07

Work, Energy and Power

IN THIS CHAPTER ....

The terms work, energy and power are frequently used in everyday language. In physics, however the word *work* converse a definite and precise meaning. *Energy* is our capacity to do work and it is related to work. In other words, an object with lots of energy can do lots of work. The term *power* is usually associated with the time in which the work is done, *i.e.* the rate at which work is done.

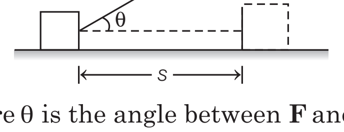
Work

The scientific meaning of work is transfer of energy by mechanical means. Work is said to be done by a force, when a body a displaced actually through some distance in the direction of applied force. The SI unit of work is joule (J) and in CGS, it is erg.

$$1 \text{ joule (J)} = 10^7 \text{ erg}$$

Work Done by a Constant Force

The work done by a constant force F on a particle which undergoes displacement s is given by



$W = \mathbf{F} \cdot \mathbf{s} = F\,s\cos\theta$, where θ is the angle between $\mathbf{F}$ and $\mathbf{s}$.

Work done is a scalar or dot product of $\mathbf{F}$ and $\mathbf{s}$.

Work can be *positive*, *negative* or even *zero*, also depending on the angle (θ) between the force vector $\mathbf{F}$ and displacement vector $\mathbf{s}$. Work done by a force is zero when $\theta = 90^\circ$, it is positive when $\theta < 90^\circ$ and negative when $\theta > 90^\circ$.

For example, when a person lifts a body, the work done by the lifting force is positive (as $\theta = 0^\circ$) but work done by the force of gravity is negative (as $\theta = 180^\circ$). Similarly, work done by centripetal force is always zero (as $\theta = 90^\circ$).

Work Done by a Variable Force

The force is said to be variable force, if it changes its direction or magnitude or both. The work done by a variable force can be calculated as

$$W = \int_{r_1}^{r_2} \mathbf{F} \cdot d\mathbf{r}$$

where integration is performed along the path of particle and $d\mathbf{r}$ is the position vector of the particle.

If the particle moves from $\mathbf{r}_1(x_1, y_1, z_1)$ to $\mathbf{r}_2(x_2, y_2, z_2)$, then the work done by force $\mathbf{F}$ is given by

$$W = \int dW = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$$

where, F_x, F_y and F_z are the rectangular components of force in x, y and z-directions, respectively.

If the motion is one dimensional, $W = \int_{x_i}^{x_f} F_x dx$ in which F_x is the component of force along motion.

Example 1. *When a force is applied on a moving body, its motion is retarded, then the work done is*

(a) positive *(b) negative*
(c) zero *(d) positive and negative*

Sol. (*b*) The angle between the displacement and the applied retarded force is 180°. $\{\because \cos 180° = -1\}$

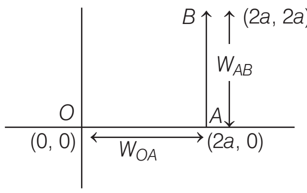
$\therefore$ Work done $= Fs \cos 180° = -Fs$
$= \text{negative}$

Example 2. *A force* $\mathbf{F} = -(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$ *acts on a particle moving in the XY-plane starting from the origin, the particle is taken along the positive X-axis to the point* $(2a, 0)$ *and then parallel to the Y-axis to the point* $(2a, 2a)$*. The total work done on the particle is*

(a) $-4a^2$ *(b)* $-2a^2$ *(c)* $4a^2$ *(d)* $2a^2$

Sol. (*a*) Here, $\mathbf{F} = -(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$

The given situation is shown in the figure. According to question, particle is taken from origin point O to A and then point A to B.



Hence, total work done,

$$W = W_{OA} + W_{AB} \quad \text{...(i)}$$

$\therefore$ $W_{OA} = \int_0^{2a} \mathbf{F} \cdot \mathbf{dx}$ [only x-coordinate is varying]

$$= \int_0^{2a} -(0 \cdot \hat{\mathbf{i}} + x\hat{\mathbf{j}})\, dx\, \hat{\mathbf{i}}$$

$$= \int_0^{2a} x dx\, (\hat{\mathbf{j}} \cdot \hat{\mathbf{i}})$$

$W_{OA} = 0$ $[\because \hat{\mathbf{j}} \cdot \hat{\mathbf{i}}] = 0$

$\because$ $W_{AB} = \int_0^{2a} -(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})\,(dy\hat{\mathbf{j}})$

[x-coordinate is constant $\therefore x = 2a$]

$$= \int_0^{2a} -(y\hat{\mathbf{i}} + 2a\hat{\mathbf{j}})\, dy\, \hat{\mathbf{j}}$$

$$= \int_0^{2a} (-y\, \hat{\mathbf{i}} \cdot \hat{\mathbf{j}})\, dy - \int_0^{2a} 2a\, dy\, (\hat{\mathbf{j}} \cdot \hat{\mathbf{j}})$$

$\Rightarrow$ $W_{AB} = 0 - 2a\,[y]_0^{2a} = -2a \cdot 2a - 4a^2$

$\therefore$ $W = W_{OA} + W_{AB} = 0 - 4a^2 = -4a^2$

Example 3. *A force* $F = (2 + x)$ *acts on a particle in x-direction, where F is in newton and x in metre. The work done by this force during a displacement from* $x = 1.0$ *m to* $x = 2.0$ *m is*

(a) 2 J *(b) 3.5 J*
(c) 4.5 J *(d) None of these*

Sol. (*b*) The work done in small displacement from x to $x + dx$ is

$$dW = Fdx = (2 + x)\, dx$$

Hence, $W = \int_1^2 dW = \int_1^2 (2 + x)\, dx = \int_1^2 2\, dx + \int_1^2 x dx$

$$= \left[2x + \frac{x^2}{2}\right]_1^2 = 3.5 \text{ J}$$

Example 4. *A body of mass 2kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. The work done by applied force in 10 s will be equal to*

(a) 800 J *(b) 825 J* *(c) 882 J* *(c) 856 J*

Sol. (*c*) Here, $m = 2$ kg, $u = 0$, $F = 7$ N, $\mu = 0.1$, $t = 10$ s

Acceleration produced by applied force,

$$a_1 = \frac{F}{m} = \frac{7}{2} = 3.5 \text{ ms}^{-2}$$

Force of friction, $f = \mu R = \mu mg = 0.1 \times 2 \times 9.8 = 1.96$ *N*

Retardation produced by friction,

$$a_2 = \frac{-f}{m} = -\frac{1.96}{2} = -0.98 \text{ ms}^{-2}$$

Net acceleration with which body moves,

$$a = a_1 + a_2 = 3.5 - 0.98 = 2.52 \text{ ms}^{-2}$$

Distance moved by the body in 10 s,

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2.52 \times 10^2 = 126 \text{ m}$$

$\therefore$ Work done by the applied force, $W = F \times s = 7 \times 126 = 882$ J

Conservative and Non-Conservative Forces

Conservative Forces

A force is said to be conservative, if work done by or against the force in moving a body depends only on the initial and final positions of the body and not the nature of path followed between the final and initial positions.

For example, gravitational force, electrostatic force, etc., are conservative forces.

Non-Conservative Forces

A force is said to be non-conservative, if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions.

For example, force of friction and viscous force are non-conservative forces.

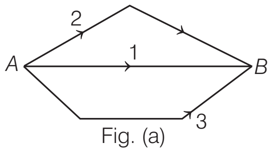


Fig. (a)

Fig. (a) Let W_1, W_2, W_3 denote the amounts of work done moving a body from A to B along three different paths 1, 2, 3, respectively. If the force is non-conservative, then $W_1 \neq W_2 \neq W_3$.

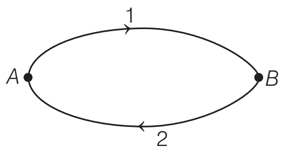


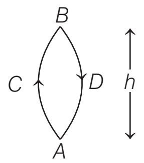
Fig. (b)

Fig. (b) Shows that a particle moving a closed path. $A \to 1 \to B \to 2 \to A$. If W_1 is work done in moving the particle from $A \to 1 \to B$ and W_2 is work done in moving the particle from $B \to 2 \to A$, then for a non-conservative force $|W_1| \neq |W_2|$.

$\therefore$ Net work done along the closed path, $A \to B \to A$ is not zero.

i.e. $$\oint \mathbf{F} \cdot \mathbf{ds} \neq 0$$

Example 5. *A particle is taken from point A to point B via the path ACB and then come back to point A via the path BDA. What is the work done by gravity on the body over this closed path, if the motion of the particle is in the vertical plane?*



(a) mgh (b) $-mgh$

(c) $\frac{1}{2}mgh$ (d) zero

Sol. (*d*) Here, displacement of the particle is **AB**, gravity is acting vertically downwards. The vertical component of **AB** is h upwards, hence

$$W_{(ACB)} = -mgh$$

For the path *BDA*, component of the displacement acting along vertical direction is h (downward).

In this case, $W_{(BDA)} = mgh$

Total work done, $W_{ACB} + W_{BDA} = 0$

Energy

Capacity or ability of a body to do work is called its energy. Like work, energy is a scalar quantity. The units of measurement of energy are same as the units of work. In SI, the unit of energy of joule (J) and in CGS, the unit of energy is erg.

There are so many types of energy. *e.g.* Kinetic, potential, electrostatic, magnetic, geothermal, elastic, solar, etc. In this chapter, we will discuss only mechanical energy.

Mechanical energy consists of kinetic energy and potential energy.

$$\text{ME} = \text{KE} + \text{PE}$$

Note KE is always positive but PE may be positive, negative or zero. Infact, when forces involved are repulsive, PE is positive and when forces involved are attractive, PE is negative.

Kinetic Energy

Kinetic energy (KE) is the capacity of a body to do work by virtue of its motion. The faster the object moves, the greater is the kinetic energy. When the object is stationary, its kinetic energy is zero.

An object of mass m moving with velocity $\mathbf{v}$, has a kinetic energy,

$$\text{KE} = \frac{1}{2}mv^2 \qquad \left[\frac{1}{2}mv^2 = \frac{1}{2}m(|\mathbf{v}|)^2\right]$$

- Kinetic energy is corelated with momentum as

$$K = \frac{p^2}{2m}$$

or $$p = \sqrt{2mK}$$

- Kinetic energy for a system of particle will be

$$K = \frac{1}{2}\sum_i m_i v_i^2$$

Example 6. *A 120 g mass has a velocity* $\mathbf{v} = (2\hat{\mathbf{i}} + 5\hat{\mathbf{j}})\, ms^{-1}$ *at a certain instant, KE of the body at that instant is*

(a) 3.0 J (b) 1.74 J

(c) 4.48 J (d) 5.84 J

Sol. (*b*) Here, $m = 120$ g = 0.12 kg,

$$\mathbf{v} = (2\hat{\mathbf{i}} + 5\hat{\mathbf{j}})\,\text{ms}^{-1}$$

$$v = |\mathbf{v}| = \sqrt{2^2 + 5^2} = \sqrt{29}\ \text{ms}^{-1}$$

$$\therefore \quad \text{KE} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.12 \times 29 = 1.74\ \text{J}$$

Example 7. *If the linear momentum of a body is increased by 50%, then the kinetic energy of that body increases by*

(a) 100% (b) 125%

(c) 225% (d) 25%

Sol. (*b*) Kinetic energy of the body,

$$K = \frac{p^2}{2m}$$

Since the mass remains constant, so $K \propto p^2$.

So, $$\frac{K_2}{K_1} = \frac{p_2^2}{p_1^2} = \left[\frac{150}{100}\right]^2 = \frac{9}{4}$$

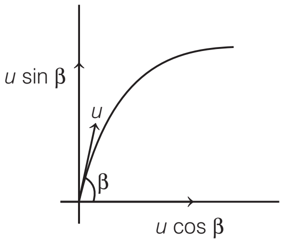
Thus, $$\left(\frac{K_2}{K_1} - 1\right) \times 100 = \left(\frac{9}{4} - 1\right) \times 100 = 125\%$$

Example 8. *A body is projected horizontally with a velocity of u ms^{-1} at an angle β with the horizontal. The kinetic energy at the highest point is (3/4)th of the initial kinetic energy. The value of β is*

(a) 30° (b) 45°
(c) 60° (d) 120°

Sol. (a) The kinetic energy at the highest point would be equal to $\frac{1}{2}m(u\cos\beta)^2$ as the vertical component of the velocity is zero.

$$\Rightarrow \quad \text{KE} = \frac{1}{2}mu^2\cos^2\beta$$



The initial kinetic energy (K) is the maximum kinetic energy.

