Time, Speed, Distance and Time & Work

Time, Speed and Distance

This section consists of problems on calculating speed, distance, time, head start and relative speed, etc.

Concepts

1. Distance = Speed × Time

2. 1 km/hr = $\frac{5}{18}$ m/sec

3. If the ratio of speed is a : b : c, then the ratio of time taken is $\frac{1}{a} : \frac{1}{b} : \frac{1}{c}$.

Speed is a relation between time and distance.

$S \propto D$, i.e. if speed is doubled, distance covered in a given time also gets doubled and $S \propto \frac{1}{T}$, i.e. if speed is doubled, time taken to cover the distance will be half.

Average speed

Average speed is defined as $\frac{\text{Total distance travelled}}{\text{Total time taken}}$.

Suppose, a man covers a distance d_1 kms at s_1 km/hr and a distance d_2 kms at s_2 km/hr, then

Average speed of the entiry journey $= \dfrac{d_1 + d_2}{\dfrac{d_1}{s_1} + \dfrac{d_2}{s_2}}$ km / hr.

If the distances are equal, then

Average speed $= \dfrac{d + d}{\dfrac{d}{s_1} + \dfrac{d}{s_2}} = \dfrac{2s_1s_2}{s_1 + s_2}$ km/hr .

Relative speed

If two bodies are moving (in the same direction or in the opposite direction), then the speed of one body with respect to the other is called its relative speed.

Relative speed is a phenomenon that we observe everyday. Suppose you are travelling in a train and there is a second train coming from the opposite direction on the parallel track, then it seems that the second train is moving much faster than actual. If both the trains are moving in the same direction on parallel tracks at same speeds, they seem to be stationary if seen from one of these trains, even though they might actually be at a speed of 100 km/hr each. So what you actually observe is your speed relative to the other.

Concepts

1. If two objects are moving in opposite directions towards each other or away from each other on a straight-line at speeds u and v, then they seem to be moving towards each other or away from each other at a relative speed = Speed of first + Speed of second = u + v.

This is also the speed at which they are moving towards each other or the speed at which they may be moving away from each other.

2. If the two objects move in the same direction with speeds u and v, then

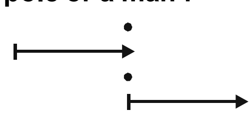
relative speed = difference of their speeds = u – v.

This is also the speed at which the faster object is either moving closer to the slower object or moving away from the slower object as the case may be.

3. If two objects start from A and B with speeds u and v respectively, and after crossing each other, take a and b hours to reach B and A respectively, then u : v $= \sqrt{\frac{b}{a}}$.

Trains

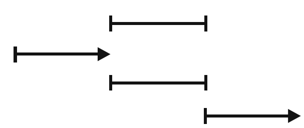
Train crossing a pole or a man :



Time taken by a train to cross a pole or a man is given by the ratio of length of the train to its speed.

$\therefore$ Time taken $= \dfrac{\text{Length of the train}}{\text{Speed of the train}}$

Train crossing a bridge or a platform:



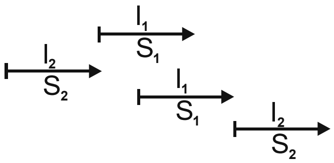
The distance covered in the time taken to cross a bridge or a platform will be equal to the sum of the length of the train and the length of the bridge or a platform being crossed.

$\therefore$ Time taken to cross a bridge or a platform

$= \dfrac{\text{Length of the train + length of the bridge or a platform}}{\text{Speed of the train}}$

Train crossing another train:

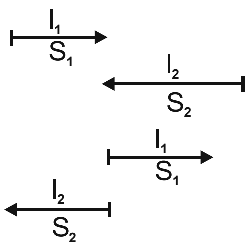
In the same direction **:**



$$\therefore \text{Time taken} = \frac{\text{Sum of lengths of the trains}}{\text{Relative speed of the trains}}$$

$$= \frac{l_1 + l_2}{S_2 - S_1}$$

In the opposite direction :



$$\therefore \text{Time taken} = \frac{\text{Sum of lengths of the trains}}{\text{Sum of the speeds of the trains}}$$

$$= \frac{l_1 + l_2}{S_1 + S_2}$$

Boats and Streams

Downstream and Upstream Motion

Downstream motion of a boat is its motion in the same direction as the flow of the river.

Upstream motion is exactly the opposite.

There are two parameters in these problems :

1. Speed of the river (R): This is the speed with which the river flows.

2. Speed of the boat in still water (B): If the river is still, this is the speed at which the boat would be moving.

The effective speed of a boat in upstream = B – R

The effective speed of a boat in downstream = B + R

3. The speed of the boat in still water is given as B $= \frac{1}{2}(d + u)$, and the speed of the river R = $\frac{1}{2}(d - u)$, where d and u are the downstream and upstream speeds, respectively.

Circular motion

The problems on circular motion deal with races on a circular track to calculate the time of meeting at the starting point and anywhere on the track.

Concepts

1. If two people A and B start from the same point, at the same time and move in the same direction along a circular track and take x minutes and y minutes respectively to come back to the starting point, then they would meet for the first time at the starting point according to the formula given below:

First time meeting of A and B at the starting point = (LCM of x and y)

Notes:

This formula would remain the same even if they move in the opposite directions.

2. If two people A and B start from the same point with speeds m km/hr and n km/hr respectively, at the same time and move in the same direction along a circular track, then the two would meet for the first time by the formula given below:

Time of the first meeting

$$= \frac{\text{Circumference of the track}}{\text{Relative speed}}.$$

Time and Work

The Time and work topic involves problems on time taken to do a certain job by certain number of workers; the change in the number of hours required to do a job if the number of workers is changed; the number of hours required to do the same job by different number of workers if their speeds are different, etc.

w = n × h × d = constant

where, w : Work

n : Number of men

h : Number of hours

d : Number of days

In general, we can say that,

If **'w_1'** work is done by **'m_1'** men working **'h_1'** hours per day in **'d_1'** days and **'w_2'** work is done by **'m_2'** men working **'h_2'** hours per day in **'d_2'** days, then

$$\frac{m_1 d_1 h_1}{w_1} = \frac{m_2 d_2 h_2}{w_2}$$

Generally, the following types of questions are asked in the examination.

