Real Numbers

Exercise 1.1

Q. 1. (a) Write any three rational numbers

(b) Explain rational number is in your own words.

Answer : (a). A rational number is any number that can be expressed as a quotient or fraction p/q of two integers, where p is a numerator and q is a denominator, provided that p is non-zero i.e., p ≠ 0.

Any three rational numbers can be 1, -6.6, 4/5.

(This is because 1 can be written as 1/1, -6.6 can be written as -66/10 or -33/5 & 4/5 is already in p/q form)

(b). Rational number can be defined easily, as basically any sort of number that can possibly be written in a p/q form, numerator is p and denominator is q (q is non-zero).

Since, a rational number is merely an integer or fraction, we can say that we can find infinite rational numbers between two distinct numbers. It might be difficult to obtain rational numbers minutely from a number line graph and hence, formulas are used to find rational numbers.

Its range is from negative numbers in an axis to positive numbers, including zero. Infact all whole numbers, all positive and negative numbers are rational numbers.

Some examples:

1/2 is a rational number (p/q form, where q ≠ 0)

0.50 is a rational number (1/2)

1 is a rational number (1/1)

2.12 is a rational number (212/100)

-6.5 is a rational number (-13/2)

Q. 2. Give one example each to the following statements.

i. A number which is rational but not an integer
ii. A whole number which is not a natural number
iii. An integer which is not a whole number
iv. A number which is natural number, whole number, integer and rational

number.
v. A number which is an integer but not a natural number.

Answer : (i) Integer is any number that can be written without a fractional component. So, write a rational number in which numerator and denominator doesn’t have any common factor.

For example: 99/98, 2/3, 57/2 etc…

(ii) Whole numbers are all positive natural numbers including zero.

Natural numbers are the set of positive integers from 1 to infinity, excluding fractional and decimal parts. Basically, they are whole numbers without 0.

This clearly implies that 0 is the only whole number which is not a natural number.

(iii) Integers are set of positive as well as negative numbers, that can be written without a fractional component. While whole numbers are natural numbers, including 0.

Thus, all negative non-fractional numbers are integers but not whole numbers.

For example: -1, -2, -3, etc…

(iv) Natural numbers are counting numbers (positive, non-fractional and non-decimal). (1, 2, 3, 4, …)

Whole numbers are all positive natural numbers, including 0. (0, 1, 2, 3, 4, …)

Integer is any number that can be written without a fractional component. (…, -4, -3, -2, -1, 0, 1, 2, 3, …)

And rational numbers are numbers that can be written in the form p/q, where p is a numerator and q ≠ 0 is a denominator. (1, 2/1, 3/2, 55/3, …)

Drawing out a common number from these set, we can say

1, 2, 3, … are numbers which are natural, whole, integer and rational numbers.

(v). Natural numbers are positive counting numbers, excluding fractions and decimals.

Integers are basically any numbers that can be written without fractional component.

So, numbers which are integers but not natural numbers are:

-3, -2, -44, -1, …

Q. 3. Find five rational numbers between 1 and 2.

Answer : We can represent a rational number between two numbers a and b as

(a + b)/2

Now, put a = 1 and b = 2.

Then,

$$\text{Rational number between 1 and 2} = \frac{1+2}{2} = \frac{3}{2}$$

$$\text{Rational number between 1 and 3/2} = \frac{\left[1+\left(\frac{3}{2}\right)\right]}{2} = \frac{2+3}{4} = \frac{5}{4}$$

$$\text{Rational number between 1 and 5/4} = \frac{\left[1+\left(\frac{5}{4}\right)\right]}{2} = \frac{4+5}{8} = \frac{9}{8}$$

$$\text{Rational number between 5/4 and 3/2} = \frac{\left[\left(\frac{5}{4}\right)+\left(\frac{3}{2}\right)\right]}{2} = \frac{5+6}{8} = \frac{11}{8}$$

$$\text{Rational number between 3/2 and 2} = \frac{\left[\left(\frac{3}{2}\right)+2\right]}{2} = \frac{3+4}{4} = \frac{7}{4}$$

Hence, five rational numbers between 1 and 2 are 3/2, 5/4, 9/8, 11/8 and 7/4.

Q. 4. Insert three rational numbers between $\frac{2}{3}$ *and* $\frac{3}{5}$.

Answer : We can represent a rational number between two rational numbers a and b as

(a + b)/2

Now, put a = 2/3 and b = 3/5.

Then,

A rational number between 2/3 and 3/5 = $\frac{[(\frac{2}{3})+(\frac{3}{5})]}{2} = \frac{10+9}{30} = \frac{19}{30}$

A rational number between 2/3 and 19/30 $= \frac{[(\frac{2}{3})+(\frac{19}{30})]}{2} = \frac{20+19}{60} = \frac{39}{60}$

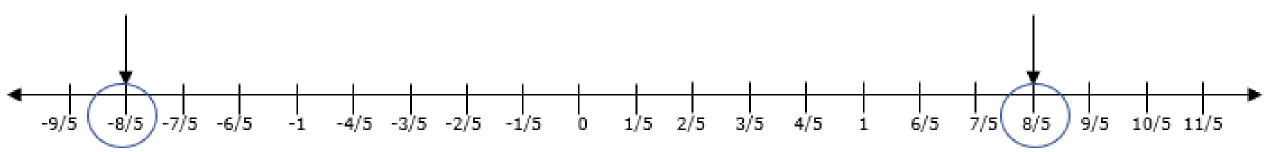
A rational number between 19/30 and 3/5 = $\frac{[(\frac{19}{30})+(\frac{3}{5})]}{2} = \frac{19+18}{60} = \frac{37}{60}$

Hence, three rational numbers between 2/3 and 3/5 are **19/30, 39/60 and 37/60**.