So, $$\text{KE} = K\cos^2\beta$$

Given, $$K\cos^2\beta = \frac{3}{4}K$$

$$\Rightarrow \quad \cos\beta = \frac{\sqrt{3}}{2}$$

So, $$\beta = 30°$$

Example 9. *At time $t = 0$, particle starts moving along the X-axis. If its kinetic energy increases uniformly with time t, the net force acting on it must be proportional to*

(a) $\sqrt{t}$ (b) constant
(c) t (d) $1/\sqrt{t}$

Sol. (d) Given, $K \propto t$

$$\Rightarrow \quad \frac{dK}{dt} = \text{constant}$$

$$\Rightarrow \quad K \propto t$$

$$\frac{1}{2}mv^2 \propto t$$

$$\Rightarrow \quad v \propto \sqrt{t}$$

Also, $P = Fv = \frac{dK}{dt} = \text{constant}$

$$\Rightarrow \quad F \propto \frac{1}{v}$$

$$\Rightarrow \quad F \propto \frac{1}{\sqrt{t}}$$

Potential Energy

Potential energy (PE) of a body is the energy stored in a body or a system by virtue of its position of configuration in a field.

The change in potential energy (dU) of a system corresponding to a conservative force is

$$dU = -\mathbf{F}\cdot d\mathbf{s} = -dW \qquad \left(\because F = -\frac{dU}{ds}\right)$$

$$\int_{U_i}^{U_f} dU = -\int_{\mathbf{s}_1}^{\mathbf{s}_2} \mathbf{F}\cdot d\mathbf{s}$$

or $$U_f - U_i = -\int_{\mathbf{s}_1}^{\mathbf{s}_2} \mathbf{F}\cdot d\mathbf{s}$$

We generally choose the reference point at infinity and assume potential energy to be zero at that point.

i.e. $\mathbf{s_1} = \infty$ and $U_i = 0$

then $$U_f = -\int_{\infty}^{\mathbf{s}} \mathbf{F}\cdot d\mathbf{s} = -W$$

Thus, potential energy of a body is negative of work done by the conservative forces in bringing it from infinity to the present position.

Potential Energy of a Spring

Whenever an elastic body (say a spring) is either stretched or compressed, work is being done against the elastic spring force. The work done is $W = \frac{1}{2}kx^2$, where k is spring constant and x is the displacement.

Elastic potential energy, $U = \frac{1}{2}kx^2$

If spring is stretched from initial position x_1 to final position x_2, then

Work done = Increment in elastic potential energy

$$= \frac{1}{2}k(x_2^2 - x_1^2)$$

Example 10. *When a body is projected vertically up, its PE is twice its KE, when it is at a height h above the ground. At what height, will its KE be twice the KE?*

(a) $2h$ (b) $\frac{h}{3}$
(c) $\frac{h}{2}$ (d) $\frac{h}{4}$

Sol. (c) Total energy,

$$E_1 = \text{PE} + \text{KE}$$

$$= \text{PE} + \frac{1}{2}\text{PE} = \frac{3}{2}\text{PE} = \frac{3}{2}mgh$$

$$E_2 = \text{PE} + \text{KE} = \text{PE} + 2\text{PE} + 3\text{PE} = 3\,mgh'$$

As, $$E_2 = E_1$$

$$3mgh' = \frac{3}{2}mgh$$

$$\Rightarrow \quad h' = \frac{h}{2}$$

Example 11. *A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m, 1000 times. Assume that, the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted into mechanical energy with a 20% of efficiency rate. (Take, g = 9.8 ms^{-2})* **[JEE Main 2016]**

(a) 2.45×10^{-3} kg (b) 6.45×10^{-3} kg
(c) 9.89×10^{-3} kg (d) 12.89×10^{-3} kg

Sol. (d) Given, potential energy burnt by lifting weight

$$= mgh = 10 \times 9.8 \times 1 \times 1000 = 9.8 \times 10^4$$

If mass lost by a person be m, then energy dissipated

$$= m \times \frac{2}{10} \times 3.8 \times 10^7 \text{ J}$$

$$\Rightarrow \quad 9.8 \times 10^4 = m \times \frac{1}{5} \times 3.8 \times 10^7$$

$$\Rightarrow \quad m = \frac{5}{3.8} \times 10^{-3} \times 9.8 = 12.89 \times 10^{-3} \text{ kg}$$

Example 12. *A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then, the work required to stretch it further by another 5 cm is*

(a) 12.50 N-m (b) 18.75 N-m
(c) 25.00 N-m (d) 6.25 N-m

Sol. (b) Work done to stretch the spring by 5 cm from mean position,

$$W_1 = \frac{1}{2} kx_1^2$$

$$= \frac{1}{2} \times 5 \times 10^3 \times (5 \times 10^{-2})^2 = 6.25 \text{ J}$$

Work done to stretch the spring by 10 cm from mean position,

$$W_2 = \frac{1}{2} k(x_1 + x_2)^2$$

$$= \frac{1}{2} \times 5 \times 10^3 \, (5 \times 10^{-2} + 5 \times 10^{-2})^2 = 25 \text{ J}$$

Net work done to stretch the spring from 5 cm to 10 cm

$$= W_2 - W_1 = 25 - 6{\cdot}25$$
$$= 18.75 \text{ J}$$
$$= 18.75 \text{ N-m}$$

Work-Energy Theorem

Work done by all the forces (conservative or non-conservative, external or internal) acting on a particle or an object is equal to the change in its kinetic energy of the particle. Thus, we can write,

$$W = \Delta K = K_f - K_i = \frac{1}{2} m(v_f^2 - v_i^2)$$

We can also write, $K_f = K_i + W$

Note In a non-inertial frame, work done by all the forces (including the pseudo forces) = change in kinetic energy in non-inertial frame.

Example 13. *A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$, where $a = 5m^{-1/2}s^{-1}$. The work done by the net force during its displacement from x = 0 to x = 2 m is*

(a) 50 J (b) 45 J
(c) 25 J (c) None of these

Sol. (a) Here, $m = 0.5$ kg,

$$v = ax^{3/2}, \; a = 5 \text{ m}^{-1/2} \text{ s}^{-1}, W = ?$$

Initial velocity (at $x = 0$), $v_1 = a \times 0 = 0$

Final velocity (at $x = 2$), $v_2 = 5 \times 2^{3/2}$

$$\text{Work done} = \text{Increase in KE}$$
$$= \frac{1}{2} m(v_2^2 - v_1^2)$$
$$W = \frac{1}{2} \times 0.5 \, [(5 \times 2^{3/2})^2 - 0] = 50 \text{J}$$

Example 14. *A particle which is experiencing a force, is given by $\mathbf{F} = 3\hat{\mathbf{i}} - 12\hat{\mathbf{j}}$, undergoes a displacement of $\mathbf{d} = 4\hat{\mathbf{i}}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement?* **[JEE Main 2019]**

(a) 9 J (b) 15 J (c) 12 J (d) 10 J

Sol. (b) We know that, work done in displacing a particle at displacement d under force F is given by

$$\Delta W = \mathrm{F} \cdot \mathrm{d}$$

By substituting given values, we get

$$\Rightarrow \quad \Delta W = (3\hat{\mathrm{i}} - 12\hat{\mathrm{j}}) \cdot (4\hat{\mathrm{i}})$$

$$\Rightarrow \quad \Delta W = 12 \text{ J} \quad \text{... (i)}$$

Now, using work-energy theorem, we get

work done (ΔW) = change in kinetic energy (ΔK)

$$\text{or} \quad \Delta W = K_2 - K_1 \quad \text{... (ii)}$$

Comparing Eqs. (i) and (ii), we get

$$K_2 - K_1 = 12 \text{ J}$$

$$\text{or} \quad K_2 = K_1 + 12 \text{ J}$$

Given, initial kinetic energy, $K_1 = 3$ J

$\therefore$ Final kinetic energy, $K_2 = 3 \text{ J} + 12 \text{ J} = 15 \text{ J}$

Example 15. *A block of mass $m = 1$ kg moving on a horizontal surface with speed $v = 2$ ms^{-1} enters a rough patch ranging from x = 0.10 m to x = 2.01 m. The retarding force F_r on the block in this range inversely proportional to x over this range*

$$F_r = -\frac{k}{x} \quad \text{for } 0.1 < x < 2.01 \text{ m}$$
$$= 0 \quad \text{for } x < 0.1 \text{ m and } x > 2.01 \text{ m}$$

where, k = 0.5 J. The final kinetic energy of the block as it crosses this patch is

(a) 5 J (b) 50 J
(c) 0.5 J (d) 500 J

Sol. (c) If K_i and K_f are initial and final kinetic energies corresponding to x_i and x_f, then

$$K_f - K_i = \int_{x_i}^{x_f} F\,dx$$

$$K_f = K_i + \int_{0.1}^{2.01} \frac{(-k)}{x}\,dx$$

$$K_f = \frac{1}{2}mv_i^2 - k\ln(x)\Big|_{0.1}^{2.01}$$

$$= 2 - 0.5\ln(20.1)$$

$$= 2 - 1.5$$

$$= 0.5\text{ J}$$

Conservation of Mechanical Energy

The mechanical energy E of a system is the sum of its kinetic energy K and its potential energy U.

$$E = K + U$$

When the forces acting on the system are conservative in nature, the mechanical energy of the system remains constant.

$$K + U = \text{constant}$$

$$\Rightarrow \quad \Delta K + \Delta U = 0$$

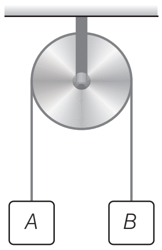
There are physical situations, where one or more non-conservative force act on the system but net work done by them is zero, then the mechanical energy of the system remains constant.

If $\Sigma W_{\text{net}} = 0$, then mechanical energy, E = constant.

Example 16. *In the arrangement shown in figure, string is light and inextensible and friction is absent everywhere.*

The speed of both the blocks after the block A has ascend a height of 1 m will be

(Given that, $m_A = 1\text{kg}$ and 2 kg)



(a) 2 ms^{-1} (b) 2.58 ms^{-1}
(c) 3 ms^{-1} (c) 3.58 ms^{-1}

Sol. (b) Since, there is no friction anywhere, so mechanical energy will be conserved.

Here, the speed of both the blocks are same. Let the speed be v ms^{-1}. Since block of 2 kg is coming down, hence the gravitational potential energy is decreasing while the gravitational potential energy of 1 kg block is increasing.

So, kinetic energy of both the blocks will increase.

Hence, $\quad m_Bgh = m_Agh + \frac{1}{2}m_Av^2 + \frac{1}{2}m_Bv^2$

or $\quad 2\times10\times1 = 1\times10\times1 + \frac{1}{2}\times1\times v^2 + \frac{1}{2}\times2v^2$

or $\quad 20 = 10 + 0.5v^2 + v^2$

or $\quad 1.5v^2 = 10$

$\therefore \quad v^2 = \frac{10}{1.5} = 6.67$

or $\quad v = 2.58\text{ ms}^{-1}$

Example 17. *The potential energy of a 1 kg particle free to move along the X-axis is given by*

$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)\text{ J}$$

The total mechanical energy of the particle is 2 J, then the maximum speed (in ms^{-1}) is **[AIEEE 2006]**

(a) $\frac{3}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 2

Sol. (a) Given, $V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$

For minimum value of V, $\frac{dV}{dx} = 0$

$$\Rightarrow \quad \frac{4x^3}{4} - \frac{2x}{2} = 0$$

$$\Rightarrow \quad x = 0,\quad x = \pm 1$$

So, $\quad V_{\min}(x = \pm 1) = \frac{1}{4} - \frac{1}{2} = \frac{-1}{4}\text{ J}$

Now, $\quad K_{\max} + V_{\min}$ = Total mechanical energy

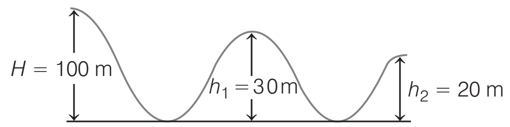
$$\Rightarrow \quad K_{\max} = \left(\frac{1}{4}\right) + 2 \quad\text{or}\quad K_{\max} = \frac{9}{4}$$

or $\quad \frac{mv^2}{2} = \frac{9}{4} \quad$ or $\quad v = \frac{3}{\sqrt{2}}\text{ ms}^{-1}$

Example 18. *A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is* **[AIEEE 2005]**

(a) 40 m/s (b) 20 m/s (c) 10 m/s (d) $10\sqrt{30}$ m/s

Sol. (a) According to conservation of energy, potential energy at height H is sum of kinetic energy and potential energy at h_2.



$$\therefore \quad mgH = \frac{1}{2}mv^2 + mgh_2$$

$$\Rightarrow \quad mg(H - h_2) = \frac{1}{2} mv^2$$

or $$v = \sqrt{2g(100 - 20)}$$

or $$v = \sqrt{2 \times 10 \times 80}$$

$$= 40 \text{ m/s}$$

Power

The time rate of doing work is called power. If an external force is applied to an object and if the work done by this force is ΔW in the time interval Δt, then the average power during this interval is defined as

$$P = \frac{\Delta W}{\Delta t}$$

The work done on the object contributes to increasing the energy of the object.

