Class XI Session 2023-24
Subject - Mathematics
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. $\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ}$ = ? **[1]**

   a) tan 52° b) tan 37°

   c) None of these d) tan 8°

2. The domain and range of real function f defined by f (x) $= \sqrt{x-1}$ is given by **[1]**

   a) Domain $= [\infty, \infty)$, Range $= [0, \infty)$ b) Domain $= [1, \infty)$, Range $= (\infty, \infty)$

   c) Domain $= [1, \infty)$, Range $= [0, \infty)$ d) Domain $= (1, \infty)$, Range $= (0, \infty)$

3. The mean and the variance of 10 observations are given to be 4 and 2 respectively. If every observation is multiplied by 2, the mean and the variance of the new series will be respectively. **[1]**

   a) 8 and 4 b) 8 and 20

   c) 8 and 8 d) 80 and 40

4. If G(x) = $\sqrt{25 - x^2}$ then $\lim\limits_{x \to 1} \frac{G(x) - G(1)}{x - 1}$ has the value **[1]**

   a) $\frac{1}{24}$ b) $-\sqrt{24}$

   c) $\frac{-1}{\sqrt{24}}$ d) $\frac{1}{5}$

5. The equations of the sides AB, BC and CA of $\triangle$ABC are y - x = 2, x + 2y = 1 and 3x + y + 5 = 0 respectively. The equation of the altitude through B is **[1]**

   a) 3x - y + 2 = 0 b) x - 3y + 4 = 0

   c) x - 3y + 1 = 0 d) None of these

6. Equation of y-axis is considered as **[1]**

a) y = 0, z = 0

b) none of these

c) z = 0, x = 0

d) x = 0, y = 0

7. If z = x + iy ; x, y ∈ R then : **[1]**

a) $z\bar{z} < |z|^2$

b) $z\bar{z} = |z|^2$

c) $z\bar{z} > |z|^2$

d) none of these

8. ${}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$ is equal to **[1]**

a) 33

b) 30

c) 31

d) 32

9. $\lim\limits_{x \to 0} \frac{\tan 2x - x}{3x - \sin x}$ is equal to **[1]**

a) $\frac{1}{2}$

b) 2

c) $\frac{1}{4}$

d) -$\frac{1}{2}$

10. If the arcs of the same length in two circles subtend angles of 60° and 75° at their respective centres, the ratio of their radii is **[1]**

a) 5 : 3

b) 3 : 5

c) 5 : 4

d) 4 : 5

11. For any two sets A and B, $A \cap (A \cup B)$ = ... **[1]**

a) none of these

b) B

c) ϕ

d) A

12. The integral part of $(\sqrt{2} + 1)^6$ is **[1]**

a) 98

b) 96

c) 99

d) 100

13. If x = 99^{50} + 100^{50} and y = $(101)^{50}$ then **[1]**

a) x < y

b) x > y

c) x = y

d) $x \geq y$

14. If x and a are real numbers such that a > 0 and |x| > a, then **[1]**

a) $x \in (-a, \infty)$

b) $x \in (-\infty, -a) \cup (a, \infty)$

c) x ∈ (-a, a)

d) $x \in [-\infty, a]$

15. Let R be set of points inside a rectangle of sides a and b (a, b > 1) with two sides along the positive direction of x-axis and y-axis. Then **[1]**

a) R = {(x, y) : 0 ≤ x ≤ a, 0 ≤ y ≤ b}

b) R = {(x, y) : 0 ≤ x < a, 0 ≤ y ≤ b}

c) R = {(x, y) : 0 < x < a, 0 < y < b}

d) R = {(x, y) : 0 ≤ x ≤ a, 0 < y < b}

16. If A + B + C = π, then $\frac{\tan A + \tan B + \tan C}{\tan A \tan B \tan C}$ is equal to **[1]**

a) 1

b) tan A tan B tan C

c) None of these

d) 0

[1]

17. $\lim_{x \to \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$ is equal to

a) 1
b) $\frac{1}{2}$
c) 0
d) 2

18. Find r if ${}^{10}P_r = 2.{}^{9}P_r$ **[1]**

a) 6
b) 4
c) 3
d) 5

19. **Assertion (A):** if A = set of letters in **Alloy** B = set of letters in **LOYAL**, then set A & B are equal sets. **[1]**

Reason (R): If two sets have exactly the same elements, they are called equal sets.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The sum of infinite terms of a geometric progression is given by $S_\infty = \frac{a}{1-r}$, provided |r| < 1. **[1]**

Reason (R): The sum of n terms of Geometric progression is $S_n = \frac{a(r^n - 1)}{r-1}$.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Let A and B be two non-empty sets such that n(A) = 5, n(B) = 6 and $n(A \cap B)$ = 3. Find **[2]**

i. $n(A \times B)$
ii. $n(B \times A)$
iii. $n\{(A \times B) \cap (B \times A)\}$

OR

Find the values of a and b, if

i. (2a - 5, 4) = (5, b + 6)
ii. (a - 3, b + 7) = (3, 7)

22. Evaluate: $\lim_{x \to 0} \frac{\sin^2 4x^2}{x^4}$. **[2]**

23. If the odds against the occurrence of an event be 4 : 7, find the probability of the occurrence of the event. **[2]**

OR

A bag contains 8 red, 3 white and 9 blue balls. If three balls are drawn at random, determine the probability that all the three balls are blue balls.

24. For any two sets A and B prove by using properties of sets that: (A ∩ B) ∪ (A - B) = A. **[2]**

25. Find the equation of a line that has y-intercept 4 and is perpendicular to the line joining (2, -3) and (4, 2). **[2]**

Section C

26. In how many ways can six persons be seated in a row? **[3]**

27. Verify that (-1, 2, 1), (1, -2, 5), (4, -7, 8) and (2, -3, 4) are the vertices of a parallelogram. **[3]**

28. Using binomial theorem, expand: $\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$ **[3]**

OR

Evaluate: $(\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$

29. Differentiate $(x^2 + 1)(x - 5)$ from first principle. **[3]**

OR

Find the derivative of the following functions from first principle. $\frac{x+1}{x-1}$

30. If A.M. and G.M. of roots of a quadratic equation are 8 and 5 respectively then obtain the quadratic equation. **[3]**

OR

If the sum of an infinite decreasing G.P. is 3 and the sum of the squares of its term is $\frac{9}{2}$, then write its first term and common difference.

31. Out of 25 members in a family, 12 like to take tea, 15 like to take coffee and 7 like to take coffee and tea both. How many like **[3]**

i. at least one of the two drinks
ii. only tea but not coffee
iii. only coffee but not tea
iv. neither tea nor coffee

Section D

32. Calculate the mean deviation about the mean of the set of first n natural numbers when n is an even number. **[5]**

33. Find the vertex, axis, focus, directrix, latus - rectum of the following parabolas. Also, draw their rough sketches: $y = x^2 - 2x + 3$. **[5]**

OR

Find the eccentricity, centre, vertices, foci, minor axis, major axis, directrices and latus-rectum of the ellipse $25x^2 + 9y^2 - 150x - 90y + 225 = 0$.

