CHAPTER

2

POLYNOMIALS

Polynomials

The word Polynomial is derived from two greek words, namely Poly (meaning "many") and Nomial (meaning "terms"), therefore meaning of polynomial = many terms.

An algebraic expression p(x) of the form $p(x) = a_0 + a_1x + a_2x^2 + ...+ a_nx^n$, where $a_0, a_1, a_2.......a_n$ are real numbers and all the powers of x are non-negative integers is called a polynomial.

For Example

(i) $x^2 + 9x$ (ii) $4x^5 - \sqrt{5}x^2 + 8$ (iii) $3x^3 + \frac{5}{8}x + 5$

Following expressions are not polynomials:

(i) $\mathbf{3x - \frac{2}{x} - 5}$, as power of x is –1 i.e., negative.

(ii) $\mathbf{4x^3 - 3\sqrt{x} + 6}$, as power of x is $\frac{1}{2}$ i.e., fraction.

Degree of a Polynomial

The highest power (exponent) of the terms of a polynomial is called degree of the polynomial.

For example:

In polynomial $4x^3 - 7x^6 + 4$:

- The power of term $4x^3$ is 3.
- The power of term $-7x^6$ is 6.
- The power of term 4 is 0.

Since, the highest power is 6, therefore degree of the polynomial $4x^3 - 7x^6 + 4$ is 6.

Let's see some more examples.

(i) The degree of polynomial $5y^2 - 6$ is 2.

(ii) The degree of polynomial $7x - 13x^5 + 4x^3$ is 5.

(iii) The degree of polynomial $13x + 7$ is 1 as $x = x^1$.

Mind it

The degree of a polynomial with more than one variable can be computed by adding the exponents of each variable in terms. For example: $7a^4 - 3ab^4$

- The power of term $7a^4$ is 4.
- The power of term $- 3ab^4$ is 5. (a has exponent 1, b has 4, so 1 + 4 = 5)

Types of Polynomials

In general, the polynomials are divided into three categories.

Types of Polynomials
- Based on number of distinct variables
- Based on degree
- Based on number of terms

Based on degree

Degree	Name	Example
Not defined	Zero	0
0	(non-zero)constant	$(p^0 = 1)$
1	Linear	$p + 1$
2	Quadratic	$p^2 + p + 13$
3	Cubic	$p^3 + 2p - p + 4$

Generally, a polynomial of degree n, for n greater than 3, is called a polynomial of degree n, although quartic and quintic polynomial are sometimes used for degree 4 and 5 respectively.

Based on number of terms

Number of non-zero terms	Name	Example
0	zero polynomial	0
1	monomial	x^2
2	binomial	$x^2 + 1$
3	trinomial	$x^2 + x + 1$

Based on number of distinct variables

Number of distinct variables	Name	Example
1	Univariate	$a + 3$
2	Bivariate	$a + b + 5$
3	Trivariate	$a + b + c + 7$

Usually, a polynomial in more than one variable is called a multivariate polynomial. In this chapter, mainly we will discuss only about the polynomials in one variable.

Ex. $P(x) = 4x^7 - 3x^2 + 5$ Trinomial of degree 7

$Q(y) = 5y^2 + 9$ Quadratic Binomial

$R(s) = 75$ Linear monomial

Note: In general, the polynomial in one variable x of degree 'n' is $a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_2x^2 + a_1x + a_0$, $a_n \neq 0$, where $a_n, a_{n-1}, \dots a_2, a_1, a_0$ all are constants.

Values and Zeroes of a Polynomial

If p(x) is a polynomial in variable x and a is any real number, then the value obtained by replacing x by a in p(x) is called **value of p(x) at x = a** and is denoted by p(a).

For Example:

For a polynomial $f(x) = 2x^3 - 5x^2 + 7$.

To find its value at $x = 5$, replace x by 5.

So, the value of $f(x) = 2x^3 - 5x^2 + 7$ at $x = 5$

$$f(5) = 2(5)^3 - 5(5)^2 + 7$$
$$= 250 - 125 + 7$$
$$= 132$$

If for $x = a$, the value of the polynomial p(x) is 0 i.e., $p(a) = 0$; then $x = a$ is a **zero of the polynomial p(x)**.

For **example:**

For polynomial $p(x) = x - 2$; $p(2) = 2 - 2 = 0$

$\therefore$ $x = 2$ or simply 2 is a zero of the polynomial $p(x) = x - 2$.

Note:

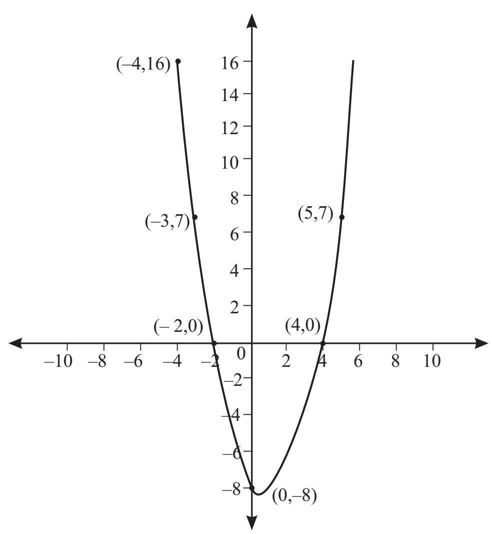
(i) A polynomial of degree n can have atmost n zeroes which means if a polynomial is of degree 5, then it can have at the most 5 zeroes.

(ii) Difference between zeroes and solutions

Let's try to draw a graph of the polynomial $f(x) = x^2 - 2x - 8$

The following table gives the values of y or f(x) for various values of x.

x	**–4**	**–3**	**–2**	**0**	**4**	**5**
$y = x^2 - 2x - 8$	16	7	0	–8	0	7



All the above values are the solutions of the polynomial $f(x) = x^2 - 2x - 8$

From these values, (–2, 0) and (4,0) are the zeroes of the polynomial f(x) as it makes the value of f(x) equal to 0.

Factor Theorem

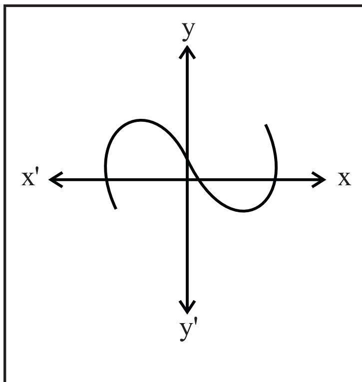
Let p(x) be a polynomial of degree $n \geq 1$ and 'a' be a real number such that $p(a) = 0$, then $(x - a)$ is a factor of p(x). Conversely, if $(x - a)$ is a factor of p(x), then $p(a) = 0$.

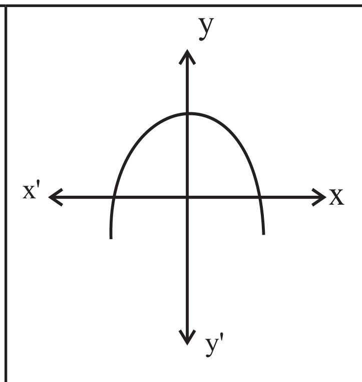
Remainder Theorem

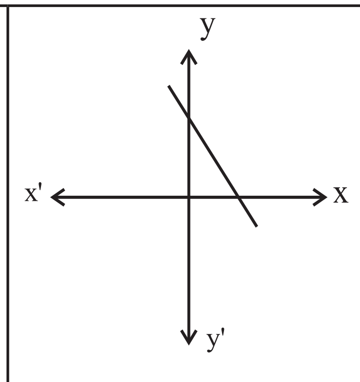
Let p(x) be any polynomial of degree $n \geq 1$ and 'a' be any real number. If p(x) is divided by $(x - a)$, then the remainder is equal to p(a) i.e., zero.

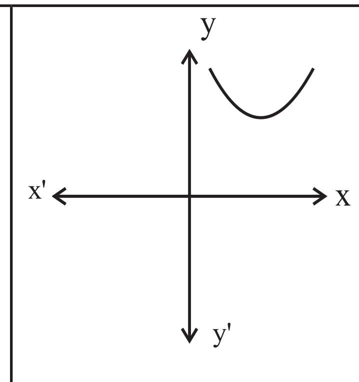
Geometrical Meaning of the Zeroes of a Polynomial

When the graph of the polynomial is drawn on a graph sheet, it will cut or touches the x-axis in as many places as there are zeroes of the polynomial. Thus, the zeroes of a polynomial f(x) are the x-coordinates of the points, where the graph of y = f(x) intersects the x-axis.





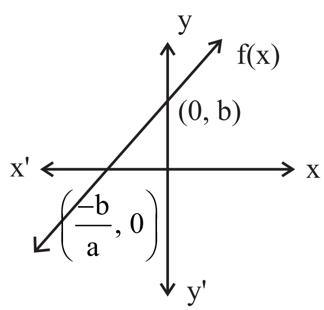




Graph Showing Three Zeroes of f(x)	Graph Showing Two Zeroes of f(x)	Graph Showing One Zero of f(x)	Graph Showing No Zero of f(x)

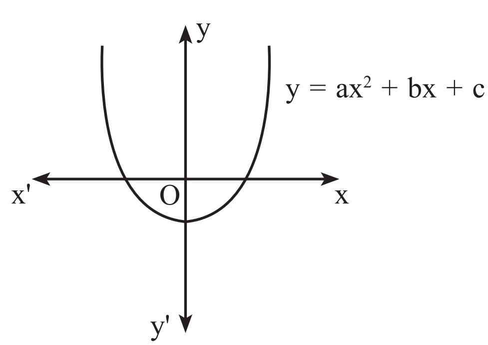
Graph of a Linear Polynomial

The graph of a linear polynomial $f(x) = ax + b, a \neq 0$ is always a straight line. To draw the graph of a straight line we need minimum two points, because a unique line will pass through two given points. The line represented by $y = ax + b$ crosses the X-axis at exactly one point, namely $\left(-\frac{b}{a}, 0\right)$.

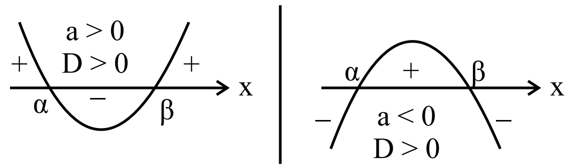


Graph of a Quadratic Polynomial

The graph of any quadratic polynomial $f(x) = ax^2 + bx + c, a \neq 0$ is a parabola. It is basically a curved shape opening up or down depending upon whether $a > 0$ or $a < 0$.



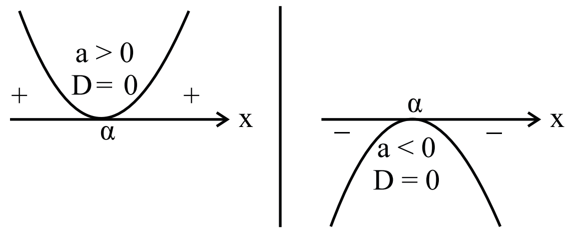
Case 1: If $D > 0$, the parabola will intersect the x-axis at two distinct points where $D = \sqrt{b^2 - 4ac}$. So, the quadratic polynomial $ax^2 + bx + c, a \neq 0$ has 2 zeroes in this case.



Note: In the above graph α and β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$. D is the discriminant of a quadratic equation about which you will study in depth in chapter 4.

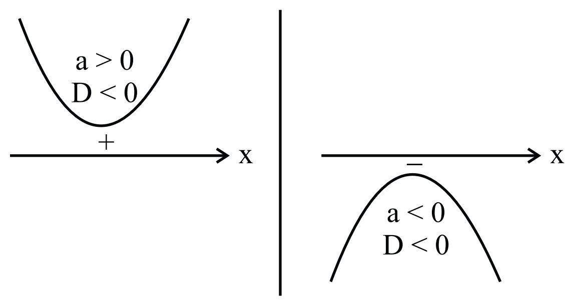
Roots are real & distinct

Case II: If $D = 0$, the parabola will just touch the x-axis at one point and vice-versa. So, the quadratic polynomial $ax^2 + bx + c = 0$, $a \neq 0$ has 1 zero in this case.



Roots are equal

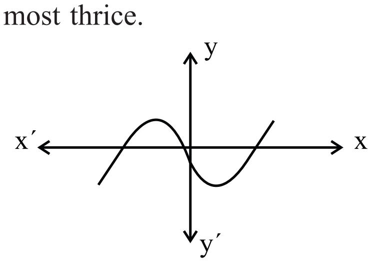
Case III: If $D < 0$, the parabola will not intersect x-axis at all and vice-versa. So, the quadratic polynomial $ax^2 + bx + c = 0$, $a \neq 0$ has no real zero in this case.

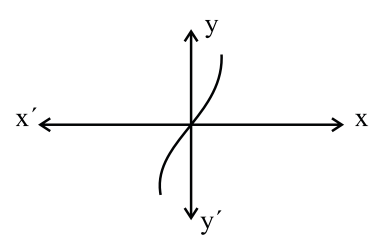


Roots are imaginary

Graph of a Cubic Polynomial

Graphs of cubic polynomial does not have a fixed standard shape. The graphs of cubic polynomial will always cross X-axis at least once and at most thrice.





Mind it

- ❒ A non-zero constant is a polynomial of degree zero, but the degree of zero polynomial is not defined.
- ❒ Zero of a polynomial is actually the solution of the curve or graph, $y = f(x)$ and the line $y = 0$.
- ❒ A polynomial of degree $n \geq 1$ can have at the most n real zeroes.
- ❒ A non-zero constant polynomial has no zeroes.
- ❒ If the sum of the co-efficients of polynomial is zero, then $(x – 1)$ is a factor of the polynomial.

Example

1. Degree of polynomial 5 is ________________?

Ans. We know the degree of the polynomial is the highest power of a variable in the polynomial. That is degree indicates the highest exponential power in the polynomial (excluding the coefficients).

We need to find the degree of polynomial 5.

5 can be written as : 5×1

We know that a non-zero number raised to the power of zero is equal to one.

That is $x^0 = 1$, where 'x' is a variable

Then we have,

$$5 = 5 \times x^0$$

We can see that the exponential power of term 'x' is zero and which is the greatest exponential. Hence, the degree of the polynomial 5 is 0.

2. Find the remainder if $f(x) = x^4 - 3x^2 + 4$ is divided by $g(x) = x - 2$

Ans. Let us denote the given polynomials as

$$f(x) = x^4 - 3x^2 + 4,$$
$$g(x) = x - 2$$

We have to find the remainder when f(x) is divided by g(x).

By the factor theorem, when f(x) is divided by g(x) the remainder is

$$f(2) = (2)^4 - 3(2)^2 + 4$$
$$= 16 - 12 + 4$$
$$= 8$$

3. Find the remainder if $f(x) = 4x^3 - 12x^2 + 14x - 3$ is divided by $g(x) = 2x - 1$

Ans. Let us denote the given polynomials as

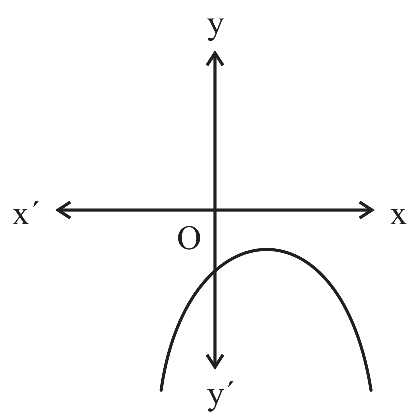
$$f(x) = 4x^3 - 12x^2 + 14x - 3,$$
$$g(x) = 2x - 1$$
$$\Rightarrow g(x) = 2\left(x - \frac{1}{2}\right)$$

We have to find the remainder when f(x) is divided by g(x).

By the factor theorem, when f(x) is divided by g(x) the remainder is

$$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3$$
$$= 4\times\frac{1}{8} - 12\times\frac{1}{4} + 14\times\frac{1}{2} - 3$$
$$= \frac{1}{2} - 3 + 7 - 3$$
$$= \frac{3}{2}$$

4. The graphical representation of quadratic polynomial $y = ax^2 + bx + c$ is shown in the given figure. Does it have (i) any solution (ii) zero(s)?



Ans. (i) There are infinite solutions because there are infinite points that satisfy the equation $y = ax^2 + bx + c$.

(ii) There is no zero because graphically there is no point of intersection of x-axis and the curve.

5. Find the remainder when $x^3 - 6x^2 + 2x - 4$ is divided by $3x - 1$.

Ans. Let us suppose $f(x) = x^3 - 6x^2 + 2x - 4$

We have, $3x - 1 = 3\left(x - \frac{1}{3}\right)$

So, by using factor theorem when f(x) is divided by $3\left(x - \frac{1}{3}\right)$, the remainder is equal to $f\left(\frac{1}{3}\right)$

Now, $f(x) = x^3 - 6x^2 + 2x - 4$

$\Rightarrow f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - 6\left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{3}\right) - 4$

$= \frac{1}{27} - \frac{6}{9} + \frac{2}{3} - 4$

$= \frac{2 - 18 + 18 - 108}{27}$

$= \frac{-107}{27}$

Hence, remainder is equal to $\frac{-107}{27}$

6. Find the values of a and b if $x^3 - ax^2 - 13x + b$ has $(x - 1)$ and $(x + 3)$ as factors.

Ans. Let us suppose $f(x) = x^3 - ax^2 - 13x + b$

We are given, $(x - 1)$ and $(x + 3)$ are the factors of f(x).

According to the factor theorem, if $(x - 1)$ and $(x + 3)$ are the factors of f(x) then the value of f(1) and f(–3) will be equal to zero.

$\therefore f(1) = 0$

$\Rightarrow (1)^3 - a(1)^2 - 13(1) + b = 0$

$\Rightarrow 1 - a - 13 + b = 0$

$\Rightarrow -a + b = 12$

$\Rightarrow a - b = -12$ ...(i)

And, $f(-3) = 0$

$\Rightarrow (-3)^3 - a(-3)^2 - 13(-3) + b = 0$

$\Rightarrow -27 - 9a + 39 + b = 0$

$\Rightarrow -9a + b = -12$

$\Rightarrow 9a - b = 12$ ...(ii)

On solving equation (i) and equation (ii) we get $a = 3$, $b = 15$.

7. The polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, find the value of a.

Ans. Let us suppose $f(z) = az^3 + 4z^2 + 3z - 4$ and $g(z) = z^3 - 4z + a$ leaves the same remainder when divided by $z - 3$. Then, using factor theorem.

$f(3) = g(3)$

$\Rightarrow a \times (3)^3 + 4 \times (3)^2 + 3 \times 3 - 4 = (3)^3 - 4 \times 3 + a$

$\Rightarrow 27a + 36 + 9 - 4 = 27 - 12 + a$

$\Rightarrow 26a + 41 = 15$

$\Rightarrow 26a = -26$

$\Rightarrow a = \frac{-26}{26} = -1$

8. Find atleast on factor of the polynomial $p(x) = x^3 - 3x^2 + x + 1$.

Ans. In the polynomial $p(x) = x^3 - 3x^2 + x + 1$, sum of the coefficients $(1 - 3 + 1 + 1)$ is equal to zero.

