Work and Energy

Exercise Solutions

Solution 1:
Initial speed = 6000kmph = 5/3 m/sec

We know, K.E. = (1/2) mv^2 = 1/2 x 90 x $(5/3)^2$ = 125 Joules

Final speed = 12 kmph = 10/3 m/s

K.E. = 1/2 x 90 x $(10/3)^2$ = 500 Joules

Therefore, the increase in K.E. = 500 - 125 = 375 Joules

Solution 2:

Given, initial velocity u = 10m/sec, acceleration a= 3m/ and time t= 5sec

Find the final velocity at the end of 5 sec.

Therefore, final velocity = v = u + at

= 10 + 15 = 25 m/s

Now, Final K.E. = 1/2 mv^2 = 1/2 x 2 x 25^2 = 625 Joules

Solution 3:
Force = F = 100 N
S = 4m and θ = 0^o
Now, ω = F.S = 100 x 4 = 400 Joules

Solution 4:
Given, m = 5 kg, S = 10 m, θ = 30^0
We know, F = mg

Therefore, work done by the force of gravity = ω = mgh

= 5 x 9.8 x 5

= 245 J

Solution 5:

Given:

Displacement = S = 2.5m, m = 15g or 0.015 kg and F = 2.50N

Also, given the initial velocity u = 0, implies initial K.E =0

Acceleration = a = force/mass = 2.5/0.015 = 500/3 m

and v = 2500/3 m/s [using equation v^2-u^2=2as]

Now, Final K.E. = 1/2 mv^2 = 1/2 x (0.015) x $(2500/3)^2$

= 25/4 J

Again, Work done = W = change in K.E. = 25/4 - 0 = 6.25 J

[Uisng work energy theorem]

Let us find the times taken for the displacement:

We know, v = u + at

[equation of uniform linear motion]

Thus, by substituting for v,u and a, we get t = 0.17sec

Now,

Power = P = Work/time = 6.25/0.17 = 36.08 watts or 36.1 watts.

Solution 6:

Displacement of the particle = r = r_1 - r_2

= (2i + 3j) - (3i + 2j)

= -i + j

From given, work done W = F.r

=> W = (5i + 5j) . (-i + j)

= (-5 + 5)

= o Joule

Solution 7:

Mass of block = 2kg

acceleration = a = $0.5 m/s^2$

Distance covered= s = 40m

Using, $v^2 - u^2 = 2as$

or $v^2 = u^2 + 2as$

Here u = 0

=> $v^2 = 40$

Now by using the work energy theorem = W = ΔK.E.

= $1/2\ mv^2 - 1/2\ mu^2$

On substituting the values, we get

W = 40 Joules

Solution 8:

Given, F = a+bx

Here body moves in +ve x-direction from x=0 to x=d.

At x=0 => F=a and

At x=d => F'= a+bd

Average of forces at both the positions: (F+F')/2

=> a + bd/2

Solution 9:

Mass = m = 0.25kg

displacement =s= 1m

Inclination=θ =37°

Also, $g=10 m/s^2$

Now,

Weight = mg= 0.25× 10=2.5N

The force against is the component of weight along the incline = mg sinθ = mg sin 37^{o}

Work done against the friction force on the block = F× s i.e. mg sin 37^{o} = 1.50 J.

Solution 10:
(a) Weight of the combined system = (m+M)g = N

acceleration =a = F/(2(M+m)) ...(Given)

Frictional force = μ N = μ(m + M)g

By equating above equations, we have

F - μ (m + M)g = (m + M)a

on substituting the value of "a" and solving we have
μ = F/(2g(M+m))

(b) Let f be the frictional force acting on the smaller block
=> Force = mass x acceleration

= mF/(2(m+M))

(c)

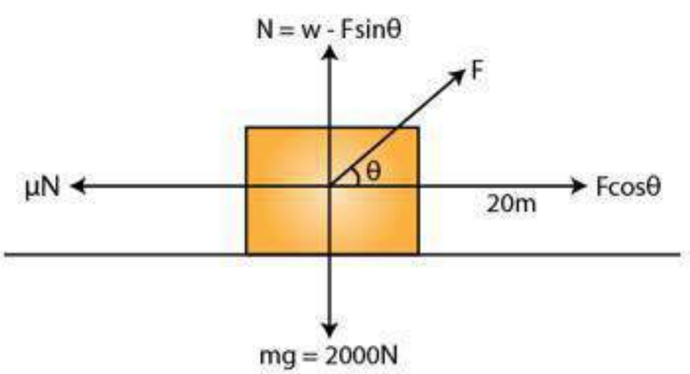
velocity of the block can be evaluated by using uniform motion equation, $v^2 - u^2 = 2as$
When d is the displacement given.

Here u = 0

=> v = √[Fd/(m+M)]

And initial K.E =0
therefore, final K.E. = 1/2 mv^2 = mFd/2(m+M)
Thus, So, work done by the frictional force on the smaller block by the larger block = change in kinetic energy = mFd/2(m+M)

Solution 11:



Here g=acceleration due to gravity=$10 m/s^2$

w = weight of the box = mg = 2000 N

Force of friction = μN

From free body diagram of the box, $N = w - F\sin\theta$

Now,

$F\cos\theta = \mu N = \mu(w - F\sin\theta)$

$F(\cos\theta + \mu\sin\theta) = \mu mg$

or $F = \mu mg/(\cos\theta + \mu\sin\theta)$

(a)
Work done horizontally: $W' = F \sin\theta \times 20$

$= \mu mg/(\cos\theta + \mu\sin\theta) \times \sin\theta \times 20$

$= 40000/(\tan\theta+5)$ Joules

Work done vertical= $w'' = 0$ Joules

[as there is no displacement in the vertical direction]

Total work done on the box = $W = W' + W''$

$= 40000/(\tan\theta+5) + 0$

$= 40000/(\tan\theta+5)$ Joules

(b) Differentiate F w.r.t. θ

$$\left(\frac{dF}{d\theta}\right)_{\theta=\theta_{min}} = 0$$

$$\left(\frac{-\mu mg(\mu\cos\theta - \sin\theta)}{(\cos\theta\ + \mu\ \sin\theta)^2}\right)_{\theta=\theta_{min}} = 0$$

Let θ_{min} be the angle for which F is minimum.