Type of question	Example	Approach to question
1. Calculate the time taken by two persons working together to finish a work.	A can complete a piece of work in 10 days which B alone can complete in 12 days. In how many days can both complete it if they work together?	$T = \frac{X \times Y}{X + Y}$, where X and Y are the time taken by A and B individually. $T = \frac{10 \times 12}{10 + 12} = \frac{120}{22} = 5\frac{5}{11}$ days
2. If A and B can finish a work in X days and A alone can complete it in Y days, find the time taken by B alone.	Two persons A and B working together can dig a trench in 8 hrs while. A alone can dig it in 12 hrs. In how many hours can B alone dig the trench?	$T = \frac{X \times Y}{Y - X} = \frac{12 \times 8}{12 - 8} = \frac{96}{4} = 24$ hrs
3. If one person is m times as good as another worker and takes n day less than the other person, in how many days can they together finish the work?	A is thrice as good as B and takes 60 days less than B for completing a job. Find the time taken by them if they work together.	$T = \frac{m \times l}{m^2 - 1} = \frac{3 \times 60}{9 - 1} = \frac{180}{8}$ $= \frac{45}{2}$ days = 22.5 days
4. One pipe can fill in a cistern in T_1 min and another pipe in T_2 min. If both the pipes are opened together, find the time taken by them to fill in the cistern.	A tap can fill in a cistern in 30 min and another can fill it in 40 min. If both the taps are opened simultaneously, find the time taken to fill the cistern completely.	$T = \frac{T_1 \times T_2}{T_1 + T_2}$, where T_1 and T_2 are the time taken by each pipe individually. $= \frac{30 \times 40}{30 + 40} = \frac{1200}{70} = 17\frac{1}{7}$ min
5. One pipe can fill in a cistern in T_1 min and another pipe can empty it in T_2 min. If both the pipes are opened together, find the time taken to fill in the cistern.	A cistern is filled in by a pipe A in 10 hr and emptied by a pipe B in 12 hr. If both the pipes are opened together, in how much time will the cistern be full?	$T = \frac{T_1 \times T_2}{T_2 - T_1}$, where T_1 and T_2 are the time taken by each pipe individually to fill and empty the cistern respectively. $= \frac{10 \times 12}{12 - 10} = \frac{120}{2} = 60$ hrs
6. A cistern takes T_1 min to be filled by the filling pipes, but takes x extra min to fill in due to a leakage in the cistern. Find out the time, in which the leak will empty the cistern.	Two pipes can fill in a cistern in 14 hrs and 16 hrs respectively. The pipes are opened simultaneously. Due to a leakage in the bottom of the cistern, 32 min extra are required to fill the cistern. If the cistern is full, in what time will the leak empty it?	$T = T_1 \times \left(1 + \frac{T_1}{x}\right)$, where T_1 is the time taken to fill in the cistern and x is the extra time taken due to a leakage. T is equal to $\frac{112}{15}\left(1 + \frac{112 \times 60}{15 \times 32}\right)$ = 112 hrs.

Concepts

1. If A can do a piece of work in 10 days, then in 1 day, A will do $\frac{1}{10}$ of the total work.

2. If A is thrice as good as B, then

(a) In a given amount of time, A will be able to do 3 times the work B does.

Ratio of work done by A and B (in the same time) = 3 : 1.

(b) For the same amount of work, B will take thrice the time as much as A takes.

Ratio of time taken by A and B (same work done) = 1 : 3.

3. Efficiency is directly proportional to the work done and inversely proportional to the time taken.

Pipes and Cisterns

This consists of problems on how long it will take for different pipes of different diameters to fill a cistern; the time taken to fill a cistern when one pipe is filling it while another empties it, etc.

Concepts

1. If a pipe can fill a tank in x hours and another pipe can fill it in y hours, then the fraction of tank filled by both pipes together in 1 hour = $\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}$.

   Or, time required to fill the tank by both the pipes $=\frac{xy}{x+y}$.

2. If a pipe can fill a tank in x hours and another pipe can empty it in y hours, then the fraction of tank filled by both the pipes together in 1 hour = $\frac{1}{x}-\frac{1}{y}=\frac{y-x}{xy}$.

Solved Examples

1. Three men can complete a piece of work in 6 days. Two days after they start, 3 more men join them. How many days will they take to complete the remaining work?

   Solution :

   Work done by 3 men in 2 days = $\frac{1}{6}\times 2=\frac{1}{3}$

   Remaining work = $\left(1-\frac{1}{3}\right)=\frac{2}{3}$

   3 men's 1 day's work = $\frac{1}{6}$

   6 men's 1 day's work = $\frac{1}{3}$

   $\frac{2}{3}$ work is done by them in $\left(\frac{3\times 2}{3}\right)$ = 2 days.

   Alternative method:

   After two days, only four days' work remains.

   If 3 more men join the existing 3 men, work will be completed in half the time.

   Therefore, the work is completed in 2 days.

2. A and B undertake a piece of work for ₹600. A alone can complete it in 6 days, while B alone can complete it in 8 days. With the help of C, they finish it in 3 days. Find B's share.

   Solution :

   C's 1 day's work = $\frac{1}{3}-\left(\frac{1}{6}+\frac{1}{8}\right)=\frac{1}{24}$

   Ratio of work done in 1 day for A : B : C

   $=\frac{1}{6}:\frac{1}{8}:\frac{1}{24}=4:3:1$

   B's share = ₹ $\left(\frac{600\times 3}{8}\right)$ = ₹ 225

   Alternative method:

   Let the total work be LCM (6, 8, 3) = 24 units.

   They finished the total work in 3 days.

   Therefore, (A + B + C)'s 1 day's work = 8 units. A's 1 day's work = 4 units.

   B's 1 day's work = 3 units.

   Hence, C's 1 day's work = 8 – 7 = 1 unit.

   Ratio of work done in 1 day for A : B : C = 4 : 3 : 1

   B's share = ₹ $\left(\frac{600\times 3}{8}\right)$ = ₹ 225

3. Two men and 3 boys can complete a piece of work in 10 days while 3 men and 2 boys can complete the same work in 8 days. In how many days can 2 men and 1 boy complete the same work?

   Solution :

   Let 1 man's 1 day's work be x.

   Let 1 boy's 1 day's work be y.

   $2x + 3y = \frac{1}{10}$ and $3x + 2y = \frac{1}{8}$

   On solving, we get $x = \frac{7}{200}$ and $y = \frac{1}{100}$

   (2 men + 1 boy's) 1 day's work

   $=2\times\frac{7}{200}+1\times\frac{1}{100}=\frac{16}{200}=\frac{2}{25}$

   Thus, 2 men and 1 boy can finish the work in $\frac{25}{2}$ days.

   Or

   Alternative method 1:

   Two men and three boys can complete a piece of work in 10 days.

   Hence, in one day, the number of men and boys required = (2m + 3b) $\times$ 10.

   Similarly, if three men and two boys can complete a piece of work in 8 days, then in one day the number of men and boys required = (3m + 2b) $\times$ 8.

   Hence, equating both, we get

   (2m + 3b) $\times$ 10 = (3m + 2b) $\times$ 8.

   $\Rightarrow$ 20m + 30b = 24m + 16b

$\Rightarrow$ 4m = 14b

$\Rightarrow$ m = $\frac{7}{2}$b

Now, 2m + 3b = 2$\times\frac{7}{2}$b + 3b = 10 b.

New requirement is 2m + 1b = 2$\times\frac{7}{2}$b + 1b = 8 b

By unitary method,

10 b $\longrightarrow$ 10 days

8 b $\longrightarrow$?

$10\times\frac{10}{8}$ = 12.5 days.

Alternative method 2

Let the total work be LCM (10, 8) = 40 units.

Let 1 man's 1 day's work be x.

Let 1 boy's 1 day's work be y.

2x + 3y = 4 and 3x + 2y = 5

On solving, we get x = $\frac{7}{5}$ and y = $\frac{2}{5}$

(2 men + 1 boy's) 1 day's work

= $\left(2\times\frac{7}{5}+1\times\frac{2}{5}\right)=\frac{16}{5}$ units.

Number of days taken to complete the work

$=40\times\frac{5}{16}=\frac{25}{2}$ days = 12.5 days.

4. Three men can complete a piece of work in 6 days. Two days after they start, 3 more men joined them. How many days will they take to complete the remaining work?

Solution :

Work done by 3 men in 2 days = $\frac{1}{6}\times2=\frac{1}{3}$

Remaining work = $\left(1-\frac{1}{3}\right)=\frac{2}{3}$

3 men's 1 day work = $\frac{1}{6}$.

6 men's 1 day work = $\frac{1}{3}$.

$\frac{2}{3}$ work is done by them in $\left(\frac{3\times2}{3}\right)$ = 2 days.

