

Airthmetic Progressions

1. In an A.P.; if $a = 8$ and $a_{10} = -19$, then value of d is: (2024)

- (a) 3 (b) $-\frac{11}{9}$ (c) $-\frac{27}{10}$ (d) -3

Answer. (d) -3

2. Two alarm clocks ring their alarms at regular intervals of 20 minutes and 25 minutes respectively. If they first beep together at 12 noon, at what time will they beep again together next time? (2024)

Answer. LCM (20, 25) = 100

$\therefore$ After 100 minutes from 12:00 noon

$\Rightarrow$ They will beep again together at 1:40 pm

3. In an A.P. if $S_n = 4n^2 - n$, then

(i) find the first term and common difference. (2024)

Answer. (i) $S_n = 4n^2 - n$

$$S_1 = 4 - 1 = 3 = a_1$$

$$S_2 = 2a + d = 14 \Rightarrow d = 14 - 6 = 8$$

(ii) write the A.P. (2024)

Answer. A.P. is 3, 11, 19, 27,

(iii) which term of the A.P. is 107? (2024)

$$\text{Answer. } 107 = 3 + (n - 1)8 \Rightarrow n = 14$$

4. In an A.P., if the first term $a = 7$, n th term $a_n = 84$ and the sum of first (2024)

$$n \text{ terms } S_n = \frac{2093}{2}, \text{ then } n \text{ is equal to :}$$

- (a) 22
(b) 24
(c) 23

(d) 26

Answer. (c) 23

5. (A) The sum of first and eighth terms of an A.P. is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms. (2024)

Answer. $a + a_8 = 32 \Rightarrow 2a + 7d = 32$ ----- (i)

$a \times a_8 = 60 \Rightarrow a(a + 7d) = 60$ ----- (ii)

Solving (i) & (ii), we get

$a = 2$ or $a = 30$

and $d = 4$ or $d = -4$

First term and common difference of A.P. are 2 and 4 or 30 and -4 respectively.

Now, for $a = 2$ & $d = 4$

$S_{20} = 10(4 + 76) = 800$

and for $a = 30$ & $d = -4$

$S_{20} = 10(60 - 76) = -160$

6. In an A.P. of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687. Determine the first term and common difference of A.P. Also, find the sum of all the terms of the A.P. (2024)

Answer. Here $n = 40$,

$S_9 = \frac{9}{2} [2a + 8d] = 153 \Rightarrow a + 4d = 17$ ---- (i)

and $S_{40} - S_{34} = 687$ or $a_{35} + a_{36} + a_{37} + a_{38} + a_{39} + a_{40} = 687$

$\Rightarrow 6a + 219d = 687$ or $2a + 73d = 229$ ---- (ii)

solving (i) and (ii) to get $a = 5$, $d = 3$

Also, $S_{40} = \frac{40}{2} (10 + 39 \times 3) = 2540$

7. Directions:

Assertion (A) is followed by a statement

of Reason (R). Select the correct option from the following options: (2024)

(a) Both, Assertion (A) and Reason (R) are true. Reason (R) explains Assertion (A) completely.

(b) Both, Assertion (A) and Reason (R) are true. Reason (R) does not explain Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Q. Assertion (A): The tangents drawn at the end points of a diameter of a circle, are parallel.

Reason (R): Diameter of a circle is the longest chord. **(2024)**

Answer. (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation for Assertion (A).

5.2 Arithmetic Progressions

MCQ

1. If a, b, c form an A.P. with common difference d , then the value of $a^2b - c$ is equal to

- (a) $2a + 4d$
- (b) 0
- (c) $-2a - 4d$
- (d) $-2a - 3d$ (2023)

2. The next term of the A.P.: $\sqrt{7}, \sqrt{28}, \sqrt{63}$ is

- (a) $\sqrt{70}$
- (b) $\sqrt{80}$
- (c) $\sqrt{97}$
- (d) $\sqrt{112}$ (2023)

3. If $k + 2, 4k - 6$ and $3k - 2$ are three consecutive terms of an A.P., then the value of k is

- (a) 3
- (b) -3
- (c) 4
- (d) -4 (2023)

4.

If $-\frac{5}{7}, a, 2$ are consecutive terms in an Arithmetic Progression, then the value of ' a ' is

- (a) $\frac{9}{7}$
- (b) $\frac{9}{14}$
- (c) $\frac{19}{7}$
- (d) $\frac{19}{14}$

(2020C) 

5. Which of the following is not an A.P.?

- (a) $-1.2, 0.8, 2.8, \dots$

(b) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \dots$

(c) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \dots$

(d) $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \dots$ (2020)

6. The value of x for which $2x$, $(x + 10)$ and $(3x + 2)$ are the three consecutive terms of an A.P., is

(a) 6

(b) -6

(c) 18

(d) -18 (2020)

7. The first three terms of an A.P. respectively are $3y-1$, $3y+5$ and $5y + 1$. Then y equals

(a) -3

(b) 4

(c) 5

(d) 2 (Delhi 2014)

8. If k , $2k - 1$ and $2k + 1$ are three consecutive terms of an A.P., the value of k is

(a) 2

(b) 3

(c) -3

(d) 5 (AI 2014) VSA (1 mark)

9. Find the common difference of the Arithmetic

Progression (A.P.) $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \dots (a \neq 0)$ (2019)

10. Write the common difference of A.P.

$\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \dots$ (A/ 2019)

11. For what value of k will $k+9$, $2k - 1$ and $2k + 7$ are the consecutive terms of an A.P.? (AI 2016)

12. For what value of k will the consecutive terms $2k + 1$, $3k + 3$ and $5k - 1$ form an A.P.? (Foreign 2016) SAI (2 marks)

13. Find a and b so that the numbers a , 7 , b , 23 are in A.P. (Term II, 2021-22)

14. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in A.P. (2020) U

5.3 n^{th} Term of an A.P.

MCQ

15. The first term of an A.P. is p and the common difference is q , then its 10^{th} term is

- (a) $q+9p$
- (b) $p-9q$
- (c) $p+9q$
- (d) $2p+9q$ (2020)

16. The next term of the A.P. $\sqrt{7}, \sqrt{28}, \sqrt{63}, \dots$ is

- (a) $\sqrt{70}$
- (b) $\sqrt{84}$
- (c) $\sqrt{97}$
- (d) $\sqrt{112}$ (Foreign 2014)

VSA (1 mark)

17. If the n^{th} term of an A.P. is $pn + q$, find its common difference. (2019C)

18. Which term of the A.P. $10, 7, 4, \dots$ is -41 ? (2019C)

19. If in an A.P., $a = 15$, $d = -3$ and $a, 0$, then find the value of n . (2019)

20. How many two digit numbers are divisible by 3? (NCERT, Delhi 2019)

21. In an A.P., if the common difference $(d) = -4$, and the seventh term (a_7) is 4, then find the first term. (2018)

22. What is the common difference of an A.P. in which $9a_1 - a_7 = 84$? (AI 2017)

23. Find the 9th term from the end (towards the first term) of the A.P. $5, 9, 13, \dots, 185$. (Delhi 2016)

24.

Find the 25^{th} term of the A.P. $-5, -\frac{5}{2}, 0, \frac{5}{2}, \dots$

(Foreign 2015)

SAI (2 marks)

For the A.P.; $a_1, a_2, a_3, \dots$ if $\frac{a_4}{a_7} = \frac{2}{3}$, then find $\frac{a_6}{a_8}$.

(Term II, 2021-22C)

26. Find the number of terms of the A.P. :

293, 285, 277, ..., 53 (Term II, 2021-22C)

27. For what value of 'n', are the n^{th} terms of the A.P's:

9, 7, 5, ... and 15, 12, 9, ... the same? (Term II, 2021-22)

28.

Which term of the A.P. $-\frac{11}{2}, -3, -\frac{1}{2}, \dots$ is $\frac{49}{2}$?

(Term II, 2021-22)

29. Determine the A.P. whose third term is 5 and seventh term is 9. (Term II, 2021-22)

30. If the 9th term of an A.P. is zero, then show that its 29th term is double of its 19th term. (2019C)

31. Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21st term? (Delhi 2019)

32. If the 17th term of an A.P. exceeds its 10th term by 7, find the common difference. (AI 2019)

33. Find how many integers between 200 and 500 are divisible by 8. (Delhi 2017)

34.

Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots$ is the first negative term?

(AI 2017) 

35. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term. (AI 2016)

36. Find the middle term of the A.P. 6, 13, 20, ..., 216. (Delhi 2015)

37. Find the middle term of the A.P.

213, 205, 197,, 37. (Delhi 2015)

38. The fourth term of an A.P. is 11. The sum of the fifth and seventh terms of the A.P. is 34. Find its common difference. (Foreign 2015)

39. The fifth term of an A.P. is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference of the A.P. (Foreign 2015)

40. The ninth term of an A.P. is -32 and the sum of its eleventh and thirteenth terms is -94. Find the common difference of the A.P. (Foreign 2015)

41. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5. (AI 2014)

SA II (3 marks)

42. How many terms are there in A.P. whose first and fifth term are -14 and 2, respectively and the last term is 62. (2023)

43. Which term of the A.P.: 65, 61, 57, 53, _____ is the first negative term? (2023)

44.

If the m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$ then

show that its $(mn)^{\text{th}}$ term is 1. (Delhi 2017) 

45. The p^{th} , q^{th} and r^{th} terms of an A.P. are a , b and c respectively. Show that $a(q - r) + b(r - p) + c(p - q) = 0$. (Foreign 2016) Ev

46. Divide 56 in four parts in A.P. such that the ratio of the product of their extremes (1^{st} and 4^{th}) to the product of means (2^{nd} and 3^{rd}) is 5:6. (Foreign 2016)

47.

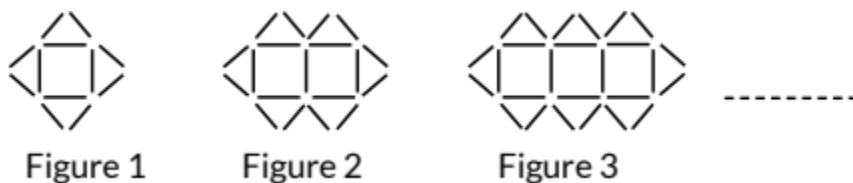
If the seventh term of an A.P. is $\frac{1}{9}$ and its ninth term is $\frac{1}{7}$, find its 63^{rd} term. (Delhi 2014)

48. The sum of the 5th and the 9th terms of an A.P. is 30. If its 25th term is three times its 8th term, find the A.P. (AI 2014)

49. The sum of the 2nd and the 7th term of an A.P. is 30. If its 15th term is 1 less than twice its 8th term, find the A.P. (AI 2014)

LA (4/5/6 marks)

50. In Mathematics, relations can be expressed in various ways. The matchstick patterns are based on linear relations. Different strategies can be used to calculate the number of matchsticks used in different figures. One such pattern is shown below. Observe the pattern and answer the following questions using Arithmetic Progression:



(a) Write the AP for the number of triangles used in the figures. Also, write the n th term of this AP.

(b) Which figure has 61 matchsticks?
(Term II, 2021-22)

51. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7: 15. Find the numbers. (2020, 2018)

52. Which term of the Arithmetic Progression $-7, -12, -17, -22, \dots$ will be -82 ? Is -100 any term of the A.P.? Give reason for your answer. (2019)

53. The sum of three numbers in A.P. is 12 and sum of their cubes is 288. Find the numbers. (Delhi 2016)

5.4 Sum of First n Terms of an A.P.

MCQ

54. Assertion (A): a, b, c are in A.P. if and only if $2b = a + c$.

Reason (R): The sum of first n odd natural numbers is n^2 .

