{-4} m

(c) 0.5×10^{-4} m (d) 4×10^{-4} m

4. Which of the following is correct for light diverging from a point source?

(a) The intensity decreases in proportion for the distance squared.

(b) The wavefront is parabolic.

(c) The intensity at the wavelength does not depend on the distance.

(d) None of these.

5. A Young's double slit experiment uses a monochromatic source of light. The shape of interference fringes formed on the screen is

(a) parabola (b) straight line

(c) circle (d) hyperbola

6. A parallel beam of sodium light of wavelength 5890 Å is incident on a thin glass plate of refractive index 1.5 such that the angle of refraction in the plate is 60°. The smallest thickness of the plate which will make it dark by reflection

(a) 3926 Å (b) 4353 Å

(c) 1396 Å (d) 1921 Å

7. Consider the following statements in case of Young's double slit experiment.

(1) A slit S is necessary if we use an ordinary extended source of light.

(2) A slit S is not needed if we use an ordinary but well collimated beam of light.

(3) A slit S is not needed if we use a spatially coherent source of light.

Which of the above statements are correct?

(a) (1), (2) and (3) (b) (1) and (2) only

(c) (2) and (3) only (d) (1) and (3) only

8. In a Young's double slit experiment an electron beam is used to obtain interference pattern. If the spread of electron is decreases then

(a) distance between two consecutive fringes remains the same

(b) distance between two consecutive fringes decreases

(c) distance between two consecutive fringes increases

(d) none of these

9. The colours seen in the reflected white light from a thin oil film are due to

(a) Diffraction (b) Interference

(c) Polarisation (d) Dispersion

10. In young's double slit experiment using monochromatic light of wavelengths λ, the intensity of light at a point on the screen with path difference is λ is M unit. The intensity of light at a point where path difference is $\lambda/3$ is

(a) $\frac{M}{2}$ (b) $\frac{M}{4}$

(c) $\frac{M}{8}$ (d) $\frac{M}{16}$

11. In a two slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the plane of slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} m. If the distance between slits is 10^{-3} m, the wavelength of light will be

(a) 3000 Å (b) 4000 Å
(c) 6000 Å (d) 7000 Å

12. In a double slit experiment the distance between slits is increased ten times whereas their distance from screen is halved then the fringe width is

(a) becomes $\frac{1}{20}$ (b) becomes $\frac{1}{90}$
(c) it remains same (d) becomes $\frac{1}{10}$

13. The fringe width in a young's double slit interference pattern is 2.4×10^{-4} m, when red light of wavelength 6400 Å is used. How much will it change, if blue light of wavelength 4000 Å is used ?

(a) 0.9×10^{-4} m (b) 1.5×10^{-4} m
(c) 4.5×10^{-4} m (d) 0.45×10^{-4} m

14. Two slits in young's double slit experiment have widths in the ratio 81 : 1. The ratio of the amplitudes of light waves is

(a) 3 : 1 (b) 3 : 2
(c) 9 : 1 (d) 6 : 1

15. In Young's double slit experiment two disturbances arriving at a point P have phase difference of $\frac{\pi}{3}$. The intensity of this point expressed as a fraction of maximum intensity I_0 is

(a) $\frac{3}{2} I_0$ (b) $\frac{1}{2} I_0$
(c) $\frac{4}{3} I_0$ (d) $\frac{3}{4} I_0$

16. Interference fringes were produced in Young's double slit experiment using light of wavelength 5000 Å. When a film of material 2.5×10^{-3} cm thick was placed over one of the slits, the fringe pattern shifted by a distance equal to 20 fringe widths. The refractive index of the material of the film is

(a) 1.25 (b) 1.33
(c) 1.4 (d) 1.5

17. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then the difference between the resultant intensities at A and B is

(a) $2I$ (b) $4I$
(c) $5I$ (d) $7I$

18. In the case of light waves from two coherent sources S_1 and S_2, there will be constructive interference at an arbitrary point P, the path difference $S_1P - S_2P$ is

(a) $\left(n + \frac{1}{2}\right)\lambda$ (b) $n\lambda$
(c) $\left(n - \frac{1}{2}\right)\lambda$ (d) $\frac{\lambda}{2}$

19. A plane wave passes through a convex lens. The geometrical shape of the wavefront that emerges is

(a) plane
(b) diversing spherical
(c) converging spherical
(d) none of these

20. In a Young's double slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case

(a) there shall be alternate interference patterns of red and blue
(b) there shall be an interference pattern for red distinct from that for blue
(c) there shall be no interference fringes
(d) there shall be an interference pattern for red mixing with one for blue

21. In a double slit experiment, the distance between the slits is d. The screen is at a distance D from the slits. If a bright fringe is formed opposite to one of the slits, its order is

(a) $\frac{d}{\lambda}$ (b) $\frac{\lambda^2}{dD}$
(c) $\frac{D^2}{2\lambda d}$ (d) $\frac{d^2}{2D\lambda}$

22. Consider sunlight incident on a slit of width 10^4 Å. The image seen through the slit shall

(a) be a fine sharp slit white in colour at the centre
(b) a bright slit white at the centre diffusing to zero intensities at the edges
(c) a bright slit white at the centre diffusing to regions of different colours
(d) only be a diffused slit white in colour

23. Two sources of light of wavelength 2500 Å and 3500 Å are used in Young's double slit experiment simultaneously. Which orders of fringes of two wavelength patterns coincide?
(a) 3^{rd} order of 1^{st} source and 5^{th} of the 2^{nd}
(b) 7^{th} order of 1^{st} and 5^{th} order of 2^{nd}
(c) 5^{th} order of 1^{st} and 3^{rd} order of 2^{nd}
(d) 5^{th} order of 1^{st} and 7th order of 2^{nd}

24. Yellow light of wavelength 6000 Å produces fringes of width 0.8 mm in Young's double slit experiment. If the source is replaced by another monochromatic source of wavelength 7500 Å and the separation between the slits is doubled then the fringe width becomes
(a) 0.1 mm (b) 0.5 mm
(c) 4.3 mm (d) 1 mm

25. In Young's double slit experiment the distance d between the slits S_1 and S_2 is 1 mm. What should the width of each slit be so as to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern?
(a) 0.9 mm (b) 0.8 mm
(c) 0.2 mm (d) 0.6 mm

26. The idea of secondary wavelets for the propagation of a wave was first given by
(a) Newton (b) Huygens
(c) Maxwell (d) Fresnel

27. When interference of light takes place
(a) energy is created in the region of maximum intensity
(b) energy is destroyed in the region of maximum intensity
(c) conservation of energy holds good and energy is redistributed
(d) conservation of energy does not hold good

28. Which of the following is the path difference for distructive interference ?
(a) $n(\lambda + 1)$ (b) $(2n+1)\dfrac{\lambda}{2}$
(c) $n\lambda$ (d) $(n+1)\dfrac{\lambda}{2}$

29. The phenomena which is not explained by Huygens construction of wavefront
(a) reflection (b) diffraction
(c) refraction (d) origin of spectra

30. In a Young's double slit experiment, (slit distance d) monochromatic light of wavelength λ is used and the fringe pattern observed at a distance D from the slits. The angular position of the bright fringes are
(a) $\sin^{-1}\left(\dfrac{N\lambda}{d}\right)$ (b) $\sin^{-1}\left(\dfrac{\left(N+\frac{1}{2}\right)\lambda}{d}\right)$
(c) $\sin^{-1}\left(\dfrac{N\lambda}{D}\right)$ (d) $\sin^{-1}\left(\dfrac{\left(N+\frac{1}{2}\right)\lambda}{D}\right)$

31. For what distance is ray optics a good approximation when the aperture is 4 mm wide and the wavelength is 500 nm?
(a) 22 m (b) 32 m
(c) 42 m (d) 52 m

32. A narrow slit of width 2 mm is illuminated by monochromatic light of wavelength 500 nm. The distance between the first minima on either side on a screen at a distance of 1 m is
(a) 5 mm (b) 0.5 mm
(c) 1 mm (d) 10 mm

33. The two coherent sources with intensity ratio β produce interference. The fringe visibility will be
(a) $\dfrac{2\sqrt{\beta}}{1+\beta}$ (b) 2β
(c) $\dfrac{2}{(1+\beta)}$ (d) $\dfrac{\sqrt{\beta}}{1+\beta}$

34. A slit of width d is illuminated by white light. The first minimum for red light (λ = 6500 Å) will fall at $\theta = 30°$ when d will be
(a) 3200 Å (b) 6.5×10^{-4} mm
(c) 1.3 micron (d) 2.6×10^{-4} cm

35. In a Young's double slit experiment, let S_1 and S_2 be the two slits, and C be the centre of the screen. If $\angle S_1CS_2 = \theta$ and λ is the wavelength, the fringe width will be
(a) $\dfrac{\lambda}{\theta}$ (b) $\lambda\theta$
(c) $\dfrac{2\lambda}{\theta}$ (d) $\dfrac{\lambda}{2\theta}$

36. Young's experiment is performed with light of wavelength 6000 Å wherein 16 fringes occupy a certain region on the screen. If 24 fringes occupy the same region with another light, of wavelength λ, then λ is
(a) 6000 Å (b) 4500 Å
(c) 5000 Å (d) 4000 Å

37. Wavefront is the locus of all points, where the particles of the medium vibrate with the same

(a) phase

(b) amplitude

(c) frequency

(d) period

38. In a Young's double slit experiment, the angular width of a fringe formed on a distant screen is 1°. The slit separation is 0.01 mm. The wavelength of the light is

(a) 0.174 nm (b) 0.174 Å

(c) 0.174 μm (d) 0.174×10^{-4} m

39. Light from two coherent sources of the same amplitude A and wavelength λ illuminates the screen. The intensity of the central maximum is I_0. If the sources were incoherent, the intensity at the same point will be

(a) $4I_0$ (b) $2I_0$

(c) I_0 (d) $\frac{I_0}{2}$

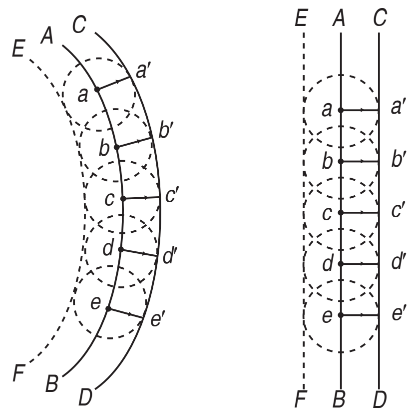
Case Based MCQs

Case I : Read the passage given below and answer the following questions from 40 to 42.

