GUIDED REVISION

(COMPLEX NUMBER) - 1 | **MATHEMATICS**

TIME : 60 MIN. | **M.M. : 66**

SECTION–I(i)

Straight Objective Type (3 Marks each, –1 for wrong answer)

1. If z is a complex number satisfying the equation $|z - (1 + i)|^2 = 2$ and $\omega = \frac{2}{z}$, then the locus traced by 'ω' in the complex plane is

(A) $x - y - 1 = 0$ (B) $x + y - 1 = 0$ (C) $x - y + 1 = 0$ (D) $x + y + 1 = 0$

2. Let z_r $(1 \le r \le 4)$ be complex numbers such that $|z_r| = \sqrt{r+1}$

and $|30 z_1 + 20 z_2 + 15 z_3 + 12 z_4| = k\,|z_1z_2z_3 + z_2z_3z_4 + z_3z_4z_1 + z_4z_1z_2|$.

Then the value of k equals

(A) $|z_1z_2z_3|$ (B) $|z_2z_3z_4|$ (C) $|z_3z_4z_1|$ (D) $|z_4z_1z_2|$

3. Let z_1 & z_2 be non zero complex numbers satisfying the equation, $z_1^2 - 2z_1z_2 + 2z_2^2 = 0$. The geometrical nature of the triangle whose vertices are the origin and the points representing z_1 & z_2 is :

(A) an isosceles right angled triangle (B) a right angled triangle which is not isosceles

(C) an equilateral triangle (D) an isosceles triangle which is not right angled.

4. If $1, \alpha_1, \alpha_2, \ldots, \alpha_{2008}$ are $(2009)^{th}$ roots of unity, then the value of $\sum_{r=1}^{2008} r(\alpha_r + \alpha_{2009-r})$ equals

(A) 2009 (B) 2008 (C) 0 (D) – 2009

5. If $w = \alpha + i\beta$ where $\beta \ne 0$ and $z \ne 1$, satisfies the condition that $\frac{w - \bar{w}z}{1-z}$ is purely real, then the set of values of z is

(A) $\{z : |z| = 1\}$ (B) $\{z : z = \bar{z})$ (C) $\{z : z \ne 1\}$ (D) $\{z : |z| = 1, z \ne 1\}$

6. A man walks a distance of 3 units from the origin towards the North-East (N 45° E) direction. From there, he walks a distance of 4 units towards the North-West (N 45° W) direction to reach a point P. Then the position of P in the Argand plane is

(A) $3e^{i\pi/4} + 4i$ (B) $(3 - 4i)e^{i\pi/4}$ (C) $(4 + 3i)e^{i\pi/4}$ (D) $(3 + 4i)e^{i\pi/4}$

7. Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$ is

(A) 48 (B) 32 (C) 40 (D) 80

8. If P and Q are respectively by the complex numbers z_1 and z_2 such that $\left|\frac{1}{z_1}+\frac{1}{z_2}\right| = \left|\frac{1}{z_1}-\frac{1}{z_2}\right|$, then the circumcentre of ΔOPQ (where O is the origin) is

(A) $\frac{z_1 - z_2}{2}$ (B) $\frac{z_1 + z_2}{2}$ (C) $\frac{z_1 + z_2}{3}$ (D) $z_1 + z_2$

9. Let complex numbers α and $\frac{1}{\overline{\alpha}}$ lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$ respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha| =$

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{7}}$ (D) $\frac{1}{3}$

10. Suppose A is a complex number & $n \in N$, such that $A^n = (A + 1)^n = 1$, then the least value of *n* is
(A) 3 (B) 6 (C) 9 (D) 12

SECTION–I(ii)

Multiple Correct Answer Type (4 Marks each, –1 for wrong answer)

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11. If the expression $(1 + ir)^3$ is of the form of $s(1 + i)$ for some real 's' where 'r' is also real and $i = \sqrt{-1}$, then the value of 'r' can be