Alternative method:

After 2 days, only 4 days' work remains. If 3 more men join the existing 3 men, work will be completed in half of the time. Therefore, the work is completed in 2 days.

5. A and B undertake to do a piece of work for ₹600. A alone can do it in 6 days, while B alone can do it in 8 days. With the help of C, they finish in 3 days. Find B's share.

Solution :

C's 1 day work = $\frac{1}{3}-\left(\frac{1}{6}+\frac{1}{8}\right)=\frac{1}{24}$.

Ratio of work done in 1 day for A, B and C

$=\frac{1}{6}:\frac{1}{8}:\frac{1}{24}=4:3:1$

B's share = ₹$\left(\frac{600\times3}{8}\right)$ = ₹225

Alternative method:

Let the total work be LCM (6, 8, 3) = 24 units.

They finished the total work in 3 days.

Therefore, (A + B + C)'s 1 day work = 8 units.

A's 1 day work = 4 units.

B's 1 day work = 3 units.

Hence, C's 1 day work = 8 – 7 = 1 unit.

B's share = $\frac{3}{8}\times600$ = ₹225

6. Two men and 3 boys can do a piece of work in 10 days, while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work?

Solution :

Let 1 man's 1 day work = x.

Let 1 boy's 1 day work = y.

2x + 3y = $\frac{1}{10}$ and 3x + 2y = $\frac{1}{8}$

On solving we get, x = $\frac{7}{200}$ and y = $\frac{1}{100}$.

(2 men + 1 boy)'s 1 day work

$=2\times\frac{7}{200}+1\times\frac{1}{100}=\frac{16}{200}=\frac{2}{25}$.

Thus, 2 men and 1 boy can finish the work in $\frac{25}{2}$ days.

Alternative method:

Let the total work be LCM (10, 8) = 40 units.

Let 1 man's 1 day work = x.

Let 1 boy's 1 day work = y.

2x + 3y = 4 and 3x + 2y = 5

On solving, we get x = $\frac{7}{5}$ and y = $\frac{2}{5}$.

(2 men + 1 boy)'s 1 day work

$= \left(2 \times \frac{7}{5} + 1 \times \frac{2}{5}\right) = \frac{16}{5}$ units.

Number of days taken to do the work

$= 40 \times \frac{5}{16} = \frac{25}{2}$ days.

7. A fort of 60 men has food for 28 days. 8 days later reinforcements arrive, leaving the number of days the food would last for 15 days. What was the strength of the reinforcement?

Solution :

Originally, the food would have lasted for 28 days.

After 8 days the food would have lasted for 20 days.

Let the reinforcement number be x.

The food that would have been consumed by 60 men in 20 days, was consumed by

(60 + x) men in 15 days.

60 × 20 = (60 + x)15

$\Rightarrow$ x = 20.

$\therefore$ The strength of the reinforcement was 20 men.

8. Two pipes A and B can fill a tank in 24 min and 32 min respectively. If both pipes are opened simultaneously, after how much time should pipe B be closed such that the tank is full in 18 min?

Solution :

Let B be closed after x minutes.

Part filled by (A + B) in x min + Part filled by A in (18 – x) min = 1

Thus, $x \times \left(\frac{1}{24} + \frac{1}{32}\right) + (18 - x)\frac{1}{24} = 1$

Solving for x, x = 8.

Hence, B must be closed after 8 min.

Alternative method:

Let B be closed after x minutes.

Let the total capacity be LCM (24, 32) = 96 units.

In 1 min, A can fill 4 units and B can fill 3 units.

7x + 4(18 – x) = 96. Solving for x, x = 8 min.

Hence, B must be closed after 8 min.

9. Two pipes A and B can fill a tank in 36 min and 45 min respectively. Pipe C can empty it in 30 min. A and B are opened and after 7 min C is also opened. In how much time will the tank be full?

Solution :

Part filled in 7 min = $7 \times \left[\frac{1}{36} + \frac{1}{45}\right] = \frac{7}{20}$

Remaining part = $1 - \frac{7}{20} = \frac{13}{20}$

Net part filled in 1 min when A, B and C are opened

$= \frac{1}{36} + \frac{1}{45} - \frac{1}{30} = \frac{1}{60}$.

$\frac{13}{20}$ part is filled in $60 \times \frac{13}{20} = 39$ min.

Total time taken to fill the tank = (39 + 7) = 46 min.

Alternative method:

Let the capacity be LCM (36, 45, 30) = 180 units.

In 1 min, A fills 5 units, B fills 4 units, C empties 6 units.

In 7 min, A and B fill (5 + 4) × 7 = 63 units.

After 7 min, A and B fill 9 units and C empties 6 units.

Effectively, only 3 units are retained in 1 min.

Tank to be filled = 180 – 63 = 117 units.

Time taken = $\frac{117}{3}$ = 39 min.

Total time taken = (7 + 39) = 46 min.

10. A scooterist covers a certain distance at 36 km/hr. How many metres does he cover in 3 min?

Solution :

Speed = 36 km/hr = 36 × $\frac{5}{18}$ × m/s = 10 m/s.

Thus, distance covered in 3 min

= (10 × 3 × 60) = 1,800 m.

11. Walking at $\frac{3}{4}$ of his usual speed a man is $1\frac{1}{2}$ hr late. Find his usual travel time.

Solution :

Let the usual time be t hours.

$\frac{4}{3} \times t = t + \frac{3}{2} \Rightarrow t = 4.5$ hr

Alternative method:

$st = \frac{3s}{4}\left(t + \frac{3}{2}\right) \Rightarrow 4st = 3st + 4.5s$

t = 4.5 hr

12. A man starts from L to M, another from M to L at the same time. After passing each other, they complete their journey in $3\frac{1}{3}$ hr and $4\frac{4}{5}$ hr respectively. Find the speed of the second man if the speed of the first is 24 km/hr.

Solution :

Speed of the first man : Speed of the second man

$\sqrt{\frac{b}{a}} = \sqrt{\frac{24}{5} \times \frac{3}{10}} = \sqrt{\frac{36}{25}} = \frac{6}{5}$

where ‘a’ and ‘b’ are the time taken by the first and the second man after passing each other.

Thus, the second man’s speed = $\frac{5}{6} \times 24$

= 20 km/hr.

13. Two cyclists cover the same distance with speeds 15 km/hr and 16 km/hr respectively. Find the distance travelled by each, if one takes 16 min longer than the other takes.

Solution :

Let the required distance be x kilometres.

$$\frac{x}{15} - \frac{x}{16} = \frac{16}{60}$$

$\Rightarrow 16x - 15x = 64 \Rightarrow x = 64.$

Hence, the required distance = 64 km.

14. A thief is spotted by a policeman at a distance of 200 m. If the speed of the thief be 10 km/hr and that of the policeman be 12 km/hr, at what distance will the policeman catch the thief?

Solution :

Relative speed of the policeman = 2 km/hr.

Time taken by the policeman to cover 200 m

$$= \frac{200}{1000} \times \frac{1}{2} \text{ hr} = \frac{1}{2} \text{ hr.}$$

In this time the thief covers a distance

$$= 10 \times \frac{1}{10} = 1 \text{ km.}$$

The policeman catch the thief at a distance of (1 km + 0.2 km) 1.2 km

15. A train running at 54 km/hr takes 20 sec to cross a platform and 12 sec to pass a man walking in the same direction at a speed of 6 km/hr. Find the lengths of the train and the platform?

Solution :

Let the length of the train = x metres and the length of the platform = y metres.