$\therefore$ $(x - 1)$ is a factor of polynomial p(x). To verify the above statement, we may find p(x) at x = 1

$p(x) = 13 - 3(1) + 1 + 1$

$= 1 - 3 + 1 + 1$

$= 0$

Relationship Between Zeroes and Coefficients of a Quadratic Polynomial

Let α and β be the zeroes of a quadratic polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, then f(x) can be written in the form of factors $(x - \alpha)$ and $(x - \beta)$ using factor theorem as

$\therefore \quad f(x) = k(x - \alpha)(x - \beta)$, where k is constant

$\Rightarrow \quad ax^2 + bx + c = k(x^2 - \beta x - \alpha x + \alpha\beta)$

$\Rightarrow \quad ax^2 + bx + c = k(x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow \quad ax^2 + bx + c = kx^2 - k(\alpha + \beta)x + k\alpha\beta$

Comparing the coefficients of x^2, x and constant terms on both sides, we get

$a = k$, $b = -k(\alpha + \beta)$ and $c = k\alpha\beta$

On dividing both b and c by a

$$\Rightarrow \frac{b}{a} = \frac{-k(\alpha+\beta)}{k} \text{ and } \frac{c}{a} = \frac{k\alpha\beta}{k}$$

$$\Rightarrow \frac{b}{a} = -(\alpha + \beta) \text{ and } \frac{c}{a} = \alpha\beta$$

$$\Rightarrow \alpha+\beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

Hence,

$$\text{Sum of the zeroes} = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of the zeroes} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Note:

If α and β are the zeroes of a quadratic polynomial f(x), then the polynomial f(x) is given by

$f(x) = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

or

$f(x) = k\{x^2 - (\text{Sum of the zeroes})\, x + \text{Product of the zeroes}\}$ where k is any non-zero real number.

Relationship Between Zeroes and Coefficients of a Cubic Polynomial

Let α, β, γ be the zeroes of a cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$. Then, by factor theorem, $(x-\alpha), (x-\beta)$ and $(x-\gamma)$ are factors of f(x).

$\therefore \quad f(x) = k(x-\alpha)(x-\beta)(x-\gamma)$

$\Rightarrow \quad ax^3 + bx^2 + cx + d = k(x-\alpha)(x-\beta)(x-\gamma)$

$\Rightarrow \quad ax^3 + bx^2 + cx + d = k\{x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma\}$

$\Rightarrow \quad ax^3 + bx^2 + cx + d = kx^3 - k(\alpha+\beta+\gamma)x^2 + k(\alpha\beta+\beta\gamma+\gamma\alpha)x - k\alpha\beta\gamma$

By comparing the coefficients of x^3, x^2, x and constant terms on both sides, we get

$a = k$, $b = -k(\alpha+\beta+\gamma)$, $c = (\alpha\beta+\beta\gamma+\gamma\alpha)$ and $d = -k(\alpha\beta\gamma)$

On solving we will get the following results.

$$\text{Sum of the zeroes} = \alpha + \beta + \gamma = -\frac{b}{a} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\Rightarrow \text{Sum of the products of the zeroes taken two at a time} = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\Rightarrow \text{Product of the zeroes} = \alpha\beta\gamma = -\frac{d}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$$

Note: Cubic polynomial having α, β, and γ as its zeroes is given by

$f(x) = k\{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$, where k is any non-zero real number.

Mind It

Quartic Polynomial: If α, β, γ, δ are zeroes of a quartic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$, then

$$\alpha + \beta + \gamma + \delta = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\delta + \alpha\gamma + \gamma\delta + \beta\delta = \frac{c}{a}$$

$$\alpha\beta\gamma + \alpha\gamma\delta + \alpha\beta\delta + \beta\gamma\delta = \frac{-d}{a}$$

$$\alpha\beta\gamma\delta = \frac{e}{a}$$

IMPORTANT FORMULAE

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $a^2 - b^2 = (a + b)(a - b)$
- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Special Case: If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$.

Example

1. Find the zeroes of each of the following quadratic polynomial and verify the relationship between the zeroes and their coefficients:

(i) $g(s) = 4s^2 - 4s + 1$ (ii) $g(x) = 6x^2 - 3 - 7x$

Ans. (i) When have, $g(s) = 4s^2 - 4s + 1$

$$= 4s^2 - 2s - 2s + 1$$

$$= 2s(2s - 1) - 1(2s - 1)$$

$$\therefore g(s) = (2s - 1)(2s - 1)$$

The zeroes of g(s) are given by

$g(s) = 0$

$(2s - 1)(2s - 1) = 0$

$(2s - 1) = 0$ and $(2s - 1) = 0$

$2s = 1$ and $2s = 1$

$s = \frac{1}{2}$ and $s = \frac{1}{2}$

Thus, the zeroes of $g(s) = 4s^2 - 4s + 1$ are

$\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$

Now, sum of the zeroes $= \alpha + \beta$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= \frac{1+1}{2}$$

$$= \frac{2}{2}$$

$$= 1$$

and

$$\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{-4}{4}$$

$$= \frac{\cancel{4}}{\cancel{4}}$$

$$= 1$$

Therefore, sum of the zeroes $= \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeroes $= \alpha\beta$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

Therefore, the product of the zeroes

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{1}{4}$$

Hence, the relationship between the zeroes and coefficient are verified.

(ii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3$

$= (3x + 1)(2x - 3)$

The value of $6x^2 - 3 - 7x$ is zero when

$3x + 1 = 0$ or $2x - 3 = 0$, i.e., $x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$

Sum of zeroes $= \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6}$

$$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

Product of zeroes $= \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6}$

$$= \frac{\text{Constent term}}{\text{Coefficient of } x^2}$$

2. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also, find the zeroes of these polynomials by factorization.

(i) $-\frac{8}{3}, \frac{4}{3}$, (ii) $-2\sqrt{3}, -9$

Ans. We know that a quadratic polynomial whose sum and product of zeroes are given is

$f(x) = k[x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}]$ ... (i)

(i) We have, sum of zeroes $= \frac{-8}{3}$ and product of zeroes $= \frac{4}{3}$

So, the required quadratic polynomial will be

$f(x) = k\left(x^2 + \frac{8}{3}x + \frac{4}{3}\right)$ [from eq. (i)]

$$= \frac{k}{3}\left(3x^2+8x+4\right)$$

$$= \frac{k}{3}\left(3x^2+6x+2x+4\right)$$

$$= \frac{k}{3}\left[3x\left(x+2\right)+2\left(x+2\right)\right]$$

$$= \frac{k}{3}\left(3x+2\right)\left(x+2\right)$$

Now, the zeroes are given by f(x) = 0

Thus, $x = -\frac{2}{3}$ and $x = -2$

(ii) We have, sum of zeroes = $-2\sqrt{3}$ and product of zeroes = –9

So, the required quadratic polynomial will be

$$f(x) = k(x^2+2\sqrt{3}x-9)$$

[from eq. (i)]

$$= k(x^2+3\sqrt{3}x-\sqrt{3}x-9)$$

$$= k\left(x+3\sqrt{3}\right)\left(x-\sqrt{3}\right)$$

Now, the zeroes are given by f(x) = 0

Thus, $x = -3\sqrt{3}$ and $x=\sqrt{3}$

3. Find a quadratic polynomial whose zeroes are 4 and 3 respectively.

Ans. Let the quadratic polynomial be $ax^2 + bx + c$ and it's zeroes be α and β

Given,

$$\alpha + \beta = 4 + 3 = 7$$

$$\alpha\beta = 4(3) = 12$$

A quadratic polynomial whose sum and product of its zeroes is given can be written as

f(x) = K [x^2 – (sum of zeroes) x + product of zeroes]

Where K is a constant

∴ Required quadratic polynomial

f(x) = K[$x^2 + 7x + 12$]

4. If α and β are the roots (zeroes) of the polynomial f(x) = $x^2 - 3x + k$ such that α – β = 1, find the value of k.

Ans. Since α and β are the roots (zeroes) of the polynomial f(x) = $x^2 - 3x + k$

$$\therefore \quad \alpha+\beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \frac{-(-3)}{1} = 3 \text{ and}$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = k$$

Given, $\alpha - \beta = 1$

Squaring both side, we get

$$\Rightarrow \quad (\alpha-\beta)^2 = (1)^2$$

$$\Rightarrow \quad \alpha^2 - 2\alpha\beta + \beta^2 = 1$$

$$\left[\begin{array}{l}(a+b)^2 = a^2+b^2+2ab \\ \Rightarrow (a+b)^2 - 2ab = a^2+b^2\end{array}\right]$$

$$\Rightarrow \{(\alpha+\beta)^2 - 2\alpha\beta\} - 2\alpha\beta = 1$$

$$\Rightarrow (\alpha+\beta)^2 - 4\alpha\beta = 1$$

$$\Rightarrow (3)^2 - 4\times k = 1$$

$$\Rightarrow 4k = 8$$

$$\Rightarrow k = 2$$

Therefore, the value of k is 2.

5. If α, β are the zeroes of the polynomial f(x) = $2x^2 + 5x + k$ satisfying the relation $\alpha^2+\beta^2+\alpha\beta = \frac{21}{4}$, then find the value of k.

Ans. Since α and β are the zeroes of the polynomial f(x) = $2x^2 + 5x + k$.

$$\therefore \quad \alpha+\beta = \frac{-5}{2} \text{ and } \alpha\beta = \frac{k}{2}$$

Now, $$\alpha^2+\beta^2+\alpha\beta = \frac{21}{4}$$

$$\Rightarrow (\alpha^2+\beta^2+2\alpha\beta) - \alpha\beta = \frac{21}{4}$$

$$\Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4}$$

$$\left[\because \alpha+\beta=\frac{5}{2} \text{ and } \alpha\beta=\frac{k}{2}\right]$$

$\Rightarrow \quad -\frac{k}{2} = -1$

$\Rightarrow \quad k = 2$

6. If both (x – 2) and $\left(\mathbf{x-\frac{1}{2}}\right)$ are factors of $\mathbf{px^2 + 5x + r}$, then show that p = r.

Ans. Let us suppose, $f(x) = px^2 + 5x + r$

Here, roots α and β are 2 and 3 respectively.

According to the question,

Product of roots $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\Rightarrow \alpha \times \beta = \dfrac{r}{p}$

$\Rightarrow 2\times\dfrac{1}{2}=\dfrac{r}{p}$

$\Rightarrow 1=\dfrac{r}{p}$

$\Rightarrow \therefore r = p$ Hence proved.

7. Find the zeroes of the quadratic polynomial $\mathbf{9x^2 – 5}$ and verify the relation between the zeroes and its coefficients.

Ans. We have, $9x^2 - 5 = 0$

or $(3x)^2-(\sqrt{5})^2 = 0$

$(3x-\sqrt{5})(3x+\sqrt{5}) = 0$ $[a^2 - b^2 = (a + b)(a - b)]$

$\Rightarrow (3x-\sqrt{5}) = 0$ and $(3x+\sqrt{5}) = 0$

or $x = \dfrac{\sqrt{5}}{3}$ and $x = \dfrac{-\sqrt{5}}{3}$

Therefore, the zeroes of $9x^2 - 5$ are $\dfrac{\sqrt{5}}{3}$ and $\dfrac{-\sqrt{5}}{3}$

Sum of the zeroes $= \dfrac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

$\Rightarrow \quad \dfrac{\sqrt{5}}{3}-\dfrac{\sqrt{5}}{3} = \dfrac{-0}{9}$

$\Rightarrow \quad 0 = 0$

Product of zeroes $= \dfrac{\text{constant term}}{\text{coefficient of } x^2}$

$\Rightarrow \quad \left(\dfrac{\sqrt{5}}{3}\right)\left(\dfrac{-\sqrt{5}}{3}\right) = \dfrac{-5}{9}$

$\Rightarrow \quad \dfrac{-5}{9} = \dfrac{-5}{9}$

Hence, relation verified.

8. If α and β are the zeroes of the quadratic polynomial $\mathbf{f(x) = ax^2 + bx + c}$, then calculate the value of $\boldsymbol{\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}}$.

Ans. $f(x) = ax^2 + bx + c$

$\therefore \quad \alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta=\dfrac{c}{a}$

We have,

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{\left(-\frac{b}{a}\right)^3-3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)}{\frac{c}{a}}$$

$$= \frac{\frac{-b^3}{a^3}+3\frac{bc}{a^2}}{\frac{c}{a}}$$

$$\therefore \quad \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{3abc-b^3}{a^2c}$$

9. If α and β are the zeroes of the quadratic polynomial $\mathbf{p(s) = 3s^2 – 6s + 4}$, then find the value of $\boldsymbol{\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}+2\left(\dfrac{1}{\alpha}+\dfrac{1}{\beta}\right)+3\alpha\beta}$.

Ans. Since α and β are the zeroes of the polynomial $p(s) = 3s^2 - 6s + 4$,

$\therefore \alpha + \beta = \dfrac{-(-6)}{3}=2$ and $\alpha\beta=\dfrac{4}{3}$

Given,

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta$$

$$=\frac{\alpha^2+\beta^2}{\alpha\beta}+2\left(\frac{\beta+\alpha}{\alpha\beta}\right)+3\alpha\beta$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}+\frac{2(\alpha+\beta)}{\alpha\beta}+3\alpha\beta$$

$$=\frac{(2)^2-2\times\frac{4}{3}}{\frac{4}{3}}+\frac{2\times 2}{\frac{4}{3}}+3\times\frac{4}{3}$$

$$=\frac{4-\frac{8}{3}}{\frac{4}{3}}+\frac{4}{\frac{4}{3}}+\frac{12}{3}$$

$= 8$

10. Verify that numbers given along side of the polynomial are their zeroes $x^4 + 2x^3 - 7x^2 - 8x + 12$; -3, –2, 1, 2.

Ans. Here the polynomial p(x) is

$x^4 + 2x^3 - 7x^2 - 8x + 12$

Value of the polynomial $x^4 + 2x^3 - 7x^2 - 8x + 12$ at $x = -3$

$P(-3) = (-3)^4 + 2(-3)^3 - 7(-3)^2 - 8(-3) + 12$

$= 81 - 54 - 63 + 24 + 12$

$= 0$

So, –3 is a zeroes of p(x).

Value of the polynomial $x^4 + 2x^3 - 7x^2 - 8x + 12$ at $x = -2$ is

$P(-2) = (-2)^4 + 2(-2)^3 - 7(-2)^2 - 8(-2) + 12$

$= 16 - 16 - 28 + 16 + 12$

$= 0$

So, –2 is a zeroes of p(x).

Value of the polynomial $x^4 + 2x^3 - 7x^2 - 8x + 12$ at $x = 1$

$P(1) = (1)^4 + 2(1)^3 - 7(1)^2 - 8(1) + 12$

$= 1 + 2 - 7 - 8 + 12$

$= 0$

So, 1 is a zeroes of p(x).

Value of the polynomial $x^4 + 2x^3 - 7x^2 + 12$ at $x = 2$

$P(2) = (2)^4 + 2(2)^3 - 7(2)^2 - 8(2) + 12$

$= 16 + 16 - 28 - 16 + 12$

$= 0$

So, 2 is a zeroes of p(x).

So, –3, –2, 1, 2 are the zeroes of given polynomial.

11. If α and β are the zeroes of the polynomial $ax^2 + bx + c$, find a polynomial whose zeroes are $\frac{1}{a\alpha+b}$ and $\frac{1}{a\beta+b}$

Ans. As α and β are the zeroes of the polynomial $ax^2 + bx + c$

$\therefore\ \alpha + \beta = \frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$ ...(i)

Since $\frac{1}{a\alpha+b}$ and $\frac{1}{a\beta+b}$ are the zeroes of the required polynomial.

$\therefore$ sum of the zeroes

$$=\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}=\frac{a\beta+b+a\alpha+b}{(a\alpha+b)(a\beta+b)}$$

$$=\frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$$

$$=\frac{a\times\left(\frac{-b}{a}\right)+2b}{a^2\times\left(\frac{c}{a}\right)+ab\times\left(\frac{-b}{a}\right)+b^2}\quad\text{[using eq. (i)]}$$

$$=\frac{\frac{-ab+2ba}{a}}{\frac{a^2c-ab^2+ab^2}{a}}$$

$$=\frac{ab}{a^2c}$$

$$=\frac{b}{ac}$$

t

Product of the zeroes

$$= \left(\frac{1}{a\alpha + b}\right)\left(\frac{1}{a\beta + b}\right)$$

$$= \frac{1}{a^2\alpha\beta + ab(\alpha + \beta) + b^2}$$

$$= \frac{1}{a^2 \times \frac{c}{a} + ab \times \left(\frac{-b}{a}\right) + b^2} \quad \text{[using eq. (i)]}$$

$$= \frac{1}{\frac{a^2c - ab^2 + b^2a}{a}}$$

$$= \frac{1}{\frac{a^2c}{a}}$$

$$= \frac{1}{ac}$$

Hence, the required polynomial is given by x^2 – (sum of zeroes) x + product of zeroes

$$= x^2 - \left(\frac{b}{ac}\right)x + \frac{1}{ac}$$

Division Algorithm for Polynomials

If f(x) and p(x) are any two polynomials such that $p(x) \neq 0$, then we can find polynomials r(x) and q(x) such that $f(x) = p(x) \times q(x) + r(x)$ i.e., Dividend = (Divisor × Quotient) + Remainder

where $r(x) = 0$ or degree of r(x) < degree of p(x).

(i) If $r(x) = 0$, p(x) is a factor of f(x).

(ii) If $\deg(f(x)) > \deg(p(x))$, then $\deg(q(x)) = \deg(f(x)) - \deg(p(x))$

(iii) If $\deg(f(x)) = \deg(p(x))$, then $\deg(q(x)) = 0$ and $\deg(r(x)) < \deg(p(x))$

Working Rule to Divide a Polynomial By Another Polynomial

Step 1: Arrange the terms of dividend and the divisor in the decreasing order of their degrees.

Step 2: To obtain the first term of quotient, divide the highest degree term of the dividend by the highest degree term of the divisor.

Step 3: To obtain the second term of the quotient, divide the highest degree term of the new dividend obtained as remainder by the highest degree term of the divisor.

Step 4: Continue this process till the degree of remainder is less than the degree of divisor.

Example

1. Check whether first polynomial is a factor of the second polynomial by applying the division algorithm.

$x^2 + 3x + 1$, $3x^4 + 5x^3 - 7x^2 + 2x + 2$

Ans. We divide $3x^4 + 5x^3 - 7x^2 + 2x + 2$ by $x^2 + 3x + 1$

$$
\begin{array}{rl}
 & 3x^2-4x+2\\
x^2+3x+1 & \overline{)\,3x^4+5x^3-7x^2+2x+2}\\
 & 3x^4+9x^3+3x^2\\
 & -\quad -\quad -\\
\hline
 & -4x^3-10x^2+2x+2\\
 & -4x^3-12x^2-4x\\
 & +\quad +\quad +\\
\hline
 & 2x^2+6x+2\\
 & 2x^2+6x^2+2\\
 & -\quad -\quad -\\
\hline
 & 0
\end{array}
$$

Since, here remainder is zero.

Hence, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$

Checking $3x^4 + 5x^3 - 7x^2 + 2x + 2$

$= (3x^2 - 4x + 2)(x^2 + 3x + 1) + 0$

$= 3x^4 + 5x^3 - 7x^2 + 2x + 2 =$ Dividend

2. Apply the division algorithm to find the quotient and remainder on dividing p(x) by g(x) as given below

$p(x) = x^4 - 3x^2 + 4x + 5$,

$g(x) = x^2 + 1 - x$

Ans. We have,

$p(x) = x^4 - 3x^2 + 4x + 5$,

$g(x) = x^2 + 1 - x$

$$
\begin{array}{rl}
 & x^2+x-3\\
x^2-x+1 & \overline{)\,x^4-3x^2+4x+5}\\
 & x^4-x^3+x^2\\
 & -\quad +\quad -\\
\hline
 & x^3-4x^2+4x+5\\
 & x^3-x^2+x\\
 & -\quad +\quad -\\
\hline
 & -3x^2+3x+5\\
 & -3x^2+3x-3\\
 & +\quad -\quad -\\
\hline
 & 8
\end{array}
$$

We stop here since degree of 8 < degree of $(x^2 + 1 - x)$. So, quotient $= x^2 + 1 - x$, remainder $= 8$

Therefore,

Quotient × Divisor + Remainder

$= (x^2 + x - 3)(x^2 - x + 1) + 8$

$= x^4 - x^3 + x^2 + x^3 - x^2 + x - 3x^2 + 3x - 3 + 8$

$= x^4 - 3x^2 + 4x + 5 =$ Dividend

Therefore the Division Algorithm is verified.

