

4. QUADRATIC EQUATIONS

Q. Find the roots of the following quadratic equations by factorization:

1. $4x^2 - 4a^2x + (a^4 - b^4) = 0$

2. $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}, \quad x \neq 0, 1$

3. $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

4. $\frac{4x-3}{2x+1} - 10\frac{2x+1}{4x-3} = 3, \quad x \neq \frac{-1}{2}, \frac{3}{4}$

5. $(12abx^2) - (9a^2 - 8b^2)x - 6ab = 0$

6. If the roots of the quadratic equation: $(a^2 + b^2)x^2 + 2(bc - ad)x + (c^2 + d^2) = 0$ are real and equal, show that $ac + bd = 0$.

1. If the roots of the quadratic equation: $p(q - r)x^2 + q(r - p)x + r(p - q) = 0$ are equal, then show that $\frac{1}{p} + \frac{1}{r} = \frac{2}{q}$.

8. If the equation $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, Prove that $c^2 = a^2(1 + m^2)$.

9. If the roots of the equation:

$$(c^2 - ab)x^2 - 2(a^2 - cb)x + b^2 - ac = 0 \text{ are equal, prove that either } a = 0 \text{ or } a^3 + b^3 + c^3 = 3abc$$

Q. Find the roots of the following quadratic equations, by applying the quadratic formula:

10. $9x^2 - 3(a + b)x + ab = 0$

11. $9x^2 - 3(a^2 + b^2)x + a^2b^2 = 0$

12. $a^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$

13. $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

14. $x^2 - 2(a^2 + b^2)x + (a^2 - b^2)^2 = 0$

Q. Solve for x:

15. $(x^2 - 5x)^2 - (x^2 - 5x) + 6 = 0$

16. $9^{x+2} - 6 \cdot 3^{x+1} + 1 = 0$

17. $4\left(x - \frac{1}{x}\right)^2 - 4\left(x + \frac{1}{x}\right) + 1 = 0$

18. $\left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0$

19. $\left(\frac{2x}{x-5}\right)^2 + \left(\frac{10x}{x-5}\right) - 24 = 0, (x \neq 5)$

ANSWER

1. $\left(\frac{a^2+b^2}{2}, \frac{a^2-b^2}{2}\right)$

2. $\left(\frac{9}{13}, \frac{4}{13}\right)$

3. $(-a, -b)$

4. $\left(\frac{-4}{3}, \frac{1}{8}\right)$

5. $\left(\frac{3a}{4b}, -\frac{2b}{3a}\right)$

10. $\left(\frac{a}{3}, \frac{b}{3}\right)$

11. $\left(\frac{a^2}{3}, \frac{b^2}{3}\right)$

12. $\left(\frac{4b^2}{a^2}, \frac{-3a^2}{b^2}\right)$

13. $\left(\frac{2a+b}{3}, \frac{a+2b}{3}\right)$

14. $((a+b)^2, (a-b)^2)$

15. $(x = 6, -1, \frac{5+\sqrt{29}}{2}, \frac{5-\sqrt{29}}{2})$

16. $(x = -2)$

17. $(x = 2, \frac{1}{2})$

18. $\left(-1, 1, \frac{3 \pm \sqrt{13}}{2}\right)$

19. $(x = 4, 15]$