34. Solve the following system of linear inequalities **[5]**

$-2 - \frac{x}{4} \geq \frac{1+x}{3}$ and $3 - x < 4(x-3)$

35. If $2 \tan\alpha = 3 \tan \beta$, prove that $\tan (\alpha - \beta) = \frac{\sin 2\beta}{5-\cos 2\beta}$. **[5]**

OR

If $\sec (x + \alpha) + \sec(x - \alpha) = 2 \sec x$, prove that $\cos x = \pm\sqrt{2}\cos \frac{\alpha}{2}$.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Method to Find the Sets When Cartesian Product is Given

For finding these two sets, we write first element of each ordered pair in first set say A and corresponding second element in second set B (say).

Number of Elements in Cartesian Product of Two Sets

If there are p elements in set A and q elements in set B, then there will be pq elements in $A \times B$ i.e. if n(A) = p and n(B) = q, then $n(A \times B) = pq$.

(i) The Cartesian product $A \times A$ has 9 elements among which are found (-1, 0) and (0, 1). Find the set A and the remaining elements of $A \times A$.

(ii) A and B are two sets given in such a way that $A \times B$ contains 6 elements. If three elements of $A \times B$ are (1, 3), (2, 5) and (3, 3), then find the remaining elements of $A \times B$.

(iii) If the set A has 3 elements and set B has 4 elements, then find the number of elements in $A \times B$.

OR

If $A \times B$ = {(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)}. Find A and B.

37. **Read the text carefully and answer the questions:** **[4]**

Four friends Dinesh, Yuvraj, Sonu, and Rajeev are playing cards. Dinesh, shuffling a cards and told to Rajeev choose any four cards.

(i) What is the probability that Rajeev getting all face card.

(ii) What is the probability that Rajeev getting two red cards and two black card.

(iii) What is the probability that Rajeev getting one card from each suit.

OR

What is the probability that Rajeev getting two king and two Jack cards.

38. **Read the text carefully and answer the questions:** **[4]**

We have, i = $\sqrt{-1}$. So, we can write the higher powers of i as follows

i. $i^2 = -1$

ii. $i^3 = i^2 \cdot i = (-1) \cdot i = -i$

iii. $i^4 = (i^2)^2 = (-1)^2 = 1$

iv. $i^5 = i^{4+1} = i^4 \cdot i = 1 \cdot i = i$

v. $i^6 = i^{4+2} = i^4 \cdot i^2 = 1 \cdot i^2 = -1$...

In order to compute i^n for n > 4, write

$i^n = i^{4q+r}$ for some q, r $\in$ N and $0 \leq r \leq 3$. Then, $i^n = i^{4q} \cdot i^r$

$=(i^4)^q \cdot i^r = (1)^q \cdot i^r = i^r$

In general for any integer k

$i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$ and $i^{4k+3} = -i$

(i) Find the value of i^{30}.

(ii) If $z = i^{-39}$, then find the simplest form of z.

Solution

Section A

1.

(b) tan 37°

Explanation: $\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \frac{1 - \tan 8^\circ}{1 + \tan 8^\circ} = \frac{\tan 45^\circ - \tan 8^\circ}{1 + \tan 45^\circ \tan 8^\circ} [\because 1 = \tan 45^\circ]$

= tan (45° - 8°) = tan 37°

2.

(c) Domain $= [1, \infty)$, Range $= [0, \infty)$

Explanation: We have, f(x) $= \sqrt{x - 1}$

Clearly, f(x) is defined if $x - 1 \geq 0$

$\Rightarrow \quad x \geq 1$

$\therefore$ Domain of f $= [1, \infty)$

Now for $x \geq 1, x - 1 \geq 0$

$\Rightarrow \quad \sqrt{x - 1} \geq 0$

$\Rightarrow$ Range of f $= [0, \infty)$

3.

(c) 8 and 8

Explanation: Let the observations be x_i' s, i = 1, 2, ..., 10 and the mean and variance of y_i' s are $\bar{x}$ = 4 and σ^2 = 2.

Now, let y_i = $2x_i'$ s and the mean and variance of y_i's and $\bar{y}$ and σ_1^2 then

$\bar{y} = \frac{\Sigma 2x_i}{10} = 2\frac{\Sigma 2x_i}{10} = 2\bar{x}$ = 8 and σ_1^2 = var(y_i's) = var ($2x_i'$s)

= 4 var (x_i's) = 4 $\times$ 2 = 8

Thus, the mean and variance of new series are 8 and 8.

4.

(c) $\frac{-1}{\sqrt{24}}$

Explanation: The equation is in the form of $\frac{0}{0}$

Using L' Hospital rule we have $\frac{\frac{1}{2\sqrt{25-x^2}} \cdot (-2x)}{1}$

Substituting x = 1 we get $\frac{-1}{\sqrt{24}}$

5.

(b) x - 3y + 4 = 0

Explanation: The equation of the sides AB, AC and CA of $\triangle$ABC are y - x = 2, x + 2y = 1 and 3x + y + 5 = 0, respectively.

Solving the equations of AB and BC, i,e, y - x = 2 and x + 2y = 1, we get

x = -1, y = 1

So, the coordinates of B are (-1, 1)

$\therefore$ Slope of AC = -3

Thus, slope of the altitude through B is $\frac{1}{3}$.

Equation of the required altitude is given below as per the general formula :

y - 1 = $\frac{1}{3}$ (x + 1)

$\Rightarrow$ x - 3y + 4 = 0.

6.

(c) z = 0, x = 0

Explanation: On y-axisis consider as x = 0 and z = 0

7.

(b) $z\bar{z} = |z|^2$

Explanation: If z = x + iy then $\bar{Z}$ = x - iy

Now $z\bar{z}$ = (x + iy) $\cdot$ (x - iy) = $x^2 + y^2$ = $|z|^2$ [$\because$ |z| = $\sqrt{x^2 + y^2}$]

8.

(c) 31

Explanation: ${}^5C_1 +^5 C_2 +^5 C_3 +^5 C_4 +^5 C_5$

= ${}^5C_1 +^5 C_2 +^5 C_2 +^5 C_1 +^5 C_5$

= $2 \times^5 C_1 + 2 \times^5 C_2 +^5 C_5$

= $2 \times 5 + 2 \times \frac{5!}{2!3!} + 1$

= 10 + 20 + 1

= 31.

9. **(a)** $\frac{1}{2}$

Explanation: Given, $\lim\limits_{x \to 0} \frac{\tan 2x - x}{3x - \sin x} = \lim\limits_{x \to 0} \frac{x\left[\frac{\tan 2x}{x} - 1\right]}{x\left[3 - \frac{\sin x}{x}\right]}$

$\lim\limits_{x \to 0} \frac{\frac{\tan 2x}{2x} \times 2 - 1}{3 - \frac{sinx}{x}} = \frac{1.2 - 1}{3 - 1} = \frac{2-1}{2} = \frac{1}{2}$

10.

