26

Properties of Triangles

Numerical 2023

Question:1

If the line $x=y=z$ intersects the line

$$x \sin A+y \sin B+z \sin C-18=0=x \sin 2A+y \sin 2B+z \sin 2C-9,$$

where A, B, C are the angles of a triangle ABC, then $80\left(\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}\right)$

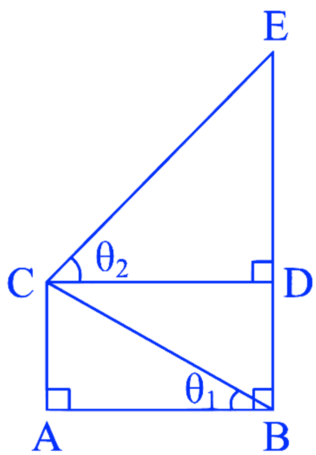
is equal to _____________.

JEE Main 2023 (Online) 15th April Morning Shift

Question:2

In the figure, $\theta_1+\theta_2=\frac{\pi}{2}$ and $\sqrt{3}(\mathrm{BE})=4(\mathrm{AB})$. If the area of $\triangle \mathrm{CAB}$ is $2\sqrt{3}-3$ unit ^{2}, when $\frac{\theta_2}{\theta_1}$ is the largest, then the perimeter (in unit) of $\triangle \mathrm{CED}$ is equal to ________.

JEE Main 2023 (Online) 10th April Evening Shift



Numerical Answers Key

1. Ans. (5)
2. Ans. (6)

Numerical Explanation

Ans.1

$x=y=z=k(\text{ let })$

$\therefore k(\sin A+\sin B+\sin C)=18$

$\Rightarrow k\left(4 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2}\right)=18$

$k(\sin 2A+\sin 2B+\sin 2C)=9$

$\Rightarrow k(4 \sin A \cdot \sin B \cdot \sin C)=9 \ldots \text{ (ii)}$

(ii)/(i)

$8 \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2}=\frac{9}{18}$

$\Rightarrow 80 \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2}=5$

Question:1

In a triangle ABC, if $\cos A + 2\cos B + \cos C = 2$ and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then $\cos A - \cos C$ is equal to

JEE Main 2023 (Online) 12th April Morning Shift

A $\frac{3}{7}$

B $\frac{9}{7}$

C $\frac{10}{7}$

D $\frac{5}{7}$

Question:2

For a triangle ABC, the value of $\cos 2A + \cos 2B + \cos 2C$ is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct?

JEE Main 2023 (Online) 1st February Morning Shift

A $\overrightarrow{MA} \, . \, \overrightarrow{MB} = -18$

B $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$

C perimeter of ΔABC is $18\sqrt{3}$

D area of ΔABC is $\frac{27\sqrt{3}}{2}$

Question:3

A straight line cuts off the intercepts $\mathrm{OA}=\mathrm{a}$ and $\mathrm{OB}=\mathrm{b}$ on the positive directions of x-axis and y axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y-axis and the area of $\triangle\mathrm{OAB}$ is $\frac{98}{3}\sqrt{3}$, then $\mathrm{a}^2-\mathrm{b}^2$ is equal to :

JEE Main 2023 (Online) 30th January Morning Shift

A $\frac{392}{3}$

B 98

C 196

D $\frac{196}{3}$

MCQ Answers Key

1. Ans. (C)
2. Ans. (D)
3. Ans. (A)

MCQ Explanation

Ans.1

$$\cos A+2\cos B+\cos C=2$$

$$\cos A+\cos C=2(1-\cos B)$$

$$2\cos\frac{A+C}{2}\cos\left(\frac{A-C}{2}\right)=2\times 2\sin^2\frac{B}{2}$$

$$\cos\frac{A-C}{2}=2\sin\frac{B}{2}$$

$$2\cos\frac{B}{2}\cos\frac{A-C}{2}=4\sin\frac{B}{2}\cos\frac{B}{2}$$

$$2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)=2\sin B$$

$$\sin A+\sin C=2\sin B$$

$$a+c=2b \quad (\because a=3, c=7)$$

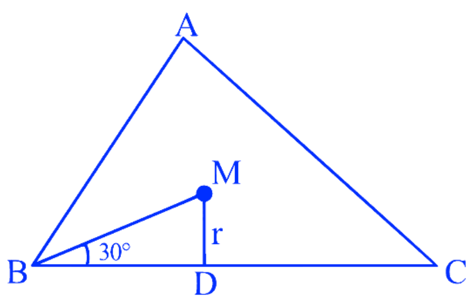
$$\Rightarrow b=5$$

$$\cos A-\cos C=\frac{b^2+c^2-a^2}{2bc}-\frac{a^2+b^2-c^2}{2\text{ab}}$$

$$=\frac{25+49-9}{70}-\frac{9+25-49}{30}$$

$$=\frac{65}{70}+\frac{1}{2}=\frac{20}{14}=\frac{10}{7}$$

Ans.2



If $\cos 2\mathrm{A}+\cos 2\mathrm{B}+\cos 2\mathrm{C}$ is minimum then $\mathrm{A}=$

$$\mathrm{B}=\mathrm{C}=60^\circ$$

So $\triangle\mathrm{ABC}$ is equilateral

Now in-radias $r=3$

So in $\triangle\mathrm{MBD}$ we have

$$\tan 30^\circ=\frac{MD}{BD}=\frac{r}{a/2}=\frac{6}{a}$$

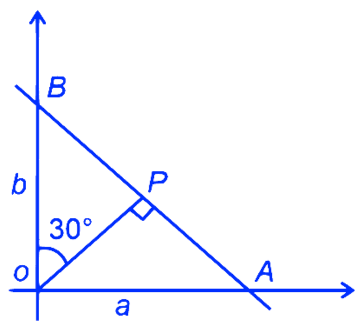
$$1/\sqrt{3} = \frac{1}{a} = a = 6\sqrt{3}$$

Perimeter of $\triangle \mathrm{ABC} = 18\sqrt{3}$

Area of $\triangle \mathrm{ABC} = \frac{\sqrt{3}}{4}a^2 = 27\sqrt{3}$

Ans.3

$\frac{1}{2}ab = \frac{98\sqrt{3}}{3}$



$\Rightarrow \sqrt{3}ab = 196$ ..... (i)

$$OP = OB\cos 30^\circ = OA\cos 60^\circ$$

$\Rightarrow \frac{b\sqrt{3}}{2} = \frac{a}{2}$

$\Rightarrow \sqrt{3}b = a$ ..... (ii)

By (i) and (ii)

$$a^2 = 196$$

$$a = 14$$

$b^2 = \frac{a^2}{3}$

$a^2 - b^2 = \frac{2a^2}{3} = \frac{392}{3}$

MCQ 2022

Question:1

The lengths of the sides of a triangle are $10 + x^2$, $10 + x^2$ and $20 - 2x^2$. If for $x = k$, the area of the triangle is maximum, then $3k^2$ is equal to :

JEE Main 2022 (Online) 27th June Morning Shift

(A) 5

(B) 8

(C) 10

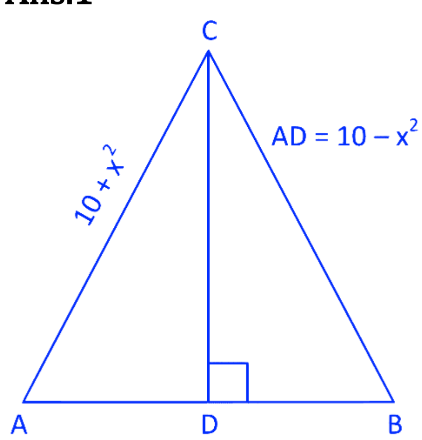
(D) 12

MCQ Answers Key

1. Ans. (C)

MCQ Explanation

Ans.1



$$CD = \sqrt{(10 + x^2)^2 - (10 - x^2)^2} = 2\sqrt{10}|x|$$

Area $= \frac{1}{2} \times CD \times AB = \frac{1}{2} \times 2\sqrt{10}|x|(20 - 2x^2)$

$$A = \sqrt{10}|x|(10 - x^2)$$

$$\frac{dA}{dx} = \sqrt{10}\frac{|x|}{x}(10 - x^2) + \sqrt{10}|x|(-2x) = 0$$

$$\Rightarrow 10 - x^2 = 2x^2$$

$$3x^2 = 10$$

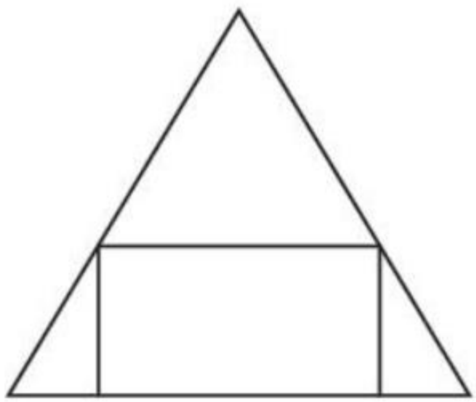
$$x = k$$

$$3k^2 = 10$$

2021

Numerical

Q.1 If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is ___________.



25th Jul Evening Shift 2021

Q.2 In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of 2R + r (in cm) is equal to __________.

16th Mar Evening Shift 2021

Q.3 Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3}\,\beta$, where α β are integers, then $\alpha + \beta$ is equal to ___________.

16th Mar Morning Shift 2021

Numerical Answer Key

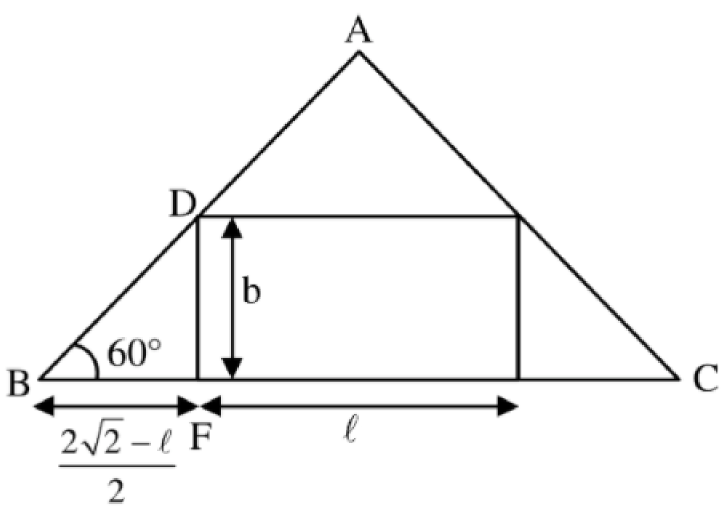
1. Ans. (3)

2. Ans. (15)

3. Ans. (1)

Numerical Explanation

Ans. 1



In ΔDBF

$$\tan 60^\circ = \frac{2b}{2\sqrt{2}-l} \Rightarrow b = \frac{\sqrt{3}\left(2\sqrt{2}-l\right)}{2}$$

