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Work and Time

If any task is given to a person and he has to do an effort for completing that task. Then the effort for which he applied to complete the task is said to be **work** and the duration (or interval) to complete that work is said to be **time**.

Important Formulae

- If A can do a piece of work in n days, then the work is completed by A in one day (i.e. efficiency of A) is $\frac{1}{n}$.
- If A is 3 times as efficient as B, then ratio of time taken by them to complete the work is 1 : 3.
- If A can complete a work in n days and B can do it in m days, then one day combine work of A and B is $\frac{1}{m}+\frac{1}{n}$. Similarly we can determine the combine work for more than two persons.

  Or A and B can complete it in $\frac{nm}{n+m}$ days.

Use the same approach to solve questions related to pipe and cistern. i.e. if we use the term leakage (or empty) then we use '–' sign.

Example 1 The ratio of working efficiencies of A and B is 4 : 5. If A alone completes the work in 15 days, then in how many days can B alone complete the work?

(a) 12 days (b) 15 days
(c) 16 days (d) 20 days

Sol. (*a*) Ratio of efficiency of A and $B = 4 : 5$

$\therefore$ Ratio of time taken by them to complete work $= 5 : 4$

But A can complete it in 15 days.

$\therefore$ B will complete it in $\frac{15}{5}\times 4 = 12$ days

Example 2 A can finish a piece of work in 12 days while B can do it in 15 days. If both work together, what time will they take to do the work ?

(a) $\frac{20}{3}$ days (b) $\frac{20}{5}$ days
(c) $\frac{20}{7}$ days (d) None of these

Sol. (*a*) We know that, if A can do a piece of work in n days and B in m days, then they both working together in $\frac{mn}{m+n}$ days.

Here, $n = 12$ days $m = 15$ days

$\therefore$ Both A and B work together $= \frac{12\times 15}{12+15}$

$= \frac{180}{27} = \frac{20}{3}$ days

Example 3 Efficiency of A is three times to that of B. If they will complete the work in 15 days by working together, then in how many days can B alone complete the work?

(a) 20 days (b) 25 days (c) 15 days (d) 60 days

Sol. (*d*) We know that, if A and B can do a work in n and m days, then they do combine work for one day is $\frac{1}{n}+\frac{1}{m}$.

Let A can do the work in x days, then B can do the same work in $3x$ days.

Then $\frac{1}{x}+\frac{1}{3x}=\frac{1}{15}$

$\Rightarrow \frac{4}{3x}=\frac{1}{15}$

$\therefore x=\frac{4\times 15}{3}=20$

Hence, B will complete the work in $20\times 3=60$ days working alone.

Example 4 Akshu can do a piece of work in 10 days and Harshal can do same work in 12 days. They started working together but Akshu left the work 2 days before completion of work, then how much time taken to complete the work?

(a) $5\frac{6}{11}$ days (b) $6\frac{5}{11}$ days

(c) $7\frac{6}{11}$ days (d) None of these

Sol. *(b)* Harshal's 2 days work $=\frac{2}{12}=\frac{1}{6}$

Remaining part of work $=1-\frac{1}{6}=\frac{5}{6}$

Akshu's and Harshal's 1 day work $=\frac{1}{10}+\frac{1}{12}=\frac{11}{60}$

[since combine work for A and B is $1/m+1/n$]

Since, Akshu and Harshal can do the whole work in $\frac{60}{11}$ days.

$\therefore$ Time taken by them to complete $\frac{5}{6}$ of work

$=\frac{5}{6}\times\frac{60}{11}=\frac{50}{11}=4\frac{5}{11}$ days.

$\therefore$ Total time taken $=2+4\frac{5}{11}=6\frac{5}{11}$ days.

Example 5 Two pipes A and B can fill a tanker in 10 h and 15 h respectively. Find the time taken to fill the tank when both the pipes are turned on simultaneously.

(a) 5h (b) 6 h (c) 7 h (d) 8 h

Sol. *(b)* We know that, if pipes A and B fill the water in m and n hrs, them combined pipes fill the water in $\frac{mn}{m+n}$ hrs.