(c) 5 : 4

Explanation: $\theta_1 = 60^\circ = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$ and $\theta_2 = 75^\circ = \left(75 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{12}\right)^c$

$\therefore l = r_1\theta_1 = r_2\theta_2$

$\Rightarrow r_1 \times \frac{\pi}{3} = r_2 \times \frac{5\pi}{12} \Rightarrow \frac{r_1}{r_2} = \left(\frac{5}{12} \times 3\right) = \frac{5}{4}$ $\Rightarrow$ $r_1 : r_2$ = 5 : 4

11.

(d) A

Explanation: Common between set A and $(A \cup B)$is set A itself

12.

(c) 99

Explanation: We have $(1 + x)^n = 1 +^n C_1(x) +^n C_2(x)^2 + ... + (x)^n$

Hence $(\sqrt{2} + 1)^6$ = 1 + ${}^6C_1(\sqrt{2}) +^6 C_2(\sqrt{2})^2 +^6 C_3(\sqrt{2})^3$ + ${}^6C_4(\sqrt{2})^4 +^6 C_5(\sqrt{2})^5 + (\sqrt{2})^6$

$\Rightarrow$ $(\sqrt{2} + 1)^6$ = 1 + 6($\sqrt{2}$) + 15 $\times$ 2 + 20 $\times$ 2($\sqrt{2}$) + 15 $\times$ 4 + 6 $\times$ 4($\sqrt{2}$) + 8

= 99 + 70$\sqrt{2}$

Hence integral part of $(\sqrt{2} + 1)^6$ = 99

13. **(a)** x < y

Explanation: Given $x = 99^{50} + 100^{50}$ and $y = (101)^{50}$

Now $y = (101)^{50} = (100 + 1)^{50} = {}^{50}C_0(100)^{50} +^{50} C_1(100)^{49} + {}^{50}C_2(100)^{48} + ... + {}^{50}C_{50}$ .....(i)

Also $(99)^{50} = (100 - 1)^{50} = =^{50} C_0(100)^{50} -^{50} C_1(100)^{49} + {}^{50}C_2(100)^{48} - .... + {}^{50}C_{50}$ ....(ii)

Now subtract equation (ii) from equation (i), we get

$(101)^{50} - (99)^{50} = 2\left[{}^{50}C_1 \quad (100)^{49} +^{50} C_3 \quad (100)^{47} + \ldots\right]$

= $2\left[50(100)^{49} + \frac{50 \times 49 \times 48}{3 \times 2 \times 1}(100)^{47} + \ldots\right]$

= $(100)^{50} + 2\left(\frac{50 \times 49 \times 48}{3 \times 2 \times 1}(100)^{47}\right)$

$\Rightarrow (101)^{50} - (99)^{50} > (100)^{50}$

$\Rightarrow (101)^{50} > (100)^{50} + (99)^{50}$

$\Rightarrow$ y > x

14.

(b) $x \in (-\infty, -a) \cup (a, \infty)$

Explanation: |x| > a

$\Rightarrow$ x < -a or x > a

$\Rightarrow$x $\in$ $(-\infty$, -a) $\cup$ (a, ∞)

15.

(c) R = {(x, y) : 0 < x < a, 0 < y < b}

Explanation: We have, R be set of points inside a rectangle of sides a and b

Since, a, b > 1

a and b cannot be equal to 0

Thus,R = {(x, y) : 0 < x < a, 0 < y < b}

16. **(a)** 1

Explanation: $\pi = 180^\circ$

Using tan (180 - A) = -tan A, we get;

C = π - (A + B)

Now,

$\frac{\tan A+\tan B+\tan C}{\tan A \tan B \tan C}$

$= \frac{\tan A+\tan B+\tan[\pi-(A+B)]}{\tan A \tan B \tan[\pi-(A+B)]}$

$= \frac{\tan A+\tan B-\tan(A+B)}{-\tan A \tan B \tan(A+B)}$

$= \frac{\tan A+\tan B-\frac{\tan A+\tan B}{1-\tan A \tan B}}{-\tan A \tan B\times\frac{\tan A+\tan B}{1-\tan A \tan B}}$

$= \frac{\tan A+\tan B-\tan^2 A\tan B-\tan A\tan^2 B-\tan A-\tan B}{-\tan^2 A\tan B-\tan A\tan^2 B}$

$= \frac{-\tan^2 A\tan B-\tan A\tan^2 B}{-\tan^2 A\tan B-\tan A\tan^2 B}$

= 1

17.

(d) 2

Explanation: Let $x - \frac{\pi}{4}$ = t

$\Rightarrow \lim_{t\to 0} \frac{\tan\left(\frac{\pi}{4}+t\right)-1}{t}$

$\Rightarrow \lim_{t\to 0} \frac{2\tan t}{(1-\tan t)(t)}$

= 2

18.

(d) 5

Explanation: Given $^{10}P_r = 2.^9 P_r$

$\Rightarrow \frac{10!}{(10-r)!} = 2 \cdot \frac{(9)!}{(9-r)!}$

$\Rightarrow \frac{10\times 9!}{(10-r)\times(9-r)!} = 2 \cdot \frac{(9)!}{(9-r)!}$

$\Rightarrow \frac{10}{(10-r)}$ = 2

$\Rightarrow$ 10 = 20 - 2r

$\Rightarrow$ 2r = 10

$\Rightarrow$ r = 5

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Both A and R are true but R is not the correct explanation of A.

Section B

21. Here we are given that , A and B are two non-empty sets such that n(A) = 5, n(B) = 6 and = 3

i. $n(A \times B) = n(A) \times n(B) = (5 \times 6) = 30$

ii. $n(B \times A) = n(B) \times n(A) = (6 \times 5) = 30$

iii. Given: $n(A \cap B) = 3$

$\therefore$ A and B have 3 elements in common

So, $(A \times B)$ and $(B \times A)$ have 3^2 = 9 elements in common.

Hence, $n\{(A \times B) \cap (B \times A)\} = 9$

OR

We know that two ordered pairs are equal if their corresponding elements are equal.

i. $(2a-5, 4) = (5, b+6) \Rightarrow 2a-5 = 5$ and $4 = b+6$ [equating corresponding elements]

$\Rightarrow 2a = 5+5$ and $4-6 = b$

$\Rightarrow 2a = 10$ and $-2 = b \Rightarrow a = 5$ and $b = -2$

ii. $(a-3, b+7) = (3, 7) \Rightarrow$ a - 3 = 3 and b + 7 = 7 [equating corresponding elements]

$\Rightarrow$ a = 3 + 3 and b = 7 - 7 $\Rightarrow$ a = 6 and b = 0

22. We have: $\lim\limits_{x \to 0}\left[\frac{\sin^2 4x^2}{x^4}\right]$

$= \lim\limits_{x \to 0}\left[\frac{\sin(4x^2)}{x^2} \times \frac{\sin(4x^2)}{x^2}\right]$

$= \lim\limits_{x \to 0}\left[\frac{\sin(4x^2)}{4x^2} \times 4 \times \frac{\sin(4x^2)}{4x^2} \times 4\right]$