The instantaneous power is the limiting value of the **average power** as Δt approaches zero.

i.e. $$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

where we have represented the infinitesimal value of the work done by dW.

$$P = \frac{dW}{dt} = \mathbf{F} \frac{d\mathbf{s}}{dt} = \mathbf{F} \cdot \mathbf{v} \qquad \left[\text{as, } \frac{d\mathbf{s}}{dt} = \mathbf{v}\right]$$

- Power is equal to the scalar product of force and velocity.
- Power is a scalar with dimensions $[ML^2T^{-3}]$. The SI unit of power is Js^{-1} and is called **watt (W)**. Practical unit of power is **horse power (HP)**.

$$1 \text{ HP} = 746 \text{ W}$$

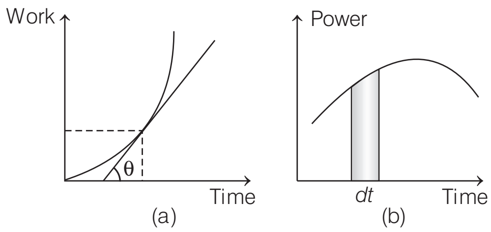
- Kilowatt-hour or watt-day are units of work or energy but not of power.

$$1 \text{ kWh} = 10^3 \times Js^{-1} \times (60 \times 60 \text{ s})$$
$$= 3.6 \times 10^6 \text{ J}$$

- The slope of work-time curve gives the instantaneous power as $P = \frac{dW}{dt} = \tan\theta$ [from Fig. (a)]

  while the area under P-t curve gives the work done.

Since, $P = \frac{dW}{dt}$, which means $W = \int P dt$ = area under P-t curve [as shown in Fig. (b)].



(a) (b)

Example 19. *An advertisement claims that a certain 1200 kg car can accelerate from rest to a speed of 25 ms^{-1} in a time of 8 s. What average power must the motor produce to cause this acceleration? (Ignore friction)*

(a) 45 kW *(b) 45.9 kW*
(c) 46.9 kW *(d) None of these*

Sol. (c) The work done in accelerating the car is given by

$$W = \Delta K = \frac{1}{2} m(v_f^2 - v_i^2) = \frac{1}{2}(1200)[(25)^2 - 0^2]$$

or $$W = 375 \text{ kJ}$$

$$\text{Power} = \frac{W}{t} = \frac{375}{8} = 46.9 \text{ kW}$$

Example 20. *A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time, is given by*

[AIEEE 2005, 04]

(a) $\frac{mv^2}{T^2} t$ *(b)* $\frac{mv^2}{T^2} t^2$
(c) $\frac{1}{2} \frac{mv^2}{T^2} t$ *(d)* $\frac{1}{2} \frac{mv^2}{T^2} t^2$

Sol. (a) A body of mass m with uniform acceleration, then force,

$$F = ma = \frac{mv}{T} \qquad \left[\therefore a = \frac{v - 0}{T}\right]$$

Instantaneous power $= Fv = mav = \frac{mv}{T} \cdot at$

$$= \frac{mv}{T} \cdot \frac{v}{T} \cdot t = \frac{mv^2}{T^2} t$$

Practice Exercise

ROUND I Topically Divided Problems

Work Done by Variable and Constant Forces

1. The work done in pulling up a block of wood weighing 2 kN for a length of 10 m on a smooth plane inclined at an angle of 15° with the horizontal is [$\sin 15° = 0.2588$]

(a) 4.36 kJ (b) 5.13 kJ
(c) 8.91 kJ (d) 9.82 kJ

2. A block of mass 10 kg slides down a rough slope which is inclined at an angle of 45° to the horizontal. The coefficient of sliding friction is 0.30. When the block has slide 5 m, the work done on the block by the force of friction is nearly

(a) 115 J (b) $-75\sqrt{2}$ J
(c) 321.4 J (d) -321.4 J

3. A position-dependent force $F = 3x^2 - 2x + 7$ acts on a body of mass 7 kg and displaces it from $x = 0$ m to $x =$ 5m. The work done on the body is x' joule. If both F and x' are measured in SI units, the value of x' is

(a) 135 (b) 235
(c) 335 (d) 935

4. Water is drawn from a well in a 5 kg drum of capacity 55 L by two ropes connected to the top of the drum. The linear mass density of each rope is 0.5 kgm^{-1}. The work done in lifting water to the ground from the surface of water in the well 20 m below is ($g = 10$ ms^{-2})

(a) 1.4×10^4 J (b) 1.5×10^4 J
(c) $9.8 \times 10 \times 6$ J (d) 18 J

5. Under the action of a force, a 2 kg body moves such that its position x as a function of time t is given by $x = t^3 / 3$, where x is in metre and t in second. The work done by the force in the first two seconds is

(a) 1.6 J (b) 16 J
(c) 160 J (d) 1600 J

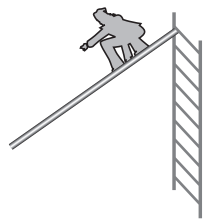
6. A mass M is lowered with the help of a string by a distance h at a constant acceleration $g/2$. The work done by the string will be

(a) $\frac{Mgh}{2}$ (b) $\frac{-Mgh}{2}$
(c) $\frac{3Mgh}{2}$ (d) $\frac{-3Mgh}{2}$

7. A 5 kg brick of 20 cm × 10 cm × 8 cm dimension lying on the largest base. It is now made to stand with length vertical. If $g = 10$ ms^{-2}, then the amount of work done is

(a) 3 J (b) 5 J (c) 7 J (d) 9 J

8. In a children's park, there is a slide which has a total length of 10 m and a height of 8.0 m. A vertical ladder is provided to reach the top. A boy weighing 200 N climbs up the ladder to the top of the slide and slides down to the ground. The average friction offered by the slide is three-tenth of his weight. The work done by the slide on the boy as he comes down is



(a) zero (b) + 600 J (c) –600 J (d) +1600 J

9. A ball is released from the top of a tower. The ratio of work done by force of gravity in Ist second, 2nd second and 3rd second of the motion of ball is

(a) 1 : 2 : 3 (b) 1 : 4 : 16 (c) 1 : 3 : 5 (d) 1 : 9 : 25

10. A plate of mass m, length b and breadth a is initially lying on a horizontal floor with length parallel to the floor and breadth perpendicular to the floor. The work done to erect it on its breadth is

(a) $mg\left[\frac{b}{2}\right]$ (b) $mg\left[a + \frac{b}{2}\right]$
(c) $mg\left[\frac{b-a}{2}\right]$ (d) $mg\left[\frac{b+a}{2}\right]$

11. A uniform chain of length L and mass M overhangs a horizontal table with its two-third part on the table. The friction coefficient between the table and the chain is μ. The work done by the friction during the period the chain slips off the table is

(a) $-\frac{1}{4}\mu MgL$ (b) $-\frac{2}{9}\mu MgL$
(c) $-\frac{4}{9}\mu\, MgL$ (d) $-\frac{6}{7}\mu MgL$

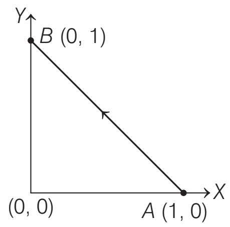
12. A uniform chain of length L and mass M is lying on a smooth table and one-third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on the table is

(a) MgL (b) $MgL/3$
(c) $MgL/9$ (d) $MgL/18$

13. A rod AB of mass 10 kg and length 4 m rests on a horizontal floor with end A fixed, so as to rotate it in vertical. Work done on the rod is 100 J. The height to which the end B be raised vertically above the floor is

(a) 1.5 m (b) 2.0 m
(c) 1.0 m (d) 2.5 m

14. Consider a force $\mathbf{F} = -x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$. The work done by this force in moving a particle from point $A(1, 0)$ to $B(0, 1)$ along the line segment is (all quantities are in SI units) **[JEE Main 2020]**

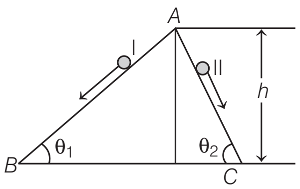


(a) $\frac{3}{2}$ (b) 2 (c) 1 (d) $\frac{1}{2}$

15. A cord is used to lower vertically a block of mass M by a distance d with constant downward acceleration $g/4$. Work done by the cord on the block is

(a) $Mg\frac{d}{4}$ (b) $3\,Mg\frac{d}{4}$
(c) $-3\,Mg\frac{d}{4}$ (d) Mgd

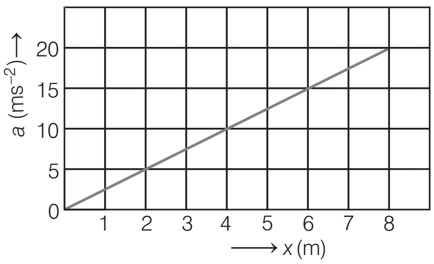
16. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure.



Which of the following statement is correct? **[NCERT Exemplar]**

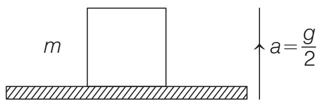
(a) Both the stones reach the bottom at the same time but not with the same speed.
(b) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.
(c) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.
(d) Both the stones reach the bottom at different times and with different speeds.

17. A 10 kg brick moves along an X-axis. Its acceleration as a function of its position is shown in figure. What is the net work performed on the brick by the force causing the acceleration as the brick moves from $x = 0$ to $x = 8.0$ m ?



(a) 4 J (b) 8 J (c) 2 J (d) 1 J

18. A block of mass m is kept on a platform which starts from rest with constant acceleration $\frac{g}{2}$ upwards as shown in figure. Work done by normal reaction on block in time t is **[JEE Main 2019]**



(a) $\frac{mg^2t^2}{8}$ (b) $\frac{3mg^2t^2}{8}$ (c) 0 (d) $-\frac{mg^2t^2}{8}$

Energy

19. A stone is dropped from the top of a tall tower. The ratio of the kinetic energy of the stone at the end of three seconds to the increase in the kinetic energy of the stone during the next three seconds is

(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 9

20. An engine pumps water continuously through a hole. Speed with which water passes through the hole nozzle is v and k is the mass per unit length of the water jet as it leaves the nozzle. Find the rate at which kinetic energy is being imparted to the water.

(a) $\frac{1}{2}kv^2$ (b) $\frac{1}{2}kv^3$
(c) $\frac{v^2}{2k}$ (d) $\frac{v^3}{2k}$

21. Two masses of 1 g and 4 g are moving with equal kinetic energies. The ratio of the magnitudes of their linear momenta is

(a) 4 : 1 (b) $\sqrt{2} : 1$
(c) 1 : 2 (d) 1 : 16

22. A mass of 5 kg is moving along a circular path of radius 1m. If the mass moves with 300 revolutions per minute, its kinetic energy (in J) would be **[NCERT Exemplar]**
(a) $250\pi^2$ (b) $100\pi^2$ (c) $5\pi^2$ (d) 0

23. A body of mass 2 kg is thrown up vertically with kinetic energy of 490 J. The height at which the kinetic energy of the body becomes half of its original value is
(a) 50 m (b) 12.25 m (c) 25 m (d) 10 m

24. Two springs have force constants k_1 and k_2. These are extended through the same distance x. If their elastic energies are E_1 and E_2, then $\frac{E_1}{E_2}$ is equal to
(a) $k_1 : k_2$ (b) $k_2 : k_1$ (c) $\sqrt{k_1} : \sqrt{k_2}$ (d) $k_1^2 : k_2^2$

25. A stone of mass 2 kg is projected upward with kinetic energy of 98 J. The height at which the kinetic energy of the body becomes half of its original value, is given by (Take, $g = 10\ \text{ms}^{-2}$)
(a) 5 m (b) 2.5 m (c) 1.5 m (d) 0.5 m

26. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is **[JEE Main 2018]**
(a) $-\frac{k}{4a^2}$ (b) $\frac{k}{2a^2}$
(c) zero (d) $-\frac{3}{2}\frac{k}{a^2}$

27. The potential energy of a particle of mass 5 kg moving in the XY-plane is given by $U = (-7x + 24y)$ J, x and y being in metre. If the particle starts from rest from origin, then speed of particle at $t = 2$ s is
(a) 5 ms^{-1} (b) 0.1 ms^{-1}
(c) 17.5 ms^{-1} (d) 10 ms^{-1}