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

- (c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true. (2023)

VSA (1 mark)

55. Find the sum of the first 100 natural numbers. (2020)

SAI (2 marks)

56. In an AP if $S_n = n(4n + 1)$, then find the AP. (Term II, 2021-22)

57. Find the common difference 'd' of an A.P. whose first term is 10 and sum of the first 14 terms is 1505. (Term II, 2021-22)

58. Find the sum of first a_{20} terms of an AP in which $d = 5$ and $a_{20} = 135$. (Term II, 2021-22)

59. Find the sum of first 20 terms of an A.P. whose n^{th} term is given as $a_1 = 5 - 2n$. (Term II, 2021-22)

60. If S, the sum of first n terms of an A.P. is given by $S_1 = 3n^2 - 4n$, find the nth term. (Delhi 2019)

61. If S, the sum of the first n terms of an A.P. is given by $S_n = 2n^2 + n$, then find its nth term. (A/ 2019)

62. Find the sum of first 8 multiples of 3. (2018)

63. How many terms of the A.P. 18, 16, 14, ... be taken so that their sum is zero? (Delhi 2016)

64. How many terms of the A.P. 27, 24, 21, ... should be taken so that their sum is zero? (Delhi 2016)

65. In an A.P., if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the A.P., where S_n denotes the sum of its first n terms. (AI 2015)

66. The first and the last terms of an A.P. are 7 and 49 respectively. If the sum of all its terms is 420, find its common difference. (Delhi 2014)

67. The first and the last terms of an A.P. are 8 and 65 respectively. If the sum of all its terms is 730, find its common difference. (Delhi 2014)

68. The sum of the first n terms of an A.P. is $3n^2 + 6n$. Find the nth term of this A.P. (Foreign 2014)

69. The sum of the first n terms of an A.P. is $5n - n^2$. Find the n th term of this A.P. (Foreign 2014)

70. The sum of the first n terms of an A.P. is $4n^2 + 2n$. Find the n th term of this A.P. (Foreign 2014) SA II (3 marks)

71. The sum of first 15 terms of an A.P. is 750 and its first term is 15. Find its 20th term. (2023)

72. Rohan repays his total loan of 1,18,000 by paying every month starting with the first instalment of 1,000. If he increase the instalment by 100 every month, what amount will be paid by him in the 30th instalment? What amount of loan has he paid after 30th instalment? (2023)

73. Find the sum of first 16 terms of an Arithmetic Progression whose 4th and 9th terms are -15 and -30 respectively. (2020C)

74. In an A.P. given that the first term $(a) = 54$, the common difference $(d) = -3$ and the n th term $(a_n) = 0$, find n and the sum of first n terms (S_n) of the A.P. (2020)

75. Find the sum : $(-5) + (-8) + (-11) + \dots + (-230)$ (2020) (Ap)

76. For an A.P., it is given that the first term $(a) = 5$, common difference $(d) = 3$, and the n th term $(a_n) = 50$. Find n and sum of first n terms (S_n) of the A.P. (2020)

77.

If m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then find

the sum of its first mn terms. (2019, Delhi 2017) 

78. Find the sum of n terms of the series

$$\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \dots \quad (\text{Delhi 2017})$$

79. The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P. (AI 2017, Delhi 2014)

80. If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first n terms of the A.P. (NCERT, Delhi 2016)

81. How many terms of the A.P. 65, 60, 55, ... be taken so that their sum is zero? (Delhi 2016)

82. If the ratio of the sum of first n terms of two A.P.'s is $(7n + 1) : (4n + 27)$, find the ratio of their m th terms. (AI 2016)

83. The digits of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number. (AI 2016)

84. The sums of first n terms of three arithmetic progressions are S_1 , S_2 and S_3 respectively. The first term of each A.P. is 1 and their common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$. (AI 2016)

85. The sum of the first n terms of three A.P.'s are S_1 , S_2 and S_3 . The first term of each is 5 and their common difference are 2, 4 and 6 respectively. Prove that $S_1 + S_3 = 2S_2$. (Foreign 2016)

86. If S_n denotes the sum of first n terms of an A.P., prove that $S_{12} = 3(S_8 - S_4)$. (Delhi 2015)

87.

If the sum of the first n terms of an A.P. is $\frac{1}{2}(3n^2 + 7n)$,
then find its n^{th} term. Hence write its 20th term.
(Delhi 2015)

88. If S_n denotes the sum of first n terms of an A.P., prove that $S_{30} = 3(S_{20} - S_{10})$ (Delhi 2015, Foreign 2014)

89. The 14th term of an A.P. is twice its 8th term. If its 6th term is -8, then find the sum of its first 20 terms. (AI 2015)

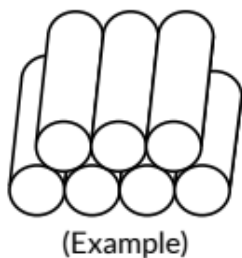
90. The 16th term of an A.P. is five times its third term. If its 10th term is 41, then find the sum of its first fifteen terms. (AI 2015)

91. The 13th term of an A.P. is four times its 3rd term. If its fifth term is 16, then find the sum of its first ten terms. (AI 2015)

92. In an A.P., if the 12th term is -13 and the sum of its first four terms is 24, find the sum of its first ten terms. (Foreign 2015)
93. The tenth term of an A.P. is -37 and the sum of its first six terms is -27. Find the sum of its first eight terms. (Foreign 2015)
94. The sum of the first seven terms of an A.P. is 182. If its 4th and the 17th terms are in the ratio 1 : 5, find the A.P. (AI 2014)
95. The sum of the first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the 28th term of this A.P. (Foreign 2014)

LA (4/5/6 marks)

96. The ratio of the 11th term to 17th term of an A.P. is $\frac{3}{4}$. Find the ratio of 5th term to 21st term of the same A.P. Also, find the ratio of the sum of first 5 terms to that of first 21 terms. (2023)
97. 250 logs are stacked in the following manner:
22 logs in the bottom row, 21 in the next row, 20 in the row next to it and so on (as shown by an example). In how many rows, are the 250 logs placed and how many logs are there in the top row? (2023)



98. Solve: $1 + 4 + 7 + 10 + \dots + x = 287$ (2020)
99. Find the sum of all odd numbers between 0 and 50. (2019C)
100. How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer. (2019C)
101. The first term of an A.P. is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the A.P. (Delhi 2019)
102. Find the sum of all the two digit numbers which leave the remainder 2 when divided by 5. (AI 2019)

103. The ratio of the sums of first m and first n terms of an A. P. is $m^2 : n^2$. Show that the ratio of its m th and n th terms is $(2m - 1) : (2n - 1)$. (Delhi 2017, Foreign 2016)

104. If the sum of first m terms of an A.P. is the same as the sum of its first n terms, show that the sum of its first $(m + n)$ terms is zero. (Delhi 2017)

105. If the ratio of the sum of the first n terms of two A.P.'s is $(7n + 1) : (4n + 27)$, then find the ratio of their 9th terms. (AI 2017)

106. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief? (Delhi 2016)

107. A thief, after committing a theft, runs at a uniform speed of 50 m/minute. After 2 minutes, a policeman runs to catch him. He goes 60 m in first minute and increases his speed by 5 m/minute every succeeding minute. After how many minutes, the policeman will catch the thief? (Delhi 2016)

108. The houses in row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses preceding the house numbered X is equal to sum of the numbers of houses following X . (AI 2016)

109. Reshma wanted to save at least 6500 for sending her daughter to school next year (after 12 months). She saved 450 in the first month and raised her savings by 20 every next month. How much will she be able to save in next 12 months? Will she be able to send her daughter to the school next year? What value is reflected in this question? (Foreign 2016)

110. Ramkali required ₹2500 after 12 weeks to send her daughter to school. She saved 100 in the first week and increased her weekly saving by ₹20 every week. Find whether she will be able to send her daughter to school after 12 weeks. What value is generated in the above situation? (Delhi 2015)

111. Find the 60th term of the A.P., 8, 10, 12, ..., if it has a total of 60 terms and hence find the sum of its last 10 terms. (AI 2015)

112. An arithmetic progression 5, 12, 19, ... has 50 terms. Find its last term. Hence find the sum of its last 15 terms. (AI 2015)
113. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 3, when divided by 4. Also find the sum of all numbers on both sides of the middle term separately. (Foreign 2015)
114. Find the middle term of the sequence formed by all numbers between 9 and 95, which leave a remainder 1 when divided by 3. Also find the sum of the numbers on both sides of the middle term separately. (Foreign 2015)
115. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 5 when divided by 7. Also find the sum of all numbers on both sides of the middle term separately. (Foreign 2015)
116. In an A.P. of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P. (Delhi 2014)
117. In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students. Which value is shown in this question? (AI 2014)

CBSE Sample Questions

5.3 nth Term of an A.P.

VSA (1 mark)

1. Which term of the A.P. 27, 24, 21, ... is zero? (2020-21)
2. In an Arithmetic Progression, if $d = 4$, $n = 7$, $a_1 = 4$, then find a_n . (2020-21)

SA I (2 marks)

3. Find the value of $a_{25} - a_{15}$ for the AP: 6, 9, 12, 15, (Term II, 2021-22)
4. If 7 times the seventh term of the AP is equal to 5 times the fifth term, then find the value of its 12th term. (Term II, 2021-22)

5.4 Sum of first n terms of an A.P.

LA (4/5/6 marks)

5. The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.



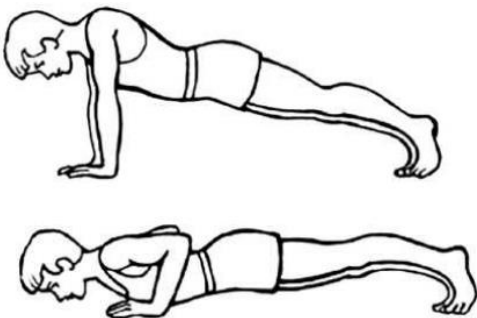
- (i) If the first circular row has 30 seats, how many seats will be there in the 10th row?
- (ii) For 1500 seats in the auditorium, how many rows need to be there?

OR

If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row?

- (iii) If there were 17 rows in the auditorium, how many seats will be there in the middle row? (2022-23)

6. Push-ups are a fast and effective exercise for building strength. These are helpful in almost all sports including athletics. While the push-up primarily targets the muscles of the chest, arms, and shoulders, support required from other muscles helps in toning up the whole body.



Nitesh wants to participate in the push-up challenge. He can currently make 3000 push-ups in one hour. But he wants to achieve a target of 3900 push-ups

in 1 hour for which he practices regularly. With each day of practice, he is able to make 5 more push-ups in one hour as compared to the previous day. If on first day of practice he makes 3000 push-ups and continues to practice regularly till his target is achieved. Keeping the above situation in mind answer the following questions:

(i) Form an A.P. representing the number of push-ups per day and hence find the minimum number of days he needs to practice before the day his goal is accomplished.

(ii) Find the total number of push-ups performed by Nitesh up to the day his goal is achieved. (Term II, 2021-22)

SOLUTIONS

Previous Years' CBSE Board Questions

1. (c): We have, a, b, c , are in A.P.

$\therefore b = a + d$, and $c = a + 2d$

Now, $a - 2b - c = a - 2(a + d) - (a + 2d)$

$= a - 2a - 2d - a - 2d = -2a - 4d$

2. (d): We have, $\sqrt{7}, \sqrt{28}, \sqrt{63}, \dots$ i.e., $\sqrt{7}, 2\sqrt{7}, 3\sqrt{7}, \dots$

Here, first term, $a = \sqrt{7}$ and common difference, $d = \sqrt{7}$

($\therefore d = a_2 - a_1$)

$\therefore$ Next term, $a_3 = a_2 + d = 3\sqrt{7} + \sqrt{7} = 4\sqrt{7} = \sqrt{112}$

3. (a): Since, $k + 2, 4k - 6$ and $3k - 2$ are three consecutive terms of A.P.

$\therefore a_2 - a_1 = a_3 - a_2$

$= (4k - 6) - (k + 2) = (3k - 2) - (4k - 6)$

$= 4k - 6 - k - 2 = 3k - 2 - 4k + 6$

$= 3k - 8 = -k + 4 \Rightarrow 4k = 12 \Rightarrow k = 3$

4.

(b): Given, $\frac{-5}{7}, a, 2$ are in A.P. therefore common difference is same.

$\therefore a_2 - a_1 = a_3 - a_2$

$a - \left(\frac{-5}{7}\right) = 2 - a \Rightarrow a + \frac{5}{7} = 2 - a \Rightarrow 2a = \frac{9}{7} \Rightarrow a = \frac{9}{14}$

5.

(c): In option (c), We have

$$a_2 - a_1 = \frac{7}{3} - \frac{4}{3} = \frac{3}{3} = 1; a_3 - a_2 = \frac{9}{3} - \frac{7}{3} = \frac{2}{3}$$

As $a_2 - a_1 \neq a_3 - a_2$, the given list of numbers does not form an A.P.