Speed of the train relative to the man

= 48 km/hr = $\frac{40}{3}$ m/s.

In passing the man, the train covers its own length with relative speed.

Length of the train = $\frac{40}{3} \times 12 = 160$ m.

Since speed of the train = 54 km/hr = 15 m/s,

$$\frac{x+y}{15} = 20 \Rightarrow x + y = 300 \Rightarrow y = 140$$

Length of the platform = 140 m.

16. A can beat B by 20 m in a 200 m race. B can beat C by 10 m in a 250 m race. By how many metres can A beat C in a 100 m race?

Solution :

A runs 200 m when B runs 180 m.

B runs 250 m when C runs 240 m.

LCM of 180 and 250 = 4500.

Thus, when A runs 5000 m, B runs 4,500 m.

When B runs 4,500 m, C runs 4,320 m.

A beats C by 680 m in a race of 5,000 m.

Hence, A beats C by $\frac{680}{5000} \times 100$

= 13.6 m in a 100 m race.

17. A hare makes 9 leaps in the same time as a dog makes 4. But the dog’s leap is $2\frac{1}{3}$ m while hare’s is only 1 m. How many leaps will the dog have to make before catching up with the hare if the hare has a start of 16 m?

Solution :

Distance covered by the dog in one minute (4 leaps)

$$= 4 \times \frac{7}{3} = \frac{28}{3} \text{ m.}$$

Distance covered by the hare in one minute (9 leaps) = 9 × 1 = 9 m.

Distance gained by the dog in one minute (4 leaps)

$$= \frac{1}{3} \text{ m.}$$

Hence, for 1 m gain he has to make 12 leaps.

Number of leaps required by the dog to gain 16 m = 12 × 16 = 192 leaps.

18. A ship, 156 km away from the shore, springs a leak which admits $\frac{7}{3}$ tonnes of water in 6.5 min. A pump throws out 15 tonnes of water in 1 hr. If 68 tonnes would be suffice to sink the ship, find the average rate of sailing so as to just reach the shore.

Solution :

Net volume of water in the ship in 1 min

$$= \frac{7}{3} \times \frac{1}{6.5} - \frac{15}{60} = \frac{17}{156} \text{ tonnes.}$$

Time for 68 tonnes of water

$$= 68 \times \frac{156}{17} = 624 \text{ min.}$$

Speed of the ship = $\frac{\text{Distance}}{\text{Time}}$

$$= \frac{156 \text{ km}}{624 \text{ min}} = \frac{1}{4} \text{ km/min} = 15 \text{ km/hr.}$$

19. A man rows 27 km downstream and 18 km upstream taking 3 hr each time. What is the velocity of the current?

Solution :

Rate downstream = $\frac{27}{3}$ = 9 km/hr.

Rate upstream = $\frac{18}{3}$ = 6 km/hr.

Velocity of the current = 0.5(9 – 6) = 1.5 km/hr.

20. A man can row upstream at 7 km/hr and downstream at 10 km/hr. Find his rate in still water and the rate of the current.

Solution :

Rate in still waters = 0.5(10 + 7) = 8.5 km/hr.

Rate of the current = 0.5(10 – 7) = 1.5 km/hr.

21. Buses take 12 hr to cover the distance of 120 km between A and B. A bus starts from A at 8 a.m. and another bus starts from B at 10 a.m. on the same day. When do the two buses meet?

Solution :

The distance between A and B is 120 km.

Speed of the buses = $\frac{120}{12}$ = 10 km/hr.

By 10 a.m., the bus from A would have covered 20 km. Hence, the distance between the buses at 10 a.m. = 120 – 20 = 100 km.

Relative speed of the buses = 20 km/hr.

Time taken to meet = $\frac{100}{20}$ = 5 hr.

22. A and B can separately do a piece of work in 20 and 15 days respectively. They worked together for 6 days, after which B was replaced by C. If the work was finished in next 4 days, then in how many days could C alone do the work?

Solution :

(A + B)'s 6 days work = $6\left(\frac{1}{20}+\frac{1}{15}\right)=\frac{7}{10}$.

(A + C)'s 4 days work = $\frac{3}{10}$;

(A + C)'s 1 day work = $\frac{3}{40}$.

A's 1 day work = $\frac{1}{20}$.

$\therefore$ C's 1 day work = $\left(\frac{3}{40}-\frac{1}{20}\right)=\frac{1}{40}$.

Hence, C alone can finish the work in 40 days.

Exercise

1. A can do a piece of work in 7 days of 9 hr each, and B can do it in 6 days of 7 hr each. How long will they take to do it working together $\frac{42}{5}$ hr a day?
   (a) 3 days (b) 4 days
   (c) 4.5 days (d) None of these
2. A can do a piece of work in 80 days. He works for 10 days and then B alone finishes the remaining work in 42 days. The two together could complete the work in
   (a) 24 days (b) 25 days
   (c) 30 days (d) 29 days
3. A and B can do a piece of work in 45 and 40 days respectively. They begin together, but A leaves after some days and B completes the remaining work in 23 days. For how many days did A work?
   (a) 6 days (b) 8 days
   (c) 9 days (d) 12 days
4. A does half as much as work as B in three-fourths of the time. If together they take 18 days to complete the work, how much time will B take to do it?
   (a) 30 days (b) 35 days
   (c) 40 days (d) None of these
5. A can do a certain job in 12 days. B is 60% more efficient than A. B can do the work alone in
   (a) 6 days (b) 6.25 days
   (c) 7.5 days (d) 8 days
6. A and B can do a job in 25 days and 20 days respectively. A started the work and was joined by B after 10 days. The number of days taken to complete the work is
   (a) 12.5 days (b) 14.22 days
   (c) 15 days (d) 16.66 days
7. A is twice as good as B, and they together finish a job in 14 days. A will finish the work alone in
   (a) 11 days (b) 21 days
   (c) 28 days (d) 42 days
8. A and B can do a piece of work in 72 days; B and C in 120 days; A and C in 90 days. In how many days can A do it alone?
   (a) 150 days (b) 120 days
   (c) 100 days (d) 80 days
9. A and B can do a job in 12 days, and B and C can do it in 16 days. After A has been working for 5 days and B for 7 days, C finishes the work in 13 days. In how many days can C do the work alone?
   (a) 16 days (b) 24 days
   (c) 36 days (d) 48 days
10. Twelve men can do a job in 8 days. Six days after they start, 4 men join them. How many more days will it take to do the job?
    (a) 2.5 days (b) 3.5 days
    (c) 1.5 days (d) 4 days
11. Excluding the stoppages, the speed of a bus is 54 km/hr, and including the stoppages, it is 45 km/hr. For how many minutes does the bus stop per hour?
    (a) 9 min (b) 10 min
    (c) 12 min (d) 20 min
12. A job is done by 10 men in 20 days and 20 women in 15 days. How many days will it take for 5 men and 10 women to finish the work?
    (a) $17\frac{1}{2}$ days (b) $17\frac{1}{7}$ days
    (c) 17 days (d) $17\frac{1}{20}$ days
13. R and S can do a job in 8 and 12 days respectively. If they work on alternate days with R beginning, in how many days will the work be finished?
    (a) $9\frac{1}{3}$ days (b) $9\frac{1}{2}$ days
    (c) $9\frac{1}{24}$ days (d) $9\frac{1}{3}$ days
14. A, B and C can do a job in 11, 20 and 55 days respectively. How soon can the work be done if A is assisted by B and C on alternate days?
    (a) 7 days (b) 8 days
    (c) 9 days (d) 10 days
15. Machines A and B produce 8,000 clips in 4 hr and 6 hr respectively. If they work alternatively for 1 hr, A starting first, then 8,000 clips will be produced in
    (a) 4.33 hr (b) 4.66 hr
    (c) 5.33 hr (d) 5.66 hr

16. Father can work on a task as fast as his two sons working together. If one does the work in 3 hr and the other in 6 hr, in how many hours can the father do the work?