Hnygen Principle

Huygen principle is the basis of wave theory of light. Each point on a wavefront acts as a fresh source of new disturbance, called secondary waves or wavelets. The secondary wavelets spread out in all directions with the speed light in the given medium.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.



40. The initial shape of the wavefront of the beam is

(a) planar (b) convex

(c) concave

(d) convex near the axis and concave near the periphery

41. According to Huygens Principle, the surface of constant phase is

(a) called an optical ray (b) called a wave

(c) called a wavefront

(d) always linear in shape

42. As the beam enters the medium, it will

(a) travel as a cylindrical beam

(b) diverge

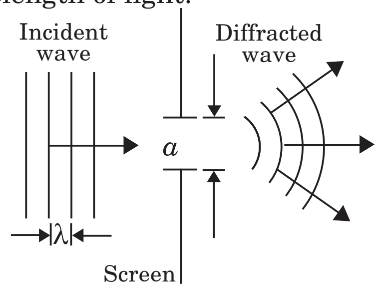
(c) converge

(d) diverge near the axis and converge near the periphery.

Case II : Read the passage given below and answer the following questions from 43 to 45.

Diffraction

The phenomenon of bending of light around the sharp corners and the spreading of light within the geometrical shadow of the opaque obstacles is called diffraction of light. The light thus deviates from its linear path. The deviation becomes much more pronounced, when the dimensions of the aperture or the obstacle are comparable to the wavelength of light.



43. In diffraction from a single slit the angular width of the central maxima does not depends on

(a) λ of light used

(b) width of slit

(c) distance of slits from the screen

(d) ratio of λ and slit width

44. For a diffraction from a single slit, the intensity of the central point is

(a) infinite

(b) finite and same magnitude as the surrounding maxima

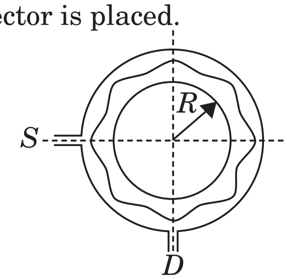
(c) finite but much larger than the surrounding maxima
(d) finite and substantially smaller than the surrounding maxima

45. In a single diffraction pattern observed on a screen placed at D metre distance from the slit of width d metre, the ratio of the width of the central maxima to the width of other secondary maxima is
(a) 2 : 1 (b) 1 : 2 (c) 1 : 1 (d) 3 : 1

Case III : Read the passage given below and answer the following questions from 46 to 49.

Young's Double Slit Experiment

A narrow tube is bent in the form of a circle of radius R, as shown in figure. Two small holes S and D are made in the tube at the positions at right angle to each other. A source placed at S generates a wave of intensity I_0 which is equally divided into two parts: one part travels along the longer path, while the other travels along the shorter path. Both the waves meet at point D where a detector is placed.



46. If a maxima is formed at a detector, then the magnitude of wavelength λ of the wave produced is given by
(a) πR (b) $\frac{\pi R}{2}$
(c) $\frac{\pi R}{4}$ (d) all of these

47. The maximum intensity produced at D is given by
(a) $4I_0$ (b) $2I_0$ (c) I_0 (d) $3I_0$

48. In a Young's double slit experiment, the intensity at a point where the path difference is $\lambda/6$ (λ – wavelength of the light) is I. If I_0 denotes the maximum intensity, then I/I_0 is equal to
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{3}{4}$

49. Two identical light waves, propagating in the same direction, have a phase difference d. After they superpose the intensity of the resulting wave will be proportional to
(a) $\cos\delta$ (b) $\cos(\delta/2)$
(c) $\cos^2(\delta/2)$ (d) $\cos^2\delta$

Assertion & Reasoning Based MCQs

For question numbers 50-58, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false and R is also false

50. Assertion (A) : The maximum intensity in interference pattern is four times the intensity due to each slit.
Reason (R): Intensity is directly proportional to square of amplitude.

51. Assertion (A) : Diffraction is common in sound but not common in light waves.
Reason (R): Wavelength of light is more than the wavelength of sound.

52. Assertion (A) : Interference obeys the law of conservation of energy.
Reason (R) : The energy is redistributed in case of interference.

53. Assertion (A) : We cannot get diffraction pattern from a wide slit illuminated by monochromatic light.
Reason (R) : In diffraction pattern, all the bright bands are not of the same intensity.

54. Assertion (A) : When a light wave travels from a rarer to a denser medium, it loses speed. The reduction in speed imply a reduction in energy carried by the light wave.
Reason (R) : The energy of a wave is proportional to velocity of wave.

55. Assertion (A) : The film which appears bright in reflected system will appear dark in the transmitted light and vice-versa.

Reason (R) : The conditions for film to appear bright or dark in reflected light are just reverse to those in the transmitted light.

56. Assertion (A) : In Young's double slit experiment, the fringes become indistinct if one of the slits is covered with cellophane paper.
Reason (R) : The cellophane paper decrease the wavelength of light.

57. Assertion (A) : One of the condition for interference is that the two source should be very narrow.
Reason (R) : One broad source is equal to large number of narrow sources.

58. Assertion (A) : When tiny circular obstacle is placed in the path of light from some distance, a bright spot is seen at the centre of the shadow of the obstacle.
Reason (R) : Destructive interference occurs at the centre of the shadow.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. Define the term 'coherent sources' which are required to produce interference pattern in Young's double slit experiment.

2. State Huygens principle of diffraction of light.

3. How does the angular separation between fringes in single-slit diffraction experiment change when the distance of separation between the slit and screen is doubled.

4. In a Young's double slit experiment, the fringe width is found to be 0.12 mm. If the whole apparatus is immersed in water of refractive index (4/3), without disturbing the geometrical arrangement, what is the new fringe width?

5. In double-slit experiment using light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is 0.1°. What is the spacing between the two slits?

6. What is the shape of the wavefront on earth for sunlight?

7. Is Huygens principle valid for longitudinal sound waves?

8. Why is the diffraction of sound waves more evident in daily experience than that of light wave?

9. If one of the slits in Young's double slit experiment is fully closed, the new pattern has ______ central maximum in angular size.

10. Two slits are made one millimetre apart and the screen is placed one metre away. What is the fringe separation when blue-green light of wavelength 500 nm is used?

Short Answer Type Questions (SA-I)

11. In a two-slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5}. If the distance between the slits is 10^{-3} m, calculate the wavelength of the light used.

12. A soap film of thickness 0.3 μm appears dark when seen by the reflected light of wavelength 580 nm. What is the index of refraction of the soap solution,if it is known to be between 1.3 and 1.5?

13. Light of wavelength 6×10^{-5} cm falls on a screen at a distance of 100 cm from a narrow slit. Find the width of the slit if the first minima lies 1 mm on either side of the central maximum.

14. The intensity of the light coming from one of the slits in a YDSE is double the intensity from the other slit. Find the ratio of maximum intensity to minimum intensity in the interference fringe pattern observed.

15. In a YDSE, the slits are 2 mm apart and are illuminated with a mixture of two wavelengths λ = 750 nm and λ' = 900 nm. At what distance from the common central bright fringe on a screen 2 m from the slits will a bright fringe from one interference pattern coincide with a bright fringe from the other?

16. Two wavelengths of sodium light 590 nm and 596 nm are used, in turn to study the diffraction taking place at a single slit of aperture 2×10^{-4} m. The distance between the slit and the screen is 1.5 m. Calculate the separation between the positions of the first maxima of the diffraction pattern obtained in the two cases.

17. Laser light of wavelength 640 nm incident on a pair of slits produces an interference pattern in which the bright fringes are separated by 7.2 mm. Calculate the wavelength of another source of light which produces interference fringes separated by 8.1 mm using same arrangement. Also find the minimum value of the order (n) of bright fringe of shorter wavelength which coincides with that of the longer wavelength.

18. A slit of width a is illuminated by light of wavelength 6000 Å. For what value of a will the

(i) First maximum fall at an angle of diffraction of 30°

(ii) First minimum fall at an angle of diffraction 30°?

19. Yellow light (l = 6000 Å) illuminates a single slit of width 1×10^{-4} m. Calculate the distance between two dark lines on either side of the central maximum, when the diffraction pattern is viewed on a screen kept 1.5 m away from the slit.

20. In a single slit diffraction experiment first minimum for λ_1 = 660 nm coincides with first maxima for wavelength λ_2. Calculate λ_2.

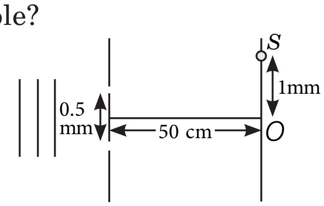
Short Answer Type Questions (SA-II)

21. A beam of light consisting of two wavelengths, 6500 Å and 5200 Å is used to obtain slit experiment ($1\ Å = 10^{-10}$ m). The distance between the slits is 2.0 mm and the distance between the plane of the slits and the screen is 120 cm.

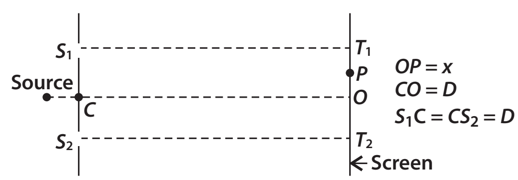
(i) Find the distance of the third bright fringe on the screen from the central maximum for the wavelength 6500 Å.

(ii) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

22. White coherent light (400 nm-700 nm) is sent through the slits of a YDSE, the separation between the slits is 0.5 mm and the screen is 50 cm away from the slits. There is a hole in the screen at a point 1 mm away (along the width of the fringes) from the central line. Which wavelength will be absent in the light coming from the hole?