=> $\mu\ \cos\theta_{min} = \sin\theta_{min}$

=> $\tan\theta_{min} = \mu$

=> $\theta_{min} = \tan^{-1}\mu$

Solution 12:

(a) Work done = w = Force along the line of displacement × displacement

=100 sin37° × 2

=120.363005

= 120 J (approx)

(b) Work done will be the same as above. This is so because the component of force along the line of displacement will still be the same i.e. 100 sin37°.

Solution 13:

m = 500 kg, u = 72 km/h or 20 m/s

s = 25 m

Using equation, $v^2 - u^2 = 2as$

or $-a = (v^2-u^2)/2s$

=> a = 400/50 = 8 m/s^2

Now, friction force = f = ma = 500x8 = 4000N

Magnitude of frictional force required =4000N

Solution 14:
Speed of the car = 0 m/s
Accelerate the car to a speed of 72km/h or 20 m/s

K.E. = (1/2) ×500×20×20 = 105 J

The change in kinetic energy = 105 m/s

The work done in this process = change in kinetic energy

= 105-0 = 105 J

Also, work done = force × displacement

=> Force = work done/ displacement = 100000/25 = 4000 N

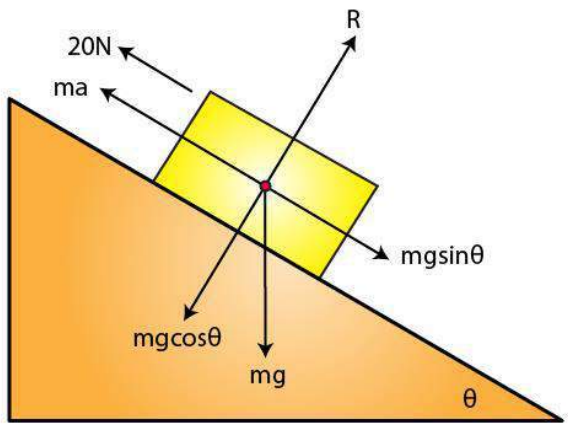
Solution 15:
Given , $v = a\sqrt{x}$
Displacement = d
At x = 0 => $v_1 = 0$ and at x = d => $v_2 = a\sqrt{d}$

$a = (v_2^2 - v_2^2)/2s = a^2/2$

Now, force = f = ma = $ma^2/2$

Work done = w = Fs cos θ = $ma^2/2$ x d = $ma^2d/2$

Solution 16:



mass = m = 2 kg
Time for which the force acts, t = 1 s
Force=F = 20 N

Acceleration due to gravity, $g = 10\ m/s^2$

Now, $F = mg \sin\theta + ma$

$\Rightarrow 20 = 2\times10\sin37° + 2\times a$
$\Rightarrow a = 4\ m/s^2$

The distance travelled by the block = $s = 1/2\ at^2$

= 0.5×4×1×1
= 2m

(a) Maximum work done = Fs
= 20 × 2
= 40 J

(b) If the work done by the applied force is 40 J, then

Distance travelled by the block = work done/ force applied

= 40/20 = 2m

Height corresponding to this distance = h = 2sin37° = 1.2 m

Work done by force of gravity = -mgh

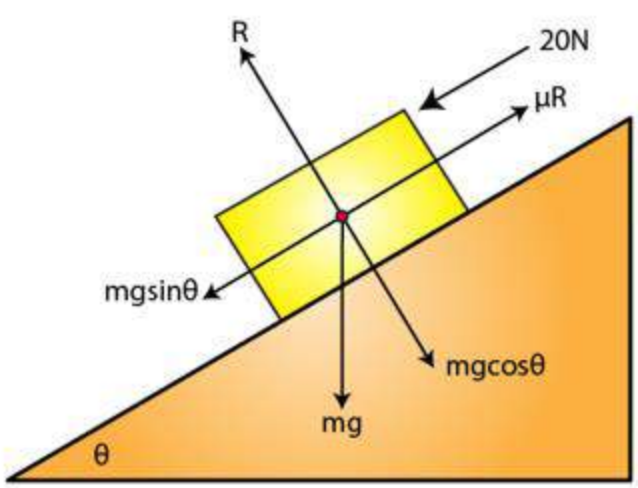
= -2 × 10 × 1.2

= -24 J

(-ve sign due to displacement is against gravity)

(c) Speed of the block after 1s = at = 4×1 = 4 m/s
Corresponding K.E. = 1/2 mv2
= 0.5 × 2 × 4 × 4
= 16 J

Solution 17:



m = 2kg
θ = 37°
F = 20N
Acceleration = a = 10 m/s^2

(a) Find work done by the applied force in the first second

Here t = 1 sec
s = ut+ 1/2 x at^2 = 5m

Work done = W = Fs cosθ = 20x5 = 100 J

(b) Work done by the weight of the box will be equal to the potential energy of the box.

W = Potential energy of the box = mass×g×height = mgh

= m×g×s×sin37°
=2×10×5×sin37°

= 60 J

(c) Frictional force = μR = mgsinθ

Work done by this force on the body = W = mgsinθ × s × cosΦ

W = 2×10×sin37°×s×cos180°

W = -60 J

Solution 18: m = 250 g or 0.25 kg

u = 40cm/s or 0.4 m/s

Initial K.E. = $mu^2/2$

= 0.25 × 0.4 × 0.4 × 0.5

= 0.02 J

Final speed of the block, v = 0 m/s
Final K.E. = $mv^2/2$

= 0.25 × 0 × 0 × 0.5

= 0 J

Let "s m" distance the block moves before coming to rest

Coefficient of friction= μ = 0.1

Acceleration due to gravity, g=9.8 m/s^2

Frictional force, F = μmg

= 0.1 × 0.25 × 9.8

= 0.245 N

Work done by the frictional force = Δ kinetic energy

W = 0-0.02
W = -0.02 J

Also, this work done by frictional force on the block = Fs cosθ

-0.02 = W = 0.245 × s × cos180°

-0.02 = -0.245s

s = 0.082 m or 8.2 cm
Which is the distance before the block stops due to the frictional force.