(a) 1 hr (b) 2 hr
(c) 3 hr (d) 4 hr

17. A sum of money is sufficient to pay A's wages for 21 days and B's wages for 28 days. It is then sufficient to pay the wages of both for

(a) 12 days (b) 14 days
(c) 12.25 days (d) 24.5 days

18. A does half as much as work as B, and C does half as much as work as A and B together in the same time. If C alone can do the work in 40 days, then all of them together will finish the work in

(a) 13.33 days (b) 15 days
(c) 20 days (d) 30 days

19. Pipes A, B and C take 20, 15 and 12 min to fill a cistern. Together they will fill the cistern in

(a) 5 min (b) 10 min
(c) 12 min (d) 15.66 min

20. Two pipes can fill a tank in 10 hr and 12 hr, while a third pipe can empty it in 20 hr. If all three pipes are opened, the cistern will be filled in

(a) 7 hr (b) 8 hr
(c) 7.5 hr (d) 8.5 hr

21. A cistern can be filled in 9 hr but it takes 10 hr due to a leak. In how much time can the leak empty the full cistern?

(a) 60 hr (b) 70 hr
(c) 80 hr (d) 90 hr

22. Taps A and B can fill a tank in 12 min and 15 min respectively. If both are opened and A is closed after 3 min, how long will it take for B to fill the tank?

(a) 7 min 45 sec (b) 7 min 15 sec
(c) 8 min 5 sec (d) 8 min 15 sec

23. If two pipes operate simultaneously, a tank will be full in 12 hr. If one pipe takes 10 hr less than the other, how much time does the other take to fill the tank?

(a) 25 hr (b) 28 hr
(c) 30 hr (d) 35 hr

24. A leak in the bottom of a tank can empty it in 6 hr. A pipe fills the tank at the rate of 4 L per min. When the tank is full, the inlet is opened but due to the leak the tank is emptied in 8 hr. What is the capacity of the tank?

(a) 5,260 L (b) 5,760 L
(c) 5,846 L (d) 6,970 L

25. A and B can fill a tank in 6 hr and 4 hr respectively. If they are opened on alternate hours and A is opened first, how many hours will it take to fill the tank?

(a) 4 hr (b) 5 hr
(c) 4.5 hr (d) 5.5 hr

26. A car can finish a journey in 10 hr at 48 km/hr. To cover the same distance in 8 hr, the speed must be increased by

(a) 6 km/hr (b) 7.5 km/hr
(c) 12 km/hr (d) 15 km/hr

27. Tarun can cover a certain distance in 1 hr 24 min by covering $\frac{2}{3}$ of the distance at 4 km/hr and the rest at 5 km/hr. The total distance is

(a) 5 km (b) 6 km
(c) 8 km (d) 9.2 km

28. Walking at $\frac{3}{4}$ of his normal speed, a man is late by 2.5 hr. The usual time is

(a) 7.5 hr (b) 3.5 hr
(c) 3.25 hr (d) $\frac{7}{8}$ hr

29. If a boy walks from his house at 4 km/hr, he reaches school 10 min early. If he walks at 3 km/hr, he reaches 10 min late. What is the distance from his house to the school?

(a) 6 km (b) 4.5 km
(c) 4 km (d) 3 km

30. If a train runs at 40 km/hr, it is late by 11 min, but if it runs at 50 km/hr, it is late by 5 min. The correct time to complete the journey is

(a) 13 min (b) 15 min
(c) 19 min (d) 21 min

31. Two trains start from opposite directions, 200 km apart, at the same time. They cross each other at a distance of 110 km from one of the stations. What is the ratio of their speeds?

(a) 11 : 20 (b) 9 : 20
(c) 11 : 9 (d) None of these

32. A thief steals a car at 2.30 p.m. and drives it at 60 km/hr. At 3 p.m. the owner sets off in another car at 75 km/hr. When will the owner overtake the thief?
(a) 4.30 p.m. (b) 4.45 p.m.
(c) 5 p.m. (d) 5.15 p.m.
33. The speeds of A and B are in the ratio 3 : 4. A takes 30 min more than B to reach the destination. How much time does A take to reach?
(a) 1.25 hr (b) 1.33 hr
(c) 2 hr (d) 2.5 hr
34. A man goes uphill at 24 km/hr and comes down at 36 km/hr. What is his average speed?
(a) 30 km/hr (b) 28.8 km/hr
(c) 32.6 km/hr (d) None of these
35. A plane travels 2,500 km, 1,200 km and 500 km at the rate of 500 km/hr, 400 km/hr and 250 km/hr respectively. The average speed is
(a) 405 km/hr (b) 410 km/hr
(c) 420 km/hr (d) 575 km/hr
36. A car has to cover 80 km in 10 hr. If it covers $\frac{1}{2}$ of the journey in $\frac{3}{5}$ of the time, what should be its speed for the remaining journey?
(a) 8 km/hr (b) 6.4 km/hr
(c) 10 km/hr (d) 20 km/hr
37. A is twice as fast as B, and B is twice as fast as C. The distance covered by C in 54 min will be covered by A in
(a) 216 min (b) 27 min
(c) 108 min (d) 13.5 min
38. The ratio of speeds of A and B is 3 : 4. If B covers a distance in 36 min, A will cover it in
(a) 27 min (b) 48 min
(c) $15\frac{3}{7}$ min (d) None of these
39. A train 110 m long is moving at 132 km/hr. How long will it take to cross a platform 165 m long?
(a) 5 sec (b) 7.5 sec
(c) 10 sec (d) 15 sec
40. A train travelling at the rate of 90 km/hr crosses a pole in 10 s. Its length is
(a) 250 m (b) 150 m
(c) 900 m (d) 100 m
41. A train 300 m long crossed a platform 900 m long in 1 min 12 sec. The speed of the train (in km/hr) is
(a) 45 (b) 50
(c) 54 (d) 60
42. A train crosses a pole in 15 sec and a platform 100 m long in 25 sec. Its length is
(a) 200 m (b) 150 m
(c) 50 m (d) Data insufficient
43. A train 110 m long is travelling at speed of 58 km/hr. What is the time in which it will pass a man walking in the same direction at a speed of 4 km/hr?
(a) 6 sec (b) 7.5 sec
(c) 7.33 sec (d) 7.33 min
44. A train 108 m long moving at the rate of 50 km/hr crosses another train of 112 m long coming from the opposite direction in 6 sec. The speed of the second train is
(a) 48 km/hr (b) 54 km/hr
(c) 66 km/hr (d) 82 km/hr
45. Two trains travel in opposite directions at 36 km/hr and 45 km/hr. A man sitting in the slower train passes the faster train in 8 sec. The length of the faster train is
(a) 80 m (b) 100 m
(c) 120 m (d) 180 m
46. A man sees a train passing over a bridge, 1 km long. The length of the train is half that of the bridge. If the train clears the bridge in 2 min, the speed of the train is
(a) 30 km/hr (b) 45 km/hr
(c) 50 km/hr (d) 60 km/hr
47. A man rows 13 km upstream and 28 km downstream in 5 hr each. The velocity of the stream is
(a) 1.5 km/hr (b) 2 km/hr
(c) 2.5 km/hr (d) 3 km/hr
48. If a man rows at 6 km/hr in still water and 4.5 km/hr against the current, then his rate of rowing along the current is
(a) 9.5 km/hr (b) 7.5 km/hr
(c) 7 km/hr (d) 5.25 km/hr
49. A river runs at 2 km/hr. If a man takes twice as long to row up the river as to row down, the speed of the man in still water is
(a) 6 km/hr (b) 4 km/hr
(c) 10 km/hr (d) 8 km/hr