23. Consider a two slit interference arrangements such that the distance of the screen from the slits is half the distance between the slits. Obtain the value of D in terms of λ such that the first minima on the screen falls at a distance D from the centre O.



24. (a) In a single slit diffraction pattern, how does the angular width of the central maximum vary, when

(i) aperture of slit is increased?

(ii) distance between the slit and the screen is decreased?

(b) How is the diffraction pattern different from the interference pattern obtained in Young's double slit experiment?

25. (a) Two monochromatic waves emanating from two coherent sources have the displacements represented by

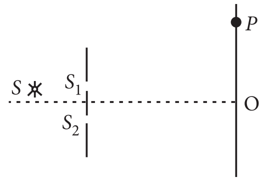
$y_1 = a \cos \omega t$ and $y_2 = a \cos (\omega t + \phi)$

where ϕ is the phase difference between the two displacements. Show that the resultant intensity at a point due to their superposition is given by $I = 4 I_0 \cos^2 \phi/2$, where $I_0 = a^2$.

(b) Hence obtain the conditions for constructive and destructive interference.

26. Why cannot two independent monochromatic sources produce sustained interference pattern? Deduce, with the help of Young's arrangement to produce interference pattern, an expression for the fringe width.

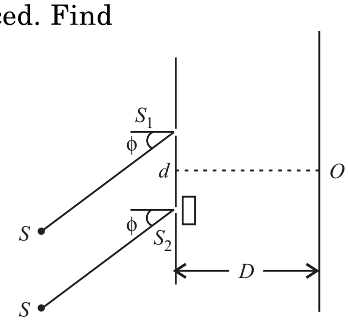
27. The figure shows a modified Young's double slit experimental set-up. Here $SS_2 - SS_1 = \lambda/4$.



(a) Write the condition for constructive interference.

(b) Obtain an expression for the fringe width.

28. The YDSE set-up is shown. On the lower slit a slab of thickness 0.1 mm and refractive index 3/2 is placed. Find



(i) Position of central maxima

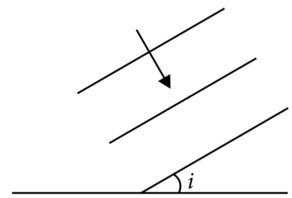
(ii) Order of maxima at O and how many fringes will cross O if slab is removed? λ = 5000 Å, $d = 50 \times 10^{-4}$ cm, $\phi = 30°$, D = 2 m

29. In a YDSE, D = 1 m, d = 1 mm, and λ = 1/2 mm.

(i) Find the distance between the first and central maxima on the screen.

(ii) Find the number of maximum and minimum obtained on the screen.

30. A plane wavefront propagating in a medium of refractive index 'μ_1' is incident on a plane surface making the angle of incidence i as shown in the figure. It enters into a medium of refraction of refractive index 'μ_2' ($\mu_2 > \mu_1$). Use Huygens' construction of secondary wavelets to trace the propagation of the refracted wavefront. Hence verify Snell's law of refraction.



31. Explain the following, giving reasons:

(i) When light travels from a rarer to a denser medium, the speed decreases. Does this decrease in speed imply a reduction in the energy carried by the wave ?

(ii) In the wave picture of light, intensity of light is determined by the square of the amplitude of the wave. What determines the intensity in the photon picture of light?

32. (a) If one of two identical slits producing interference in Young's experiment is covered with glass, so that the light intensity passing through it is reduced to 50%, find the ratio of the maximum and minimum intensity of the fringe in the interference pattern.

(b) What kind of fringes do you expect to observe if white light is used instead of monochromatic light?

Long Answer Type Questions (LA)

33. (a) Use Huygen's geometrical construction to show how a plane wave-front at t = 0 propagates and produces a wave-front at a later time.

(b) Verify, using Huygen's principle, Snell's law of refraction of a plane wave propagating from a denser to a rarer medium.

(c) When monochromatic light is incident on a surface separating two media, the reflected and refracted light both have the same frequency. Explain why.

34. (a) What is the effect on the interference fringes in a Young's double slit experiment when

(i) the separation between the two slits is decreased?

(ii) the width of the source slit is increased?

Justify your answer in each case.

(b) The intensity at the central maxima in Young's double slit experimental set-up is I_0. Show that the intensity at a point where the path difference is $\lambda/3$ is $I_0/4$.

35. (a) Using Huygens' construction of secondary wavelets explain how a diffraction pattern is obtained on a screen due to a narrow slit on which a monochromatic beam of light is incident normally.

(b) Show that the angular width of the first diffraction fringe is half that of the central fringe.

(c) Explain why the maxima at $\theta = \left(n+\frac{1}{2}\right)\frac{\lambda}{a}$ become weaker and weaker with increasing n.

36. (a) Describe briefly how a diffraction pattern is obtained on a screen due to a single narrow slit illuminated by a monochromatic source of light. Hence obtain the conditions for the angular width of secondary maxima and secondary minima.

(b) Two wavelengths of sodium light of 590 nm and 596 nm are used in turn to study the diffraction taking place at a single slit of aperture 2×10^{-6} m. The distance between the slit and the screen is 1.5 m. Calculate the separation between the positions of first maxima of the diffraction pattern obtained in the two cases.

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. **(a) :** Angular width of fringe,

$$\theta = 0.1° = 0.1 \times \frac{\pi}{180} \text{ rad}$$

$$= \frac{3.14}{1800} \text{ rad}$$

$$\therefore \quad d = \frac{\lambda}{\theta} = \frac{600 \times 10^{-9}}{3.14/1800} = 3.44 \times 10^{-4} \text{ m}$$

2. **(d) :** When red light is replaced by blue light the diffraction bands become narrow and crowded.

3. **(b) :** The position of n^{th} minima in the diffraction pattern is

$$x_n = \frac{nD\lambda}{d}$$

$$\therefore \quad x_3 - x_1 = (3-1)\frac{D\lambda}{d} = \frac{2D\lambda}{d}$$

or $$d = \frac{2D\lambda}{x_3 - x_1} = \frac{2 \times 0.50 \times 6000 \times 10^{-10}}{3 \times 10^{-3}} = 2 \times 10^{-4} \text{ m}$$

4. **(a)**

5. **(d)**

6. **(a) :** The condition for minimum thickness corresponding to a dark band is, $2\mu t \cos r = \lambda$

$$\therefore \quad t = \frac{\lambda}{2\mu \cos r} = \frac{5890 \times 10^{-10}}{2 \times 1.5 \times \cos 60°}$$

$$\Rightarrow \quad 3926 \times 10^{-10} \text{ m} = 3926 \text{ Å}$$

7. **(a)**

8. **(c) :** Fringe width, $\beta = \frac{\lambda D}{d}$

Also, $\lambda = \frac{h}{mv}$

Here h is planck's constant. This wavelength is inversily proportional to the velocity. Hence, the fringe width increases with decrease in electron speed.

9. **(b) :** The colours of a thin oil film are due to interference.

10. **(b) :** Resultant intensity

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$$

If path difference = λ, phase difference 2π

$$\therefore \quad I_R = I + I + 2\sqrt{I \times I}\cos 2\pi \quad (\because I_1 = I_2 = I)$$

$$= 4I = M \qquad ..(i)$$

If path difference $\frac{\lambda}{3}$, phase difference $\phi = \frac{2\pi}{3}$ rad

$$I_R' = I + I + 2\sqrt{I \times I}\cos\frac{2\pi}{3}$$

$$= 2I + 2I\left(-\frac{1}{2}\right) = I = \frac{M}{4} \qquad \text{[(Using (i)]}$$

11. **(c) :** Fringe width, $\beta = \frac{\lambda D}{d}$

where λ is the wavelength of light, D is the distance between screen and the slits and d is the distance between two slits

$$\therefore \quad \Delta\beta = \frac{\lambda}{d}\Delta D \qquad \text{(As } \lambda \text{ and } d \text{ are constants)}$$

or $$\lambda = \frac{\Delta\beta d}{\Delta D}$$

Substituting the given values, we get

$$\lambda = \frac{(3 \times 10^{-5}\text{ m})(10^{-3}\text{ m})}{(5 \times 10^{-2}\text{ m})} = 6 \times 10^{-7} \text{ m} = 6000 \text{ Å}$$

12. **(a) :** Fringe width,

$$\beta = \frac{D\lambda}{d} \qquad ...(i)$$

According to the question,

$$D' = \frac{D}{2} \text{ and } d' = 10\, d$$

$$\therefore \quad \beta' = \frac{D'\lambda}{d'} = \frac{(D/2)\lambda}{10d} = \frac{1}{20}\frac{D\lambda}{d}$$

$$\Rightarrow \quad \beta' = \frac{\beta}{20} \qquad \text{(Using (i))}$$

13. **(a) :** Here, $\beta_1 = 2.4 \times 10^{-4}$ m,

$\lambda_1 = 6400$ Å, $\lambda_2 = 4000$ Å

$\because \quad \frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1} = \frac{4000}{6400} = \frac{5}{8}$

or $\quad \beta_2 = \frac{5}{8} \times \beta_1$

$= \frac{5}{8} \times 2.4 \times 10^{-4} = 1.5 \times 10^{-4}$ m

Decrease in fringe width

$\Delta\beta = \beta_1 - \beta_2 = (2.4 - 1.5) \times 10^{-4} = 0.9 \times 10^{-4}$ m

14. (c) : Width ratio, $\frac{\beta_1}{\beta_2} = \frac{I_1}{I_2} = \frac{81}{1}$

$\therefore$ Amplitude ratio, $\frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{81}{1}} = 9 : 1$

15. (d) : The resultant intensity

$I = I_0 \cos^2 \frac{\phi}{2}$

Here, I_0 is the maximum intensity

and $\phi = \frac{\pi}{3}$

$\therefore \quad I = I_0 \cos^2\left(\frac{\pi}{3 \times 2}\right) = I_0 \cos^2 \frac{\pi}{6} = I_0\left(\frac{\sqrt{3}}{2}\right)^2$

$I = \frac{3}{4} I_0$

16. (c) : Fringe width, $\beta = \frac{\lambda D}{d}$ ...(i)

where D is the distance between the screen and slit and d is the distance between two slits.