A = Area of rectangle = l $\times$ b

$$A = l \times \frac{\sqrt{3}}{2}\left(2\sqrt{2} - l\right)$$

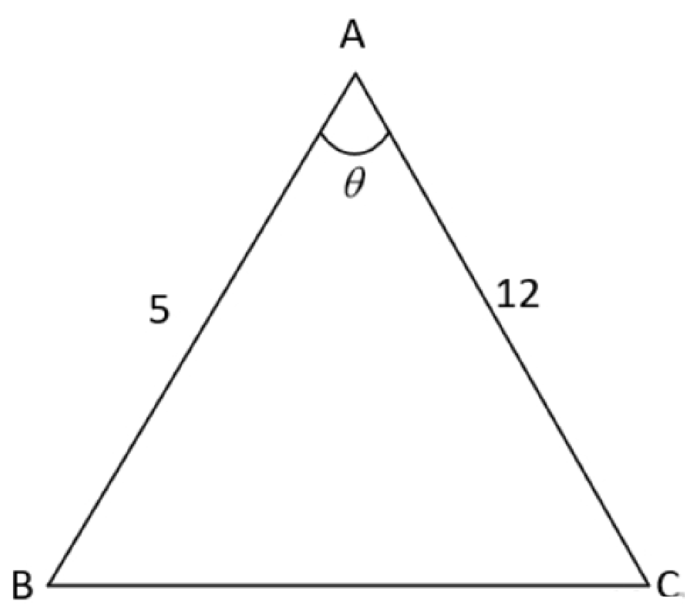
$$\frac{dA}{dl} = \frac{\sqrt{3}}{2}\left(2\sqrt{2} - l\right) - \frac{l.\sqrt{3}}{2} = 0$$

$$l = \sqrt{2}$$

$$A = l \times b = \sqrt{2} \times \frac{\sqrt{3}}{2}\left(\sqrt{2}\right) = \sqrt{3}$$

$\Rightarrow A^2 = 3$

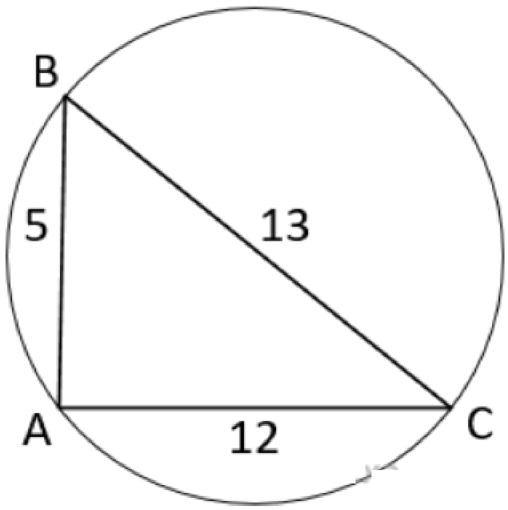
Ans. 2



Area = $\frac{1}{2}(5)(12)\sin\theta = 30$

$\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$

Δ is right angle Δ



$r = (s-a)\tan\frac{A}{2}$

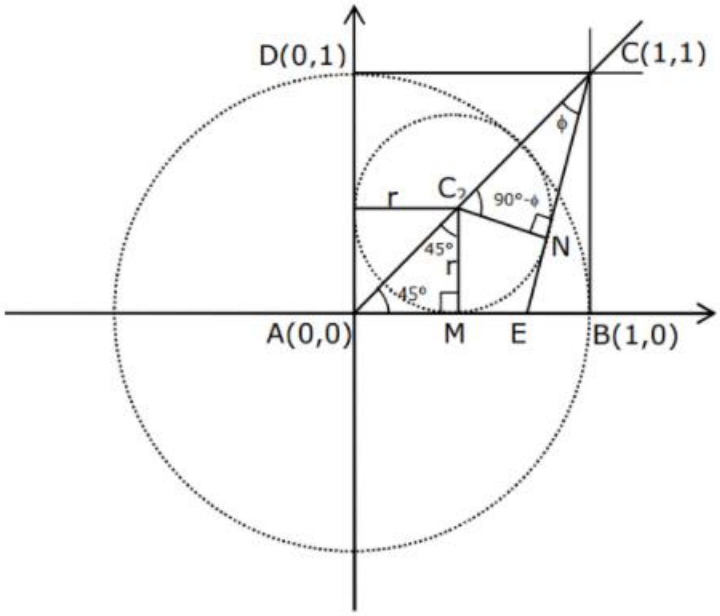
$r = (s-a)\tan\frac{90}{2}$

$r = (s-a)$

$2R + r = s$, (As $a = 2R$)

$2R + r = \frac{5+12+13}{2} = 15$

Ans. 3



(i) $\sqrt{2}r + r = 1$

$r = \frac{1}{\sqrt{2}+1}$

$r = \sqrt{2} - 1$

(ii) $CC_2 = 2\sqrt{2} - 2 = 2\left(\sqrt{2} - 1\right)$

From $\Delta CC_2N = \sin\phi = \frac{\sqrt{2}-1}{2\left(\sqrt{2}-1\right)}$

$\phi = 30^{\circ}$

(iii) In ΔACE apply sine law

$$\frac{AE}{\sin\phi} = \frac{AC}{\sin 105^\circ}$$

$$AE = \frac{1}{2} \times \frac{\sqrt{2}}{\sqrt{3}+1}.2\sqrt{2}$$

$$AE = \frac{2}{\sqrt{3}+1} = \sqrt{3} - 1$$

$$\therefore EB = 1 - \left(\sqrt{3} - 1\right)$$

$$= 2 - \sqrt{3}$$

$$\therefore \alpha = 2, \beta = -1 \Rightarrow \alpha + \beta = 1$$

MCQ (Single Correct Answer)

Q.1. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

A $12\sqrt{15}$

B $12\sqrt{10}$

C $8\sqrt{10}$

D $6\sqrt{10}$

31st Aug Morning Shift 2021

Q.2. Two poles, AB of length a metres and CD of length a + b (b ≠ a) metres are erected at the same horizontal level with bases at B and D. If BD = x and tan ∠ACB = ½, then :

(A) $x^2 + 2(a + 2b)x - b(a + b) = 0$

(B) $x^2 + 2(a + 2b)x + a(a + b) = 0$

(C) $x^2 - 2ax + b(a + b) = 0$

(D) $x^2 - 2ax + a(a + b) = 0$

27th Aug Evening Shift 2021

Q.3.

Let $\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$, where A, B, C are angles of triangle ABC. If the lengths of the sides opposite these angles are a, b, c respectively, then :

(A) $b^2 - a^2 = a^2 + c^2$

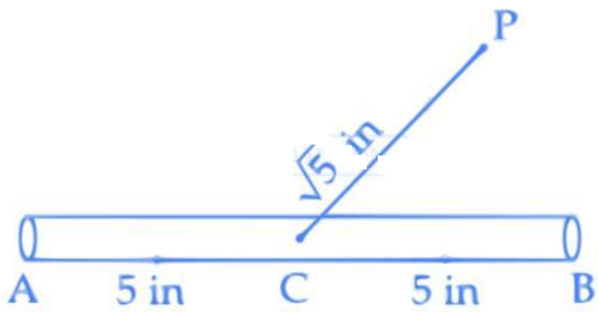
(B) b^2, c^2, a^2 are in A.P.

(C) c^2, a^2, b^2 are in A.P.

(D) a^2, b^2, c^2 are in A.P.

27th Aug Morning Shift 2021

Q.4. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that PC = $\sqrt{5}$ inches and ∠PCB = $\tan^{-1}(2)$. The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is :



A $\tan^{-1}\left(\frac{3}{4}\right)$

B $\tan^{-1}(1)$

C $\tan^{-1}\left(\frac{4}{3}\right)$

D $\tan^{-1}\left(\frac{1}{2}\right)$

26th Aug Evening Shift 2021

Q.5. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is :

A $8(2+2\sqrt{3}+\sqrt{2})$

B $8(\sqrt{6}+\sqrt{2}+2)$

C $8(\sqrt{2}+2+\sqrt{3})$

D $8(\sqrt{6}-\sqrt{2}+2)$

25th Jul Morning Shift 2021

Q.6. Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then sinθ is equal to :

A. $\frac{\sqrt{5}+1}{4}$

B. $\frac{\sqrt{5}-1}{2}$

C. $\frac{\sqrt{2}-1}{2}$

D. $\frac{\sqrt{5}-1}{4}$

20th Jul Evening Shift 2021

Q.7. If in a triangle ABC, AB = 5 units, ∠B = $\cos^{-1}$ (3/5) and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is :

A. $10 + 6\sqrt{2}$

B. $8 + 2\sqrt{2}$

C. $6 + 8\sqrt{3}$

D. $4 + 2\sqrt{3}$

20th Jul Morning Shift 2021

Q.8. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be π3. If the radius of the circumcircle of ΔABC is 2, then the height of the pole is equal to :

A. $\frac{1}{\sqrt{3}}$

B. $2\sqrt{3}$

C. $\sqrt{3}$

D. $\frac{2\sqrt{3}}{3}$

18th Mar Evening Shift 2021

Q.9. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is :

A An equilateral triangle having each of its side of length $\sqrt{3}$r.

B An equilateral triangle of height $\frac{2r}{3}$.

C A right angle triangle having two of its sides of length 2r and r.

D An isosceles triangle with base equal to 2r.

26th Feb Evening Shift 2021

Q.10. A man is observing, from the top of a tower, a boat speeding towards the lower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is :

A $10(\sqrt{3}+1)$

B $10(\sqrt{3}-1)$

C 10

D $10\sqrt{3}$

25th Feb Morning Slot 2021

Q.11. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is :

(A) $3600\sqrt{3}$ m

(B) $1200\sqrt{3}$ m

(C) $1800\sqrt{3}$ m

(D) $2400\sqrt{3}$ m

24th Feb Evening Slot 2021

Q.12. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is :

(A) 30

(B) 25

(C) $20\sqrt{3}$

(D) $25\sqrt{3}$

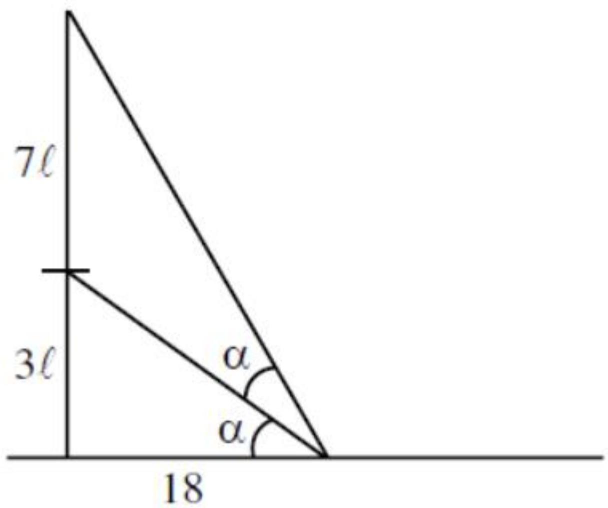
MCQ Answer Key

1. Ans. (B)
2. Ans. (C)
3. Ans. (B)
4. Ans. (A)
5. Ans. (B)
6. Ans. (B)
7. Ans. (C)
8. Ans. (B)
9. Ans. (A)

10. Ans. (A)
11. Ans. (B)
12. Ans. (D)

MCQ Explanation

Ans 1.