3. If the polynomial $p(x) = 2x^5 - 15x^4 + 40x^3 - 34x^2 + ax + b$ is divisible by $g(x) = x^2 - 4x + 6$. Then find the value of a + b.

Ans. We have, $p(x) = 2x^5 - 15x^4 + 40x^3 - 34x^2 + ax + b$ and $g(x) = x^2 - 4x + 6$

As p(x) is divisible by g(x), the remainder will be zero.

Now, applying long division method,

$$
\begin{array}{rll}
x^2-4x+6 & \overline{)\,2x^5-15x^4+40x^3-34x^2+ax+b} & (2x^3-7x^2+8\\
 & 2x^5-8x^4+12x^3 & \\
 & -\quad +\quad - & \\
\hline
 & -7x^4+28x^3-34x^2+ax+b & \\
 & -7x^4+28x^3-42x^2 & \\
 & +\quad -\quad + & \\
\hline
 & 8x^2+ax+b & \\
 & 8x^2-32x+48 & \\
 & -\quad +\quad - & \\
\hline
 & (32+a)x+(b-48) &
\end{array}
$$

As remainder must be zero

$(32 + a)x + (b - 48) = 0x + 0$

$\therefore \quad 32 + a = 0 \Rightarrow a = -32$

And $\quad b - 48 = 0 \Rightarrow b = 48$

$\therefore \quad a + b = -32 + 48 = 16$

4. If $x + y + z = 1$, $x^2 + y^2 + z^2 = 21$ and $xyz = 8$, then find the value of $(1 - x)(1 - y)(1 - z)$.

Ans. Given, $x + y + z = 1$, ...(i)

$x^2 + y^2 + z^2 = 21$ ...(ii)

and $xyz = 8$ ...(iii)

By squaring we get eq. (i), we get

$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 1$

$[(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac]$

$\Rightarrow 21 + 2(xy + yz + zx) = 1$ [Using (ii)]

$\Rightarrow xy + yz + zx = -10$

Now, $(1 - x)(1 - y)(1 - z)$

$= (1 - x)(1 - y - z + yz)$

$= (1 - y - z + yz - x + xy + xz - xyz)$

$= 1 - (x + y + z) + (xy + yz + zx) - xyz$

$= 1 - 1 - 10 - 8 = -18$ [Using (ii) and (iii)]

5. Find the value of b for which the polynomial $2x^3 + 9x^2 - x - b$ is exactly divisible by $2x + 3$.

Ans.

$$\begin{array}{r|l}
 & x^2 + 3x - 5 \\
2x + 3 & 2x^3 + 9x^2 - x - b \\
 & \underline{2x^3 + 3x^2} \\
 & \quad\ -\quad\ - \\
 & \qquad 6x^2 - x - b \\
 & \qquad \underline{6x^2 + 9x} \\
 & \qquad\ -\quad\ - \\
 & \qquad\quad -10x - b \\
 & \qquad\quad \underline{-10x - 15} \\
 & \qquad\qquad -b + 15
\end{array}$$

For the dividend to be exactly divisible by the divisor the remainder must be zero.

$\therefore \quad -b + 15 = 0 \Rightarrow b = 15$

6. Find the value that must be subtracted from the expression $8x^4 + 14x^3 - 2x^2 + 8x - 12$, so that it is exactly divisible by $4x^2 + 3x - 2$.

Ans.

$$\begin{array}{r|l|l}
4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 8x - 12 & 2x^2 + 2x - 1 \\
 & \underline{8x^4 + 6x^3 - 4x^2} & \\
 & \ -\qquad -\qquad + & \\
 & \qquad 8x^3 + 2x^2 + 8x - 12 & \\
 & \qquad \underline{8x^3 + 6x^2 - 4x} & \\
 & \qquad\ -\qquad -\qquad + & \\
 & \qquad\quad -4x^2 + 12x - 12 & \\
 & \qquad\quad \underline{-4x^2 - 3x + 2} & \\
 & \qquad\quad\ +\qquad +\qquad - & \\
 & \qquad\qquad 15x - 14 &
\end{array}$$

$\because$ We get a remainder equal to $15x - 14$. For exact divisibility remainder must be equal to zero.

$\therefore$ The expression that must be subtracted

$= 15x - 14$

7. If $x^n - py^n + qz^n$ is exactly divisible by $x^2 - (ay + bz)x + abyz$, then find the value of $\frac{p}{a^n} - \frac{q}{b^n}$

Ans. On factorizing $x^2 - ayx - bzx + abyz = x(x - ay) - bz(x - ay)$

$= (x - ay)(x - bz)$

Let, $f(x) = x^n - py^n + qz^n$ is exactly divisible by $(x - ay)(x - bz)$

$\therefore f(ay) = f(bz) = 0$ (Using factor theorem)

$f(ay) = a^n y^n - py^n + qz^n = 0$ ...(i)

$f(bz) = b^n z^n - py^n + qz^n = 0$ ...(ii)

Subtracting (ii) from (i)

$a^n y^n - b^n z^n = 0$

$$\Rightarrow \quad y^n = \frac{b^n z^n}{a^n} \quad \text{...(iii)}$$

Now substituting the value of y^n in eq (ii), we get

$$b^n z^n - p\left(\frac{b^n z^n}{a^n}\right) + qz^n = 0$$

Dividing all the terms of this expresssion by $b^n z^n$, we have

$$1 - \frac{p}{a^n} + \frac{q}{b^n} = 0$$

$$\Rightarrow \frac{p}{a^n} - \frac{q}{b^n} = 1$$

8. Let a, b, c, x, y and z be numbers such that $a = \frac{b+c}{x-2}, b = \frac{c+a}{y-2}, c = \frac{a+b}{z-2}$. If xy + yz + zx = 67 and x + y + z = 2010, find the value of xyz + 5900.

Ans. Given, $a = \frac{b+c}{x-2}, b = \frac{c+a}{y-2}, c = \frac{a+b}{z-2}$

$\Rightarrow x - 2 = \frac{b+c}{a}$

Adding 1 on both sides, we get

$\Rightarrow x - 1 = \frac{b+c}{a} + 1$

$\Rightarrow x - 1 = \frac{a+b+c}{a}$

$\Rightarrow \frac{1}{x-1} = \frac{a}{a+b+c}$ ...(i)

Similarly, $\frac{1}{y-1} = \frac{b}{a+b+c}$ and ...(ii)

$\frac{1}{z-1} = \frac{c}{a+b+c}$ ...(iii)

Adding (i), (ii) and (iii)

$$\frac{1}{x-1} + \frac{1}{y-1} + \frac{1}{z-1} = \frac{a}{a+b+c} + \frac{b}{a+b+c} + \frac{c}{a+b+c}$$

$$\frac{(y-1)(z-1)+(x-1)(z-1)+(x-1)(y-1)}{(x-1)(y-1)(z-1)} = 1$$

$\Rightarrow$ yz – y – z + 1 + xz – x – z + 1 + xy – y – x + 1 = xyz – xy – zx – yz + x + y + z – 1

$\Rightarrow$ 2(xy + yz + zx) – 3(x + y + z) + 4 = xyz

$\Rightarrow$ xyz = – 5892

[Using the values given in the question]

xyz + 5900 = –5892 + 5900 = 8

Summary

POLYNOMIALS

An algebraic expression f(x) of the form $f(x) = a_0 + a_1x + a_2x^2 + \ldots a_nx^n$ is a polynomial, where $a_0, a_1, \ldots a_n$ are real numbers and all the powers of x are non-negative integers.

- **Degree of a polynomial** is the highest exponent of the variable in the polynomial.
- **Zero of a polynomial** is a number for which the value of that polynomial is zero.
- **Zeroes of a polynomial** are points where graph of the polynomial intersect x-axis.

- $f(x) = ax + b$; where a, b are real and $a \neq 0$, is a **linear polynomial.**
- The graph of a linear polynomial is a **straight line.**

- $f(x) = ax^2 + bx + c$, where a, b, c are real, and $a \neq 0$ is a **quadratic polynomial.**
- The graph of a quadratic polynomial is a **parabola.**
- **Parabola opens upwards,** if a is +ve.
- **Parabola opens downwards,** if a is –ve

- If $f(x) = ax^2 + bx + c$ is a quadratic polynomial, then:
  - (i) Sum of zeroes $(\alpha + \beta) = \dfrac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\dfrac{b}{a}$
  - (ii) Product of zeroes $(\alpha \times \beta) = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{c}{a}$ [where α, β are the zeroes of the polynomial]
- **Quadratic polynomial** is x^2 – (sum of zeroes)x + (product of zeroes)

- $f(x) = ax^3 + bx^2 + cx + d$ is a cubic polynomial with α, β, γ as zeroes of polynomial.
- Sum of zeroes $(\alpha + \beta + \gamma) = \dfrac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\dfrac{b}{a}$
- Product of zeroes taken two at a time $(\alpha\beta + \beta\gamma + \gamma\alpha) = \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \dfrac{c}{a}$
- Product of zeroes $(\alpha\beta\gamma) = \dfrac{\text{-Constant term}}{\text{Coefficient of } x^3} = -\dfrac{d}{a}$

Division Algorithm:

Dividend = Divisor × Quotient + Remainder

$f(x) = g(x) \times q(x) + r(x)$

or $$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$$

Quick Recall

Fill in the blanks

1. The degree of cubic polynomial is _________
2. A polynomial of degree n has at the most _________ zeroes.
3. _________ is not equal to zero when the divisor is not a factor of dividend.
4. Degree of remainder is always _________ than degree of divisor.
5. Degree of a zero polynomial is _________
6. A linear polynomial is represented by a _________
7. The highest power of a variable in a polynomial is called its _________
8. The graph of quadratic polynomial opens upward when 'a' is _________
9. A _________ is a polynomial of degree 0.
10. The zeroes of a polynomial p(x) are precisely the x – coordinates of the points, where the graph of y = p(x) intersects the _________ axis.

True and False Statements

1. $3, -1, \frac{1}{3}$ are the zeroes of the cubic polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$
2. $(z - 1)$ is a factor of $g(z) = 2z^3 - 2$.
3. $x^3 - 2\sqrt{x}$ is a polynomial.
4. A cubic polynomial has atleast one zero.
5. The degree of the sum of two polynomials each of degree 3 is always 3.
6. Sum of zeroes of quadratic polynomial $= -\frac{(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$
7. Graph of a quadratic polynomial is a parabola.
8. The degree of the product of two polynomials each of degree 5 is always 5.
9. Product of zeroes of quadratic polynomial is $= -\frac{(\text{constant term})}{(\text{coefficient of } x^2)}$
10. Zeroes of quadratic polynomial $x^2 + 6x + 9$ are 2 and 7.

Match The Followings

1. Match the following with correct response.

Zeroes	Quadratic polynomials
(1) 3 and –3	(A) $x^2 + x - 42$
(2) $5+\sqrt{2}$ and $5-\sqrt{2}$	(B) $x^2 - 9$
(3) –9 and $\frac{1}{9}$	(C) $x^2 + \left(\frac{80}{9}\right)x - 1$
(4) –7 and 6	(D) $x^2 - 10x + 21$

a. 1-A, 2-C, 3-B, 4-D b. 1-C, 2-B, 3-D, 4-A
c. 1-B, 2-D, 3-C, 4-A d. 1-D, 2-A, 3-C, 4-B

2. Match the following with correct response.

Polynomial	Remainder
(1) $\frac{x^3 - 3x^2 + x + 2}{x^2 - x + 1}$	(A) 8
(2) $\frac{3x^3 + 5x^2 + 2}{x^2 + 2x + 1}$	(B) $41x - 18$
(3) $\frac{x^4 + 7x^3 + 3x + 2}{x^2 + 3x - 2}$	(C) $-x + 3$
(4) $\frac{x^4 - 3x^2 + 4x + 5}{x^2 - x + 1}$	(D) $-2x + 4$

a. 1-D, 2-C, 3-B, 4-A b. 1-C, 2-B, 3-D, 4-A
c. 1-A, 2-C, 3-B, 4-D d. 1-B, 2-D, 3-A, 4-C

Answers

Fill in the Blanks:

1. Three
2. n
3. Remainder
4. Less
5. Not defined
6. Straight line
7. Degree
8. Positive
9. Constant
10. x

True and False:

1. True
2. True
3. False, because the exponent of the variable is not a whole number.
4. True
5. False
6. True
7. True
8. False
9. False
10. False

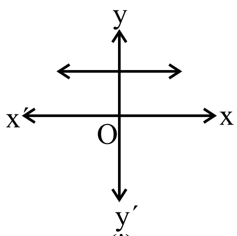
Match the Followings

1. (c) **2.** (a)

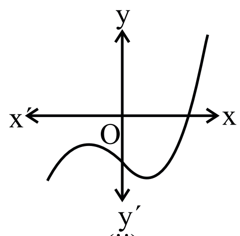
NCERT Exercise

Exercise-I

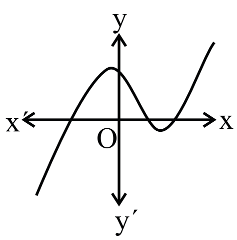
1. The graphs of y = p(x) are given in Fig. below, for some polynomials p(x). Find the number of zeroes of p(x) in each case.



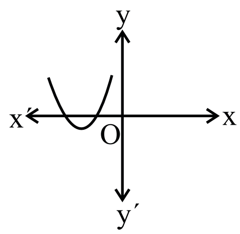
(i)



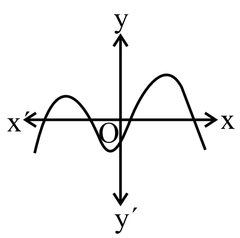
(ii)



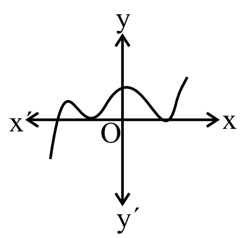
(iii)



(iv)



(v)



(vi)

Exp. Do it yourself.

Hint: Total number of zeroes in any polynomial
= total number of times the curve intersects x-axis.

Exercise-II

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

(ii) $4s^2 - 4s + 1$

(iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$

(v) $t^2 - 15$

(vi) $3x^2 - x - 4$

Exp. **(i)** $x^2 - 2x - 8$

Using middle term splitting and factorizing, we get

$$\Rightarrow x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = 0$$

$$= (x-4)(x+2) = 0$$

$\Rightarrow x - 4 = 0$ and $x + 2 = 0$

$\Rightarrow x = +4$, and -2

Hence, zeroes of polynomial equation $x^2 - 2x - 8$ are $(4, -2)$

$$\text{Sum of zeroes} = 4 - 2$$

$$(\alpha + \beta) = 2$$

$$= \frac{-(-2)}{1}$$

$$= \frac{-(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$$

$$\text{Product of zeroes} = 4 \times (-2)$$

$$(\alpha \times \beta) = -8$$

$$= \frac{-(8)}{1}$$

$$= \frac{(\text{Constant term})}{(\text{Coefficient of } x^2)}$$

(ii) $4s^2 - 4s + 1 = (2s-1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, i.e., $s = \frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$

$$\text{Sum of zeroes} = \frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$$

$$\frac{1}{2} + \frac{1}{2} = \frac{-(4)}{4}$$

$$\Rightarrow \quad 1 = 1$$

$$\text{Product of zerores} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{1}{4} = \frac{1}{4}$$

(iii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3$

$= (3x + 1)(2x - 3)$

The value of $6x^2 - 3 - 7x$ is zero when

$3x + 1 = 0$ and $2x - 3 = 0$

$\Rightarrow x = \frac{-1}{3}$ and $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$

$$\text{Sum of zeroes} = \frac{-(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$$

$$\Rightarrow \quad \frac{-1}{3} + \frac{3}{2} = \frac{-(-7)}{6}$$

$$\Rightarrow \quad \frac{7}{6} = \frac{7}{6}$$

$$\text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\Rightarrow \quad \frac{-1}{3} \times \frac{3}{2} = \frac{-3}{6}$$

$$\Rightarrow \quad \frac{-1}{2} = \frac{-1}{2}$$

(iv) Do it yourself

(v) Do it yourself

(vi) Do it yourself

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$

(ii) $\sqrt{2}, \frac{1}{3}$

(iii) $0, \sqrt{5}$

(iv) 1, 1

(v) $-\frac{1}{4}, \frac{1}{4}$

(vi) 4, 1

Exp. (i) We know,

$$\text{Sum of zeroes} = \alpha + \beta = \frac{1}{4}$$

$$\text{Product of zeroes} = \alpha\beta = -1$$

$\Rightarrow$ If sum and product of zeroes of any quadratic polynomial is given, then the quadratic polynomial equation can be written directly as:

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$$x^2 - \frac{1}{4}x + (-1) = 0$$

$$4x^2 - x - 4 = 0$$

Thus, $4x^2 - x - 4$ is the required polynomial.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$

If $a = 3$, then $b = -3\sqrt{2}, c = 1$

Then the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) Do it yourself.

(iv) Do it yourself.

(v) $-\frac{1}{4}, \frac{1}{4}$.

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha + \beta = -\frac{1}{4} = \frac{-b}{a}$$

$$\alpha \times \beta = \frac{1}{4} = \frac{c}{a}$$

if $a = 4$, then $b = 1$, $c = 1$

Then the quadratic polynomial is $4x^2 + x + 1$.

(vi) Do it yourself.

Exercise-III

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following :

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

Exp. **(i)** Given,

Dividend = $p(x) = x^3 - 3x^2 + 5x - 3$

Divisor = $g(x) = x^2 - 2$

$$
\begin{array}{r|l}
 & x-3 \\
\hline
x^2-2 & x^3-3x^2+5x-3 \\
 & -(x^3+0x^2-2x) \\
\hline
 & -3x^2+7x-3 \\
 & -(-3x^2+0x+6) \\
\hline
 & 7x-9
\end{array}
$$

Therefore, upon division we get,

Quotient = $x - 3$

and remainder = $7x - 9$

(ii) Do it yourself.

(iii) Do it yourself.

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3$, $2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1$, $3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

Exp. **(i)** $t^2 - 3$, $2t^4 + 3t^3 - 2t^2 - 9t - 12$

Given,

First polynomial = $t^2 - 3$ = divisor

Second polynomial = $2t^4 + 3t^3 - 2t^2 - 9t - 12$ = dividend

$$
\begin{array}{r|l}
 & 2t^2+3t+4 \\
\hline
t^2-3 & 2t^4+3t^3-2t^2-9t-12 \\
 & -(2t^4+0t^3-6t^2) \\
\hline
 & 3t^3+4t^2-9t-12 \\
 & -(3t^3+0t^2-9t) \\
\hline
 & 4t^2+0t-12 \\
 & -(4t^2+0t-12) \\
\hline
 & 0
\end{array}
$$

Since, remainder as 0. Therefore, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) Do it yourself.

(iii) Do it yourself.

3. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Exp. Since it is a quartic polynomial, it can have at most 4 zeroes.