50. A stream runs at 1 km/hr. A boat goes 35 km upstream and back again in 12 hr. The speed of the boat in still water is
    (a) 6 km/hr (b) 7 km/hr
    (c) 8 km/hr (d) 8.5 km/hr
51. Ajay completes one lap of a circular track of radius 21 m in 6 minute while Bijay completes it in 8 minutes. If they start together from the same point in the same direction after how much time (in minutes) will they meet for the first time?
    (a) 24 (b) 16
    (c) 12 (d) 20
52. In the above questions after how many minute they meet for the second time at the starting point?
    (a) 24 (b) 36
    (c) 48 (d) 40
53. Salma and Nagma are standing on a circular track diametrically opposite to each other. They start to run in anticlockwise direction. If the initial distance between them is 70 metres and speed of Salma is 5 m/s and that of Nagma is 6m/s, then how many times will they meet each other in first 6 minutes
    (a) 1 (b) 2
    (c) 3 (d) 4
54. In the above Question, if Nagma runs in clockwise direction then, after how many seconds they will meet for the third time?
    (a) 70 (b) 40
    (c) 50 (d) 90

Answer Key

1. (a)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (d)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (c)
11. (b)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (a)	**20.** (c)
21. (d)	**22.** (d)	**23.** (c)	**24.** (b)	**25.** (b)	**26.** (c)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (c)
31. (c)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (c)	**36.** (c)	**37.** (d)	**38.** (b)	**39.** (b)	**40.** (a)
41. (d)	**42.** (b)	**43.** (c)	**44.** (d)	**45.** (d)	**46.** (b)	**47.** (a)	**48.** (b)	**49.** (a)	**50.** (a)
51. (a)	**52.** (c)	**53.** (b)	**54.** (c)						

Explanations

1. a A takes 7 × 9 = 63 hr.

$\therefore$ In 1 hr A does $\frac{1}{63}$ of work.

B takes 6 × 7 = 42 hr.

$\therefore$ In 1 hr B does $\frac{1}{42}$ of work.

(A + B)'s work in 1 hr = $\frac{1}{63}+\frac{1}{42}$.

$\therefore$ (A + B) in $\frac{42}{5}$ hr = $\left(\frac{1}{63}+\frac{1}{42}\right)\times\frac{42}{5}$

$=\frac{105}{63\times 42}\times\frac{42}{5}=\frac{1}{3}$

$\therefore$ Number of days = $\frac{1}{\left(\frac{1}{3}\right)}$ = 3 days.

2. c A's work in one day = $\frac{1}{80}$.

$\therefore$ 10 days work $=\frac{10}{80}=\frac{1}{8}$.

Work to be done by B = $\left(1-\frac{1}{8}\right)=\frac{7}{8}$.

$x\times 42=\frac{7}{8},\ x=\frac{7}{42\times 8}=\frac{7}{336}.$

Number of days required by B = $\frac{336}{7}$ = 48 days.

A + B = $\frac{1}{80}+\frac{1}{48}=\frac{128}{80\times 48}$

$\therefore$ Number of days = $\frac{80\times 48}{128}=30$ days.

3. c B's work lasts 23 days.

$\therefore$ Work done in 23 days by B = $\frac{23}{40}$.

Remaining work = $\left(1-\frac{23}{40}\right)=\frac{17}{40}$.

(A + B) in one day = $\left(\frac{1}{45}+\frac{1}{40}\right)^{th}$ of work.

Number of days required to finish $\frac{17}{40}$ of work

$=\frac{17}{40}\times\frac{(40\times 45)}{(45+40)}=\frac{17\times 40\times 45}{40\times 85}=9$ days.

4. a Work done A : B = 1 : 2.

Time taken A : B = 3 : 4.

$\therefore$ A works 1 unit in 3 hr.

$\therefore$ In 1 hr = $\frac{1}{3}$ unit of work.

$\therefore$ B works 2 units in 4 hr.

$\therefore$ In 1 hr = $\frac{2}{4}$ unit of work.

$\therefore$ Work done per hour = A : B = $\frac{1}{3}:\frac{1}{2}=2:3$.

$\therefore$ A + B can do 5 units in 1 day.

$\therefore$ A + B can do 5 × 18 units in 18 days.

$\therefore$ Total work = 90 units.

Time taken by B to complete the work = $\frac{90}{3}$ = 30 days.

5. c A : B work done in same time = 10 : 16 = 5 : 8.

(As B is 60% more efficient)

A can finish work in 12 days.

$\therefore$ Units of work = 12 × 5 = 60.

$\therefore$ B can finish work in $\frac{60}{8}$ days = 7.5 days.

6. d A's 10 days' work $=\frac{1}{25}\times 10=\frac{2}{5}$.

Remaining work $=\left(1-\frac{2}{5}\right)=\frac{3}{5}$

$\left(\frac{1}{25}+\frac{1}{20}\right)$ work is done by (A + B) in 1 day

$\frac{3}{5}$ work is done by them in $=\frac{100}{9}\times\frac{3}{5}=6.66$ days.

Number of days = 10 + 6.66 = 16.66 days

7. b Let A can finish a job in x days and B can do it in 2x days

$\Rightarrow\frac{1}{2x}+\frac{1}{x}=\frac{1}{14}$

$\therefore$ x = 21 days.

8. b A + B = 72 days, B + C = 120 days, A + C = 90 days.

Assume units of work = 360 = [(LCM(72, 120, 90)].

(A + B)'s work in 1 day = $\frac{360}{72}$ = 5 units.

(B + C)'s units of work in 1 day = $\frac{360}{120}$ = 3 units.

(A + C)'s of work in 1 day = $\frac{360}{90}$ = 4 units.

$\therefore$ 2(A + B + C) units in 1 day = 5 + 3 + 4 = 12.

(A + B + C)'s work in 1 day = $\frac{12}{2}$ = 6.

$\therefore$ A's work in 1 day = 6 – 3 = 3 units.

$\therefore$ A will finish 360 units in $\frac{360}{3}$ = 120 days.

9. b Assume a units of work = LCM of (12, 16) = 48.

(A + B)'s work in 1 day = $\frac{48}{12}$ = 4 units.

(B + C)'s work in 1 day = $\frac{48}{16}$ = 3 units.

(A + B) worked for 5 days = 4 × 5 = 20 units done.

(B + C) worked for 2 days = 3 × 2 = 6 units done.

Remaining work = 48 – 26 = 22 units.

C finishes in (13 – 2) days = 11 days.

∴ Units of work in 1 day by C = $\frac{22}{11}$ = 2.

∴ Number of days required for C = $\frac{48}{2}$ = 24 days.

10. c Total work = 12 × 8 = 96.

12 men finish 6 days of work = 12 × 6 = 72.

∴ Work left = 96 – 72 = 24.

Now number of men = 12 + 4 = 16.

∴ Time = $\frac{24}{16} = 1\frac{1}{2}$ = 1.5 days.

11. b Due to stoppage, it covers 9 km less.