Let height of pole = $10l$

$\tan\alpha = \frac{3l}{18} = \frac{l}{6}$

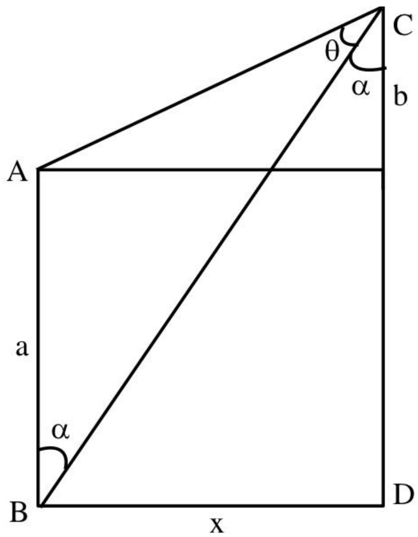
$\tan 2\alpha = \frac{10l}{18}$

$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{10l}{18}$

use $\tan\alpha = \frac{l}{6} \Rightarrow l = \sqrt{\frac{72}{5}}$

height of pole = $10l = 12\sqrt{10}$

Ans 2.



$$\tan\theta = \frac{1}{2}$$

$$\tan(\theta+\alpha) = \frac{x}{b}, \tan\alpha = \frac{x}{a+b}$$

$$\Rightarrow \frac{\frac{1}{2}+\frac{x}{a+b}}{1-\frac{1}{2}\times\frac{x}{a+b}} = \frac{x}{b}$$

$$\Rightarrow x^2 - 2ax + ab + b^2 = 0$$

Ans 3.

$$\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$$

As A, B, C are angles of triangle.

A + B + C = π

A = π − (B + C) ...... (1)

Similarly sinB = sin(A + C) ..... (2)

From (1) and (2)

$\frac{\sin(B+C)}{\sin(A+C)} = \frac{\sin(A-C)}{\sin(C-B)}$

$\sin(C+B).\sin(C-B) = \sin(A-C)\sin(A+C)$

$\sin^2 C - \sin^2 B = \sin^2 A - \sin^2 C$

$\because \{\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y\}$

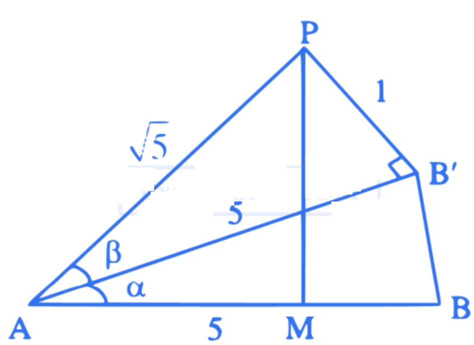
$2\sin^2 C = \sin^2 A + \sin^2 B$

By sine rule

$2c^2 = a^2 + b^2$

$\Rightarrow b^2, c^2$ and a^2 are in A.P.

Ans 4.



From figure,

$$\sin\beta = \frac{1}{\sqrt{5}}$$

$$\therefore \tan\beta = \frac{1}{2}$$

$$\tan(\alpha+\beta) = 2$$

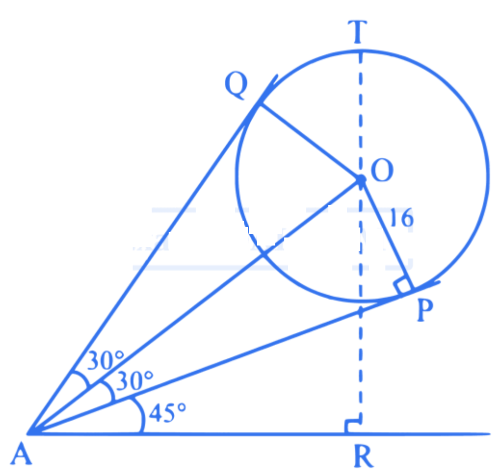
$$\frac{\tan\alpha + \tan\beta)}{1 - \tan\alpha.\tan\beta} = 2$$

$$\frac{\tan\alpha + \frac{1}{2}}{1 - \tan\alpha\left(\frac{1}{2}\right)} = 2$$

$$\tan\alpha = \frac{3}{4}$$

$$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$$

Ans 5.



O $\rightarrow$ centre of sphere

P, Q $\rightarrow$ point of contact of tangents from A

Let T be top most point of balloon & R be foot of perpendicular from O to ground.

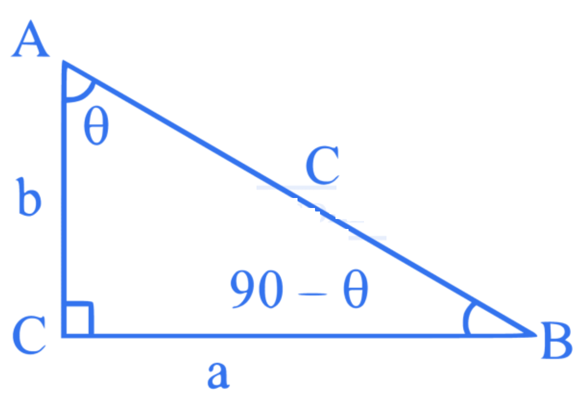
From triangle OAP, OA = 16cosec30° = 32

From triangle ABO, OR = OA sin75° = $32\frac{(\sqrt{3}+1)}{2\sqrt{2}}$

So level of top most point = OR + OT

$= 8\left(\sqrt{6}+\sqrt{2}+2\right)$

Ans 6.



Let a ΔABC having C = 90° and A = θ

$\frac{\sin\theta}{a}=\frac{\cos\theta}{b}=\frac{1}{c}$ ..... (i)

Also for triangle of reciprocals

$$\cos A=\frac{\left(\frac{1}{c}\right)^2+\left(\frac{1}{b}\right)^2-\left(\frac{1}{a}\right)^2}{2\left(\frac{1}{c}\right)\left(\frac{1}{b}\right)}$$

$$\frac{1}{c^2}+\frac{1}{(c\cos\theta)^2}=\frac{1}{(c\sin\theta)^2}$$

$$\Rightarrow 1+\sec^2\theta=\cos ec^2\theta$$

$$\Rightarrow \frac{1}{4}=\frac{\cos^2\theta}{4\sin^2\theta\cos^2\theta}$$

$$\Rightarrow \frac{1}{4}=\frac{\cos^2\theta}{\sin^2 2\theta}$$

$$\Rightarrow 1-\cos^2 2\theta=4\cos 2\theta$$

$$\cos^2 2\theta+4\cos 2\theta-1=0$$

$$\cos 2\theta=\frac{-4\pm\sqrt{16+4}}{2}$$

$$\cos 2\theta=-2\pm\sqrt{5}$$

$$\cos 2\theta = \sqrt{5} - 2 = 1 - 2\sin^2\theta$$

$$\Rightarrow 2\sin^2\theta = 3 - \sqrt{5}$$

$$\Rightarrow \sin^2\theta = \frac{3-\sqrt{5}}{2}$$

$$\Rightarrow \sin\theta = \frac{\sqrt{5}-1}{2}$$

Ans 7.

Image

As, $\cos B = \frac{3}{5} \Rightarrow B = 53^\circ$

As, $R = 5 \Rightarrow \frac{c}{\sin c} = 2R$

$$\Rightarrow \frac{5}{10} = \sin c \Rightarrow C = 30^\circ$$

Now, $\frac{b}{\sin B} = 2R \Rightarrow b = 2(5)\left(\frac{4}{5}\right) = 8$

Now, by cosine formula

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\Rightarrow \frac{3}{5} = \frac{a^2+25-64}{2(5)a}$$

$$\Rightarrow a^2 - 6a - 3g = 0$$

$\therefore a = \frac{6 \pm \sqrt{192}}{2} = \frac{6 \pm 8\sqrt{3}}{2}$

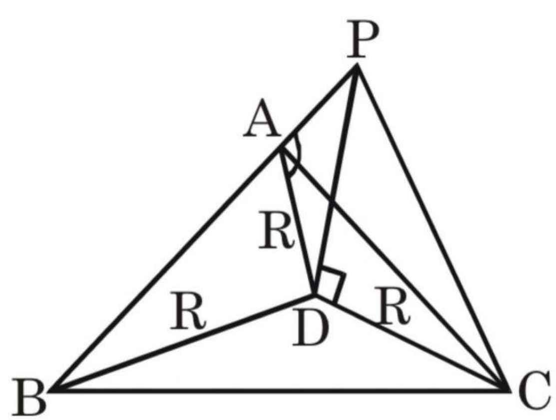
$\Rightarrow 3 + 4\sqrt{3}$ (Reject $a = 3 - 4\sqrt{3}$)

Now, $\Delta = \frac{abc}{4R} = \frac{(3+4\sqrt{3})(8)(5)}{4(5)} = 2(3 + 4\sqrt{3})$

$\Rightarrow \Delta = (6 + 8\sqrt{3})$

$\Rightarrow$ Option (3) is correct.

Ans 8.



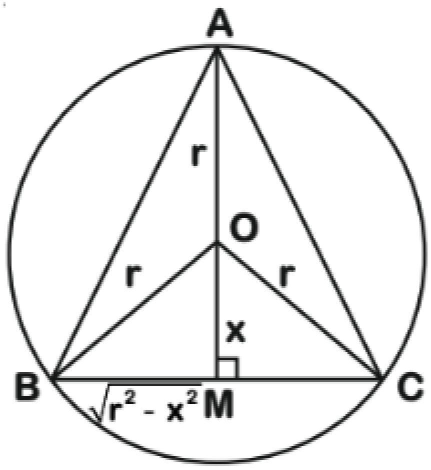
Let PD = h, R = 2

As angle of elevation of top of pole from A, B, C are equal. So D must be circumcentre of ΔABC

$\tan\left(\frac{\pi}{3}\right) = \frac{PD}{R} = \frac{h}{R}$

$h = R\tan\left(\frac{\pi}{3}\right) = 2\sqrt{3}$

Ans 9.