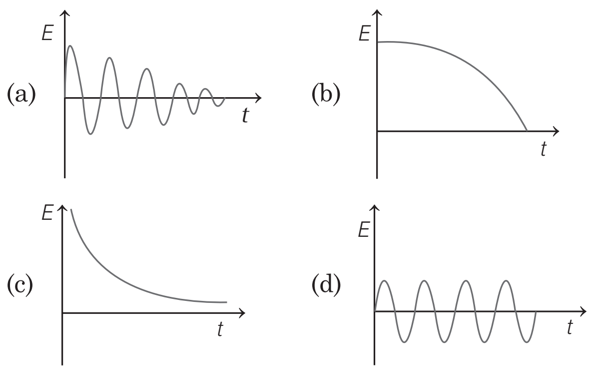
28. An elastic string of unstretched length L and force constant k is stretched by a small length x. It is further stretched by another small length y. The work done in the second stretching is
(a) $\frac{1}{2}ky^2$ (b) $\frac{1}{2}k(x^2 + y^2)$
(c) $\frac{1}{2}k(x + y)^2$ (d) $\frac{1}{2}ky(2x + y)$

29. If the potential energy between two molecules is given by $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation between molecules and the potential energy are **[JEE Main 2020]**
(a) $\left(\frac{B}{2A}\right)^{1/6}, -\frac{A^2}{2B}$ (b) $\left(\frac{B}{A}\right)^{1/6}, 0$
(c) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{4B}$ (d) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{2B}$

30. A bomb of mass 3.0 kg explodes in air into two pieces of masses 2.0 kg and 1.0 kg. The smaller mass goes at a speed of 80 m/s. The total energy imparted to the two fragments is
(a) 1.07 kJ (b) 2.14 kJ
(c) 2.4 kJ (d) 4.8 kJ

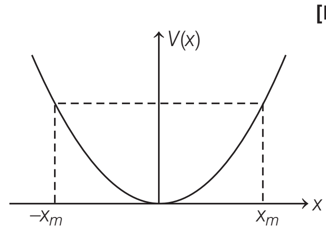
31. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table? **[AIEEE 2004]**
(a) 7.2 J (b) 3.6 J
(c) 120 J (d) 1200 J

32. Which of the diagrams shown in figure represents variation of total mechanical energy of a pendulum oscillating in air as function of time? **[NCERT Exemplar]**



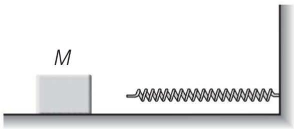
33. A ball is projected vertically upwards with a certain initial speed. Another ball of the same mass is projected at an angle of 60° with the vertical with the same initial speed. At highest point of their journey, the ratio of their potential energies will be
(a) 1 : 1 (b) 2 : 1 (c) 3 : 2 (d) 4 : 1

34. The potential energy function for a particle executing linear SHM is given by $V(x) = \frac{1}{2}kx^2$, where k is the force constant of the oscillator. For $k = 0.5$ N/m, the graph of $V(x)$ *versus* x is shown in the figure. A particle of total energy E turns back when it reaches $x = \pm x_m$. If V and K indicate the PE and KE respectively of the particle at $x = \pm x_m$, then which of the following is correct? **[NCERT Exemplar]**



(a) $V = 0, K = E$ (b) $V = E\ K = 0$
(c) $V < E, K = 0$ (d) $V = 0, K < 0$

35. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is **[AIEEE 2005]**



(a) $\sqrt{MkL}$ (b) $\frac{kL^2}{2M}$ (c) zero (d) $\frac{ML^2}{k}$

Work-Energy Theorem

36. A spring of spring constant $5 \times 10^3 \text{ Nm}^{-1}$ is stretched initially by 5 cm from the unstretched position, then the work required to stretch it further by another 5 cm is

(a) 12.50 N-m (b) 18.75 N-m
(c) 25.00 N-m (d) 6.25 N-m

37. A body of mass 4 kg is moving with momentum of 8 kg-ms^{-1}. A force of 0.2 N acts on it in the direction of motion of the body for 10 s. The increase in kinetic energy (in J) is

(a) 10 (b) 8.5
(c) 4.5 (d) 4

38. A force acts on a 30 g particle in such a way that the position of the particle as function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metre and t is in second. The work done during the first 4 s is

(a) 5.28 J (b) 450 mJ
(c) 490 mJ (d) 530 mJ

39. The upper half of an inclined plane with inclination ϕ is perfectly smooth, while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom, if the coefficient of friction for the lower half is given by **[AIEEE 2005]**

(a) $2\sin\phi$ (b) $2\cos\phi$
(c) $2\tan\phi$ (d) $\tan\phi$

40. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 s will be **[JEE Main 2017]**

(a) 22 J (b) 9 J (c) 18 J (d) 4.5 J

41. When a rubber band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$, where a and b are constants. The work done in stretching the unstretched rubber band by L is **[JEE Main 2014]**

(a) $aL^2 + bL^3$ (b) $\frac{1}{2}(aL^2 + bL^3)$

(c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

42. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest, assuming that it faces constant resistance to motion? **[AIEEE 2005]**

(a) 3.0 cm (b) 2.0 cm
(c) 1.5 cm (d) 1.0 cm

43. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10000 N/m. The spring compresses by **[AIEEE 2007]**

(a) 5.5 cm (b) 2.5 cm
(c) 11.0 cm (d) 8.5 cm

44. A body of mass M is dropped from a height h on a sand floor. If the body penetrates x cm into the sand, the average resistance offered by the sand to the body is

(a) $Mg\left(\frac{h}{x}\right)$ (b) $Mg\left(1+\frac{h}{x}\right)$

(c) $Mgh + Mgx$ (d) $Mg\left(1-\frac{h}{x}\right)$

45. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is

(a) $\sqrt{u^2 - 2gl}$

(b) $\sqrt{2gl}$

(c) $\sqrt{u^2 - gl}$

(d) $\sqrt{2(u^2 - gL)}$

46. A uniform cable of mass M and length L is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)$th part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be **[JEE Main 2019]**

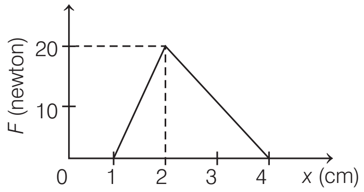
(a) $\frac{2MgL}{n^2}$ (b) $nMgL$

(c) $\frac{MgL}{n^2}$ (d) $\frac{MgL}{2n^2}$

47. In a shotput event, an athlete throws the shotput of mass 10 kg with an inital speed of 1 m s^{-1} at 45° from a height 1.5 m above ground. Assuming, air resistance to be negligible and acceleration due to gravity to be 10 ms^{-2}, the kinetic energy of the shotput when it just reaches the ground will be **[NCERT Exemplar]**

(a) 2.5 J (b) 5.0 J
(c) 52.5 J (d) 155.0 J

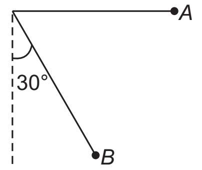
48. The graph between the resistive force F acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 kg and initial velocity is 2 m/s. When the distance covered by the body is 4 m, its kinetic energy would be



(a) 50 J (b) 40 J (c) 20 J (d) 10 J

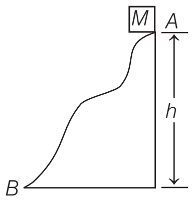
49. A 0.5 kg ball is thrown up with an initial speed 14 m/s and reaches a maximum height of 8.0 cm. How much energy is dissipated by air drag acting on the ball during the time of ascent?
(a) 19.6 J (b) 4.9 J (c) 10 J (d) 9.8 J

50. A simple pendulum is released from A as shown in figure. If M and l represent the mass of the bob and length of the pendulum respectively, the gain in kinetic energy at B is



(a) $\frac{Mgl}{2}$ (b) $\frac{\sqrt{3}}{2}Mgl$ (c) $\frac{Mgl}{\sqrt{2}}$ (d) $\frac{2}{\sqrt{3}}mgl$

51. In the given curved road, if particle is released from A, then



(a) kinetic energy at B must be mgh
(b) kinetic energy at B may be zero
(c) kinetic energy at B must be less than mgh
(d) kinetic energy at B must not be equal to zero

52. A 50 g bullet moving with a velocity of 10 ms^{-1} gets embedded into a 950 g stationary body. The loss in kinetic energy of the system will be
(a) 95% (b) 100% (c) 5% (d) 50%

53. Given that, the position of the body in metre is a function of time as follows

$$x = 2t^4 + 5t + 4$$

The mass of the body is 2 kg. What is the increase in its kinetic energy, one second after the start of motion?
(a) 168 J (b) 169 J (c) 32 J (d) 144 J

Power

54. A one kilowatt motor is used to pump water from a well 10 m deep. The quantity of water pumped out per second is nearly
(a) 1 kg (b) 10 kg
(c) 100 kg (d) 1000 kg

55. A car manufacturer claims that his car can be accelerated from rest to a velocity of 10 ms^{-1} in 5 s. If the total mass of the car and its occupants is 1000 kg, then the average horse power developed by the engine is
(a) $\frac{10^3}{746}$ (b) $\frac{10^4}{746}$
(c) $\frac{10^5}{746}$ (d) 8

56. A 10 HP motor pump take out water from a well of depth 20 m and falls a water tank of volume 22380 L at a height of 10 m from the ground. The running time of the motor to fill the empty water tank is (Take, $g = 10\,ms^{-2}$)
(a) 5 min (b) 10 min
(c) 15 min (d) 20 min

57. Ten litre of water per second is lifted from well through 20 m and delivered with a velocity of 10 ms^{-1}, then the power of the motor is
(a) 1.5 kW (b) 2.5 kW
(c) 3.5 kW (d) 4.5 kW

58. A dam is situated at a height of 550 m above sea level and supplies water to a power house which is at a height of 50 m above sea level. 2000 kg of water passes through the turbines per second. What would be the maximum electrical power output of the power house, if the whole system were 80% efficient?
(a) 8 MW (b) 10 MW
(c) 12.5 MW (d) 16 MW

59. An automobile weighing 120 kg climbs up a hill that rises 1 m in 20 s. Neglecting frictional effects, the minimum power developed by the engine is 9000 W. If $g = 10\,ms^{-2}$, then the velocity of the automobile is
(a) 36 kmh^{-1}
(b) 54 kmh^{-1}
(c) 72 kmh^{-1}
(d) 90 kmh^{-1}

60. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to (Take, 1 HP = 746 W and $g = 10\,ms^{-2}$) **[JEE Main 2020]**
(a) 2.0 ms^{-1} (b) 1.5 ms^{-1}
(c) 1.9 ms^{-1} (d) 1.7 ms^{-1}

61. A 500 kg car, moving with a velocity of 36 kmh^{-1} on a straight road unidirectionally, doubles its velocity in one minute. The power delivered by the engine for doubling the velocity is

(a) 750 W (b) 1050 W (c) 1150 W (d) 1250 W

62. A quarter horse power motor runs at a speed of 600 rpm. Assuming 40% efficiency, the work done by the motor in one rotation will be

(a) 7.46 J (b) 7400 J (c) 7.46 erg (d) 74.6 J

63. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$. The power is

(a) $2\pi mk^2r^2t$ (b) mk^2r^2t (c) $\frac{mk^4r^2t^5}{3}$ (d) zero

64. The power supplied by a force acting on a particle moving in a straight line is constant. The velocity of the particle varies with the displacement x as

(a) $x^{1/2}$ (b) x (c) x^2 (d) $x^{1/3}$

65. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to

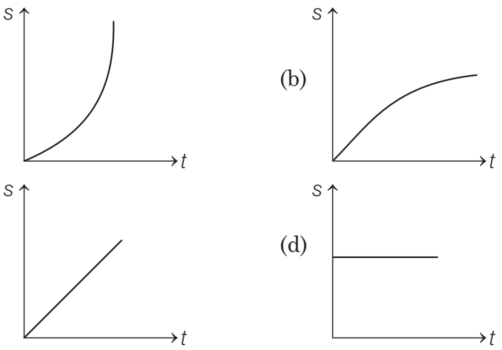
(a) $t^{3/4}$ (b) $t^{3/2}$ (c) $t^{1/4}$ (d) $t^{1/2}$

66. An elevator in a building can carry a maximum of 10 persons with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N.

If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator must be at least (Take, $g = 10\ \text{m/s}^2$) **[JEE Main 2020]**

(a) 62360 W (b) 48000 W (c) 56300 W (d) 66000 W

67. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s)-time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) **[JEE Main 2020]**



(a) (b) (c) (d)

ROUND II Mixed Bag

Only One Correct Option

1. A body of mass 3 kg is under a force which causes a displacement is given by $s = \frac{t^3}{3}$ (in m). Find the work done by the force in first 2 s.