Q. 5. Represent $\frac{8}{5}$ and $\frac{-8}{5}$ **on a number line**

Answer : 8/5 = 1.6 and -8/5 = -1.6

Thus, 1.6 lies between 1 and 2; -1.6 lies between -2 and -1.



Step 1: Divide 0 to -2 and 0 to 2 into equal parts. (…, -9/5, -8/5, …, 0, 1/5, 2/5, …, 1, 6/5, 7/5, …, 11/5, …)

Step 2: Mark -8/5 and 8/5 on the number lines.

Q. 6. Express the following rational numbers as decimals numbers

I. i. $\frac{242}{1000}$ **ii.** $\frac{354}{500}$

iii. $\frac{2}{5}$ **iv.** $\frac{115}{4}$

II. i. $\frac{2}{3}$ **ii.** $-\frac{25}{36}$

iii. $\frac{22}{7}$ **iv.** $\frac{11}{9}$

Answer : I. (i) 242/1000 = 242 × 10^{-3}

⇒ 242/1000 = 0.242

(∵ 242 ÷ 1000 = 0.242)

(ii) 354/500 = (354/5) × 10^{-2}

⇒ 354/500 = 70.8 × 10^{-2}

Or 354/500 = 70.8 ÷ 100

⇒ 354/500 = 0.708

(iii) 2/5 = 2 ÷ 5

⇒ 2/5 = 0.4

(iv) 115/4 = 115 ÷ 4

⇒ 115/4 = 28.75

II. (i) 2/3 = 2 ÷ 3

⇒ 2/3 = 0.6666666…

This is clearly a repeating or recurring decimal. Thus, round-off the decimal in 2-digit or so after the decimal point.

⇒ 2/3 = 0.67

(ii) -25/36 = -25 ÷ 36

⇒ -25/36 = -0.6944444…

This is clearly a repeating or recurring decimal. Thus, by rounding off in 4-digit or so after the decimal point.

⇒ -25/36 = -0.6944

(iii) 22/7 = 22 ÷ 7

⇒ 22/7 = 3.142857143…

This clearly is non-repeating, non-terminating decimal, which is a decimal number which continues endlessly with no group of digits repeating endlessly.

Thus, take only 2 or 3 digit after decimal point.

⇒ 22/7 = 3.14

This number is also called π (pi).

(iv) 11/9 = 11 ÷ 9

⇒ 11/9 = 1.2222222…

This is a repeating or recurring decimal. Thus, round it off to 2 or so digits after the decimal point.

⇒ 11/9 = 1.22

Q. 7. Express each of the following decimals in $\frac{p}{q}$ form where q ≠ 0 and p, q are integers

i) 0.36 ii) 15.4
iii) 10.25 iv) 3.25

Answer : (i) 0.36 = 36/100

Simplify it in the same p/q form,

⇒ 0.36 = 18/50

On further simplifying it,

⇒ 0.36 = 9/25

This cannot be further simplified in p/q form.

9/25 is in p/q form, where p = 9 and q = 25 are integers and q = 25 ≠ 0.

Hence, the answer is 9/25.

(ii) 15.4 = 154/10

Simplifying it in the same p/q form,

⇒ 15.4 = 77/5

This cannot be further simplified in p/q form.

77/5 is in p/q form, where p = 77 and q = 5 are integers and q = 5 ≠ 0.

Hence, the answer is **77/5**.

(iii) 10.25 = 1025/100

Simplifying it in the same p/q form,

⇒ 10.25 = 205/20

On further simplifying it,

⇒ 10.25 = 41/4

This cannot be further simplified in p/q form.

41/4 is in the form p/q, where p = 41 and q = 4 are integers and q = 4 ≠ 0

Hence, the answer is **41/4**.

Q. 8. Express each of the following decimal number in the $\frac{p}{q}$ form

i) 0.5 ii) $3.\overline{8}$

iii) $0.\overline{36}$ iv) $3.12\overline{7}$

Answer : (i) 0.5 = 5/10

Simplifying it in the same p/q form,

⇒ 0.5 = 1/2

It cannot be further simplified in p/q form.

1/2 is in the form p/q, where p = 1 and q = 2 are integers and q = 2 ≠ 0.

Hence, the answer is **1/2**.

(ii) $3.\overline{8} = 3.8888888...$

This has to be converted in p/q form.

Let $x = 3.\overline{8}$,

That is, x = 3.88888… …(i)

Multiply 10 on both sides,

10 × x = 10 × 3.88888…

⇒ 10x = 38.8888… …(ii)

Subtracting equations (i) from (ii), we get

10x – x = 38.8888… - 3.88888…

⇒ 9x = 35

⇒ x = 35/9

35/9 is in the form of p/q, where p = 35 and q = 9 are integers and q = 9 ≠ 0.

Thus, $3.\overline{8} = \frac{35}{9}$.

(iii) $0.\overline{36} = 0.363636...$

This has to be converted in p/q form.

Let $x = 0.\overline{36}$,

That is, x = 0.363636… …(i)

Multiply 100 on both sides,

100 × x = 100 × 0.363636…

⇒ 100x = 36.363636… …(ii)

Subtracting equations (i) from (ii), we get

100x – x = 36.363636… - 0.363636…

⇒ 99x = 36

⇒ x = 99/36

Simplifying it further,

⇒ x = 11/4

11/4 is in the form of p/q, where p = 11 and q = 4 are integers and q = 4 ≠ 0.

Thus, $\mathbf{0.\overline{36}} = \frac{\mathbf{11}}{\mathbf{4}}$.

(iv) $3.12\overline{7} = 3.127777\ldots$

This has to be converted in p/q form.