$A = \frac{1}{2} \times BC \times AM$

$= \frac{1}{2} \times 2\sqrt{r^2 - x^2} \times (r + x)$

$A = (r + x)\sqrt{r^2 - x^2}$

$\frac{dA}{dx} = \sqrt{r^2 - x^2} - \frac{x}{\sqrt{r^2 - x^2}} \times (r + x)$

$= \frac{r^2 - x^2 - rx - x^2}{\sqrt{r^2 - x^2}} = \frac{r^2 - rx - 2x^2}{\sqrt{r^2 - x^2}} = \frac{-(x+r)(2x-r)}{\sqrt{r^2 - x^2}}$

$\frac{dA}{dx} = 0 \Rightarrow x = \frac{r}{2}$

Sign change of $\frac{dA}{dx}$ at $x = \frac{r}{2}$

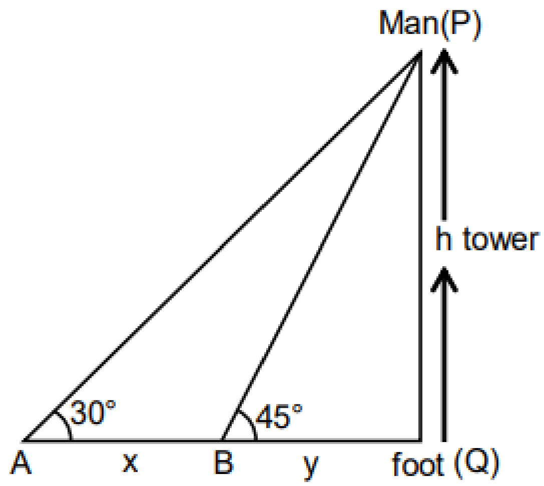
$\Rightarrow$ A has maximum at $x = \frac{r}{2}$

$BC = 2\sqrt{r^2 - x^2} = \sqrt{3}r,$

$AM = r + \frac{1}{2}r = \frac{3}{2}r$

$\Rightarrow AB = AC = \sqrt{3}r$

Ans 10.



$\frac{h}{x+y} = \tan 30^\circ$

$x + y = \sqrt{3}h$ ...... (1)

Also,

$\frac{h}{y} = \tan 45^\circ$

$h = y$ ..... (2)

put in (1)

$x + y = \sqrt{3}y$

$x=\left(\sqrt{3}-1\right)y$

$\frac{x}{20}='v'$ speed

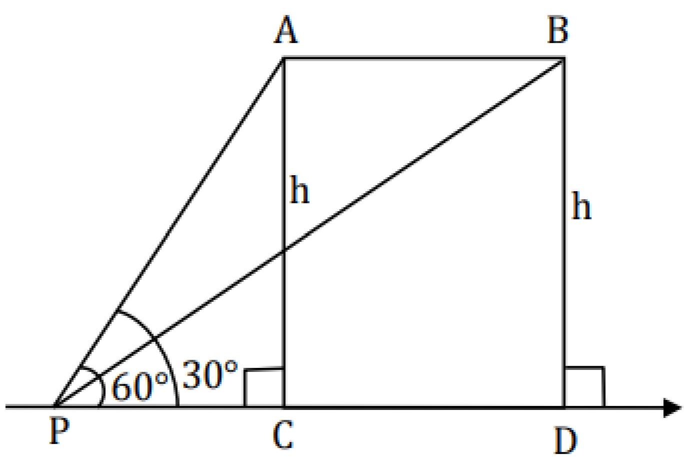
$\therefore$ time taken to reach Foot from B

$=\frac{y}{V}$

$=\frac{x}{\left(\sqrt{3}-1\right).x}\times 20$

$=10\left(\sqrt{3}+1\right)$

Ans 11.



$v=432\times\frac{1000}{60\times 60}$ m/sec = 120 m/sec

Distance AB = v $\times$ 20 = 2400 meter

In ΔPAC

$\tan 60^{\circ} = \frac{h}{PC} \Rightarrow PC = \frac{h}{\sqrt{3}}$

In ΔPBD

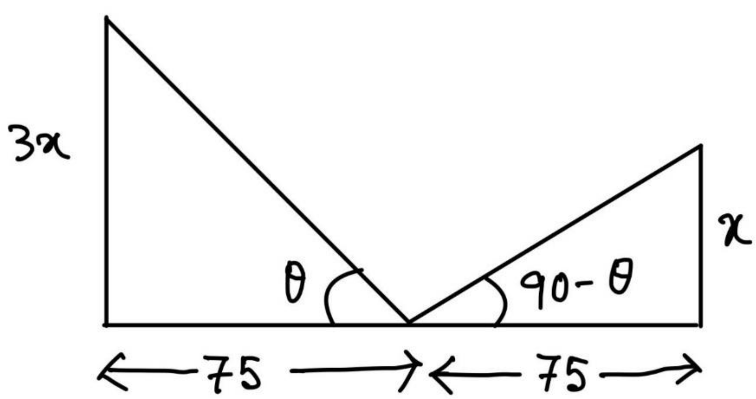
$\tan 30^{\circ} = \frac{h}{PD} \Rightarrow PD = \sqrt{3}h$

PD = PC + CD

$\sqrt{3}h = \frac{h}{\sqrt{3}} + 2400 \Rightarrow \frac{2h}{\sqrt{3}} = 2400$

$h = 1200\sqrt{3}$ meter

Ans 12.



$\therefore \tan(90-\theta) = \frac{x}{75}$

and $\tan\theta = \frac{3x}{75}$

As $\cot\theta . \tan\theta = 1$

$$\therefore \frac{x}{75} \cdot \frac{3x}{75} = 1$$

$$\Rightarrow x = 25\sqrt{3}$$

TOPIC 1 Properties of Triangle, Solutions of Triangles, Inscribed & Enscribed Circles, Regular Polygons

1. Let A(3, 0, –1), B(2, 10, 6) and C (1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then cos (∠GOA) (O being the origin) is equal to : **[April 10, 2019 (I)]**

(a) $\frac{1}{2\sqrt{15}}$ (b) $\frac{1}{\sqrt{15}}$

(c) $\frac{1}{6\sqrt{10}}$ (d) $\frac{1}{\sqrt{30}}$

2. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If $c = 4$ cm, then the area (in sq.cm) of this triangle is : **[April 10, 2019 (II)]**

(a) $\frac{2}{\sqrt{3}}$ (b) $4\sqrt{3}$

(c) $2\sqrt{3}$ (d) $\frac{4}{\sqrt{3}}$

3. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is : **[April 08, 2019 (II)]**

(a) 5 : 9 : 13 (b) 4 : 5 : 6

(c) 3 : 4 : 5 (d) 5 : 6 : 7

4. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. if $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is : **[Jan. 11, 2019 (I)]**

(a) $\frac{3}{2}y$ (b) $\frac{c}{\sqrt{3}}$

(c) $\frac{c}{3}$ (d) $\frac{y}{\sqrt{3}}$

5. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value : **[Jan. 11, 2019 (II)]**

(a) (7, 19, 25) (b) (3, 4, 5)

(c) (5, 12, 13) (d) (19, 7, 25)

6. With the usual notation, in ΔABC, if ∠A + ∠B = 120°, $a = \sqrt{3}+1$ and $b = \sqrt{3}-1$, then the ratio ∠A : ∠B, is: **[Jan. 10, 2019 (II)]**

(a) 7 : 1 (b) 5 : 3

(c) 9 : 7 (d) 3 : 1

7. In a ΔABC, $\frac{a}{b} = 2+\sqrt{3}$ and ∠C = 60°. Then the ordered pair (∠A, ∠B) is equal to : **[Online April 10, 2015]**

(a) (45°, 75°) (b) (105°, 15°)

(c) (15°, 105°) (d) (75°, 45°)

8. ABCD is a trapezium such that AB and CD are parallel and BC ⊥ CD. If ∠ADB = θ, BC = p and CD = q, then AB is equal to : **[2013]**

(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (b) $\frac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$

(c) $\frac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$ (d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

9. If in a triangle ABC, $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$, then cosA is equal to **[2012]**

(a) 5/7 (b) 1/5

(c) 35/19 (d) 19/35

10. In a ΔPQR, If 3 sin P + 4 cos Q = 6 and 4 sin Q + 3 cos P = 1, then the angle R is equal to : **[2012]**

(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

11. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A ***false*** statement among the following is **[2010]**

(a) There is a regular polygon with $\frac{r}{R}=\frac{1}{\sqrt{2}}$

(b) There is a regular polygon with $\frac{r}{R}=\frac{2}{3}$

(c) There is a regular polygon with $\frac{r}{R}=\frac{\sqrt{3}}{2}$

(d) There is a regular polygon with $\frac{r}{R}=\frac{1}{2}$

12. If in a ΔABC, the altitudes from the vertices A, B, C on opposite sides are in H.P, then $\sin A$, $\sin B$, $\sin C$ are in **[2005]**

(a) G.P. (b) A. P.
(c) A.P-G.P. (d) H. P

13. In a triangle ABC, let $\angle C=\frac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle ABC, then $2(r+R)$ equals **[2005]**

(a) $b+c$ (b) $a+b$
(c) $a+b+c$ (d) $c+a$

14. The sides of a triangle are $\sin\alpha, \cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0<\alpha<\frac{\pi}{2}$. Then the greatest angle of the triangle is **[2004]**

(a) 150° (b) 90°
(c) 120° (d) 60°

15. If in a ΔABC $a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$, then the sides a, b and c **[2003]**

(a) satisfy $a+b=c$ (b) are in A.P
(c) are in G.P (d) are in H.P

16. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle DAB=\frac{\pi}{6}$ and $\angle ABE=\frac{\pi}{3}$, then the area of the ΔABC is **[2003]**

(a) $\frac{64}{3}$ (b) $\frac{8}{3}$
(c) $\frac{16}{3}$ (d) $\frac{32}{3\sqrt{3}}$

17. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is **[2003]**

(a) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (b) $a\cot\left(\frac{\pi}{n}\right)$

(c) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (d) $a\cot\left(\frac{\pi}{2n}\right)$.