(a) 2 J (b) 3.8 J (c) 5.2 J (d) 24 J

2. A man of mass m, standing at the bottom of the staircase of height L climbs it and stands at its top. **[NCERT Exemplar]**

(a) Work done by all forces on man is equal to the rise in potential energy mgL
(b) Work done by all forces on man is zero
(c) Work done by the gravitational force on man is mgL
(d) The reaction force from a step works as the point of application of the force does not move while the force exists

3. A force acts on a 2 kg object, so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 s? **[JEE Main 2019]**

(a) 850 J (b) 900 J (c) 950 J (d) 875 J

4. A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box? **[JEE Main 2020]**

(a) 5250 J (b) 2780 J (c) 3280 J (d) 5690 J

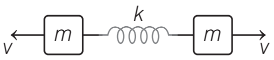
5. If a man speeds up by 1 ms^{-1}, his kinetic energy increases by 44%. His original speed in ms^{-1} is

(a) 1 (b) 2 (c) 5 (d) 4

6. A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of 20 ms^{-1}. The ball gets deflected by an obstacle on the way. After deflection, it moves with 5% of its initial kinetic energy. What is the speed of the ball now? **[JEE Main 2021]**

(a) 19.0 ms^{-1} (b) 4.47 ms^{-1} (c) 14.41 ms^{-1} (d) 1.00 ms^{-1}

7. Two blocks of mass m each are connected to a spring of spring constant k as shown in figure. The maximum displacement in the block is



(a) $\sqrt{\dfrac{2mv^2}{k}}$ (b) $\sqrt{\dfrac{mv^2}{k}}$

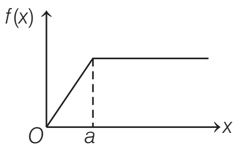
(c) $2\sqrt{\dfrac{mv^2}{k}}$ (d) $2\sqrt{\dfrac{k}{mv^2}}$

8. Power supplied to a particle of mass 2 kg varies with time as $P = \dfrac{3t^2}{2}$ watt, here t is in second. If the velocity of particle at $t = 0$ is $v = 0$, the velocity of particle at time $t = 2$ s will be

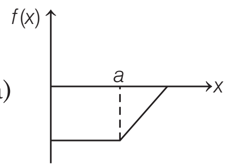
(a) 1 ms^{-1} (b) 4 ms^{-1}

(c) 2 ms^{-1} (d) $2\sqrt{2}$ ms^{-1}

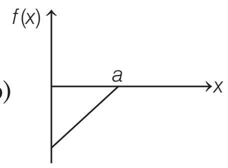
9. The potential energy of a system represent in the first figure. The force acting on the system will be represent by



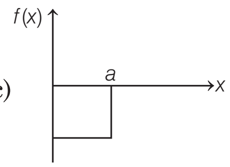
(a)



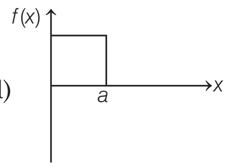
(b)



(c)



(d)



10. A gun of mass 20 kg has bullet of mass 0.1 kg in it. The gun is free to recoil 804 J of recoil energy are released on firing the gun. The speed of bullet (ms^{-1}) is

(a) $\sqrt{804 \times 2010}$ (b) $\sqrt{\dfrac{2010}{804}}$

(c) $\sqrt{\dfrac{804}{2010}}$ (d) $\sqrt{804 \times 4 \times 10^3}$

11. A particle moves on a rough horizontal ground with some initial velocity v_0. If (3/4)th of its kinetic energy is lost due to friction in time t_0, the coefficient of friction between the particle and the ground is

(a) $\dfrac{v_0}{2\,gt_0}$ (b) $\dfrac{v_0}{4\,gt_0}$ (c) $\dfrac{3\,v_0}{4\,gt_0}$ (d) $\dfrac{v_0}{gt_0}$

12. An ideal spring with spring constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched, then the maximum extension in the spring is

(a) $\dfrac{4\,Mg}{k}$ (b) $\dfrac{2\,Mg}{k}$

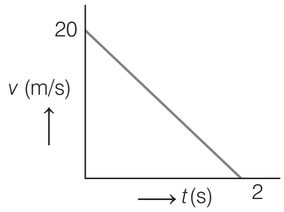
(c) $\dfrac{Mg}{k}$ (d) $\dfrac{Mg}{2\,k}$

13. A bullet when fired at a target with velocity of 100 ms^{-1} penetrates 1 m into it. If the bullet is fired at a similar target with a thickness 0.5 m, then it will emerge from it with a velocity of

(a) $50\sqrt{2}$ m/s (b) $\dfrac{50}{\sqrt{2}}$ m/s

(c) 50 m/s (d) 10 m/s

14. Velocity-time graph of a particle of mass 2 kg moving in a straight line is as shown in figure. Work done by all forces on the particle is



(a) 400 J (b) – 400 J

(c) – 200 J (d) 200 J

15. A box of mass 50 kg is pulled up on an incline 12 m long and 2 m high by a constant force of 100 N from rest. It acquires a velocity of 2 ms^{-1} on reaching the top. Work done against friction ($g = 10$ ms^{-2}) is

(a) 50 J (b) 100 J

(c) 150 J (d) 200 J

16. A particle is released from a height s. At certain height, its kinetic energy is three times its potential energy. The height and speed of the particle at that instant are respectively

(a) $\dfrac{s}{4}, \dfrac{3\,gs}{2}$ (b) $\dfrac{s}{4}, \sqrt{\dfrac{3\,gs}{2}}$

(c) $\dfrac{s}{2}, \sqrt{\dfrac{3\,gs}{2}}$ (d) $\dfrac{s}{8}, \sqrt{\dfrac{3\,gs}{2}}$

17. A man running has half the kinetic energy of a boy of half his mass. The man speeds up by 1 ms^{-1} and then has kinetic energy as that of the boy. What were the original speeds of man and the boy?

(a) $\sqrt{2}$ ms^{-1}; $2\sqrt{2} - 1$ ms^{-1}

(b) $(\sqrt{2} - 1)$ ms^{-1}, $2(\sqrt{2} - 1)$ ms^{-1}

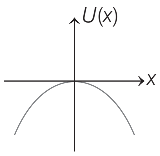
(c) $(\sqrt{2} + 1)$ ms^{-1}; $2(\sqrt{2} + 1)$ ms^{-1}

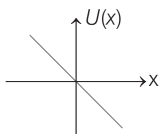
(d) None of the above

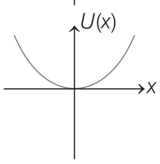
18. If the kinetic energy of a body is directly proportional to time t, the magnitude of the force acting on the body is
(a) directly proportional to $\sqrt{t}$
(b) inversely proportional to $\sqrt{t}$
(c) directly proportional to the speed of the body
(d) inversely proportional to the root of speed of the body

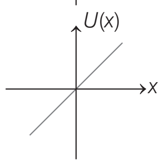
19. The kinetic energy k of a particle moving along a circle of radius R depends upon the distance s as $k = as^2$. The force acting on the particle is
(a) $2a\dfrac{s^2}{R}$ (b) $2as\left[1+\dfrac{s^2}{R^2}\right]^{1/2}$
(c) $2as$ (d) $2a$

20. A particle is placed at the origin and a force $F = kx$ is acting on it (where k is a positive constant). If $U(0) = 0$, the graph of $U(x)$ *versus* x will be (where U is the potential energy function) **[UP SEE 2004]**

(a) 

(b) 

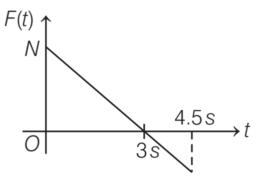
(c) 

(d) 

21. A block of mass 5 kg is resting on a smooth surface. At what angle can a force of 20 N be acted on the body so that it will acquired a kinetic energy of 40 J after moving 4 m
(a) 30° (b) 45° (c) 60° (d) 120°

22. A variable force given by the two-dimensional vector $\mathbf{F} = (3x^2\,\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ acts on a particle. The force is in newton and X is in metre. What is the change in the kinetic energy of the particle as it moves from the point with coordinates (2, 3) to (3, 0)? (The coordinates are in metres)
(a) –7 J (b) zero (c) +7 J (d) +19 J

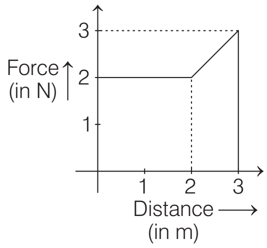
23. A block of mass 2 kg is free to move along the X-axis its is at rest and form $t = 0$ onwards it is subjected to a time dependent force $F(t)$ in the x-direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 s is



(a) 4.50 J (b) 7.50 J (c) 5.06 J (d) 14.06 J

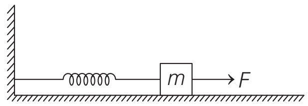
24. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to
(a) x^2 (b) e^x
(c) x (d) $\log_e x$

25. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is **[JEE Main 2019]**



(a) 4 J (b) 2.5 J (c) 6.5 J (d) 5 J

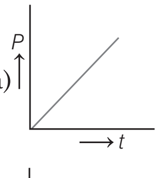
26. A block of mass m lying on a smooth horizontal surface is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is **[JEE Main 2019]**

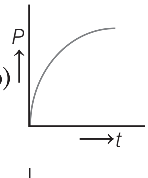


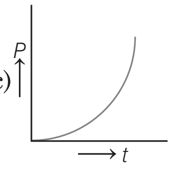
(a) $\dfrac{\pi F}{\sqrt{mk}}$ (b) $\dfrac{F}{\sqrt{mk}}$ (c) $\dfrac{2F}{\sqrt{mk}}$ (d) $\dfrac{F}{\pi\sqrt{mk}}$

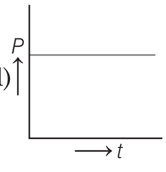
27. A car of mass m is driven with an acceleration a along a straight level road against a constant external resistive force R. When the velocity of the car is v, the rate at which engine of the car is doing work, will be
(a) $R \cdot v$ (b) $ma \cdot v$
(c) $(R + ma) \cdot v$ (d) $(ma - R) \cdot v$

28. A motor drives a body along a straight line with a constant force. The power P developed by the motor must vary with time t as shown in figure

(a) 

(b) 

(c) 

(d) 

29. A bob of mass m accelerates uniformly from rest to v_1 in time t_1. As a function of t, the instantaneous power delivered to the body is

(a) $\dfrac{mv_1 t}{t_2}$ (b) $\dfrac{mv_1 t}{t_1}$

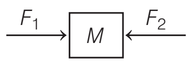
(c) $\dfrac{mv_1 t^2}{t_1}$ (d) $\dfrac{mv_1^2 t}{t_1^2}$

30. A car of mass 1000 kg moves at a constant speed of 20 ms^{-1} up an incline. Assume that the frictional force is 200 N and that $\sin\theta = 1/20$, where, θ is the angle of the incline to the horizontal. The $g = 10\ ms^{-2}$. Find the power developed by the engine.

(a) 14 kW (b) 4 kW

(c) 10 kW (d) 28 kW

31. A body of mass M is moving with a uniform speed of 10 m/s on frictionless surface under the influence of two forces F_1 and F_2. The net power of the system is



(a) $10\, F_1 F_2 M$

(b) $10\,(F_1 + F_2)\, M$

(c) $(F_1 + F_2)\, M$

(d) zero

32. An engine pumps water through a hose pipe. Water passes through the pipe and leaves with a velocity of 2 m/s. The mass per unit length of water in the pipe is 100 kg/m. What is the power of the engine?

(a) 800 W (b) 400 W

(c) 200 W (d) 100 W

33. Power applied to a particle varies with time as $P = (3t^2 - 2t + 1)$ watt, where t is in second. Find the change in its kinetic energy between $t = 2$ s and $t = 4$ s.

(a) 32 J (b) 46 J

(c) 61 J (d) 100 J

34. An engine pumps up 100 kg of water through a height of 10 m in 5 s. Given that, the efficiency of the engine is 60%. If $g = 10\ m/s^2$, the power of the engine is

(a) 3.3 kW (b) 0.33 kW

(c) 0.033 kW (d) 33 kW

35. Water falls from a height of 60 m at the rate of 15 kg/s to operate a turbine. The losses due to frictional force are 10% of energy. How much power is generated by the turbine? (Take, $g = 10\ m/s^2$) **[CBSE PMT 2008]**

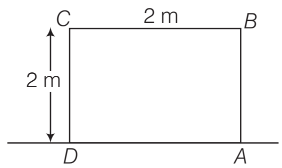
(a) 12.3 kW (b) 7.0 kW

(c) 8.1 kW (d) 10.2 kW

Numerical Value Questions

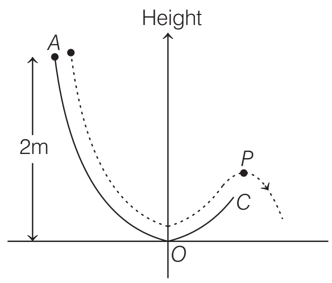
36. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine, so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m, while launching the ball, the value of F (in N) is .... (Take, $g = 10\ ms^{-2}$) **[JEE Main 2020]**

37. A boy wishes to move the block slowly shown in the figure, a distance of 2 m to the right by either sliding or by tiping over one corner with the least amount of work. The mass of block is 100 kg and coefficient of friction is 0.3. If the least amount of work is 207 nJ. The value of n is ....