Time taken to cover 9 km = $\left(\frac{9}{54} \times 60\right)$ min = 10 min.

12. b 10 men in 20 days = 20 women in 15 days.

∴ 5 men in 40 days and 10 women in 30 days.

(5 men + 10 women) in $\frac{1}{\left(\frac{1}{40}+\frac{1}{30}\right)}$ days

= $\frac{1200}{70}$ days = $17\frac{1}{7}$ days.

13. b Assume the units of work done = 24 = [LCM (12, 8)].

Units of work done by R in a day = $\frac{24}{8}$ = 3.

Units of work done by S in a day = $\frac{24}{12}$ = 2.

In 2 days units of work done = 5.

In 8 days units of work done = 20.

In 9 days units of work done = 23.

On 10th day, work to be done = 24 – 23 = 1 unit.

∴ Time required by S = $\frac{1}{2}$ days

∴ Total time = $9\frac{1}{2}$ days.

14. b Units of work done = 220 = LCM (11, 20, 55).

Units of work in a day by A, B and C = 20, 11, 4.

1st day = (A + B) = 31,

2nd day = (A + C) = 24,

work done in 2 days = 55.

work done in 8 days = 220.

15. b Assume units of work = 12 [LCM(4,6)]

A's 1 hour work = $\frac{12}{4}$ = 3 units

B'S 1 hour work = $\frac{12}{6}$ = 2 units

∴ (A + B)'s 2 hour work = 3 + 2 = 5 units

∴ In 4 hours 10 unit of work will be completed.

Next 2 unit of work will take $\frac{2}{3}$ hr = 0.66 hours.

∴ Total time = 4 + 0.66 = 4.66 hours.

16. b Assume the units of work = 18.

Father in 1 hr = 9 units. (6 + 3)

∴ Time taken = 2 hr.

17. a Assume total wages = 21 × 28.

A's wages for 1 day = 21 × $\frac{28}{21}$ = 28.

B's wages for 1 day = 28 × $\frac{21}{28}$ = 21.

A + B's wages of 1 day = 49.

A + B's wages will last for 21 × $\frac{28}{49}$ = 12 days.

18. a A : B = 1 : 2

C : (A + B) = 1.5 : 3

If C does 1.5 units of work per day, A does 1 and B does 2.

Total units done by C = 40 × 1.5 = 60.

∴ Number of units done by (A + B + C) in 1 day = 4.5, Hence all three will take $\frac{60}{4.5}$ = 13.33 days.

19. a Units to fill up = LCM(20, 15, 12) = 60.

Units filled up by A, B, C in 1 min = 3, 4, 5

Time taken by (A + B + C) to fill up = $\frac{60}{12}$ = 5 min.

20. c Say, capacity in units = LCM(10, 12, 20) = 60.

Rate of A, B, C per hour = $\frac{60}{10}, \frac{60}{12}, \frac{60}{20}$ = 6, 5 and 3 units.

∴ A + B = 11 units, C = 3 units.

∴ Per hour intake = 11 – 3 = 8 units.

Time to fill up = $\frac{60}{8}$ = 7.5 hr.

21. d Capacity in units = LCM(9, 10) = 90.

Filling rate = 10 units/hr (i.e. 90/9).

Similarly, filling rate with leak = 9 units/hr (i.e. 90/10).

∴ Resultant outflow = 10 – 9 = 1 unit/hr.

Total outflow = 90 units.

So time required = $\frac{90}{1}$ = 90 hr.

22. d Say, capacity of tank = LCM(12, 15) = 60 units.

In 1 min total intake in units = $\frac{60}{12}+\frac{60}{15}$ = 9.

In 3 minutes total intake in units = 9 × 3 = 27.

Units to be filled = 60 – 27 = 33.

Time taken by B = $\frac{33}{4}$ = 8.25 (8 min 15 sec).

23. c Let two pipes A and B are there

$\therefore \frac{1}{A}+\frac{1}{B}=\frac{1}{12}$, say A takes x hours to fill. Then B will take x – 10 hr to fill the tank.

$\frac{1}{x}+\frac{1}{x-10}=\frac{1}{12} \Rightarrow \frac{x-10+x}{x(x-10)}=\frac{1}{12}$

$\Rightarrow \frac{2x-10}{x(x-10)}=\frac{1}{12}$

$\Rightarrow 24x - 120 = x^2 - 10x$

$\Rightarrow x^2 - 34x + 120 = 0$

$\Rightarrow x^2 - 30x - 4x + 120 = 0$

$\Rightarrow x(x - 30) - 4(x - 30) = 0$

$\Rightarrow (x - 4)(x - 30) = 0$

$\Rightarrow x = 4, 30$

$\therefore x = 30$ hr ($\because$ x cannot be 4)

24. b 4 L per minute = 240 L per hour.

Let's assume that capacity of tank = LCM(6, 8) = 24 units.

When inlet is also opened, net outflow = $\frac{24}{8}$

= 3 units per hour.

$\therefore$ 4 – 3 = 1 unit per hour of water contributed by the pipe.

$\therefore$ 1 unit = 240 L.

24 units = 24 × 240 = 5,760 L.

25. b Say capacity of tank = 12 units.

A can fill = $\frac{12}{6}$ = 2 units per hour.

B can fill = $\frac{12}{4}$ = 3 units per hour.

A + B alternately in 2 hr = 2 + 3 = 5 units.

A + B alternately in 4 hr = 5 × 2 = 10 units.

In the 5th hour, A is opened.

$\therefore$ A fills the remaining 2 units.

$\therefore$ Total time = 5 hr.

26. c (a) Distance = Speed × Time = 48 × 10 = 480 km.

(b) To cover the same distance in 8 hr.

Speed = $\frac{d}{t}=\frac{480}{8}$ = 60 km/hr.

$\therefore$ Speed must be increased by 60 – 48 = 12 km/hr.

27. b Total time = 60 + 24 = $\frac{84}{60}$ hr.

At 4 km/hr, time $t_1=\frac{2/3d}{4}=\frac{1}{6}d$ hr.

At 5 km/hr, time $t_2=\frac{1/3d}{5}=\frac{1}{15}d$ hr.

Total time = $\frac{d}{6}+\frac{d}{15}=\frac{84}{60} \Rightarrow d = 6$ km.

28. a Let his usual time be t hours and his usual speed be s km/hr.

Distance d = st = $\frac{3}{4}s \times (t + 2.5)$

$\Rightarrow 4t = 3t + 7.5 \Rightarrow t = 7.5$ hr.

29. c Let his normal speed be s km/hr.

Let his normal time be t hours.

$d = st \Rightarrow 4\left(t-\frac{1}{6}\right)=3\left(t+\frac{1}{6}\right)$

$4t-\frac{4}{6}=3t+\frac{3}{6},\ t=\frac{7}{6}$ hr.

Distance = $4\times\left(\frac{7}{6}-\frac{1}{6}\right)=4$ km.

30. c Let the normal speed be s km/hr and normal time be t hours.

$d = st \Rightarrow 40\left(t+\frac{11}{60}\right)=50\left(t+\frac{5}{60}\right)$

$4t+\frac{44}{60}=5t+\frac{25}{60}$

$t=\frac{19}{60}$ hr = 19 min

31. c By the time the trains cross each other, let one cover 110 km, second cover 90 km.

Ratio of speeds = Ratio of the distances covered

= $\frac{110}{90}=11:9$

32. c Thief starts at 2.30 p.m.

Owner starts at 3 p.m.