$\sqrt{\left(\frac{5}{3}\right)}$ and $-\sqrt{\left(\frac{5}{3}\right)}$ are zeroes of the polynomial f(x).

$$\therefore \left(x - \sqrt{\left(\frac{5}{3}\right)}\right)\left(x + \sqrt{\left(\frac{5}{3}\right)}\right) = x^2 - \left(\frac{5}{3}\right) = \frac{3x^2 - 5}{3}$$

[$(a - b)(a + b) = a^2 - b^2$]

$(3x^2 - 5) = 0$ is a factor of given polynomial f(x). [Factor theorem]

$$
\begin{array}{r|l}
 & x^2+2x+1 \\
\hline
3x^2-5 & 3x^4+6x^3-2x^2-10x-5 \\
 & -(3x^4-5x^2) \\
\hline
 & +6x^3+3x^2-10x-5 \\
 & -(6x^3-10x) \\
\hline
 & 3x^2-5 \\
 & -(3x^2-5) \\
\hline
 & 0
\end{array}
$$

Now since D = dq + r

Therefore, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1)$

Now, on further factorizing $(x^2 + 2x + 1)$ we get,

$x^2 + 2x + 1 = x^2 + x + x + 1$ [splitting the middle term]

$= x(x + 1) + 1(x + 1)$

$= (x + 1)(x + 1)$

$(x + 1)(x + 1) = 0$

So, its zeroes are given by: $x = -1$ and $x = -1$.

Therefore, all four zeroes of given polynomial are:

$\sqrt{\left(\frac{5}{3}\right)}, -\sqrt{\left(\frac{5}{3}\right)}$, −1 and −1.

4. On dividing $p(x) = x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x).

Exp. Given,

Dividend, $p(x) = x^3 - 3x^2 + x + 2$

Quotient $= x - 2$

Remainder $= -2x + 4$

As we know,

Dividend = Divisor × Quotient + Remainder

$\therefore\ x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$x^3 - 3x^2 + x + 2 - (-2x + 4) = g(x) \times (x - 2)$

Therefore, $g(x) \times (x - 2) = x^3 - 3x^2 + 3x - 2$

Now, for finding g(x) we will divide $x^3 - 3x^2 + 3x - 2$ from $(x - 2)$

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x - 2 & x^3 - 3x^2 + 3x - 2 \\ & \underset{-}{x^3} \underset{+}{-\,2x^2} \\ \hline & -x^2 + 3x - 2 \\ & \underset{+}{-x^2} \underset{-}{+\,2x} \\ \hline & x - 2 \\ & \underset{-}{x} \underset{+}{-\,2} \\ \hline & 0 \end{array}$$

Therefore, $g(x) = (x^2 - x + 1)$

5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x)

(ii) deg q(x) = deg r(x)

(iii) deg r(x) = 0

Exp. **(i)** deg p(x) = deg q(x)

Degree of dividend is equal to degree of quotient, only when the divisor is a constant term.

$p(x) = 3x^2 + 3x + 6$

$q(x) = x^2 + x + 2$

$g(x) = 3$ and $r(x) = 0$

Quotient × Divisor + Remainder

$= g(x)\, q(x) + r(x)$

$= 3\,(x^2 + x + 2) + 0$

$= 3x^2 + 3x + 6$

= Dividend

= P(x)

Division algorithm satisfied.

As, you can see, the degree of quotient is equal to the degree of dividend here.

(ii) deg q(x) = deg r(x)

$p(x) = x^2 + x$

$g(x) = x$

$q(x) = x + 1$ and $r(x) = 0$

As, you can see, the degree of quotient is equal to the degree of remainder here.

Quotient × Divisor + Remainder

$= g(x)\, q(x) + r(x)$

$= x(x + 1) + 0$

$= x^2 + x$

= Dividend

= p(x)

(iii) deg r(x) = 0

The degree of remainder is 0 only when the remainder left after division algorithm is a constant.

$p(x) = x^2 + 1$

$g(x) = x$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of remainder here is 0.

Also, division algorithm is satisfied here.

Exercise-IV

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2;\ \frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2;\ 2, 1, 1$

Exp. **(i)** Given, $p(x) = 2x^3 + x^2 - 5x + 2$

And zeroes for p(x) are $= \frac{1}{2}, 1, -2$

$$\therefore \quad p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$= 0$$

$$p(1) = 2(1)^3 + (1)^2 - 5(1) + 2$$

$$= 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$

$$= 0$$

Hence, $\frac{1}{2}$, 1, –2 are the zeroes of $2x^3 + x^2 - 5x + 2$.

Now, comparing the given polynomial with general expression, we get

$ax^3 + bx^2 + cx + d = 2x^3 + x^2 - 5x + 2$

$\Rightarrow a = 2, b = 1, c = -5$ and $d = 2$

Let α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, and we take $\alpha = \frac{1}{2}$, $\beta = 1$ and $\gamma = -2$, then

$$\text{Sum of zeroes} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\Rightarrow \quad \alpha + \beta + \gamma = \frac{-1}{2}$$

$$\Rightarrow \quad \frac{1}{2} + 1 + (-2) = \frac{-1}{2}$$

$$\Rightarrow \quad \frac{3}{2} - 2 = \frac{-1}{2}$$

$$\Rightarrow \quad \frac{-1}{2} = \frac{-1}{2} \text{ Hence verified.}$$

$$\text{Sum of product of zeroes} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\Rightarrow \quad \alpha\beta + \beta\alpha + \gamma\alpha = \frac{-5}{2}$$

$$\Rightarrow \left(\frac{1}{2}\times 1\right) + (1\times -2) + \left(-2\times\frac{1}{2}\right) = \frac{-5}{2}$$

$$\Rightarrow \frac{1}{2} - 2 - 1 = \frac{-5}{2}$$

$$\Rightarrow \frac{1}{2} - 3 = \frac{-5}{2}$$

$$\Rightarrow \frac{-5}{2} = \frac{-5}{2} \text{ Hence Verified.}$$

$$\text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^3}$$

$$\Rightarrow \quad \alpha\beta\gamma = \frac{(-2)}{2}$$

$$\Rightarrow \quad \frac{1}{2}\times(1)\times(-2) = -1$$

$$\Rightarrow \quad -1 = -1 \text{ Hence verified.}$$

Hence, the relationship between the zeroes and the coefficients are satisfied.

(ii) Do it yourself.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Exp. Let α, β and γ be the zeroes of the required polynomial, then

$$\alpha + \beta + \gamma = 2$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = -7$$

$$\alpha\beta\gamma = -14$$

∴ Cubic polynomial can be written as

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$$

$$x^3 - 2x^2 - 7x + 14 = 0$$

Hence, $x^3 - 2x^2 - 7x + 14 = 0$ is the required cubic polynomial.

3. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a and b.

Exp. Let $p(x) = x^3 - 3x^2 + x + 1$

And zeroes are given as $a - b, a, a + b$

$$\text{Sum of zeroes} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$a - b + a + a + b = \frac{-(-3)}{1}$$

$$\Rightarrow \quad 3a = 3$$

$$\Rightarrow \quad a = 1$$

Thus, the zeroes are $1 - b, 1, 1 + b$.

Now, product of zeroes = $\dfrac{\text{Constant term}}{\text{Coefficient of } x^3}$

$\Rightarrow (1 - b) \times 1 \times (1 + b) = -\dfrac{1}{2}$

$\Rightarrow (1 - b)(1 + b) = -1$

$\Rightarrow (1 - b^2) = -1 \quad [(1 + a)(1 - a) = 1 - a^2]$

$\Rightarrow b^2 = 2$

$\Rightarrow b = \sqrt{2}$

Hence, $1-\sqrt{2}$, 1, $1+\sqrt{2}$ are the zeroes of the polynomial $x^3 - 3x^2 + x + 1$.

4. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Exp. Since this is a polynomial equation of degree 4, hence there will be total 4 zeroes.

Let $f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

Since $2+\sqrt{3}$ and $2-\sqrt{3}$ are zeroes of given polynomial f(x).

$\therefore [x - (2+\sqrt{3})]\,[x-(2-\sqrt{3})] = 0$

$x^2 - 4x + 1 = 0$

$x^2 - 4x + 1$ is a factor of a given polynomial f(x).

Now we divide the given polynomial by $x^2 - 4x + 1$.

$$\begin{array}{r|l}
 & x^2 - 2x - 35 \\
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & \underline{- \quad + \quad -} \\
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & \underline{+ \quad - \quad +} \\
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & \underline{+ \quad - \quad +} \\
 & 0
\end{array}$$

So, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

Now on further factorizing $(x^2 - 2x - 35)$, we get

$x^2 - (7 - 5)x - 35 = x^2 - 7x + 5x + 35 = 0$

$x(x - 7) + 5(x - 7) = 0$

$(x + 5)(x - 7) = 0$

So, its zeroes are given by: $x = -5$ and $x = 7$.

Hence, the other zeroes of given polynomial are: -5 and 7.

5. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x - 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

Exp. By division algorithm,

Dividend = Divisior × Quotient + Remainder

Dividend – Remainder = Divisor × Quotient

$x^4 - 6x^3 + 16x^2 - 25x - 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ will be perfectly divisible by $x^2 - 2x + k$.

let us divide $x^4 - 6x^3 + 16x^2 + 26x + 10 - a$ by $x^2 - 2x + k$

$$\begin{array}{r|l}
 & x^2 - 4x + (8 - k) \\
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 26x + 10 - a \\
 & x^4 - 2x^3 + kx^2 \\
 & \underline{- \quad + \quad -} \\
 & -4x^3 + (16 - k)x^2 - 26x \\
 & -4x^3 + 8x^2 - 4kx \\
 & \underline{+ \quad - \quad +} \\
 & (8 - k)x^2 - (26 - 4k)x + 10 - a \\
 & (8 - k)x^2 - (16 - 2k)x + (8k - k^2) \\
 & \underline{- \quad + \quad -} \\
 & (-10 + 2k)x + (10 - a - 8k + k^2)
\end{array}$$

It can be observed that $(-10 + 2k) = 0$ and $(10 - a - 8k + k^2) = 0$

For $(-10 + 2k) = 0$,

$2k = 10$

And thus, $k = 5$

For $(10 - a - 8k + k^2) = 0$

$10 - a - 8 \times 5 + 25 = 0$

$10 - a - 40 + 25 = 0$

$-5 - a = 0$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$

Subjective Questions

Very Short Answer Type Questions

1. Form the quadratic polynomial whose zeroes are 4 and 6.
2. Form the quadratic polynomial whose sum and product of zeroes are 6 and 7.
3. If α, β and γ are the zeroes of polynomial $x^3 - 3x^2 + x + 1$, find the value of $\alpha + \beta + \gamma$.
4. If $x = 2$ & $x = 0$ are the zeroes of the polynomial $f(x) = 2x^3 - 5x^2 + ax + b$, then find the value of $a + b$.
5. Find the remainder when $f(x) = x^3 - 6x^2 + 2x - 4$ is divided by $g(x) = 1 - 2x$.
6. If $x = 3$ is a root of $f(x) = x^3 - 4x^2 + 6k$, then find the value of k.
7. Find the remainder when $f(x) = x^3 + x^2 + 2x + 3$ is divided by $x - 1$.
8. Find the remainder when the polynomial $P(x) = x^4 + 6x^3 + 13x^2 + 12x + 6$ is divided by $g(x) = x + 2$
9. Find the value of 'a' for which $(x - 3)$ is a factor of $f(x) = x^2 - ax + 9$.
10. Show that $(x + a)$ is always a factor of $x^n + a^n$ when n is an odd positive integer.

Short Answer Type Questions

1. Find the zeroes of the given quadratic polynomial $6x^2 - 13x + 6$ and then verify the relation between the zeroes and its coefficients.
2. Verify the relation between the zeroes and the coefficients of polynomial $ax^2 + bx + c$, $a \neq 0$.
3. If α and β are the zeroes of the polynomial $ax^2 + bx + c$, then find the value of
   (i) $\alpha - \beta$
   (ii) $\alpha^2 + \beta^2$
4. If α and β are the zeroes of the quadratic polynomial $ax^2 - bx + c$, then find the value of $a^2 - \beta^2$.
5. Find a cubic polynomial whose zeroes are 1, 2 and –3.
6. If α and β are the zeroes of the polynomial $ax^2 + bx + c$, then form the polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.
7. Divide the polynomial $p(x) = 3x^3 + 4x + 11$ by $g(x) = x^2 - 3x + 2$ to find the quotient and remainder.
8. On dividing $3x^3 + 4x^2 + 5x - 13$ by a polynomial g(x), the quotient and remainder were $3x + 10$ and $16x - 43$, respectively. Find g(x).
9. For a given polynomial $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$, the sum of two zeroes is 4. Find the sum of other two zeroes.
10. Find the sum of real values of y satisfying the equation $x^2 + x^2y^2 + x^2y^4 = 525$ and $x + xy + xy^2 = 35$.

Long Answer Type Questions

1. Verify that $\frac{1}{2}, 1, -2$ are zeroes of cubic polynomial $2x^3 + x^2 - 5x + 2$. Also, verify the relationship between the zeroes and their coefficients.
2. If α and β are the zeroes of the polynomial $x^2 + 4x + 3$, form the polynomial whose zeroes are $1+\frac{\beta}{\alpha}$ and $1+\frac{\alpha}{\beta}$.
3. Using factor theorem, factorize : $p(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$.

4. Find the value that must be added to $f(x) = 3x^4 + 13x^3 - 29x^2 + 100x - 77$, so that the resulting polynomial is completely divisible by $g(x) = x^2 + 6x - 7$.

5. Simplify:

$$\frac{(x-y)^2}{(y-z)(z-x)} + \frac{(y-z)^2}{(x-y)(z-x)} + \frac{(z-x)^2}{(x-y)(y-z)}$$

Integer Type Questions

1. Find the value of "p" for which 2 is one of the zeroes of $f(x) = x^2 + 3x + p$.

2. Find the value of "x" in the polynomial $2a^2 + 2xa + 5a + 10$ for which $(a + x)$ is one of its factors.

3. If the sum of zeroes of the quadratic polynomial $3x^2 - mx + 6$ is 3, then find the value of k.

4. Find the value of p for which one zero of polynomial $3x^2 - 4x + p$ is reciprocal to the other.

5. If $(x - 1)$ is one of the factor of the quadratic polynomial $x^2 + 3x + m$, then find the value of m.

6. If 'm' and 'n' are the zeroes of polynomial $x^2 + mx + n$, find the value of $m - n$.

7. Find the value of $a^4b^3 + a^3b^4$, where a and b are the zeroes of $f(x) = x^2 - 4x + 3$.

8. Find the value of $\frac{1}{p} + \frac{1}{q} + \frac{1}{r}$, where p, q and r are zeroes of polynomial $6x^3 + 3x^2 - 5x + 1$.

9. If the HCF of the polynomials $x^2 + 2mx - 15$ and $(x - 3)(x + n)$ is $(x + 5)$, then find the value of $m + n$.

10. Find the degree of polynomial $\frac{3x^3 + 10x^2 + x - 14}{3x^2 + 13x + 14}$.

Multiple Choice Questions

Level-I

1. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then

a. c and a have opposite signs

b. c and b have same sign

c. c and a have same sign

d. c and b have opposite signs

2. Find the number of polynomials whose zeroes are –2 and 5.

a. 1 b. 2

c. 6 d. Infinite

3. Find the degree of the polynomial $(x + 1)(x^2 - x - x^4 + 1)$.

a. 2 b. 3

c. 1 d. 5

4. The zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then find the value of 'a' and 'b'.

a. $a = -7, b = -6$ b. $a = 0, b = -1$

c. $a = 2, b = -6$ d. $a = 0, b = -6$

5. If one of the zeroes of the quadratic polynomial $(a - 1)x^2 + ax + 1$ is –3, then find the value of a

a. $\frac{4}{3}$ b. $-\frac{4}{3}$

c. $\frac{2}{3}$ d. $-\frac{7}{3}$

6. Find a quadratic polynomial, whose zeroes are –3 and 4.

a. $x^2 - x + 12$ b. $\frac{x^2}{2} + \frac{x}{2} + 12$

c. $\frac{x^2}{2} - \frac{x}{2} - 6$ d. $\frac{x^2}{2} + \frac{x}{2} - 6$

7. If $(x - 2)$ is a factor of the polynomial $x^3 + 4x^2 - px + 8$, then the value of $\sqrt{p}$ is

a. 4 b. 3

c. 5 d. 16

8. The sum of the squares of zeroes of the quadratic polynomial $6x^2 + x + c$ is 25/36, then the value of c is

a. 4 b. –4

c. –3 d. –2

9. Find the value of $\frac{1}{m} + \frac{1}{n} - mn$, where m and n are the zeroes of $x^2 - 4x + 1$.

a. 3 b. $\frac{1}{3}$

c. –5 d. –3

10. Find the zeroes of polynomial $x^2 - 2x - 8$.

a. (2, –4) b. (4, –2)

c. (–2, –2) d. (4, –4)

11. What is the quadratic polynomial whose sum and the product of zeroes is $\left(\sqrt{2}, \frac{1}{3}\right)$ respectively?

a. $3x^2 - 3\sqrt{2}x + 1$ b. $6x^2 - 6\sqrt{2}x + 2$

c. $3x^2 + 3\sqrt{2}x - 1$ d. Both a and b

12. The degree of the polynomial, $5x^4 - 9x^2 + x^9$ is

a. 2 b. 4

c. 1 d. 9

13. –1 is one of the zeroes of cubic polynomial $x^3 + ax^2 + bx + c$, then product of other two zeroes is

a. $b - a - 1$ b. $b - a + 1$

c. $a - b + 1$ d. $a - b - 1$

14. If p(x) is a polynomial of degree one and p(a) = 0, then a is said to be

a. Zero of p(x) b. Constant of p(x)

c. Value of p(x) d. None of these

15. Number of zeroes of polynomial is equal to number of points where the graph of polynomial

a. Intersects x-axis

b. Intersects y-axis

c. Intersects y-axis or x-axis

d. None of the above

16. A polynomial of degree n has

a. At least n zeroes b. Only one zero

c. More than n zeroes d. Atmost n zeroes

17. Zeroes of $p(x) = x^2 - 8$ are

a. $\pm 9\sqrt{3}$ b. $\pm 2\sqrt{2}$

c. $\pm 4\sqrt{2}$ d. None of the above

18. If (x – 2) is a factor of $(x^2 - mx + 2)$, then the value of m is

a. 2 b. 3
c. 1 d. 0

19. If x + 2 is a factor of $\{(x + 1)^5 + (2x + k)^3\}$, then the value of 'k' is

a. 2 b. 3
c. 4 d. 5

20. Which of the following does NOT represent a zero polynomial?

a. p(x) = 0 b. $p(x) = x^0 - 1$
c. $p(x) = x^0$ d. $p(x) = 0.x^0$

21. The degree of a quartic polynomial is:

a. 2 b. 1
c. 4 d. 3

22. If $f(x) = 2x^3 - 13x^2 + 17x + 12$, then find out the remainder when f(x) is divided by (x + 2).

a. –90 b. –85
c. –70 d. 90

23. Find the value of $ax^2 + bx + c$ at $x = \frac{-b}{a}$.

a. a b. $b^2 - 4ac$
c. c d. b

24. If $p(x) = x^2 - 2\sqrt{2}x + 1$, then find the value of remainder when p(x) is divided by $\left(x - 2\sqrt{2}\right)$.