Let $x = 3.12\overline{7}$,

That is, x = 3.127777… …(i)

Multiply 100 by equation (i),

100 × x = 100 × 3.127777…

⇒ 100x = 312.7777… …(ii)

Multiply 1000 by equation (ii),

1000 × x = 1000 × 3.127777…

⇒ 1000x = 3127.7777… …(iii)

Subtract equation (ii) from (iii), we get

1000x – 100x = 3127.7777… - 312.7777…

⇒ 900x = 2815

⇒ x = 2815/900

⇒ x = 563/180

563/180 is in the form of p/q, where p = 563 and q = 180 are integers and q = 180 ≠ 0.

Thus, $\mathbf{3.12\overline{7}} = \frac{\mathbf{563}}{\mathbf{180}}$.

Exercise 1.2

Q. 1. Classify the following numbers as rational or irrational.

i. $\sqrt{27}$
ii. $\sqrt{441}$
iii. 30.232342345…
iv. 7.484848…
v. 11.2132435465
vi. 0.3030030003.....

Answer : (i) On prime factorization $27 = 3^3$

$\sqrt{(3^3)} = 3\sqrt{3}$

$\sqrt{3}$ is an irrational number and so $\sqrt{27}$ is an irrational number

(ii) On prime factorization $441 = 7^2 \times 3^2$

$\sqrt{(7^2 \times 3^2)} = 21$

Since 441 is a perfect square so square root of it is a rational number

(iii) The decimal is not terminating in case of this number.

So it can never be expressed in the form of fraction i.e. numerator by denominator.

So the number is an irrational number.

(iv) The decimal is not terminating in case of this number.

So it can never be expressed in the form of fraction i.e. numerator by denominator.

So the number is an irrational number.

(v) The decimal is a terminating decimal number.

So it can be expressed in the form of fraction i.e. numerator by denominator.

So the number is an rational number.

So it can never be expressed in the form of fraction i.e. numerator by denominator.

So the number is an irrational number.

(vi) The decimal is not terminating in case of this number.

So it can never be expressed in the form of fraction i.e. numerator by denominator.

So the number is an irrational number.

Q. 2. Explain with an example how irrational numbers differ from rational numbers?

Answer : The rational numbers can be expressed in the form of fraction i.e. numerator by denominator.

The irrational numbers doesn’t have a terminating decimal number, so they can never be expressed in the form of fraction i.e. numerator by denominator.

For example:

i) $4.43 = \frac{443}{100}$

The number 4.43 can be expressed in the form of fraction i.e. numerator by denominator. So it is a rational number.

ii) $\sqrt{5} = 2.236067977\ldots$

This number doesn’t have a terminating decimal number, so they can never be expressed in the form of fraction i.e. numerator by denominator.

So it is an irrational number.

Q. 3. Find an irrational number between $\frac{5}{7}$ and $\frac{7}{9}$. How many more there may be?

Answer :

$$\frac{5}{7} = 0.714285714\ldots$$

$$\frac{7}{9} = 0.77777777\ldots$$

When two numbers are irrational then their mean is also irrational

$$\frac{5}{7} + \frac{7}{9} = \frac{45 + 49}{63}$$

$$\text{Mean} = \frac{47}{63}$$

$\Rightarrow$ Mean = 0.746031746…

So $\frac{47}{63}$ is an irrational number between the two given numbers.

Q. 4. Find two irrational numbers between 0.7 and 0.77

Answer : First we square the two given numbers

$0.7^2 = 0.49$

$0.77^2 = 0.5929$

Square root of any two non-perfect square numbers between 0.49 and 0.5929 will give an irrational number since they are not perfect squares.

$\sqrt{0.5} = 0.7071067811\ldots$

$\sqrt{0.53} = 0.728010988\ldots$

So $\sqrt{0.5}$ and $\sqrt{0.53}$ are the two irrational numbers between 0.7 and 0.77

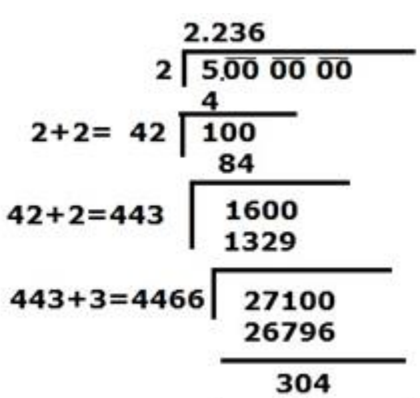
Q. 5. Find the value of √5 upto 3 decimal places.

Answer : Step 1: Since 5 is not a perfect square so we have to take the next greatest perfect square just less than 5 which is 4.

Step 2: Since the remainder is less than the quotient so a decimal is added and two zeroes are added. This becomes the dividend for the next division.

Step 3: The quotient in the previous step is now added with the divisor. A single digit is added at the end and this resulting number is multiplied with the same single digit such that it is less than the dividend for the next division.

Step 4: The remainder in this step is again added with two zeroes and this process continues until we get the number of digits required after decimal point.