6. (a): Given, $2x$, $(x + 10)$ and $(3x+2)$ are in A.P.

$$\therefore (x+10) - 2x = (3x+2) - (x + 10)$$

$$= -x+10=2x-8=-3x=-18 \Rightarrow x=6$$

7. (c): Given, $3y - 1$, $3y+5$ and $5y + 1$ are in A.P.

$$\therefore 3y+5-(3y-1)=5y + 1 - (3y+5)$$

$$= 3y+5-3y+1=5y+1-3y-5$$

$$\Rightarrow 6 = 2y - 4 \Rightarrow y = \frac{10}{2} = 5$$

8. (b): k , $2k - 1$ and $2k + 1$ are three consecutive terms of an A.P.

$$\therefore (2k-1) - (k) = (2k + 1) - (2k - 1)$$

$$= k-3 = k-1-2$$

9.

$$\text{Given A.P. is } \frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}$$

$$\therefore a_2 - a_1 = \frac{3-a}{3a} - \frac{1}{a} = \frac{3-a-3}{3a} = \frac{-a}{3a} = \frac{-1}{3}$$

10. Given A.P. is, $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}$,

or $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}, \dots$

$$\therefore d = \text{common difference} = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$$

11. Given, $k + 9$, $2k - 1$ and $2k + 7$ are in A.P.

$$\therefore (2k-1) - (k+9) = (2k+7) - (2k - 1)$$

$$= 2k-1-k-9=2k + 7 - 2k + 1 \Rightarrow k-10-8 \quad k = 18$$

12. Given, $2k + 1$, $3k + 3$ and $5k - 1$ are in A.P.

$$\therefore 2(3k+3)=2k+1+5k - 1 \Rightarrow 6k + 6 = 7k \Rightarrow k=6$$

13. Since, $a, 7, b, 23$ are in A.P.

$\therefore$ Common difference is same.

$$\therefore 7-a=b-7=23-b$$

Taking second and third terms, we get

$$b-7=23-b \Rightarrow 2b = 30$$

$$\Rightarrow b=15$$

Taking first and second terms, we get

$$7-a=b-7$$

$$= 7-a=15-7$$

$$= 7-a=8$$

$$= a = -1$$

Hence, $a = -1, b = 15$.

14. Let $a_1 = (a - b)^2$, $a_2 = (a^2 + b^2)$ and $a_3 = (a + b)^2$

$$\text{Now, } a_2 - a_1 = (a^2 + b^2) - (a - b)^2$$

$$= a^2 + b^2 - (a^2 + b^2 - 2ab)$$

$$= a^2 + b^2 - a^2 - b^2 + 2ab = 2ab$$

$$\text{Again } a_3 - a_2 = (a + b)^2 - (a^2 + b^2)$$

$$= a^2 + b^2 + 2ab - a^2 - b^2 = 2ab$$

$$\therefore a_2 - a_1 = a_3 - a_2$$

So, $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in A.P.

15. (c): Given, first term, $a = p$ and common difference,

$$d = q$$

$$\therefore \text{10th term, } a_{10} = a + (10-1)d = p + 9q$$

16. (d): First term, $a = \sqrt{7}$ and common difference,

$$d = \sqrt{28} - \sqrt{7} = 2\sqrt{7} - \sqrt{7} = \sqrt{7}$$

$\therefore$ Fourth term of the A.P. is $(a_4) = a + 3d$

$$= \sqrt{7} + 3\sqrt{7} = 4\sqrt{7} = \sqrt{112}$$

17. Given, $a, p_n + q$

$$= a + (n-1)d = p_n + q$$

$$= (n-1)d = p_n + q - a$$

$$\Rightarrow d = \frac{p_n + q - a}{n-1}$$

18. Let n th term of A.P. 10, 7, 4, is - 41.

$$\therefore a_n = a + (n-1)d$$

$$= -41 = 10 + (n-1)(-3) \quad [\because d = 7 - 10 = -3]$$

$$= -41 = 10 - 3n + 3$$

$$= -41 = 13 - 3n$$

$$= 3n = 54 \Rightarrow n = 18$$

$\therefore$ 18th term of given A.P. is -41.

19. Given, $a = 15$, $d = -3$ and $a_n = 0$

$$\therefore a + (n-1)d = 0$$

$$= 15 + (n-1)(-3) = 0$$

$$= 15 - 3n + 3 = 0 \Rightarrow 18 - 3n = 0$$

$$= -3n = -18 \Rightarrow n = 6$$

20. Two-digit numbers which are divisible by 3 are 12, 15, 18, ..., 99, which forms an A.P. with first term

(a) = 12, common difference (d) = $15 - 12 = 3$ and last term

(1) or n th term (a_n) = 99

$$\therefore a + (n-1)d = 99$$

$$= 12 + (n-1)3 = 99 \Rightarrow 3n = 99 - 12 = 87$$

$$\Rightarrow n = \frac{87}{3} = 29$$

Thus, there are 29 two-digit numbers which are divisible by 3.

21. Let a be the first term of A.P.

Here, common difference (d) = - 4,

seventh term (a_7) = 4, $n = 7$

$$\therefore a_n = a + (n-1)d$$

$$\Rightarrow a_7 = a + (7-1) \times (-4) = 4$$

$$\Rightarrow a + 6 \times (-4) = 4 \Rightarrow a - 24 = 4 \Rightarrow a = 28$$

22. Let a be the first term and d be the common difference of the A.P.

$$\text{Given, } a_{21} - a_7 = 84$$

...(i)

$$\text{Now, } a_n = a + (n - 1)d$$

$$\therefore a_{21} = a + 20d \text{ and } a_7 = a + 6d$$

$$(a+20d) - (a+6d) = 84 \text{ [From (i)]}$$

$$\Rightarrow 14d=84 \Rightarrow d=6$$

23. Given A.P. is 5, 9, 13, ..., 185.

Here, / = last term = 185

$$d = \text{common difference} = 9-5 = 4$$

$$\therefore 9^{\text{th}} \text{ term from the end} = 1-(9-1)d = 1-8d$$

$$= 185-8 \times 4 = 185 - 32 = 153$$

24.

Given, $-5, -\frac{5}{2}, 0, \frac{5}{2}, \dots$ are in A.P.

$$\Rightarrow a = -5, d = \left(\frac{-5}{2}\right) - (-5) = \frac{5}{2}$$

We know that, $a_n = a + (n-1)d$

$$\therefore a_{25} = a + (25-1)d = (-5) + 24 \times \left(\frac{5}{2}\right) = -5 + 60 = 55$$

25. Let a be the first term and d be the common difference of given A.P.

$$\text{We have, } \frac{a_4}{a_7} = \frac{2}{3}$$

$$\Rightarrow \frac{a+3d}{a+6d} = \frac{2}{3} \quad [\because a_n = a + (n-1)d]$$

$$\Rightarrow 3(a+3d) = 2(a+6d)$$

$$\Rightarrow 3a+9d = 2a+12d$$

$$\Rightarrow 3a-2a = 12d-9d \Rightarrow a = 3d$$

$$\text{Now, } \frac{a_6}{a_8} = \frac{a+5d}{a+7d} = \frac{3d+5d}{3d+7d} = \frac{8d}{10d} = \frac{4}{5}$$

$$\therefore \frac{a_6}{a_8} = \frac{4}{5}$$

26. Given, 293, 285, 277, ..., 53 be an A.P.

$$a = 293, d = 285-293 = -8$$

We know, $a_n = a + (n-1)d$

$$= 53 \Rightarrow 293 + (n-1)(-8)$$

$$= 53 \Rightarrow 293 + (n-1)(-8)$$

$$= -240 = (n-1)(-8) \Rightarrow 30 = n-1 \Rightarrow n=31$$

27.

Handwritten solution for problem 27:

9, 7, 5 — and 15, 12, 9 —

$$a_n = a + (n-1)d \quad a'_n = a' + (n-1)d'$$

$$= 9 + (n-1)(-2) \quad = 15 + (n-1)(-3)$$

$$= 9 - 2n + 2 \quad = 15 - 3n + 3$$

$$= 11 - 2n \quad = 18 - 3n$$

Since $a_n = a'_n$

$$11 - 2n = 18 - 3n$$

$$11 - 18 = -3n + 2n$$

$$-7 = -n$$

$$\boxed{n=7}$$

[Topper's Answer, 2022]

28.

Let n^{th} term of the given A.P. be $\frac{49}{2}$.

Here, $a = -\frac{11}{2} \Rightarrow a_1 = -\frac{11}{2}$ and $a_2 = -3$

$$\therefore d = a_2 - a_1 = -3 - \left(-\frac{11}{2}\right) = -3 + \frac{11}{2} = \frac{-6+11}{2} = \frac{5}{2}$$

And, $a_n = a + (n-1)d$

$$\Rightarrow \frac{49}{2} = -\frac{11}{2} + (n-1) \times \frac{5}{2} \Rightarrow \frac{49}{2} + \frac{11}{2} = (n-1) \times \frac{5}{2}$$

$$\Rightarrow 30 = (n-1) \times \frac{5}{2} \Rightarrow n-1 = 12 \Rightarrow n = 13$$

Thus, 13th term of the given A.P. is $\frac{49}{2}$.

29. Let the first term and common difference of an A.P. be a and d , respectively.

Given, $a_3 = 5$ and $a_7 = 9$

$$\Rightarrow a + (3-1)d = 5 \text{ and } a + (7-1)d = 9 \quad [a_n = a + (n-1)d]$$

$$\Rightarrow a + 2d = 5 \dots (i)$$

$$\text{and } a + 6d = 9 \dots (ii)$$

On subtracting (i) from (ii), we get

$$4d = 4 \Rightarrow d = 1$$

$$\text{From (i), } a + 2(1) = 5 \Rightarrow a + 2 = 5 \Rightarrow a = 3$$

So, required A.P. is $a, a + d, a + 2d, a + 3d, \dots$

i.e., $3, 3+1, 3+2(1), 3+3(1), \dots$ i.e., $3, 4, 5, 6, \dots$

30. Given, $a = 0$, we have to show that $a_{29} = 2919$
 $= a + 8d = 0 + 8d = 8d$

Now, $a_{19} = a + 18d = -8d + 18d = 10d$ [$a = -8d$]

$$a_{29} = a + 28d = -8d + 28d = 20d = 2(10d) = 2a_{19}$$

Hence, $a_{29} = 2a_{19}$

31. We have, first term, $a = 3$, common difference,
 $d = 15 - 3 = 12$

n^{th} term of an A.P. is given by $a_n = a + (n-1)d$

$$\therefore a_{21} = 3 + (20) \times 12 = 3 + 240 = 243$$

Let the r^{th} term of the A.P. be 120st more than the 21st term.

$$= a + (r-1)d = 243 + 120$$

$$= 3 + (r-1)12 = 12363$$

$$= (r-1)12 + 360 \Rightarrow r-1 = 30 \Rightarrow r = 31$$

32. According to question, $a_{17} - a_{10} = 7$

$$\text{i.e., } a + 16d - (a + 9d) = 7$$

where a = first term, d = common difference

$$7d = 7. d = 1$$

33. Numbers divisible by 8 between 200 and 500 are
208, 216, 224,, 496 which forms an A.P.

First term (a) = 208, common difference (d) = 8

n^{th} term of an A.P., $a_n = a + (n-1)d$

$$= 496 - 208 + (n-1)8$$

$$= 288 = (n-1)8 \Rightarrow n-1 = 36 \Rightarrow n = 37$$

34. Given sequence is an A.P. in which $a = 20$ and

$$d = 19\frac{1}{4} - 20 = \frac{77}{4} - 20 = \frac{-3}{4}$$

Let n^{th} term of the given A.P. be the first negative term.

$$\text{i.e., } a_n < 0 \Rightarrow a + (n-1)d < 0$$

$$\Rightarrow 20 + (n-1)\left(\frac{-3}{4}\right) < 0 \Rightarrow 80 - 3n + 3 < 0$$

$$\Rightarrow 83 - 3n < 0$$

$$\Rightarrow 3n > 83 \Rightarrow n > 27\frac{2}{3} \Rightarrow n = 28$$

35. Let the first term and common difference of the A.P. be a and d respectively.

Since, $a_n = a + (n-1)d$

$$\therefore a_4 = a + (4-1)d = 0$$

$$\Rightarrow a + 3d = 0 \Rightarrow a = -3d \dots (i)$$

$$\text{Now, } a_{25} = a + 24d$$

$$\Rightarrow a_{25} = -3d + 24d \text{ [using (i)]}$$

$$\Rightarrow a_{25} = 21d$$

$$\text{Now, } a_{11} = a + 10d = -3d + 10d \text{ [using (i)]}$$

$$\therefore a_{11} = 7d$$

Multiply both sides by 3, we get

$$3a_{11} = 21d \Rightarrow 3a_{11} = a_{25}$$

36. We have, first term, $a = 6$, common difference $d = 13 - 6 = 7$

Last term, $216 =$

$$\text{Now, } a_n = a + (n-1)d$$

$$= 216 = 6 + (n-1)7 \Rightarrow 216 - 6 = 7n - 7 \Rightarrow 210 + 7 = 7n$$

$$\Rightarrow n = \frac{217}{7} = 31, \text{ which is odd.}$$

$$\therefore \text{ Middle term} = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ term} = \left(\frac{31+1}{2} \right)^{\text{th}} \text{ term}$$

$$= 16^{\text{th}} \text{ term}$$

$$a_{16} = a + (16-1)d = 6 + 15 \times 7 = 111$$

37. We have, first term (a) = 213,
common difference (d) = $205 - 213 = -8$

Last term (l) = 37

$$\text{Now, } l = a + (n-1)d$$

$$= 37 = 213 + (n-1)(-8)$$

$$= 37 - 2138(n-1)$$

$$= -176 - 8(n-1)$$

$$= n-1 = 22 \Rightarrow n = 23, \text{ which is odd.}$$

$$\therefore \text{ Middle term} = \left(\frac{23+1}{2} \right)^{\text{th}} \text{ term} = 12^{\text{th}} \text{ term}$$

$$\text{So, } a_{12} = a + 11d = 213 + 11(-8) \\ = 213 - 88 = 125$$

$$\therefore \text{Middle term} = \left(\frac{23+1}{2}\right)^{\text{th}} \text{ term} = 12^{\text{th}} \text{ term}$$

$$\begin{aligned}\text{So, } a_{12} &= a + 11d = 213 + 11(-8) \\ &= 213 - 88 = 125\end{aligned}$$

