CHAPTER 14

Tacheometric Surveying

14.1 Introduction

- This is another method of surveying wherein the horizontal distances and the difference in elevations are determined indirectly by an instrument called as **tacheometer.**
- Tacheometer is an optical instrument consisting of lenses and thus tacheometry is also one of the methods of **Optical Distance Measurement (ODM).**
- It is a quick method of surveying.
- Tacheometry is not a very accurate method of measurement and the error involved in measurements increase as the distance between the tacheometer and the staff station increases. The relative error in tacheometry under the most favourable conditions normally does not exceed 1 in 1000.
- Limitations :
  (a) Staff man should be able to reach at the point whose elevation is required to be determined.
  (b) The staff station must be clearly visible from the tacheometric station.

14.2 Advantages of Tacheometric Surveying

(a) This method is suitable for preparation of topographic maps wherein both the horizontal distances and elevations of the points are required.

(b) This tacheometric method of surveying is very useful for rough terrains where direct methods of measurement are difficult.

(c) Because the tacheometric method is quick to perform and is thus very suitable for reconnaissance survey.

(d) For survey of water bodies like the rivers, wet lands, oceans etc. i.e. in hydrographic surveying, tacheometry is a very suitable method.

(e) Tacheometry is used for filling in the details of a traverse.

(f) Tacheometry can also be used as a check where the distances and also the levels have already been measured with other methods like chain, tape, dumpy level etc.

(g) Because this method is quick and thus it saves time and resources.

14.3 Tacheometer

- Tacheometer is nothing but a transit theodolite fitted with a stadia diaphragm.
- The stadia diaphragm generally consists of two stadia hairs, one above and one below and equidistant from a central horizontal hair.
- Now a days, the stadia lines are etched on the diaphragm on which the horizontal and vertical cross hair lines are also etched.

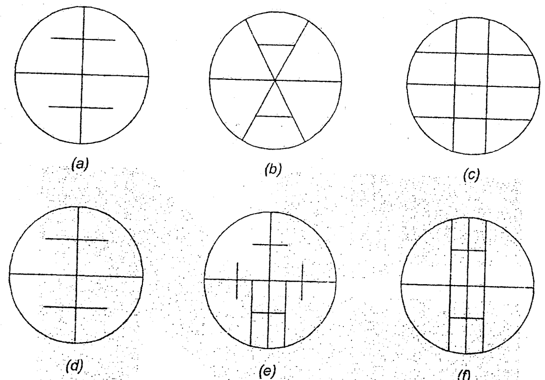


Fig. 14.1 *Different patterns of stadia lines on diaphragm*

14.4 Major Characteristics of a Tacheometer

- The multiplying constant (*k*) of the tacheometer is usually a round figure and mostly it is 100.
- The additive constant (*C*) of the tacheometer is kept very small and mostly it is kept zero.
- The telescope is fitted with an anallactic lens which makes the additive constant zero.
- The magnifying power of the eye-piece is kept high to make the staff graduations clearly visible even at large distances.
- The aperture of the objective is kept usually at 35 mm to 45 mm to make the image sharp.

14.5 Stadia Rod

- The ordinary levelling staff with 5 mm graduations can be used for short distances only.
- For long distances, special large staff called the stadia rod is used.
- This stadia rod is in one piece 3 m to 5 m long and about 50 mm to 150 mm wide. The graduations are made bold and simple so that it is easy to take the readings even from a long distance.

Limitations: Because stadia rods are large and bulky, they pose inconvenience in transportation.

- To overcome this, stadia design is generally printed on a strip of coated woven fabric and this strip is tacked on board when needed.

14.6 Systems of Tacheometric Measurements

(a) Stadia system (b) Tangential system (c) Subtense bar system

14.6.1 Stadia System

- In the stadia system of tacheometry, the tacheometer is set up at station *A* and staff at station *B* as shown in **Fig. 14.2.**
- The staff intercept between the upper and lower stadia is measured along with vertical angle θ made with the horizontal.
- The horizontal distance *D* between the instrument station A and staff station *B* and difference of elevations between stations *A* and *B* are determined from the staff intercept (*s*) and the vertical angle (θ).

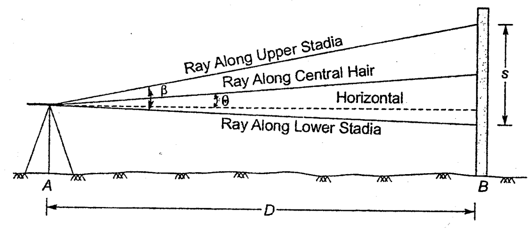


Fig. 14.2 *Stadia system of tacheometry*

The stadia system of tacheometry is further classified as:

(i) Fixed hair system

(ii) Movable hair system

(i) **Fixed hair system:** In this system, the vertical distance between the upper and lower stadia hair is fixed and this fixed distance is called as **stadia interval (*i*).**

Stadia interval is not changed during the measurement. When the tacheometer is targeted at staff then staff intercept (s) is obtained from the difference of readings of upper hair and lower hair.

(ii) **Movable hair system:** In this method of tacheometry, the distance between the upper hair and lower hair (i.e. the stadia interval, *i*) is varied by moving the stadia hairs vertically by the micrometer screws.

Here the staff intercept (s) is fixed. Here the staff used has two vanes called as **targets** which are fixed at a constant spacing (say 2m or 3m).

While taking the reading, the stadia interval is varied and is measured corresponding to the staff intercept.

It is quite difficult to measure the stadia interval accurately, and thus movable hair method is sparsely used. Fixed hair system is most commonly used in practice.

In general, unless otherwise specified, the stadia system refers to fixed hair system.

14.6.2 Tangential System

- Here in this system of tacheometry, the hairs are not at all required.
- This method can even be used when telescope is not provided with a diaphragm.
- The staff has two targets at a fixed distance (*s*) apart.
- The vertical angles θ_1 and θ_2 are measured to the two targets.
- Now these vertical angles and the fixed distance on staff are used to determine the horizontal distance and difference of elevations.