Answer is 2.236

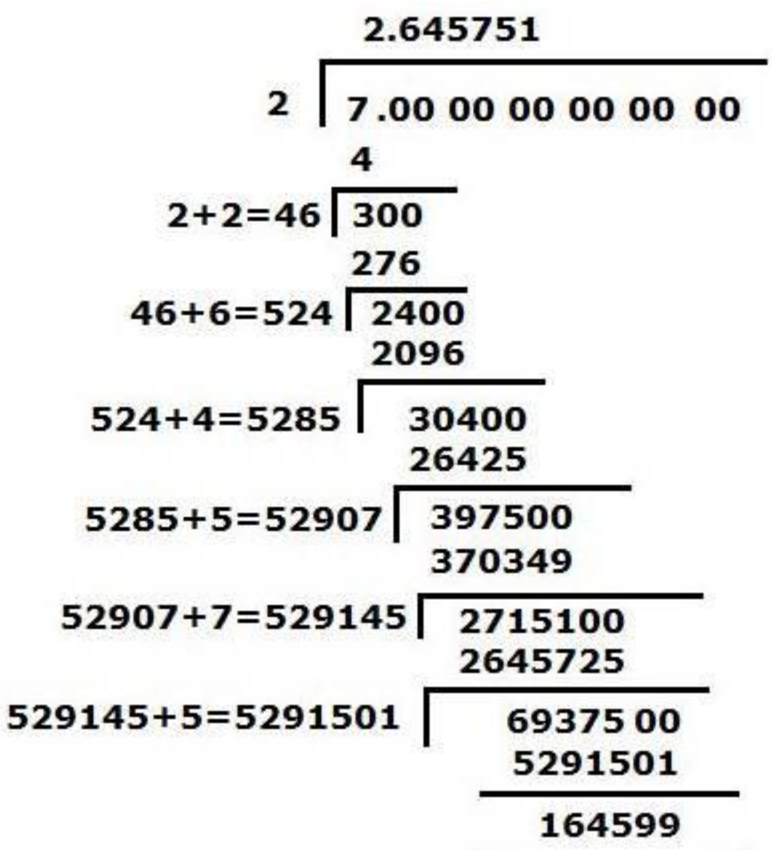
Q. 6. Find the value of √7 up to six decimal places by long division method.

Answer : Step 1: Since 7 is not a perfect square so we have to take the next greatest perfect square just less than 7 which is 4.

Step 2: Since the remainder is less than the quotient so a decimal is added and two zeroes are added. This becomes the dividend for the next division.

Step 3: The quotient in the previous step is now added with the divisor. A single digit is added at the end and this resulting number is multiplied with the same single digit such that it is less than the dividend for the next division.

Step 4: The remainder in this step is again added with two zeroes and this process continues until we get the number of digits required after decimal point.



Square root is 2.645751

Q. 7. Locate √10 on number line.

Answer : According to Pythagoras theorem:

Hypotenuse = √ (Perpendicular^2 + Base^2)

$\sqrt{10} = \sqrt{(3^2 + 1^2)}$

To plot √ 10 we can use the above concept

Step 1: Draw a number line and mark a distance of 3 units from 0 to 3 and mark the point as C

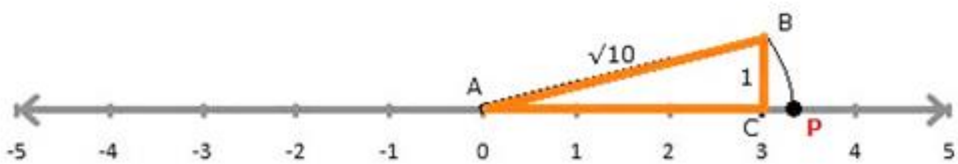
Step 2: Draw a perpendicular of 1 unit from C and mark it as B. Mark the point 0 as A.

Step 3: Join AB

Step 4: Δ ABC becomes a right angled triangle with AB = √10 units from Pythagoras theorem.

Step 5: Taking AB as radius draw an arc taking A as centre until it meets the number line. Mark the point as P.

Step 6: The point P is the required point √10 to be plot on the number line



Q. 8. Find at least two irrational numbers between 2 and 3.

Answer : First we square the two given numbers

$2^2 = 4$

$3^2 = 9$

Square root of any two numbers between 4 and 9 will give an irrational number since they are not perfect squares.

$\sqrt{5} = 2.236067977\ldots$

$\sqrt{7} = 2.645751311\ldots$

So √5 and √7 are the two irrational numbers between 2 and 3

Q. 9. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.
(ii) Every rational number is a real number.
(iii) Every real number need not be a rational number
(iv) √n is not irrational if n is a perfect square.
(v) √n is irrational if n is not a perfect square.
(vi) All real numbers are irrational.

Answer : (i) The statement is true.

Reason: Each and every irrational number can be plotted in the real number line and hence they are all real.

(ii) The statement is true.

Reason: Each and every rational number can be plotted in the real number line and hence they are all real.

(iii) The statement is true.

Reason: The set of rational number is a subset of real number. The set of real number is composed of both rational and irrational number.

(iv) The statement is true

Reason: Square root of a positive real number is either a rational number if it's a perfect square or else it is an irrational number.

(v) The statement is true

Reason: Square root of a positive real number is either a rational number if it's a perfect square or else it is an irrational number.

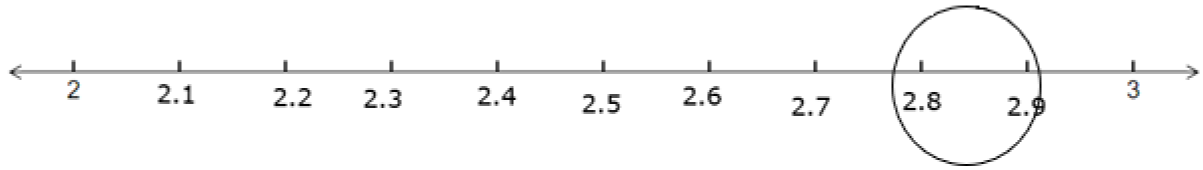
(vi) The statement is false.

Reason: The set of irrational number is a subset of real number. The set of real number is composed of both rational and irrational number.