38. Let the first term be a and d be the common difference of an A.P.

$$\text{We have, } a_4 = 11 \Rightarrow a + 3d = 11 \dots(i)$$

$$\therefore \text{According to question, } a_5 + a_7 = 34$$

$$= (a + 4d) + (a + 6d) = 34$$

$$= 2a + 10d = 34 \Rightarrow a + 5d = 17 \dots(ii)$$

Subtracting (i) from (ii), we get

$$2d = 6 \Rightarrow d = 3$$

39. Let the first term be a and d be the common difference of the A.P.

$$\text{Given, } a_5 = 20 \Rightarrow a + 4d = 20 \dots(i)$$

$$\text{Also, } a_7 + a_{11} = 64$$

$$= a + 6d + a + 10d = 64 \Rightarrow 2a + 16d = 64$$

$$\Rightarrow a + 8d = 32 \dots(ii)$$

Subtracting (i) from (ii), we have

$$4d = 12 \Rightarrow d = 3$$

40. Let the first term be a and d be the common difference of the A.P.

$$\text{Given, } a_9 = -32 \Rightarrow a + 8d = -32 \dots(i)$$

$$\text{Also, } a_{11} + a_{13} = -94$$

$$\Rightarrow a + 10d + a + 12d = -94 \Rightarrow 2a + 22d = -94$$

$$\Rightarrow a + 11d = -47 \dots(ii)$$

Subtracting (ii) from (i), we have

$$-3d = 15 \Rightarrow d = -5$$

41. Natural numbers between 101 and 999 which are divisible by both 2 and 5 are 110, 120, ..., 990, which forms an A.P.

$$\text{Here, } a = 110, d = 10, a_n = 990$$

$$\text{Now, } a + (n-1)d = a_n$$

$$\Rightarrow 110 + (n-1)10 = 990$$

$$\Rightarrow (n - 1)10 - 880 \Rightarrow n - 1 = 88 \Rightarrow n = 89$$

Hence, there are 89 numbers between 101 and 999 which are divisible by both 2 and 5.

42. We have

First term, $a_1 = -14$

Fifth term, $a_5 = 2$

Last term, $a_n = 62$

Let d be the common difference and n be the number of terms.

$$\therefore a_5 = 2$$

$$-14 + (5-1)d = 2$$

$$= 4d = 16$$

$$= d = 4$$

Now, $a_n = 62$

$$= -14 + (n-1)4 = 62$$

$$= 4n - 4 = 76$$

$$= 4n = 80$$

$$n = 20$$

$\therefore$ There are 20 terms in A.P.

43. Given, A.P. is 65, 61, 57, 53, ...

Here, first term $a = 65$ and common difference, $d = -4$

Let the n^{th} term is negative.

$$\text{Last term, } a_n = a + (n - 1)d = 65 + (n-1)(-4)$$

$$= 65 - 4n + 4$$

$$= 69 - 4n, \text{ which will be negative when } n = 18$$

So, 18th term is the first negative term.

44. Let a be the first term and d be the common difference of the given A.P.

r^{th} term of A.P, $a_r = a + (r - 1)d$

According to question,

$$a_m = a + (m-1)d = \frac{1}{n} \quad \dots(i)$$

$$\text{and } a_n = a + (n-1)d = \frac{1}{m} \quad \dots(ii)$$

Subtracting (ii) from (i), we get

$$(m-n)d = \frac{m-n}{mn} \Rightarrow d = \frac{1}{mn}$$

Putting $d = \frac{1}{mn}$ in (i), we get

$$a + (m-1)\frac{1}{mn} = \frac{1}{n} \Rightarrow a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n}$$

$$\Rightarrow a = \frac{1}{mn}$$

$$\therefore a_{mn} = a + (mn-1)d = \frac{1}{mn} + (mn-1)\frac{1}{mn} = \frac{1+mn-1}{mn} = 1$$

45. Let A be the first term and D be the common difference of given A.P.

$$\therefore T_p = A + (p-1)D = a \quad \dots(i)$$

$$T_q = A + (q-1)D = b \quad \dots(ii)$$

$$T_r = A + (r-1)D = c \quad \dots(iii)$$

Now, $a(q-r) + b(r-p) + c(p-q)$

$$= [A + (p-1)D](q-r) + [A + (q-1)D](r-p) + [A + (r-1)D](p-q) \quad [\text{Using (i), (ii) and (iii)}]$$

$$= A[q-r+r-p+p-q] + D[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)]$$

$$= 0 + D[pq - rp - q + r + qr - pq - r + p + rp - qr - p + q] = 0$$

46. Let the four parts that are in A.P. be $a - 3d$, $a - d$, $a + d$, $a + 3d$

$$\therefore a - 3d + a - d + a + d + a + 3d = 56$$

$$\Rightarrow 4a = 56 \Rightarrow a = 14$$

According to question,

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6} \Rightarrow \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{5}{6}$$

$$\Rightarrow 6(196 - 9d^2) = 5(196 - d^2) \quad [\because a = 14]$$

$$= 1176 - 54d^2 = 980 - 5d^2$$

$$= 49d^2 = 196 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$$

So, the four parts are 8, 12, 16 and 20 if $d = 2$ and 20, 16, 12 and 8 if $d = -2$.

47. Let a be the first term and d be the common difference of the A.P.

$$\text{Given, } a_7 = \frac{1}{9}, a_9 = \frac{1}{7}$$

$$a_7 = a + (7-1)d = \frac{1}{9} \Rightarrow a + 6d = \frac{1}{9} \quad \dots(i)$$

$$a_9 = a + (9-1)d = \frac{1}{7} \Rightarrow a + 8d = \frac{1}{7} \quad \dots(ii)$$

Subtracting (i) from (ii), we get

$$2d = \frac{2}{63} \Rightarrow d = \frac{1}{63}$$

Putting $d = \frac{1}{63}$ in (i), we get

$$a + \frac{6}{63} = \frac{1}{9} \Rightarrow a = \frac{1}{63}$$

$$\text{So, } a_{63} = a + 62d = \frac{1}{63} + \frac{62 \times 1}{63} = \frac{63}{63} = 1$$

48. Let a be the first term and d be the common difference of the A.P.

$$\text{Now, } a_n = a + (n-1)d$$

$$a_5 + a_9 = 30 \text{ (Given)}$$

$$a_5 + 4d + a + 8d = 30$$

$$2a + 12d = 30 \quad \dots(i)$$

$$\text{Also, } a_{25} = 3a_8$$

$$a + 24d = 3(a + 7d) \Rightarrow a + 24d = 3a + 21d$$

$$\Rightarrow 2a - 3d = 0$$

On solving (i) and (ii), we get $a = 3, d = 2$

∴ A.P. is 3, 5, 7,

49. Let a be the first term and d be the common difference of the A.P.

$$\text{Now, } a_n = a + (n-1)d$$

$$\Rightarrow a_2 + a_7 = 30 \Rightarrow a + d + a + 6d = 30$$

$$\Rightarrow 2a + 7d = 30 \quad \dots(i)$$

$$\text{Also, } a_{15} = 2a_8 - 1$$

$$\Rightarrow a + 14d = 2(a + 7d) - 1 \Rightarrow a + 14d = 2a + 14d - 1$$

$$\Rightarrow a = 1 \quad \dots(ii)$$

Substituting (ii) in (i), we get

$$2+7d=30 \quad 7d=28 \Rightarrow d=4$$

Hence, the A.P. is formed as 1, 5, 9,....

50. (a) Number of triangles in figure 1 = 4

Number of triangles in figure 2 = 6

Number of triangles in figure 3 = 8

∴ Required A.P. is 4, 6, 8

$$a=4, d=6-4=2$$

$$\therefore a_n = a + (n-1)d$$

$$= a_n = 4 + (n-1)(2) = a_n = 4 + 2n - 2 = 2n + 2$$

$$= a_n = 2(n + 1) = a_9 = 2(9 + 1) = 2(10) = 20$$

(b) Numbers of matchsticks used in figure 1 = 12

Number of matchsticks used in figure 2 = 19

Number of matchsticks used in 3 = 26

Thus, required A.P. be 12, 19, 26.....

$$A = 12, D = 19 - 12 = 7$$

$$\therefore A_n = A + (N-1)D$$

$$\Rightarrow 61 = 12 + (N-1)(7) \Rightarrow 61 = 12 + 7N - 7$$

$$\Rightarrow 61 = 5 + 7N \quad 7N = 56 \Rightarrow N = 8$$

Hence, figure 8 has 61 matchsticks.

51. Let the four consecutive numbers be $(a - 3d)$, $(a - d)$, $(a + d)$, $(a + 3d)$.

Sum of four numbers = 32 [Given]

$$\Rightarrow (a-3d) + (a-d) + (a+d) + (a+3d) = 32$$

$$\Rightarrow 4a = 32 \Rightarrow a = 8$$

$$\text{Also, } \frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

$$\Rightarrow \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15} \Rightarrow 15a^2 - 135d^2 = 7a^2 - 7d^2$$

$$\Rightarrow 8a^2 = 128d^2 \Rightarrow d^2 = \frac{8a^2}{128} = \frac{8 \times 64}{128} = 4 \quad \therefore d = \pm 2$$

If $d = 2$, then the numbers are $(8-6)$, $(8-2)$, $(8+2)$ and $(8+6)$ i.e., 2, 6, 10, 14.

If $d = -2$, then the numbers are $(8+6)$, $(8+2)$, $(8-2)$, $(8-6)$ i.e., 14, 10, 6, 2.

Hence, the numbers are 2, 6, 10, 14 or 14, 10, 6, 2.

52. Given, A.P. is -7, -12, -17, -22,

and nth term of given A.P. is -82.

$$\therefore a_n = a + (n-1)d$$

$$= -82 = -7 + (n-1)(-5) \quad [\because d = -12 - (-7) = -12 + 7 = -5]$$

$$= -82 - 7 - 5n + 5 = -82 - 5n - 2$$

$$= 5n = 82 - 2 \Rightarrow 5n = 80 \Rightarrow n = 16$$

$\therefore$ 16th term of given A.P. is -82

$$\therefore 17^{\text{th}} \text{ term} = -82 - 5 = -87$$

$$18^{\text{th}} \text{ term} = -87 - 5 = -92$$

$$19^{\text{th}} \text{ term} = -92 - 5 = -97$$

$$20^{\text{th}} \text{ term} = -97 - 5 = -102$$

Hence, -100 is not any term of given A.P.

53. Let the three numbers in A.P. are $a - d$, a , $a + d$.

According to question, $a - d + a + a + d = 12$

$$\rightarrow 3a = 12 \Rightarrow a = 4$$

$$\text{Also, } (a-d)^3 + a^3 + (a+d)^3 = 288$$

$$\Rightarrow 64 - 48d + 12d^2 - d^3 + 64 + 48d + 12d^2 + d^3 = 288$$

$$\Rightarrow 24d^2 + 192 = 288 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$$

The numbers will be $a - d$, a , $a + d$

$$4 + 2, 4, 4 - 2 = 6, 4, 2, \text{ if } d = -2$$

$$\text{or } 4 - 2, 4, 4 + 2 = 2, 4, 6, \text{ if } d = 2$$

54. (b): Since, a , b , c are in A.P., then $b - a = c - b$

$$\Rightarrow 2b = a + c$$

First n odd natural numbers be 1, 3, 5, ... $(2n-1)$.

which form an A.P. with $a = 1$ and $d = 2$

$$\text{Sum of first } n \text{ odd natural numbers} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 + (n-1)2] = n^2$$

Hence, assertion and reason are true but reason is not the correct explanation of assertion.