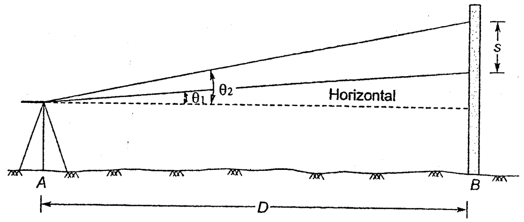


Fig. 14.3 *Tangential system of tacheometry*

14.6.3 Subtense Bar System

- Here in this system of tacheometry, a bar of fixed length is used which is known as subtense bar as shown in **Fig. 14.4.**

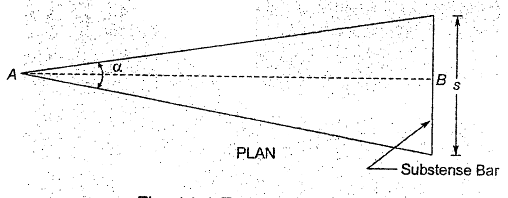


Fig. 14.4 *Tangential system*

- The subtense bar has two targets at the ends at a fixed distance (s) apart.
- The horizontal angle subtended between the instrument station A and the two targets on the subtense bar is measured.

14.7 Principle of Stadia Method of Tacheometry

As shown in **Fig. 14.5**, let there be an external focusing telescope with horizontal axis. Also the staff is held vertical at station *Q*.

Let staff intercept = *AB* = *s*

Thus light rays coming from A and *B* i.e. the rays *AO* and *BO* meet at objective center *O*. This objective forms an inverted image *ba* of *AB*.

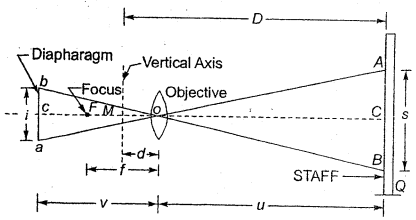


Fig. 14.5 *Stadia method of tacheometry*

Now in ΔAOB and ΔaOb

$$\angle AOB = \angle aOb$$

(Vertically opposite angles)

$$\angle abO = \angle ABO \quad \text{(ab // AB)}$$

Thus, $\Delta AOB \sim \Delta aOb$

So,

$$\frac{AB}{ab} = \frac{OC}{oc} = \frac{u}{v}$$

$$\Rightarrow \quad u = \left(\frac{AB}{ab}\right)v \qquad \text{...(14.1)}$$

Here ab = Stadia interval

u = Horizontal distance of staff from optical center of objective lens

v = Horizontal distance of cross hairs from optical center of objective lens

If *f* is the focal length of objective (=*OF*) then the distances *u* and *v* are related by the lens formula as,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \qquad \text{...(14.2)}$$

Multiplying both the sides of the Eq. (14.2) by *uf*,

$$u = f + \frac{uf}{v}$$

$$u = f + \frac{f}{v}\left(\frac{AB}{ab}v\right)$$

$$u = f + \left(\frac{AB}{ab}\right)f$$

Now, AB = Staff intercept = s

ab = Stadia interval = i

$$u = f + \left(\frac{f}{i}\right)s \qquad \text{...(14.3)}$$

If *d* is the horizontal distance between vertical axis of the instrument from the optical center *O* of objective then the horizontal distance (*D*) is equal to,

$$D = u + d$$

$$D = \left(\frac{f}{i}\right)s + (f + d)$$

$$D = ks + C \qquad \text{...(14.4)}$$

where the constant (*k*) is (*f/i*) which is called as **multiplying constant** and the constant (*C*) is (*f* + *d*) which is called as **additive constant.**

- Usually the value of multiplying constant (k) is 100 and additive constant (*C*) is kept to the minimum usually equal to zero. The value of *C* varies from 0.3 m to 0.6 m in external focusing telescope and 0.08 m to 0.2 m in internal focusing telescope.
- **RL of staff station *Q*:** Since the line of sight is assumed to be horizontal and thus the elevation of staff station *Q* is given by,

  Elevation of staff station *Q* = Elevation of line of collimation – Central hair reading (*CQ*)

  = (RL of BM + BS) – Central hair reading (*CQ*)

14.8 Tacheometric Measurement with Inclined Line of Sight and Staff Vertical

The staff may be held either vertical or inclined to the line of sight. Generally staff is held vertical to the line of sight as it is easier to do so.

Case I: Angle of Elevation

As shown in Fig. 14.6 the line of sight OC is inclined to the horizontal at an angle θ.

Let staff intercept $= AB = s$

The two rays OA and OB make an angle of β with each other i.e.

$$\angle AOB = \beta$$

Draw a line $A'CB'$ normal to the line of sight OC at C.

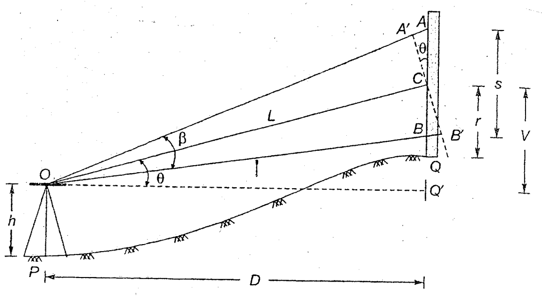


Fig. 14.6 *Angle of elevation with inclined line of sight and staff held vertical*

Now, $\angle AA'C = 90° + \frac{\beta}{2}$

$\angle BB'C = 90° - \frac{\beta}{2}$

Assuming angle β to be quite small, $\angle AA'C$ and $\angle BB'C$ can be taken as 90°.

In $\Delta AA'C$, $A'C = AC \cos\theta$

Now point C is the mid-point of AB and thus,

$$A'C = \left(\frac{AB}{2}\right)\cos\theta$$

But $2A'C = A'B'$

Thus, $A'B' = AB \cos\theta$

or $s' = s \cos\theta$

Thus staff intercept normal to the line of sight is equal to the staff intercept held vertical multiplied with $\cos\theta$.