When a film of thickness t and refractive index μ is placed over one of the slit, the fringe pattern is shifted by distance S and is given by

$S = \frac{(\mu - 1)tD}{d}$ ...(ii)

Given : $S = 20\beta$ ...(iii)

From equations (i), (ii) and (iii) we get,

$(\mu - 1)t = 20\lambda$

or $\quad (\mu - 1) = \frac{20\lambda}{t} = \frac{20 \times 5000 \times 10^{-8} \text{ cm}}{2.5 \times 10^{-3} \text{ cm}}$

$\mu - 1 = 0.4$ or $\mu = 1.4$

17. (b) : Here, $I_1 = I$, $I_2 = 4I$, $\phi_1 = \frac{\pi}{2}$, $\phi_2 = \pi$

$I_A = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi_1$

$= I + 4I + 2\sqrt{I \times 4I}\cos\frac{\pi}{2} = 5I$

$I_B = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi_2$

$= I + 4I + 2\sqrt{I \times 4I}\cos\pi\,\pi = 5I - 4I = I$

$\therefore \quad I_A - I_B = 5I - I = 4I$

18. (b) : Constructive interference occurs when the path difference ($S_1P - S_2P$) is an integral multiple of λ.

or $\quad S_1P - S_2P = n\lambda$

where $n = 0, 1, 2, 3, ...$

19. (c) : Converging spherical.

20. (c) : The light from two slits of Young's double slit experiment is of different colours/wavelengths/frequencies, Hence, there shall be no interference fringes.

21. (d) : When a bright fringe is formed opposite to one of the slits, $x = \frac{d}{2}$

path difference $= \frac{xd}{D} = \frac{d}{2} \times \frac{d}{D} = \frac{d^2}{2D}$

If it is n^{th} order bright fringe,

path differnce $n\lambda = \frac{d^2}{2D}$ or $\quad n = \frac{d^2}{2D\lambda}$

22. (a) : Visible range, λ = 3900 Å to 7000 Å

Given width of the slit, $a = 10^4$ Å = 10000 Å

Thus $a > \lambda$, Hence, no diffraction occurs.

The image seen through the slit shall be a fine sharp slit white in colour at the centre.

23. (b) : Let n^{th} fringe of 2500 Å coincide with $(n - 2)^{th}$ fringe of 3500 Å.

$\therefore \quad 3500(n - 2) = 2500 \times n$

$1000n = 7000$, or $n = 7$

$\therefore$ 7[th] order fringe of 1[st] source will coincide with 5[th] order fringe of 2[nd] source.

24. (b) : Fringe width in first case

$\beta_1 = \frac{D\lambda_1}{d}$ ...(i)

Fringe width in second case

$\beta_2 = \frac{D\lambda_2}{2d}$ ...(ii)

Divide equation (ii) by (i),

$\therefore \quad \frac{\beta_2}{\beta_1} = \frac{D\lambda_2/2d}{D\lambda_1/d} = \frac{1}{2}.\frac{\lambda_2}{\lambda_1}$ or $\quad \beta_2 = \frac{1}{2}.\frac{\lambda_2}{\lambda_1}.\beta_1$

$= \frac{1}{2} \times \frac{7500 \text{ Å}}{6000 \text{ Å}} \times 0.8 \text{ mm} = 0.5 \text{ mm} = 0.5$ mm

25. (c) : The linear separation between n bright fringes in an interference pattern on the screen is $x_n = \frac{n\lambda D}{d}$

As $x_n << D$, the angular separation between n bright fringes

$\theta_n = \frac{x_n}{D} = \frac{n\lambda}{d}$

For 10 bright fringes,

$$\theta_{10} = \frac{10\lambda}{d}$$

The angular width of the central maximum in the diffraction pattern due to slit of width *a* is

$$2\theta_1 = \frac{2\lambda}{a}$$

Now $\frac{10\lambda}{d} < \frac{2\lambda}{a}$ or $a \le \frac{d}{5} = \frac{1}{5}$ mm = 0. 2 mm

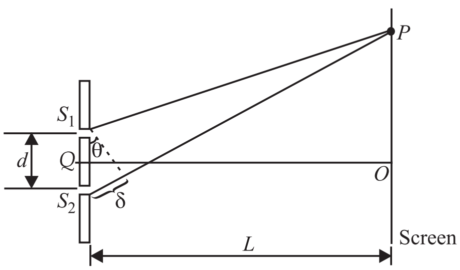
26. (b)

27. (c)

28. (b) : For destructive interference, the path difference should be an odd multiple of $\frac{\lambda}{2}$.

29. (d) : The Huygen's construction of wavefront does not explain the phenomena of origin of spectra.

30. (a) :



The condition for bright fringes is,
path difference, $\delta = d\sin\theta_{bright} = N\lambda$
where $N = 0, \pm 1, \pm 2,$
The angular position of the bright fringes is

$$\theta_{bright} = \sin^{-1}\left(\frac{N\lambda}{d}\right)$$

31. (b) : Fresnel distance,

$$z_F = \frac{a^2}{\lambda} = \frac{(4\times10^{-3})^2}{500\times10^{-9}} = \frac{4\times4\times10^{-6}}{5\times10^{-7}}$$

$\therefore \quad z_F = 32$ m

32. (b) : Here, $a = 2$ mm $= 2 \times 10^{-3}$ m
$\lambda = 500$ nm $= 500 \times 10^{-9}$ m $= 5 \times 10^{-7}$ m
$D = 1$ m
The distance between the first minima on either side on a screen is

$$= \frac{2\lambda D}{a} = \frac{2\times5\times10^{-7}\times1}{2\times10^{-3}}$$

$= 5 \times 10^{-4}$ m $= 0.5 \times 10^{-3}$ m $= 0.5$ mm

33. (a) : $\frac{I_1}{I_2} = \frac{a^2}{b^2} = \beta \quad \therefore \quad \frac{a}{b} = \sqrt{\beta}$

Fringe visibility is given by

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(a+b)^2 - (a-b)^2}{(a+b)^2 + (a-b)^2}$$

$$= \frac{4ab}{2(a^2+b^2)} = \frac{2(a/b)}{\left(\frac{a^2}{b^2}+1\right)} = \frac{2\sqrt{\beta}}{\beta+1}$$

34. (c) : For first minimum, $a\sin\theta = 1\lambda$

$$a = \frac{\lambda}{\sin\theta} = \frac{6.5\times10^{-7}}{\sin 30°} = 13 \times 10^{-7} = 1.3 \text{ micron}$$

35. (a) : Fringe width, $\beta = \frac{\lambda D}{d}$ and $\theta = \frac{d}{D} \quad \therefore \quad \beta = \frac{\lambda}{\theta}$

36. (d) : $n_1\lambda_1 = n_2\lambda_2$

$$\lambda_2 = \frac{n_1}{n_2}\lambda_1 = \frac{16\times6000 \text{ Å}}{24} = 4000 \text{ Å}$$

37. (a) : Wavefront is the locus of all points, where the particles of the medium vibrate with the same phase.

38. (c) : Angular fringe width $\theta = \frac{\lambda}{d}$

$\therefore \quad \lambda = \theta d$
Here, $d = 0.01$ mm $= 10^{-5}$ m,
$\theta = 1° = \frac{\pi}{180}$ radians

$$\lambda = \frac{\pi}{180}\times10^{-5} = 1.74\times10^{-7} \text{ m}$$

$$= 0.174\times10^{-6} \text{ m} = 0.174\ \mu\text{m}$$

39. (d) : If sources are coherent

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$I_0 = I + I + 2I\cos 0° = 4I$
If sources are incoherent,

$$I_R = I_1 + I_2 = 2I = \frac{4I}{2} = \frac{I_0}{2}$$

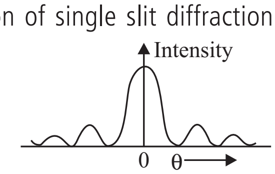
40. (a): As the beam is initially parallel, the shape of wavefront is planar.

41 (c) : According to Huygens Principle, the surface of constant phase is called a wavefront.

42. (c)

43. (c) : Angular width of central maxima, $2\theta = 2\lambda/e$.
Thus, θ does not depend on screen *i.e.*, distance between the slit and the screen.

44. (c) : The intensity distribution of single slit diffraction pattern is shown in the figure. From the graph it is clear that the intensity of the central point is finite but much larger than the surrounding maxima.



45. (a): Width of central maxima = $2\lambda D/e$

width of other secondary maxima = $\lambda D/e$

$\therefore$ Width of central maxima : width of other secondary maxima = 2 : 1

46. (d) : Path difference produced is

$$\Delta x = \frac{3}{2}\pi R - \frac{\pi}{2}R = \pi R$$

For maxima: $\Delta x = n\lambda$

$\therefore \quad n\lambda = \pi R$

$\Rightarrow \quad \lambda = \frac{\pi R}{n}$, $n = 1, 2, 3, ...$

Thus, the possible values of λ are $\pi R, \frac{\pi R}{2}, \frac{\pi R}{3}, ...$

47. (b): Maximum intensity, $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

Here, $I_1 = I_2 = \frac{I_0}{2}$ (given)

$$\therefore \quad I_{max} = \left(\sqrt{\frac{I_0}{2}} + \sqrt{\frac{I_0}{2}}\right)^2 = 2I_0$$

48. (d) : Phase difference $\phi = \frac{2\pi}{\lambda} \times$ Path difference

$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3} = 60°$ As $I = I_{max}\cos^2\frac{\phi}{2}$

$$\therefore \quad I = I_0\cos^2\frac{60°}{2} = I_0 \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}I_0 \Rightarrow \frac{I}{I_0} = \frac{3}{4}$$

49. (c) : Here $A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

$\because \; a_1 = a_2 = a$

$\therefore \; A^2 = 2a^2(1+\cos\delta) = 2a^2\left(1 + 2\cos^2\frac{\delta}{2} - 1\right)$

or $A^2 \propto \cos^2\frac{\delta}{2}$

Now $I \propto A^2 \quad \therefore I \propto A^2 \propto \cos^2\frac{\delta}{2} \Rightarrow I \propto \cos^2\frac{\delta}{2}$

50. (b) : Suppose the amplitude of waves from each slit is a^2. Therefore, intensity due to each slit = a^2. When interference is destructive, the resultant amplitude = $a - a = 0$.