(A) $\cot\frac{\pi}{8}$ (B) $\sec\pi$ (C) $\tan\frac{\pi}{12}$ (D) $\tan\frac{5\pi}{12}$

12. Let z_1 and z_2 be two distinct complex numbers and let $z = (1 - t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg(w) denotes the principal argument of a nonzero complex number w, then

(A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$ (B) $\text{Arg}(z - z_1) = \text{Arg}(z - z_2)$

(C) $\begin{vmatrix} z - z_1 & \overline{z} - \overline{z}_1 \\ z_2 - z_1 & \overline{z}_2 - \overline{z}_1 \end{vmatrix} = 0$ (D) $\text{Arg}(z - z_1) = \text{Arg}(z_2 - z_1)$

13. Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $P^2 \neq 0$, when n =

(A) 57 (B) 55 (C) 58 (D) 56

SECTION–I(iii)

Linked Comprehension Type (Single Correct Answer Type) (3 Marks each, –1 for wrong answer)

Paragraph for question nos. 14 to 16

Let A, B, C be three sets of complex numbers as defined below.

$A = \{z : |z+1| \le 2 + \text{Re}(z)\}$, $B = \{z : |z-1| \ge 1\}$ and $C = \left\{z : \left|\dfrac{z-1}{z+1}\right| \ge 1\right\}$

14. The number of point(s) having integral coordinates in the region $A \cap B \cap C$ is

(A) 4 (B) 5 (C) 6 (D) 10

15. The area of region bounded by $A \cap B \cap C$ is

(A) $2\sqrt{3}$ (B) $\sqrt{3}$ (C) $4\sqrt{3}$ (D) 2

16. The real part of the complex number in the region $A \cap B \cap C$ and having maximum amplitude is

(A) –1 (B) $\dfrac{-3}{2}$ (C) $\dfrac{1}{2}$ (D) – 2

SECTION–I(v)

Matching list type (4 × 4) (Single option correct) (3 Marks each, –1 for wrong answer)

17. Let $z_k = \cos\left(\dfrac{2k\pi}{10}\right) + i\sin\left(\dfrac{2k\pi}{10}\right)$; k = 1, 2, ... 9.

	List-I		List-II
P.	For each z_k there exists a z_j such that $z_k \cdot z_j = 1$	1.	True
Q.	There exists a $k \in \{1, 2, \ldots, 9\}$ such that $z_1 \cdot z = z_k$ has no solution z in the set of complex numbers.	2.	False
R.	$\dfrac{\|1-z_1\|\|1-z_2\|\ldots\|1-z_9\|}{10}$ equals	3.	1
S.	$1 - \sum_{k=1}^{9} \cos\left(\dfrac{2k\pi}{10}\right)$ equals	4.	2

Codes :

	P	Q	R	S
(A)	1	2	4	3
(B)	2	1	3	4
(C)	1	2	3	4
(D)	2	1	4	3

SECTION–II (i)

Numerical Grid Type (Single digit Ranging from 000 to 999) (4 Marks each, –1 for wrong answer)

18. The minimum value of the expression $E = |z|^2 + |z-3|^2 + |z-6i|^2$ (where $z = x + iy$, $x, y \in R$), is

19. Let $z_1, z_2 \in C$ such that $z_1^2 + z_2^2 \in R$. If $z_1\left(z_1^2 - 3z_2^2\right) = 10$ and $z_2\left(3z_1^2 - z_2^2\right) = 30$.

The value of $\left(z_1^2 + z_2^2\right)$, is

20. If the biquadratic $x^4 + ax^3 + bx^2 + cx + d = 0$ $(a, b, c, d \in R)$ has 4 non real roots, two with sum $3 + 4i$ and the other two with product $13 + i$. The value of 'b' is

(COMPLEX NUMBER-01) **MATHEMATICS**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	D	A	D	D	D	A	B	C	B
	Q.	11	12	13	14	15	16	17			
	A.	B,C,D	A,C,D	B,C,D	B	A	B	C			
SECTION-II	Q.	18	19	20							
	A.	030	010	051							