$= 4 \times 4 \left[\because \lim\limits_{x \to 0}\frac{\sin x}{x} = 1\right]$

= 16

23. We know that,

If odds in favour of the occurrence an event are a: b, then the probability of an event to occur is $\frac{a}{a+b}$,

similarly, if odds are <u>not</u> in the favor of the occurrence an event are a: b, then the probability of <u>not</u> occurrence of the event is $\frac{a}{a+b}$

that is the probability of not occurring = $\frac{a}{a+b}$

We also know that,

Probability of occurring = 1 - the probability of not occurring

= 1 - $\frac{a}{a+b}$

= $\frac{b}{a+b}$

Given a = 4 and b = 7

Probability of occurrence = $\frac{7}{4+7}$

= $\frac{7}{11}$

OR

We have to find the probability that all the three balls are blue balls

Given: bag which contains 8 red, 3 white, 9 blue balls

Formula: $\mathrm{P(E)} = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

three balls are drawn at random therefore

Total possible outcomes of selecting two persons is $^{20}C_3$

Therefore n(S) = $^{20}C_3$ = 1140

let E be the event that all the balls are blue

E = {B, B, B}

n(E) = $^{9}C_3$ = 84

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{84}{1140} = \frac{7}{95}$

24. We can write, (A ∩ B) ∪ (A - B) = (A ∩ B) ∪ (A ∩ B')

= X ∪ (A ∩ B'), where X = A ∩ B

= (X ∪ A) ∩ (X ∪ B')

= A ∩ (A ∪ B') [∵ X ∪ A = (A ∩ B) ∪ A = A] [∵ A ∩ B ⊂ A]

=X ∪ B' = (A ∩ B) ∪ B'

⇒(A ∪ B') ∩ (B ∪ B')

⇒ (A ∪ B') ∩ U = A ∪ B'

= A [∵ A ⊂ A ∪ B']

25. Let m be the slope of the required line.

Since the required line is perpendicular to the line joining A (2, -3) and B (4, 2).

∴ m × Slope of AB = - 1 ⇒ m × $\frac{2+3}{4-2}$ = -1 ⇒ m = -$\frac{2}{5}$

The required line cuts off an intercept of length 4 on y-axis. So, c = 4

Substituting these values in y = mx + c, we obtain that the equation of the required line is

y = -$\frac{2}{5}$x + 4

or, 2x+ 5y - 20 = 0

which is the required equation of line.

Section C

26. Given: Six persons are to be arranged in a row.

Assume six seats, now in the first seat, any one of six members can be seated, so the total number of possibilities is $^6C_1$

Similarly, in the second seat, any one of five members can be seated, so the total number of possibilities is $^5C_1$

In the third seat, any one of four members can be seated, so the total number of possibilities is $^4C_1$

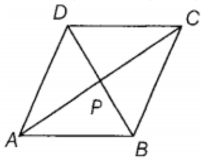
In the fourth seat, any one of three members can be seated, so the total number of possibilities is $^3C_1$

In the fifth seat, any one of two members can be seated, so the total number of possibilities is $^2C_1$

In the sixth seat, only one remaining person can be seated, so the total number of possibilities is $^1C_1$

Hence the total number of possible outcomes = $^6C_1 \times {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1 = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

27. Let A (-1, 2, 1), B(1, - 2, 5), C(4, - 7, 8) and D (2, - 3 , 4) are the vertices of a quadrilateral ABCD.



Then, mid-point of

$AC = \left(\frac{-1+4}{2}, \frac{2-7}{2}, \frac{1+8}{2}\right) = \left(\frac{3}{2}, \frac{-5}{2}, \frac{9}{2}\right)$ $\left[\because \text{coordinates of mid-point} \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)\right]$

Similarly, mid-point of BD $= \left(\frac{3}{2}, -\frac{5}{2}, \frac{9}{2}\right)$

Mid-points of both the diagonals are the same (i.e., they bisect each other).

Hence, ABCD is a parallelogram.

28. To find: Expansion of $\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$ by means of binomial theorem

Formula used: ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

$(a + b)^n = {}^n C_0 a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \ldots\ldots + {}^n C_{n-1} a b^{n-1} + nC_n b^n$

Now here We have, $\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$

$= \left[6c_0\left(\frac{2x}{3}\right)^{6-0}\right] + \left[6c_1\left(\frac{2x}{3}\right)^{6-1}\left(-\frac{3}{2x}\right)^1\right] + \left[6c_2\left(\frac{2x}{3}\right)^{6-2}\left(-\frac{3}{2x}\right)^2\right]$

$+ \left[6c_3\left(\frac{2x}{3}\right)^{6-3}\left(-\frac{3}{2x}\right)^3\right] + \left[6\mathrm{C}_4\left(\frac{2\mathrm{x}}{3}\right)^{6-4}\left(-\frac{3}{2\mathrm{x}}\right)^4\right]$

$+ \left[6c_5\left(\frac{2x}{3}\right)^{6-5}\left(-\frac{3}{2x}\right)^5\right] + \left[6c_6\left(-\frac{3}{2x}\right)^6\right]$

$= \left[\frac{6!}{0!(6-0)!}\left(\frac{2x}{3}\right)^6\right] - \left[\frac{6!}{1!(6-1)!}\left(\frac{2x}{3}\right)^5\left(\frac{3}{2x}\right)\right] + \left[\frac{6!}{2!(6-2)!}\left(\frac{2x}{3}\right)^4\left(\frac{9}{4x^2}\right)\right] - \left[\frac{6!}{3!(6-3)!}\left(\frac{2x}{3}\right)^3\left(\frac{27}{8x^3}\right)\right]$

$+ \left[\frac{6!}{4!(6-4)!}\left(\frac{2x}{3}\right)^2\left(\frac{81}{16x^4}\right)\right] - \left[\frac{6!}{5!(6-5)!}\left(\frac{2x}{3}\right)^1\left(\frac{243}{32x^5}\right)\right] + \left[\frac{6!}{6!(6-6)!}\left(\frac{729}{64x^6}\right)\right]$

$= \left[1\left(\frac{64x^6}{729}\right)\right] - \left[6\left(\frac{32x^5}{243}\right)\left(\frac{3}{2x}\right)\right] + \left[15\left(\frac{16x^4}{81}\right)\left(\frac{9}{4x^2}\right)\right] - \left[20\left(\frac{8x^3}{27}\right)\right.$
$\left.\left(\frac{27}{8x^3}\right)\right] + \left[15\left(\frac{4x^2}{9}\right)\left(\frac{81}{16x^4}\right)\right] - \left[6\left(\frac{2x}{3}\right)\left(\frac{243}{32x^5}\right)\right] + \left[1\left(\frac{729}{64x^6}\right)\right]$

$= \frac{64}{729}x^6 - \frac{32}{27}x^4 + \frac{20}{3}x^2 - 20 + \frac{135}{4}\frac{1}{x^2} - \frac{243}{8}\frac{1}{x^4} + \frac{729}{64}\frac{1}{x^6}$

OR

To find: Value of $(\sqrt{3}+1)^5 - (\sqrt{3}-1)^5$

Formula used: ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(a+1)^5 = {}^5C_0a^5 + {}^5C_1a^{5-1}1 + {}^5C_2a^{5-2}1^2 + {}^5C_3a^{5-3}1^3 + {}^5C_4a^{5-4}1^4 + {}^5C_51^5$