$\therefore$ Minimum intensity = 0

When interference is constructive, the resultant amplitude = $a + a = 2a$.

$\therefore$ Maximum intensity = $(2a)^2 = 4a^2$

= 4 times the intensity due to each slit.

51. (c) : For diffraction of a wave, size of an obstacle or aperture should be comparable to the size of wavelength of the wave. As wavelength of light is of the order of 10^{-6} m and obstacle / aperture of this size are rare, therefore, diffraction is not common in light waves. On the contrary, wavelength of sound is of the order of 1 m and obstacle / aperture of this size are readily available, therefore, diffraction is common in sound.

52. (a) : In case of interference, intensity of maxima is $I_{max} = \left(\sqrt{I_a} + \sqrt{I_b}\right)^2$ and intensity of minima is $I_{min} = \left(\sqrt{I_a} - \sqrt{I_b}\right)^2$. Thus, whatever energy disappears at the minimum is actually appearing at the maximum. So, the law of conservation of energy holds good in the phenomenon of interference because in interference energy is neither created nor destroyed but is redistributed.

53. (b) : When slit is wide (*i.e.* $a >> \lambda$), bending of light becomes so small that it cannot be detected upto a certain distance of screen from the slit. Hence, practically, no diffraction occurs.

54. (d) : When a light wave travel from a rarer to a denser medium it loses speed, but energy carried by the wave does not depend on its speed. Instead, it depends on the amplitude of wave. The frequency also remain constant.

55. (a) : For reflected system of the film, the maxima or constructive interference is $2\mu t\cos r$ = while the maxima for transmitted system of film is given by equation

$2\mu t\cos r = n\lambda$

where t is thickness of the film and r is angle of refraction.

From these two equations we can see that condition for maxima in reflected system and transmitted system are just opposite.

56. (c) : When one of slits is covered with cellophane paper, the intensity of light emerging from the slit is decreased (because this medium is translucent). Now the two interfering beam have different intensities or amplitudes. Hence, intensity at minima will not be zero and fringes will become indistinct.

57. (a) : As a broad source is equivalent to a large number of narrow sources lying side by side. Each set of these sources will produce an interference pattern of its own which will overlap on another to such an extent that all traces of a fringe system is lost and results in general illumination. Because of this reason, for interference a narrow slit should be used.

58. (c) : The waves diffracted from the edges of circular obstacle, placed in the path of light, interfere constructively at the centre of the shadow resulting in the formation of a bright spot.

SUBJECTIVE TYPE QUESTIONS

1. Two sources are said to be coherent, if they emit light waves of same frequency or wavelength and of a constant phase difference.

2. According to Huygens' principle, each point on a wavefront is a source of secondary waves, which add up to give a wavefront at any later time.

3. In a single slit diffraction separation between fringes

$$\theta \propto \frac{n\lambda}{a}$$

So, there is no effects on angular separation 2θ by changing of the distance of separation 'D' between slit and the screen.

4. As $\beta = \frac{\lambda D}{d} = 0.12 \text{mm}$

$\Rightarrow$ by changing μ; λ becomes $\frac{\lambda}{\mu}$

$\Rightarrow \beta' = \frac{3}{4} \times 0.12 = 0.9 \text{mm}.$

5. Angular width, $\theta = \frac{\lambda}{d}$

$$0.1 = \frac{0.1}{180}\pi \text{ rad} = \frac{6\times10^{-7}}{d}$$

$$\therefore\ d = \frac{6\times10^{-7}\times180}{0.1\times\pi} = 3.44 \times 10^{-4} \text{ m}$$

6. There would be spherical wave front on earth for sunlight which is treated as point source, but radius is very large as compared to radius of earth, so it is almost a plane wavefront.

7. Yes, Huygen's principle is valid for longitudinal as well as transverse waves and for all wave phenomena.

8. The diffraction effect is more pronounce if the size of the aperture or the obstacle is of the order of wavelength of wave. As wavelength of light ($\simeq 10^{-6}$ m) is much more smaller than size of object around us so diffraction of light is not easily seen but sound wave has large wavelength (15 mm < λ < 15 m), they get easily diffracted by objects around us.

9. In young's double slit experiment, if one slit is fully closed, the new pattern has larger central maximum in angular size.

10. Here, $d = 1$ mm $= 1 \times 10^{-3}$ m

$D = 1$ m, $\lambda = 500$ nm $= 5 \times 10^{-7}$ m

Fringe spacing,

$$\beta = \frac{\lambda D}{d} = \frac{5\times10^{-7}\times1}{1\times10^{-3}} = 5 \times 10^{-4} \text{ m} = 0.5 \text{ mm}$$

11. Initial fringe width $\beta_i = \lambda \times (D_i/d)$

Final fringe width $\beta_f = \lambda \times (D_f/d)$

$$\Rightarrow\ \Delta\beta = (D_f - D_i)\frac{\lambda}{d}\ \Rightarrow\ \lambda = \frac{3\times10^{-5}\times10^{-3}}{5\times10^{-2}} = 600 \text{ nm}$$

12. The path difference is $2\mu t$.

Now for destructive interference it can be

$$2\mu t = \frac{\lambda}{2} \text{ or } \frac{3\lambda}{2} \text{ or } \frac{5\lambda}{2} \text{ and so on ......}$$

$$\mu = \frac{\lambda}{4t}, \frac{3\lambda}{4t}, \frac{5\lambda}{4t} \ldots = \frac{580\times10^{-9}}{4\times0.3\times10^{-6}}, \frac{3\times580\times10^{-9}}{4\times0.3\times10^{-6}} \ldots$$

$= 0.4833,\ 3 \times 0.4833$ ....

So only $\mu = 3 \times 0.4833 = 1.45$ is the answer $\{1.3 < \mu < 1.5\}$

13. Here $n = 1$, $\lambda = 6 \times 10^{-5}$ cm

Distance of screen from slit = 100 cm

Distance of first minimum from central maxima = 0.1 cm

$$\sin\theta = \frac{\text{Distance of 1}^{st}\text{ minima from the central maxima}}{\text{Distance of the screen from the slit}}$$

$$\theta_1 = \frac{0.1}{100} = \frac{1}{1000}$$

We know that $a\sin\theta = n\lambda \Rightarrow a = \frac{\lambda}{\theta_1} = 0.06$ cm

14. $$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}$$

Now, $I_1 = 2I_2$

$$\Rightarrow \frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{2I_2}+\sqrt{I_2}}{\sqrt{2I_2}-\sqrt{I_2}}\right)^2$$

$$= \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 \simeq 34$$

15. The n^{th} bright fringe of the λ pattern and the n'^{th} bright fringe of the λ' pattern are situated at $y_n = n\cdot\frac{D\lambda}{d}$ and $y_{n'} = n'\frac{D\lambda'}{d}$.

As this coincide, $y_n = y_{n'}$

$$\Rightarrow \frac{nD\lambda}{d} = \frac{n'D\lambda'}{d} \Rightarrow \frac{n}{n'} = \frac{\lambda'}{\lambda} = \frac{900}{750} = \frac{6}{5}$$

hence the first position where overlapping occur is

$$y_{5'} = y_6 = \frac{nD\lambda}{d} = \frac{6(2\text{m})(750\times10^{-9}\text{m})}{(2\times10^{-3}\text{m})} = 4.5 \text{ mm.}$$

16. Given that: Wavelength of the light beam,

$\lambda_1 = 590$ nm $= 5.9 \times 10^{-7}$ m

Wavelength of another light beam,

$\lambda_2 = 596$ nm $= 5.96 \times 10^{-7}$ m

Distance of the slits from the screen $= D = 1.5$ m

Slits width $= a = 2 \times 10^{-4}$ m

For the first secondary maxima,

$$\sin\theta = \frac{3\lambda_1}{2a} = \frac{x_1}{D}$$

$$x_1 = \frac{3\lambda_1 D}{2a} \text{ and } x_2 = \frac{3\lambda_2 D}{2a}$$

$\therefore$ Separation between the positions of first secondary maxima of two sodium lines,

$$x_2 - x_1 = \frac{3D}{2a}(\lambda_2 - \lambda_1)$$

$$= \frac{3\times1.5}{2\times2\times10^{-4}}(5.96 \times 10^{-7} - 5.9 \times 10^{-7})$$

$= 6.75 \times 10^{-5}$ m

17. Fringe width $\beta = \frac{D\lambda}{d}$; $\beta \propto \lambda$

$\therefore \quad \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}$ or $\lambda_2 = \frac{\beta_2}{\beta_1}\lambda_1 = \frac{8.1}{7.2} \times 640$ nm

$\lambda_2 = 720$ nm

$\because \quad x = n_1\beta_1 = n_2\beta_2$

or $\quad \frac{n_1 D\lambda_1}{d} = \frac{n_2\lambda_2 D}{d}$ or $n_1\lambda_1 = n_2\lambda_2$

$\because$ Bright fringes coincides at least distance *x*, if

$n_1 = n_2 + 1$

$\Rightarrow \quad n_1 \times 640 = (n_1 - 1) \times 720$

$\frac{n_1 - 1}{n_1} = \frac{640}{720}$ or $n_1 = 9$

18. $\lambda = 6000$ Å $= 6000 \times 10^{-10}$ m $= 6 \times 10^{-7}$ m, $\theta_1 = 30°$, $m = 1$

(i) For first maximum, $\sin\theta_m = \frac{\left(m + \frac{1}{2}\right)\lambda}{a}$

$\sin\theta_1 = \frac{3\lambda}{2a}$ or $a = \frac{3\lambda}{2\sin\theta_1} = \frac{3 \times 6 \times 10^{-7}}{2 \times \sin 30°}$

$= 1.8 \times 10_{-6}$ m $= 1.8$ μm

(ii) For first minimum,

$\sin\theta_m = \frac{m\lambda}{a} \therefore \sin\theta_1 = \frac{\lambda}{a} \Rightarrow a = \frac{\lambda}{\sin\theta_1} = \frac{6 \times 10^{-7}}{\sin 30°}$