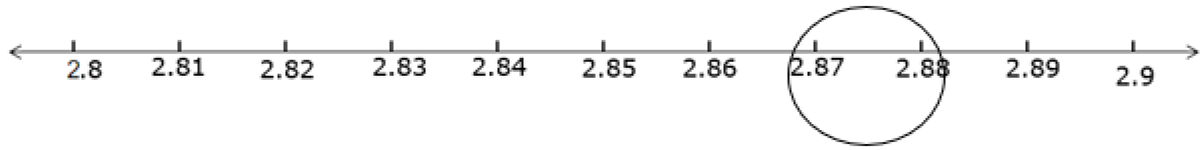
Exercise 1.3

Q. 1. Visualise 2.874 on the number line, using successive magnification.

Answer : We know that 2.874 lies between 2 and 3. So, let us decide the part of the number line between 2 and 3 into 10 equal parts and look at the portion between 2.8 and 2.9 through a magnifying glass.



Now, 2.874 lies between 2.8 and 2.9. Now, we imagine to divide this again into 10 equal parts. The first mark will represent 2.81, then next 2.82 and so on.



Again, 2.875 lies 2.87 and 2.88. Let us imagine and divide into 10 equal parts.

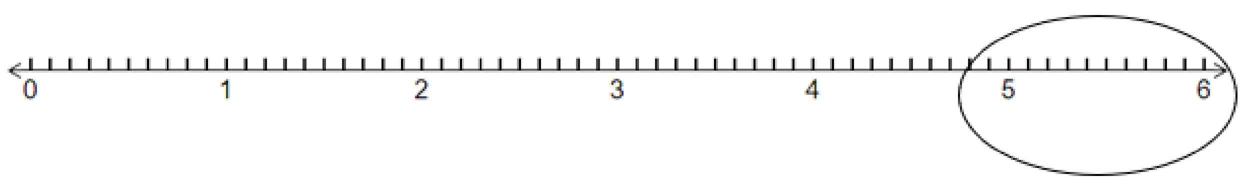
This is representation of numbers on the number line through a magnification glass.

Thus, we can visualize that 2.871 is the first mark 2.874 is the 4th mark in these subdivisions.

Q. 2. Visualilse $5.\overline{28}$ on the number line, upto 3 decimal places.

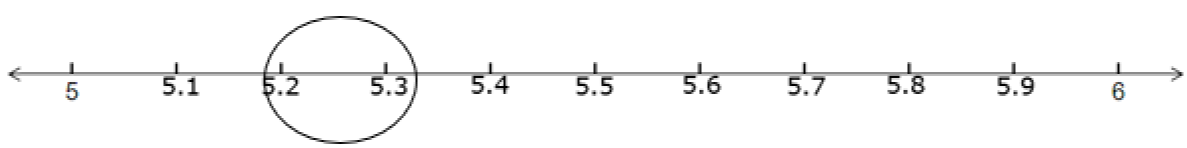
Answer : <u>Step 1:</u>

Here, the number $5.\overline{28}$ lies between 5 and 6. First, draw the number line.



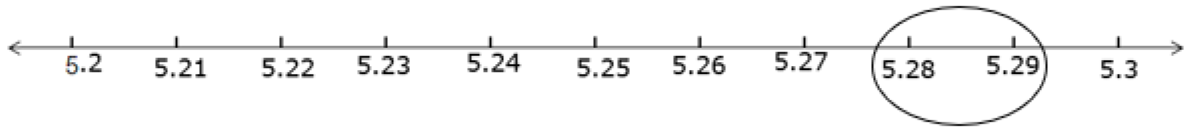
Step 2:

Divide the above part into 10 equal parts and make first point to the right of 5 as 5.1, the second 5.2 and so on.



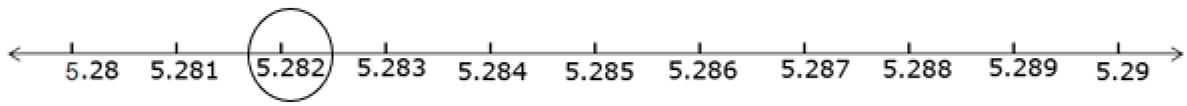
Step 3:

Now, 5.28 lies between 5.2 and 5.3. So, by focusing on this portion, divide it into 10 equal parts and mark first point to the right of 5.2 as 5.21, the second 5.22 and so on.



Step 4:

Now, 5.282 lies between 5.28 and 5.29. So, focus on this portion and again divide it into 10 equal portions and mark first point to the right of 5.28 as 5.281 and the second one as 5.282.



The number 5.282 is the 2^{nd} mark in these subdivisions.

Exercise 1.4

Q. 1. Simplify the following expressions.

i. $\left(5+\sqrt{7}\right)\left(2+\sqrt{5}\right)$

ii. $\left(5+\sqrt{5}\right)\left(5-\sqrt{5}\right)$

iii. $\left(\sqrt{3}+\sqrt{7}\right)^2$

iv. $\left(\sqrt{11}-\sqrt{7}\right)\left(\sqrt{11}+\sqrt{7}\right)$

Answer : i. (5 + √7) (2 + √5)

<u>We know that (a + √b) (c + √d) = ac + a√d + √b c + √b (√d).</u>

Comparing with the given expression,

⇒ a = 5; b = 7; c = 2; d = 5

$\therefore (5 + \sqrt{7})(2 + \sqrt{5}) = 5(2) + 5(\sqrt{5}) + \sqrt{7}(2) + \sqrt{7}(\sqrt{5})$

$= 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$

ii. $(5 + \sqrt{5})(5 - \sqrt{5})$

<u>We know that $(a + \sqrt{b})(a - \sqrt{b}) = a^2 – b$.</u>

Comparing with the given expression,

$\Rightarrow a = 5; b = 5$

$\therefore (5 + \sqrt{5})(5 - \sqrt{5}) = 5^2 – (\sqrt{5})^2$

$= 25 – 5 = 20$

$\therefore (5 + \sqrt{5})(5 - \sqrt{5}) = 20$

iii. $(\sqrt{3} + \sqrt{7})^2$

<u>We know that $(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{(ab)} + b$.</u>