And work done by the frictional force on the block = -0.02 Joule

Solution 19:
Height = h = 50 m and Mass = 1.8 × 10^5 kg

Mass of water falling per sec, m = (18x10^5)/(60x60) = 50 kg

Gravitational potential energy released per sec = mgh = 50 × 9.8 × 50 = 24500 J

Electrical energy generated per sec = (Gravitational potential energy)/2

= 24500/2 = 12250 J
Therefore, number of 100 W lamps that can be lit from this power = 12250/100 or 122 lamps(approx)

Solution 20:
Mass = m = 6.0 kg

Initial height = 2.0 m

Final height = 0.0 m

Acceleration due to gravity, g = 9.8 m/s^2

Change in potential energy of the bucket = mg(final height - initial height)

= 6.0 × 9.8 × (0.0-2.0)
= -117.6 J

Loss in gravitational potential energy = 117.6 J = 118 J (approx.)

Solution 21:
Height = h = 40 m

Initial speed = u = 50 m/s

Final speed = v m/s

Acceleration due to gravity, g = 10 m/s^2

We know, v^2 - u^2 = 2gh [equation of motion] or $v^2 = u^2 + 2gh$

=> $v^2 = 50^2 + 2\times10\times40$

=> $v^2 = 3300$

=> v = 57.45 m/s

Solution 22:

Distance covered = d= 200 m
Time in which she covered the 200m distance = t = 1 minute 57.56 seconds or 117.56 seconds

Power = 460 W or J/s

Work done = W = Power x Time

= 460 x 117.56

= 54077.6 J

Work done = W = Force × distance covered

Magnitude of this force of resistance = W/d
= 54077.6/200

= 270.388 N = 270 N (approx)

Solution 23:
m = 50 kg
v = 100/10.54 = 9.48766603 or 9.4877 m/s

(a) Kinetic energy of Griffith-Joyner at this speed = $1/2\ mv^2$

= 50×9.4877×9.4877×0.5

= 2250.411 or 2250 J

(b) Force of resistance= F = weight of the athlete / 10
= 50 × 9.8 × 0.1
= 49 N

Her, acceleration a = 0 m/s^2

=> Force in the direction of motion =F = ma = 0

Total work done by resistance = Fscosθ

= 49 × 100 × cos 180°

= -4900 J

(c) Power = work done / time for which the work is done

= 4900/10.54

= 464.89
= 465 W (approx)

Solution 24:
h = 10 m
Flow Rate = 30 kg/minute or 0.5 kg per sec

Now,

Power = Work done/Time

= [mass × acceleration due to gravity × height]/ time = mgh/t

= 0.5 × 9.8 × 10

= 49 W
So, horse power = Power/746 = 49/746 horsepower = 6.6×10^{-2} hp

Solution 25:
Mass = m = 200 g = 0.2 kg
h = 150 cm = 1.5 m
v = 3 m/s and t = 1 sec

Total Work done = Kinetic energy + Potential energy

= $1/2 \times mv^2$ + mgh

= 0.5 ×0.2 × 32 + 0.2 × 9.8 × 1.5

= 3.84 J

Power = Work done/ time

= 3.84/1

= 3.84 W

Horsepower used = 3.84/746 hp = 5.14×10^{-3} hp

Solution 26:
Mass = m = 2000 kg
Height = h = 12 m
Time = t = 1 min or 60 s

So, power = Work done in lifting/Time taken

= mgh/t

= (2000×10×12)/60

= 3920 W

= 3920/746 hp

= 5.3 hp (approx)

Solution 27:

m = 95 kg

Maximum speed=v = 60 km/h or 50/3 m/s

t = 5 s

Engine power required to achieve this speed in given time = $mv^2/2t$

= 95 x $(50/3)^2$ x 1/10

= 3.537 hp

little higher than the given maximum power.

An engine of 3.5 hp can not exactly produce 60 km/h speed in 5 s. Hence, these specifications are somewhat over claimed.

Solution 28:

Here, s = 2 m, m = 30 kg and u = 40 cm/s = 0.4 m/s

Final speed of the block = v = 0 m/s

Work done on the block = change in kinetic energy of the block

= $1/2 \times m \times (v^2 - u^2)$

= 0.5 × 30 × [0 – (0.4×0.4)]

= -2.4 J

Let T be the tension produced in the string.

Net force on the block = Tension in the string - Weight of the block

F = T - mg

= T - 30 × 9.8

= T - 294

Work done on the block= W = Fs = (T-294) x 2

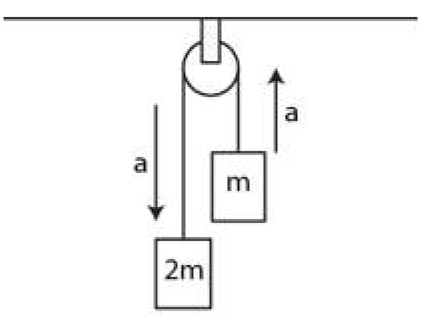
-2.4 = (T-294) × 2

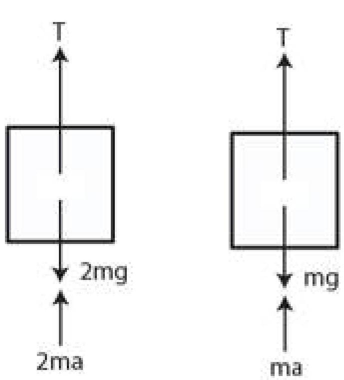
T = 292.8 N

Work done on the block = Ts cos180°

= 292.8 × 2 × -1
= -586 J (approx)

Solution 29: Mass of heavier block = 2m
Tension in the string, T = 16 N





from the free body diagram,

$T - 2mg + 2ma = 0$ ...(1)

$T - mg - ma = 0$ ...(2)

From (1) and (2)

