23

DEFINITE INTEGRATION

MCQs with One Correct Answer

1. Let $p(x)$ be a polynomial of least degree whose graph has three points of inflection (–1, –1), (1, 1) and a point with abscissa 0 at which the curve is inclined to the axis of abscissa at an angle of 60°. Then $\int_0^1 p(x)dx$ is equal to

(a) $\frac{3\sqrt{3}+4}{14}$ (b) $\frac{3\sqrt{3}}{7}$

(c) $\frac{\sqrt{3}+\sqrt{7}}{14}$ (d) $\frac{\sqrt{3}+2}{7}$

2. $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2 \, dx$; where p, q are integers; is equal to

(a) $-\pi$ (b) 0 (c) π (d) 2π

3. The value of the definite integral $\int_0^{2\pi} e^{\cos\theta} \cos\theta(\sin\theta)d\theta$ is

(a) 0 (b) π (c) 2π (d) e^{π}

4. If p, q, r, s are in arithmetic progression and

$$f(x) = \begin{vmatrix} p+\sin x & q+\sin x & p-r+\sin x \\ q+\sin x & r+\sin x & -1+\sin x \\ r+\sin x & s+\sin x & s-q+\sin x \end{vmatrix}$$

such that $\int_0^2 f(x)dx = -4$, then the common difference of the progession is

(a) ± 1 (b) $\frac{1}{2}$

(c) ± 2 (d) None of these

5. If $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} = \frac{\pi}{16}$. Then minimum value of $a\cos x + b\sin x$ is

(a) -4 (b) -8

(c) -2 (d) $-2\sqrt{2}$

6. Let $I = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$, then I belongs to

(a) $\left(\frac{\sqrt{3}}{8}, \frac{\sqrt{2}}{6}\right)$ (b) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}\right)$

(c) $\left(\frac{1}{2}, \frac{\sqrt{2}}{2}\right)$ (d) None of these

7. Let $f(x)$ be positive, continuous and differentiable on the interval (a, b) and $\lim_{x\to a^+} f(x) = 1$, $\lim_{x\to b^-} f(x) = 3^{1/4}$. If $f'(x) \geq f^3(x) + \frac{1}{f(x)}$ then the greatest value of $b - a$ is

(a) 1 (b) $3^{1/4}$

(c) $(3^{1/4} - 1)\frac{\pi}{24}$ (d) $\frac{\pi}{24}$

8. Let a, b, c be non-zero real numbers such that

$$\int_0^1 (1+\cos^8 x)(ax^2+bx+c)\,dx = \int_0^2 (1+\cos^8 x)(ax^2+bx+c)\,dx.$$

Then the quadratic equation $ax^2+bx+c=0$ has

(a) no root in (0, 2)
(b) at least one root in (0, 2)
(c) a double root in (0, 2)
(d) two imaginary roots

9. Let $f(x)=\int_x^{x^2}(t-1)dt$. Then the value of $|f'(\omega)|$ where ω is a complex cube root of unity is

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $3\sqrt{3}$ (d) $4\sqrt{3}$

10. $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{\int_2^{\sec^2 x} f(t)dt}{x^2-\frac{\pi^2}{16}}$ equals

(a) $\frac{8}{\pi}f(2)$ (b) $\frac{2}{\pi}f(2)$
(c) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ (d) $4f(2)$

11. The value of the integral $\int_0^{\pi}(1-|\sin 8x|)dx$ is

(a) 0 (b) $\pi-1$
(c) $\pi-2$ (d) $\pi-3$

12. Let S be the set of real numbers p such that there is no non-zero continuous function $f: R \to R$ satisfying $\int_0^x f(t)dt = p\,f(x)$ for all $x \in R$. Then, S is

(a) the empty set
(b) the set of all rational numbers
(c) the set of all irrational numbers
(d) the whole set R.