Now $A'B'$ is normal to the line of sight and thus equations developed in earlier sections can be applied.

$$L = ks' + C$$

$$L = ks \cos\theta + C \quad \text{...(14.5)}$$

The horizontal distance D is given by,

$$D = L \cos\theta$$

$$D = ks \cos^2\theta + C \cos\theta \quad \text{...(14.6)}$$

The vertical intercept (V) between the horizontal line OQ' through O and the point C is given by,

$$V = CQ'd = L \sin\theta$$

$$V = (ks\cos\theta + C) \sin\theta$$

$$V = ks \sin\theta.\cos\theta + C \sin\theta$$

$$V = \left(\frac{1}{2}\right) ks \sin 2\theta + C \sin\theta \quad \text{...(14.7)}$$

For an external focusing telescope, the additive constant $C = 0$ and thus,

$$V = \left(\frac{1}{2}\right) ks \sin 2\theta$$

Also $\tan\theta = \frac{V}{D}$

The RL of staff station Q is given by,

RL of Q = RL of station P + Height of instrument + $V - r$

= RL of station $P + h + V - r$

= RL of BM + $BS + V - r$

Case II: Angle of Depression

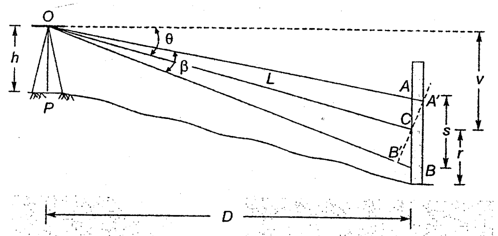


Fig. 14.7 *Angle of depression with inclined line of sight and staff held vertical*

As shown in Fig. 14.7 the line of sight OC is inclined to the horizontal at an angle θ. Here also the earlier expressions of horizontal distance D and vertical intercept V hold good i.e.,

$$D = ks \cos^2\theta + C \cos\theta$$

$$V = \left(\frac{1}{2}\right) ks \sin 2\theta + C \sin\theta \quad \text{...(14.8)}$$

Now the RL of staff station Q is given by,

RL of Q = RL of station P + Height of instrument $- V - r$

= RL of station $P + h - V - r$

= RL of BM + $BS - V - r$

14.9 Inclined Line of Sight with Staff Normal to the Line of Sight

- Here the staff is held normal to the line of sight and the line of sight is inclined to the horizontal.
- A staff man cannot judge by itself that whether the staff is normal to the line of sight or not.
- Thus the staff is provided with a sighting device to enable the staff man to hold the staff normal to the line of sight.
- Often a small sighting tube, a diopter tube, is provided with the staff at right angles to it for this purpose.

Case I: Angle of Elevation

Staff intercept $= AB = s$

Thus, the inclined distance OC is given by,

$$OC = L = ks + C$$

The horizontal distance D between instrument station P and staff station Q is,

$$D = L \cos\theta + C'Q'$$

$$D = L \cos\theta + r \sin\theta$$

$$D = (ks + C)\cos\theta + r\sin\theta \quad ...(14.9)$$

Here, θ = Vertical angle which the middle line makes with the horizontal

r = Staff reading of central hair

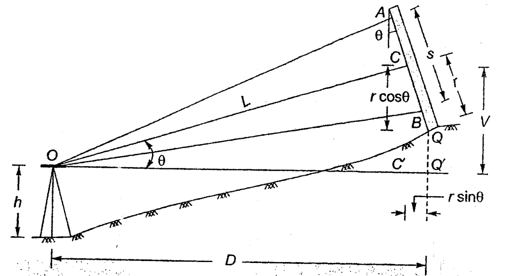


Fig. 14.8 *Angle of elevation with inclined line of sight and staff held normal to the line of sight*

The vertical intercept between O and C is given by,

$$V = L\sin\theta$$

$$V = (ks + C)\sin\theta \quad ...(14.10)$$

For a telescope fitted with an anallactic lens, $C = 0$

Thus,

$$D = ks\cos\theta + r\sin\theta$$

$$V = ks\sin\theta \quad ...(14.11)$$

Thus RL of staff station Q = RL of instrument station P + Height of instrument + $V - r\cos\theta$

= RL of station $P + h + V - r\cos\theta$

Case II: Angle of Depression

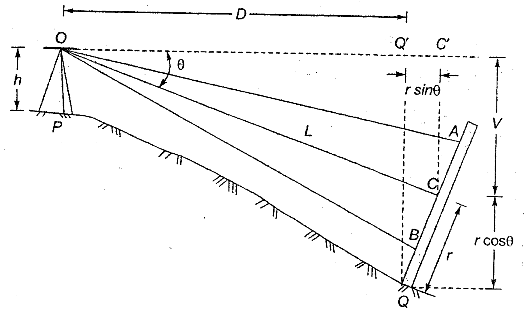


Fig. 14.9 *Angle of depression with inclined line of sight and staff held normal to the line of sight*

As shown in **Fig. 14.9**, the line of sight OC is inclined to the horizontal at an angle θ.

The horizontal distance D between instrument station P and Q staff station is given by,

$$D = L\cos\theta - C'Q'$$

$$D = (ks + C)\cos\theta - r\sin\theta \quad ...(14.12)$$

The vertical distance V is given by,

$$V = L\sin\theta$$

$$V = (ks + C)\sin\theta \quad ...(14.13)$$

RL of staff station Q = RL of instrument station P + Height of instrument $- V - r\cos\theta$

RL of staff station Q = RL of $P + h - V - r\cos\theta$

14.10 Advantages of Holding the Staff Vertical

(a) It is more convenient to hold the staff vertical than normal to the line of sight.

(b) The verticality of staff can be ensured more easily.