$= {}^5C_0a^5 + {}^5C_1a^4 + {}^5C_2a^3 + {}^5C_3a^2 + {}^5C_4a + {}^5C_5$ ..... (i)

$(a - 1)^5 = \left[{}^5\mathrm{C}_0\mathrm{a}^5\right] + \left[{}^5\mathrm{C}_1\mathrm{a}^{5-1}(-1)^1\right] + \left[{}^5\mathrm{C}_2\mathrm{a}^{5-2}(-1)^2\right] + \left[{}^5\mathrm{C}_3\mathrm{a}^{5-3}(-1)^3\right] + \left[{}^5C_4a^{5-4}(-1)^4\right] + \left[{}^5C_5(-1)^5\right]|$

$= {}^5C_0a^5 - {}^5C_1a^4 + {}^5C_2a^3 - {}^5C_3a^2 + {}^5C_4a - {}^5C_5$ ..... (ii)

Subtracting (ii) from (i)

$(a+1)^5 - (a-1)^5 = [{}^5C_0a^5 + {}^5C_1a^4 + {}^5C_2a^3 + {}^5C_3a^2 + {}^5C_4a + {}^5C_5] - [{}^5C_0a^5 - {}^5C_1a^4 + {}^5C_2a^3 - {}^5C_3a^2 + {}^5C_4a - {}^5C_5]$

$= 2[{}^5C_1a^4 + {}^5C_3a^2 + {}^5C_5]$

$= 2\left[\left(\frac{5!}{11(5-1)!}a^4\right) + \left(\frac{5!}{3!(5-3)!}a^2\right) + \left(\frac{5!}{5!(5-5)!}\right)\right]$

$= 2[(5)a^4 + (10)a^2 + (1)]$

$= 2[5a^4 + 10a^2 + 1] = (a+1)^5 - (a-1)^5$

Putting the value of a, = $\sqrt{3}$ in the above equation we get..

$(\sqrt{3}+1)^5 - (\sqrt{3}-1)^5 = 2\left[5(\sqrt{3})^4 + 10(\sqrt{3})^2 + 1\right]$

$= 2[(5)(9) + (10)(3) + 1]$

$= 2[45 + 30 + 1]$

$= 152$

29. We need to find the derivative of $f(x) = (x^2 + 1)(x - 5)$

Derivative of a function f(x) from first principle is given by

$f'(x) = \lim_{h\to 0} = \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

$\therefore$ derivative of $f(x) = (x^2 + 1)(x - 5)$ is given as

$f'(x) = \lim_{h\to 0} = \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{\left\{(x+h)^2+1\right\}(x+h-5)-(x^2+1)(x-5)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{\left\{(x+h)^3+x+h-5(x+h)^2-5\right\}-(x^3-5x^2+x-5)}{h}$

Using $(a + b)^2 = a^2 + 2ab + b^2$ and $(a + b)^3 = a^3 + 3ab(a + b) + b^3$ we have:

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{\left\{x^3+3x^2h+3h^2x+h^3+x+h-5x^2-10hx-5h^2-5\right\}-(x^3-5x^2+x-5)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{\left\{3x^2h+3h^2x+h^3+h-10hx-5h^2\right\}}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{h\left\{3x^2+3hx+h^2+1-10x-5h\right\}}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \{3x^2 + 3hx + h^2 + 1 - 10x - 5h\}$

$\Rightarrow f'(x) = 3x^2 + 3(0)x + 0^2 + 1 - 10x - 5(0)$

$\Rightarrow f'(x) = 3x^2 - 10x + 1$

OR

Here $f(x) = \frac{x+1}{x-1}$

Then $f(x+h) = \frac{x+h+1}{x+h-1}$

We know that $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}}{h}$

$= \lim_{h\to 0} \frac{(x+h+1)(x-1)-(x+1)(x+h-1)}{h(x+h-1)(x-1)}$

$= \lim_{h\to 0} \frac{x^2+x+xh-h+x-1-x^2-xh+x-x-h+1}{h(x+h-1)(x-1)}$

$= \lim_{h\to 0} \frac{-2h}{h(x+h-1)(x-1)} = \frac{-2}{(x-1)^2}$

30. Let a and b be the roots of required quadratic equation.

Then A.M. = $\frac{a+b}{2}$ = 8

$\Rightarrow$

a + b = 16

And G.M. = $\sqrt{ab}$ = 5

$\Rightarrow$ ab = 25

Now, Quadratic equation x^2 - (Sum of roots) x + (Product of roots) = 0

$\Rightarrow x^2 - (a + b)x + ab = 0$

$\Rightarrow x^2 - 16x + 25 = 0$

Therefore, required equation is $x^2 - 16x + 25 = 0$

OR

Let us take a G.P. whose first is a and common difference is r.

$\therefore S_\infty = \frac{a}{1-r}$

$\Rightarrow \frac{a}{1-r}$ =3 ...(i)

And, sum of the terms of the G.P. a^2, $(ar)^2$, $(ar^2)^2$, ... ∞

$S_\infty = \frac{a^2}{1-r^2}$

$\Rightarrow \frac{a^2}{1-r^2} = \frac{9}{2}$ ...(ii)

$\Rightarrow 2a^2 = 9(1 - r^2)$

$\Rightarrow 2[3(1 - r)]^2 = 9 - 9r^2$ [From (i)]

$\Rightarrow 18(1 + r^2 - 2r) = 9 - 9r^2$

$\Rightarrow 18 - 9 + 18r^2 + 9r^2 - 36r = 0$

$\Rightarrow 27r^2 - 36r + 9 = 0$

$\Rightarrow 3(9r^2 - 12r + 3) = 0$

$\Rightarrow 9r^2 - 12r + 3 = 0$

$\Rightarrow 9r^2 - 9r - 3r + 3 = 0$

$\Rightarrow 9r(r - 1) -3(r - 1) = 0$

$\Rightarrow (9r - 3)(r - 1) = 0$

$\Rightarrow r = \frac{1}{3}$ and r = 1

But, r = 1 is not possible.

$\therefore r = \frac{1}{3}$

Now, substituting $r = \frac{1}{3}$ in $\frac{a}{1-r}$ = 3

a = $3\left(1 - \frac{1}{3}\right)$

$\Rightarrow a = 3 \times \frac{2}{3}$ = 2

Therefore the first term is 2 and common difference is $\frac{1}{3}$

31. Given that, n(T) = 12

n(C) = 15

n(T ∩ C) = 7

i. n(T ∪ C) = n(T) + n(C) - n(T ∩ C)
   = 12 + 15 - 7
   n(T ∪ C) = 20
   20 members like at least one of the two drinks.
ii. Only tea but not coffee
   = n(T) - n(T ∩ C)
   = 12 - 7
   = 5
iii. Only coffee but not tea
   = n(C) - n(T ∩ C)
   = 15 - 7
   = 8
iv. Neither tea nor coffee
   = n(U) - n(T ∪ C)
   = 25 - 20
   = 5

Section D

32. Given: set of first n natural numbers when n is an even number.

To find: the mean deviation about the mean

We know first n natural numbers are 1, 2, 3 ..., n. And given n is even number.