Here, $m=10$ hr and $n=15$ hr

$\therefore$ A and B can fill the tank in $\frac{10\times 15}{10+15}\text{ h}=\frac{150}{25}=6\text{ h}$

Practice Exercise

1. Ravi alone does a piece of work in 2 days and Rajesh does it in 6 days. In how many days will they to do it together?

(a) $1\frac{1}{2}$ days (b) 2 days

(c) 3 days (d) 4 days

2. A is twice as good a workman as B and together they finish a piece of work in 14 days. In how many days can A alone finish the work ?

(a) 11 days

(b) 21 days

(c) 28 days

(d) 42 days

3. A and B can do a piece of work in 18 days. B and C in 24 days, C and A in 36 days. In how many days can they do it all working together?

(a) 16 days (b) 17 days

(c) 15 days (d) None of these

4. A alone can complete a work in 18 days and B alone in 15 days. B alone worked at it for 10 days and then left the work. In how many more days, will A alone complete the remaining work?

(a) 5 (b) $5\frac{1}{2}$

(c) 6 (d) $6\frac{1}{2}$

5. A does half as much work as B is three-fourth of the time. If together they take 18 days to complete the work, how many days will B take to do it ?
(a) 40 days (b) 30 days
(c) 45 days (d) None of these

6. A man can do a piece of work in 5 days, but with the help of his son, he can do it in 3 days. In what time can the son do it alone.
(a) $6\frac{1}{2}$ days (b) 7 days
(c) $7\frac{1}{2}$ days (d) 8 days

7. If 12 men and 16 boys can do a piece of work in 5 days and 13 men and 24 boys can do it in 4 days, compare the daily work done by a man with that done by a boy.
(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1

8. 7 men and 8 boys can do a piece of work in 2 days. 4 men and 12 boys can do $\frac{29}{56}$ of the same work in 1 day. In how many days will 1 man do this work ?
(a) 24 days (b) 25 days (c) 28 days (d) 30 days

9. 8 men can dig a pit in 20 days. If a man works one and a half as much again as a boy, then
4 men and 9 boys can dig it in
(a) 10 days (b) 12 days (c) 15 days (d) 16 days

10. If 1 man or 2 women or 3 boys can do a piece of work in 44 days, then the same piece of work will be done by 1 man, 1 woman and 1 boy in
(a) 21 days (b) 24 days (c) 26 days (d) 33 days

11. If 3 men or 5 women or 8 boys can finish a work in 38 days, then the number of days taken by 6 men, 10 women and 6 boys to finish the work in
(a) 4 days (b) 6 days (c) 8 days (d) 10 days

12. If 3 men and 5 boys can do as much in 17 days as 5 men and 3 boys can do in 15 days, compare the rates of working of a man and a boy.
(a) 2 : 5 (b) 3 : 5 (c) 5 : 3 (d) 5 : 2

13. 36 workmen are employed to finish a certain work in 48 days, but it is found that in 24 days only $\frac{2}{5}$ work is done. How many more men must be taken in finish the work in time ?
(a) 16 (b) 18
(c) 20 (d) 22

14. A cistern which has a leak in the bottom due to which it is filled in 15 h. Had there been no leak, it could have been filled in 12 h. If the cistern is full, the leak can empty it in
(a) 3 h (b) 12 h
(c) 15 h (d) 60 h

15. A tank can be filled by one tap in 20 min and by another in 25 min. Both the taps are kept open for 5 min and then the second is turned off. In how many minutes more is the tank completely filled?
(a) 6 min (b) 11 min
(c)15 min (d) $17\frac{1}{2}$ min

16. A tank can be filled in 10 h but owing to a leakage in its bottom it requires 5 h more to fill it. If the cistern is full, in what time can the leak empty it ?
(a) 10 h (b) 20 h
(c) 30 h (d) 40 h

17. A cistern can be filled by two taps in 20 min and 30 min respectively and be emptied by a third in 48 min. They are all turned on at once. When will the cistern be half-full?
(a) 16 min (b) 8 min
(c) 10 min (d) 12 min