$= 1.2 \times 10^{-6}$ m $= 1.2$ μm

19. (i) Here $a = 1 \times 10^{-4}$ m, $D = 1.5$ m

$\lambda = 6000$ Å $= 6000 \times 10^{-10}$ m

The distance between the two dark bands on each side of central band is equal to width of the central bright band,

i.e., $\frac{2D\lambda}{a} = \frac{2 \times 1.5 \times 6000 \times 10^{-10}}{1 \times 10^{-4}} = 18$ mm

20. For minima in diffraction pattern, $d\sin\theta = n\lambda$

For first minima, $d\sin\theta_1 = (1)\lambda_1 \Rightarrow \sin\theta_1 = \frac{\lambda_1}{d}$

For first maxima, $d\sin\theta_2 = \frac{3}{2}\lambda_2 \Rightarrow \sin\theta_2 = \frac{3\lambda_2}{2d}$

The two will coincide if, $\theta_1 = \theta_2$ or $\sin\theta_1 = \sin\theta_2$

$\therefore \quad \frac{\lambda_1}{d} = \frac{3\lambda_2}{2d} \Rightarrow \lambda_2 = \frac{2}{3}\lambda_1 = \frac{2}{3} \times 660 \text{ nm} = 440 \text{ nm}$

21. (i) 3rd bright fringe will be at

$= \frac{3 \times \lambda D}{d} = \frac{3 \times (6500 \times 10^{-10}) \times 1.2}{2 \times 10^{-3}}$

$= 0.117 \text{cm} = 1.17 \text{mm}$

(ii) Say m^{th} bright fringe of 6500 Å coincides with n^{th} bright fringe of 5200 Å

$\Rightarrow \quad n\beta_1 = m\beta_2 \quad \Rightarrow \quad \frac{n}{m} = \frac{5}{4}$

Hence, for least distance, 5th bright fringe of 5200 Å coincides with 4th bright fringe of 6500 Å.

$\Rightarrow y' = 4\beta_1 = \frac{4 \times (6500 \times 10^{-10}) \times 1.2}{2 \times 10^{-3}} = 0.156$ cm

$= 1.56$ mm

22. At the hole, wavelength which satisfy the minima condition will be absent.

$\Rightarrow \quad y = \left(n + \frac{1}{2}\right)\frac{D\lambda}{d}$ or $\lambda = \frac{2y \cdot d}{D(2n+1)}$

$= \frac{2(1 \times 10^{-3}\text{m})(0.5 \times 10^{-3}\text{m})}{(0.5\text{m})(2n+1)}$

$= \frac{2 \times 10^{-6}\text{ m}}{2n+1} = \frac{2000\text{nm}}{(2n+1)}$

Now, 400 nm $\leq \lambda \leq$ 700 nm or $400 \leq \frac{2000}{2n+1} \leq 700$

$n = 0, 1, 2, 3$

$\lambda = 667$ nm, 400 nm.

667 nm and 400 nm have a complete destructive interference at the hole and consequently will be absent in the light coming from that hole.

23. From diagram

$T_1P = T_1O - OP = (D - x)$

$T_2P = T_2O + OP = (D + x)$

Now $S_1P = \sqrt{(S_1T_1)^2 + (T_1P)^2}$

$= \sqrt{D^2 + (D - x)^2}$

$S_2P = \sqrt{(S_2T_2)^2 + (T_2P)^2} = \sqrt{D^2 + (D + x)^2}$

Path difference, $S_2P - S_1P = \frac{\lambda}{2}$; for first minimum to occur

$\sqrt{D^2 + (D + x)^2} - \sqrt{D^2 + (D - x)^2} = \frac{\lambda}{2}$

The first minimum falls at a distance *D* from the center, *i.e.*, $x = D$.

$[D^2 + 4D^2]^{1/2} - D = \frac{\lambda}{2}$

$D(\sqrt{5} - 1) = \frac{\lambda}{2}$

$D(2.236 - 1) = \frac{\lambda}{2}$; $\quad D = \frac{\lambda}{2.472}$

24. (a) The angular width of central maximum is given by

$2\theta_0 = \frac{2\lambda}{a}$, ...(i)

where the letters have their usual meanings.

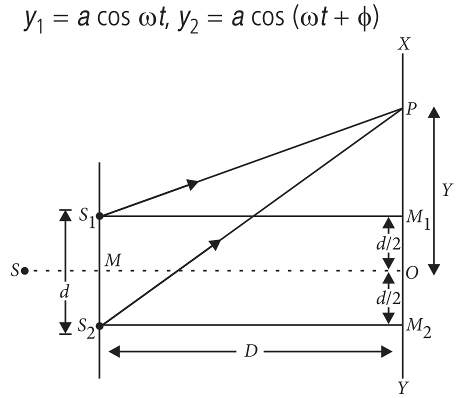
(i) Effect of slit width : From the equations (i), it follows that $\beta_0 \propto \frac{1}{a}$. Therefore, as the slit width is increased, the width of the central maximum will decrease.

(ii) Effect of distance between slit and screen (D) : From the equation (i), it follows that $2\theta_0$ is independent of D. So the angular width will remain same whatever the value of D.

(b) Difference between interference and diffraction experiment to observe diffraction pattern

Interference	Diffraction
1. Interference is caused by superposition two waves starting from two coherent sources.	1. Diffraction is caused by superposition of a number of waves starting from the slit.
2. All bright and dark fringes are of equal width.	2. Width of central bright fringe is double of all other maxima.
3. All bright fringes are of same intensity.	3. Intensity of bright fringes decreases sharply as we move away from central bright fringe.
4. Dark Fringes are perfectly dark.	4. Dark fringes are not perfectly dark.

25. (a) $y_1 = a\cos\omega t$, $y_2 = a\cos(\omega t + \phi)$



where ϕ is phase difference between them. Resultant displacement at point P will be,

$y = y_1 + y_2 = a\cos\omega t + a\cos(\omega t + \phi)$

$= a[\cos\omega t + \cos(\omega t + \phi)]$

$$= a\left[2\cos\frac{(\omega t + \omega t + \phi)}{2}\cos\frac{(\omega t - \omega t - \phi)}{2}\right]$$

$$y = 2a\cos\left(\omega t + \frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right) \quad \text{...(i)}$$

Let $y = 2a\cos\left(\frac{\phi}{2}\right) = A$, the equation (i) becomes

$$y = A\cos\left(\omega t + \frac{\phi}{2}\right)$$

where A is amplitude of resultant wave,

Now, $A = 2a\cos\left(\frac{\phi}{2}\right)$

On squaring, $A^2 = 4a^2\cos^2\left(\frac{\phi}{2}\right)$

Hence, resultant intensity,

$$I = 4I_0\cos^2\left(\frac{\phi}{2}\right)$$

(b) Condition for constructive interference,

$\cos\Delta\phi = +1$

$$2\pi\frac{\Delta x}{\lambda} = 0, 2\pi, 4\pi\ldots$$

or $\Delta x = n\lambda$; $n = 0, 1, 2, 3, \ldots$

Condition for destructive interference, $\cos\Delta\phi = -1$

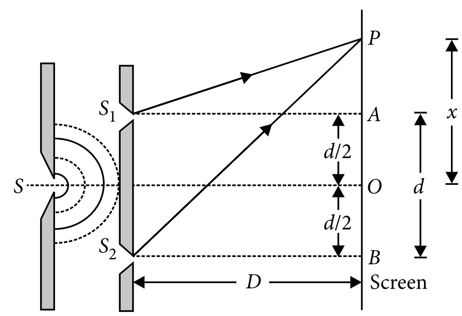
$$2\pi\frac{\Delta x}{\lambda} = \pi, 3\pi, 5\pi.$$

or $\Delta x = (2n - 1)\lambda/2$

where $n = 1, 2, 3\ldots$

26. (i) Two independent monochromatic sources cannot produce sustained interference pattern because the phase difference between the light waves from two independent sources keeps on changing continuously.

(ii)



Consider a point P on the screen at distance x from the centre O. The nature of the interference at the point P depends on path difference,

$p = S_2P - S_1P$

From right-angled ΔS_2BP and ΔS_1AP,

$(S_2P)^2 - (S_1P)^2 = [S_2B^2 + PB^2] - [S_1A^2 + PA^2]$

$$= \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

or $(S_2P - S_1P)(S_2P + S_1P) = 2xd$

or $$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

In practice, the point P lies very close to O, therefore $S_1P \simeq S_2P \simeq D$. Hence

$$p = S_2P - S_1P = \frac{2xd}{2D}$$

or $$p = \frac{xd}{D}$$

Positions of bright fringes : For constructive interference,

$$p = \frac{xd}{D} = n\lambda$$

or $x = \frac{nD\lambda}{d}$ where $n = 0, 1, 2, 3, \ldots.$

Positions of dark fringes : For destructive interference,

$$p = \frac{xd}{D} = (2n-1)\frac{\lambda}{2}$$

or $\quad x = (2n-1)\frac{D\lambda}{2d}$ where n = 1, 2, 3

Width of a dark fringe = Separation between two consecutive bright fringes

$$= x_n - x_{n-1} = \frac{nD\lambda}{d} - \frac{(n-1)D\lambda}{d} = \frac{D\lambda}{d}$$

Width of bright fringe = Separation between two consecutive dark fringes

$$= x'_n - x'_{n-1} = (2n-1)\frac{D\lambda}{2d} - [2(n-1)-1]\frac{D\lambda}{2d} = \frac{D\lambda}{d}$$

Clearly, both the bright and dark fringes are of equal width. Hence the expression for the fringe width in Young's double slit experiment can be written as

$$\beta = \frac{D\lambda}{d}$$

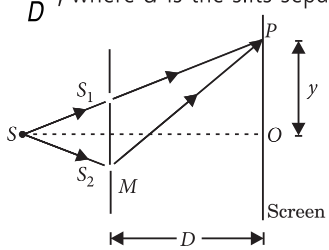
27. (a) Given : $SS_2 - SS_1 = \frac{\lambda}{4}$

Now path difference between the two waves from slit S_1 and S_2 on reaching point P on screen is

$\Delta x = (SS_2 + S_2P) - (SS_1 + S_1P)$

or $\quad \Delta x = (SS_2 - SS_1) + (S_2P - S_1P)$

or $\quad \Delta x = \frac{\lambda}{4} + \frac{yd}{D}$, where d is the slits separation.