So mean is,

$\bar{x} = \frac{1+2+3+\cdots+n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{(n+1)}{2}$

The deviations of numbers from the mean are as shown below,

1 - $\frac{(n+1)}{2}$, 2 - $\frac{(n+1)}{2}$, 3 - $\frac{(n+1)}{2}$, ..., $\frac{(n-2)}{2}-\frac{(n+1)}{2}$, $\frac{(n)}{2}-\frac{(n+1)}{2}$, $\frac{(n+2)}{2}-\frac{(n+1)}{2}$, ..., n - $\frac{(n+1)}{2}$

Or,

$\frac{2-(n+1)}{2}, \frac{4-(n+1)}{2}, \frac{6-(n+1)}{2}, ..., \frac{n-2-(n+1)}{2}, \frac{n-(n+1)}{2}, \frac{(n+2)-(n+1)}{2}, ..., \frac{2n-(n+1)}{2}$

$\frac{2-(n+1)}{2}, \frac{4-(n+1)^2}{2}, \frac{6-(n+1)}{2}, ..., \frac{-3}{2}, \frac{-1}{2}, \frac{1}{2}, ..., \frac{2n-(n+1)}{2}$

Or,

$\frac{1-n}{2}, \frac{3-n}{2}, \frac{5-n}{2}, ..., \frac{-3}{2}, \frac{-1}{2}, \frac{-1}{2}, ..., \frac{n-1}{2}$

So the absolute values of deviation from the mean is

$|x_i - \bar{x}| = \frac{(n-1)}{2}, \frac{(n-3)}{2}, \frac{(n-5)}{2}, ..., \frac{3}{2}, \frac{1}{2}, \frac{1}{2}, ..., \frac{n-1}{2}$

The sum of absolute values of deviations from the mean, is

$\Sigma |x_i - \bar{x}| = \frac{(n-1)}{2} + \frac{(n-3)}{2} + \frac{(n-5)}{2}$ + ... + $\frac{3}{2} + \frac{1}{2} + \frac{1}{2}$ + ... + $\frac{n-1}{2}$

$\sum |x_i - \bar{x}| = \left(\frac{1}{2} + \frac{3}{2} + \cdots + \frac{(n-1)}{2}\right)\left(\frac{n}{2}\right)$

Now we know sum of first n natural numbers = n^2

Therefore, mean deviation about the mean is

$\mathrm{M.D} = \frac{\sum|\mathrm{x_i}-\bar{\mathrm{x}}|}{\mathrm{n}} = \frac{\left(\frac{1}{2}+\frac{3}{2}+\cdots+\frac{(\mathrm{n}-1)}{2}\right)\left(\frac{\mathrm{n}}{2}\right)}{\mathrm{n}}$

$M.D = \frac{\sum|x_i-\bar{x}|}{n} = \frac{\left(\frac{n}{2}\right)^2}{n}$

$\mathrm{M \cdot D} = \frac{\sum|\mathrm{x_i}-\bar{\mathrm{x}}|}{\mathrm{n}} = \frac{\mathrm{n}^2}{4\mathrm{n}} = \frac{\mathrm{n}}{4}$

33. Here the given equation are;

y = x^2 - 2x + 3

$\Rightarrow$ x^2 - 2x = y - 3

$\Rightarrow$ x^2 - 2x + 1 = y - 3 + 1

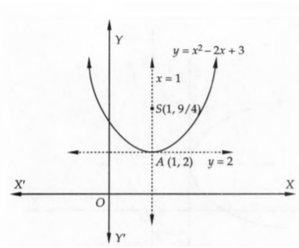
$\Rightarrow$ $(x - 1)^2$ = y - 2 .... (i)

Now, shifting the origin to the point (1, 2) without rotating the axes and denoting new coordinates with respect to these axes by X and Y, we get,

x = X + 1, y = Y + 2 .... (ii)

Using these relations, equation (i) reduces to

X^2 = Y .... (iii)



This is of the form X^2 = 4aY

Comparing, we get,

4a = 1 i,e, a = 1/4

Vertex: Coordinates of the vertex with respect to the new axes are (X = 0, Y = 0).

So, the coordinates of the vertex with respect to the old axes are (1, 2) [Put X = 0, Y = 0 in (ii)]

Axis: The equation of the axis of the parabola with respect to the new axes is X = 0

So, the equation of the axis with respect to the old axes is x = 1 [Put X = 0 in (ii)]

Focus: The coordinates of the focus with respect to the new axes are (X = 0, Y = a) i.e.

(X = 0, Y = 1/4)

So, the coordinates of the focus S with respect to the old axes are

(1, 9/4) [Put X = 0, Y = $\frac{1}{4}$ in (ii)]

Directrix: The equation of the directrix with respect to the new axes is Y = - a i.e. Y = - 1/4

So, the equation of the directrix with respect to the old axes is

y = - $\frac{1}{4} + 2$ or y = $\frac{7}{4}$ [Put Y = - $\frac{1}{4}$ in (ii)]

Latus-rectum: Length of the latus-rectum of given parabola is 4a = 1

OR

The equation of the ellipse is

$25x^2 + 9y^2 - 150x - 90y + 225 = 0$

$\Rightarrow 25x^{2} - 150x + 9y^2 - 90y = -225$

$\Rightarrow 25 (x^2 - 6x) + 9 (y^2 - 10y) = -225$

$\Rightarrow 25(x^2 - 6x + 9) + (y^2 - 10y + 25) = -225 + 225 + 225$

$\Rightarrow 25 (x - 3)^2 + 9 (y - 5)^2 = 225$

$\Rightarrow \frac{(x-3)^2}{9}+\frac{(y-5)^2}{25}=1$ ...(i)

Shifting the origin at (3, 5) without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we have .. (ii)

x = X + 3 and y = Y + 5

Using these relations, equation (i) reduces to

$\frac{X^2}{3^2}+\frac{Y^2}{5^2}=1$ ...(iii)

Comparing equation (iii) with standard form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we get

$a^2 = 32$ and $b^2 = 52$.

$\Rightarrow$ a = $4\sqrt{2}$ and b = $\sqrt{52}$

Clearly, a < b. So, equation (iii) represents an ellipse whose major and minor axes along Y and X axes respectively.

Eccentricity:

$e=\sqrt{1-\frac{a^2}{b^2}}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

Centre:

The coordinates of the centre with respect to new axes are (X = 0, Y = 0).

So, the coordinates of the centre with respect to old axes are (3, 5).