$T = 4\ ma$

$\Rightarrow a = 16/4m = (4/m)\ m/s^2$

Again, $s = ut + 1/2\ at^2$

$\Rightarrow s = 2/m$
[Here $u = 0$]

Now, net mass = $2m - m = m$
Decrease in P.E. = $mgh = mg \times 2/m = 19.6$ J
[using $g = 10 m/s^2$]

Solution 30:

Mass of the heavier block = m_1 = 3 kg

Mass of the lighter block = m_2 = 2 kg

Let the tension in the string be T and acceleration in the system be a.

and t = during the 4th second

from the free body diagram,

$T - 3g + 3a = 0$ ...(1)

$T - 2g - 2a = 0$ ...(2)

From (1) and (2)

$a = g/5\ m/s^2$

Distance travelled in t sec,

$s_t = a/2\,(2n - 1) = 6.86$ m

[Here n = 4 and $g = 9.8\ m/s^2$]

Now,

Work done by gravity = $(m_2 - m_1) \times g \times h$

$= (3-2) \times 9.8 \times 6.86$

= 67 N (approx)

Solution 31:

Mass of the block on the table = m_1 = 4 kg

Mass of the block hanging from the table = m_2 = 1 kg

Initial speed = u = 0 m/s
Distance travelled = s = 1 m
Speed at a later time = v = 0.3 m/s

using equation, $2as = v^2 - u^2$

=> $2 \times a \times 1 = 0.3 \times 0.3 - 0$

=> $a = 0.045\ m/s^2$

Let 2T be the tension in the string.

$m_2a = m_2g - 2T$

$2T = 1 \times (0.045 + 9.8)$

T = 4.9225 N

Let the force of friction F and tension in the string will be T for the 4kg block on the table

$T - F = 2m_1a$

Now, $F = \mu m_1 g$
Where, μ = Coefficient of friction between the block and the table

$T - \mu m_1 g = 2m_1 a$

$4 \times \mu \times 9.8 = 4.9225 - 2 \times 4 \times 0.045$

or μ = 0.12 (approx)

Solution 32:
mass = m = 100 g or 0.1 kg
h = diameter of the block = 2×10 = 20 cm or 0.2 m

Work done by gravity = W = -mgh
= 0.1 × 9.8 × 0.2

= 0.196 J

Initial kinetic energy = 1/2 × m v^2

= 0.5 × 0.1 × 5^2

= 1.25 J

Final kinetic energy = 0 J

Change in kinetic energy of the block = 0 - 1.25 = -1.25 J

Now,
Work done by the tube = change in kinetic energy - work done by gravity

= -1.25 – 0.196

= -1.45 J

Solution 33:
Mass =m = 1400 kg

Height = h = 10 m

Work done by gravity = -mgh
= -1400×9.8×10

= -137200

Initial speed = 54 km/h = 15 m/s

Initial kinetic energy = 1/2 × m v^2

= 0.5 × 1400 × 15 × 15

= 157500 J

Final kinetic energy = 0 m/s

Change in kinetic energy of the car = 0 – 157500 = - 157500 J

Also, Work done againest friction = Change in kinetic energy – work done by gravity

= -157500 – (-137200)

= -157500 + 137200

= -20300 J

Therefore, Work done against friction = 20300 J

Solution 34:
Mass = m = 200 g = 0.2 kg
Height of the incline plane = h = 3.2 m
Length of the incline plane = L = 10 m
Acceleration due to gravity, g = 10 m/s^2

(a) Required work for lifting the block to the top of the incline = mgh

= 0.2×10×3.2
= 6.4 J

(b) Work done against friction = 0 J

Required work for sliding the block up = Work done against gravity
= 6.4 J

(c) Speed of the block when it is at rest on the top, u = 0 m/s

According to equation of motion, $v^2 - u^2 = 2gh$
=> $v^2 - 0 = 2×10×3.2$

=> v = 8 m/s

(d) When the block slides down,

Change in KE = work done by gravity + work done by friction

=> $1/2\ mv^2 = mgh + 0$

=> $v^2 = 2\times10\times3.2 = 64$
or v = 8 m/s

Solution 35:

(a) Work done by the boy on the ladder as he goes up = 0 J
[As he goes against gravity]

(b) Weight = 200 N

Frictional force = (3/10) of the weight of the boy

= 60 N

Work done = Frictional force × Length of the ladder × $\cos\theta$

= $60 \times 10 \times \cos 180°$

= -600 J

(c) the work done by forces inside the body by the boy.

Work done against gravity = Weight of the boy × height of the ladder

= 200×8

= 1600 J

Solution 36:

Let m be the mass of the particle.

Acceleration due to gravity = g = 9.8 m/s^2

The height of the particle at point A = h = 1.0 m (Given)
The height of the particle at point where it terminates into straight horizontal section, h= 0.5 m (Given)

Now, total energy:

(Kinetic energy)_initial + (Potential energy)_initial = (Kinetic energy)_final + (Potential energy)_final

=> 0 + mgh at A = (1/2) mv^2 + mgh at point of termination

=> m×9.8×1 = 0.5×m×v^2 + m×9.8×0.5

=> 9.8 = 0.5v^2 + 4.9

=> v = 3.13 m/s

The particle starts sliding from rest, from the equation of motion

=> h = 1/2 gt^2

=> 0.5 = 0.5 × 9.8 × t2

=> t = 0.32 s
Which is the time taken by particle to reach the termination point .

Again, The horizontal distance that the particle will travel = speed × time

= 3.13 × 0.32
=1 m (approx)

Solution 37:
Coefficient of friction = μ = 0.20
Height of the point A = h = 1.0 m
Weight of the block = 10 N

Frictional force = F = μ × weight of the block

= 0.20 × 10
= 2 N

Let s be the displacement of the box from point A.

Loss in potential energy = Work done by the frictional force = Fs

Weight of the block × h = 2 × s

10 × 1 = 2 × s

s = 5 m
Thus, the block will move 5 m before stopping on the rough surface.