(c) The procedure for arriving at horizontal distance D and vertical intercept V from the staff intercept s and vertical angle θ is easier in this case as compared to staff held normal to the line of sight.

14.11 Advantages of Holding the Staff Normal to The Line of Sight

(a) The error incurred due to staff not being perfectly normal to the line of sight is less than the error incurred due to staff not being perfectly vertical.

(b) The normality of the staff can be ensured by the personnel standing at instrument station by looking at the staff director through the theodolite.

14.12 Tangential Method of Tacheometry

- In the absence of stadia hairs, the tangential method of tacheometry is resorted to.
- This method is used when the staff is held too far from the instrument station and it is difficult to read the staff.
- Here the staff is fitted with two targets spaced at a fixed vertical distance (s) apart (say 2 m or 3 m). By sighting the two targets, the vertical angles θ_1 and θ_2 are measured.
- From above three observations viz. s, θ_1 and θ_2, the horizontal distance D and vertical intercept V is arrived at as per the following three cases as described below.

Case I: Both the Angles are at Elevation

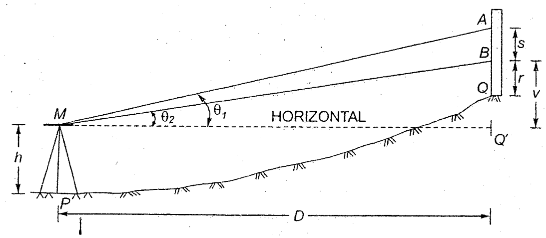


Fig. 14.10 *Tangential method of tacheometry with both the angles in elevation*

As shown in **Fig. 14.10** both the angles θ_1 and θ_2 are in elevation. The horizontal distance (D) between the instrument station P and staff station Q is given as below.

From $\Delta BMQ'$ $V = D \tan \theta_2$ ...(14.14)

From $\Delta AMQ'$ $V + s = D \tan \theta_1$ ...(14.15)

From these Eqs. (14.14) and (14.15)

$$s = D \tan\theta_1 - D \tan\theta_2$$

$$s = D (\tan\theta_1 - \tan\theta_2)$$

$$D = \frac{s}{\tan\theta_1 - \tan\theta_2}$$

$$D = \frac{s \cos\theta_1 \cos\theta_2}{\sin(\theta_1 - \theta_2)} \quad ...(14.16)$$

Also, $V = D \tan \theta_2$

$$V = \frac{s \cos\theta_1 \cos\theta_2}{\sin(\theta_1 - \theta_2)} \times \tan\theta_2$$

$$V = \frac{s \cos\theta_1 \sin\theta_2}{\sin(\theta_1 - \theta_2)} \quad ...(14.17)$$

Thus, RL of staff station Q = RL of instrument station $P + h + V - r$

Case II: Both the Angles are at Depression

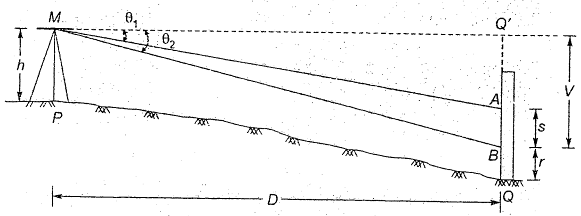


Fig. 14.11 *Tangential method of tacheometry with both the angles in depression*

As shown in Fig. 14.11, both the angles θ_1 and θ_2 are in depression.

From $\Delta MQ'B$ $V = Q'B = D \tan \theta_2$ ...(14.18)

From $\Delta MQ'A$ $V - s = Q'A = D \tan \theta_1$ ...(14.19)

From Eqs. (14.18) and (14.19)

$$D = \frac{s}{\tan\theta_2 - \tan\theta_1}$$

$$D = \frac{s \cos\theta_1 \cos\theta_2}{\sin(\theta_2 - \theta_1)} \quad ...(14.20)$$

Also, $V = D \tan\theta_2$

$$V = \frac{s \cos\theta_1 \cos\theta_2}{\sin(\theta_2 - \theta_1)} \times \tan\theta_2$$

$$V = \frac{s \cos\theta_1 \sin\theta_2}{\sin(\theta_2 - \theta_1)} \quad ...(14.21)$$

Thus, RL of staff station Q = RL of instrument station $P + h - V - r$

Case III: One Angle is in Depression and Other at Elevation

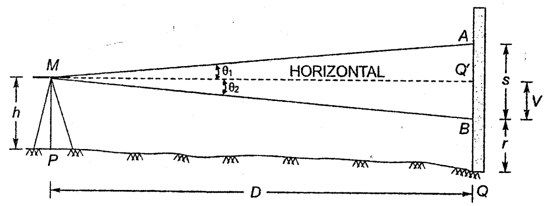


Fig. 14.12 *Tangential method of tacheometry with one angle in depression and other at elevation*

As shown in Fig. 14.12, angle θ_1 is in elevation and θ_2 is in depression.

From $\Delta MQ'B$ $V = Q'B = D \tan\theta_2$ ...(14.22)

From $\Delta MQ'A$ $s - V = Q'A = D \tan\theta_1$ ...(14.23)

From Eqs. (14.22) and (14.23)

$$s = D(\tan \theta_1 + \tan \theta_2)$$

$$D = \frac{s}{\tan\theta_1 + \tan\theta_2}$$

$$D = \frac{s \cos\theta_1 \cos\theta_2}{\sin(\theta_1 + \theta_2)} \quad ...(14.24)$$

Also, $V = D \tan\theta_2$

$$V = \frac{s \cos\theta_1 \cos\theta_2}{\sin(\theta_1 + \theta_2)} \times \tan\theta_2$$

$$V = \frac{s \cos\theta_1 \sin\theta_2}{\sin(\theta_1 + \theta_2)} \quad ...(14.25)$$

Thus, RL of staff station Q = RL of instrument station $P + h - V - r$

14.13 Disadvantages of Tangential Method of Tacheometry

(a) Here two vertical angles are to be measured and this takes more time as compared to stadia method.