55. First 100 natural numbers are 1, 2, 3,..... 100 which form an A.P. with $a = 1$, $d = 1$.

Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{100}{2}[2 \times 1 + (100-1) \times 1] = 50[2 + 99] = 50 \times 101 = 5050$$

56. Given $S_1 = n(4n + 1)$

$$S_1 = 1(4+1) = 5 = a_1$$

$$S_2 = 2(8+1) = 2(9) = 18$$

$$S_3 = 3(4(3)+1) = 3(13) = 39$$

$$S_4 = 4(4(4)+1) = 4(17) = 68$$

We know that $a_n = S_n - S_{n-1}$

$$\therefore a_2 = S_2 - S_1 = 18 - 5 = 13$$

$$a_3 = S_3 - S_2 = 39 - 18 = 21$$

$$\therefore a_4 = S_4 - S_3 = 68 - 39 = 29$$

The required A.P. is 5, 13, 21, 29,

27.

Handwritten solution for problem 27:

$$a = 10$$

$$S_{14} = \frac{n}{2}(2a + (n-1)d)$$

$$1505 = \frac{14}{2}(2(10) + 13d)$$

$$1505 = 7(20 + 13d)$$

$$1505 = 20 + 13d$$

$$\frac{1505}{7} = \frac{20 + 13d}{7}$$

$$215 = 20 + 13d$$

$$195 = 13d$$

$$d = 15$$

[Topper's Answer, 2022]

58. Given, $a_{20} = 135$, $d = 5$

Let the first term of an A.P. be a and n^{th} term be a_n .

$$\therefore a_{20} = a + (20-1)d \quad [\because a_n = a + (n-1)d]$$

$$= 135 = a + 19(5)$$

$$= 135 = a + 95 \Rightarrow a = 135 - 95 = 40$$

Now, sum of first n terms of an A.P. is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$\Rightarrow S_{20} = \frac{20}{2}[2(40) + 19(5)] = 10[80 + 95]$$
$$= 10 \times 175 = 1750$$

Hence, the sum of first 20 terms of an A.P. is 1750.

59. Given, nth term of the A.P. series is

$$a_n = 5 - 2n \dots (i)$$

$$\text{Put } n = 1, a_1 = 5 - 2(1) = 5 - 2 = 3$$

$$\text{Put } n = 2, a_2 = 5 - 2(2) = 5 - 4 = 1$$

$$\text{Put } n = 3, a_3 = 5 - 2(3) = 5 - 6 = -1$$

$$\text{Put } n = 4, a_4 = 5 - 2(4) = 5 - 8 = -3$$

So, the series becomes 3, 1, -1, -3,

$$\text{Here, } a = 3 \text{ and } d = a_2 - a_1 = 1 - 3 = -2$$

We know that, sum of n terms of an A.P. is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$\therefore \text{Sum of first 20 terms of an A.P. is}$$
$$S_{20} = \frac{20}{2}[2(3) + (20-1)(-2)]$$
$$= 10[6 + 19 \times -2] = 10[6 - 38]$$
$$= 10 \times -32 = -320$$

Hence, the required sum of 20 terms of given A.P. is -320.

$$60. \text{ Given, } S_1 = 3n^2 - 4n.$$

$$\text{We know that } S_n - S_{n-1} = a_n$$

$$= 3n^2 - 4n - \{3(n-1)^2 - 4(n-1)\} = a_n$$

$$= 3n^2 - 4n - \{3(n^2 + 1 - 2n) - 4n + 4\} = a_n$$

$$= 3n^2 - 4n - \{3n^2 + 3 - 6n - 4n + 4\} = a_n$$

$$= 3n^2 - 4n - 3n^2 - 7 + 10n = a_n \Rightarrow 6n - 7 = a_n$$

$$61. \text{ We have, } S_1 = 2n^2 + n$$

$$\therefore S_{n-1} = 2(n-1)^2 + (n-1) = 2(n^2 + 1 - 2n) + n - 1$$

$$= 2n^2 + 2 - 4n + n - 1 = 2n^2 - 3n + 1$$

$$\text{Now, } n^{\text{th}} \text{ term of the A.P., } a_n = S_n - S_{n-1}$$

$$= (2n^2 + n) - (2n^2 - 3n + 1) = 4n - 1$$

62. Multiples of 3 are 3, 6, 9, 12, _____

These numbers are in A.P. such that $a = 3$, $d = 6 - 3 = 3$,
 $n = 8$

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_8 = \frac{8}{2}[2 \times 3 + (8-1) \times 3]$$

$$\Rightarrow S_8 = 4[6 + 7 \times 3] = 4[6 + 21] = 4 \times 27 = 108$$

$$\therefore S_8 = 108$$

63. Let a be the first term and d be the common difference of A.P.

$$\text{Sum of } n \text{ terms, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\text{Here, } a = 18, d = 16 - 18 = -2$$

$$\therefore \frac{n}{2}[2(18) + (n-1)(-2)] = 0$$

$$\Rightarrow n[18 - n + 1] = 0 \Rightarrow n = 19$$

$\therefore$ Sum of 19 terms of the A.P. is zero.

($n+0$)

64. Let a be the first term and d be the common difference of A.P.

$$\text{Sum of } n \text{ terms is given as } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\text{Here, } a = 27, d = 24 - 27 = -3$$

According to question,

$$0 = \frac{n}{2}[2(27) + (n-1)(-3)]$$

$$\Rightarrow n[54 - 3n + 3] = 0$$

$$\Rightarrow 3n = 57 \Rightarrow n = 19 \quad (\because n \neq 0)$$

65. Let a be the first term and d be the common difference of the A.P.

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\text{According to question, } S_5 + S_7 = 167$$

$$\Rightarrow \frac{5}{2}[2a + (5-1)d] + \frac{7}{2}[2a + (7-1)d] = 167$$

$$\Rightarrow \frac{5}{2}(2a + 4d) + \frac{7}{2}(2a + 6d) = 167$$

$$\Rightarrow 5a + 10d + 7a + 21d = 167$$

$$\Rightarrow 12a + 31d = 167 \quad \dots(i)$$

$$\text{Also, } S_{10} = 235$$

$$\Rightarrow \frac{10}{2}[2a + (10-1)d] = 235$$

$$= 5(2a + 9d) = 235 \Rightarrow 2a + 9d = 47$$

Multiplying both sides by 6, we get

$$12a + 54d = 282 \dots(ii)$$

Subtracting (i) from (ii), we get

$$23d - 115d = -5$$

Substituting the value of d in (i), we get

$$5 = 167$$

$$12a + 31$$

$$= 12a + 155 = 167$$

$$= 12a = 12 \Rightarrow a = 1$$

Hence, the A.P. will be 1, 6, 11,...

66. Let a and d denote first term and common difference respectively of the A.P.

$$\text{Given, } a = 7 \text{ and } 49 = a + (n-1)d$$

$$= 49 = 7 + (n-1)d \Rightarrow (n-1)d = 42 \dots(i)$$

$$S_n = \frac{n}{2}[7 + 49] = 420 \Rightarrow \frac{n}{2}(56) = 420 \Rightarrow n = \frac{420}{28} = 15$$

Putting $n = 15$ in (i), we get

$$14d = 42 \Rightarrow d = 3$$

67. Let a and d denote the first term and common difference respectively of the A.P.

$$\text{Given, } a = 8 \text{ and } 65 = a + (n-1)d$$

$$= 65 = 8 + (n-1)d \Rightarrow 57 = (n-1)d \dots(i)$$

$$S_n = 730 \Rightarrow \frac{n}{2}(a + l) = 730$$

$$\Rightarrow n[8 + 65] = 1460 \Rightarrow n = \frac{1460}{73} = 20$$

Putting value of n in (i), we get $57 = (20-1)d$

$$= 57 = 19d \Rightarrow d = 3$$

$$68. \text{ We have, } S_1 = 3n^2 + 6n$$

$$\therefore S_{n-1} = 3(n-1)^2 + 6(n-1) = 3(n^2 + 1 - 2n) + 6n - 6$$

$$= 3n^2 + 3 - 6n + 6n - 6 = 3n^2 - 3$$

$$\text{nth term of A.P., } a_n = S_n - S_{n-1}$$

$$= (3n^2 + 6n) - (3n^2 - 3) = 6n + 3$$

69. We have, $S_1 = 5n - n^2$

$$\therefore S_{n-1} = 5(n-1) - (n-1)^2$$

$$= 5n - 5 - (n^2 + 1 - 2n) = -n^2 + 7n - 6$$

n th term of A.P., $a_n = S_n - S_{n-1}$

$$= 5n - n^2 - (-n^2 + 7n - 6)$$

$$= 5n - n^2 + n^2 - 7n + 6 = -2n + 6$$

70. We have, $S_1 = 4n^2 + 2n$

$$\therefore S_{n-1} = 4(n-1)^2 + 2(n-1)$$

$$= 4(n^2 + 1 - 2n) + 2n - 2$$

$$= 4n^2 + 4 - 8n + 2n - 2 = 4n^2 - 6n + 2$$

n th term of the A.P., $a_n = S_n - S_{n-1}$

$$= (4n^2 + 2n) - (4n^2 - 6n + 2) = 8n - 2$$

71. Here, $a = 15$ and $S_{15} = 750$

$$\therefore S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore S_{15} = \frac{15}{2} [2 \times 15 + (15-1)d] = 750$$

$$\Rightarrow 15(15 + 7d) = 750$$

$$\Rightarrow 15 + 7d = 50$$

$$\Rightarrow 7d = 35$$

$$\Rightarrow d = 5$$

$$\Rightarrow d = 5$$

Now, 20th term $= a + (n-1)d$

$$= 15 + (20-1)5$$

$$= 15 + 95$$

$$= 110$$

72. Total amount of loan Rohan takes = ₹ 1,18,000

First instalment paid by Rohan = 1000

Second instalment paid by Rohan = 1000 + 100 = 1100

Third instalment paid by Rohan = 1100 + 100 = ₹ 1200 and so on.

Let its 30th instalment be n .

Thus, we have 1000, 1100, 1200, ..., which forms an A.P.

with first term $(a) = 1000$

and common difference $(d) = 1100 - 1000 = 100$

n th term of an A.P., $a_n = a + (n-1)d$

For 30th instalment, $a_{30} = a + (30-1)d$
 $= 1000 + (29)100 = 1000 + 2900 = 3900$
 So, 3900 will be paid by Rohan in the 30th instalment.
 Now, we have $a = 1000$, last term $(l) = 3900$

$$\therefore \text{Sum of 30 instalments, } S_{30} = \frac{30}{2}[a+l]$$

$$S_{30} = \frac{30}{2}(1000 + 3900) = 73500$$

Total amount he still have to pay after the 30th instalment
 $= (\text{Amount of loan}) - (\text{Sum of 30 instalments})$
 $= 1,18,000 - 73,500 = 44,500$
 Hence, 44,500 still have to pay after the 30th instalment.

73. Given, $a_4 = -15$ and $a_{16} = -30$

$$\therefore a + 3d = -15 \dots (i) \quad [a + (n-1)d]$$

$$a + 15d = -30 \dots (ii)$$

On subtracting (ii) from (i), we have

$$-5d = 15 \Rightarrow d = -3$$

Put $d = -3$ in (i), we have

$$a + 3(-3) = -15 \Rightarrow a - 9 = -15 \Rightarrow a = -6$$

$$\text{Now, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\begin{aligned} \Rightarrow S_{16} &= \frac{16}{2}[2(-6) + (16-1)(-3)] \\ &= 8[2(-6) + (15)(-3)] = 8[-12 - 45] = -456 \end{aligned}$$

74. Given, $d = 3$, $a = 54$ and $a_n = 0$

Since $a_n = a + (n-1)d$

$$\therefore 0 = 54 + (n-1)(3) \Rightarrow 0 = 54 + 3n - 3 \Rightarrow 3n = -51$$

$$\Rightarrow n = -17$$

$$\text{Now, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\begin{aligned} &= \frac{-17}{2}[2 \times 54 + (-17-1)(3)] \\ &= \frac{-17}{2}[108 - 54] = \frac{-17}{2} \times 54 = -459 \end{aligned}$$

75. Let $S_n = (-5) + (-8) + (-11) + \dots + (-230)$

Here, $a = -5$,

$$\text{Now, } -8 - (-5) = -8 + 5 = -3$$

$$-11 - (-8) = -11 + 8 = -3$$

So, common difference, $d = -3$ and last term, $a_n = -230$

$$\text{Since, } a_n = a + (n - 1)d$$

$$= -230 - 5 + (n - 1)(-3)$$

$$= -230 - 5 - 3n + 3 - 230 = -3n - 2$$

$$\Rightarrow 3n = 228 \Rightarrow n = \frac{228}{3} = 76$$

By using sum formula, we have

$$\begin{aligned} S_{76} &= \frac{76}{2} [2(-5) + (76 - 1)(-3)] \\ &= 38 [-10 + 75(-3)] = 38 [-10 - 225] = -8930 \end{aligned}$$