Solution 38:
Mass of the part of chain on the table = 2m/3 (given)

=> Mass of the part hanging from the table = m - 2m/3 = m/3

So, length of the part of chain = 2l/3

Length of the part of chain hanging from the table = l/3

=> Centre of the mass m/3 will be at l/6

Potential energy = m/3 x g x 1/6 = mgl/18

Therefore, Work done to put the hanging part back on table is mgl/18

Solution 39:

A uniform chain of length L and mass M overhangs a horizontal table with its two third part on the table. (Given)

small element of the chain having length dx and mass dM.

then small mass will be dM = m/L dx

Force of friction = dF = μ g dM = μ g m/L dx

Now,

Work done on this small element by friction =

dW = dF . x = μ g m/L dx

Where x is the displacement of the small element by friction.

Again,

Total work done = W

$$W = \int_0^{2L/3} dW$$

$$= \int_0^{2L/3} \mu g \frac{m}{L} x dx$$

$$= \mu g \frac{m}{L} \int_0^{2L/3} x dx$$

$$= \mu g \frac{m}{L} \frac{4L^2}{9}$$

$$= \frac{2}{9} \mu m g L$$

Solution 40:

Work done by the frictional force = Change in potential energy of block

= Mass × acceleration due to gravity × change in height

= 1 × 10 × (0.8 – 1)

= –2J

Solution 41:
Let h be the height to which the block will rise.
Mass of the block, m = 5 kg
Upward speed of the block = v = 2 m/s

Now, mgh = 1/2 mv^2

=> h = 0.2 m

Therefore, The block will rise up to 0.2 m or 20 cm.

Solution 42:
Mass of the block = 250 g or 0.250 kg (Given)
A block of mass 250 g is kept on a vertical spring of spring constant 100 N/m (Given)
Height of the block rising is 20 cm
gravity due to acceleration = g = 10 m/s^2.

We know, formula of the energy stored in spring compression:

Energy = 1/2 kx^2

and energy stored in the block

Energy' = mgh
Applying the conservation of the mass, between mass of block and spring compression we have the height,

mgh = 1/2 kx^2

$$h = \frac{\frac{1}{2}kx^2}{mg}$$

$$h = \frac{100 \times 0.1^2}{2 \times 0.25 \times 10} = 20\text{cm}$$

The required height is 20 cm.

Solution 43:

coefficient of friction = 0.5 and the spring constant = k = 1000 N/m
angle of inclination of the plane = 37^o

Find energy stored in spring compression:

Energy = (1/2) kx^2

And, energy stored in the block:

Energy' = μ mgh

By applying, conservation of energy we get

$mg \sin 37^o h = \mu Rh + 1/2\, kx^2$

[The block on the inclined plane is derived in two forms of derivatives with vertical mg sinθ and horizontal mg cosθ].

=> $mg \sin 37^o (0.2+4.8) = \mu R(0.2+4.8) + 1/2\, kx^2$

=> $60 - 80\mu = 0.02k$ ...(i)

Again, For the upward return motion

$mg \sin 37^o (1) = \mu R(1) + 1/2\, kx^2$

=> $12 - 16\mu = 0.02k$ ...(ii)

Adding (i) and (ii)

$\mu = 0.5$

(i) => k = 1000 N/m

Solution 44:

A block of mass m moving at a speed u compresses a spring through a distance x before its speed is halved. (Given)

Find energy stored in spring compression:

Energy = $1/2\ kx^2$

And, energy stored in the block:

Energy' = $(1/2)\ mu^2$

The velocity of the block "u" becomes u/2 after compression, making the initial kinetic energy as

$K = (1/2)\ mu^2$

The total energy or the initial kinetic energy is equal to final kinetic energy and the spring compression energy.

=> $(1/2)\ mu^2 = (1/2)\ m(u/2)^2 + (1/2)\ kx^2$

=> $kx^2 = m(3/4\ u^2)$

or

$$k = m\frac{\left(\frac{3}{4}u^2\right)}{x^2}$$

Which is the spring constant.

Solution 45:

Find energy stored in spring compression:

Energy = 1/2 kx^2

And, energy stored in the block:

Energy' = mgh

In the potential energy of the block is given as x, equating the potential energy with the spring energy is

$(1/2)\ kx^2 = mgh$
if h = x

=> $(1/2)\ kx^2 = mgx$

=> x = 2mg/k
The maximum elongation of the spring is x = 2mg/k

Solution 46:
A block of mass “m” is attached with two spring constants k_1 and k_2 and the block is released on the right.

Energy stored in spring compression:

Energy = $(1/2)\ kx^2$

Where, value of spring constant = k, the compression distance = x.

And, energy stored in the block:

Energy' = $(1/2)\ mv^2$

Where, the mass of the block = m, and velocity of the block = v

The conservation of the energy between spring energy and potential energy is given as

$(1/2)\ k_1 x^2 + (1/2)\ k_2 x^2 = (1/2)\ mv^2$

$mv^2 = x^2 (k_1 + k_2)$

or

$$v = x\sqrt{\frac{(k_1 + k_2)}{m}}$$

Which is the speed of the block.

Solution 47:

The sliding velocity [vector v] of the block with spring constant k. (Given)

Energy stored in spring compression:

Energy = (1/2) kx^2
Where, spring constant = k, the compression distance = x.

Where, the mass of the block = m, and velocity of the block = v
Also, The kinetic energy of the block = K = (1/2) mv^2

On equating the energies together, we get the compression distance as

(1/2) kx^2 = (1/2) mv^2

$$x = \sqrt{\frac{mv^2}{k}} = v.\sqrt{\frac{m}{k}}$$

Which is compression distance of the spring.

Solution 48:

The block of mass = 100 g
Compressing the spring about 5 cm.
The height of the spring is 2 cm.
The spring constant = k = 100N/m.

Energy stored in spring compression:

Energy = (1/2) kx^2

Where, spring constant = k, the compression distance = x.

And, energy stored in the block:

Energy' = (1/2) mv^2

Where, the mass of the block = m, and velocity of the block = v

Stored spring compression energy is calculated as

Energy = (1/2) kx^2= 1/2 (100) $(0.05)^2$ = 0.125 Joules

After the expansion of the spring, the compression energy turns into kinetic energy.