For constructive interference at point P, path difference,

$\Delta x = n\lambda$ or $\frac{\lambda}{4} + \frac{yd}{D} = n\lambda$

or $\quad \frac{yd}{D} = \left(n - \frac{1}{4}\right)\lambda \quad$ ...(i)

where n = 0, 1, 2, 3,...,

(b) From equation (i), $y_n = \left(n - \frac{1}{4}\right)\frac{\lambda D}{d}$

and $y_{n-1} = \left(n - 1 - \frac{1}{4}\right)\frac{\lambda D}{d}$

The fringe width is given by separation of two consecutive bright fringes.

$$\beta = y_n - y_{n-1} = \left(n - \frac{1}{4}\right)\frac{\lambda D}{d} - \left(n - 1 - \frac{1}{4}\right)\frac{\lambda D}{d} = \frac{\lambda D}{d}$$

28. **(i)** $\Delta x_2 = (\mu - 1)t = (3/2 - 1)(0.01) = 5.0 \times 10^{-3}$ cm

$\Delta x_1 = d\sin\phi = (50 \times 10^{-4}) \sin 30° = 2.5 \times 10^{-3}$ cm

Since $\Delta x_2 > \Delta x_1$, central maxima will be obtained below O.

$d\sin\theta + \Delta x_1 = \Delta x_2$

$50 \times 10^{-4}\sin\phi + 2.5 \times 10^{-3} = 5.0 \times 10^{-3}$

$\sin\phi = \frac{1}{2}$ or $\phi = 30°$

(ii) At O

$\Delta x = \Delta x_1 = 2.5 \times 10^{-3} = n\lambda$

$$n = \frac{2.5 \times 10^{-3}}{500 \times 10^{-3}} = 50$$

Number of fringes that will cross O if slab is removed

$$= \frac{\text{Path difference due to slab}}{\lambda} = \frac{5 \times 10^{-3}}{500 \times 10^{-7}} = 100$$

29. (i) $D >> d$

Hence, path difference at any angular position θ on the screen

$\Delta x = d\sin\theta$

The path difference for first maxima

$\Delta x = d\sin\theta = \lambda \Rightarrow \sin\theta = \frac{\lambda}{d} = \frac{1}{2} \Rightarrow \theta = 30°$

Hence, distance between central maxima and first maxima

$y = D\sin\theta = \frac{1}{2}$ m

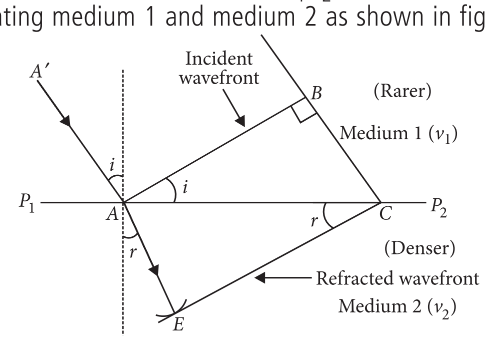
(ii) Maximum path difference, $\Delta x_{max} = d = 1$ mm

$\Rightarrow$ Highest order maxima, $n_{max} = \left[\frac{d}{\lambda}\right] = 2$ and highest order minimum $n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] = 2$

Total number of maxima = $2n_{max} + 1 = 5$

Total number of minima = $2n_{min} = 4$

30. Snell's law of refraction : Let P_1P_2 represents the surface separating medium 1 and medium 2 as shown in figure.



Let v_1 and v_2 represents the speed of light in medium 1 and medium 2 respectively. We assume a plane wavefront AB propagating in the direction $A'A$ incident on the interface at an angle i. Let t be the time taken by the wavefront to travel the distance BC.

$\therefore \quad BC = v_1t \quad$ [$\because$ distance = speed × time]

In order to determine the shape of the refracted wavefront, we draw a sphere of radius v_2t from the point A in the second medium (the speed of the wave in second medium is v_2).

Let CE represents a tangent plane drawn from the point C. Then

$AE = v_2 t$

$\therefore$ CE would represent the refracted wavefront.

In ΔABC and ΔAEC, we have

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC} \quad \text{and} \quad \sin r = \frac{AE}{AC} = \frac{v_2 t}{AC}$$

where i and r are the angles of incident and refraction respectively.

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{v_1 t}{AC} \cdot \frac{AC}{v_2 t}$$

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

If c represents the speed of light in vacuum, then

$$\mu_1 = \frac{c}{v_1} \text{ and } \mu_2 = \frac{c}{v_2}$$

$$\Rightarrow \quad v_1 = \frac{c}{\mu_1} \text{ and } v_2 = \frac{c}{\mu_2}$$

Where μ_1 and μ_2 are the refractive indices of medium 1 and medium 2.

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{c/\mu_1}{c/\mu_2} \Rightarrow \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \Rightarrow \mu_1 \sin i = \mu_2 \sin r$$

This is the Snell's law of refraction.

31. (i) Energy carried by a wave depends on the frequency of the wave, not on the speed of wave propagation, hence energy remains same.

(ii) For a given frequency, intensity of light in the photon picture is determined by

$$I = \frac{\text{Energy of photons}}{\text{area} \times \text{time}} = \frac{n \times h\upsilon}{A \times t}$$

where n is the number of photons incident normally on crossing area A in time t.

32. (a) We know, $\dfrac{I_{max}}{I_{min}} = \dfrac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$

According to question, $I_2 = 50\%$ of I_1

$I_2 = 0.5 I_1$; $a_2^2 = 0.5\, a_1^2$ $\quad (\because I \propto a^2)$

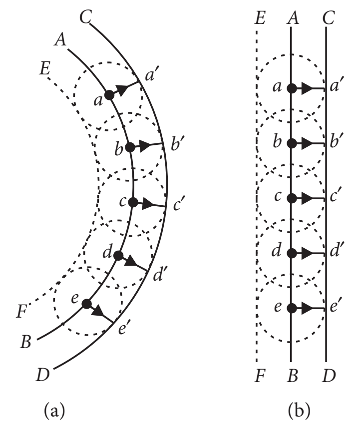
$$a_2 = \frac{a_1}{\sqrt{2}}$$

Hence,

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_1/\sqrt{2})^2}{(a_1 - a_1/\sqrt{2})^2} = \frac{(1 + 1/\sqrt{2})^2}{(1 - 1/\sqrt{2})^2} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 \approx 34$$

(b) The central fringes are white. On the either side of the central white fringe the coloured bands (few coloured maxima and minima) will appear. This is because fringes of different colours overlap.

33. (a) Consider a spherical or plane wavefront moving towards right. Let AB be its position at any instant of time. The region on its left has received the wave while region on the right is undisturbed.



Huygens geometrical construction for the propagation of (a) spherical, (b) plane wavefront.

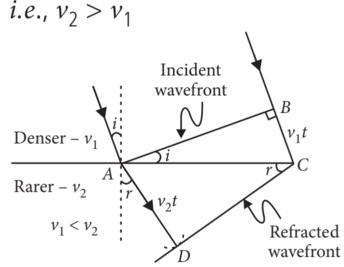
According to Huygens principle, each point on AB becomes a source of secondary disturbance, which takes with the same speed c. To find the new wavefront after time t, we draw spheres of radii ct, from each point on AB.

The forward envelope or the tangential surface CD of the secondary wavelets gives the new wavefront after time t.

The lines aa', bb', cc', etc., are perpendicular to both AB and CD. Along these lines, the energy flows from AB to CD. So these lines represent the rays. Rays are always normal to wavefronts.

(b) A source of light sends the disturbance in all the directions and continuous locus of all the particles vibrating in same phase at any instant is called as wavefront.

Given figure shows the refraction of a plane wavefront at a rarer medium *i.e.*, $v_2 > v_1$



The incident and refracted wavefronts are shown in figure.

Let the angles of incidence and refraction be i and r respectively.

From right ΔABC, we have,

$$\sin \angle BAC = \sin i = \frac{BC}{AC}$$

From right ΔADC, we have,

$$\sin \angle DCA = \sin r = \frac{AD}{AC}$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1 t}{v_2 t} \text{ or } \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = {}^1\mu_2$$

(a constant)

This verifies Snell's law of refraction. The constant ${}^1\mu_2$ is called the refractive index of the second medium with respect to first medium.

(c) Reflection and refraction arise through interaction of incident light with atomic constituents of matter which vibrate with the same frequency as that of the incident light. Hence frequency remains unchanged.

34. (a) (i) Fringe width $(\beta) = \dfrac{\lambda D}{d}$

If d decreases then β increases.

(ii) For interference fringe, the condition is $\dfrac{s}{D} < \dfrac{\lambda}{d}$

where s = size of source, D = distance of source from slits.

If the source slit width increases, fringe pattern gets less sharp or faint and fringe width decrease.

When the source slit is made wide which does not fullfil the above condition and interference pattern not visible.