Comparing with the given expression,

$\Rightarrow a = 3; b = 7$

$\therefore (\sqrt{3} + \sqrt{7})^2 = 3 + 2(\sqrt{3})(\sqrt{7}) + 7$

$= 3 + 2(\sqrt{21}) + 7$

$= 10 + 2\sqrt{21}$

$\therefore (\sqrt{3} + \sqrt{7})^2 = 10 + 2\sqrt{21}$

iv. $(\sqrt{11} - \sqrt{7})(\sqrt{11} + \sqrt{7})$

<u>We know that $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a – b$.</u>

Comparing with the given expression,

$\Rightarrow a = 11; b = 7$

$\therefore (\sqrt{11} - \sqrt{7})(\sqrt{11} + \sqrt{7}) = 11 – 7$

$= 4$

Q. 2. Classify the following numbers as rational or irrational.

i. $5-\sqrt{3}$

ii. $\sqrt{3}+\sqrt{2}$

iii. $\left(\sqrt{2}-2\right)^2$

iv. $\frac{2\sqrt{7}}{7\sqrt{7}}$

v. 2π

vi. $\frac{1}{\sqrt{3}}$

vii. $\left(2+\sqrt{2}\right)\left(2-\sqrt{2}\right)$

Answer : i. 5 - √3

Here, 5 is a rational number and √3 is an irrational number.

We know that subtraction of a rational number and an irrational number always gives an irrational number.

∴ 5 - √3 is an irrational number.

ii. √3 + √2

Here, both √3 and √2 are irrational numbers.

We know that sum of two irrational number is always an irrational number.

∴ √3 + √2 is an irrational number.

iii. $(\sqrt{2} - 2)^2$

We know that $(\sqrt{a} - b)^2 = a - 2\sqrt{a}(b) + b^2$.

Comparing with the given expression,

⇒ a = 2; b = 2

On simplification, we get

$\Rightarrow (\sqrt{2} - 2)^2 = 2 - 2(\sqrt{2})(2) + 2^2$

$= 2 - 4\sqrt{2} + 4$

$= 6 - 4\sqrt{2}$

Here, 6 is a rational number and $4\sqrt{2}$ is an irrational number.

We know that subtraction of a rational number and an irrational number always gives an irrational number.

∴ $6 - 4\sqrt{2}$ is an irrational number.

iv. $\frac{2\sqrt{7}}{7\sqrt{7}}$

On simplification, we get

$\Rightarrow \frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$

Which is a rational number.

v. 2π

Here, 2 is a rational number and π is an irrational number.

We know that the product of a rational number and an irrational number is always an irrational number.

∴ 2π is an irrational number.

vi. $\frac{1}{\sqrt{3}}$

Here, 1 is a rational number and $\sqrt{3}$ is an irrational number.

We know that division of a rational number and an irrational number gives an irrational number.

∴ $\frac{1}{\sqrt{3}}$ is an irrational number.

vii. $(2 + \sqrt{2})(2 - \sqrt{2})$

We know that $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$.

Comparing with the given expression,

⇒ a = 2; b = 2

∴ $(2 + \sqrt{2})(2 - \sqrt{2}) = 2^2 - (\sqrt{2})^2$

= 4 – 2 = 2

Here, 2 is a rational number.

∴ $(2 + \sqrt{2})(2 - \sqrt{2})$ is rational.

Q. 3. In the following equations, find whether variables x, y, z etc. represent rational or irrational numbers.

i. $x^2 = 7$ ii. $y^2 = 16$

iii. $z^2 = 0.02$ iv. $u^2 = \frac{17}{4}$

v. $w^2 = 27$ vi. $t^4 = 256$

Answer : i. $x^2 = 7$

⇒ $x = \sqrt{7}$

Here, $\sqrt{7}$ is an irrational number.

∴ x is an irrational number.

ii. $y^2 = 16$

⇒ $y = \sqrt{16} = \pm 4$

Here, 4 is a rational number.

∴ y is a rational number.

iii. $z^2 = 0.02$

⇒ $z = \sqrt{0.02}$

Here, $\sqrt{0.02}$ is an irrational number.

∴ z is an irrational number.

iv. $u^2 = \frac{17}{4}$

$\Rightarrow u = \frac{\sqrt{17}}{\sqrt{4}} = \frac{\sqrt{17}}{2}$

Here, √17 is an irrational number and 2 is a rational number.

We know that division of a rational number and an irrational number gives an irrational number.

∴ u is an irrational number.

v. $w^2 = 27$

⇒ w = √27 = 3√3

Here, √3 is an irrational number.

∴ w is an irrational number.

vi. $t^4 = 256$

$\Rightarrow t = \sqrt[4]{256} = \sqrt[4]{4 \times 4 \times 4 \times 4} = 4$

Here, 4 is a rational number.

∴ t is a rational number.

Q. 4. The ratio of circumference to the diameter of a circle $\frac{c}{d}$ is represented by p. But we say that π is an irrational number. Why?

Answer : When we measure the length with a scale, we only get an approximate rational value, so we may not realize that in c and d which one is irrational.

So, either c or d is irrational and hence c/d is irrational i.e. π is irrational.

∴ π is an irrational number.

Q. 5. Rationalise the denominators of the following:

i. $\frac{1}{3+\sqrt{2}}$

ii. $\frac{1}{\sqrt{7}-\sqrt{6}}$

iii. $\frac{1}{\sqrt{7}}$

iv. $\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}$

Answer : i. $\frac{1}{3+\sqrt{2}}$

Here, the conjugate of the denominator (3 + √2) is (3 - √2).

By rationalizing,

$\Rightarrow \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$

We know that (a + √b) (a - √b) = a^2 – b.