0.125 = (1/2) mv^2

0.125 = 1/2 x $(0.1)v^2$

or v = 1.58 m/s

Let's evaluate time taken for the block to touch the ground:

s = ut + 1.2 gt^2

2 = ½ x $(9.8)t^2$ [Here u = 0]

or t = 0.63 sec

Horizontal distance covered by the block = S = ut = 1.58 x 0.63 = 1 m

Solution 49:

Let m is the mass of the object, v is the velocity, and assume that the lower end the potential energy of the block is zero.
Also g is the acceleration due to gravity and h is the height at which the object is kept.

Now, total energy = (1/2) mv^2 + 0

At the maximum point the potential energy = (mg)2l.

By law of conservation of energy, we have
=> (1/2) mv^2 = mg x 2l

or v = 2√(gl), is the minimum horizontal velocity.

Solution 50:

The masses of the blocks = 320 g
Spring constant = 40 N/m
Block B is attached at a height of 40cm from the horizontal surface, with a gravity of 10 m/s^2.
We know, T = kx and T = mg

where, T = tension, k = spring constant, m = mass, x = compression distance and g = gravity.

Force/tension, applied on the block A due to spring is given as,

T = kx ...(1)
where x = h(coseθ - 1)

By equating the y-axis direction of the T=mg force is

T sinθ = mg ...(2)

Placing the T as kh(coseθ - 1), we get

(2)=> kh(coseθ - 1)sinθ = mg

0.4 x 40(coseθ - 1)sinθ = 0.32 x 9.8

sinθ = 0.8

Again,

(1) => kh(coseθ - 1) = kx

x = 0.1 m

Now, the difference in Kinetic Energy is equivalent to the force applied on the block.

$v^2 = gs – (1/2) kx^2$

$= gh \cot\theta - (1/2) kx^2$

$= 3 - 0.2/0.32$
$= 2.375$

or v = 1.54 m/s, which is velocity of the block A.

Solution 51:
The spring with a spring constant = k, and at a height of h forming at an angle of 37°

The conservation equation using conservation of static and dynamic energy such as spring energy and kinetic energy.

$(½) mv^2 = (1/2) kx^2$

Where, m = mass of the object, v = velocity, k = spring constant and x = elongation distance.

Now, speed of the block moved from the resting position
$(1/2) mv^2 = (1/2) kx^2$

or $v = x \sqrt{(k/m)}$

Again, height, h, is formed at angle 37°

$h \cos 37^\circ - h = x$

and, if height of the horizontal plane and block as h' = h/4, we get

$v = (h/4) \sqrt{(k/m)}$, which is the velocity of the block.

Solution 52:

We are given that, the length of the tight rod is given as “l”, the gap between the final and initial position of the ring is given as “h”

The total energy in terms of kinetic and potential energy:

Total energy = E_T =(½) mv^2 + mgl ...(1)

Where, m = mass of the object, l = length of the object and g = acceleration in terms of gravity.

Also, the potential energy of the block = E_p = mgh ...(2)

Equating (1) and (2), to find the gap of the final and initial position of the ring that is "h".

(½) mv^2 + mgl = mgh

or h = $v^2/2g$ + l
At v = 0, we have

h = l, which is the length of the height.

Solution 53:
Equation of motion for the particle:
T - mg cosθ = mv^2/l, and V_A = √(10 gl)

(a) As per law of conservation of energy:

$E_A = E_B$

$$\frac{1}{2}mV_A^2 = \frac{1}{2}mV_B^2 + mgl$$

$$\frac{1}{2} \times 10gl = \frac{1}{2}V_B^2 + gl$$

$$V_B = \sqrt{8gl}$$

Also,

$$T_B = \frac{mV_B^2}{l} + mg\cos(90^o)$$

$$T_B = \frac{m \times 8gl}{l}$$

$$T_B = 8mg$$

(b) Again, from law of conservation of energy:
$E_A = E_C$

$$\frac{1}{2}mV_A^2 = \frac{1}{2}mV_C^2 + 2mgl$$

$$5gl = \frac{1}{2}V_C^2 + 2gl$$

$$V_C = \sqrt{6gl}$$

Also,

$$T_C = \frac{mV_C^2}{l} + mg\cos(180^o)$$

$$T_C = \frac{m \times 6gl}{l} - mg$$

$$T_C = 5mg$$

(c) Again, from law of conservation of energy:
$E_A = E_D$

$$\frac{1}{2}mV_A^2 = \frac{1}{2}mV_D^2 + mg \times \frac{3l}{2}$$

$$5gl = \frac{1}{2}V_D^2 + \frac{3}{2}gl$$

$$\frac{V_D^2}{2} = \frac{7}{2}gl$$

$$V_D = \sqrt{7gl}$$

Also,

$$T_D = \frac{mV_D^2}{l} + mg\cos(120^o)$$

$$T_D = \frac{13}{2}mg$$

Solution 54:

The length of the bob = 50 cm
Ball is pulled at a distance of 60 cm making the angle of 37^o. (Given)

Now, total energy in terms of kinetic and potential energy is,

E_T = (½) mv^2 + mgl

and
$T = mv^2/r + mg$

Let E_A be the energy at the point which is lowest in the bob circulation and the position of bob at an angle of 37^o, from the normal is given as point E_B.

Now, $1/2\ mv^2 = mg\ (1 - l\cos\theta)$

Here $\theta = 37^0$ and $l = 0.5$

=> $(1/2)\ mv^2 = mg(0.5 - 0.4)$

or $v = 1.414$

And tension is, $T = mv^2/r + mg$

$= (0.1x2)/0.5 + 10$

$= 1.4$ N

Solution 55:

Using the conservation of static and dynamic energy such as centripetal and kinetic energy, we have the conservation equation, $(mv^2/r)\ R = (1/2)\ kx^2$

Where, m = mass of the object, v = velocity, k = spring constant and x = elongation distance, R = radius of the circular part, and r = radius of the object.