18. In a triangle with sides $a, b, c, r_1>r_2>r_3$ (which are the ex-radii) then **[2002]**

(a) $a>b>c$ (b) $a<b<c$
(c) $a>b$ and $b<c$ (d) $a<b$ and $b>c$

19. The sides of a triangle are $3x+4y$, $4x+3y$ and $5x+5y$ where $x, y>0$ then the triangle is **[2002]**

(a) right angled (b) obtuse angled
(c) equilateral (d) none of these

TOPIC 2 Heights & Distances

20. A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30° on the line $x=1$ at the point A. The ray gets reflected on the line $x=1$ and meets x-axis at the point B. Then, the line AB passes through the point: **[Sep. 06, 2020 (I)]**

(a) $\left(3,-\frac{1}{\sqrt{3}}\right)$ (b) $\left(4,-\frac{\sqrt{3}}{2}\right)$

(c) $(3,-\sqrt{3})$ (d) $(4,-\sqrt{3})$

21. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is ______. **[Sep. 06, 2020 (I)]**

22. The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climbing up on km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then the height (in km) of the summit from the ground is: **[Sep. 06, 2020 (II)]**

(a) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(c) $\frac{1}{\sqrt{3}-1}$ (d) $\frac{1}{\sqrt{3}+1}$

23. Two vertical poles $AB = 15$ m and $CD = 10$ m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is : **[Sep. 04, 2020 (I)]**

(a) 20/3 (b) 5
(c) 10/3 (d) 6

24. The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30°. If the angle of depression of the image of C in the lake from the point P is 60°, then PC (in m) is equal to : **[Sep. 04, 2020 (II)]**

(a) 100 (b) $200\sqrt{3}$
(c) 400 (d) $400\sqrt{3}$

25. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\text{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is : **[April 10, 2019 (I)]**

(a) $\frac{100}{3\sqrt{3}}$ (b) $10\sqrt{5}$
(c) 20 (d) 25

26. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in meters) from the surface of the lake is: **[Jan. 12, 2019 (II)]**

(a) 60 (b) 50
(c) 45 (d) 42

27. Consider a triangular plot ABC with sides AB = 7 m, BC = 5 m and CA = 6 m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is: **[Jan. 10, 2019 (I)]**

(a) $\frac{3}{2}\sqrt{21}$ (b) $\frac{2}{3}\sqrt{21}$
(c) $2\sqrt{21}$ (d) $7\sqrt{3}$

28. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is : **[2018]**

(a) 50 (b) $100\sqrt{3}$
(c) $50\sqrt{2}$ (d) 100

29. A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizontal road. If it takes 18 min, for the angle of depression of the car to change from 30° to 45°, then after this, the time taken (in min) by the car to reach the foot of the tower, is. **[Online April 16, 2018]**

(a) $9(1+\sqrt{3})$ (b) $\frac{9}{2}(\sqrt{3}-1)$
(c) $18(1+\sqrt{3})$ (d) $18(\sqrt{3}-1)$

30. An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}km$ above it, is observed at an elevation of 60° from a point on the ground. If, after five seconds, its elevation from the same point, is 30°, then the speed (in km/hr) of the aeroplane is **[Online April 15, 2018]**

(a) 1500 (b) 750
(c) 720 (d) 1440

31. A tower T_1 of height 60 m is located exactly opposite to a tower T_2 of height 80 m on a straight road. From the top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road between the feet of the towers T_1 and T_2 is **[Online April 15, 2018]**

(a) $20\sqrt{2}$ (b) $10\sqrt{2}$
(c) $10\sqrt{3}$ (d) $20\sqrt{3}$

32. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to : **[2017]**

(a) $\frac{4}{9}$ (b) $\frac{6}{7}$
(c) $\frac{1}{4}$ (d) $\frac{2}{9}$

33. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is: **[2016]**

(a) 20 (b) 5
(c) 6 (d) 10

34. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45°. The angle of elevation of the top of the same tower from a point B, due south of A is 30°. If the distance between A and B is $54\sqrt{2}$ m, then the height of the tower (in metres), is : **[Online April 10, 2016]**

(a) 108 (b) $36\sqrt{3}$
(c) $54\sqrt{3}$ (d) 54

(which are the ex-radii) then **[2002]**

35. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB : BC, is : **[2015]**

(a) $1:\sqrt{3}$ (b) $2:3$
(c) $\sqrt{3}:1$ (d) $\sqrt{3}:\sqrt{2}$

36. Let 10 vertical poles standing at equal distances on a straight line, subtend the same angle of elevation at a point O on this line and all the poles are on the same side of O. If the height of the longest pole is 'h' and the distance of the foot of the smallest pole from O is 'a'; then the distance between two consecutive poles, is : **[Online April 11, 2015]**

(a) $\dfrac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$ (b) $\dfrac{h\sin\alpha + a\cos\alpha}{9\sin\alpha}$

(c) $\dfrac{h\cos\alpha - a\sin\alpha}{9\cos\alpha}$ (d) $\dfrac{h\sin\alpha - a\cos\alpha}{9\cos\alpha}$

37. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is $45°$. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to $30°$. Then the speed (in m/s) of the bird is **[2014]**

(a) $20\sqrt{2}$ (b) $20\left(\sqrt{3}-1\right)$

(c) $40\left(\sqrt{2}-1\right)$ (d) $40\left(\sqrt{3}-\sqrt{2}\right)$

38. The angle of elevation of the top of a vertical tower from a point P on the horizontal ground was observed to be α. After moving a distance 2 metres from P towards the foot of the tower, the angle of elevation changes to β. Then the height (in metres) of the tower is: **[Online April 11, 2014]**

(a) $\dfrac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$ (b) $\dfrac{\sin\alpha\sin\beta}{\cos(\beta-\alpha)}$

(c) $\dfrac{2\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$ (d) $\dfrac{\cos(\beta-\alpha)}{\sin\alpha\sin\beta}$

39. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD =7 m. From D the angle of elevation of the point A is 45°. Then the height of the pole is **[2008]**

(a) $\dfrac{7\sqrt{3}}{2}\dfrac{1}{\sqrt{3}-1}m$ (b) $\dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)m$

(c) $\dfrac{7\sqrt{3}}{2}(\sqrt{3}-1)m$ (d) $\dfrac{7\sqrt{3}}{2}\dfrac{1}{\sqrt{3}+1}m$

40. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A and B is 30°. The height of the tower is **[2007]**

(a) $a/\sqrt{3}$ (b) $a\sqrt{3}$

(c) $2a/\sqrt{3}$ (d) $2a\sqrt{3}$

41. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is **[2004]**

(a) 60m (b) 30 m

(c) 40m (d) 20 m

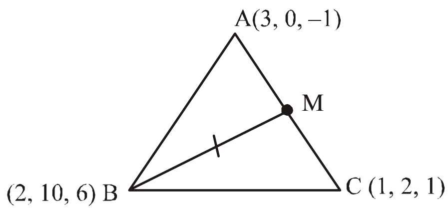
42. The upper $\dfrac{3}{4}$ th portion of a vertical pole subtends an angle $\tan^{-1}\dfrac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is **[2003]**

(a) 80 m (b) 20 m

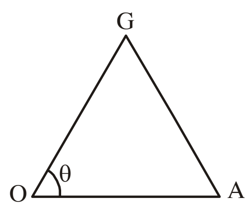
(c) 40 m (d) 60 m.

Hints & Solutions

1. **(b)** G is the centroid of ΔABC.



$$\Rightarrow G \equiv \left(\frac{3+2+1}{3}, \frac{0+10+2}{3}, \frac{-1+6+1}{3}\right) \equiv (2, 4, 2)$$



$OG = \sqrt{4+16+4}$, $OA = \sqrt{9+1}$, $AG = \sqrt{1+16+9}$

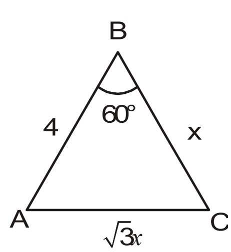
$$\therefore \cos\theta = \frac{(OG)^2+(OA)^2-(AG)^2}{2(OG)(OA)} = \frac{24+10-26}{2\sqrt{24}\sqrt{10}}$$

$$= \frac{8}{2\sqrt{8\times3\times2\times5}} = \frac{4}{4\sqrt{15}} = \frac{1}{\sqrt{15}}$$

2. **(c)** Given that A, B, C, are in A.P. $\Rightarrow 2B = A + C$

Now, $A + B + C = \pi \Rightarrow B = \frac{\pi}{3}$

Area $= \frac{1}{2}(4x)\sin 60° = \sqrt{3}x$



Now $\cos 60° = \frac{16+x^2-3x^2}{8x}$

$\Rightarrow 4x = 16 - 2x^2 \Rightarrow x^2 + 2x - 8 = 0$

$\Rightarrow x = 2$ [$\because$ x can't be negative]

Hence, area $= 2\sqrt{3}$ sq. cm

3. **(b)** Let the sides of triangle are $a > b > c$ where

Given $A = 2C$

$\because$ $A + B + C = \pi$ and $A = 2C$

$\Rightarrow B = \pi - 3C$ ...(i)

$\because$ a, b, c are in A.P. $\Rightarrow a + c = 2b$

$\Rightarrow \sin A + \sin C = 2 \sin B$ ...(ii)

$\Rightarrow \sin A = \sin(2C)$ and $\sin B = \sin 3C$

From (ii),

$\sin 2C + \sin C = 2 \sin 3C$

$\Rightarrow (2\cos C + 1)\sin C = 2 \sin C\,(3 - 4\sin^2 C)$

$\Rightarrow 2\cos C + 1 = 6 - 8(1 - \cos^2 C)$

$\Rightarrow 8\cos^2 C - 2\cos C - 3 = 0$

$\Rightarrow \cos C = \frac{3}{4}$ or $\cos C = -\frac{1}{2}$

$\because$ C is acute angle

$\Rightarrow \cos C = \frac{3}{4} \Rightarrow \sin C = \frac{\sqrt{7}}{4}$

and $\sin A = 2 \sin C \cos C = 2 \times \frac{\sqrt{7}}{4} \times \frac{3}{4} = \frac{3\sqrt{7}}{8}$

$\sin B = \frac{3\sqrt{7}}{4} - \frac{4\sqrt{7}}{4} \times \frac{7}{16} = \frac{5\sqrt{7}}{16}$

$\Rightarrow \sin A : \sin B : \sin C :: a : b : c$ is $6 : 5 : 4$

4. **(b)** Let two sides of triangle are a and b.

$a + b = x$

$ab = y$

$x^2 - c^2 = y \Rightarrow (a + b)^2 - c^2 = ab$

$\Rightarrow (a + b - c)(a + b + c) = ab$

$\Rightarrow 2(s - c)(2s) = ab$

$\Rightarrow 4s(s - c) = ab$

$\Rightarrow \frac{s(s-c)}{ab} = \frac{1}{4}$

$\Rightarrow \cos^2\frac{c}{2} = \frac{1}{4}$

$\Rightarrow \cos c = -\frac{1}{2} \Rightarrow c = 120°$

$\therefore$ Area of triangle is,

$\Delta = \frac{1}{2}ab(\sin 120°) = \frac{\sqrt{3}}{4}ab$

$\because \quad R=\dfrac{abc}{4\Delta}$

$\because \quad R=\dfrac{abc}{\sqrt{3}\,ab}=\dfrac{c}{\sqrt{3}}$

5. **(a)** Let $\dfrac{b+c}{11}=\dfrac{c+a}{12}=\dfrac{a+b}{13}=k$ (Say).

$\therefore \quad b+c=11k,\ c+a=12k,\ a+b=13k$

$\therefore \quad a+b+c=18k$

$\therefore \quad a=7k,\ b=6k$ and $c=5k$

$\therefore \quad \cos A=\dfrac{36k^2+25k^2-49k^2}{2.30k^2}=\dfrac{1}{5}$

and $\cos B=\dfrac{49k^2+25k^2-36k^2}{2.35k^2}=\dfrac{19}{35}$

and $\cos C=\dfrac{49k^2+36k^2-25k^2}{2.42k^2}=\dfrac{5}{7}$

$\therefore \quad \cos A:\cos B:\cos C=7:19:25$

$\therefore \quad \dfrac{\cos A}{7}=\dfrac{\cos B}{19}=\dfrac{\cos C}{25}$

Hence, required ordered triplet is (7, 19, 25).