a. 0 b. 1
c. $4\sqrt{2}$ d. $8\sqrt{2} + 1$

25. Find the value of the polynomial $(3x^3) \times (4x^2) + 7x^5$ at x = 3.

a. 4617 b. 4628
c. 4611 d. 4619

26. Find the roots of the polynomial $x^3 - 3x - 2 = 0$.

a. –1, –1, 2 b. –1, 1, –2
c. –1, 2, 3 d. 1, 1, –2

27. Let $f(x) = x^2 + 3x + 2$, the remainder when f(x) is divided by (x − 1) will be

a. 3 b. 5
c. 6 d. 4

28. Let $f(x)=2x^2 + 3x + 1$, the remainder when f(x) is divisible by (x + 1) is

a. 6 b. 0
c. –6 d. 1

29. The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by (x – 4) leaves remainder R_1 & R_2 such that $2R_1 - R_2 = 0$, then the value of a is

a. $-\frac{18}{127}$ b. $\frac{18}{127}$
c. $\frac{17}{127}$ d. $\frac{127}{18}$

30. Simplify: $(l + m)^2 - 4lm$

a. $(l - m)^2$ b. $4l^2m^2$
c. $(l + 2m)^2$ d. None of these

31. Simplify the following expression: $(s - r)^2 - (s + r)^2$

a. 2sr b. $2s^2 + 2r^2$
c. –4sr d. None of these

32. If (2x + 3) is a factor of polynomial $2x^3 + 9x^2 - x - b$ then b will be

a. 15 b. –15
c. 25 d. –10

33. Find the value of c for which the polynomial $2x^2 + kx + \sqrt{2}$ is exactly divisible by (x – 1).

a. $2+\sqrt{2}$ b. $2-\sqrt{2}$
c. $-(2+\sqrt{2})$ d. $-2+\sqrt{2}$

34. Find the value of polynomial f(x) = 3x (4x – 5) + 3 at $x = \frac{1}{2}$.

a. $-\frac{3}{2}$ b. $-\frac{1}{2}$
c. $-\frac{3}{7}$ d. $-\frac{3}{8}$

35. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by x – a.

a. 2a b. 5a
c. 6a d. 9a

Level-II

1. The zeroes of the quadratic polynomial $x^2 + 99x - 127$ are

a. Both positive
b. Both negative
c. One positive and one negative
d. Can't say

2. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

a. $\frac{-c}{a}$ b. $\frac{c}{a}$
c. 0 d. $\frac{-b}{a}$

3. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

a. Has no linear term and the constant term is negative.

b. Has no linear term and the constant term is positive.

c. Can have a linear term but the constant term is negative.

d. Can't say

4. If α, β are zeroes of $x^2 - 6x + p$ then what is the value of p if $3\alpha + 2\beta = 20$?

a. –16 b. 5

c. 2 d. –8

5. Find the value of p for which $(x - 2)$ is a factor of polynomial $x^4 - x^3 + 2x^2 - px + 4$.

a. 10 b. –9

c. 4 d. –10

6. If 2 and 3 are zeroes of the quadratic polynomial $x^2 + ax + b$, the values of $a + b$ is

a. –3 b. 5

c. 1 d. 6

7. If $f(x) = 2x^2 - x + 1$ and $g(x) = x^3 - 3x + 1$, then the value of $f(1) + g(-1)$ is

a. 2 b. 1

c. –5 d. 5

8. The value of the polynomial $4x^2 + 3x - 7$ at $x = 1$ is

a. 1 b. 2

c. 7 d. None of these

9. The function $f(x) = x^3 - 6x^2 + ax + b$ where a and b are constants is exactly divisible by $(x - 3)$ and leaves a remainder of –55 when divided by $(x + 2)$ Find the value of $a + b$.

a. –30 b. 10

c. 7 d. 13

10. If $(x + \lambda)$ is a factor of polynomial $p(x) = x^2 + 5x + 6$, then the value of λ is

a. 2 b. 3

c. Both (a) and (b) d. None of these

11. If $x^4 + \frac{1}{x^4} = 119$, then the value of $x^3 - \frac{1}{x^3}$ is equal to

a. 9 b. 4

c. 25 d. 36

12. When $6x^9 + 3x^{16} - p$ is divided by $x + 1$, the remainder is 20. The value of p is

a. –23 b. –12

c. 8 d. 23

13. A quadratic polynomial is divisible by $(2x - 1)$ and $(x + 3)$ and leaves remainder 12 on division by $(x - 1)$. The polynomial is

a. $6x^2 + 15x + 9$ b. $2x^2 + 15x - 3$

c. $6x^2 + 15x - 9$ d. Both b and c

14. If the zeroes of the polynomial $x^3 - 6x^2 - 45x + 162$ are $a - b, -a, a + b$, then the value of a is

a. 3 b. –6

c. 6 d. 5

15. Find the remainder when the polynomial $ax^3 + bx^2 + cx + d$ is divided by $ax + b$.

a. $ad - bc$ b. $\frac{1}{a}(ad - bc)$

c. $\frac{a - bc}{d}$ d. $\frac{a + b + cd}{2}$

Assertion & Reason Type Questions

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

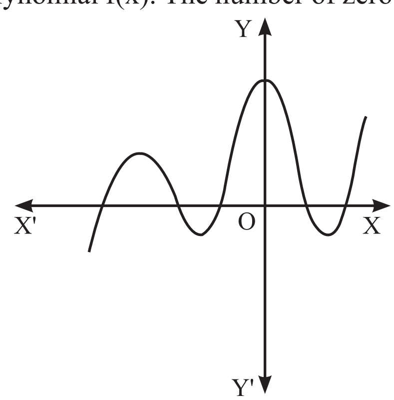
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

1. Assertion: The graph $y = f(x)$ is shown in figure, for the polynomial f(x). The number of zeroes of f(x) is 5.



Reason: The number of zero of the polynomial f(x) is the number of point of which f(x) cuts or touches the y-axis.

2. Assertion: $(2 - \sqrt{3})$ is one zero of the quadratic polynomial then other zero will be $(2 + \sqrt{3})$.

Reason: Irrational zeroes (roots) always occurs in pairs.

3. **Assertion:** The sum and product of the zeroes of a quadratic polynomial are $-\frac{1}{4}$ and $\frac{1}{4}$ respectively. Then the quadratic polynomial is $2x^2 + \frac{x}{2} + \frac{1}{2}$.

   Reason: The quadratic polynomial whose sum and product of zeroes are given is x^2 – (sum of zeroes). x + product of zeroes.

4. **Assertion:** If α, β, γ are the zeroes of $x^3 - 2x^2 + qx - r$ and $\alpha + \beta = 0$, then $2q = r$.

   Reason: If α, β, γ are the zeroes of $ax^3 + bx^2 + cx + d$

   then $\alpha + \beta + \gamma = \frac{b}{a}$

   $$\alpha\beta + \beta\gamma + \gamma\alpha = -\frac{c}{a}$$

   $$\alpha\beta\gamma = -\frac{d}{a}$$

5. **Assertion:** If one zero of polynomial $p(x) = (k + 4)x^2 + 13x + 3k$ is reciprocal of other, then $k = 2$.

   Reason: If $(x - \alpha)$ is a factor of p(x), then $p(\alpha) = 0$ i.e. α is a zero of p(x).

Comprehension Type

Passage–I: If α, β and γ be the zeroes of the polynomial $ax^3 + bx^2 + cx + d$, then the value of

1. $\alpha^2 + \beta^2 + \gamma^2$

   a. $\frac{b^2 - ac}{a^2}$ b. $\frac{b^2 - 2ac}{a}$

   c. $\frac{b^2 + 2ac}{b^2}$ d. $\frac{b^2 - 2ac}{a^2}$

2. $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ is

   a. $\frac{c^2 - ab}{a^2}$ b. $\frac{b^2 - 2ac}{a}$

   c. $\frac{c^2 + 2bd}{d^2}$ d. $\frac{c^2 - 2bd}{d^2}$

3. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

   a. $-\frac{b}{a}$ b. $-\frac{c}{d}$

   c. $-\frac{b}{c}$ d. $-\frac{c}{a}$

Passage–II: Cricket is the most popular sport in India. Usually, cricket is played outdoors on a large field. The projectile of a cricket ball is in the form of a parabola respresenting quadratic polynomial.

1. The graph of parabola opens downwards, if _____

   a. $a = 0$ b. $a > 0$

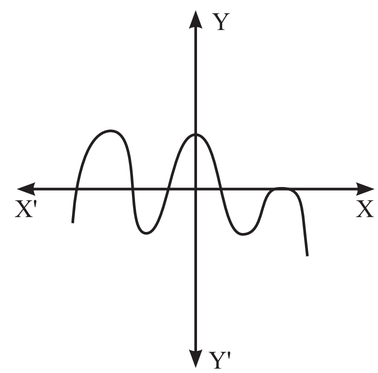
   c. $a < 0$ d. None of these

2. In a polynomial if sum of zeroes = $\alpha + \beta = -8$ and product of zeroes = $\alpha\beta = 6$, then find the polynomial whose zeroes are, $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

   a. $6x^2 + 8x + 1$ b. $6x^2 - 8x - 1$

   c. $6x^2 - 4x + 6$ d. $6x^2 - 8x + 1$

3. In the graph, how many zeroes are there for the polynomial?



   a. 1 b. 4

   c. 5 d. 3

Multi Correct MCQ's

1. If $2x^2 + xy - 3y^2 + x + ay - 10 = (2x + 3y + b)(x - y - 2)$, then the value of a and b are

   a. –|11| and |5| b. 1 and –5

   c. –1 and –5 d. –11 and 5

2. If $a - b = 3$ and $a^3 - b^3 = 117$, then a + b is

   a. 5 b. 7

   c. $21 \div \sqrt[3]{27}$ d. 11

3. If a three digit prime number is such that the digit in the units place is equal to the sum of the other two, then the total number of such primes is

   a. 6

   b. 4

   c. 2

   d. Value of the smallest composite number

4. If $x^2 - 1$ is a factor of $ax^4 + bx^3 + cx^2 + dx + e$, then

   a. $a + c + e = 0$ b. $a + b + c = 0$

   c. $b + d = 0$ d. None of these

Olympiad & NTSE Type Questions

1. An unknown polynomial yields a remainder 2 upon division by $x - 1$, and remainder of 1 upon division by $x - 2$, then find the remainder if this polynomial is divided by $(x - 1)(x - 2)$

a. $x - 3$ b. $2x - 5$

c. $-x + 3$ d. None of these

2. Factorisation of $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$ is

a. $3(a + b)^2 (b + c)^2 (c + a)^2$

b. $3(a^2 + b^2) (b^2 + c^2) (c^2 - a^2)$

c. $3(a^2 - b^2) (b^2 - c^2) (c^2 - a^2)$

d. None of these

3. P(x) is a degree 2 polynomial in x. It leaves remainder 5 on division by $(x - 1)$ and remainder 2 on division by $(x + 2)$. When P(x) is divided by $(x - 1)(x + 2)$, then the remainder will be

a. $2x - 7$ b. $x - 5$

c. $4 - x$ d. $x + 4$

4. If $(x - k)$ is the H.C.F. of $x^2 + x - 12$ and $2x^2 - kx - 9$, then the value of k is

a. -3 b. 3

c. Both (a) and (b) d. None of these

5. If the polynomial $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$, then the value of $a + b$ is

a. 7 b. 4

c. 8 d. 3

6. The zeroes of the polynomial $f(x) = x^3 - 12x^2 + 39x - 28$, if it is given that the zeroes are in A.P. are

a. 1, 3, 5 b. 1, 5, 7

c. 1, 7, 9 d. None of these

7. The value of a and b, if $(x^2 - 2x - 3)$ is a factor of the polynomail $x^3 - 3x^2 + ax - b$ are

a. $a = 1, b = 3$ b. $a = -1, b = 3$

c. $a = -1, b = -3$ d. $a = 3, b = 4$

8. $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ leaves remainder 5 and 19 on division by $(x - 1)$ and $(x + 1)$ respectively. If f(x) divided by $(x - 3)$, the remainder is

a. 0 b. 23

c. 47 d. -47

9. A cubic polynomial f(x) is such that $f(1) = 1$, $f(2) = 4$, $f(3) = 9$ and $f(4) = 20$, then the value of f(7) is

a. 16 b. 139

c. 68 d. 129

10. If the polynomial $3x^4 - 7x^3 - 7x^2 + 21x - 6$ has one of its zero as $-\sqrt{3}$, then find the values of other zeroes.

a. $\sqrt{3}, 2, \frac{1}{3}$ b. $\sqrt{3}, \frac{1}{\sqrt{3}}, -2$

c. $-\sqrt{3}, 2, -\frac{1}{3}$ d. $\sqrt{2}, 2, -\frac{1}{3}$

11. If the value of $x - \frac{1}{x} = 2$, then find the value of $x^4 + \frac{1}{x^4}$.

a. 4 b. 38

c. 36 d. 34

12. If $a + 2b + 3c = 0$, then $a^3 + 8b^3 + 27c^3$ is equal to

a. 9 abc b. 6 abc

c. 21 abc d. 18 abc

13. The value of p and q, such that $x^4 + px^3 + 2x^2 - 3x + q$ is divisible by $(x^2 - 1)$ are

a. $-3, -3$ b. $-3, 3$

c. $3, -3$ d. 4, 2

14. If $f(x) = x + x^9 + x^{25} + x^{49} + x^{81}$ is divided by $(x^3 - x)$, then the remainder is

a. x^{27} b. $x^2 + 5x + 1$

c. $5x^2$ d. $5x$

15. If $a = x + \frac{1}{x}, b = x - \frac{1}{x}$, then find the value of $a^2 + 2ab - 3b^2$.

a. $4x^2\left(\frac{2}{x^2} - 1\right)$ b. $\frac{4}{x^2}\left(2x^2 - 1\right)$

c. $4\left(1 - \frac{2}{x^2}\right)$ d. $8\left(\frac{1}{x^2} - \frac{1}{2}\right)$

Explanations

Subjective Questions

Very Short Answer Type Questions

1. Here, zeroes are 4 and 6

Sum of the zeroes = 4 + 6 = 10

Product of the zeroes = 4 × 6 = 24

Hence the polynomial formed

= x^2 – (sum of zeroes) x + Product of zeroes

= $x^2 - 10x + 24$

2. Here, sum and product of zeroes are given.

Sum of the zeroes = 6

Product of the zeroes = 7

Hence the polynomial formed

= x^2 – (sum of zeroes) x + Product of zeroes

= $x^2 - 6x + 7$

3.
$$\text{Sum of zeroes} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$
$$= -\frac{(-3)}{1}$$
$$= 3$$

4. If 2 and 0 are the zeroes of the polynomial, then by using factor theorem

$f(2) = f(0) = 0$

$f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow 16 - 20 + 2a + b = 0$

$\Rightarrow \quad 2a + b = 4 \quad$ ...(i)

Also $\quad f(0) = 2(0)^3 - 5(0)^2 + a(0) + b$

$\Rightarrow \quad 0 = b$

By putting b = 0 in (i), we get

$2a = 4$

$\Rightarrow \quad a = 2$

So, $\quad a + b = 2 + 0 = 2$

5. Using factor theorem

$1 - 2x = 0$

$\Rightarrow \quad 2x = 1$

$\Rightarrow \quad x = \frac{1}{2}$

By using the factor theorem the value of polynomial f(x) at $x = \frac{1}{2}$ is the remainder when f(x) is divided by g(x) = 1 – 2x.

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 4$$
$$= \frac{1}{8} - \frac{3}{2} + 1 - 4$$
$$= \frac{1 - 12 + 8 - 32}{8} = -\frac{35}{8}$$

6. Given, x = 3 is a root of f(x)

$\Rightarrow \quad f(3) = 0$

$\Rightarrow (3)^3 - (4).(3)^2 + 6k = 0$

$\Rightarrow \quad -9 + 6k = 0$

Or $\quad k = \frac{9}{6} = \frac{3}{2}$

7. When f(x) is divided by x – 1, then using factor theorem remainder is f(1)

$$f(1) = (1)^3 + (1)^2 + 2(1) + 3$$
$$= 1 + 1 + 2 + 3$$
$$= 7$$

∴ The value of remainder is 7.

8. When P(x) is divided by x + 2, then remainder is P(–2).

$\therefore \quad P(-2) = 16 - 6(8) + 13(4) - 12(2) + 6$

$= 74 - 72$

$= 2$

Hence remainder is 2.

9. If (x – 3) is a factor of f(x), then f(3) = 0

$\Rightarrow \quad f(3) = (3)^2 - a(3) + 9$

$\Rightarrow \quad 18 - 3a = 0$

$\Rightarrow \quad a = 6$

10. $f(x) = x^n + a^n$

If x + a is a factor of f(x) then f(–a) should be zero.

$$f(-a) = (-a)^n + a^n$$
$$= (-1)^n a^n + a^n$$
$$= [(-1)^n + 1]a^n$$
$$= (-1 + 1)a^n$$

$(\because$ n odd $\Rightarrow (-1)^n = -1)$

$\therefore \quad f(-a) = 0$

Hence proved.

Short Answer Type Questions

1. We shall use middle term splitting to factorise the given polynomial and find its zero.

$$\begin{aligned}6x^2 - 13x + 6 &= 6x^2 - 4x - 9x + 6\\ &= 2x\,(3x - 2) - 3\,(3x - 2)\\ &= (3x - 2)\,(2x - 3)\end{aligned}$$

Therefore, the zeroes of $6x^2 - 13x + 6$ are $\frac{2}{3}$ and $\frac{3}{2}$

$$\begin{aligned}\text{Sum of the zeroes} &= \frac{2}{3}+\frac{3}{2}\\ &= \frac{13}{6}\\ &= \frac{-(-13)}{6}\\ &= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}\end{aligned}$$

$$\begin{aligned}\text{Product of the zeroes} &= \frac{2}{3}\times\frac{3}{2}\\ &= \frac{6}{6}\\ &= \frac{\text{constant term}}{\text{coefficient of } x^2}\end{aligned}$$

Hence, verified.

2. Let α and β be the zeroes of polynomial

$ax^2 + bx + c$, then according to factor theorem, $(x - \alpha)$, $(x - \beta)$ are the factors of the polynomial $ax^2 + bx + c$.

$\Rightarrow ax^2 + bx + c = k\,(x - \alpha)\,(x - \beta)$

$\Rightarrow ax^2 + bx + c = k\,\{x^2 - (\alpha + \beta)\,x + \alpha\beta\}$

$\Rightarrow ax^2 + bx + c = kx^2 - k\,(\alpha + \beta)\,x + k\alpha\beta$ ...(1)

By comparing the coefficients of x^2, x and constant terms of (1) of both sides, we get

$a = k$, $b = -k\,(\alpha + \beta)$ and $c = k\alpha\beta$

$$\Rightarrow \quad \alpha + \beta = -\frac{b}{k} \text{ and } \alpha\beta = \frac{c}{k}$$

$$\alpha + \beta = \frac{-b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

[$\because$ k = a]

$$\text{Sum of the zeroes} = \frac{-b}{a} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of the zeroes} = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, verified.