18. Two pipes A and B can fill a water tank in 20 min and 24 min respectively and a third pipe C can empty at the rate of 3 gallons per minute. If A, B and C are opened together, they can fill it in 15 min. The capacity of tank is
(a) 180 gallons (b) 150 gallons
(c) 120 gallons (d) 60 gallons

Answers

1	(a)	2	(b)	3	(a)	4	(c)	5	(b)	6	(c)	7	(c)	8	(c)	9	(d)	10	(b)
11	(c)	12	(c)	13	(b)	14	(d)	15	(b)	16	(c)	17	(b)	18	(c)				

Hints & Solutions

1. (a) Ravi's 1 day's work $= \frac{1}{2}$

and Rajesh's 1 day's work $= \frac{1}{6}$

Both of them 1 day's work $= \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

[$\because$ combine work for A and B is $\frac{1}{m} + \frac{1}{n}$.]

$\therefore$ Both will take $\frac{3}{2}$ days or $1\frac{1}{2}$ days to complete the work.

2. (b) Ratio of 1 day's work of A and B = 2 : 1

(A + B)'s 1 day's work $= \frac{1}{14}$.

$\therefore$ A's 1 day's work $= \frac{1}{14} \times \frac{2}{3} = \frac{1}{21}$

Hence, A alone can finish the work in 21 days.

3. (a) (A + B)'s 1 day's work $= \frac{1}{18}$ ...(i)

(B + C)'s 1 day's work $= \frac{1}{24}$ ...(ii)

and (C + A)'s 1 day's work $= \frac{1}{36}$ ...(iii)

On adding Eqs. (i), (ii) and (iii), we get

2 (A + B + C)'s 1 day's work $= \frac{1}{18} + \frac{1}{24} + \frac{1}{36}$

$= \frac{9}{72} = \frac{1}{8}$

$\therefore$ (A + B + C)'s 1 day's work $= \frac{1}{16}$

Hence, all of them can finish the work in 16 days.

4. (c) B can complete the work in 15 days, but he worked for 10 days.

$\therefore$ Part of work done $= \frac{10}{15} = \frac{2}{3}$

Remaining part of work done $= 1 - \frac{2}{3} = \frac{1}{3}$

A can do the whole work in 18 days.

$\therefore$ A can do $\frac{1}{3}$ of the work in $\frac{1}{3} \times 18$ days $= 6$ days

5. (b) Suppose, B takes x days.

Then, A takes $= 2 \times \frac{3}{4}x = \frac{3x}{2}$ days

Now, (A + B)'s 1 day's work $= \frac{1}{18}$

$\therefore \quad \frac{1}{x} + \frac{2}{3x} = \frac{1}{18} \Rightarrow \frac{5}{3x} = \frac{1}{18}$

$\Rightarrow \quad x = 30$ days

6. (c) Son's 1 day's work $= \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

$\therefore$ Son alone can do the work in $\frac{15}{2}$ days

$= 7\frac{1}{2}$ days

7. (c) Let 1 man's daily work be $\frac{1}{x}$ and that of 1 boy's be $\frac{1}{y}$.

Then, $\frac{12}{x} + \frac{16}{y} = \frac{1}{5}$ ...(i)

and $\frac{13}{x} + \frac{24}{y} = \frac{1}{4}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = 100$ and $y = 200$

$\therefore \quad \frac{1}{x} : \frac{1}{y} = \frac{1}{100} : \frac{1}{200}$

$= 2 : 1$

8. (c) Let the man does it in a days and the boy in b days.

Then $\frac{7}{a} + \frac{8}{b} = \frac{1}{2}$ ...(i)

and $\frac{4}{a} + \frac{12}{b} = \frac{29}{56}$ ...(ii)

From Eqs. (i) and (ii),

$$\frac{52}{b}=\frac{29}{8}-2=\frac{13}{8}$$

$\Rightarrow$ $b = 32$

On putting the value of b in Eq. (i), we get

$$\frac{7}{a}+\frac{8}{32}=\frac{1}{2} \Rightarrow \frac{7}{a}=\frac{1}{4}$$

$\Rightarrow$ $a = 28$

Hence, a man will do it in 28 days.