(b) Fringe width $(\beta) = \dfrac{\lambda D}{d}$

$$y = \frac{\beta}{3} = \frac{\lambda D}{3d}$$

Path difference $(\Delta p) = \dfrac{yd}{D} \Rightarrow \Delta p = \dfrac{\lambda D}{3d} \cdot \dfrac{d}{D} = \dfrac{\lambda}{3}$

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot \Delta p = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{3} = \frac{2\pi}{3}$$

Intensity at point $P = I_0 \cos^2 \Delta\phi$

$$= I_0 \left[\cos \frac{2\pi}{3}\right]^2 = I_0 \left(\frac{1}{2}\right)^2 = \frac{I_0}{4}$$

Here D = 120 cm = 1.20 m

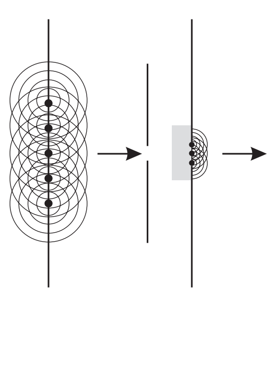
and d = 2 mm = 2×10^{-3} m

$\therefore$ Least distance,

$$y_{\min} = \frac{nD\lambda_1}{d} = \frac{4 \times 1.2 \times 650 \times 10^{-9}}{2 \times 10^{-3}}$$

$= 1.56 \times 10^{-3}$ m = 1.56 mm

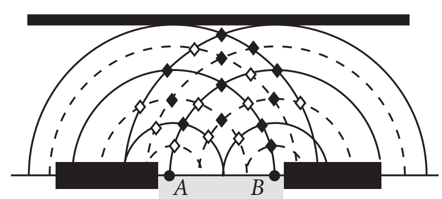
35. (a) Waves diffract when they encounter obstacles. A wavefront impinging on a barrier with a slit in it, only the points on the wavefront that move into the slit can continue emitting forward moving waves but because a lot of the wavefront has been blocked by the barrier, the points on the edges of the hole emit waves that bend round the edges.



Before the wavefront strikes the barrier the wavefront generates another forward moving wavefront. Once the barrier blocks most of the wavefront the forward moving wavefront bends around the slit because the secondary waves they would need to interfere with to create a straight wavefront have been blocked by the barrier.

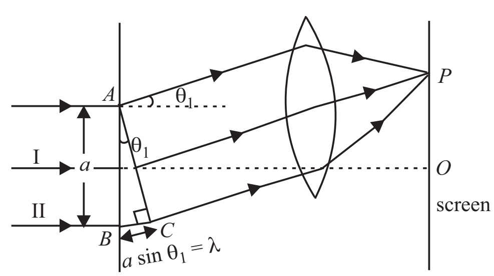
According to Huygen's principle, each point on the wavefront moving through the slit acts like a point source. We can think about some of the effect of this if we analyse what happens when two point sources are close together and emit wavefronts with the same wavelength and frequency. These two point sources represent the point sources on the two edges of the slit and we can call the source A and source B as shown in the figure.

Each point source emits wavefronts from the edge of the slit. In the diagram we show a series of wavefronts emitted from each point source. The continuous lines show peaks in the waves emitted by the point sources and the dotted lines represent troughs. We label the places where constructive interference (peak meets a peak or trough meets a trough) takes place with a solid diamond and places where destructive interference (trough meets a peak) takes place with a hollow diamond. When the wavefronts hit a barrier there will be places on the barrier where constructive interference takes place and places where destructive interference happens.



The measurable effect of the constructive or destructive interference at a barrier depends on what type of waves we are dealing with.

(b) Condition for n^{th} secondary dark fringe :



Light rays which on passing through the slit of width 'a' get diffracted by an angle θ_1, such that the path difference between extreme rays on emerging from slit is $a \sin \theta_1 = \lambda$

Then the waves from first half and second half of slit have a path difference of $\lambda/2$, so they interfere destructively at point P on screen, forming first secondary dark fringe.

Thus condition for first secondary dark fringe or first secondary minimum is

$$\sin\theta_1 = \frac{\lambda}{a}$$

Similarly, condition for n^{th} secondary dark fringe or n^{th} secondary minimum is

$$\sin\theta_n = \frac{n\lambda}{a}$$

where n = 1, 2, 3, 4,..........

Angular width of first diffraction fringe, $\theta_1 = \frac{\lambda}{a}$

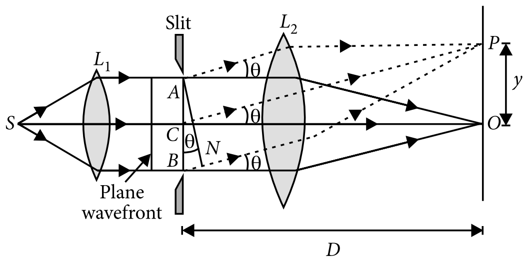
Angular width of central maxima, θ

$$\theta_1 + \theta_1 = 2\theta_1 = 2\frac{\lambda}{a}$$

$$\therefore \quad \theta_1 = \frac{1}{2}\theta$$

(c) On increasing the value of n, the part of slit contributing to the maxima decreases. Hence, the maxima become weaker.

36. (a) Diffraction of light due to a narrow single slit



Consider a set of parallel rays from a lens L_1 falling on a slit, form a plane wavefront. According to Huygens principle, each point on the unblocked point of plane wavefront AB sends out secondary wavelets in all directions. The secondary waves from points equidistant from the centre C of the slit lying in the portion CA and CB of the wavefront travel the same distance in reaching at O and hence the path difference between them is zero. These secondary waves reinforce each other, resulting maximum intensity at point O.

Position of secondary minima : The secondary waves travelling in the direction making an angle θ with CO, will reach a point P on the screen. The intensity at P will depend on the path difference between the secondary waves emitted from the corresponding points of the wavefront. The wavelets from points A and B will have a path difference equal to BN. If this path difference is λ, then P will be a point of minimum intensity. This is because the whole wavefront can be considered to be divided into two equal halves CA and CB. If the path difference between secondary waves from A and B is λ, then the path difference between secondary waves from A and C will be $\lambda/2$ and also the path difference between secondary waves from B and C will again be $\lambda/2$. Also for every point in the upper half AC, there is a corresponding point in the lower half CB for which the path difference between secondary waves reaching P is $\lambda/2$. Thus, at P destructive interference will take place.

From the right-angled ΔANB given in figure

$BN = AB \sin\theta$

$BN = a \sin\theta$

Suppose $BN = \lambda$ and $\theta = \theta_1$

$\therefore \quad \lambda = a \sin\theta_1$

$$\sin\theta_1 = \frac{\lambda}{a}$$

Such a point on the screen will be the position of the first secondary minimum.

If $BN = 2\lambda$ and $\theta = \theta_2$, then,

$2\lambda = a \sin\theta_2$

$$\sin\theta_2 = \frac{2\lambda}{a}$$

Such a point on the screen will be the position of the second secondary minimum.

In general, for n^{th} minimum at point P.

$$\sin\theta_n = \frac{n\lambda}{a}$$

For small $\theta_n, \theta_n = \frac{n\lambda}{a}$ ...(i)

Position of secondary maxima :

If any other P' is such that path difference at point is given by

$$a\sin\theta = \frac{3\lambda}{2}$$

Then P_1 will be position of first secondary maximum. Here, we can consider the wavefront to be divided into three equal parts, so that the path difference between secondary waves from corresponding points in the 1^{st} two parts will be $\lambda/2$. This will give rise to destructive interference. However, the secondary waves from the third part remain unused and therefore, they will reinforce each other and produce first secondary maximum.

Similarly if the path difference at that points given by

$$a\sin\theta_5 = \frac{5\lambda}{2}$$

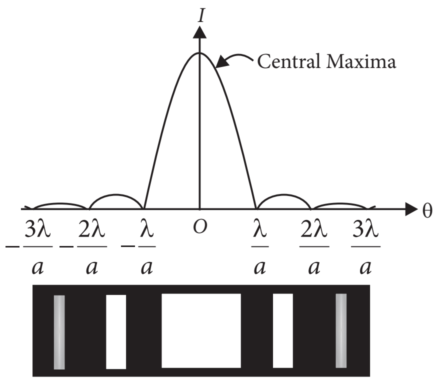
We get second secondary maximum of lower intensity.

In general, for n^{th} secondary maximum, we have

$$a\sin\theta_n = (2n+1)\frac{\lambda}{2}$$

For small $\theta_n, \theta_n = (2n+1)\frac{\lambda}{2a}$

The diffraction pattern on the screen is shown below along with intensity distribution of fringes



Width of the secondary maximum,

$$\beta = y_n - y_{n-1} = \frac{nD\lambda}{a} - \frac{(n-1)D\lambda}{a}$$

$$\beta = \frac{D\lambda}{a} \qquad \text{...(i)}$$

$\therefore$ β is independent of n, all the secondary maxima are of the same width β.

If $BN = \frac{3\lambda}{2}$ and $\theta = \theta_1'$, from above equation, we have

$$\sin\theta_1' = \frac{3\lambda}{2a}$$

Such a point on the screen will be the position of the first secondary maximum.

Corresponding to path difference,

$BN = \frac{5\lambda}{2}$ and $\theta = \theta_2'$, the second secondary maximum is produced. In general, for the n^{th} maximum at point P,

$$\sin\theta_n' = \frac{(2n+1)\lambda}{2a} \qquad \text{...(ii)}$$

If y'_n is the distance of n^{th} maximum from the centre of the screen, then the angular position of the n^{th} maximum is given by

$$\tan\theta_n' = \frac{y_n'}{D} \qquad \text{...(iii)}$$

In case θ'_n is small,

$$\sin\theta_n' \approx \tan\theta_n'$$

$$\therefore \quad y_n' = \frac{(2n+1)D\lambda}{2a}$$

Width of the secondary minimum,

$$\beta' = \frac{D\lambda}{a} \qquad \text{...(iv)}$$

Since β′ is independent of n, all the secondary minima are of the same width β′.

(b) Here, $\lambda_1 = 590$ nm $= 590 \times 10^{-9}$ m,

$\lambda_2 = 596$ nm $= 596 \times 10^{-9}$ m, $d = 2 \times 10^{-6}$ m, $D = 1.5$ m

Distance of first secondary maximum from the centre of the screen is

$$x = \frac{3}{2}\frac{\lambda D}{d}$$

For the two wavelengths,

$$x_1 = \frac{3}{2}\frac{D\lambda_1}{d} \text{ and } x_2 = \frac{3}{2}\frac{D\lambda_2}{d}$$

Spacing between the first two maximum of sodium lines,

$$x_2 - x_1 = \frac{3D}{2d}(\lambda_2 - \lambda_1)$$

$$= \frac{3\times1.5}{2\times2\times10^{-6}}(596\times10^{-9} - 590\times10^{-9})$$

$$= \frac{3\times1.5\times6\times10^{-3}}{4} = 6.75\times10^{-3}\text{m} = 6.75 \text{ mm}$$

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