Vertices:

The vertices of the ellipse with respect to the new axes are (X = 0, Y = $\pm$ b) i.e. (X = 0, Y = $\pm$ 5). So, the vertices with respect to the old axes are

(3, 5 $\pm$ 5) i.e. (3, 0) and (3, 10) [Putting X = 0, Y = $\pm$ 5 in (ii)]

Foci:

The coordinates of the foci with respect to the old axes are (X = 0, Y = $\pm$ be) i.e. (X = 0, Y = $\pm$ 4). So, the coordinates of the foci with respect to the old axes are

(3, $\pm$ 4 + 5) i.e. (3, 1) and (3, 9) [Putting X = 0,Y = $\pm$ 4 in (ii)]

Directrices:

The equations of the directrices with respect to the new axes are $\mathrm{Y}=\pm\frac{b}{e}$ i.e. $\mathrm{Y}=\pm\frac{25}{4}$

So, the equations of the directrices with respect to the old axes are

y = $\pm\frac{25}{4}$ + 5 i.e. y = $-\frac{5}{4}$ and y = $\frac{45}{4}$ [Putting Y = $\pm\frac{25}{4}$ in (ii)]

Axes:

Lengths of the major axis = 2b - 10,

Lengths of the Minor axis = 2a = 6.

Equation of the major axis with respect to the new axes is X = 0. So, the equation of the major axis with respect to the old axes is x = 3. [Putting X = 0 in (ii)]

The equation of the minor axis with respect to the new axes is Y = 0. So, the equation of the minor axis with respect to the old axes is y = 5. [Putting Y = 0 in (ii)]

Latus-rectum: The length of the latus-rectum $=\frac{2a^2}{b}=\frac{2\times 9}{5}=\frac{18}{5}$

The equations of the latus-rectum with respect to the new axes are Y = $\pm$ ae i.e. y = + Y $\pm$ 4. So, the equations of the latus-recttum with respect to the old axes are

y = $\pm$ 4 + 5 i.e. y = 1 and y = 9. [Putting Y = $\pm$ 4 in (ii)]

34. The given system of linear inequalities is

- 2 - $\frac{x}{4}\geq\frac{1+x}{3}$ ... (i)

and 3 - x < 4 (x - 3) ... (ii)

From inequality (i), we get

- 2 - $\frac{x}{4}\geq\frac{1+x}{3}$

$\Rightarrow$ - 24 - 3x $\geq$ 4 + 4x [multiplying both sides by 12]

$\Rightarrow$ - 24 - 3x - 4 $\geq$ 4 + 4x - 4 [subtracting 4 from both sides]

$\Rightarrow$ - 28 - 3x $\geq$ 4x

$\Rightarrow$ - 28 - 3x + 3x $\geq$ 4x + 3x [adding 3x on both sides]

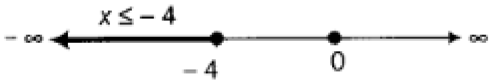
$\Rightarrow$ - 28 $\geq$ 7x

$\Rightarrow$ - $\frac{28}{7} \geq \frac{7x}{7}$ [dividing both sides by 7]

$\Rightarrow$ - 4 $\geq$ x or x $\leq$ - 4 ... (iii)

Thus, any value of x less than or equal to - 4 satisfied the inequality.

So, solution set is $x \in (-\infty, -4]$



From inequality (ii), we get

3 - x < 4 (x - 3)

$\Rightarrow$ 3 - x < 4x - 12

$\Rightarrow$ 3 - x + 12 < 4x - 12 + 12 [adding 12 on both sides]

$\Rightarrow$ 15 - x < 4x

$\Rightarrow$ 15 - x + x < 4x + x [adding x on both sides]

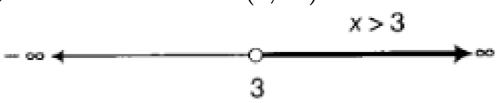
$\Rightarrow$ 15 < 5x

$\Rightarrow$ 3 < x [dividing both sides by 3]

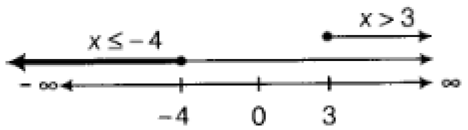
or x > 3 ... (iv)

Thus, any value of x greater than 3 satisfies the inequality.

So, the solution set is $x \in (3, \infty)$



The solution set of inequalities (i) and (ii) are represented graphically on number line as given below:



As no region is common, hence the given system has no solution.

35. LHS = tan (α - β) = $\frac{\sin 2\beta}{5-\cos 2\beta}$

$= \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$= \frac{\frac{3}{2}\tan\beta-\tan\beta}{1+\frac{3}{2}\tan\beta\tan\beta}$ ... [$\because$ 2 tanα = 3 tanβ $\Rightarrow$ tanα = $\frac{3}{2}$tanα]

$= \frac{\tan\beta\left(\frac{3}{2}-1\right)}{1+\frac{3}{2}\tan^2\beta}$

$= \frac{\frac{1}{2}\tan\beta}{1+\frac{3}{2}\tan^2\beta}$

$= \frac{\frac{1}{2}\frac{\sin\beta}{\cos\beta}}{1+\frac{3}{2}\cdot\left(\frac{\sin\beta}{\cos\beta}\right)^2}$ ... [$\because$ tanβ = $\frac{\sin\beta}{\cos\beta}$]

$= \frac{\frac{\sin\beta}{2\cos\beta}}{1+\frac{3\sin^2\beta}{2\cos^2\beta}}$

$= \frac{\frac{\sin\beta}{2\cos\beta}}{\frac{2\cos^2\beta+3\sin^2\beta}{2\cos^2\beta}}$

$= \frac{2\cos^2\beta\sin\beta}{2\cos\beta(2\cos^2\beta+3\sin^2\beta)}$

$= \frac{2\cos\beta\sin\beta}{2(2\cos^2\beta+3\sin^2\beta)}$

$= \frac{\sin 2\beta}{2(2\cos^2\beta)+3(2\sin^2\beta)}$ .. {$\because$ sin 2x = 2(sin x)(cos x)}

= $\frac{\sin 2\beta}{2(1+\cos 2\beta)+3(1-\cos 2\beta)}$ ... {$\because$ 2 $\cos^2$ x = 1 + cos 2x and 2 $\sin^2$ x = 1 - cos 2x}

= $\frac{\sin 2\beta}{2+2\cos 2\beta+3-3\cos 2\beta}$

= $\frac{\sin 2\beta}{5-\cos 2\beta}$

LHS = RHS

Hence Proved

OR

We have to prove that cos x = $\pm\sqrt{2}\cos\frac{\alpha}{2}$.