$\Rightarrow \frac{3-\sqrt{2}}{9-2} = \frac{3-\sqrt{2}}{7}$

$\therefore \frac{1}{3+\sqrt{2}} = \frac{3-\sqrt{2}}{7}$

ii. $\frac{1}{\sqrt{7}-\sqrt{6}}$

Here, the conjugate of the denominator (√7 - √6) is (√7 + √6).

By rationalizing,

$\Rightarrow \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$

We know that (√a + √b) (√a - √b) = a – b.

$\Rightarrow \dfrac{\sqrt{7}+\sqrt{6}}{7-6} = \dfrac{\sqrt{7}+\sqrt{6}}{1} = \sqrt{7}+\sqrt{6}$

$\therefore \frac{1}{\sqrt{7}-\sqrt{6}} = \sqrt{7}+\sqrt{6}$

iii. $\frac{1}{\sqrt{7}}$

By rationalizing the denominator,

$\Rightarrow \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$

$\therefore \frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{7}$

iv. $\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}$

Here, the conjugate of the denominator (√3 - √2) is (√3 + √2).

By rationalizing,

$\Rightarrow \frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

<u>We know that (√a + √b) (√a - √b) = a – b.</u>

$\Rightarrow \dfrac{\sqrt{6}(\sqrt{3}+\sqrt{2})}{3-2}$

$= \dfrac{\sqrt{6}\left(\sqrt{3}+\sqrt{2}\right)}{1}$

$= \sqrt{6}\left(\sqrt{3}+\sqrt{2}\right)$

$= 3\sqrt{2} + 2\sqrt{3}$

$\therefore \frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} = 3\sqrt{2}+2\sqrt{3}$

Q. 6. Simplify each of the following by rationalising the denominator:

i. $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

ii. $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$

iii. $\frac{1}{3\sqrt{2}-2\sqrt{3}}$

iv. $\frac{3\sqrt{5}-\sqrt{7}}{3\sqrt{3}+\sqrt{2}}$

Answer :

i. $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Rationalizing the denominator by its conjugate,

$\Rightarrow \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}}$

We know that (a - √b) (a + √b) = a^2 – b.

We know that (a - √b)^2 = a^2 – 2a√b + b.

$\Rightarrow \frac{36-2(6)4\sqrt{2}+32}{36-32}$

$\Rightarrow \frac{68-48\sqrt{2}}{4}$

$\Rightarrow \frac{4(17-12\sqrt{2})}{4}$

⇒ 17 – 12√2

ii. $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$

Rationalizing the denominator, we get

$\Rightarrow \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

We know that (√a - √b) (√a + √b) = a – b.

We know that (√a - √b)^2 = a - 2√(ab) + b.

$\Rightarrow \frac{7-2\sqrt{7}(\sqrt{5})+5}{7-5}$

$\Rightarrow \frac{12-2(\sqrt{35})}{2}$

$\Rightarrow \frac{2(6-\sqrt{35})}{2}$

⇒ 6 - √35

iii. $\frac{1}{3\sqrt{2}-2\sqrt{3}}$

Rationalizing the denominator,

$\Rightarrow \frac{1}{3\sqrt{2}-2\sqrt{3}} = \frac{1}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

We know that (√a - √b) (√a + √b) = a – b.

⇒

$$\frac{1}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = \frac{3\sqrt{2}+2\sqrt{3}}{18-12}$$

$$= \frac{3\sqrt{2}+2\sqrt{3}}{6}$$

iv. $\frac{3\sqrt{5}-\sqrt{7}}{3\sqrt{3}+\sqrt{2}}$

Rationalizing the denominator,

$\Rightarrow \frac{3\sqrt{5}-\sqrt{7}}{3\sqrt{3}+\sqrt{2}} = \frac{3\sqrt{5}-\sqrt{7}}{3\sqrt{3}+\sqrt{2}} \times \frac{3\sqrt{3}-\sqrt{2}}{3\sqrt{3}-\sqrt{2}}$

<u>We know that (√a - √b) (√a + √b) = a – b.</u>

$\Rightarrow \frac{3\sqrt{5}-\sqrt{7}}{3\sqrt{3}+\sqrt{2}} \times \frac{3\sqrt{3}-\sqrt{2}}{3\sqrt{3}-\sqrt{2}} = \frac{(3\sqrt{5}-\sqrt{7})(3\sqrt{3}-\sqrt{2})}{27-2}$

$= \frac{9\sqrt{15}-3\sqrt{10}-3\sqrt{21}+\sqrt{14}}{25}$

Q. 7. Find the value of $\frac{\sqrt{10}-\sqrt{5}}{2\sqrt{2}}$ upto three decimal places. (Take √2 = 1.414 and √5 = 2.236)

Answer : Given √2 = 1.414 and √5 = 2.236

Given $\frac{\sqrt{10}-\sqrt{5}}{2\sqrt{2}}$

Rationalizing the denominator,

$\Rightarrow \frac{\sqrt{10}-\sqrt{5}}{2\sqrt{2}} = \frac{\sqrt{10}-\sqrt{5}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\Rightarrow \frac{\sqrt{2}(\sqrt{10}-\sqrt{5})}{4}$

$\Rightarrow \frac{\sqrt{2}(\sqrt{10})-\sqrt{2}(\sqrt{5})}{4} = \frac{2(\sqrt{5})-\sqrt{2}(\sqrt{5})}{4}$

$\Rightarrow \frac{2(2.236)-(1.414)(2.236)}{4}$

$\Rightarrow \frac{4.472-3.161}{4}$

$\Rightarrow \frac{1.311}{4} = 0.327$

Q. 8. Find:

i. $64^{\frac{1}{6}}$ **ii.** $32^{\frac{1}{5}}$
iii. $625^{\frac{1}{4}}$ **iv.** $16^{\frac{3}{2}}$
v. $243^{\frac{2}{5}}$ **vi.** $(46656)^{\frac{-1}{6}}$

Answer : i. $64^{\frac{1}{6}}$

$\Rightarrow 64^{\frac{1}{6}} = (2^6)^{\frac{1}{6}}$

<u>We know that $(a^m)^n = a^{mn}$.</u>

$\Rightarrow (2^6)^{\frac{1}{6}} = 2^{6\times\frac{1}{6}} = 2$

∴ $64^{\frac{1}{6}}$ = 2

ii. $32^{\frac{1}{5}}$

⇒ $32^{\frac{1}{5}} = (2^5)^{\frac{1}{5}}$

We know that $(a^m)^n = a^{mn}$.