(b) It may happen that instrument may get disturbed in between the readings for the two angles and thus error may creep in.

(c) Changes in atmospheric refraction in between the two readings may induce error.

14.14 Errors in Tacheometric Surveying

14.14.1 Errors Due to Personnel Reasons

(a) Error due to faulty centering, levelling and bisections: If the tacheometer is not centered and levelled properly then errors may creep in.

(b) **Error due to non-verticality of staff:** Often the staff may not be held perfectly vertical due to which error incurs. A plumb bob provided with the staff ensures the vertically of staff.

(c) **Error due to wrong reading of staff:** The staff intercept if not read correctly then this introduces error. Thus it is always advisable to read the staff readings for all the three stadia wires and to check whether difference of readings of upper and middle stadia is equal to difference of readings of middle and lower stadia.

(d) **Error due to faulty reading of vertical angle:** Tacheometry also involves the measurement of vertical angle and the same should be done precisely otherwise appreciable errors may occur in the results.

14.14.2 Instrumental Errors

(a) **Error due to faulty instrument constants:** With continuous use of instrument (i.e., tacheometry), the instrument constants k and C may get change. The erroneous effect due to this is cumulative in nature. Thus it is quite essential to verify the instrument constants regularly.

(b) **Error due to imperfect adjustment of instrument:** Certain adjustments of the instrument are done in factory only and thus if instrument is having fault in its factory production, then error may get introduced.

(c) **Error due to stadia rod:** Faulty divisions in stadia rod and improper standardization lead to errors.

14.14.3 Error Due to Natural Causes

(a) **Wind:** Very strong wind currents disturb the work and readings in field cannot be taken correctly. Apart from that, strong winds make it difficult to hold the staff truly vertical.

(b) **Sun:** In hot sunny days unequal expansion of instrument parts take place and is a source of error.

(c) **Atmospheric refraction:** Atmospheric refraction changes the path of line of sight thereby giving wrong staff readings. Thus for very long sights, read only the top and middle hair and multiply the difference of top and middle hair readings by two to get the staff intercept (S).

Illustrative Examples

Example 14.1 The stadia readings with sight horizontal taken on a vertical staff 60 m away from the tacheometer were 1.280 m and 1.785 m. The focal length of the object lens was 30 cm and distance between object lens and vertical axis of tacheometer was 20 cm. Find the stadia interval.

Solution:

Tacheometric constants are k and c where

$$c = f + d$$
$$= 30 + 20 = 50 \text{ cm} = 0.5 \text{ m}$$
$$k = \frac{f}{i} \text{ where } i = \text{stadia interval}$$

Staff intercept (s) $= 1.785 - 1.280 = 0.505$ m

Now $D = ks + C$

$\Rightarrow$ $D = \frac{f}{i}s + C$

$\Rightarrow$ $60 = \frac{0.3}{i}(0.505) + 0.5$

$\Rightarrow$ $i = 2.546 \times 10^{-3}\text{m} = 2.546 \text{ mm} = 2.55 \text{ mm}$

Example 14.2 Distance between the two points P and Q was required to be determined from tacheometer fitted with an anallactic lens (k = 100, C = 0). With the instrument at P and staff at Q, the observations made were a vertical angle of + 8.43′ and staff intercept of 1.805 m. What is the horizontal distance PQ? At a later stage, it was found that constants of the tacheometer were 100.5 and 0.7. What would be the percentage error in the computed distance PQ?

Solution:

$\because$ $k = 100$ and $C = 0$, $\theta = +8°43'$, $s = 1.805$ m

$D = ks\cos^2\theta + C \cdot \cos\theta$

$= 100(1.805)\cos^2 8°43' + 0.\cos 8°43' = 176.35$ m

When $k = 100.5$ and $C = 0.7$

$D = ks\cos^2 8°43' + 0.\cos 8°43'$

$= (100.5)(1.805)\cos^2 8°43' + 0.7\cos 8°43'$

$= 179.307 + 0.6919 = 179.9989 \text{ m} = 180$ m

$\therefore$ Percentage error $= \frac{180 - 176.35}{176.35} \times 100 = 2.07\%$

Example 14.3 A levelling staff is held vertical at distance of 104 m and 307 m from the tacheometer axis and staff intercepts for horizontal sights are 0.850 m and 2.750 m respectively. Find the instrument constants.

When instrument was set up at P and staff at Q, the telescope was depressed at an angle of 8.5° with the horizontal and the staff readings were 2.780 m, 1.845 m and 0.955 m. Find the R.L. of Q and its horizontal distance from P. The height of instrument at P is 1.25 m and R.L. of P is 435 m.

Solution:

$D = ks + C$

Given, $D_1 = 104$ m and $D_2 = 307$ m

$S_1 = 0.850$ m, $S_2 = 2.750$ m

$\therefore$ $D_1 = ks_1 + C$, $D_2 = ks_2 + C$

$\Rightarrow$ $104 = k(0.85) + C$ ...(i)

$\Rightarrow$ $307 = k(2.75) + C$ ...(ii)

Solving (i) and (ii),

$k = 106.842$

$C = 13.1843$

Thus instrument constants are:

$k = 106.842$

$C = 13.1843$

In the second part of the question,

Staff intercept (s) $= 2.780 - 0.955 = 1.825$ m

Angle of depression, $\theta = 8.5°$

Now, horizontal distance $D = ks\cos^2\theta + C\cos\theta$

$= (106.842)(1.825)\cos^2 8.5° + 13.1843\cos 8.5°$

$= 190.727 + 13.0395 = 203.77$ m

and vertical distance, $V = \frac{ks}{2}\sin 2\theta + c\cdot\sin\theta$

$= \frac{(106.842)1.825}{2}\sin(2\times 8.5°) + 13.1843\sin 8.5°$

$= 190.727 + 13.0395 = 203.77$ m

and vertical distance, $V = \frac{ks}{2}\sin 2\theta + c\cdot\sin\theta$

$= \frac{(106.842)1.825}{2}\sin(2\times 8.5°) + 13.1843\sin 8.5°$

$= 28.504 + 1.949 = 30.453$ m $= 30.45$ m

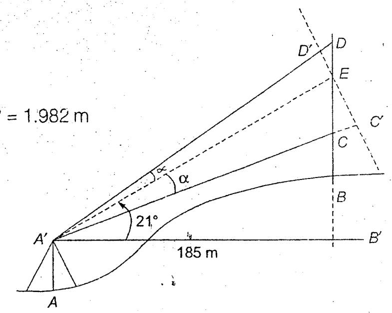
$\therefore$ R.L. of Q = R.L. of $P + HI - V - h$

$= 435 + 1.25 - 30.45 - 1.845 = 403.955$ m

$\therefore$ R.L. of $Q = 403.955$ m

Example 14.4 A tacheometer is fitted with an anallactic lens $k = 100$ and $c = 0$. The reading corresponding to the cross-wire on a staff held vertical at point B was 2.455 m when sighted from A. If the vertical angle was + 21° and horizontal distance AB was 185 m. Find the stadia wire readings and thus show that the two intercepts are equal. Find the R.L. of B if R.L. of A is 103.5 m and height of instrument is 1.16 m.

Solution:



$D = ks\cos^2\theta + C$

$\Rightarrow 185 = 100(s)\cos^2 21° + 0$

$\Rightarrow s = 2.123$ m

$C'D' = s\cos 21° = 2.123\cos 21° = 1.982$ m

From $\Delta A'B'E$,

$\cos 21° = \frac{A'B'}{A'E}$

$\Rightarrow A'E = A'B'\sec 21°$

$= 185\sec 21° = 198.16$ m

$\therefore 2\alpha = \frac{C'D'}{A'E} = \frac{1.982}{198.16} = 36''$

$\Rightarrow \alpha = 18''$

$B'C = D\tan(21° - \alpha) = 185\tan(21° - 18'') = 70.996 \text{ m} \simeq 71$ m

$B'E = D\tan 21°$

$= 185\tan 21° = 71.02$ m

$B'D = D\tan(21° + 18'')$

$= 185\tan(21° + 18'') = 71.033$ m

$\therefore$ Stadia intercept interval $E'C = B'E - B'C = 71.02 - 71 = 0.02$ m

Stadia intercept interval $ED = B'D - B'E$

$= 71.033 - 71.02$

$= 0.013$ m (very near to 0.02 m)

$\simeq 0.02$ m

Thus staff intercepts are equal

$S = EC + ED$

$= 0.02 + 0.02 = 0.04$ m

Middle cross-wire reading = 2.455 m

Lower stadia wire reading = 2.455 − 0.02 = 2.435 m

Upper stadia wire reading = 2.455 + 0.02 = 2.475 m

$B'E = V = D\tan 21° = 185\tan 21° = 71.01$ m

R.L. of $B = 103.5 + 1.16 + 71.01 - 2.455 = 173.215$ m

Example 14.5 Find upto what vertical angle in stadia work, a sloping distance may be assumed to be horizontal so that error may not exceed 1 in 350? The instrument is fitted with anallactic lens and staff is held vertical.

Solution:

Let the required vertical angle be θ

True horizontal distance $D = ks\cos^2\theta + c\cdot\cos\theta$

$= ks\cos^2\theta$

$\therefore \frac{\text{Sloping distance}}{\text{Horizontal distance}} = \frac{ks}{ks\cos^2\theta} = \sec^2\theta$

Permissible error = 1 in 350

$\therefore \frac{L}{D} = \frac{1+350}{300} = \frac{351}{300} = \sec^2\theta$

$\therefore \theta = 22.41°$

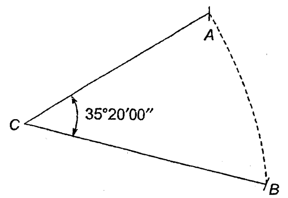
Example 14.6 In order to determine the gradient between the two points A and B, a tacheometer was set up at C and following observations were taken with a vertical staff:

Staff	Vertical Angle	Stadia readings
A	04°20′ 00″	1.3, 1.61, 1.92
B	00°10′ 40″	1.1, 1.41, 1.72

The plan angle ACB is 35° 20′ 00″. What is the gradient between A and B? Take $k = 100$ and $c = 0$.

Solution:

For staff reading at A



$s = 1.92 - 1.3 = 0.62$ m

$Ac = d = ks\cos^2\theta$

$= 100(0.62)\cos^2 4°20' = 61.646$ m

$V = \frac{ks\sin^2\theta}{2}$

$= \frac{100(0.62)\sin 8°40'}{2} = 4.671$ m

$\therefore$ Level difference between A and $C = 4.671 - 1.61 = 3.061$ m

$\therefore$ A is above C

For staff reading at B

$$s = 1.72 - 1.1 = 0.62 \text{ m}$$

$$CB = d = ks\cos^2\theta = 100(0.62)\cos^2(00°\ 10'40'') = 62 \text{ m}$$

$$v = \frac{ks}{2}\sin^2\theta = \frac{100(0.62)\sin(00°20'80'')\sin^2}{2} = 0.1924 \text{ m}$$

∴ Level difference between B and $C = 0.1924 - 1.41 = -1.2176$ m

∴ C is higher than B

Level difference between A and $B = 3.061 - (-1.2176) = 4.2786$ m

In ΔABC

$$\cos\angle ACB = \frac{AC^2 + BC^2 - AB^2}{2(AC)(BC)}$$

$$\Rightarrow \quad \cos(35°\ 30'\ 00'') = \frac{61.646^2 + 62^2 - AB^2}{2(61.646)(62)}$$