6. **(a)** $\angle A+\angle B=120°$ ...(1)

$\Rightarrow \quad \angle C=180°-120°=60°$

$\tan\left(\dfrac{A-B}{2}\right)=\dfrac{a-b}{a+b}\cot\left(\dfrac{C}{2}\right)$

$=\dfrac{2}{2\sqrt{3}}(\cot 30°)=\dfrac{1}{\sqrt{3}}\times\sqrt{3}=1$

$\Rightarrow \quad \dfrac{\angle A-\angle B}{2}=\dfrac{\pi}{4}$ ($\angle$ is angle)

$\Rightarrow \quad \angle A-\angle B=90°$ ...(2)

From eqn (1) and (2)

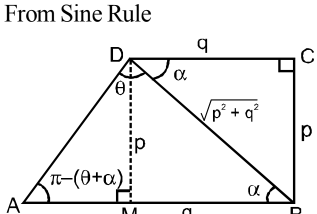
$\angle A=105°,\ \angle B=15°$

Then, $\angle A:\angle B=7:1$

7. **(b)** $\dfrac{\sin A}{\sin B}=2+\sqrt{3}$

$\dfrac{\sin(105°)}{\sin(15°)}=2+\sqrt{3} \quad \dfrac{\cos 15°}{\sin 15°}=2+\sqrt{3}$

8. **(a)** From Sine Rule



$\dfrac{AB}{\sin\theta}=\dfrac{\sqrt{p^2+q^2}}{\sin(\pi-(\theta+\alpha))}$

$AB=\dfrac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cos\alpha+\cos\theta\sin\alpha}$

$=\dfrac{(p^2+q^2)\sin\theta}{q\sin\theta+p\cos\theta}$

$\left(\because \cos\alpha=\dfrac{q}{\sqrt{p^2+q^2}} \text{ and } \sin\alpha=\dfrac{p}{\sqrt{p^2+q^2}}\right)$

9. **(b)** In a triangle ABC.

Let $\dfrac{b+c}{11}=\dfrac{c+a}{12}=\dfrac{a+b}{13}=K$

$\Rightarrow b+c=11K,\ c+a=12K,\ a+b=13K$

On solving these equations, we get

$a=7K,\ b=6K,\ c=5K$

Now we know,

$\cos A=\dfrac{b^2+c^2-a^2}{2bc}=\dfrac{36K^2+25K^2-49K^2}{2(6K)(5K)}=\dfrac{1}{5}$

10. **(b)** Given that $3\sin P+4\cos Q=6$ ...(i)

$4\sin Q+3\cos P=1$ ...(ii)

Squaring and adding (i) & (ii) we get

$9\sin^2 P+16\cos^2 Q+24\sin P\cos Q$
$+16\sin^2 Q+9\cos^2 P+24\sin Q\cos P$
$=36+1=37$

$\Rightarrow \quad 9(\sin^2 P+\cos^2 P)+16(\sin^2 Q+\cos^2 Q)$
$+24(\sin P\cos Q+\cos P\sin Q)=37$

$\Rightarrow \quad 9+16+24\sin(P+Q)=37$

$[\because \sin^2\theta+\cos^2\theta=1$ and $\sin A\cos B+\cos A\sin B=\sin(A+B)]$

$\Rightarrow \quad \sin(P+Q)=\dfrac{1}{2}$

$\Rightarrow \quad P+Q=\dfrac{\pi}{6}$ *or* $\dfrac{5\pi}{6}$

$\Rightarrow \quad R=\dfrac{5\pi}{6}$ or $\dfrac{\pi}{6}$ $\quad (\because P+Q+R=\pi)$

If $\quad R=\dfrac{5\pi}{6}$ then $0<Q,\ P<\dfrac{\pi}{6}$

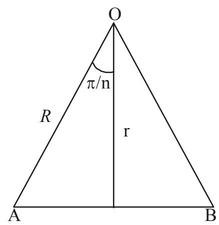
$\Rightarrow \quad \cos Q<1$ and $\sin P<\dfrac{1}{2}$

$\Rightarrow \quad 3\sin P+4\cos Q<\dfrac{11}{2}$

But given that $3\sin P+4\sin Q=6$

So, $R=\dfrac{\pi}{6}$

11. (b) Let O is centre of polygon of n sides and AB is one of the side, then by figure



$$\cos\frac{\pi}{n} = \frac{r}{R}$$

$$\Rightarrow \quad \frac{r}{R} = \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}$$

for $n = 3, 4, 6$ respectively.

12. (b) Let altitudes from A, B and C be p_1, p_2 and p_3 resp.

$$\therefore \quad \Delta = \frac{1}{2}p_1 a = \frac{1}{2}p_2 b = \frac{1}{2}p_3 b$$

Given that, p_1, p_2, p_3, are in H.P.

$\Rightarrow \quad \frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c}$ are in H.P.

$\Rightarrow \quad \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in H.P.

$\Rightarrow \quad a, b, c$ are in A.P.

By sine formula

$\Rightarrow \quad K\sin A, K\sin B, K\sin C$ are in AP

$\Rightarrow \quad \sin A, \sin B, \sin C$ are in A.P.

13. (b) We know that for the circle circumscribing a right triangle, hypotenutse is the diameter

$\because \quad \angle C = 90°$

$$\therefore \quad 2R = c \Rightarrow R = \frac{c}{2}$$

$$\text{also } r = \frac{\Delta}{s} = \frac{\frac{1}{2} \times a \times b}{\frac{a+b+c}{2}}$$

$$\Rightarrow \quad r = \frac{ab}{a+b+c}$$

$$\therefore \quad 2r + 2R = \frac{2ab}{a+b+c} + c = \frac{2ab + ac + bc + c^2}{a+b+c}$$

$$= \frac{2ab + ac + bc + a^2 + b^2}{a+b+c} \qquad (\because c^2 = a^2 + b^2)$$

$$= \frac{(a+b)^2 + (a+b)c}{a+b+c} = (a+b)$$

14. (c) Let $a = \sin\alpha$, $b = \cos\alpha$ and

$c = \sqrt{1 + \sin\alpha\cos\alpha}$

Clearly a and $b < 1$ but $c > 1$ as $\sin\alpha > 0$ and $\cos\alpha > 0$

$\therefore$ c is the greatest side and greatest angle is C.

We know that, $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$$= \frac{\sin^2\alpha + \cos^2\alpha - 1 - \sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha} = -\frac{1}{2}$$

$\therefore C = 120°$

15. (b) Given that, $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$

$a[\cos C + 1] + c[\cos A + 1] = 3b$

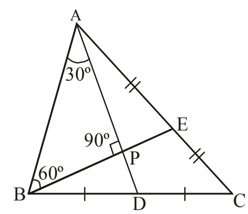
$(a + c) + (a\cos C + c\cos B) = 3b$

We know that, $b = a\cos C + c\cos B$

$a + c + b = 3b$ or $a + c = 2b$

or a, b, c are in A.P.

16. (d)



We know that median divides each other in ratio 2 :1

$AP = \frac{2}{3}AD = \frac{8}{3}$; $PD = \frac{4}{3}$; Let $PB = x$

$\tan 60^o = \frac{8/3}{x}$ or $x = \frac{8}{3\sqrt{3}}$

Area of $\Delta ABD = \frac{1}{2} \times 4 \times \frac{8}{3\sqrt{3}} = \frac{16}{3\sqrt{3}}$

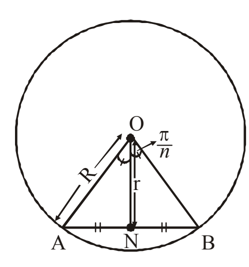
$\therefore$ Area of $\Delta ABC = 2 \times \frac{16}{3\sqrt{3}} = \frac{32}{3\sqrt{3}}$

[$\because$ Median of a Δ divides it into two Δ's of equal area.]

17. (c) We know that, $\tan\left(\frac{\pi}{n}\right) = \frac{a}{2r}$; $\sin\left(\frac{\pi}{n}\right) = \frac{a}{2R}$

$$\Rightarrow \quad r = \frac{a}{2}\cot\frac{\pi}{n}; \ R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n}$$

$$r + R = \frac{a}{2}\left[\cot\frac{\pi}{n} + \operatorname{cosec}\frac{\pi}{n}\right]$$



$$= \frac{a}{2}\left[\frac{\cos\frac{\pi}{n} + 1}{\sin\frac{\pi}{n}}\right] = \frac{a}{2}\left[\frac{2\cos^2\frac{\pi}{2n}}{2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}}\right] = \frac{a}{2}\cot\frac{\pi}{2\pi}$$

18. **(a)** We know that, $r_1 = \dfrac{\Delta}{s-a}, r_2 = \dfrac{\Delta}{s-b}$ and $r_3 = \dfrac{\Delta}{s-c}$

Given that,

$r_1 > r_2 > r_3 \Rightarrow \dfrac{\Delta}{s-a} > \dfrac{\Delta}{s-b} > \dfrac{\Delta}{s-c}$;

$\Rightarrow \quad s-a < s-b < s-c$

$\Rightarrow \quad -a < -b < -c \Rightarrow a > b > c$

19. **(b)** Let $a = 3x+4y, b = 4x+3y$ and $c = 5x+5y$

as $x, y > 0$, $c = 5x+5y$ is the largest side

$\therefore$ C is the largest angle . Now

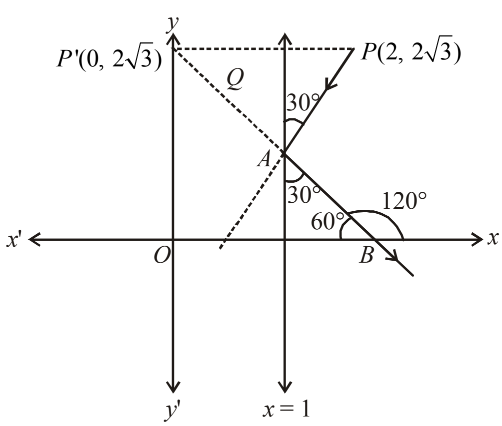
$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$\cos C = \frac{(3x+4y)^2+(4x+3y)^3-(5x+5y)^2}{2(3x+4y)(4x+3y)}$$

$$= \frac{-2xy}{2(3x+4y)(4x+3y)} < 0$$

$\therefore$ C is obtuse angle $\Rightarrow \Delta$ABC is obtuse angled

20. **(c)**



Slope of $AB = \tan 120° = -\sqrt{3}$

$\therefore$ Equation of line AB (i.e. BP') :

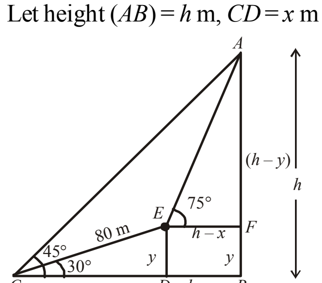
$y - 2\sqrt{3} = -\sqrt{3}(x-0)$

$\Rightarrow \sqrt{3}x + y = 2\sqrt{3}$

$\therefore$ Point $(3, -\sqrt{3})$ lies on line AB.