⇒ $(2^5)^{\frac{1}{5}} = 2^{5\times\frac{1}{5}} = 2$

∴ $32^{\frac{1}{5}}$ = 2

iii. $625^{\frac{1}{4}}$

⇒ $625^{\frac{1}{4}} = (5^4)^{\frac{1}{4}}$

We know that $(a^m)^n = a^{mn}$.

⇒ $(5^4)^{\frac{1}{4}} = 5^{4\times\frac{1}{4}} = 5$

∴ $625^{\frac{1}{4}}$ = 5

iv. $16^{\frac{3}{2}}$

⇒ $16^{\frac{3}{2}} = (4^2)^{\frac{3}{2}}$

We know that $(a^m)^n = a^{mn}$.

⇒ $(4^2)^{\frac{3}{2}} = 4^{2\times\frac{3}{2}} = 4^3 = 64$

∴ $16^{\frac{3}{2}}$ = 64

v. $243^{\frac{2}{5}}$

$\Rightarrow 243^{\frac{2}{5}} = (3^5)^{\frac{2}{5}}$

We know that $(a^m)^n = a^{mn}$.

$\Rightarrow (3^5)^{\frac{2}{5}} = 3^{5\times\frac{2}{5}} = 3^2 = 9$

$\therefore 243^{\frac{2}{5}} = 9$

vi. $(46656)^{\frac{-1}{6}}$

$\Rightarrow \frac{1}{(46656)^{\frac{1}{6}}} = \frac{1}{(6^6)^{\frac{1}{6}}}$

We know that $(a^m)^n = a^{mn}$.

$\Rightarrow \frac{1}{(6^6)^{\frac{1}{6}}} = \frac{1}{6^{6\times\frac{1}{6}}} = \frac{1}{6}$

$\therefore (46656)^{\frac{-1}{6}} = \frac{1}{6}$

Q. 9. Simplify :

$\sqrt[4]{81} - 8\sqrt[3]{343} + 15\sqrt[5]{32} + \sqrt{225}$

Answer : Given $\sqrt[4]{81} - 8\sqrt[3]{343} + 15\sqrt[5]{32} + \sqrt{225}$

It can also be written as

$\Rightarrow 81^{\frac{1}{4}} - 8(343)^{\frac{1}{3}} + 15(32)^{\frac{1}{5}} + (225)^{\frac{1}{2}}$

$\Rightarrow (3^4)^{\frac{1}{4}} - 8(7^3)^{\frac{1}{3}} + 15(2^5)^{\frac{1}{5}} + (15^2)^{\frac{1}{2}}$

We know that $(a^m)^n = a^{mn}$.

$\Rightarrow 3^{4\times\frac{1}{4}} - 8(7^{3\times\frac{1}{3}}) + 15(2^{5\times\frac{1}{5}}) + 15^{2\times\frac{1}{2}}$

⇒ 3 – 8(7) + 15(2) + 15

⇒ 3 – 56 + 30 + 15 = -8

$\therefore \sqrt[4]{81} - 8\sqrt[3]{343} + 15\sqrt[5]{32} + \sqrt{225} = -8$

Q. 10. If 'a' and 'b' are rational numbers, find the value of a and b in each of the following equations.

i. $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}$

ii. $\dfrac{\sqrt{5}+\sqrt{3}}{2\sqrt{5}-3\sqrt{3}} = a - b\sqrt{15}$

Answer :

i. Given $a + b\sqrt{6} = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Rationalizing the denominator,

$\Rightarrow a + b\sqrt{6} = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

<u>We know that (√a - √b) (√a + √b) = a – b.</u>

<u>We know that $(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{(ab)} + b$.</u>

$\Rightarrow a + b\sqrt{6} = \frac{3+2\sqrt{6}+2}{3-2}$

$= \frac{5+2\sqrt{6}}{1}$

= 5 + 2√6

Comparing it with a + b√6, we get

⇒ a = 5 and b = 2

ii. Given a – b√15 = $\frac{\sqrt{5}+\sqrt{3}}{2\sqrt{5}-3\sqrt{3}}$

Rationalizing the denominator,

⇒ a – b√15 = $\frac{\sqrt{5}+\sqrt{3}}{2\sqrt{5}-3\sqrt{3}} \times \frac{2\sqrt{5}+3\sqrt{3}}{2\sqrt{5}+3\sqrt{3}}$

We know that (√a - √b) (√a + √b) = a – b.

⇒ a – b√15 = $\frac{(\sqrt{5}+\sqrt{3})(2\sqrt{5}+3\sqrt{3})}{20-27}$

= $\frac{10+3\sqrt{15}+2\sqrt{15}+9}{-7}$

= $\frac{19+5\sqrt{15}}{-7}$

= $\frac{-19}{7} - \frac{5\sqrt{15}}{7}$

Comparing with a – b√15, we get

⇒ a = $\frac{-19}{7}$ and b = $\frac{5}{7}$