76. Given, $a = 5$, $d = 3$ and $a_n = 50$

$$\text{Since, } a_n = a + (n - 1)d$$

$$50 = 5 + (n - 1)3 \Rightarrow 50 = 5 + 3n - 3$$

$$\Rightarrow 3n = 50 - 2 = 48 \Rightarrow n = \frac{48}{3} = 16$$

$$\text{Now, sum of } n \text{ terms } (S_n) = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{16}{2} [2 \times 5 + (16 - 1)3] = 8[10 + 15 \times 3]$$

$$= 8[10 + 45] = 8 \times 55 = 440$$

77. Let a be first term and d be the common difference of an A.P. Since, we have,

$$a_m = a + (m - 1)d = \frac{1}{n} \quad \dots(i)$$

$$a_n = a + (n - 1)d = \frac{1}{m} \quad \dots(ii)$$

Subtracting (ii) from (i), we get

$$(m - 1)d - (n - 1)d = \frac{1}{n} - \frac{1}{m}$$

$$\Rightarrow d(m - n) = \frac{m - n}{mn}$$

$$\Rightarrow d = \frac{1}{mn} \quad \dots(iii)$$

On substituting (iii) in (i), we get

$$a + (m - 1) \times \frac{1}{mn} = \frac{1}{n} \Rightarrow a = \frac{1}{n} - \frac{m - 1}{mn}$$

$$\Rightarrow a = \frac{1}{mn}$$

Sum of first mn terms is

$$\begin{aligned} S_{mn} &= \frac{mn}{2} \left[2 \left(\frac{1}{mn} \right) + (mn-1) \frac{1}{mn} \right] \\ &= \frac{mn}{2} \left[\frac{2}{mn} + 1 - \frac{1}{mn} \right] = \frac{mn}{2} \left[\frac{1}{mn} + 1 \right] = \frac{1+mn}{2} \end{aligned}$$

78.

We have, $\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \dots$ which forms an A.P. where first term $(a) = \left(4 - \frac{1}{n}\right)$

Common difference $(d) = \left(4 - \frac{2}{n}\right) - \left(4 - \frac{1}{n}\right) = -\frac{1}{n}$

and last term $(l) = \left(4 - \frac{n}{n}\right) = (4-1) = 3$ ($\because$ Series has n terms)

$\therefore$ Sum of n terms $(S_n) = \frac{n}{2}(a+l)$

$$= \frac{n}{2} \left(4 - \frac{1}{n} + 3 \right) = \frac{n}{2} \left(7 - \frac{1}{n} \right) = \frac{7n}{2} - \frac{1}{2} = \left(\frac{7n-1}{2} \right)$$

79. Let a , n and d be first term, number of terms and common difference of the A.P. respectively.

We have, first term $(a) = 5$; last term $(l) = a_n = 45$

Sum of all terms $(S_n) = 400$

$$\Rightarrow \frac{n}{2}(a+l) = 400 \Rightarrow \frac{n}{2}(5+45) = 400 \Rightarrow n = \frac{800}{50} = 16$$

$$\text{Now, } a_n = a + (n-1)d$$

$$\Rightarrow 45 = 5 + (16-1)d$$

$$\Rightarrow 40 = 15d \Rightarrow d = \frac{40}{15} = \frac{8}{3}$$

80. Let a be the first term and d be the common difference of the A.P.

Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

We have, $S_7 = 49 \Rightarrow \frac{7}{2}[2a + 6d] = 49$

$$\Rightarrow 14a + 42d = 98 \Rightarrow a + 3d = 7 \quad \dots(i)$$

and $S_{17} = 289 \Rightarrow \frac{17}{2}[2a + 16d] = 289$

$$\Rightarrow 34a + 272d = 578 \Rightarrow a + 8d = 17 \quad \dots(ii)$$

On solving (i) and (ii), we get $a = 1, d = 2$

$$\therefore S_n = \frac{n}{2}[2 + (n-1)2] = n^2$$

81. Let a be the first term and d be the common difference of A.P.

Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Here, $a = 65, d = 60 - 65 = -5$

$\therefore$ According to question,

$$0 = \frac{n}{2}[2(65) + (n-1)(-5)] \Rightarrow n[130 - 5n + 5] = 0$$

$$\Rightarrow 5n = 135 \Rightarrow n = 27 \quad (\because n \neq 0)$$

82. Let a_1, d_1 and a_2, d_2 be the first term and common difference of the two A.P.'s respectively.

$$\frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{7n+1}{4n+27} \quad [\text{Given}]$$

$$\Rightarrow \frac{a_1 + \frac{(n-1)}{2}d_1}{a_2 + \frac{(n-1)}{2}d_2} = \frac{7n+1}{4n+27}$$

$$\text{Put } \frac{n-1}{2} = m-1 \Rightarrow n-1 = 2m-2$$

$$\Rightarrow n = 2m - 2 + 1 = 2m - 1$$

$$\therefore \frac{a_1 + (m-1)d_1}{a_2 + (m-1)d_2} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-7+1}{8m-4+27} = \frac{14m-6}{8m+23}$$

83. Let the required three digit number be xyz .

$\therefore$ Digits are in A.P. $x = y-d$ and $z = y+d$ [where d is common difference]

According to question,

$$(y-d) + y + (y+d) = 15$$

$$= 3y=15 \Rightarrow y=5$$

Since, number obtained by reversing the digits (z y x) i.e.,
 $100z + 10y + x$ is 594 less than original number.

$$= (100x + 10y + z) - (100z + 10y + x) = 594$$

$$= (z - 100z) + (100x - x) = 594$$

$$= 99x - 99z = 594 \Rightarrow x - z = 6$$

$$= (y - d) - (y + d) = 6$$

$$= -2d = 6d = -3$$

$$\text{So, } x = y - d = 5 - (-3) = 8 \text{ and } z = y + d = 5 + 3 = 2$$

$\therefore$ The number is $xyz = 852$.

84.

We have, $a = 1, d_1 = 1, d_2 = 2$ and $d_3 = 3$

$$S_1 = \frac{n}{2} \{2(1) + (n-1)1\} = \frac{n}{2} \{2 + n - 1\} = \frac{n}{2}(n+1)$$

$$S_2 = \frac{n}{2} \{2(1) + (n-1)2\} = \frac{n}{2} (2 + 2n - 2) = n^2$$

$$S_3 = \frac{n}{2} \{2(1) + (n-1)3\} = \frac{n}{2} \{2 + 3n - 3\} = \frac{n(3n-1)}{2}$$

$$\text{Now, } S_1 + S_3 = \frac{n}{2}(n+1) + \frac{n}{2}(3n-1)$$

$$= \frac{n}{2}(n+1+3n-1) = \frac{n}{2} \times 4n = 2n^2 = 2S_2$$

85.

We have, $a = 5, d_1 = 2, d_2 = 4, d_3 = 6$

$$\therefore S_1 = \frac{n}{2} [10 + (n-1)2] = n(5 + n - 1) = n(n+4)$$

$$S_2 = \frac{n}{2} [10 + (n-1)4] = n(5 + 2n - 2) = n(2n+3)$$

$$S_3 = \frac{n}{2} [10 + (n-1)6] = n(5 + 3n - 3) = n(3n+2)$$

$$\text{Now, } S_1 + S_3 = n(n+4) + n(3n+2)$$

$$= n[n+4+3n+2] = n[4n+6]$$

$$= 2n(2n+3) = 2S_2$$

Hence proved.

86. Let a be the first term and d be the common difference of the A.P.

$$\therefore S_{12} = \frac{12}{2}\{2a + (12-1)d\} \quad \left[\because S_n = \frac{n}{2}\{2a + (n-1)d\} \right]$$

$$= 6\{2a + 11d\} = 12a + 66d$$

$$S_8 = \frac{8}{2}\{2a + (8-1)d\} = 4\{2a + 7d\} = 8a + 28d$$

$$S_4 = \frac{4}{2}\{2a + (4-1)d\} = 2\{2a + 3d\} = 4a + 6d$$

$$\begin{aligned} \text{Now, } 3(S_8 - S_4) &= 3(8a + 28d - 4a - 6d) \\ &= 3(4a + 22d) = 12a + 66d = S_{12} \end{aligned}$$

87.

$$\text{We have, } S_n = \frac{1}{2}(3n^2 + 7n)$$

$$S_{n-1} = \frac{1}{2}\{3(n-1)^2 + 7(n-1)\} = \frac{1}{2}\{3(n^2 - 2n + 1) + 7n - 7\}$$

$$= \frac{1}{2}\{3n^2 - 6n + 3 + 7n - 7\} = \frac{1}{2}\{3n^2 + n - 4\}$$

We know that, $a_n = S_n - S_{n-1}$

$$= \frac{1}{2}(3n^2 + 7n) - \frac{1}{2}(3n^2 + n - 4)$$

$$= \frac{1}{2}(3n^2 + 7n - 3n^2 - n + 4) = \frac{1}{2}(6n + 4)$$

$$\Rightarrow a_n = 3n + 2$$

$$\therefore a_{20} = 3 \times 20 + 2 = 60 + 2 = 62$$

88. Let a be the first term and d be the common difference of the A.P.

$$\text{Now, sum of } n \text{ terms, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{30} = \frac{30}{2}[2a + (30-1)d] = 15[2a + 29d] = 30a + 435d$$

$$S_{20} = \frac{20}{2}[2a + (20-1)d] = 10[2a + 19d] = 20a + 190d$$

$$S_{10} = \frac{10}{2}[2a + (10-1)d] = 5[2a + 9d] = 10a + 45d$$

$$\begin{aligned} \therefore 3[S_{20} - S_{10}] &= 3[20a + 190d - 10a - 45d] \\ &= 30a + 435d = S_{30} \end{aligned}$$

89. Let a be the first term and d be the common difference of the A.P.. $a_n = a + (n-1)d$

Here, $a_{14} = 2 \times a_8$

$$\Rightarrow a + (14-1)d = 2[a + (8-1)d]$$

$$\Rightarrow a + 13d = 2a + 14d \Rightarrow a + d = 0 \dots (i)$$

$$\text{Also, } a_6 = -8 \Rightarrow a + 5d = -8 \dots (ii)$$

Subtracting (i) from (ii), we get

$$4d - 8d = -2$$

From equation (i), we have $a = 2$

$$\text{Now, } S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore S_{20} = \frac{20}{2} [2 \times 2 + 19(-2)] = 10[4 - 38] = -340$$

90. Let a be the first term and d be the common difference of the A.P. $\therefore a_n = a + (n-1)d$

Here, $a_{16} = 5 \times a_3$

$$= a + (16-1)d = 5[a + (3-1)d]$$

$$= a + 15d = 5a + 10d$$

$$\Rightarrow 4a = 5d \Rightarrow a = \frac{5d}{4}$$

$$\text{Also, } a_{10} = a + 9d = 41$$

$$\Rightarrow 41 = \frac{5d}{4} + 9d \Rightarrow 41 = \frac{5d + 36d}{4} \Rightarrow d = 4$$

$$\therefore a = \frac{5 \times 4}{4} = 5$$

Hence, A.P. is 5, 9, 13,

$$\text{Now, sum of } n \text{ terms, } S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore S_{15} = \frac{15}{2} [2 \times 5 + 14 \times 4] = \frac{15}{2} [10 + 56] = \frac{15}{2} \times 66 = 495$$

91. Let a be the first term and d be the common difference of the A.P.

n th term of an A.P., $a_n = a + (n-1)d$

Given, $a_{13} = 4 \times a_3$

$$\Rightarrow a + 12d = 4(a + 2d) \Rightarrow a + 12d = 4a + 8d$$

$$\Rightarrow 3a - 4d = 0 \quad \dots(i)$$

Also, $a_5 = 16$

$$\Rightarrow a + 4d = 16 \quad \dots(ii)$$

Adding (i) and (ii), we get

$$4a = 16 \Rightarrow a = 4$$

From (ii), we have

$$4 + 4d = 16 \Rightarrow 4d = 12 \Rightarrow d = 3$$

$$\text{Now, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{10} = \frac{10}{2}[2 \times 4 + (10-1)3] = 5[8 + 27] = 5 \times 35 = 175$$