21. **(80)**

Let height $(AB) = h$ m, $CD = x$ m and $ED = y$ m



In rt. ΔCDE,

$$\sin 30° = \frac{y}{80} \Rightarrow y = 40$$

$$\cos 30° = \frac{x}{80} \Rightarrow x = 40\sqrt{3}$$

Now, in ΔAEF,

$$\tan 75° = \frac{h-y}{h-x}$$

$$\Rightarrow (2+\sqrt{3}) = \frac{h-40}{h-40\sqrt{3}}$$

$$\Rightarrow (2+\sqrt{3})(h-40\sqrt{3}) = h-40$$

$$\Rightarrow 2h - 80\sqrt{3} + \sqrt{3}h - 120 = h - 40$$

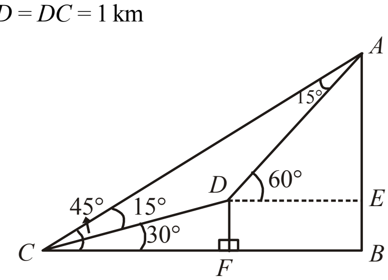
$$\Rightarrow h + \sqrt{3}h = 80 + 80\sqrt{3}$$

$$\Rightarrow (\sqrt{3}+1)h = 80(\sqrt{3}+1)$$

$\therefore h = 80$ m

22. **(c)** $\because \angle DCA = \angle DAC = 30°$

$\therefore AD = DC = 1$ km



In ΔDEA,

$$\frac{AE}{AD} = \sin 60° \Rightarrow AE = \frac{\sqrt{3}}{2} \text{ km}$$

In ΔCDF, $\sin 30° = \dfrac{DF}{CD} \Rightarrow DF = \dfrac{1}{2}$ km

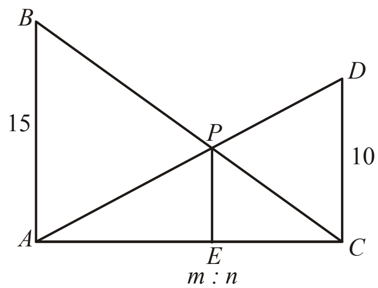
$\therefore EB = DF = \dfrac{1}{2}$ km

$\therefore$ Height of mountain $= AE + EB$

$$= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \left(\frac{\sqrt{3}+1}{2}\right) \text{ km}$$

$$= \frac{1}{\sqrt{3}-1} \text{ km}$$

23. (d)



Let $PE \perp AC$ and $\frac{AE}{EC} = \frac{m}{n}$

$\because \Delta AEP \sim \Delta ACD$, $\frac{m}{PE} = \frac{m+n}{10}$

$\Rightarrow PE = \frac{10m}{m+n}$ ...(i)

$\because \Delta CEP \sim \Delta CAB$, $\frac{n}{PE} = \frac{m+n}{15}$

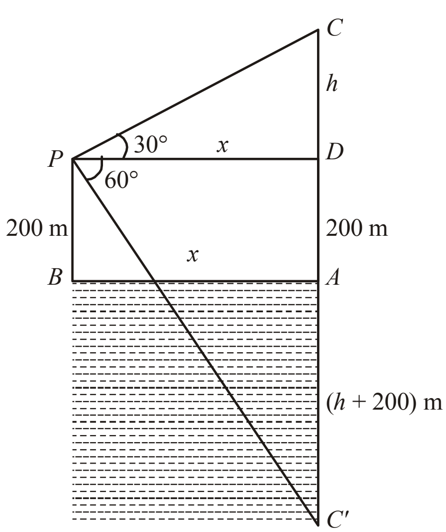
$\Rightarrow PE = \frac{15n}{m+n}$ ...(ii)

From (i) and (ii),

$10m = 15n \Rightarrow m = \frac{3}{2}n$

So, $PE = 6$

24. (c)



Here in ΔPCD,

$\sin 30° = \frac{h}{PC} \Rightarrow PC = 2h$ ...(i)

$\tan 30° = \frac{h}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow x = \sqrt{3}h$ ...(ii)

Now, in right $\Delta PC'D$

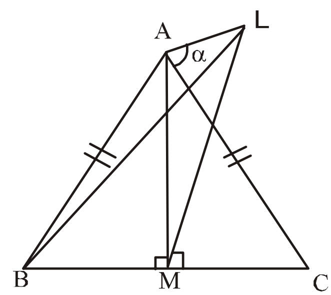
$\tan 60° = \frac{h+400}{x}$

$\Rightarrow \sqrt{3}x = h + 400 \Rightarrow 3h = h + 400$ [From (ii)]

$\Rightarrow h = 200$

So, $PC = 400$ m [From (i)]

25. (3) Let the height of the vertical tower situated at the mid point of BC be h.



In ΔALM,

$\cot A = \frac{AM}{LM}$

$\Rightarrow 3\sqrt{2} = \frac{AM}{h} \Rightarrow AM = 3\sqrt{2}h$

In ΔBLM,

$\cot B = \frac{BM}{LM} \Rightarrow \sqrt{7} = \frac{BM}{h} \Rightarrow BM = \sqrt{7}h$

In ΔABM by Pythagoras theorem

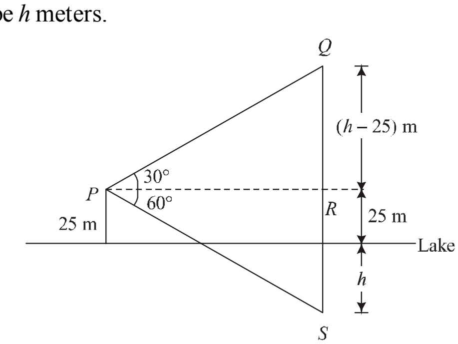
$AM^2 + MB^2 = AB^2$

$\therefore AM^2 + MB^2 = (100)^2$

$\Rightarrow 18h^2 + 7h^2 = 100 \times 100$

$\Rightarrow h^2 = 4 \times 100 \Rightarrow h = 20$

26. (2) Let height of the cloud from the surface of the lake be h meters.



$\therefore$ In ΔPRQ:

$$\tan 30° = \frac{h-25}{PR}$$

$\therefore\quad PR = (h-25)\sqrt{3}$ ...(i)

and in ΔPRS : $\tan 60° = \frac{h+25}{PR}$

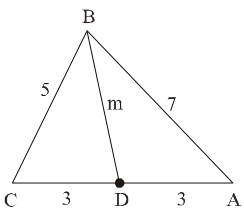
$PR = \frac{h+25}{\sqrt{3}}$ ...(ii)

Then, from eq. (i) and (ii),

$$(h-25)\sqrt{3} = \frac{h+25}{\sqrt{3}}$$

$\therefore\quad h = 50\,m$

27. **(b)** Let the height of the lamp-post is h.



By Appolonius Theorem,

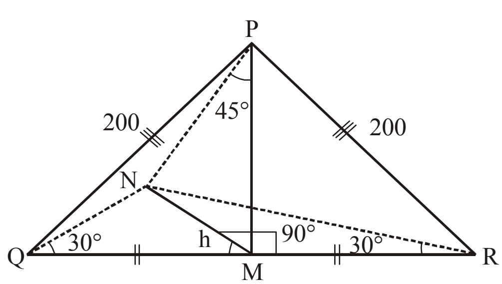
$$2\left(BD^2 + \left(\frac{AC}{2}\right)^2\right) = BC^2 + AB^2$$

$\Rightarrow\quad 2(m^2 + 3^2) = 25 + 49 \Rightarrow m = 2\sqrt{7}$

$\tan 30° = \frac{h}{BD}$

$\Rightarrow\quad h = 2\sqrt{7} \times \frac{1}{\sqrt{3}} = \frac{2\sqrt{21}}{3}$

28. **(d)**



Let height of tower MN = h

In ΔQMN we have

$\tan 30° = \frac{MN}{QM}$

$\therefore\quad QM = \sqrt{3}h = MR$ ...(1)

Now in ΔMNP

MN = PM ...(2)

In ΔPMQ we have :

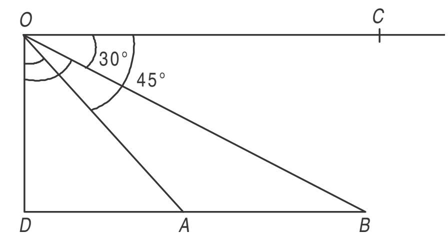
$MP = \sqrt{(200)^2 - (\sqrt{3}\,h)^2}$

$\therefore$ From (2), we get :

$\sqrt{(200)^2 - (\sqrt{3}\,h)^2} = h \Rightarrow h = 100m$

29. **(a)** Here; $\angle DOA = 45°$; $\angle DOB = 60°$

Now, let height of tower = h.



In ΔDOA, $\tan(\angle DOA) = \frac{DA}{OD}$

$\Rightarrow\quad \tan 45° = \frac{DA}{h} \qquad \Rightarrow h = DA$

Now, in Δ DOB

$\tan(\angle DOB) = \frac{BD}{OD}$

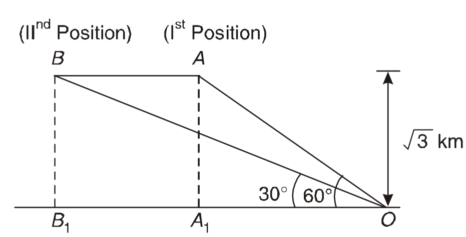
$\Rightarrow \tan 60° = \frac{BD}{h} \qquad \Rightarrow BD = \sqrt{3}\,h.$

$\therefore$ speed for the distance BA $= \frac{BD - AD}{18} = \frac{(\sqrt{3}-1)\,h}{18}$

$\therefore$ required time taken

$$= \frac{AD}{\text{speed}} = \frac{h \times 18}{(\sqrt{3}-1)\,h} = \frac{18}{\sqrt{3}-1} = 9(\sqrt{3}+1)$$

30. **(d)** For Δ OA, A, $OA_1 = \frac{\sqrt{3}}{\tan 60°} = 1$ km.