3. As α and β are the zeroes of the polynomial $ax^2 + bx + c$.

$$\therefore \quad \alpha + \beta = -\frac{b}{a};\ \alpha\beta = \frac{c}{a}$$

(i)
$$\begin{aligned}(\alpha - \beta)^2 &= (\alpha^2 + \beta^2 - 2\alpha\beta)\\ &= (\alpha^2 + \beta^2 + 2\alpha\beta - 4\alpha\beta)\\ &= (\alpha + \beta)^2 - 4\alpha\beta\\ &= \left(-\frac{b}{a}\right)^2 - \frac{4c}{a}\\ &= \frac{b^2}{a^2} - \frac{4c}{a}\\ &= \frac{b^2 - 4ac}{a^2}\\ \alpha - \beta &= \pm\frac{\sqrt{b^2 - 4ac}}{a}\end{aligned}$$

(ii)
$$\begin{aligned}\alpha^2 + \beta^2 &= \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta\\ &= (\alpha + \beta)^2 - 2\alpha\beta\\ &= \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)\\ &= \frac{b^2 - 2ac}{a^2}\end{aligned}$$

4. Since α and β are the zeroes of polynomial $ax^2 - bx + c$

$$\therefore \quad \alpha + \beta = -\left(\frac{-b}{a}\right) = \frac{b}{a},\ \alpha\beta = \frac{c}{a}$$

$$\begin{aligned}\alpha^2 - \beta^2 &= (\alpha + \beta)\,(\alpha - \beta)\\ &= \frac{b}{a}\sqrt{(\alpha-\beta)^2}\\ &= \frac{b}{a}\sqrt{\alpha^2+\beta^2-2\alpha\beta}\\ &= \frac{b}{a}\sqrt{\alpha^2+\beta^2+2\alpha\beta-4\alpha\beta}\\ &= \frac{b}{a}\sqrt{(\alpha+\beta)^2-4\alpha\beta}\\ &= \frac{b}{a}\sqrt{\left(\frac{b}{a}\right)^2-4\frac{c}{a}}\\ &= \frac{b}{a}\sqrt{\frac{b^2-4ac}{a^2}}\\ &= \frac{b\sqrt{b^2-4ac}}{a^2}\end{aligned}$$

5. Given 1, 2 and –3 are the zeroes of the cubic polynomial.

Sum of zeroes = $1 + 2 + (-3) = 0$

Sum of product of zeroes taken two at a time = $1(2) + 2(-3) + (-3)(1) = -7$

Product of zeroes = $1 \times 2 \times (-3) = -6$

We know that cubic polynomial can be written as-

x^3 – (sum of zeroes)x^2 + (sum of product of zeroes taken two at a time)x – (product of zeroes)

$= x^3 - 0x^2 + (-7)x - (-6)$

$x^3 - 7x + 6$ is the required cubic polynomial.

6. Since α and β are the zeroes of $ax^2 + bx + c$

$$\therefore\ \alpha + \beta = -\frac{b}{a},\ \alpha\beta = \frac{c}{a}$$

Sum of the zeroes $= \dfrac{1}{\alpha} + \dfrac{1}{\beta}$

$$= \frac{\beta + \alpha}{\alpha\beta}$$

$$= \frac{\frac{-b}{a}}{\frac{c}{a}}$$

$$= \frac{-b}{c}$$

Product of the zeroes $= \dfrac{1}{\alpha}.\dfrac{1}{\beta}$

$$= \frac{1}{\frac{c}{a}}$$

$$= \frac{a}{c}$$

The required polynomial can be written as

x^2 – (sum of zeroes)x + Product of zeores

$$\Rightarrow\ x^2 - \left(\frac{-b}{c}\right)x + \left(\frac{a}{c}\right) = x^2 + \frac{b}{c}x + \frac{a}{c}$$

7. Given,

$p(x) = 3x^3 + 4x + 11$, $g(x) = x^2 - 3x + 2$

$$\begin{array}{r|l}
 & 3x + 9 \\ \hline
x^2 - 3x + 2 & 3x^3 + 0x^2 + 4x + 11 \\
 & 3x^3 - 9x^2 + 6x \\
 & -\quad +\quad - \\ \hline
 & \quad 9x^2 - 2x + 11 \\
 & \quad 9x^2 - 27x + 18 \\
 & \quad -\quad +\quad - \\ \hline
 & \qquad 25x - 7 \\ \hline
\end{array}$$

Hence, we get quotient = $3x + 9$ and remainder = $25x - 7$.

8. $p(x) = 3x^3 + 4x^2 + 5x - 13$

$q(x) = 3x + 10$ and $r(x) = 16x - 43$

By Division Algorithm, we know that

$p(x) = q(x) \times g(x) + r(x)$

Therefore,

$3x^3 + 4x^2 + 5x - 13 = (3x + 10) \times g(x) + (16x - 43)$

$\Rightarrow 3x^3 + 4x^2 - 11x + 30 = 3x + 10 \times g(x)$

$$\Rightarrow g(x) = \frac{3x^3 + 4x^2 - 11x + 30}{3x + 10}$$

$$\begin{array}{r|l}
 & x^2 - 2x + 3 \\ \hline
3x + 10 & 3x^3 + 4x^2 - 11x + 30 \\
 & 3x^3 + 10x^2 \\
 & -\quad - \\ \hline
 & \quad -6x^2 - 11x + 30 \\
 & \quad -6x^2 - 20x \\
 & \quad +\quad + \\ \hline
 & \qquad 9x + 30 \\
 & \qquad 9x + 30 \\
 & \qquad -\quad - \\ \hline
 & \qquad\quad 0 \\ \hline
\end{array}$$

Hence, $g(x) = x^2 - 2x + 3$

9. The given polynomial

$p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$ is a quartic polynomial $ax^4 + bx^3 + cx^2 + dx + e = 0$ i.e., it has a degree 4. let α, β, γ and δ be its zeroes

We know that,

$$\alpha + \beta + \gamma + \delta = \frac{-b}{a} = -\frac{(-6)}{1} = 6$$

∴ Sum of other two zeroes = 6 – 4 = 2

10. We have, $x^2 + x^2y^2 + x^2y^4 = 525$

$\Rightarrow\quad x^2(1 + y^2 + y^4) = 525$ ...(i)

And $\quad x + xy + xy^2 = 35$

$\Rightarrow\quad x(1 + y + y^2) = 35$ ...(ii)

squaring the equation (ii)

$x^2(1 + y + y^2)^2 = 1225$ ...(iii)

divide eq. (iii) by (i)

$$\frac{x^2\left(1 + y + y^2\right)^2}{x^2\left(1 + y^2 + y^4\right)} = \frac{1225}{525}$$

$$\Rightarrow \frac{\left(1 + y + y^2\right)^2}{\left(1 + y^2 + y^4\right)} = \frac{7}{3}$$

$$\Rightarrow \frac{1 + 2y + 3y^2 + 2y^3 + y^4}{1 + y^2 + y^4} = \frac{7}{3}$$

[using $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$]

$\Rightarrow 3 + 6y + 9y^2 - 6y^3 + 3y^4 = 7 + 7y^2 + 7y^4$

$\Rightarrow 4y^4 - 6y^3 - 2y^2 - 6y + 4 = 0$

$\Rightarrow 2y^4 - 3y^3 - y^2 - 3y + 2 = 0$ ...(iv)

Using hit and trial method 2 and 1/2 are the roots of the above equation.

$$\frac{2y^4 - 3y^3 - y^2 - 3y + 2}{(y-2)\left(y-\frac{1}{2}\right)} = \frac{2y^4 - 3y^3 - y^2 - 3y + 2}{y^2 - \frac{5}{2}y + 1}$$

$$= (2y^2 - 5y + 2)$$

Hence from equation (iv)

$$2y^4 - 3y^3 - y^2 - 3y + 2 = (y-2)\left(y-\frac{1}{2}\right)(2y^2 - 5y + 2) = 0$$

$$\Rightarrow (y-2)\left(y-\frac{1}{2}\right)(2y^2 - 5y + 2) = 0$$

The term $(2y^2 - 5y + 2)$ has complex roots.

$\therefore$ sum of real values of $y = \frac{1}{2} + 2 = \frac{5}{2}$

Long Answer Type Questions

1. $f(x) = 2x^3 + x^2 - 5x + 2$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$

$$= 0$$

$$f(1) = 2(1)^3 + (1)^2 - 5(1) + 2$$

$$= 2 + 1 - 5 + 2$$

$$= 0$$

$$f(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$

$$= -16 + 4 + 10 + 2$$

$$= 0$$

Let $\alpha = \frac{1}{2}$, $\beta = 1$ and $\gamma = -2$

Now, Sum of zeroes $= \alpha + \beta + \gamma$

$$= \frac{1}{2} + 1 - 2 = -\frac{1}{2}$$

Also,

sum of zeroes $= -\dfrac{(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3} = -\dfrac{1}{2}$

sum of zeroes $= \alpha + \beta + \gamma = -\dfrac{(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$

Sum of product of zeroes taken two at a time

$= \alpha\beta + \beta\gamma + \gamma\alpha$

$$= \frac{1}{2} \times 1 + 1 \times (-2) + (-2) \times \frac{1}{2} = -\frac{5}{2}$$

Also, $\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \dfrac{-5}{2}$

So, sum of product of zeroes taken two at a time

$$= \alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

Now,

Product of zeroes $= \alpha\beta\gamma$

$$= \left(\frac{1}{2}\right)(1)(-2) = -1$$

Also,

Product of zeroes $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^3}$

$$= \frac{-2}{2} = -1$$

$\therefore$ Product zeroes $= \alpha\beta\gamma = -\dfrac{\text{Constant term}}{\text{Coefficient of } x^3}$

2. Since α and β are the zeroes of the polynomial $x^2 + 4x + 3$.

Then, $\therefore \alpha + \beta = -4$, $\alpha\beta = 3$

For the required polynomial, Sum of the zeroes

$$= 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta}$$

$$= \frac{\alpha\beta + \beta^2 + \alpha\beta + \alpha^2}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$$

Product of the zeroes

$$= \left(1 + \frac{\beta}{\alpha}\right)\left(1 + \frac{\alpha}{\beta}\right)$$

$$= 1 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{\alpha\beta}{\alpha\beta}$$

$$= 2 + \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{2\alpha\beta + \alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$$

The required polynomial is x^2 – (sum of zeroes) x + product of zeroes

or $x^2 - \dfrac{16}{3}x + \dfrac{16}{3}$ or $k\left(x^2 - \dfrac{16}{3}x + \dfrac{16}{3}\right)$

3. Factors of 45 = $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45$

If we put x = 1 in p(x)

$$p(1) = 2(1)^4 - 7(1)^3 - 13(1)^2 + 63(1) - 45$$
$$p(1) = 2 - 7 - 13 + 63 - 45$$
$$= 65 - 65 = 0$$

$\therefore$ x = 1 or x – 1 is a factor of p(x).

If we put x = –1 in p(x)

$$p(-1) = 2(-1)^4 - 7(-1)^3 - 13(-1)^2 + 63(-1) - 45$$
$$= 2 + 7 - 13 - 63 - 45 = -112$$

$\therefore$ x = –1 is not a factor of p(x).

Similarly if we put x = 3 in p(x)

$$p(3) = 2(3)^4 - 7(3)^3 - 13(3)^2 + 63(3) - 45$$
$$p(3) = 162 - 189 - 117 + 189 - 45$$
$$= 162 - 162$$
$$= 0$$

Hence, x = 3 or (x – 3) = 0 is the factor of p(x).

Now we shall factorise the given polynomial using is that (x – 1) and (x – 3)

$$p(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$$
$$\therefore\ p(x) = 2x^3(x-1) - 5x^2(x-1) - 18x(x-1) + 45(x-1)$$
$$\Rightarrow p(x) = (x-1)(2x^3 - 5x^2 - 18x + 45)$$
$$\Rightarrow p(x) = (x-1)[2x^2(x-3) + x(x-3) - 15(x-3)]$$
$$\Rightarrow p(x) = (x-1)(x-3)(2x^2 + x - 15)$$
$$\Rightarrow p(x) = (x-1)(x-3)(2x^2 + 6x - 5x - 15)$$
$$\Rightarrow p(x) = (x-1)(x-3)[2x(x+3) - 5(x+3)]$$
$$\Rightarrow p(x) = (x-1)(x-3)(x+3)(2x-5).$$

4. We know that,

$$f(x) = g(x) \times q(x) + r(x)$$
$$f(x) - r(x) = g(x) \times q(x)$$
$$f(x) + \{-r(x)\} = g(x) \times q(x)$$

LHS is completely divisible by g(x).

Hence we can say, if we add –r(x) to f(x), then the resulting polynomial is divisible by g(x).

$$\begin{array}{r|l}
 & 3x^2 - 5x + 22 \\
\hline
x^2 + 6x - 7 & 3x^4 + 13x^3 - 29x^2 + 100x - 77 \\
 & \underline{\overset{-}{}3x^4 \overset{-}{+} 18x^3 \overset{+}{-} 21x^2} \\
 & -5x^3 - 8x^2 + 100x - 77 \\
 & \underline{\overset{+}{}-5x^3 \overset{+}{-} 30x^2 \overset{-}{+} 35x} \\
 & 22x^2 + 65x - 77 \\
 & \underline{\overset{-}{}22x^2 \overset{-}{+} 132x \overset{+}{-} 154} \\
 & -67x + 77
\end{array}$$

$\therefore$ $r(x) = -67x + 77$

Therefore, we should add –r(x) = 67x – 77 to f(x), such that resulting polynomial is divisible by g(x).

5. Let

$$P(x) = \frac{(x-y)^2}{(y-z)(z-x)}$$

Multiplying numerator and denominator by (x – y), we get

$$P(x) = \frac{(x-y)^3}{(x-y)(y-z)(z-x)} \quad \text{...(i)}$$

let $$Q(x) = \frac{(y-z)^2}{(x-y)(z-x)}$$

Multiplying numerator and denominator by (y – z), we get

$$Q(x) = \frac{(y-z)^3}{(x-y)(y-z)(z-x)} \quad \text{...(ii)}$$

Let $$R(x) = \frac{(z-x)^2}{(x-y)(y-z)}$$

multiplying numerator and denominator by (z – x), we get

$$R(x) = \frac{(z-x)^3}{(x-y)(y-z)(z-x)} \quad \text{...(iii)}$$

Adding (i), (ii) and (iii)

$$= \frac{(x-y)^3}{(x-y)(y-z)(z-x)} + \frac{(y-z)^3}{(x-y)(y-z)(z-x)} + \frac{(z-x)^3}{(x-y)(y-z)(z-x)}$$

$$= \frac{(x-y)^3 + (y-z)^3 + (z-x)^3}{(x-y)(y-z)(z-x)} \quad \text{...(iv)}$$

Special case: If a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$

As x – y + y – z + z – x = 0

$$\therefore (x-y)^3 + (y-z)^3 + (z-x)^3 = 3(x-y)(y-z)(z-x)$$

$$= \frac{3(x-y)(y-z)(z-x)}{(x-y)(y-z)(z-x)} = 3$$

Integer Type Questions

1. Given, 2 is the zero of the polynomial.

We know that if α is a zero of the polynomial p(x), then $p(\alpha) = 0$

Substituting x = 2 in $x^2 + 3x + p$,

$$2^2 + 3(2) + p = 0$$
$$\Rightarrow \quad 4 + 6 + p = 0$$
$$\Rightarrow \quad 10 + p = 0$$
$$\Rightarrow \quad p = -10$$

2. As, $(a + x)$ is a factor of $2a^2 + 2xa + 5a + 10$

$$\therefore \quad f(-x) = 2x^2 - 2x^2 - 5x + 10 = 0$$

$$\Rightarrow \quad -5x + 10 = 0$$

$$\Rightarrow \quad x = 2$$

3. Here $a = 3$, $b = -m$, $c = 6$

Sum of the zeroes, $(\alpha + \beta) = -\dfrac{b}{a}$

$$\Rightarrow \quad \frac{-(-m)}{3} = 3$$

$$\Rightarrow \quad m = 9$$

4. Let us suppose m be one of zero, then according to the question 1/m will be another zero

Now $\quad m \times \dfrac{1}{m} = \dfrac{p}{a}$

$$\Rightarrow \quad p = 3$$

5. As 1 is zero of the quadratic polynomial

$$f(1) = 1^2 + 3(1) + m = 0$$

$$\Rightarrow \quad m = -4$$

6. We have given m and n are the zeroes of polynomial $x^2 + mx + n$. Then,

Sum of zeroes $= -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$\Rightarrow m + n = \frac{-m}{1}$$

$$\Rightarrow 2m + n = 0 \qquad \text{...(i)}$$

Product of zeroes $= \dfrac{\text{Constent term}}{\text{Coefficient of } x^2}$

$$\Rightarrow m \times n = \frac{n}{1}$$

$$\Rightarrow m = 1$$

By putting the value of m in equation (i),

$2 \times 1 + n = 0$

$\Rightarrow n = -2$

$\therefore m - n = 1 + 2$

$= 3$

7. As, a and b are the zeroes of polynomial $x^2 - 4x + 3$

Sum of zeroes $= \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$a + b = \frac{-(-4)}{1} = 4$$

And, product of zeroes $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$ab = \frac{3}{1} = 3$$

$$a^4b^3 + a^3b^4 = a^3b^3(a + b)$$

$$= (ab)^3 (a + b)$$

$$= 27 \times 4 \quad [\because ab = 3, (a + b) = 4]$$

$$= 108$$

8. As p, q and r are the zeroes of polynomial

$6x^3 + 3x^2 - 5x + 1$

Sum of the product of zeroes taken two at a time

$= \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$$pq + qr + pr = \frac{-5}{6}$$

Product of the zeroes $= \dfrac{-\text{Constant term}}{\text{Coefficient of } x^3}$

$$pqr = -\frac{1}{6}$$

Now

$$\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{pq + qr + pr}{pqr} = \frac{\frac{-5}{6}}{\frac{-1}{6}} = 5$$

9. Given HCF $= x + 5$, which means $x + 5$ is a factor of both the polynomials.

As $(x + 5)$ is a factor of $x^2 + 2mx - 15$,

$$f(-5) = (-5)^2 + 2m(-5) - 15$$

$$= 0$$

$$\Rightarrow \quad 25 - 15 - 10m = 0$$

$$\Rightarrow \quad m = 1$$

One of the factor of $(x - 3)(x + n)$ is $x - 3$ and the other factor is $(x + 5)$.

$$\therefore \quad (x - 3)(x + n) = (x - 3)(x + 5)$$

$$\Rightarrow \quad x + n = x + 5$$

$$\Rightarrow \quad n = 5$$

So, $\quad m + n = 1 + 5$

$$= 6$$

10. To find the degree of such polynomials, divide the highest power variable in numerator by highest power variable in denominator.

$$\frac{3x^3 + 10x^2 + x - 14}{3x^2 + 13x + 14} = x^{3-2} = x^1$$

$\therefore$ The degree is 1.

Multiple Choice Questions

Level-I

1. **(c)** Let m be the zero of the polynomial $ax^2 + bx + c = 0$

$$\therefore \quad m + m = -\frac{b}{a}$$

Also, $m \times m = \frac{c}{a} \Rightarrow m^2 = \frac{c}{a}$

$m^2 > 0$ (Since square of any number cannot be negative) which is only possible when a and c have same sign.

2. **(d)** The polynomial with zeroes –2 and 5 is

$$f(x) = x^2 - (-2 + 5)x + (-2)5$$

$$\Rightarrow \quad f(x) = x^2 - 3x - 10$$

But as we can multiply this polynomial with any number, The number of polynomials having zeroes as –2 and 5 can be infinite.

For example:

(i) $2(x^2 - 3x - 10) = 2x^2 - 6x - 20$

(ii) $3(x^2 - 3x - 10) = 3x^2 - 9x - 30$.

3. **(d)** Multiply the highest degree variable in first bracket with the highest degree variable in second bracket. Hence, the highest power we obtain is 5.