The energy of the block in form of kinetic energy is equated with the centripetal energy of the surface making the equation

$mv^2/r = (1/2)\ kx^2$ ...(1)

Using $v = gR$

Putting value of v in (1), we have

$x = \sqrt{(3mgR/k)}$

Solution 56:

$V = \sqrt{(3gl)}$ … (Given)

$(½) mv^2 – (1/2) mu^2 = -mgh$

or $v^2 = 3gl - 2gl(1 + \cos\theta)$ ...(i)

Again, $mv^2/l = mg \cos \theta$

or $v^2 = lg \cos \theta$ ...(ii)

From (i) and (ii), we have

$\theta = \cos^{-1} (1/3)$

Thus, angle rotating before the string become slack

$\theta = 180^o - \cos^{-1} (1/3)$

=> angle reached before slack = $\theta = \cos^{-1} (1/3)$

Solution 57:
The length string = 15 cm
Horizontal velocity of $\sqrt{57}$ m/s
Acceleration due to gravity = g = 10 m/s^2.

Total energy in terms of kinetic and potential energy:

$E_T = (1/2) mv^2 + mgl$
and
Tension = $T = mv^2/r + mg$

(a) angle made before it becomes slack

$(1/2) mv^2 = (1/2) mu^2 + mgh$

or $v^2 = u^2 + gh$

We know $u^2 = 57$ and $h = -3(1+\cos \theta)$

$\Rightarrow v^2 = 57 - 3g(1 + \cos\theta)$ ....(1)

Using the equation for the bob in terms of centripetal force in (1) i.e., $v^2 = lg \cos\theta$

(1)=> $lg \cos\theta = 57 - 3g(1 + \cos\theta)$

or $\cos \theta = 3/5$

or $\theta = \cos^{-1} (3/5) = 53^o$

(b) speed of the particle present at a particular time

$v^2 = 57 - 3g(1 + \cos\theta)$

or v = 3 m/s
[On putting values]

(c) string becomes slack after losing velocity making the maximum height, h

$$H = 1.5\cos\theta + \frac{u^2 \sin^2\theta}{2g}$$

$$H = 1.5 \times \cos 53 + \frac{u^2 \sin^2\theta}{2g}$$

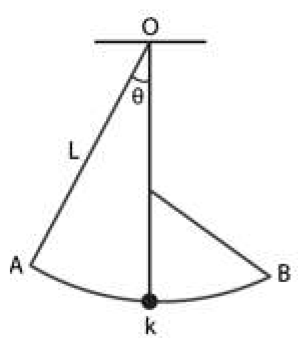
$$H = 1.5 \times \frac{3}{5} \times \frac{9 \times (0.8)^2}{20} = 1.2m$$

Solution 58:

The pendulum has a mass "m" attached to length "l" which hits the peg hanging at "x" situated at an angle of θ. (Given)

Total energy in terms of kinetic and potential energy:

$E_T = (1/2)\, mv^2 + mgl$

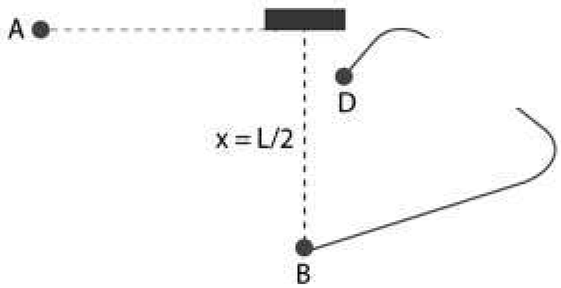


(a) When the height of the bob is less than the peg than the total potential energy of the bob at point A is equal to the potential energy of the bob at point B.

K.E. at both places is zero.

$PE_A = PE_B$ = maximum height of the bob is equivalent to the initial height.

(b)



When the particle is released at angle θ and x = L/2, the path of the bob travelling will slack at point C making a projectile motion.

$$\frac{1}{2}mv_c^2 = mg\left(\frac{L}{2}\right)(1 - \cos\theta)$$

$$v_c^2 = gL(1 - \cos\theta)$$

$$gL(1 - \cos\alpha) = \frac{gL}{2}\cos\alpha$$

$$1 - \cos\alpha = \frac{1}{2}\cos\alpha$$

Distance after slack at point D:

BD = L/2 + L/2 cosθ = 5/6 L
[When cos θ = 2/3]

(c) The velocity of the bob at point D

$v_D^2 = g(L-x)$ ...(1)

The conservation of the force in the bob is

$mg = m\,v_D^2/(L-x)$
using (1)

$mg = mg(L-x)/(L-x)$

$x/L = 3/5 = 0.6$

Solution 59:

The particle moving portrays a centripetal force of mv^2/r and a force of mg.

On equating the forces, we have

$mg \cos \theta - N = mv^2/r$

Find the value of velocity:

$mg \cos\theta = mv^2/r$

$v^2 = rg \cos \theta$

and $r - r \cos \theta$ is the real height.

Let PE be the potential energy due to height is $PE = mg(r - r \cos \theta)$

Let us place kinetic and potential energy together, then

$g(r - r \cos \theta) = (1/2)\ mr \cos\theta$

or $3/2 \cos \theta = 1$

or $\theta = \cos^{-1}(2/3)$

Solution 60:

Using conservation of static and dynamic energy such as potential and centripetal energy, we have the conservation equation as

$(mv^2/r)\ R = mgh$

(a) The mass of the particle when horizontal

$N = mg \cos 30^0 = (\sqrt{3}/2)\, mg$ = force exerted by the sphere

(b) The distance travelled by the particle in terms of radian/degree

The change in potential energy due to the angle of 30^0

$mg R \cos 30^0 = R \cos(\theta + 30^0)$

Let us equate kinetic and potential energy together to find the value of velocity

$(1/2)\, mv^2 = mg R \cos 30^o - R \cos(\theta + 30^o)$ and $mv^2/R = mg \cos\theta$

$\Rightarrow gR\cos(\theta + 30^o) = 2gR \cos 30^o - R \cos(\theta + 30^o)$

On solving above equation, we have

or $\theta = 25^o$ or $\theta = 0.43$ radian

Solution 61:

The radius of the sphere on which the particle is kept is R and the horizontal speed is taken as v. ...(Given)

Using the conservation of static and dynamic energy such as centripetal and potential energy,

$(mv^2/r)\, R - N = mgh$

(a) When a particle is kept on the top of the sphere a downward force of "mg" and an upward force of "N" is applied on the block giving the equation of forces as

$mg - N = mv^2/R$

the normal force of the particle = $N = mg - mv^2/R$

(b) When a particle is at minimum velocity, the reactionary force becomes zero

$mg - N = mv^2/R$

Here $N = 0$

=> $v = \sqrt{gR}$

(c) when velocity as half : $v = 1/2 \times \sqrt{gR}$

=> $v^2 = (1/4)\, gR$

(d) Find the value of angle

$mv'^2/R = mg \cos\theta$

Let v' is the velocity of the particle leaving the sphere

=> $1/2\, mv'^2 - 1/2\, mv^2 = mgR(1 - \cos\theta)$

Putting $v^2 = (1/4)\, gR$ and $v' = Rg \cos\theta$

we have,

$(1/2)\, Rg \cos^2\theta - (1/8)\, gR^2 = gR(1 - \cos\theta)$

or $\theta = \cos^{-1} (3/4)$

Solution 62:

The formula for the total energy in terms of kinetic and potential energy:

$E_T = (1/2)\, mv^2 + mgl$

(a) height of the particle

$H = l \sin\theta + h$

$H = l \sin\theta + (R - R \cos\theta)$

Now, the PE at the top of the sphere

$mgH = mg(l \sin\theta + (R - R \cos\theta))$

Total energy experienced on the particle = T = PE + KE

Initial PE = 0 and initial KE = (1/2) mv^2_0

=> Total energy = (1/2) m v^2_0

Now, equation of total energy, we have

$$v_o^2 = \sqrt{2g(l\sin\theta + (R - R\cos\theta))}$$

(b) If initial speed is $2v_0$ and final velocity is v

Total energy = PE + KE

(1/2) mv^2 + mgH = (1/2)m(2 v^2_0)

$v^2 = 4v_0 - 2gH$

Again, centripetal acceleration = v^2/R

Therefore, the force acting = F = m/R x ($4v_o$ - 2gH)

= (6gR(1-cosθ) + l sinθ)

=> F = 6mg(1 - cos θ + lsinθ/R)

(c) If the speed is doubled
velocity = $2v_o$

KE = (1/2) mv^2

and PE = mg(R - R cos θ)

gR cosθ = 2gR(1 - cosθ)
or 3 cos θ = 2

or θ = $\cos^{-1}(2/3)$

Solution 63:

The formula for the total energy in terms of kinetic and potential energy:

$E_T = (1/2)\ mv^2 + mgl\ \sin\theta$

(a) Let α be the angle formed by the chain, and l be the length

Angle can be written as $\alpha = l/R$

Length of the chain in terms of radius = $l = Rd\theta$

Force derivative of the chain = $F = (m/l)\ R\ d\theta$

The potential energy is calculated as

Now, the P.E. = $(mR^2g/l)\cos\theta\ d\theta$

The PE after integration = $(mR^2g/l)\sin(l/R)$

(b) On equating kinetic energy and the potential energy of the chain, we have

$$\mathrm{P.E.} = \left(\frac{mgR^2}{l}\right)\left(\sin\left(\theta + \frac{l}{R}\right) - \sin\theta\right)$$

Initial P.E.

$$\mathrm{P.E.} = \left(\frac{mgR^2}{l}\right)\sin\left(\frac{l}{R}\right)$$

Change in P.E.

$$\left(\frac{mgR^2}{l}\right)\sin\left(\frac{l}{R}\right) - \left(\sin\left(\theta + \frac{l}{R}\right) - \sin\theta\right)$$

(c) find the tangential velocity
on differentiating u with respect to t, we have

$$\frac{du}{dt} = \frac{1}{2}\left(2u.\frac{du}{dt}\right) = \frac{mgR^2}{l}\left(\left(-\cos\left(\theta + \frac{l}{R}\right)\right)\frac{d\theta}{dt} + \cos\theta\frac{d\theta}{dt}\right)$$

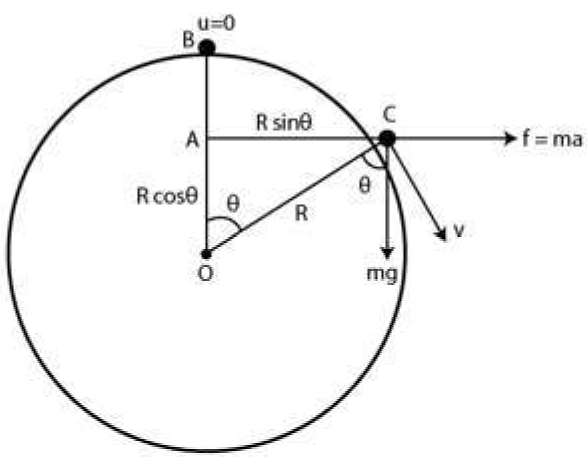
$$\frac{du}{dt} = \left(\frac{Rg}{l}\right)\left(1 - \cos\left(\frac{l}{R}\right)\right)$$

Solution 64: The radius of the smooth sphere is R, the constant acceleration is α.

A force f acts on the sphere towards left, so the particle experience pseudo force = f = ma towards right.

Initial kinetic energy of the particle = zero.
Let the speed of the particle at point C be v.
From work-energy theorem, $W_g + W_f = \Delta K.E. = K.E._f$

Where W_g = work done by the gravity and W_f= work done by pseudo force.



From figure, AB = R − Rcosθ = R(1−cosθ). Also, AC = Rsinθ

$mgR(1 - \cos\theta) + ma R \sin\theta = (1/2) mv^2$

at a = g

$mgR(1 + \sin \theta - \cos\theta) = (1/2) mv^2$

or

$$v = \sqrt{2gR(1 + \sin\theta - \cos\theta)}$$