It is given that sec (x + α) + sec(x - α) = 2 sec x

$\Rightarrow \frac{1}{\cos(x+\alpha)}+\frac{1}{\cos(x-\alpha)} = \frac{2}{\cos x}$ ... [$\because$ sec x = $\frac{1}{\cos x}$]

$\Rightarrow \frac{\cos(x-\alpha)+\cos(x+\alpha)}{\cos(x+\alpha)\cos(x-\alpha)}=\frac{2}{\cos x}$ ... [$\because$ cos A + cos B = 2 cos $\frac{A+B}{2}\cos\frac{A-B}{2}$]

$\Rightarrow \frac{2\cos\left(\frac{x+\alpha+x-\alpha}{2}\right)\cos\left(\frac{x+\alpha-x+\alpha}{2}\right)}{\cos(x+\alpha)\cos(x-\alpha)}=\frac{2}{\cos x}$

$\Rightarrow \frac{2\cos\left(\frac{2x}{2}\right)\cos\left(\frac{2\alpha}{2}\right)}{2\cos(x+\alpha)\cos(x-\alpha)} = \frac{1}{\cos x}$ ... {$\because$ 2 cos A cos B = cos (A + B) + cos (A - B)}

$\Rightarrow \frac{2\cos x\cos\alpha}{\cos(x+\alpha+x-\alpha)+\cos(x+\alpha-x+\alpha)} = \frac{1}{\cos x}$

$\Rightarrow \frac{2\cos x\cos\alpha}{\cos 2x+\cos 2\alpha}=\frac{1}{\cos x}$

$\Rightarrow$ 2 $\cos^2$x cosα = cos 2x + cos 2α

$\Rightarrow$ 2 $\cos^2$x cosα = 2 $\cos^2$x - 1 + cos 2α ... {$\because$ cos 2x = 2 $\cos^2$ x - 1}

$\Rightarrow$ 2 $\cos^2$x cosα - 2 $\cos^2$x = cos 2α - 1

$\Rightarrow$ 2 $\cos^2$x (cosα - 1) = 2 cos 2α - 1 - 1 ... {$\because$ cos 2x = 2 $\cos^2$ x - 1}

$\Rightarrow$ 2 $\cos^2$x = $\frac{2\cos^2\alpha-2}{\cos\alpha-1}$

$\Rightarrow$ 2 $\cos^2$x = $\frac{2\left(\cos^2\alpha-1\right)}{\cos\alpha-1}$

$\Rightarrow$ 2 $\cos^2$x = $\frac{(\cos\alpha-1)(\cos\alpha+1)}{\cos\alpha-1}$

$\Rightarrow$ 2 $\cos^2$x = cosα + 1

$\Rightarrow$ 2 $\cos^2$x = 2 $\cos^2\frac{\alpha}{2}$ - 1 + 1 ... [$\pm\sqrt{2}\cos\frac{\alpha}{2}$ cos x = 2 $\cos^2\frac{x}{2}$ - 1]

$\Rightarrow$ 2 $\cos^2$x = 2 $\cos^2\frac{\alpha}{2}$

$\Rightarrow$ cos x = $\pm\sqrt{2\cos^2\frac{\alpha}{2}}$

$\Rightarrow$ cos x = $\pm\sqrt{2}\cos\frac{\alpha}{2}$

Hence Proved.

Section E

36. **Read the text carefully and answer the questions:**

Method to Find the Sets When Cartesian Product is Given

For finding these two sets, we write first element of each ordered pair in first set say A and corresponding second element in second set B (say).

Number of Elements in Cartesian Product of Two Sets

If there are p elements in set A and q elements in set B, then there will be pq elements in A $\times$ B i.e. if n(A) = p and n(B) = q, then n(A $\times$ B) = pq.

(i) n(A $\times$ A) = 9

$\Rightarrow$ n(A) $\subset$ n(A) = 9 $\Rightarrow$ n(A) = 3

(-1,0) $\in$ A $\times$ A $\Rightarrow$ -1 $\in$ A, 0 $\in$ A

(0,1) $\in$ A $\times$ A $\Rightarrow$ 0 $\in$ A, 1 $\in$ A

$\Rightarrow$ -1, 0, 1 $\in$ A

Also, n(A) = 3 $\Rightarrow$ A = (-1, 0, 1)

Hence, A = {-1, 0, 1}

Also, A $\times$ A = {-1, 0, 1} $\times$ {-1, 0, 1}

= {(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)}

Hence, the remaining elements of A $\times$ A are

(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0) and (1, 1).

(ii) Given, $n(A \times B) = 6$ and $(A \times B) = \{(1, 3), (2, 5), (3, 3)\}$

We know that Cartesian product of set A = {a, b} & B = {c, d} is $A \times B = \{(a, c), (a, d), (b, c), (b, d)\}$

Therefore, A = {1, 2, 3} & B = {3, 5}

$\Rightarrow A \times B = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}$

Thus, remaining elements are $A \times B = \{(1, 5), (2, 3), (3, 5)\}$

(iii) If the set A has 3 elements and set B has 4 elements, then the number of elements in $A \times B = 12$

OR

Clearly, A is the set of all first entries in ordered pairs in $A \times B$ and B is the set of all second entries in ordered pairs in $A \times B$

$\therefore$ A = {a, b} and B = {1, 2, 3}

37. **Read the text carefully and answer the questions:**

Four friends Dinesh, Yuvraj, Sonu, and Rajeev are playing cards. Dinesh, shuffling a cards and told to Rajeev choose any four cards.

(i) Total number of possible outcomes = ${}^{52}C_4$

We know that there are 12 face cards

$\therefore$ Number of favourable outcomes = ${}^{12}C_4$

$\therefore$ Required probability $= \frac{{}^{12}C_4}{{}^{52}C_4}$

(ii) Total number of possible outcomes = ${}^{52}C_4$

We know that there are 26 red and 26 black cards.

$\therefore$ Number of favourable outcomes = ${}^{26}C_2 \times {}^{26}C_2$

$\therefore$ Required probability $= \frac{\left({}^{26}C_2\right)^2}{{}^{52}C_4}$

(iii) Total number of possible outcomes = ${}^{52}C_4$

$\therefore$ Number of favourable outcomes = $\left({}^{13}C_1\right)^4$

$\therefore$ Required probability = $\frac{(13)^4}{{}^{52}C_4}$

OR

Total number of possible outcomes = ${}^{52}C_4$

In playing cards there are 4 king and 4 jack cards.

$\because$ Number of favourable outcomes = $\left({}^{4}C_2 \times {}^{4}C_2\right)$

$= 6 \times 6 = 36$

$\therefore$ Required probability = $\frac{36}{{}^{52}C_4}$

38. **Read the text carefully and answer the questions:**

We have, $i = \sqrt{-1}$. So, we can write the higher powers of i as follows

i. $i^2 = -1$

ii. $i^3 = i^2 \cdot i = (-1) \cdot i = -i$

iii. $i^4 = (i^2)^2 = (-1)^2 = 1$

iv. $i^5 = i^{4+1} = i^4 \cdot i = 1 \cdot i = i$

v. $i^6 = i^{4+2} = i^4 \cdot i^2 = 1 \cdot i^2 = -1$...

In order to compute i^n for $n > 4$, write

$i^n = i^{4q+r}$ for some $q, r \in N$ and $0 \leq r \leq 3$. Then, $i^n = i^{4q} \cdot i^r$

$= (i^4)^q \cdot i^r = (1)^q \cdot i^r = i^r$

In general for any integer k

$i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$ and $i^{4k+3} = -i$

(i) $i^{30} = (i)^{4\times7} i^2 = -1$

(ii) $i^{-39} = i(i^{-40})$

$= i((i^2)^{-20}) = i((-1)^{-20})$ $[\because i^2 - 1]$

$= i(\frac{1}{(-1)^{20}}) = i(\frac{1}{1}) = i = 0 + i(1)$

Comparing with a + ib,

a = 0, b = 1

0 + i