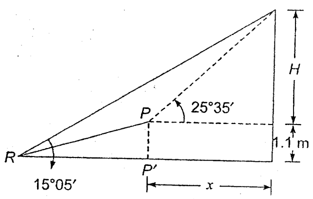
$$\Rightarrow \quad AB = 37.525 \text{ m}$$

$$\text{Gradient } AB = \frac{\text{Level difference between } A \text{ and } B}{\text{Distance } AB}$$

$$= \frac{4.2486}{37.525} = 0.114 = 1 \text{ in } 8.77$$

Example 14.7 It is required to determine the elevation of top Q of a signal on a hill, observations were made from two stations P and R. The stations P, R and Q are in the same plane. If angle of elevation of the top Q of the signal measured at P and R were 25° 35′ 00″ and 15° 05′ 00″ respectively. Find the elevation of the foot of the signal if the height of signal above base was 4 m. The staff reading on a BM (RL = 105.42 m) were respectively 2.755 m and 3.855 m when the instrument was at P and R. The distance between P and R is 120 m.

Solution:



$$PR = 120 \text{ m}$$

$$PP' = \sqrt{RP^2 - PP^2}$$

$$= \sqrt{120^2 - 1.1^2} = 119.995 \text{ m}$$

$$H + 1.1 = (x + RP')\tan 15°\ 05'$$

Also

$$H = x\tan 25°\ 15'$$

∴ $x\tan 25°\ 15' + 1.1 = (x + 119.995)\tan 15°\ 05'$

⇒ $x = 154.559$ m

∴ $H = x\tan 25°\ 15' = 154.559\tan 25°\ 15' = 72.895$ m

∴ Elevation of foot $= 105.42 + 72.895 - 4 + 2.755$

$= 177.07$ m

Example 14.8 There is a closed traverse $ABCD$. Determine the elevations of points C and D and distance between these points based on the following observations recorded on a vertically held staff from the traverse stations A and B. Take tacheometric constants as $k = 100$, $c = 0$.

Traverse station	RL (m)	HI (m)	Co-ordinates Lat.	Co-ordinates Dep.	Staff stn.	Web Bearing	Vertical Angle	Staff readings (m)
A	105.75	1.50	560	1400	C	12°30′	+7°05′	1.20 1.35 2.5
B	106.32	1.52	210	2000	D	278°43′	+3°15′	1.45 2.1 2.75

Solution:

When staff is at station C,

Staff intercept $(s_1) = 1.5 - 1.2 = 0.3$ m

∴ Horizontal distance $AC = ks_1\cos^2\theta_1 = 100\,(0.3)\cos^2 7°05' = 29.54$ m

$$\text{Vertical distance } v_1 = \frac{1}{2}ks_1\sin 2\theta_1 = \frac{1}{2}(100)(0.3)\sin(2\times 7°05') = 3.67 \text{ m}$$

When staff is at station D,

Staff intercept $(s_2) = 2.75 - 1.45 = 1.3$ m

∴ Horizontal distance $BD = ks_2\cos^2\theta_2 = 100\,(1.3)\cos^2 3°15' = 129.58$ m

$$\text{Vertical distance } v_2 = \frac{1}{2}ks_2\sin 2\theta_2 = \frac{1}{2}(100)(1.3)\sin(2\times 3°15') = 7.36 \text{ m}$$

Now, bearing of $AC = 12°30'$

∴ Latitude of $AC = (AC)\cos 12°30' = 29.54\cos 12°30' = 28.84$ m

Departure of $AC = (AC)\sin 12°30' = (29.54)\sin 12°30' = 6.39$ m

Bearing of $BD = 278°43' = \text{N } 81°\ 17'\ \text{W}$

∴ Latitude of $BD = (BD)\cos 81°17' = (129.58)\cos 81°17' = -128.08$ m

Latitude of $AB = 560 - 210 = 350$ m

Departure of $AB = 2000 - 1400 = 700$ m

In a closed traverse, $\Sigma L = 0$ and $\Sigma D = 0$

$\Sigma L = 0$

⇒ $L_{AC} + L_{CD} + L_{DB} + L_{BA} = 0$

⇒ $28.84 + L_{CD} + 19.64 + 350 = 0$

⇒ $L_{CD} = -398.48$ m

$\Sigma D = 0$

⇒ $D_{AC} + D_{CD} + D_{DB} + D_{BA} = 0$

⇒ $6.39 + D_{CD} + (-128.08) + 700 = 0$

⇒ $D_{CD} = -578.31$ m

$$\therefore \quad \text{Distance } CD = \sqrt{(-398.48)^2 + (-578.31)^2} = 702.3 \text{m}$$

$$\text{Bearing of } CD = \tan^{-1}\frac{D_{CD}}{L_{CD}} = \tan^{-1}\left(\frac{-578.31}{-398.48}\right) = 55.43°$$

Example 14.9 There is a fort on a hill in Jaipur. Adjacent to the hill, Delhi-Jaipur NH8 is passing through. In order to determine the height of fort roof from its plinth and also to determine the distance of fort from road, tacheometer observations were taken with instrument constants as 100 and 0.

Instantaneous station	HI (m)	Staff Station	Vert. Angle	Staff readings (m)
A point on NH8	1.4	Plinth of fort	+12°47′	3.250 3.525 3.200
		Roof top of fort	+18°35′	2.180 2.630 3.080

Solution:

When staff is at plinth of fort,

$$\theta_1 = 12°47' = 12.783°$$

$$\text{Staff intercept } (s_1) = 3.8 - 3.25 = 0.55 \text{ m}$$

$$\therefore \quad \text{Horizontal distance } D = ks_1 \cos^2\theta_1 = 100\,(0.55)\cos^2 12.783° = 52.31 \text{ m}$$

$$\text{Vertical height of plinth } (v_1) = \frac{1}{2}ks_1 \sin 2\theta_1 = \frac{1}{2}(100)(0.55)\sin(2\times 12.783°) = 11.87 \text{ m}$$

$$\text{Vertical height of roof top } (v_2) = \frac{1}{2}ks_2 \sin 2\theta_2$$

$$\text{Staff intercept } (s_2) = 3.080 - 2.180 = 0.9 \text{ m}$$

$$\theta_2 = 18°35' = 18.583°$$

$$\therefore \quad v_2 = \frac{1}{2}(100)(0.9)\sin(2\times 18.583°) = 27.19 \text{ m}$$

$$\therefore \quad \text{Height of roof top from plinth} = 27.19 - 11.87 = 15.32 \text{ m}$$