4. **(d)** Sum of zeroes is

$$2 + (-3) = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\Rightarrow \quad -1 = -\frac{a+1}{1}$$

$$\Rightarrow \quad a + 1 = 1$$

$$\Rightarrow \quad a = 0$$

Product of zeroes is

$$2 \times (-3) = \frac{\text{constant term}}{\text{coefficent of } x^2}$$

$$\Rightarrow \quad -6 = \frac{b}{1}$$

$$\Rightarrow \quad b = -6$$

5. **(a)** As –3 is a zero of the given polynomial, so

$$P(-3) = (a - 1)(-3)^2 + a(-3) + 1$$

$$0 = 9a - 9 - 3a + 1$$

$$0 = 6a - 8$$

$$\Rightarrow \quad a = \frac{4}{3}$$

6. **(c)** The polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$

$$= x^2 - (-3 + 4)x + (-3)4$$

$$= x^2 - x - 12$$

By dividing the whole polynomial with 2, we get

$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

7. **(a)** Since $(x - 2)$ is one of the factor, so $f(2) = 0$

$$\Rightarrow \quad f(2) = (2)^3 + 4(2)^2 - p(2) + 8$$

$$\Rightarrow \quad 0 = 8 + 16 - 2p + 8$$

$$\Rightarrow \quad 2p = 32$$

$$\Rightarrow \quad p = 16$$

$$\therefore \sqrt{p} = 4$$

8. **(d)** Let γ and δ are the zeroes of given polynomial, then

$$\therefore \quad \gamma + \delta = -\frac{1}{6}$$

$$\text{Also} \quad \gamma\delta = \frac{c}{6}$$

$$\text{Given} \quad \gamma^2 + \delta^2 = \frac{25}{36}$$

$$\Rightarrow \quad (\gamma + \delta)^2 - 2\gamma\delta = \frac{25}{36}$$

$$\Rightarrow \quad \left(\frac{-1}{6}\right)^2 - 2\left(\frac{c}{6}\right) = \frac{25}{36}$$

$$\Rightarrow \quad \frac{-2c}{6} = \frac{24}{36}$$

$$\Rightarrow \quad c = -2$$

9. **(a)** As m and n are zeroes of polynomial $x^2 - 4x + 1$,

$$\therefore \quad m + n = \frac{-(-4)}{1} = 4$$

$$\text{Also,} \quad mn = 1$$

Therefore,

$$\frac{1}{m} + \frac{1}{n} - mn = \frac{m+n}{mn} - mn$$

$$= \frac{4}{1} - 1 = 3$$

10. **(b)** $x^2 - 2x - 8 = x^2 - 4x + 2x - 8$

$$= x(x - 4) + 2(x - 4)$$

$$= (x - 4)(x + 2)$$

$$\text{Therefore,} \quad x = 4, -2.$$

11. **(d)** Explanation: Sum of zeroes $= \alpha + \beta = \sqrt{2}$

$$\text{Product of zeroes} = \alpha\beta = \frac{1}{3}$$

$\therefore$ If α and β are zeroes of any quadratic polynomial, then the polynomial can be written as

$$k[x^2 - (\alpha + \beta)x + \alpha\beta] = kx^2 - (\sqrt{2})x + \left(\frac{1}{3}\right)$$

$$= k\left[3x^2 - 3\sqrt{2}x + 1\right]$$

12. **(d)** Degree is the highest power of the variable in any polynomial. Therefore, degree of given polynomial is 9.

13. **(b)** Since one zero is –1, $\therefore$ P(–1) = 0

$$P(x) = x^3 + ax^2 + bx + c$$
$$P(-1) = (-1)^3 + a(-1)^2 + b(-1) + c$$
$$0 = -1 + a - b + c$$
$$c = 1 - a + b$$

Product of zeroes,

$$\alpha\beta\gamma = -\frac{\text{constant term}}{\text{coefficient of } x^3}$$

$$\Rightarrow \quad (-1)\beta\gamma = -\frac{c}{1}$$

$$\Rightarrow \quad \beta\gamma = b - a + 1 \ [\because c = 1 - a + b]$$

14. **(a)** If P(a) = 0 for any polynomial, then 'a' is the zero of that polynomial.

15. **(a)** The number of zeroes is equal to the number of times graph touches or intersect x-axis.

16. **(d)** Maximum number of zeroes of a polynomial = Degree of the polynomial

17. **(b)** Given,

$$x^2 - 8 = 0$$
$$\Rightarrow \quad x^2 = 8$$
$$\Rightarrow \quad x = \pm\sqrt{8}$$
$$\Rightarrow \quad x = \pm 2\sqrt{2}$$

18. **(b)** Let $P(x) = x^2 - mx + 2$

As, (x − 2) is a factor of P(x), $\therefore$ P(2) = 0

$$\Rightarrow \quad 2^2 - m(2) + 2 = 0$$
$$\Rightarrow \quad 4 - 2m + 2 = 0$$
$$\Rightarrow \quad 6 = 2m$$
$$\Rightarrow \quad m = 3$$

19. **(d)** Let $P(x) = (x + 1)^5 + (2x + k)^3$

As (x + 2) is a factor of P(x), $\therefore$ P(–2) = 0

$$P(-2) = (-2 + 1)^5 + [(2(-2) + k)]^3 = 0$$
$$\Rightarrow (-1)^5 + [-4 + k]^3 = 0$$
$$\Rightarrow \quad -1 + [-4 + k]^3 = 0$$
$$\Rightarrow \quad (-4 + k)^3 = 1$$
$$\Rightarrow \quad -4 + k = 1$$
$$\Rightarrow \quad k = 5$$

20. **(c)** In the polynomial, $p(x) = ax^2 + bx + c$, if a = b = c = 0, then the expression becomes a zero polynomial.

Here, $p(x) = x^0$ or p(x) = 1 is a constant polynomial but not a zero polynomial as $c \neq 0$.

21. **(c)** Quartic polynomial is a polynomial in which highest power of the variable is 4.

Thus, degree of Quartic polynomial is 4.

22. **(a)** If a polynomial f(x) is divided by (x − a) then remainder will be equal to f(a)

$$f(x) = 2x^3 - 13x^2 + 17x + 12$$
$$f(-2) = 2(-2)^3 - 13(-2)^2 + 17(-2) + 12$$
$$= -16 - 52 - 34 + 12$$
$$= -90$$

23. **(c)** Let $P(x) = ax^2 + bx + c$

$$\therefore \quad P\left(\frac{-b}{a}\right) = a\left(\frac{-b}{a}\right)^2 + b\left(\frac{-b}{a}\right) + c$$
$$= \frac{a \times b^2}{a^2} + \frac{b \times -b}{a} + c$$
$$= \frac{b^2}{a} - \frac{b^2}{a} + c$$
$$= c$$

24. **(b)** Given, $p(x) = x^2 - 2\sqrt{2}x + 1$

When p(x) is divided by $\left(x - 2\sqrt{2}\right)$, the remainder will be $p(2\sqrt{2})$

$$\therefore \quad p(2\sqrt{2}) = (2\sqrt{2})^2 - 2\sqrt{2}(2\sqrt{2}) + 1$$
$$= 8 - 8 + 1$$
$$= 1$$

Thus, the remainder is 1.

25. **(a)** $3x^3(4x^2) + 7x^5 = 12x^5 + 7x^5 = 19x^5$

To find the value of the expression for x=3, we have to substitute x = 3.

So, $\quad 19(3)^5 = 19(243)$

$$= 4617$$

26. **(b)** $x^3 - 3x - 2 = 0$

Using hit and trial, x = −1 is one solution of the equation, hence (x + 1) is a factor of the equation

On dividing this polynomial by x + 1 and then factorising the quotient, we shall find the other two roots as follows:

$$\begin{array}{r|l}
 & x^2 - x - 2 \\ \hline
x + 1 & x^3 + 0x^2 - 3x - 2 \\
 & x^3 + x^2 \\
 & - \quad - \\ \hline
 & \quad -x^2 - 3x - 2 \\
 & \quad -x^2 - x \\
 & \quad + \quad + \\ \hline
 & \qquad -2x - 2 \\
 & \qquad -2x - 2 \\
 & \qquad + \quad + \\ \hline
 & \qquad\quad 0
\end{array}$$

Now factorising the quotient

$$x^2 - x - 2 = 0$$

$\Rightarrow \quad x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x + 2) - 1\ (x + 2) = 0$

$\Rightarrow \quad (x - 1)\ (x + 2) = 0$

Factors

$(x + 1)(x - 1)(x + 2) = 0$

Roots = −1, 1, −2

27. **(c)** Let

$$f(x) = x^2 + 3x + 2.$$

When f(x) is divided by (x – 1), the remainder will be f(1).

$$f(1) = (1)^2 + 3(1) + 2$$
$$= 1 + 3 + 2$$
$$= 6$$

Thus, the remainder obtained is 6.

28. **(b)** Do it yourself.

29. **(b)** Let $p(x) = ax^3 + 3x^2 - 3$ and $f(x) = 2x^3 - 5x + a$

According to factor theorem. If p(x) is divided by (x – 4), then R_1 will be

$$p(4) = ax^3 + 3x^2 - 3$$
$$= (4)^3\ a + 3(4)^2 - 3$$
$$= 64a + 48 - 3$$
$$= 64a + 45 \text{ ------} R_1$$

Similarly if f(x) is divided by (x – 4), then R_2 will be

$$f(4) = 2x^3 - 5x + a$$
$$= 2(4)^3 - 5(4) + a$$
$$= 128 - 20 + a$$
$$= 108 + a \text{ ------} R_2$$

Given, $\quad 2(R_1) - R_2 = 0$

$2(64a + 45) - 108 - a = 0$

$\Rightarrow 128a + 90 - 108 - a = 0$

$\Rightarrow \quad 127a = 18$

$\Rightarrow \quad a = \dfrac{18}{127}$

30. **(a)** $(l + m)^2 - 4lm = l^2 + 2\ lm + m^2 - 4\ lm$

$$= l^2 - 2\ lm + m^2$$
$$= (l - m)^2$$

31. **(c)** $(s - r)^2 - (s + r)^2 = s^2 - 2sr + r^2 - s^2 - 2sr - r^2$

$$= -4sr$$

32. **(a)** Given, 2x + 3 is a factor of the polynomial $p(x)= 2x^3 + 9x^2 - x - b$

Therefore, by factor theorem $p\left(-\dfrac{3}{2}\right) = 0$

$$\Rightarrow 2\left(-\frac{3}{2}\right)^3 + 9\left(-\frac{3}{2}\right)^2 - \left(-\frac{3}{2}\right) - b = 0$$

$$\Rightarrow -\frac{27}{4} + \frac{81}{4} + \frac{3}{2} - b = 0$$

$$\Rightarrow \frac{-27 + 81 + 6}{4} - b = 0 \Rightarrow b = \frac{60}{4} = 15$$

$\therefore\ b = 15$

33. **(c)** Let $p(x) = 2x^2 + kx + \sqrt{2}$

If (x – 1) is a factor of p(x), then p(1) = 0

$2(1)^2 + k(1) + \sqrt{2} = 0$

$\Rightarrow 2 \times 1 + k + \sqrt{2} = 0$

$\Rightarrow \quad 2 + k + \sqrt{2} = 0$

$\Rightarrow \quad k = -2 - \sqrt{2}$

$\Rightarrow \quad k = -(2 + \sqrt{2}\,)$

34. **(a)** $3x(4x - 5) + 3 = 3x \times 4x - 3x \times 5 + 3$

$$= 12x^2 - 15x + 3$$

$$f\left(\frac{1}{2}\right) = 3\times\frac{1}{2}\left(4\times\frac{1}{2} - 5\right) + 3$$

$$= \frac{3}{2}\times -3 + 3 = -\frac{9}{2} + 3 = -\frac{3}{2}$$

35. **(b)** Given, $p(x) = x^3 - ax^2 + 6x - a$

When p(x) is divided by x – a, the value of f(a) will be the remainder

$\therefore \quad p(a) = a^3 - a^3 + 6a - a$

$\quad p(a) = 5a$

Thus, the required remainder is 5a

Level-II

1. **(c)** In a quadratic equation, $ax^2 + bx + c$, if a and c are of opposite sign then the roots of the equation will always be of opposite sign.

Hence, both the zeroes of given polynomial $x^2 + 99x - 127$ are of opposite signs.

2. **(b)** Let $P(x) = ax^3 + bx^2 + cx + d$

Since one of the zero of the cubic polynomial is zero, therefore P(0) = 0

$\Rightarrow \quad P(x) = ax^3 + bx^2 + cx + d$

$\Rightarrow \quad P(0) = a(0)^3 + b(0)^2 + c(0) + d$

$\Rightarrow \quad d = 0$

Now polynomial reduces to $ax^3 + bx^2 + cx$

Let the zeroes be α, β and γ and α = 0 (given), then

$$\alpha \times \beta + \beta \times \gamma + \gamma \times \alpha = \frac{\text{coefficent of x}}{\text{coefficent of } x^3}$$

$$\Rightarrow \qquad \beta\gamma = \frac{c}{a}$$

So the product of other two zeroes is $\frac{c}{a}$.

3. (a) If one of the zeroes (say α) of a quadratic polynomial $x^2 + ax + b$ is the negative of the other, then sum of zeroes is:

$$\alpha + (-\alpha) = -a$$

$\Rightarrow$ a = 0, which means polynomial has no linear term.

Product of zeroes is:

$$\alpha(-\alpha) = b$$

$$\Rightarrow \qquad b = -\alpha^2$$

b has to be negative, which means constant term is negative.

Therefore polynomial has no linear term and the constant term is negative.

4. (a) Let, $f(x) = x^2 - 6x + p$

If α, β are zeroes, then

$$(\alpha + \beta) = 6 \qquad ...(1)$$
$$\alpha\beta = p \qquad ...(2)$$
$$3\alpha + 2\beta = 20 \qquad \text{(given)}$$
$$\alpha + 2\alpha + 2\beta = 20$$
$$\alpha + 2(\alpha + \beta) = 20$$
$$\alpha + 2 \times 6 = 20$$
$$\alpha = 8$$

As α is one of the zeroes of f(x), f(α) = 0

$$\therefore \qquad f(8) = 8^2 - 6(8) + p$$
$$64 - 48 + p = 0$$
$$\Rightarrow \qquad p = -16$$

5. (a) $p(x) = x^4 - x^3 + 2x^2 - px + 4$

Since x − 2 is a factor of p(x), p(2) = 0

So, $p(2) = 2^4 - 2^3 + 2(2)^2 - p(2) + 4 = 0$

$$\Rightarrow \quad 16 - 8 + 8 - 2p + 4 = 0$$
$$\Rightarrow \qquad 2p = 20$$
$$\Rightarrow \qquad p = 10$$

6. (c) As 2 and 3 are the zeroes of the quadratic polynomial, we know

$$\text{Sum of zeroes} = 2 + 3$$
$$= -a$$
$$\Rightarrow \qquad a = -5$$

Similarly,

$$\text{product of zeroes} = b$$
$$= 6$$

$$\therefore \qquad a = -5,$$
$$b = 6$$

So, $\qquad a + b = 1$

7. (d)

$$f(x) = 2x^2 - x + 1$$
$$\therefore \qquad f(1) = 2 \times 1^2 - 1 + 1$$
$$= 2$$
$$g(x) = x^3 - 3x + 1$$
$$\therefore \qquad g(-1) = (-1)^3 - 3 \times (-1) + 1$$
$$= 3$$
$$\therefore \qquad f(1) + g(-1) = 2 + 3$$
$$= 5$$

8. (d) $\qquad f(x) = 4x^2 + 3x - 7$

So, $\qquad f(1) = 4(1)^2 + 3(1) - 7$

$$f(1) = 4 + 3 - 7$$
$$= 0$$

9. (c) Given,

$$f(x) = x^3 - 6x^2 + ax + b$$

Since it is exactly divisible by (x – 3), f(3) = 0

$$\Rightarrow \qquad f(3) = 3^3 - 6 \times 3^2 + 3a + b$$
$$= 0$$
$$\Rightarrow \qquad 3a + b = 27 \qquad ...(1)$$

And since it leaves a remainder of −55 when divided by (x + 2)

i.e., $\qquad f(-2) = -55$

$$\Rightarrow \qquad (-2)^3 - 6 \times (-2)^2 - 2a + b = -55$$
$$\Rightarrow \qquad -2a + b = -23 \qquad ...(2)$$

Solving equations (1), (2), we get a = 10, b = −3

Hence, the value of a + b is 7.

10. (c) $\qquad P(x) = x^2 + 5x + 6$

Since x + λ is a factor of p(x), by factor theorem

$$P(-\lambda) = 0$$
$$\lambda^2 - 5\lambda + 6 = 0$$
$$(\lambda - 3)(\lambda - 2) = 0$$
$$\Rightarrow \qquad \lambda = 2, 3$$

11. (d) $x^4 + \frac{1}{x^4} = 119$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 119$$

$$\Rightarrow \qquad \left(x^2 + \frac{1}{x^2}\right)^2 = 121$$

$$\Rightarrow \qquad \left(x^2 + \frac{1}{x^2}\right) = 11$$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 + 2 = 11$$

$$\Rightarrow \quad \left(x-\frac{1}{x}\right)^2 = 9$$

$$\Rightarrow \quad x-\frac{1}{x} = 3$$

Now, $\quad x^3-\frac{1}{x^3} = \left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x}.x+\frac{1}{x^2}\right)$

$= 3(11 + 1) = 36$

12. **(a)** When $6x^9 + 3x^{16} - p$ is divisible by $x + 1$, the remainder is 20.

Let $\quad f(x) = 6x^9 + 3x^{16} - p$

$\Rightarrow \quad f(-1) = 20$

$\Rightarrow \quad 6(-1)^9 + 3(-1)^{16} - p = 20$

$\Rightarrow \quad 6(-1) + 3\ (1) - p = 20$

$\Rightarrow \quad -p = 23$

$\Rightarrow \quad p = -23$

13. **(c)** Let $f(x) = ax^2 + bx + c$ be the quadratic polynomial which is divisible by $(2x - 1)$ and $(x + 3)$.

$\therefore\ ax^2 + bx + c = k(2x - 1)(x + 3)$

f(x) leaves remainder 12 on division by $(x - 1)$

$\Rightarrow \quad f(1) = 12$

$\Rightarrow k(2 - 1)(1 + 3) = 12$

$\Rightarrow \quad k = 3$

$\therefore \quad f(x) = 3(2x - 1)(x + 3)$

$= 6x^2 + 15x - 9$

14. **(c)** Given, $f(x) = x^3 - 6x^2 - 45x + 162$

Roots are $a - b, -a, a + b$

$$\text{sum of zeroes} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^3}$$

$$\Rightarrow \quad a - b - a + a + b = -\frac{(-6)}{1}$$

$$\Rightarrow \quad a = 6$$

15. **(b)** Here, $p(x) = ax^3 + bx^2 + cx + d$ and factor is $ax + b$

$$\therefore \quad p\left(-\frac{b}{a}\right) = a\left(-\frac{b}{a}\right)^3+b\left(-\frac{b}{a}\right)^2+c\left(-\frac{b}{a}\right)+d$$

$$p\left(-\frac{b}{a}\right) = -\frac{b^3}{a^2}+\frac{b^3}{a^2}-\frac{bc}{a}+d$$

$$p\left(-\frac{b}{a}\right) = -\frac{bc}{a}+d$$

$$p\left(-\frac{b}{a}\right) = \frac{1}{a}(ad-bc)$$

$\therefore$ The remainder is $\frac{1}{a}(ad-bc)$

Assertion Reason Type

1. **(c)** Assertion (A) is true but reason (R) is false.