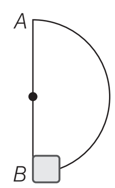


38. A particle of mass 1 kg is moving on a circular path of radius 1 m. Its kinetic energy is $K = bt^4$, where $b = 1$ J/s. The force acting on the particle at $t = 1$ s is $(2x)^{1/2}\ m/s^2$. The value of x is ....

39. A particle ($m = 1$ kg) slides down a frictionless track AOC starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy of the particle (in J) is .... (Figure drawn is schematic and not to scale) (Take, $g = 10\ ms^{-2}$) **[JEE Main 2020]**



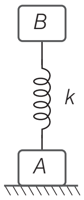
40. A sleeve of mass m is moving along perimeter by applying a constant force of 10N always directed toward point A. The radius of semi-circle is 10 cm. The work done (in J) by applied force F during motion of sleeve from B to A is ....



41. A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 s, the body has moved a distance (in m) .... **[JEE Main 2020]**

42. A ball of mass 4 kg is moving with a velocity of $10\,\text{ms}^{-1}$, collides with a spring of length 8 m and force constant $100\ \text{Nm}^{-1}$. The length of the compressed spring is x m. The value of x, to the nearest integer, is .... **[JEE Main 2021]**

43. Two blocks A and B of equal masses ($m = 10$ kg) are connected by a light spring of spring constant $k = 150$ N/m. The system is in equilibrium. The minimum value of initial downward velocity v_0 of the block B for which the block A bounce up is $\frac{20}{\sqrt{3\,n}}$ m/s. The value of n is ....



Answers

Round I

1. (b)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (c)
11. (b)	**12.** (d)	**13.** (b)	**14.** (c)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (b)
21. (c)	**22.** (a)	**23.** (b)	**24.** (a)	**25.** (b)	**26.** (c)	**27.** (d)	**28.** (d)	**29.** (c)	**30.** (d)
31. (b)	**32.** (c)	**33.** (d)	**34.** (b)	**35.** (a)	**36.** (b)	**37.** (c)	**38.** (a)	**39.** (c)	**40.** (d)
41. (c)	**42.** (d)	**43.** (a)	**44.** (b)	**45.** (d)	**46.** (d)	**47.** (d)	**48.** (d)	**49.** (d)	**50.** (b)
51. (b)	**52.** (a)	**53.** (d)	**54.** (b)	**55.** (b)	**56.** (a)	**57.** (b)	**58.** (a)	**59.** (b)	**60.** (c)
61. (d)	**62.** (a)	**63.** (b)	**64.** (d)	**65.** (b)	**66.** (d)	**67.** (a)			

Round II

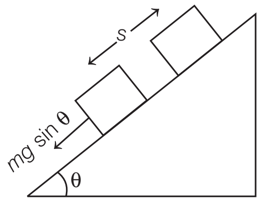
1. (d)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (d)
11. (a)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (a)
21. (c)	**22.** (c)	**23.** (c)	**24.** (a)	**25.** (c)	**26.** (b)	**27.** (c)	**28.** (a)	**29.** (d)	**30.** (a)
31. (d)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (c)	**36.** 150	**37.** 2	**38.** 6	**39.** 10	**40.** 2
41. 18	**42.** 6	**43.** 5							

Solutions

Round I

1. $W = mg\sin\theta \times s$

$= 2\times 10^3 \times \sin 15^\circ \times 10 = 5.13$ kJ



2. As, $F = \mu mg\cos\theta$

or $\quad F = 0.30\times 10\times 10\cos 45^\circ$

or $\quad F = \dfrac{30}{\sqrt{2}}$ N

$$W = F\times s\times \cos 180^\circ = -\frac{30}{\sqrt{2}}\times 5$$

$$= -\frac{150}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = -75\sqrt{2}\text{ J}$$

3. This is the case of work done by a variable force

$$W = \int_0^5 (3x^2 - 2x + 7)\,dx$$

$$W = |x^3 - x^2 + 7x|_0^5$$

or $\quad W = (5\times 5\times 5 - 5\times 5 + 7\times 5)$

or $\quad W = (125 - 25 + 35) = 135$ J

$\therefore \quad x' = 135$

4. Work done in lifting water and drum $= (60\times 10\times 20)$ J

$= 12000$ J

Total mass of ropes $= (4\times 0.5)$ kg $= 20$ kg

Work done in the case of ropes $= 20\times 10\times 10 = 2000$ J

Total work done $= 12000 + 2000 = 14000 \text{ J} \simeq 1.4\times 10^4$ J

5. $v = \dfrac{dx}{dt} = \dfrac{d}{dt}\left(\dfrac{t^3}{3}\right) = t^2$

When $t = 0$, then $v = 0$ and when $t = 2$, then

$v = 4$ m/s

According to work-energy theorem,

$$W = \frac{1}{2}m[(4)^2 - (0)^2] = \frac{1}{2}\times 2\times 16 = 16\text{J}$$

6. Tension in the string,

$$T = M(g - a) = M\left(g - \frac{g}{2}\right) = \frac{Mg}{2}$$

$$W = T\times h\times \cos 180^\circ$$

$$= -\frac{Mgh}{2}$$

7. Initial height of CG $= 4$ cm

Final height of CG $= 10$ cm

Increase in height $= (10 - 4)$ cm $= 6$ cm $= 0.06$ m

Work done $= 5\times 10\times 0.06 = 3$ J

8. Average friction, $F = \dfrac{3}{10}mg$

As, $\quad W = -Fs$ or $W = -\dfrac{3}{10}mgs$

or $\quad W = \left(-\dfrac{3}{10}\times 200\times 10\right)\text{J} = -600$ J

9. Initial velocity of ball is zero, *i.e.* $u = 0$.

$\therefore$ Displacement of ball in t^{th} second,

$$s = gt - \frac{1}{2}g = g\left(t - \frac{1}{2}\right)$$

$$s \propto \left(t - \frac{1}{2}\right)$$

or $\quad s_1 : s_2 : s_3 = \left(1 - \dfrac{1}{2}\right) : \left(2 - \dfrac{1}{2}\right) : \left(3 - \dfrac{1}{2}\right) = 1:3:5$

Now, $\quad W = mgs,\ W \propto s$

$\therefore \quad W_1 : W_2 : W_3 = 1:3:5$

10. Initial height of CG $= \dfrac{a}{2}$

Final height of CG $= \dfrac{b}{2}$

$$\text{Work done} = mg\left[\frac{b}{2} - \frac{a}{2}\right] = mg\left(\frac{b-a}{2}\right)$$

11. $dW = -\mu\left[\dfrac{M}{L}\right]gl\,dl$

$$W = \int_0^{\frac{2L}{3}} -\frac{\mu Mg}{L}\,l\,dl$$

or $\quad W = -\dfrac{\mu Mg}{L}\left[\dfrac{l^2}{2}\right]_0^{\frac{2L}{3}}$

or $\quad W = -\dfrac{\mu Mg}{2L}\dfrac{4L^2}{9} - 0$ or $W = -\dfrac{2}{9}\mu MgL$

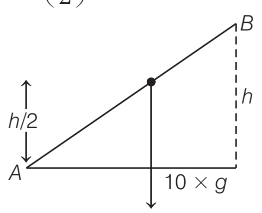
12. The weight of hanging part $\left(\dfrac{L}{3}\right)$ of chain is $\left(\dfrac{1}{3}Mg\right)$.

This weight acts at centre of gravity of the hanging part which is at a depth of $L/6$ from the table.

As, $\quad$ work done = force $\times$ distance

$$\therefore \quad W = \frac{Mg}{3}\times\frac{L}{6} = \frac{MgL}{18}$$

13. Work done $= mg\left(\dfrac{h}{2}\right)$



$$100 = \frac{10\times 10\times h}{2}$$

$$h = 2.0\text{ m}$$

14. Work done by a variable force on the particle,

$$W = \int \mathbf{F} \cdot d\mathbf{r}$$
$$= \int \mathbf{F} \cdot (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$$

$\therefore$ In two dimension, $d\mathbf{r} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}$

and it is given $\mathbf{F} = -x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$

$\therefore$
$$W = \int (-x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) \cdot (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$$
$$= \int -x\,dx + y\,dy$$
$$= \int -x\,dx + \int y\,dy$$

As particle is displaced from $A(1, 0)$ to $B(0, 1)$, so x varies from 1 to 0 and y varies from 0 to 1.

So, with limits, work will be

$$W = \int_1^0 -x\,dx + \int_0^1 y\,dy$$
$$= \left[\frac{-x^2}{2}\right]_1^0 + \left[\frac{y^2}{2}\right]_0^1$$
$$= \frac{1}{2}[(0-(-1)^2] + [(1)^2 - 0)]$$
$$= 1 \text{ J}$$

15. When the block moves vertically downward with acceleration $\frac{g}{4}$, then tension in the cord,

$$T = M\left(g - \frac{g}{4}\right) = \frac{3}{4}Mg$$

Work done by the cord $\mathbf{F} \cdot \mathbf{s} = Fs\cos\theta = Td\cos 180°$

$$= \left(-\frac{3}{4}Mg\right) \times d \qquad [\because \cos 180° = -1]$$
$$= -3\,Mg\frac{d}{4}$$

16. As both surfaces I and II are frictionless and two stones slide from the same height, therefore both the stones reach the bottom with same speed $\left(\frac{1}{2}mv^2 = mgh\right)$. As acceleration down the plane II is larger ($a_2 = g\sin\theta_2$ greater than $a_1 = g\sin\theta_1$), therefore stone II reaches the bottom earlier than stone I.

17. According to the graph, the acceleration a varies linearly with the coordinate x. We may write $a = \alpha x$, where α is the slope of the graph.

$$\alpha = \frac{20}{8} = 2.5 \text{ s}^{-2}$$

The force on the brick is in the positive x- direction and according to Newton's second law, its magnitude is given by

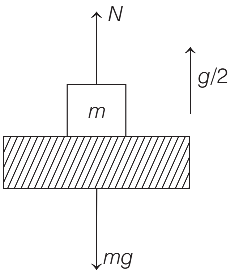
$$F = \frac{a}{m} = \frac{\alpha}{m}x$$

If x_f is the final coordinate, the work done by the force is

$$W = \int_0^{x_f} F dx = \frac{\alpha}{m}\int_0^{x_f} x\,dx$$
$$= \frac{\alpha}{2m}x_f^2 = \frac{2.5}{2 \times 10} \times (8)^2 = 8 \text{ J}$$

18. Normal reaction force on the block,

$$N = ma_{\text{net}}$$



where, a_{net} = net acceleration of block.

$$= g + a$$
$$= g + \frac{g}{2} = \frac{3g}{2}$$

$\Rightarrow$
$$N = m\left(g + \frac{g}{2}\right)$$
$$= \frac{3mg}{2}$$

Now, in time t, block moves by a displacement s given by

$$s = 0 + \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{g}{2}\right)t^2 \qquad (\because u = 0)$$

Here, $a = \frac{g}{2}$ (given)

$\therefore$ Work done = Force $\times$ Displacement

$\Rightarrow$
$$W = \frac{3mg}{2} \times \frac{gt^2}{4}$$
$$= \frac{3mg^2t^2}{8}$$

19. As,
$$E = \frac{1}{2}mv^2$$
$$= \frac{1}{2}m(gt)^2 \qquad [\because v = 0 + gt = gt]$$
$$= \frac{1}{2}mg^2t^2$$
$$\frac{E_1}{E_2} = \frac{\frac{1}{2}mg^2 \times 3^2}{\frac{1}{2}mg^2(6^2 - 3^2)} = \frac{9}{9 \times 3} = \frac{1}{3}$$

20. Velocity of water = v

Mass flowing per unit length = k

Mass flowing per second = kv

$\therefore$ Rate of kinetic energy or KE per second

$$= \frac{1}{2} \times \text{mass flowing per second} \times v^2$$
$$= \frac{1}{2} \times kv \times v^2 = \frac{1}{2}kv^3$$

21. $p = \sqrt{2mE_K}$

or
$$p \propto \sqrt{m} \qquad [\because E_k \text{ is given to be constant}]$$

$\therefore$
$$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

22. Here, $m = 5$ kg, $r = 1$ m

$$\omega = \frac{300}{60} \text{ rps} = 5 \text{ rps} = 5 \times 2\pi \text{ rad s}^{-1}$$

$$\text{KE} = \frac{1}{2} mv^2 = \frac{1}{2} m (r\omega)^2$$

$$= \frac{1}{2} \times 5 (1 \times 10\pi)^2$$

$$= 250\pi^2 \text{ J}$$

23. Potential energy at the required height $= \frac{490}{2} = 245\text{J}$

Again, $245 = 2 \times 10 \times h$

or $h = \frac{245}{20} \text{ m} = 12.25 \text{ m}$

24. $E = \frac{1}{2} kx^2$

$$F \propto k$$

$$\frac{E_1}{E_2} = \frac{k_1}{k_2}$$

25. As, $K = \frac{1}{2} mv^2$

$$v^2 = \frac{98 \times 2}{2} = 98$$

$$h = \frac{v^2}{2g} = \frac{98}{2 \times 9.8} = 5$$

$$K_1 = \frac{1}{2} mv^2 = \frac{1}{2} m \times 2gh$$

$\therefore$ $\frac{K_2}{K_1} = \frac{h_2}{h_1}$

Given $K_2 = \frac{K_1}{2} = \frac{K_1}{2K_1} = \frac{h_2}{5}$

$\therefore$ $h_2 = 2.5$ m

26. $\therefore$ Force $= -\frac{dU}{dr}$

$\Rightarrow$ $F = -\frac{d}{dr}\left(\frac{-k}{2r^2}\right) = -\frac{k}{r^3}$

As particle is on circular path, this force must be centripetal force.