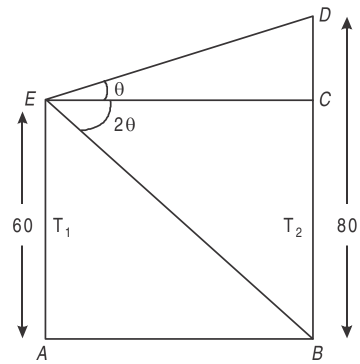


For Δ OB_1, B, $OB_1 = \frac{\sqrt{3}}{\tan 30°} = 3$ *km.*

As, a distance of 3 – 1 = 2 km is covered in 5 seconds. Therefore the speed of the plane is

$$\frac{2 \times 3600}{5} = 1440 \text{ km / hr}$$

31. (d) Let the distance between T_1 and T_2 be x



From the figure

$EA = 60$ m (T_1) and $DB = 80$ m (T_2)

$\angle DEC = \theta$ and $\angle BEC = 2\theta$

Now in ΔDEC,

$$\tan\theta = \frac{DC}{AB} = \frac{20}{x}$$

and in ΔBEC,

$$\tan 2\theta = \frac{BC}{CE} = \frac{60}{x}$$

We know that

$$\tan 2\theta = \frac{2\tan\theta}{1-(\tan\theta)^2}$$

$$\Rightarrow \frac{60}{x} = \frac{2\left(\frac{20}{x}\right)}{1-\left(\frac{20}{x}\right)^2}$$

$$\Rightarrow x^2 = 1200 \quad \Rightarrow x = 20\sqrt{3}$$

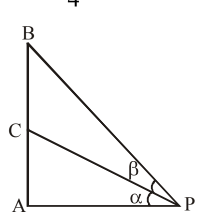
32. (d) Since $AP = 2AB \Rightarrow \frac{AB}{AP} = \frac{1}{2}$ ...(i)

Let $\angle APC = \alpha$

$$\therefore \quad \tan\alpha = \frac{AC}{AP} = \frac{1}{2}\frac{AB}{AP} = \frac{1}{4}$$

($\because$ C is the mid point) $\left(\therefore AC = \frac{1}{2}AB\right)$

$$\Rightarrow \tan\alpha = \frac{1}{4}$$



As $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

$$\Rightarrow \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{1}{2}$$

$$\left[\begin{array}{l}\because \tan(\alpha+\beta) = \frac{AB}{AP} \\ \tan(\alpha+\beta) = \frac{1}{2}\ [\text{From}(1)]\end{array}\right]$$

$$\Rightarrow \frac{\frac{1}{4}+\tan\beta}{1-\frac{1}{4}\tan\beta} = \frac{1}{2} \quad \therefore \tan\beta = \frac{2}{9}$$

33. (b) $\tan 30° = \frac{h}{x+a}$

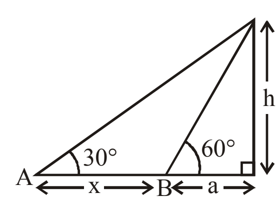
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+a} \Rightarrow \sqrt{3}h = x+a$$

...(1)

$$\tan 60° = \frac{h}{a} \Rightarrow \sqrt{3} = \frac{h}{a}$$

$$\Rightarrow h = \sqrt{3}a$$

...(2)



From (1) and (2)

$3a = x + a \Rightarrow x = 2a$

Here, the speed is uniform

So, time taken to cover x = 2 (time taken to cover a)

$\therefore$ Time taken to cover $a = \frac{10}{2}$ minutes = 5 minutes

34. (d) Let $AP = x$

$BP = y$

$$\tan 45° = \frac{H}{x} \Rightarrow H = x$$

$$\tan 30° = \frac{H}{y} \Rightarrow y = \sqrt{3}H$$

$$x^2 + (54\sqrt{2})^2 = y^2$$

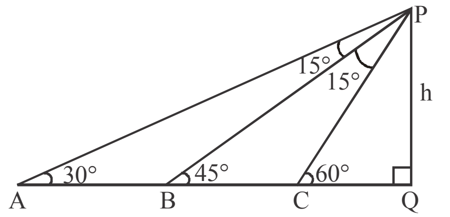
$$H^2 + (54\sqrt{2})^2 = 3H^2$$

$$(54\sqrt{2})^2 = 2H^2$$

$$54\sqrt{2} = \sqrt{2}H$$

$$54 = H$$

35. (c)



$\because$ PB bisects $\angle APC$, therefore

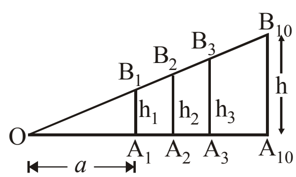
$AB : BC = PA : PC$

Also in ΔAPQ, $\sin 30° = \frac{h}{PA} \Rightarrow PA = 2h$

and in ΔCPQ, $\sin 60° = \frac{h}{PC} \Rightarrow PC = \frac{2h}{\sqrt{3}}$

$\therefore \quad AB : BC = 2h : \frac{2h}{\sqrt{3}} = \sqrt{3} : 1$

36. (a)



$\Delta OA_1B_1, \Delta OA_2B_2, \Delta OA_3B_3, ..., \Delta OA_{10}B_{10}$ all are similar triangles.

$\Rightarrow \quad \frac{h_1}{a_1} = \frac{h_2}{a_2} = \frac{h_3}{a_3} = ... = \frac{h_{10}}{a_{10}} = \tan\alpha$.

Since, $h_{10} = h = a_{10} \tan\alpha$ ...(1)

and $a_1 = a \Rightarrow h_1 = a \tan\alpha$ ...(2)

$\Rightarrow$ h = (a + 9d) tan α where d is distance between poles

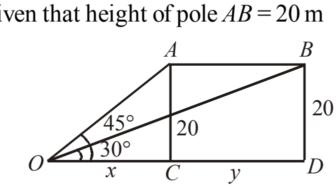
$(\because a_{10} = a + 9d)$

$\Rightarrow \quad h = a\tan\alpha + 9d\tan\alpha$

$$\Rightarrow \quad \frac{h - a\tan\alpha}{9\tan\alpha} = d \Rightarrow \frac{h - \frac{a\sin\alpha}{\cos\alpha}}{9\frac{\sin\alpha}{\cos\alpha}} = d$$

$$\Rightarrow \quad d = \frac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$$

37. (b) Given that height of pole $AB = 20$ m



Let O be the point on the ground such that $\angle AOC = 45°$

Let $OC = x$ and $CD = y$

In right ΔAOC, $\tan 45° = \frac{20}{x}$ ...(i)

In right ΔBOD, $\tan 30° = \frac{20}{x+y}$ ...(ii)

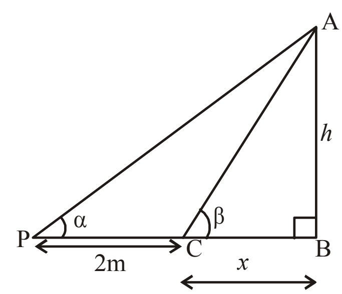
From (i) and (ii), we have $x = 20$ and $\frac{1}{\sqrt{3}} = \frac{20}{x+y}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{20}{20+y} \Rightarrow 20 + y = 20\sqrt{3}$$

So, $y = 20(\sqrt{3}-1)$ m and time = 1 s (Given)

Hence, speed $= 20(\sqrt{3}-1)$ m/s

38. (a) Let AB be the tower of height 'h'.



Given : In ΔABP

$\tan\alpha = \frac{AB}{PB}$

or $\frac{\sin\alpha}{\cos\alpha} = \frac{h}{x+2}$

$\Rightarrow \quad (x + 2)\sin\alpha = h\cos\alpha$

$\Rightarrow \quad h = \frac{x\sin\alpha + 2\sin\alpha}{\cos\alpha}$ ...(1)

Now, In ΔABC, $\tan\beta = \frac{AB}{BC}$

$\Rightarrow \quad \frac{\sin\beta}{\cos\beta} = \frac{h}{x} \Rightarrow x = \frac{h\cos\beta}{\sin\beta}$ ...(2)

Putting the value of x in eq. (2) to eq. (1), we get

$$h = \frac{\frac{h\cos\beta\sin\alpha}{\sin\beta} + \frac{2\sin\alpha}{1}}{\cos\alpha}$$

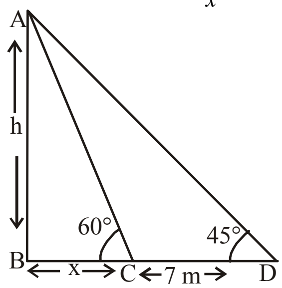
$$\Rightarrow \quad h = \frac{h\cos\beta.\sin\alpha + 2\sin\alpha\sin\beta}{\sin\beta.\cos\alpha}$$

$\Rightarrow \quad h(\sin\beta.\cos\alpha - \cos\beta.\sin\alpha) = 2\sin\alpha.\sin\beta$

$\Rightarrow \quad h[\sin(\beta - \alpha)] = 2\sin\alpha.\sin\beta$

$$\Rightarrow \quad h = \frac{2\sin\alpha.\sin\beta}{\sin(\beta - \alpha)}$$

39. **(b)** In right, ΔABC $\tan 60° = \dfrac{h}{x} = \sqrt{3}$



$\Rightarrow \quad x = \dfrac{h}{\sqrt{3}}$ ....(i)

In right, $\Delta ABD = \tan 45° = \dfrac{h}{x+7} = 1$

$\Rightarrow \quad h = x + 7 \Rightarrow h - \dfrac{h}{\sqrt{3}} = 7$ [From (i)]

$\Rightarrow \quad h = \dfrac{7\sqrt{3}}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow \quad h = \dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)m$

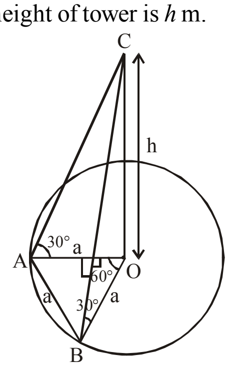
40. **(a)** In the ΔAOB given that $\angle$AOB = 60°
and OA = OB = radius

$\therefore \quad \angle OBA = \angle OAB = 60°$

$\therefore \quad \Delta$ AOB is a equilateral triangle.

$\Rightarrow \quad OA = OB = AB = a$

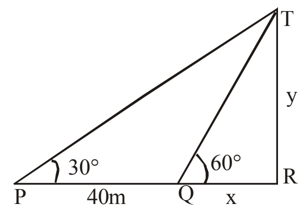
Let the height of tower is h m.



In ΔOAC, $\tan 30° = \dfrac{h}{a} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{a}$

$\Rightarrow h = \dfrac{a}{\sqrt{3}}$

41. **(d)**



In right ΔQTR

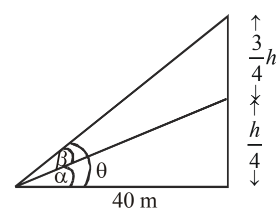
$\tan 60° = \dfrac{y}{x} \Rightarrow y = \sqrt{3}x$ ....(1)

In right ΔPTR

$\tan 30° = \dfrac{y}{x+40} \Rightarrow y = \dfrac{x+40}{\sqrt{3}}$ ....(2)

From (1) and (2), $\sqrt{3}x = \dfrac{x+40}{\sqrt{3}} \Rightarrow x = 20m$

42. **(c)**



$\theta = \alpha + \beta, \beta = \tan^{-1}\left(\dfrac{3}{5}\right)$

or $\quad \beta = \theta - \alpha$

$\Rightarrow \quad \tan\beta = \dfrac{\tan\theta - \tan\alpha}{1 + \tan\theta . \tan\alpha}$

or $\quad \dfrac{3}{5} = \dfrac{\dfrac{h}{40} - \dfrac{h}{160}}{1 + \dfrac{h}{40} . \dfrac{h}{160}}$

$h^2 - 200h + 6400 = 0$

$\Rightarrow \quad h = 40$ or 160 metre

$\therefore \quad$ possible height = 40 metre