Objective Brain Teasers

Q.1 The stadia method of tacheometry is used to determine:
(a) Vertical angles
(b) Horizontal distances
(c) Horizontal angles
(d) Horizontal and vertical distances

Q.2 The standard of accuracy in tacheometric distance measurement is:
(a) 1 : 5000 (b) 1 : 10000
(c) 1 : 10 (d) 1 : 100

Q.3 A subtense bar is used for:
(a) Measurement of horizontal distances in flat terrains
(b) Measurement of angles
(c) Levelling
(d) Measurement of horizontal distances in undulated terrains.

Q.4 The multiplying constant of tacheometer is:
(a) f/i (b) fi
(c) i/f (d) f/i^2

Q.5 The method of tacheometry commonly used is:
(a) Subtense bar system
(b) Movable stadia system
(c) Fixed stadia system
(d) Tangential system

Q.6 A tacheometer differs from an ordinary theodolite in the sense that its diaphragm is provided with:
(a) Two horizontal hairs in addition
(b) Two vertical hairs in addition
(c) Three vertical hairs in addition
(d) None of the above

Q.7 If the intercept on the vertical staff is 0.75 m when observed through a tacheometer with horizontal line of sight. The distance between the tacheometer and the staff is:
(a) 75 m (b) 7.5 m
(c) 750 m (d) 0.75 m

Q.8 The intercept of a staff:
(a) is minimum, if staff is held truly normal to the line of sight
(b) is maximum, if staff is held truly normal to the line of sight
(c) decreases if staff is tilted towards normal
(d) decreases if staff is tilted away from the normal

Q.9 In tangential tacheometry, an ordinary level staff is used which:
(a) is vertical on all cases
(b) is equipped with two targets a fixed distance apart
(c) leans away from the instrument for inclined sight downwards
(d) leans towards the instrument for inclined sight upwards

Q.10 Choose the incorrect statement:
(a) Tacheometry is very suitable for preliminary location surveys.
(b) Tacheometry is preformed over changing on rough terrain
(c) Subtense bar is used to measure short distances
(d) Vertical holding of staff is better than normal holding since the error due to improper staff holding is less

Q.11 The most commonly practiced method of tacheometry is
(a) Fixed stadia system
(b) Tangential system
(c) Subtense bar system
(d) Movable stadia system

Q.12 While computing distance from tacheometric method, the standard of accuracy is
(a) 1 : 500 (b) 1 : 10000
(c) 1 : 5000 (d) 1 : 100

Q.13 Subtense bar can be used for
(a) short distances upto 200 m
(b) long distances of upto 10 km
(c) very long distance of upto 1000 km
(d) very short distance of upto 5 cm

Q.14 Match **List-I** (Instrument) with **List-II** (Usage) and choose the correct answer

List-I
A. Sextant
B. Tangent clinometer
C. Range finder
D. Subtense bar

List-II
1. To determine horizontal distance
2. To measure angles
3. To determine difference in elevation between the points
4. To establish right angles

Codes:

	A	B	C	D
(a)	4	2	1	3
(b)	1	2	3	4
(c)	3	1	2	4
(d)	2	3	4	1

Q.15 In an ordinary stadia telescope, the focal length is 20 cm. The tacheometric constants are 100 and 15 cm. An error of 0.0035 cm exists in stadia interval. What will be the numerical error in computed horizontal distance if 's' is the stadia interval?
(a) $\frac{7}{5}s$ (b) $\frac{2}{3}s$
(c) s (d) $\frac{5}{6}s$

Answers

1. (d)	2. (b)	3. (d)	4. (a)	5. (c)
6. (a)	7. (a)	8. (a)	9. (b)	10. (d)
11. (a)	12. (b)	13. (a)	14. (d)	15. (a)

Hint and Solution:

15. (a)

$$D = ks + C = \left(\frac{f}{i}\right)s + C$$

Differentiating,

$$\delta D = -\frac{f}{i^2}s.\delta i$$

$$\text{Now,} \quad \frac{f}{i} = 100$$

$$\Rightarrow \quad i = \frac{f}{100} = \frac{20}{100}\text{cm} = 0.2 \text{ cm}$$

$$\therefore \quad \delta D = -\frac{20}{(0.2)^2}.s(0.0035)$$

$$= 1.75s = \frac{7}{4}s$$

Student's Assignments

Ex.1 Determine the tacheometer constants from the following readings:

Distance of staff from vertical axis of tacheometer	Stadia readings	
	Lower Wire	Upper Wire
50 m	1.115	1.350
135 m	1.215	2.315

Ans. 98.27, 26.91

Ex.2 Stadia readings with staff vertical and line of sight horizontal on a vertical staff held 70 m away from a tacheometer were 1.755 and 2.550. The focal length of the objective is 250 mm. The distance between the objective and the trunnion axis of the tacheometer was 150 mm. What is the stadia interval?

Ans. 2.86 mm

Ex.3 It is required to determine the gradient between the two points *A* and *B*. For doing so, a tacheometer was set up at station *C* and the following observations were taken with staff vertical.

Staff at	Vertical angle	Stadia Readings		
A	+3° 20′ 00″	1.350	1.655	1.950
B	+0° 15′ 35″	1.100	1.650	1.750

Determine the average gradient between A and *B* if horizontal angle *ACB* = 33° 35′. Take tacheometer constants as 100 and 0.0.

Ans. 1 in 11.44