In 30 min $\left(\text{or } \frac{1}{2}\text{ hr}\right)$, the thief covers

$60\times\frac{1}{2}$ = 30 km

At that time

distance between them = 30 km,

Relative speed = 75 – 60 = 15 km/hr.

$\therefore$ Owner will overtake the thief in $\frac{30}{15}$ = 2 hr,

i.e. at 5 p.m.

33. c Let speed of A = 3x km/hr and speed of B = 4x km/hr.

Time taken by A = $t_B + \frac{1}{2}$ hr.

Time taken by B = t_B hr

$d \Rightarrow s \times t \Rightarrow 3x \times \left(t_B + \frac{1}{2}\right) = 4x \times t_B$

$\Rightarrow 3t_B = \frac{3}{2} = 4t_B \Rightarrow t_B = \frac{3}{2},\ t_A = \frac{1}{2} + \frac{3}{2} = 2$ hr.

Short cut:

Speed is inversely proportional to time taken.

34. b Let the distance be d.

Uphill travelling time = $\frac{d}{24}$.

Downhill travelling time = $\frac{d}{36}$.

$\therefore$ Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

$= \frac{2d}{\frac{d}{24} + \frac{d}{36}} = \frac{2 \times 24 \times 36}{24 + 36} = \frac{144}{5} = 28.8$ km/hr

35. c Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

$= \frac{2500 + 1200 + 500}{\frac{2500}{500} + \frac{1200}{400} + \frac{500}{250}} = \frac{4200}{10} = 420$ km/hr

36. c d = s × t

$\frac{1}{2}$ journey (= 40 km) in $\frac{3}{5}$ of time (= 6 hr)

Total distance = 80 km.

Total time = 10 hr.

80 = 40 + S_2 × 4,

$\therefore S_2$ = 10 km/hr

37. d A = 2B, B = 2C. C takes 54 min. So, B would cover the same distance in half of the time = $\frac{54}{2} = 27$ min.

So, time taken by A = $\frac{27}{2}$ min = 13.5 min.

38. b $\frac{S_A}{S_B} = \frac{3}{4}$

Speed of A = 3x km/min.

Speed of B = 4x km/min.

Distance is same.

d = $S_A \times t_A = S_B \times t_B$

3x × t_A = 4x × 36;

t_A = 48 min

39. b To cross the platform the train has to cover 165 + 110 = 275 m.

132 km/hr = 132 × $\frac{5}{18} = \frac{110}{3}$ m/sec

t = $\frac{275}{110} \times 3 = 7.5$ sec

40. a t = 10 sec.

Speed = 90 km/hr = 90 × $\frac{5}{18}$ = 25 m/s.

Length = s × t = 25 × 10 = 250 m.

41. d Total length to be covered

= Length of train + Length of platform

= 900 + 300 = 1200 m

Total time = 60 + 12 = 72 sec

Speed = $\frac{1200}{72} = \frac{50}{3}$ m/s = $\frac{50}{3} \times \frac{18}{5}$ = 60 km/hr.

42. b Let length of the train be l m and its speed be x m/s.

It crosses a pole in 15 sec.

$\therefore$ l = 15 x … (i)

It crosses a platform of 100 m long in 25 sec.

$\therefore \frac{100 + l}{x} = 25$ … (ii)

Solving (i) and (ii),

100 + 15 x = 25 x

$\therefore x = \frac{100}{10} = 10$ m/s $\Rightarrow l = 150$ m

43. c Length of the train = 110 m.

Speed of the train = 58 × $\frac{5}{18}$ m/s = 16.11 m/sec.

Speed of man = 4 km/hr = 4 × $\frac{5}{18}$ = 1.11 m/sec.

Time = $\frac{110\text{ m}}{16.11 - 1.11} = \frac{110}{15}$ = 7.33 sec.

44. d Length of the first train = 108 m. Its speed = 50 km/hr.

Length of the second train = 112 m.

Let its speed = x km/hr.

Time to cross = 6 s

$\Rightarrow 6 = \frac{108 + 112}{(50 + x) \times \frac{5}{18}} = \frac{220 \times 18}{5(50 + x)}$

250 + 5x = 660 $\Rightarrow$ 5x = 410

x = 82 km/hr

45. d Length of the faster train

= (36 + 45) × $\frac{5}{18}$ × 8 m = 81 × $\frac{5}{18}$ × 8 = 180 m.

46. b Length of the bridge = 1 km.
Length of the train = 0.5 km.
Time to clear the bridge = 2 min = $\frac{2}{60}$ hr.
Speed = $\frac{1+0.5}{2/60} = \frac{1.5}{2} \times 60 = 45$ km/hr.

47. a Let speed of the boat be x km/hr and speed of the stream be y km/hr.
Speed upstream = $\frac{13}{5} = x - y$ ... (i)
Speed downstream = $\frac{28}{5} = x + y$ ... (iii)
Solving for (y) $\Rightarrow$ y = 1.5 km/hr.

48. b Speed of the boat in still water be 'x' = 6 km/hr.
Let speed of the stream be 'y'.
x – y = 4.5 km/hr
$\therefore$ y = 1.5 km/hr
$\therefore$ Rate along the stream = (x + y) = 1.5 + 6
= 7.5 km/hr.

49. a Let x be the speed of the man in still water.
Speed of the river = 2 km/hr = 'y'.
Speed upstream = x – y km/hr.
Speed downstream = x + y km/hr.
(x – y)2t = (x + y)t
2(x – 2) = x + 2 $\Rightarrow$ x = 6 km/hr.

50. a Speed of the stream = 1 km/hr.
Let speed of the boat in still water = x km/hr.
Total time = 12 hr.
$12 = \frac{35}{x-1} + \frac{35}{x+1} = 35\left[\frac{1}{x-1} + \frac{1}{x+1}\right] = 35\left[\frac{2x}{x^2-1}\right]$
$12x^2 - 70x - 12 = 0$
$\Rightarrow 6x^2 - 35x - 6 = 0$
$\Rightarrow 6x^2 - 36x + x - 6 = 0$
$\Rightarrow 6x(x-6) + 1(x-6) = 0$
$\Rightarrow (x-6)(6x+1) = 0$
$\Rightarrow$ x = 6 km/hr

51. a Length of the track $2\pi r = 2 \times \frac{22}{7} \times 21 = 132$ metres
$\therefore$ Speed of Ajay is $\frac{132}{6}$ m/minute and that of Bijay is $\frac{132}{8}$ m/minute
$\because$ They are running in same direction,
$\therefore$ Relative Speed = $\frac{132}{6} - \frac{132}{8} = \frac{132}{24}$ m/minute
$\therefore$ Time taken to meet first time = $\frac{132}{\frac{132}{24}} = 24$ minutes

52. c Time taken to meet first time at starting point = LCM of 6 and 8 = 24 minutes
So they will meet second time after 2 × 24 = 48 minutes

53. b The length of the track = 220 metres
Two situations will arise in this, for meeting first time it will take $\frac{1}{2}\left(\frac{220}{6-5}\right)$ seconds = 110 seconds and after first meet it will take 220 second for each consecutive meet, Because for first meeting, they will have to fill the gap of half of the length of the track.
So in 6 minutes (i.e. 360 seconds) they will meet two times.

54. c They are running in opposite directions
Relative speed = 6 + 5 = 11 m/s
For meeting third time the distance they will have to cover = $2\frac{1}{2}$ times of length of the track
= $\frac{5}{2} \times 220 = 550$ metres
(half for first and two for next two meetings)
Time taken = $\frac{550}{11} = 50$ seconds