9. (d) Given, 1 man $\equiv \frac{3}{2}$ boys

So, 8 men $\equiv$ 12 boys

and 4 men + 9 boys $\equiv$ 15 boys

Here, 12 boys can dig it in 20 days.

$\therefore$ 15 boys can dig it in $\frac{20\times12}{15}=16$ days

10. (b) 1 man $\equiv$ 3 boys and 1 woman $\equiv \frac{3}{2}$ boys.

$\therefore$ (1 man + 1 woman + 1 boy)

$$=\left(3+\frac{3}{2}+1\right)\text{boy}=\frac{11}{2}\text{ boys}$$

Now, 3 boys can do the work in 44 days.

$\therefore \frac{11}{2}$ boys can do it in $\frac{44\times3}{\frac{11}{2}}=24$ days

11. (c) 3 men $\equiv$ 5 women $\equiv$ 8 boys

$\Rightarrow$ 6 men $\equiv$ 16 boys and 10 women $\equiv$ 16 boys

$\therefore$ 6 men + 10 women + 6 boys $\equiv$ (16+16+6) boys

$\equiv$ 38 boys

Now, 8 boys can finish the work in 38 days.

$\therefore$ 38 boys would finish it in $\left(\frac{38\times8}{38}\right)=8$ days

12. (c) Let one day's work of a man and a boy be x and y respectively.

Then, $3x + 5y = \frac{1}{17}$

$\Rightarrow$ $17(3x + 5y) = 1$

Similarly, $15(5x + 3y) = 1$

$\therefore$ $17(3x + 5y) = 15(5x + 3y)$

$\Rightarrow$ $x : y = 5 : 3$

13. (b) 36 men can complete $\frac{2}{5}$ of work in 24 days.

$\therefore$ Time taken by them to complete $\frac{3}{5}$ of work

$$=\frac{24}{2}\times5\times\frac{3}{5}\text{ days}=36\text{ days}$$

But, the remaining work must be completed in 24 days.

Number of men required to complete work in 36 days = 36

Number of men required to complete work in 1 day = 36×36

Number of men required to complete work in 24 days = $\frac{36\times36}{24}=54$

$\therefore$ Required men $= 54 - 36 = 18$

14. (d) Work done by the leak in 1 h $=\frac{1}{12}-\frac{1}{15}=\frac{1}{60}$

$\therefore$ Leak can empty the full cistern in 60 h.

15. (b) Work done by both in 5 min $=\left(\frac{1}{20}+\frac{1}{25}\right)5$

$$=\frac{9}{20}$$

Remaining part $=\left(1-\frac{9}{20}\right)=\frac{11}{20}$

Now, $\frac{1}{20}$ part is filled in 1 min so, $\frac{11}{20}$ part will be filled in 11 min.

Hence, the tank will be full in 11 min more.

16. (c) Work done by the leak boys in 1 h

$$=\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$$

Hence, the leak will empty it in 30 h.

17. (b) The part of the cistern filled by the taps in 1 min

$$=\frac{1}{20}+\frac{1}{30}-\frac{1}{48}$$

$$=\frac{12+8-5}{240}=\frac{1}{16}$$

$\therefore$ They fill the complete cistern in 16 min.

$\therefore \frac{1}{2}$ of the cistern can be filled in 8 min.

18. (c) Let pipe C can empty the tank in x min.

$\therefore$ $\frac{1}{20}+\frac{1}{24}-\frac{1}{x}=\frac{1}{15}$

$\Rightarrow$ $\frac{1}{x}=\frac{1}{20}+\frac{1}{24}-\frac{1}{15} \Rightarrow \frac{1}{x}=\frac{6+5-8}{120}$

$\therefore$ $x = 40$ min.

$\therefore$ Rate of flow of water = 3 gallons per minute

$\therefore$ Capacity of tank $= 40\times3$

$= 120$ gallons