13. Suppose the limit $L = \lim\limits_{n\to\infty}\sqrt{n}\int_0^1 \frac{1}{(1+x^2)^n}dx$ exists and is larger than $\frac{1}{2}$. Then,

(a) $\frac{1}{2}<L<2$ (b) $2<L<3$
(c) $3<L<4$ (d) $L\geq 4$

14. Suppose a continuous function $f: [0, \infty) \to \mathbf{R}$ satisfies

$$f(x)=2\int_0^x tf(t)dt+1 \text{ for all } x\geq 0.$$

Then $f(1)$ equals

(a) e (b) e^2 (c) e^4 (d) e^6

15. Let $J=\int_0^1 \frac{x}{1+x^8}dx.$

Consider the following assertions:

I. $J>\frac{1}{4}$ II. $J<\frac{\pi}{8}$

Then

(a) only I is true
(b) only II is true
(c) both I and II are true
(d) neither I nor II is true

16. For $x \in \mathbf{R}$, let $f(x)=|\sin x|$ and $g(x)=\int_0^x f(t)\,dt$.

Let $p(x)=g(x)-\frac{2}{\pi}x$. Then

(a) $p(x+\pi)=p(x)$ for all x
(b) $p(x+\pi)\neq p(x)$ for at least one but finitely many x
(c) $p(x+\pi)\neq p(x)$ for infinitely many x
(d) p is a one-one function

17. Let f: [0, 1] → [0, 1] be a continuous function such that

$x^2+(f(x))^2\leq 1$ for all $x\in[0, 1]$ and $\int_0^1 f(x)\,dx=\frac{\pi}{4}$.

Then $\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{f(x)}{1-x^2}dx$ equals

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{15}$

(c) $\frac{\sqrt{2}-1}{2}\pi$ (d) $\frac{\pi}{10}$

Numeric Value Answer

18. Let $f: R \to R$ be a differentiable function and $f(1) = 4$. Then the value of $\lim_{x\to 1}\frac{\int_4^{f(x)} 2t\,dt}{x-1}$, if $f'(1)=2$ is

19. If $f(x)=\begin{vmatrix} \sin x & \sec x & x^2-1 \\ \operatorname{cosec} x & x\sin x & \cos x \\ \tan x & x\tan x & x^2+1 \end{vmatrix}$ then $\int_{-\pi/3}^{\pi/3} f(x)dx =$

20. $\lim_{x\to 0}\left[\frac{1}{x^5}\int_0^x e^{-t^2}dt-\frac{1}{x^4}+\frac{1}{3x^2}\right]=$

21. If $I(n)=\int_0^{\pi/2}\theta\sin^n\theta d\theta,\ n\in N,\ n>3,$ then $\frac{1}{1005}[2010I(2010)\ -2009I(2008)]^{-1}$ is equal to

22. Let $f:(0,\infty)\to R$ be a differentiable function such that

$$x\int_0^x (1-t)f(t)dt=\int_0^x t\,f(t)dt\ \forall x\in(0,\infty)$$ and

$f(1)=1$. The value of the limit $\lim_{x\to\infty} f(x)$ is equal to

23. If p and q are different roots of the equation $\tan x = x$ then $\int_0^1 2\sin(pt)\sin(qt)dt$ is equal to

24. Find the value $\int_0^3\left[\sqrt{x}\right]dx$, when $[x]$ is the greatest integral value of x.

25. Let $f: \mathbb{R}\to\mathbb{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1)=\frac{1}{2}$. Suppose that $F(x)=\int_{-1}^x f(t)dt$ for all $x\in[-1, 2]$ and $G(x)=\int_{-1}^x t|f(f(t))|dt$ for all $x\in[-1, 2]$. If $\lim_{x\to 1}\frac{F(x)}{G(x)}=\frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is

ANSWER KEY																	
1	(a)	**4**	(a)	**7**	(d)	**10**	(a)	**13**	(a)	**16**	(a)	**19**	(0)	**22**	0	**25**	(7)
2	(d)	**5**	(a)	**8**	(b)	**11**	(c)	**14**	(a)	**17**	(a)	**20**	(0.30)	**23**	0		
3	(c)	**6**	(a)	**9**	(c)	**12**	(d)	**15**	(a)	**18**	(16)	**21**	(2)	**24**	(2)		