As the number zero of polynomial f(x) is the number of points at which f(x) cuts (intersects) the x -axis and number of zero in the given figure is 5. So A is correct but R is incorrect.

2. **(a)** Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

As irrational roots/zeroes always occurs in pairs therefore, if one zero is $(2-\sqrt{3})$ then other will be $(2+\sqrt{3})$.

3. **(a)** Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Sum of zeroes $= -\frac{1}{4}$ and

product of zeroes $= \frac{1}{4}$

Quadratic polynomial be $x^2-\left(-\frac{1}{4}\right)x+\frac{1}{4}$

$$\Rightarrow x^2+\frac{1}{4}x+\frac{1}{4} \Rightarrow \frac{1}{2}\left(2x^2+\frac{x}{2}+\frac{1}{2}\right)$$

Quadratic polynomial be $4x^2 + x + 1$. So, both A and R are correct and R explains A.

4. **(c)** Assertion is true but reason (R) is false.

Clearly, Reason is false. [Standard Result]

Sum of zeroes $= \alpha + \beta + \gamma$

$= -(-2)$

$= 2$

$0 + \gamma = 2$

$\gamma = 2 \quad$ ...(i)

Product of zeroes $= \alpha\beta\gamma$

$= -(-r)$

$= r$

$\alpha\beta(2) = r \quad$ (using (i))

$\alpha\beta = \frac{r}{2} \quad$ ...(ii)

Sum of product of zeroes $= \alpha\beta + \beta\gamma + \gamma\alpha = q$

$\frac{r}{2}+\gamma(\alpha+\beta) = q \quad$ (using (ii))

$\frac{r}{2}+\gamma(0) = q$

$r = 2q$ (Assertion is true).

5. **(b)** Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Reason is true.

Let α, $\frac{1}{\alpha}$ be the zeroes of p(x), then

$$\alpha \times \frac{1}{\alpha} = \frac{3k}{k+4}$$

$$\Rightarrow \quad 1 = \frac{3k}{k+4}$$

$$\Rightarrow \quad k + 4 = 3k$$

$$\Rightarrow \quad k = 2$$

Assertion is true but, Reason is not correct explanation for Assertion.

Case Based Type Questions

Case-Based-I

1. **(d)** If α, β and γ are the zeroes of the polynomial $ax^3 + bx^2 + cx + d$

Then

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

Using identity,

$$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$\alpha^2 + \beta^2 + \gamma^2 = \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$$

$$\alpha^2 + \beta^2 + \gamma^2 = \frac{b^2}{a^2} - \frac{2c}{a}$$

$$\alpha^2 + \beta^2 + \gamma^2 = \frac{b^2 - 2ac}{a^2}$$

Therefore, the value of $\alpha^2 + \beta^2 + \gamma^2 = \frac{b^2 - 2ac}{a^2}$

2. **(c)** $ax^3 + bx^2 + cx + d$

since α, β, γ are the roots of the equation,

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2}{\alpha^2\beta^2\gamma^2}$$

$$= \frac{(\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2(\alpha\beta^2\gamma + \alpha\beta\gamma^2 + \alpha^2\beta\gamma)}{\alpha^2\beta^2\gamma^2}$$

[Using $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow \quad a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$]

$$= \frac{(\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)}{(\alpha\beta\gamma)^2}$$

$$= \frac{\left[\frac{c}{a}\right]^2 - 2\left[-\frac{d}{a}\right]\left[-\frac{b}{a}\right]}{\left[-\frac{d}{a}\right]^2}$$

$$= \frac{\frac{c^2}{a^2} - \frac{2bd}{a^2}}{\frac{d^2}{a^2}}$$

$$= \frac{c^2 - 2bd}{d^2}$$

3. **(b)** α, β and γ are the zeroes of a polynomial

$$P(x) = ax^3 + bx^2 + cx + d$$

$$\Rightarrow \quad \alpha \,.\, \beta \,.\, \gamma = -\frac{d}{a} \qquad ...(i)$$

$$\text{and} \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \qquad ...(ii)$$

Now,

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}$$

$$= \frac{\frac{c}{a}}{-\frac{d}{a}} = -\frac{c}{d}$$

Case-Based-II

1. **(c)** The graph of parabola opens downward when $a < 0$.

2. **(a)** Sum of zeroes $= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-8}{6} = \frac{-4}{3}$

Product of zeroes $= \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{6}$

The required polynomial can be written as x^2 – (sum of zeroes) x + product of zeroes

$$\therefore \quad x^2 - \left(\frac{-4}{3}\right)x + \frac{1}{6} \text{ i.e., } 6x^2 + 8x + 1$$

3. **(c)** As the given graph touches or intersects the x-axis at 5 places. So, there are total 5 zeroes.

Multi Correct MCQs

1. **(a, d)**

We are given

$2x^2 + xy - 3y^2 + x + ay - 10$

$= (2x + 3y + b)(x - y - 2)$

$= 2x(x - y - 2) + 3y(x - y - 2) + b(x - y - 2)$

$= 2x^2 - 2xy - 4x + 3xy - 3y^2 - 6y + bx - by - 2b$

$\therefore 2x^2 + xy - 3y + x + 2y - 10$

$= 2x^2 + xy - 3y^2 + (b - 4)x + (- b - 6)y - 2b$

Equating both sides.

$-2ab = -10$

$\Rightarrow b = 5$

And $a = - b - 6 = -5 - 6 = -11$

2. **(b, c)**

Given, $a - b = 3$ and $a^3 - b^3 = 117$

As, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

On putting the value of $a - b$ and $a^3 - b^3$, we get

$\therefore \quad a^2 + ab + b^2 = 39$

$\Rightarrow a^2 + ab + b^2 - 2ab + 2ab = 39$

$\Rightarrow a^2 - 2ab + b^2 + 3ab = 39$

$\Rightarrow \quad (a - b)^2 + 3ab = 39$

$\Rightarrow \quad ab = 10$

$$\begin{aligned}\therefore \quad (a + b)^2 &= a^2 + b^2 + 2ab \\ &= a^2 + b^2 - 2ab + 4ab \\ &= (a - b)^2 + 4ab \\ &= 9 + 40 \\ &= 49\end{aligned}$$

$\therefore \quad (a + b) = 7$

or $\quad (a + b) = 21 \div \sqrt[3]{27}$

3. **(b, d)**

For a 3-digit number to be a prime, last digit has to be odd i.e., 1, 3, 5, 7 or 9.

Last digit can't be 1 as it doesn't satisfy the option of summing.

If we choose last digit as 3, possible numbers are 123 and 321 which are not a prime number

Last digit 5 is not a prime i.e., 235 and 325 are not prime numbers.

Similarly for last digit 9, numbers are 189 and 819 which are not prime.

$\therefore$ Only possibility is 7.

Thus numbers are 167, 617, 347, 437, 257 and 527 out of which 527 and 437 are not prime.

Hence, there are total 4 such prime numbers.

As we know that 4 is the smallest composite number. Hence option (d) is also correct.

4. **(a, c)**

Given $x^2 - 1 = (x + 1)(x - 1)$ is a factor of $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, then by using factor theorem.

$f(+1) = 0$

$\Rightarrow a + b + c + d + e = 0$ ...(i)

And,

$f(-1) = 0$

$\Rightarrow a - b + c - d + e = 0$ ...(i)

Adding, (i) and (ii), we get

$a + c + e = 0$

Subtracting (ii) from (i), we get

$b + d = 0$

Olympiad & NTSE Type

1. **(c)** Let the unknown polynomial be $p(x)$ and let $q(x)$ be quotient and $r(x) = ax + b$, the remainder

Dividend = Divisor × Quotient + Remainder

$\Rightarrow \quad p(x) = (x - 1)(x - 2)q(x) + ax + b$ ...(i)

By substituting $x = 1$ and $x = 2$, successively, in (i), and applying the remainder theorem we obtain the two equations,

$p(1) = a + b = 2$ ...(ii)

$p(2) = 2a + b = 1$ ...(iii)

Solving (ii) and (iii), we get $a = -1$, $b = 3$

By substituting a & b in $ax + b$ we get, $-x + 3$.

Hence, the remainder is $-x + 3$.

2. **(c)** Here, consider $a^2 - b^2 = x$, $b^2 - c^2 = y$, $c^2 - a^2 = z$

Now, $x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$\Rightarrow$ if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

In given question,

$x + y + z = a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$

$\Rightarrow \quad x^3 + y^3 + z^3 = 3xyz$

$\therefore$ Factors of $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

$= 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

3. **(d)** Do it yourself.

4. (b) Let $p(x) = x^2 + x - 12$

Since $(x - k)$ is a factor of $p(x)$ then using factor theorem

$$P(k) = 0$$

$$\Rightarrow \quad k^2 + k - 12 = 0$$

$$\Rightarrow \quad (k + 4)(k - 3) = 0$$

$$\Rightarrow \quad k = -4$$

$$\text{or} \quad k = 3 \quad \text{...(i)}$$

Let $q(x) = 2x^2 - kx - 9$,

Since $(x - k)$ is a factor of $q(x)$, then using Factor theorem $q(k) = 0$

$$\Rightarrow \quad 2k^2 - k^2 - 9 = 0$$

$$\Rightarrow \quad k^2 = 9$$

$$\Rightarrow \quad k = \pm 3 \quad \text{...(ii)}$$

From equation (i) and equation (ii), we get

$k = 3$ as the only common solution, therefore the value of k will be 3.

5. (c) $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$

$\therefore$ Remainder $= 0$

$$\begin{array}{r|l} & x^2 + x + 7 \\ \hline x^2 + 1 & x^4 + x^3 + 8x^2 + ax + b \\ & \underline{\underset{-}{x^4} \qquad \underset{-}{+} x^2} \\ & \quad x^3 + 7x^2 + ax + b \\ & \quad \underline{\underset{-}{x^3} \qquad \underset{-}{+} x} \\ & \qquad 7x^2 + x(a - 1) + b \\ & \qquad \underline{\underset{-}{7x^2} \qquad \underset{-}{+} 7} \\ & \qquad x(a - 1) + b - 7 \end{array}$$

$$x(a - 1) + (b - 7) = 0$$

$$\Rightarrow \quad a - 1 = 0, \ b - 7 = 0$$

$$\Rightarrow \quad a = 1, \ b = 7$$

So, $a + b = 8$

6. (d) Let $\alpha = a - d$, $\beta = a$, $\gamma = a + d$ be the zeroes of the polynomial $f(x) = x^3 - 12x^2 + 39x - 28$

$$\text{Sum of zeroes} = -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$$

$$\alpha + \beta + \gamma = -\left(\frac{-12}{1}\right) = 12$$

$a - d + a + a + d = 12$

$3a = 12$

$a = 4 \quad \text{...(i)}$

And,

$$\text{Product of zeroes} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$$

$$\alpha\beta\gamma = \frac{-(-28)}{1}$$

$\Rightarrow (a - d)(a)(a + d) = 28$

$\Rightarrow (4 - d)\, 4\, (4 + d) = 28$ [from eq. (i) $a = 4$]

$\Rightarrow 4(16 - d^2) = 28$

$\Rightarrow 16 - d^2 = 7$

$\Rightarrow d = \pm 3$

Now $\alpha = a - d$

$= 4 - 3$ or $4 + 3$

$= 1$ or 7

$\beta = a = 4$

$\gamma = a + d = 4 + 3$ or $4 - 3$

$= 7$ or 1

Hence zeroes of the polynomial are 1, 4 and 7.

7. (c) Given, $x^2 - 2x - 3$ is a factor of $x^3 - 3x^2 + ax - b$

On factorising, $x^2 - 2x - 3$, we have

$\Rightarrow x^2 - 3x + x - 3 = x(x - 3) + 1(x - 3)$

$= (x + 1)(x + 3)$

$(x - 3)$ and $(x + 1)$ are factors of $f(x)$

$$f(3) = 3^3 - 3(3^2) + a(3) - b$$

$$\text{or,} \quad 3a - b = 0 \quad \text{...(i)}$$

$$f(-1) = (-1)^3 - 3(-1)^2 + a(-1) - b = 0$$

$$-1 - 3 - a - b = 0$$

$$\text{or,} \quad a + b = -4 \quad \text{...(ii)}$$

By solving eq^n (i) and (ii), we get

$a = -1$ and $b = -3$

8. (c) $f(1) = 5$ [using factor theorem]

$\Rightarrow (1)^4 - 2(1)^3 + 3(1)^2 - a(1) + b = 5$

$\Rightarrow 1 - 2 + 3 - a + b = 5$

$\Rightarrow 2 - a + b = 5$

$\Rightarrow -a + b = 3 \quad \text{...(i)}$

And $f(-1) = 19$ [using factor theorem]

$\Rightarrow (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 19$

$\Rightarrow 1 + 2 + 3 + a + b = 19$

$\Rightarrow a + b = 13 \quad \text{...(ii)}$

On adding equation (i) and (ii)

$2b = 16$

$\Rightarrow b = 8$

On putting the value of b in equation (i)

$-a + 8 = 3$

$\Rightarrow -a = -5$

$\Rightarrow a = 5$

Hence $a = 5$ and $b = 8$

By putting the value of a and b in $f(x)$

$= x^4 - 2x^3 + 3x^2 - ax + b$ we get

$f(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$

When f(x) is divided by (x – 3), then using factor theorem.

Remainder = f(3)

$= (3)^4 - 2(3)^3 + 3(3)^2 - 5(3) + 8$

$= 81 - 2 \times 27 + 3 \times 9 - 15 + 8$

$= 81 - 54 + 27 - 15 + 8$

$= 47$

Hence remainder is 47 when $f(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$ is divided by (x – 3).

9. (d)

Let the cubic polynomial f(x)= k(x – 1) (x – 2) (x – 3) + remainder

As $f(1) = 1 = 1^2$

$f(2) = 4 = 2^2$

$f(3) = 9 = 3^2$

$\therefore$ remainder for $f(x) = x^2$

$\Rightarrow$ required polynomial = $k(x - 1)(x - 2)(x - 3) + x^2$

Now, $f(4) = k(4 - 1)(4 - 2)(4 - 3) + 16$

$\Rightarrow \quad 20 = k \times 3 \times 2 \times 1 + 16$

$\Rightarrow \quad k = \frac{4}{6} = \frac{2}{3}$

$\therefore \quad f(7) = \frac{2}{3}(7-1)(7-2)(7-3) + 49 = 129$

$= \frac{2}{3} \times 6 \times 5 \times 4 + 49$

$= 80 + 49$

$= 129$

10. (a)

Let $f(x) = 3x^4 - 7x^3 - 7x^2 + 21x - 6$

As one of its zeroes are $-\sqrt{3}$ and $\sqrt{3}$

$\therefore$ Using factor theorem $(x - \sqrt{3})\ (x + \sqrt{3}) = x^2 - 3$

will be the factor of $f(x) = 3x^4 - 7x^3 - 7x^2 + 21x - 6$

Now, applying long division method

$$\begin{array}{r|l}
 & 3x^2 - 7x + 2 \\
x^2 - 3 & 3x^4 - 7x^3 - 7x^2 + 21x - 6 \\
 & \underline{\overset{-}{}3x^4 \quad \overset{+}{-}9x^2} \\
 & -7x^3 + 2x^2 + 21x - 6 \\
 & \underline{\overset{+}{-}7x^3 \quad \overset{-}{+}21x} \\
 & 2x^2 \quad -6 \\
 & \underline{\overset{-}{}2x^2 \quad \overset{+}{-}6} \\
 & 0
\end{array}$$

$\therefore f(x) = (x^2 - 3)(3x^2 - 7x + 2)$

$= (x + \sqrt{3})(x - \sqrt{3})(3x^2 - 6x - x + 2)$

$= (x + \sqrt{3})(x - \sqrt{3})[3x(x - 2) - 1(x - 2)]$

$= (x + \sqrt{3})(x - \sqrt{3})(3x - 1)(x - 2)$

$\therefore$ Other roots are $\sqrt{3}$, 2, and $\frac{1}{3}$.

11. (c)

Given, $x - \frac{1}{x} = 2$

On squaring both sides

$x^2 + \frac{1}{x^2} - 2 = 4$ $\quad [(a-b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow x^2 + \frac{1}{x^2} = 6$

Again

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 36$

$\Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 \times x^2 \times \frac{1}{x^2} = 36$ $\quad [(a + b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 36$

$\Rightarrow x^4 + \frac{1}{x^4} = 36 - 2$

$\therefore x^4 + \frac{1}{x^4} = 34$

12. (d) As we know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

$\therefore (a)^3 + (2b)^3 + (3c)^3 - 3(a)(2b)(ac)$

$= (a + 2b + 3c)[(a)^2 + (2b)^2 + (3c)^2 - (a)(2b) - (2b)(3c) - (a)(3c)]$

$\Rightarrow a^3 + 8b^3 + 27c^3 - 18abc = 0$

[Given a + 2b + 3c = 0]

$\Rightarrow a^3 + 8b^3 + 27c^3 = 18abc$

Hence the value of $a^3 + 8b^3 + 27c^3$ is equal to 18 abc.

13. (c) Given, $x^4 + px^3 + 2x^2 - 3x + q$ is divisible by $x^2 - 1 = (x - 1)(x + 1)$ $\quad [\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta)]$

$\therefore$ (x – 1) and (x + 1) are the factors of given polynomial, i.e., f(1) = f(–1) = 0

$f(1) = 1 + p + 2 - 3 + q = 0$

$\Rightarrow \quad p + q = 0 \quad$ ...(i)

$f(-1) = 1 - p + 2 + 3 + q = 0$

$\Rightarrow \quad -p + q = -6 \quad$ ...(ii)

By solving eq^n (i) and (ii), we get

p = 3 and q = –3

14. **(d)** We know that the degree of remainder is always less than the degree of divisor. Hence, the remainder will be a quadratic.

Let remainder $R(x) = ax^2 + bx + c$

Given, $f(x) = x + x^9 + x^{25} + x^{49} + x^{81}$

$f(x) = Q(x).(x^3 - x) + R(x)$

According to the question,

$f(x) = Q(x).x(x + 1)(x - 1) + R(x)$

So, $f(0) = R(0)$

$= c$

$= 0$

$f(1) = R(1)$

$= a + b + c$

$5 = a + b + c$

$\Rightarrow \quad a + b = 5 \quad$ ...(i)

$f(-1) = R(-1)$

$-5 \Rightarrow a - b + c$

$\Rightarrow \quad a - b = -5 \quad$ ...(ii)

By solving eq^n (i) and (ii), we get

$a = 0,\ b = 5$

$\therefore \quad R(x) = ax^2 + bx + c$

$= ox^2 + 5x + o = 5x$

15. **(b)** Given, $a = x + \dfrac{1}{x}$ and $b = x - \dfrac{1}{x}$

Squaring both sides and simplifying we get

$\therefore a^2 = x^2 + \dfrac{1}{x^2} + 2$ and $b^2 = x^2 + \dfrac{1}{x^2} - 2$

Now, $a^2 + 2ab - 3b^2$

$= x^2 + \dfrac{1}{x^2} + 2 + 2\left(x + \dfrac{1}{x}\right)\left(x - \dfrac{1}{x}\right) - 3\left(x^2 + \dfrac{1}{x^2} - 2\right)$

$[\because (a + b)(a - b) = a^2 - b^2]$

$= x^2 + \dfrac{1}{x^2} + 2 + 2x^2 - \dfrac{2}{x^2} - 3x^2 - \dfrac{3}{x^2} + 6$

$= \dfrac{-4}{x^2} + 8 = \dfrac{4}{x^2}\left(2x^2 - 1\right)$