92. Let the first term be a and d be the common difference of the A.P.

Given, $a_{12} - a_{13} = -13 \dots(i)$

Also, $S_4 = 24$

$$\Rightarrow \frac{4}{2}[2a + 3d] = 24 \quad \left[\because S_n = \frac{n}{2}[2a + (n-1)d] \right]$$

$$\Rightarrow 2a + 3d = 12 \quad \dots(ii)$$

Multiply (i) by 2 and then subtracting (ii) from it, we get

$$19d = -38$$

$$\Rightarrow d = -2$$

$\therefore$ Put $d = -2$ in (i), we have

$$a + 11(-2) = -13$$

$$\Rightarrow a = -13 + 22 \Rightarrow a = 9$$

$$\therefore S_{10} = \frac{10}{2}[2(9) + 9(-2)] = \frac{10}{2}[18 - 18] = 0$$

93. Let first term be a and common difference be d . Given, $a_{10} = -37$

$$a + 9d = -37 \dots(i)$$

$$\text{Also, } S_6 = -27 \Rightarrow \frac{6}{2}(2a + 5d) = -27$$

$$\Rightarrow 2a + 5d = -9 \quad \dots(ii)$$

Multiply (i) by 2 and then subtracting (ii) from it, we get

$$13d = -65 \Rightarrow d = -5$$

Put $d = -5$ in (i), we get,

$$a + 9(-5) = -37 \Rightarrow a - 45 = -37 \Rightarrow a = 8$$

$$\therefore \text{Sum of first eight terms, } S_8 = \frac{8}{2}[2a + 7d]$$

$$= 4[2(8) + 7(-5)] = 4[16 - 35] = 4(-19) = -76$$

94. Given, sum of first seven terms of an A.P., $S_7 = 182$

$$\text{i.e., } 182 = \frac{7}{2}[2a + (7-1)d]$$

$$\Rightarrow 364 = 14a + 42d \Rightarrow 26 = a + 3d \quad \dots(i)$$

$$\text{Also, } \frac{a_4}{a_{17}} = \frac{1}{5} \Rightarrow \frac{a+3d}{a+16d} = \frac{1}{5}$$

$$= 5(a+3d) = a + 16d \Rightarrow 5a + 15d = a + 16d$$

$$= 4a - d = 0 \Rightarrow d = 4a \dots(ii)$$

Substituting (ii) in (i), we get

$$26 = a + 3(4a) \Rightarrow 13a = 26 \Rightarrow a = 2$$

$$\therefore d = 4(2) = 8$$

Hence, the A.P. is formed as 2, 10, 18, ...

95. Let a be the first term and d be the common difference of the A.P.

Given, $S_7 = 63$ and $S_{14} = 63 + 161 = 224$

$$S_7 = 63 \Rightarrow \frac{7}{2}[2a + (7-1)d] = 63 \quad \left[\because S_n = \frac{n}{2}\{2a + (n-1)d\} \right]$$

$$\Rightarrow \frac{7}{2}(2a + 6d) = 63 \Rightarrow a + 3d = 9 \quad \dots(i)$$

$$\text{Also, } \frac{14}{2}[2a + (14-1)d] = 224$$

$$\Rightarrow 7(2a + 13d) = 224$$

$$\Rightarrow 2a + 13d = 32 \quad \dots(ii)$$

Solving (i) and (ii), we get

$$a = 3, d = 2$$

$$\text{Now, } a_{28} = a + 27d = 3 + 27(2) = 57$$

96.

$$\text{Given, } \frac{a_{11}}{a_{17}} = \frac{3}{4}$$

$$\Rightarrow \frac{a+10d}{a+16d} = \frac{3}{4}$$

$$\Rightarrow 4(a+10d) = 3(a+16d)$$

$$\Rightarrow 4a+40d = 3a+48d$$

$$\Rightarrow 4a+40d-3a-48d = 0 \Rightarrow a = 8d$$

$$\text{Also, } \frac{a_5}{a_{21}} = \frac{a+4d}{a+20d}$$

$$= \frac{8d+4d}{8d+20d} \quad (\because a = 8d)$$

$$= \frac{12d}{28d} = \frac{3}{7} \text{ i.e., } 3:7$$

$$\text{Required ratio} = \frac{S_5}{S_{21}}$$

$$= \frac{\frac{5}{2}[2a+(5-1)d]}{\frac{21}{2}[2a+(21-1)d]} = \frac{5[2a+4d]}{21[2a+20d]}$$

$$= \frac{5[2(8d)+4d]}{21[2(8d)+20d]} \quad (\because a = 8d)$$

$$= \frac{5 \times 20d}{21 \times 36d} = \frac{25}{189} \text{ i.e., } 25:189$$

97. Number of logs in 1st row = 22

Number of logs in 2nd row = 21

Number of logs in 3rd row = 20

The number of logs i.e., 22, 21, 20, ..., forms an A.P, where

$$a = 22, d = a_2 - a_1 = 21 - 22 = -1$$

Let the number of rows be n.

$$\text{Now, } S_n = \frac{n}{2}[2(22) + (n-1)(-1)]$$

$$\Rightarrow 250 = \frac{n}{2}[44 - (n-1)] \quad [\because S_n = 250 \text{ (Given)}]$$

$$\Rightarrow 250 \times 2 = 44n - n(n-1)$$

$$\Rightarrow 500 = 44n - n^2 + n \Rightarrow n^2 - 45n + 500 = 0$$

$$\Rightarrow (n-20)(n-25) = 0 \Rightarrow n = 20 \text{ or } n = 25$$

$$T_n = 0 \Rightarrow a + (n-1)d = 0$$

$$\Rightarrow 22 + (n-1)(-1) = 0 \Rightarrow 22 - (n-1) = 0$$

$$\Rightarrow 22 - n + 1 = 0 \Rightarrow n = 23$$

$$\Rightarrow n = 23 \text{ i.e., } 23^{\text{rd}} \text{ term becomes 0.}$$

$$\therefore n = 25 \text{ is not required.}$$

$$\therefore \text{Number of rows} = 20$$

$$\text{Now, } T_{20} = a + (20-1)d$$

$$= 22 + 19(-1)$$

$$= 22 - 19 = 3$$

$$\therefore \text{Number of logs in the } 20^{\text{th}} \text{ (top) row is 3.}$$

98. We have, $1 + 4 + 7 + 10 + \dots + x = 287$

It is an A.P. with $a = 1$, $d = 4 - 1 = 3$

Let n be the number of terms.

$$\therefore \frac{n}{2}[2a + (n-1)d] = 287$$

$$\Rightarrow \frac{n}{2}[2 + 3(n-1)] = 287 \Rightarrow n(2 + 3n - 3) = 574$$

$$\Rightarrow n(3n-1) = 574 \Rightarrow 3n^2 - n - 574 = 0$$

$$\Rightarrow 3n^2 - 42n + 41n - 574 = 0$$

$$\Rightarrow 3n(n-14) + 41(n-14) = 0$$

$$\Rightarrow (n-14)(3n+41) = 0$$

$$\Rightarrow n = 14$$

$$\text{Now, } x = a + (n-1)d = 1 + 13 \times 3 = 40.$$

99. Odd numbers between 0 and 50 are 1, 3, 5, 7, ..., 49

$$a = 1, d = 3 - 1 = 2, a_n = 49$$

$$a_n = a + (n-1)d$$

$$49 = 1 + (n-1)(2) \Rightarrow 49 = 1 + 2n - 2$$

$$\Rightarrow 49 - 2n = 1 - 2 \Rightarrow 50 = 2n \Rightarrow n = 25$$

$$\text{Now, } S_n = \frac{n}{2}(a+l) \Rightarrow S_n = \frac{25}{2}(1+49) = \frac{25}{2} \times 50 = 625$$

100. Given, A.P. is 45, 39, 33.....

$$a = 45, d = 39 - 45 = -6$$

$$S_1 = 180$$

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow 180 = \frac{n}{2}[2 \times 45 + (n-1)(-6)] \Rightarrow 180 = \frac{n}{2}[90 - 6n + 6]$$

$$\Rightarrow 180 = \frac{n}{2}(96 - 6n) \Rightarrow 180 = \frac{n}{2} \times 2(48 - 3n)$$

$$\Rightarrow 180 = n(48 - 3n) \quad 180 = 48n - 3n^2$$

$$\Rightarrow 3n^2 - 48n + 180 = 0$$

$$\Rightarrow 3(n^2 - 16n + 60) = 0$$

$$\Rightarrow (n-6)(n-10) = 0 \Rightarrow n = 6 \text{ or } 10$$

The sum of given A.P. remains same either we are taking $n = 6$ or $n = 10$ as they are repeated with reverse sign.

101. Let a , n and d be the first term, number of terms and common difference of the A.P. respectively.

Given, first term, $a = 3$, last term, $a_n = 83$

Also, sum of all terms, $S_n = 903$ (Given)

$$\Rightarrow \frac{n}{2}[a+l] = 903 \Rightarrow \frac{n}{2}[3+83] = 903$$

$$\Rightarrow 43n = 903 \Rightarrow n = 21$$

$$\text{Now, } a_n = a + (n-1)d$$

$$\Rightarrow 83 = 3 + (21-1)d$$

$$\Rightarrow 80 = 20d \Rightarrow d = 4$$

102. Two digit numbers which leave the remainder

2, when divided by 5 are 12, 17, 22, 27, ..., 92, 97.

This is an A.P. with first term, $a = 12$, common difference, $d = 5$ and last term, $l = 97$

$$\text{Here, } a_n = 97 \quad a + (n-1)d = 97$$

$$12 + (n-1)5 = 97 \Rightarrow 12 + 5n - 5 = 97$$

$$\Rightarrow 5n = 90 \Rightarrow n = 18$$

Now, $S_n = \frac{n}{2}(a+l)$

$\therefore S_{18} = \frac{18}{2}(12+97)=981$

103. Let a be the first term and d be the common difference of the given A.P. Then, the sums of m terms and n terms are respectively given by

$$S_m = \frac{m}{2}\{2a+(m-1)d\} \text{ and } S_n = \frac{n}{2}\{2a+(n-1)d\}$$

Also, $\frac{S_m}{S_n} = \frac{m^2}{n^2}$ (Given)

$$\Rightarrow \frac{\frac{m}{2}\{2a+(m-1)d\}}{\frac{n}{2}\{2a+(n-1)d\}} = \frac{m^2}{n^2} \Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

$$\Rightarrow \{2a+(m-1)d\}n = \{2a+(n-1)d\}m$$

$$\Rightarrow 2a(n-m) = d\{(n-1)m - (m-1)n\}$$

$$\Rightarrow 2a(n-m) = d(n-m) \Rightarrow d = 2a$$

$$\therefore \frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)2a}{a+(n-1)2a} = \frac{2m-1}{2n-1}$$

104. Let a be the first term and d be the common difference of the A.P. Sum of m and n terms of A.P. are

$$S_m = \frac{m}{2}[2a+(m-1)d]$$

and $S_n = \frac{n}{2}[2a+(n-1)d]$

Given that $S_m = S_n$

$$\therefore \frac{m}{2}[2a+(m-1)d] = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow 2a(m-n) + \{m(m-1) - n(n-1)\}d = 0$$

$$\Rightarrow 2a(m-n) + \{(m^2 - n^2) - (m-n)\}d = 0$$

$$\Rightarrow (m-n)\{2a + (m+n-1)d\} = 0$$

$$\Rightarrow 2a + (m+n-1)d = 0 \quad [\because m-n \neq 0] \quad \dots(i)$$

$$\begin{aligned}\text{Now, } S_{m+n} &= \frac{m+n}{2} [2a + (m+n-1)d] \\ &= \frac{m+n}{2} [0] = 0 \quad [\text{Using (i)}]\end{aligned}$$

105. Let a_1, a_2 be the first terms and d_1, d_2 be common differences of the two A.P.'s respectively.

$$\text{Given, ratio of sum of first } n \text{ terms of two A.P.} = \frac{7n+1}{4n+27}$$

$$\therefore \frac{\frac{n}{2} \{2a_1 + (n-1)d_1\}}{\frac{n}{2} \{2a_2 + (n-1)d_2\}} = \frac{7n+1}{4n+27} \quad \dots(i)$$

$$\Rightarrow \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+27}$$

Putting $\frac{n-1}{2} = 8$, we get

$$\frac{a_1 + 8d_1}{a_2 + 8d_2} = \frac{7(17)+1}{4(17)+27} \quad \left\{ \because \frac{n-1}{2} = 8 \Rightarrow n=17 \right\}$$

$$\Rightarrow \frac{a_1 + 8d_1}{a_2 + 8d_2} = \frac{120}{95} = \frac{24}{19}$$

$$\therefore \text{Ratio of 9}^{\text{th}} \text{ terms} = \frac{a_1 + 8d_1}{a_2 + 8d_2} = \frac{24}{19}$$

106. Let the total number of minutes be n .

$\therefore$ Distance covered (in m) by thief = $100n$

Distance covered (in m) by policeman

= $100 + 110 + 120 + \dots + (n-1)$ terms

According to question,

$$100n = \frac{n-1}{2} [200 + (n-2)10]$$

$$100n = 5n^2 + 105n - 10n - 210$$

$$5n^2 - 15n - 210 = 0$$

$$n^2 - n - 42 = 0$$

$$(n-7)(n+6) = 0 \Rightarrow n = 7 \quad [\therefore n = -6]$$

The policeman will catch the thief after 5 minutes.