$\Rightarrow$ $|F| = \frac{mv^2}{r}$

So, $\frac{k}{r^3} = \frac{mv^2}{r}$

$\Rightarrow$ $\frac{1}{2} mv^2 = \frac{k}{2r^2}$

$\therefore$ Total energy of particle = KE + PE

$$= \frac{k}{2r^2} - \frac{k}{2r^2} = 0$$

Total energy = 0

27. $F = \frac{\partial U}{\partial x} \hat{\mathbf{i}} - \frac{\partial U}{\partial y} \hat{\mathbf{j}} = -7\,\hat{\mathbf{i}} + 24\,\hat{\mathbf{j}}$

$\therefore$ $a_x = \frac{F_X}{m} = -\frac{7}{5} = -1.4 \text{ ms}^{-2}$ along negative X- axis

$a_y = \frac{F_Y}{m} = \frac{24}{5} = 4.8 \text{ ms}^{-2}$ along positive Y-axis

$\therefore$ $v_x = a_x t = 1.4 \times 2 = 2.8 \text{ ms}^{-2}$

and $v_y = 4.8 \times 2 = 9.6 \text{ ms}^{-1}$

$\therefore$ $v = \sqrt{v_x^2 + v_y^2} = 10 \text{ ms}^{-1}$

28. Elastic force in string is conservative in nature.

$$W = -\Delta V_1$$

where, W = work done by elastic force of string

$$W = -(V_f - V_i) = V_i - V_f$$

or $W = \frac{1}{2} kx^2 - \frac{1}{2} k(x + y)^2$

or $W = \frac{1}{2} kx^2 - \frac{1}{2} k(x^2 + y^2 + 2xy)$

$$= \frac{1}{2} kx^2 - \frac{1}{2} kx^2 - \frac{1}{2} ky^2 - \frac{1}{2} k(2xy)$$

$$= -kxy - \frac{1}{2} ky^2 = \frac{1}{2} ky(-2x - y)$$

The work done against elastic force,

$$W_{\text{ext}} = -W = \frac{ky}{2}(2x + y)$$

29. Potential energy between two molecules is given by

$$U = -\frac{A}{r^6} + \frac{B}{r^{12}}$$

From the relation between force and potential energy,

Force acting between them, $F = -\frac{dU}{dr}$

$\therefore$ $F = -\frac{d}{dr}\left(-\frac{A}{r^6} + \frac{B}{r^{12}}\right)$

$$= -\left(\frac{6A}{r^7} - \frac{12B}{r^{13}}\right)$$

$$= \frac{6}{r^7}\left(-A + \frac{2B}{r^6}\right)$$

At equilibrium, $F = 0$

$\Rightarrow$ $\frac{6}{r^7}\left(-A + \frac{2B}{r^6}\right) = 0$

$\Rightarrow$ $-A + \frac{2B}{r^6} = 0$ $\quad (\because r \neq 0)$

$\Rightarrow$ $r^6 = \frac{2B}{A}$

or $r = \left(\frac{2B}{A}\right)^{\frac{1}{6}}$

The above calculated value of r is the separation between molecules at equilibrium.

Now, putting this value in the expression of potential energy, we get

$$U = -\frac{A}{\left\{\left(\frac{2B}{A}\right)^{\frac{1}{6}}\right\}^6} + \frac{B}{\left\{\left(\frac{2B}{A}\right)^{\frac{1}{6}}\right\}^{12}} = -\frac{A^2}{2B} + \frac{A^2}{4B}$$

$\therefore$ $U = -\frac{A^2}{4B}$

30. Both fragments will possess the equal linear momentum.

i.e. $m_1 v_1 = m_2 v_2$

$\Rightarrow \quad 1 \times 80 = 2 \times v_2$

$\Rightarrow \quad v_2 = 40 \text{ m/s}$

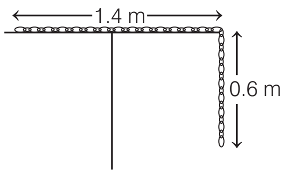
$\therefore$ Total energy of system $= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$$= \frac{1}{2} \times 1 \times (80)^2 + \frac{1}{2} \times 2 \times (40)^2$$

$$= 4800 \text{ J}$$

$$= 4.8 \text{ kJ}$$

31. Mass per unit length $= \frac{M}{L} = \frac{4}{2} = 2$ kg/m



The mass of 0.6 m of chain $= 0.6 \times 2 = 1.2$ kg

The height of centre of mass of hanging part,

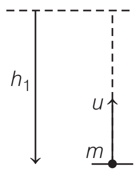
$$h = \frac{0.6 + 0}{2} = 0.3 \text{ m}$$

Hence, work done in pulling the chain on the table = Work done against gravity force

i.e. $W = mgh = 1.2 \times 10 \times 0.3 = 3.6$ J

32. When a pendulum oscillates in air, it loses energy continuously in overcoming resistance due to air. Therefore, total mechanical energy of the pendulum decreases continuously with time. The variation of total mechanical energy E with time t is shown correctly by curve (c).

33. For first ball, $mgh_1 = \frac{1}{2} mu^2$



i.e. $h_1 = \frac{u^2}{2g}$

For second ball,

$$mgh_2 = mg \frac{u^2 \cos^2 \theta}{2g}$$

$$= \frac{1}{2} mu^2 \cos^2 \theta$$

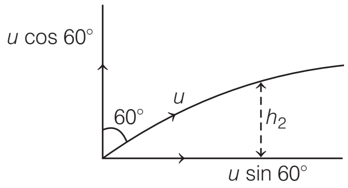
$$= \frac{1}{2} mu^2 \cos^2 60°$$

$$= \frac{1}{2} mu^2 \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} mu^2 \left(\frac{1}{4}\right)$$

$\Rightarrow \quad h_2 = \frac{u^2}{8g}$

$$\frac{h_1}{h_2} = \frac{u^2}{2g} \times \frac{8g}{u^2}$$



$\Rightarrow \quad \frac{h_1}{h_2} = \frac{4}{1}$

$\therefore$ At highest point, $U_1 : U_2 = h_1 : h_2 = 4 : 1$

34. At $x = +x_m$, the particle turns back. Therefore, its velocity at this point is zero.Therefore, kinetic energy $K = 0$. The total energy E is in the form of potential energy, *i.e.* $V = E$.

35. Momentum would be maximum when KE would be maximum and this is the case when total elastic PE is converted into KE.

According to conservation of energy,

$$\frac{1}{2} kL^2 = \frac{1}{2} Mv^2$$

$\Rightarrow \quad kL^2 = \frac{(Mv)^2}{M}$

or $\quad MkL^2 = p^2 \qquad [\because p = Mv]$

$\Rightarrow \quad p = L\sqrt{Mk}$

36. As, $W_1 = \frac{1}{2} kx_1^2 = \frac{1}{2} \times 5 \times 10^3 \times (5 \times 10^{-2})^2 = 6.25$ J

$$W_2 = \frac{1}{2} k (x_1 + x_2)^2$$

$$\frac{1}{2} \times 5 \times 10^3 (5 \times 10^{-2} + 5 \times 10^{-2})^2 = 25 \text{ J}$$

Net work done $= W_2 - W_1 = 25 - 6.25$

$$= 18.75 \text{ J} = 18.75 \text{ N-m}$$

37. Initially, $4u = 8 \Rightarrow u = 2$ m/s

Now, $\quad mv - mu = Ft$

$$mv - 8 = 0.2 \times 10$$

or $\quad v = (5/2) \text{ ms}^{-1}$

Increase in KE $= \frac{1}{2} m(v^2 - u^2)$

$$= \frac{1}{2} \times \left[4 \left(\frac{5}{2}\right)^2 - (2)^2 \right] = 4.5 \text{ J}$$

38. As, $v = \frac{dx}{dt} = 3 - 8t + 3t^2$

At $t = 0$, $v_0 = 3 - 8 \times 0 + 3 \times 0^2 = 3$ m/s

and at $t = 4$ s, $v_4 = 3 - 8 \times 4 + 3 \times 4^2 = 19$ m/s

$$W = \frac{1}{2} m (v_4^2 - v_0^2)$$

(According to work-energy theorem)

$$= \frac{1}{2} \times 0.03 \times (19^2 - 3^3) = 5.28 \text{ J}$$

39. According to work-energy theorem,

Work done = Change in kinetic energy

$$W = \Delta K = 0$$

$\Rightarrow$ Work done by friction + Work done by gravity $= 0$

$$\Rightarrow \quad -(\mu\, mg \cos\phi)\frac{l}{2} + mgl \sin\phi = 0$$

or $$\frac{\mu}{2}\cos\phi = \sin\phi$$

or $$\mu = 2\tan\phi$$

40. From Newton's second law,

$$\frac{\Delta p}{\Delta t} = F$$

$\Rightarrow$ $$\Delta p = F\Delta t$$

$\therefore$ $$p = \int dp = \int_0^1 F\,dt$$

$\Rightarrow$ $$p = \int_0^1 6\,t\,dt = 3\text{ kg}\left(\frac{\text{m}}{\text{s}}\right)$$

Also, change in kinetic energy,

$$\Delta K = \frac{\Delta p^2}{2m} = \frac{3^{\,2}}{2\times 1} = 4.5\text{ J}$$

From work-energy theorem,

Work done = Change in kinetic energy

So, work done $= \Delta K = 4.5$ J

41. Given, $F = ax + bx^2$

According to work-energy theorem, work done in stretching the rubber band by t is

$$|dW| = Fdx$$

$$|W| = \int_0^L (ax + bx^2)\,dx = \left[\frac{ax^2}{2}\right]_0^L + \left[\frac{bx^3}{3}\right]_0^L$$

$$= \left[\frac{aL^2}{2} - \frac{a\times(0)^2}{2}\right] + \left[\frac{b\times L^3}{3} - \frac{b\times(0)^3}{3}\right]$$

$\therefore$ $$|W| = \frac{aL^2}{2} + \frac{bL^3}{3}$$

42. According to work-energy theorem,

Total work done = Change in kinetic energy

$$W = \Delta K$$

***Case* I** $$F\times 3 = \frac{1}{2}m\left(\frac{v_0}{2}\right)^2 - \frac{1}{2}mv_0^2$$

where, F is resistive force and v_0 is initial speed.

***Case* II** Let the further distance travelled by the bullet before coming to rest be s.

$\therefore$ $$-F(3+s) = K_f - K_i = -\frac{1}{2}mv_0^2$$

$\Rightarrow$ $$-\frac{1}{8}mv_0^2(3+s) = -\frac{1}{2}mv_0^2$$

or $$\frac{1}{4}(3+s) = 1$$

or $$\frac{3}{4} + \frac{s}{4} = 1$$

or $$s = 1\text{ cm}$$

43. According to work-energy theorem,

Loss in kinetic energy = Work done against friction + Potential energy of spring

$$\frac{1}{2}mv^2 = f\,x + \frac{1}{2}kx^2$$

$\Rightarrow$ $$\frac{1}{2}\times 2\,(4)^2 = 15\,x + \frac{1}{2}\times 10000\,x^2$$

$\Rightarrow$ $$5000\,x^2 + 15x - 16 = 0$$

$\therefore$ $$x = 0.055\text{ m} = 5.5\text{ cm}$$

44. If the body strikes the sand floor with a velocity v, then

$$Mgh = \frac{1}{2}mv^2$$

With this velocity v, when body passes through the sand floor it comes to rest after travelling a distance x. Let F be the resisting force acting on the body. Net force in downwards direction $= Mg - F$

Work done by all the forces is equal to change in KE

$$(Mg - F)\,x = 0 - \frac{1}{2}Mv^2$$

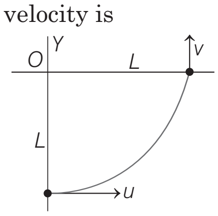
$$(Mg - F)\,x = -Mgh$$

or $$Fx = Mgh + Mgx$$

or $$F = Mg\left(1 + \frac{h}{x}\right)$$

45. In this case, motion of stone is in vertical circle of radius L and centre at O.

The change in velocity is



$$\Delta\mathbf{v} = \mathbf{v} - \mathbf{u} = v\hat{\mathbf{j}} - u\hat{\mathbf{i}}$$

$$|\Delta\mathbf{v}| = \sqrt{(v)^2 + (-u)^2} = \sqrt{v^2 + u^2}$$

According to work-energy theorem,

$$W = \Delta K$$

or $$W_T + W_g = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \qquad \ldots\text{(i)}$$

W_T = work done by the force of tension = 0

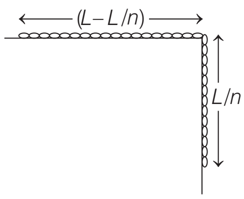
W_g = work done by the force of gravity $= mgL$ (path independent)

From Eq. (i), $0 - mgL = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$\because$ $$v^2 = u^2 - 2gL$$

$\therefore$ $$|\Delta\mathbf{v}| = \sqrt{v^2 + u^2} = \sqrt{2\,(u^2 - gl)}$$

46.