108.

$$1, 2, 3, \dots, X-1, X, X+1, \dots, 49$$

$\underbrace{\hspace{10em}}_S$
 $\underbrace{\hspace{10em}}_{S'}$

$$S = 1 + 2 + 3 + \dots + (X - 1)$$

$$= \left(\frac{X-1}{2} \right) [2 + X - 2] = \left(\frac{X-1}{2} \right) (X)$$

$$S' = (X + 1) + (X + 2) + \dots + 49$$

$$= \left(\frac{49-X}{2} \right) (X+1+49) = \frac{49-X}{2} (X+50)$$

For $S = S'$, we have

$$X^2 - X = 49X + 49 \times 50 - X^2 - 50X$$

$$\Rightarrow 2X^2 = 49 \times 50 \Rightarrow X^2 = 49 \times 25$$

$$\therefore X = 35$$

109. Amount saved by Reshma in first month = ₹450

Amount saved by her in second month = $(450 + 20) = 470$

Continuing in this manner, we have following A.P. as 450, 470, 490,

Here, $a = 450$, $d = 20$

$$\begin{aligned} \therefore S_{12} &= \frac{12}{2} [2 \times 450 + (12-1)20] \\ &= 6[900 + 220] = 6 \times 1120 = 6720 \end{aligned}$$

Hence, Reshma will save ₹6720 in next 12 months. Yes, she will be able to send her daughter to the school next year.

So, we observe that small and regular savings can minimize the problems in our daily life.

110. Saving of first week = ₹100

Saving of second week = $100 + ₹20 = ₹120$

Saving of third week = $120 + 20 = ₹140$

So, 100, 120, 140, [forms an A.P.]

Here, $a = 100$ and $d = 120 - 100 = 20$, $n = 12$

$$\therefore S_{12} = \frac{12}{2} \{2 \times 100 + (12-1)20\} \quad \left[\because S_n = \frac{n}{2} [2a + (n-1)d] \right]$$

$$= 6 \{200 + 220\} = 2520$$

Since $2520 > 2500$

So, she would be able to send her daughter to school after 12 weeks. Values generated are awareness, responsibility and we learnt that small savings can fulfill our big desires.

111. Here, $a = 8$, $d = 10 - 8 = 2$, $n = 60$

$$a_n = a + (n-1)d$$

$$\therefore a_{60} = 8 + (60-1) \times 2 = 8 + 59 \times 2 = 126$$

10th term from the last = $(60 - 10 + 1)$ th term from the beginning = 51st term from the beginning

$$a_{51} = a + (51-1)d = 8 + 50 \times 2 = 108$$

$$\text{Sum of } n \text{ term, } S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore \text{Sum of last 10 terms} = \frac{10}{2} [2a_{51} + (10-1)d]$$

$$= 5 [2 \times 108 + 9 \times 2] = 5 \times 234 = 1170$$

112. Here, $a = 5$, $d = 12 - 5 = 7$, $a_n = a + (n-1)d$

$$\therefore \text{Last term} = a_{50} = 5 + (50-1) \times 7 = 5 + 49 \times 7$$

$$= 5 + 343 = 348$$

15th term from the last = $(50 - 15 + 1)$ th term from the beginning = 36th term from the beginning

$$\therefore a_{36} = a + (36-1)d = 5 + 35 \times 7 = 250$$

$$\text{Now, } S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore \text{Sum of last 15 terms} = \frac{15}{2} [2a_{36} + (15-1)d]$$

$$= \frac{15}{2} [2 \times 250 + 14 \times 7] = \frac{15}{2} \times 598 = 4485$$

113. The three digit numbers which leave a remainder 3, when divided by 4 are

103, 107, 111, ..., 999

It forms an A. P. with $a = 103$ and $d = 107 - 103 = 4$

Last term, a_n , 999

$$= 999 = 103 + (n-1)4 \quad (\because a_n = a + (n-1)d)$$

$$= 999 = 103 + 4n - 4$$

$$\Rightarrow 999 = 99 + 4n \Rightarrow n = \frac{900}{4}$$

$$\Rightarrow n = 225, \text{ an odd number}$$

$$\therefore \text{Middle term} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} = \left(\frac{225+1}{2}\right)^{\text{th}} \text{ term}$$

$$= 113^{\text{th}} \text{ term}$$

$$\therefore a_{113} = a + 112d = 103 + 112 \times 4$$

$$= 103 + 448 = 551$$

$$\text{We know that, } S_n = \frac{n}{2} \{2a + (n-1)d\}$$

$$\therefore S_{112} = \frac{112}{2} \{2 \times 103 + (112-1)4\}$$

$$= 56 \{1110 + 444\} = 56 \times 1554 = 87024$$

114. Numbers between 9 and 95 which leaves a remainder 1 when divided by 3 are 10, 13, 16, 19, ..., 94.

It forms an A.P. with first term, $a = 10$ and common difference, $d = 13 - 10 = 3$

$$\text{Now, } a_n = 94 = a + (n-1)d = 94$$

$$= 10 + (n-1)3 = 94 \Rightarrow (n-1)3 = 84$$

$$\Rightarrow n-1 = \frac{84}{3} \Rightarrow n = 29 \text{ (odd number)}$$

$$\text{So, middle term} = \left(\frac{29+1}{2}\right)^{\text{th}} \text{ term} = 15^{\text{th}} \text{ term}$$

$$\therefore a_{15} = a + 14d = 10 + 14 \times 3 = 52$$

$$\text{We know that, } S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore \text{Sum of first 14 terms} = \frac{14}{2} [2 \times 10 + (14-1)3]$$

$$= 7[20 + (13 \times 3)] = 7 \times 59 = 413$$

For last 14 terms, first term is $a_{16} = a + 15d$

$$= 10 + 15(3) = 55$$

$$\therefore \text{Sum of last 14 terms} = \frac{14}{2} [2 \times 55 + (14-1)3]$$

$$= 7[110 + (13 \times 3)] = 7 \times 149 = 1043$$

115. All three-digit numbers which leave a remainder 5, when divided by 7 are 103, 110, 117, ..., 999.

It forms an A.P. with first term, $a = 103$ and

$$d = 110 - 103 = 7$$

Now, $a_n = 999$

$$= 103 + (n-1)7 = 999$$

$$= (n-1)7 + 103 = 999$$

$$\Rightarrow n-1 = \frac{896}{7} = 128$$

$$\Rightarrow n = 129, \text{ which is an odd number}$$

$$\therefore \text{Middle term} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} = \left(\frac{129+1}{2}\right)^{\text{th}} \text{ term}$$

$$= 65^{\text{th}} \text{ term}$$

$$a_{65} = a + 64d = 103 + 64 \times 7 = 103 + 448 = 551$$

$$\text{We know that, } S_n = \frac{n}{2} \{2a + (n-1)d\}$$

$$\therefore \text{Sum of first 64 terms} = \frac{64}{2} [2 \times 103 + (63 \times 7)]$$

$$= 32(206 + 441) = 32 \times 647 = 20704$$

For sum of last 64 terms, i.e., first term = a_{66}

$$\Rightarrow a_{66} = 103 + 65 \times 7 = 558$$

$$\therefore \text{Sum of last 64 terms} = \frac{64}{2} [2 \times 558 + 63 \times 7]$$

$$= 32[1116 + 441] = 32 \times 1557 = 49824$$

116. Let a and d be the first term and the common difference of the A. P., respectively.

$$\text{Given, sum of the first 10 terms} = \frac{10}{2} [2a + 9d]$$

$$\Rightarrow 210 = 5[2a + 9d] \Rightarrow 42 = 2a + 9d \dots (i)$$

15th term from the last = $(50 - 15 + 1)^{\text{th}}$ term from the beginning = 36th term from the beginning

$$\text{Now, } a_{36} = a + 35d$$

$$\therefore \text{Sum of the last 15 terms} = \frac{15}{2} [2(a + 35d) + 14d]$$

$$= 2565 = 15[a + 42d] \Rightarrow 171 = a + 42d \dots(ii)$$

Solving (i) and (ii), we get $d = 4$, $a = 3$

Thus, A.P. is 3, 7, 11, 15,

117. Number of trees planted by 1st class

$$= 1 \times 2 + 1 \times 2 = 4$$

Number of trees planted by 2nd class $= 2 \times 2 + 2 \times 2 = 8$

Number of trees planted by 3rd class $= 3 \times 2 + 3 \times 2 = 12$

Number of trees planted by 12th class $= 12 \times 2 + 12 \times 2 = 48$

This will form an A.P. 4, 8, 12, ..., 48 with first term $a = 4$,
common difference, $d = 8 - 4 = 4$

$\therefore$ Number of plants planted are

$$S_{12} = \frac{12}{2} [2(4) + (11)4] = 6[8 + 44] = 6(52) = 312$$

Hence, 312 plants were planted by the students. Value shown is that we should take care of our environment.

CBSE Sample Questions

1. We have, $a = 27$, $d = -3$ and $a_n = 0$.

Since, $a_n = a + (n - 1)d$

$$= 0 = 27 + (n - 1)(-3) \quad (1/2)$$

$$\Rightarrow 30 = 3n$$

$n = 10^{\text{th}}$ Thus, 10th term of A.P. will be zero. (1/2)

2. Since $a_n = a + (n - 1)d$

$$\Rightarrow 4 = a + 6 \times (-4) \quad [d = -4, n = 7, a_n = 4 \text{ (Given)}] \quad (1/2)$$

$$\Rightarrow a = 28 \quad (1/2)$$

3. Here, a (first term) $= 6$,

d (common difference) $= 9 - 6 = 3$

Using $a_n = a + (n - 1)d$

$$\text{So, } a_{25} = 6 + 24(3) = 78 \quad (1)$$

$$\Rightarrow a_{15} = 6 + 14(3) = 48$$

$$\Rightarrow a_{25} - a_{15} = 78 - 48 = 30 \quad (1)$$

4. Given that, $7a_7 = 5a_5$

$$\Rightarrow 7(a + 6d) = 5(a + 4d)$$

$$\Rightarrow 2a + 22d = 0 \Rightarrow a + 11d = 0 \Rightarrow a_{12} = 0$$

5. (i) Since each row is increasing by 10 seats, so it is an A.P. with first term $a = 30$, and common difference $d = 10$.

In an A.P. n^{th} term a_n is

$$a_n = a + (n-1)d,$$

where, a is first term, n is number of terms, d is common difference.

So, number of seats in 10^{th} row $= a_{10} = a + 9d$

$$= 30 + 9 \times 10 = 120 \quad (1)$$

$$(ii) \quad S_n = \frac{n}{2} (2a + (n-1)d) \quad (1/2)$$

$$\Rightarrow 1500 = \frac{n}{2} (2 \times 30 + (n-1)10)$$

$$\Rightarrow 3000 = 50n + 10n^2$$

$$\Rightarrow n^2 + 5n - 300 = 0 \quad (1/2)$$

$$\Rightarrow n^2 + 20n - 15n - 300 = 0$$

$$\Rightarrow (n+20)(n-15) = 0$$

$$\Rightarrow n = -20, 15$$

$$\therefore n = 15$$

Hence, required number of rows is 15. (1)

OR

No. of seats already put up to the 10^{th} row $= S_{10} \quad (1/2)$

$$S_{10} = \frac{10}{2} \{2 \times 30 + (10-1)10\}$$

$$= 5(60 + 90) = 750$$

$$= 5(60 + 90) = 750 \quad (1)$$

So, the number of seats still required to be put are

$$1500 - 750 = 750 \quad (1/2)$$

(iii) If no. of rows $= 17$

then the middle row is the 9^{th} row

$$a_9 = a + 8d$$

$$= 30 + 80 = 110 \text{ seats} \quad (1)$$

6. (i) Let n be the number of days to reach his goal.

;- Required A.P. will be: 3000, 3005, 3010, ..., 3900 (1)

Here, $a = 3000$, $d = 3005 - 3000 = 5$, $l = 3900$

$$\therefore a_n = a + (n-1)d$$

$$\Rightarrow 3900 = 3000 + (n-1)5$$

$$900 = 5n - 5 \Rightarrow 5n = 905 \Rightarrow n = 181$$

Minimum number of days of practice = $n - 1 = 180$ days. (1)

$$(ii) \quad S_n = \frac{n}{2}(a+l) \quad (1)$$

$$= \frac{181}{2} \times (3000 + 3900) = 624450 \text{ push-ups} \quad (1)$$