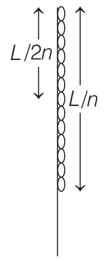


Given, mass of the cable is M.

So, mass of $\frac{1}{n}$th part of the cable, *i.e.* hanged part of the cable is M/n. ...(i)

Now, centre of mass of the hanged part will be its middle point.

So, its distance from the top of the table will be $L/2n$.



$\therefore$ Initial potential energy of the hanged part of cable,

$$U_i = \left(\frac{M}{n}\right)(-g)\left(\frac{L}{2n}\right)$$

$$\Rightarrow \quad U_i = -\frac{MgL}{2n^2} \quad \text{...(ii)}$$

When the whole cable is on the table, its potential energy will be zero.

$$\therefore \quad U_f = 0 \quad \text{...(iii)}$$

Now, using work-energy theorem,

$$W_{\text{net}} = \Delta U = U_f - U_i$$

$$\Rightarrow \quad W_{\text{net}} = 0 - \left(-\frac{MgL}{2n^2}\right)$$

[using Eqs. (ii) and (iii)]

$$\Rightarrow \quad W_{\text{net}} = \frac{MgL}{2n^2}$$

47. As the shotput reaches the ground, its KE

= PE of shotput when it is thrown + KE given

$= mgh + \frac{1}{2}mv^2$

$= 10 \times 10 \times 1.5 + \frac{1}{2} \times 10\,(1)^2$

$= 150 + 5 = 155$ J

48 Initial KE of the body $= \frac{1}{2}mv^2 = \frac{1}{2} \times 25 \times 4 = 50$ J

Work done against resistive force

= Area between F-x graph

$= \frac{1}{2} \times 4 \times 20 = 40$ J

Final KE = Initial KE – Work done against resistive force

$= 50 - 40 = 10$ J

49. If there is no air drag, then maximum height,

$$H = \frac{u^2}{2g} = \frac{14 \times 14}{2 \times 9.8} = 10 \text{ m}$$

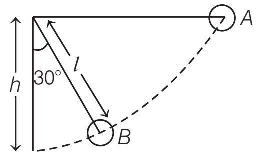
But due to air drag ball reaches upto height 8 m only, so loss of energy

$= Mg\,(10 - 8) = 0.5 \times 9.8 \times 2 = 9.8$ J

50. Vertical height, $h = l\cos\theta = l\cos 30°$

Loss of potential energy $= Mgh$

$$= Mgl\cos 30° = \frac{\sqrt{3}}{2}Mgl$$



$\therefore$ Kinetic energy gained $= \frac{\sqrt{3}}{2}Mgl$

51. (a) If the surface is smooth, then the kinetic energy at B never be zero.

(b) If the surface is rough, the kinetic energy at B be zero. Because, work done by force of friction is negative. If work done by friction is equal to mgh, then net work done on body will be zero. Hence, net change in kinetic energy is zero. Hence, (b) is correct.

(c) If the surface is rough, the kinetic energy at B must be lesser than mgh. If surface is smooth, the kinetic energy at B is equal to mgh.

(d) The reason is same as in (a) and (b)

52. Applying principle of conservation of linear momentum, velocity of the system (v) is

$$m_1v_1 = (m_1 + m_2)v$$

$$\Rightarrow \quad v = \frac{m_1v_1}{m_1 + m_2} = \frac{50 \times 10 \times 100}{(50 + 950) \times 100} = \frac{1}{2}\text{ms}^{-1}$$

Initial KE, $E_1 = \frac{1}{2}m_1v_1^2$

$$= \frac{1}{2} \times \left(\frac{50}{1000}\right) \times 10^2 = 2.5 \text{ J}$$

Final KE, $E_2 = \frac{1}{2}(m_1 + m_2)\,v^2$

$$= \frac{1}{2}\frac{(50 + 950)}{1000} \times \left(\frac{1}{2}\right)^2 = 0.125\text{J}$$

Percentage loss in KE,

$$\frac{E_1 - E_2}{E_1} \times 100 = \frac{2.5 - 0.125}{2.5} \times 100 = 95\%$$

53. $x = 2t^4 + 5t + 4 \Rightarrow v = \frac{dx}{dt} = 8t^3 + 5$

At $t = 0$, $v = 5$ m/s

At $t = 1$ s, $v = 8 \times 1 + 5 = 13$ m/s

Increase in KE $= \frac{1}{2}m\,[(13)^2 - (5)^2]$

$= \frac{1}{2} \times 2 \times 144 = 144$ J

54. As, $P = \frac{mgh}{t}$

or $\frac{m}{t} = \frac{P}{gh}$

or $\frac{m}{t} = \frac{1000}{10 \times 10}$ kg $= 10$ kg

55. As, $a = \frac{10-0}{5}$ $ms^{-2} = 2$ ms^{-2}

$\because$ $F = ma$

$\therefore$ $F = (1000 \times 2)$ N $= 2000$ N

Average velocity $= \frac{0+10}{2} ms^{-1} = 5$ ms^{-1}

Average power $= (2000 \times 5) W = 10^4 W$

Required horse power is $\frac{10^4}{746}$.

56. Volume of water of raise $= 22380$ L $= 22380 \times 10^{-3}$ m^3

$$P = \frac{mgh}{t} = \frac{V\rho gh}{t}$$

$$\Rightarrow \quad t = \frac{V\rho gh}{p}$$

$$t = \frac{22380 \times 10^{-3} \times 10^3 \times 10 \times 10}{10 \times 746} = 5 \text{ min}$$

57. As power, $P = \frac{\text{total energy}}{t} = \frac{mgh + \frac{1}{2}mv^2}{t}$

$$= \frac{10 \times 10 \times 20 + \frac{1}{2} \times 10 \times 10 \times 10}{1}$$

$= 2000 + 500 = 2500$ W

$= 2.5$ kW

58. Given, $h = 500$ m, $\frac{dm}{dt} = 2000$ kgs^{-1}

$\therefore$ Power output $= \frac{80}{100} \times \frac{dm}{dt} gh$

$= \frac{4}{5} \times 2000 \times 10 \times 500$ W

$= 8 \times 10^6 W = 8$ MW

59. Minimum force $mg \sin \theta$, so minimum power is given by

$$P = mg \sin \theta\, v \qquad (\because F = mg \sin \theta)$$

or $\quad v = \frac{P}{mg \sin \theta}$

or $\quad v = \frac{9000 \times 2}{1200 \times 10 \times 1} = 15$ ms^{-1}

$= 15 \times \frac{18}{5} = 54$ kmh^{-1}

60. At maximum load, force provided by motor to pull the lift,

F = weight carried + friction $= mg + f$

$= (2000 \times 10) + 4000 = 24000$ N

Power delivered by motor at speed v of load, $P = F \times v$

$\Rightarrow \quad v = \frac{P}{F} = \frac{60 \times 746}{24000} = 1.865 = 1.9$ ms^{-1}

61. Given, $u = 36$ km/h $= 36 \times \frac{5}{18}$ ms^{-1}

$= 10 ms^{-1}$ and $v = 20$ ms^{-1}

$\therefore$ Work done = Increase in kinetic energy

$= \frac{1}{2} \times 500$ $[20^2 - 10^2]$

$= \frac{500 \times 30 \times 10}{2}$

Power $= \frac{500 \times 30 \times 10}{2 \times 60} = 1250$ W

62. Motor makes 600 revolution per minute,

$\therefore$ $n = 600 \frac{\text{revolution}}{\text{min}} = 10 \frac{\text{rev}}{\text{s}}$

$\therefore$ Time required for one revolution $= \frac{1}{10}$ s

Energy required for one revolution = Power $\times$ Time

$= \frac{1}{4} \times 746 \times \frac{1}{10} = \frac{746}{40}$ J

But work done = 40% of input

$= 40\% \times \frac{746}{40}$

$= \frac{40}{100} \times \frac{746}{40} = 7.46$ J

63. Here $a_c = \frac{v^2}{r} = k^2rt^2$

$\therefore$ $v = krt$

The tangential acceleration, $a_t = \frac{dv}{dt}$

$= \frac{d(krt)}{dt} = kr$

The work done by centripetal force will be zero.

So, power is delivered to the particle by only tangential force which acts in the same direction of instantaneous velocity.

$\therefore$ Power $= F_t v = ma_t krt$

$= m\,(kr)(krt)$

$= mk^2r^2t$

64. Since, power, $P = \frac{W}{t} = F \times v$

$\Rightarrow \quad P = F \times v = ma \times v \qquad [\because F = ma]$

$\Rightarrow \quad P = mav \qquad \ldots(i)$

Again, by equation of motion,

$v^2 - u^2 = 2ax$

$\Rightarrow \quad v^2 - 0 = 2ax$

$\Rightarrow \quad a = \frac{v^2}{2x} \qquad \ldots(ii)$

From Eqs. (i) and (ii), we have

$P = m \cdot \frac{v^2}{2x} \cdot v$

$P = \frac{mv^3}{2x}$

$\Rightarrow \quad v^3 = \left(\frac{2P}{m}\right)x$

Since, P is constant, hence

$v^3 \propto x$

$\Rightarrow \quad v \propto x^{1/3}$

65. Delivering power of a machine,

P = constant

$$P = F \cdot v \qquad \left[\because \text{Power} = \frac{W}{t} = F \cdot \frac{s}{t} = F \cdot v\right]$$

$$= mav = m\frac{dv}{dt}v$$

$$P = mv\frac{dv}{dt}$$

$$vdv = \frac{P}{m}dt$$

Integrating on both sides, we get

$$\int_0^v v\,dv = \int_0^t \frac{P}{m}\,dt$$

$$\frac{v^2}{2} = \frac{Pt}{m} \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$v = \frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$ds = \left(\frac{2Pt}{m}\right)^{1/2} dt$$

$$\int_0^s ds = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}\,dt$$

$$s = \sqrt{\frac{2P}{m}}\,\frac{t^{3/2}}{3/2}$$

$$\therefore \qquad s \propto t^{3/2}$$

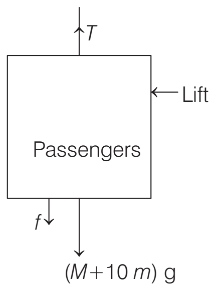
66. Mass of elevator, $M = 920$ kg

Mass of all 10 passengers carried by elevator $= 10 \times m$

$$= 10 \times 68 = 680 \text{ kg}$$

Total weight of elevator and passengers

$$= (M + 10m)g$$
$$= (920 + 680) \times 10$$
$$= 16000 \text{ N}$$



Force of friction $= 6000$ N

Total force (T) applied by the motor of elevator

$$= 16000 + 6000 = 22000 \text{ N}$$

Power delivered by elevator's motor,

$$P = F \cdot v = 22000 \times 3 \qquad [\because v = 3\text{ms}^{-1}]$$
$$= 66000 \text{ W}$$

67. As, power $= \dfrac{d}{dt}$ (KE) = constant

$$\Rightarrow \qquad \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = P$$

$$\Rightarrow \qquad \frac{1}{2}m \cdot 2v\frac{dv}{dt} = P$$

$$\Rightarrow \qquad vdv = \frac{P}{m} \cdot dt$$

Integrating both sides, we get

$$\int v\,dv = \frac{P}{m}\int dt$$

$$\Rightarrow \qquad \frac{v^2}{2} = \frac{P}{m} \cdot t$$

$$\Rightarrow \qquad v = \sqrt{\frac{2P}{m}} \cdot t^{1/2}$$

$$\Rightarrow \qquad \frac{ds}{dt} = \sqrt{\frac{2P}{m}} \cdot t^{1/2}$$

$$\Rightarrow \qquad ds = \sqrt{\frac{2P}{m}} \cdot t^{1/2}dt$$

Again integrating both sides, we get

$$s = \int ds = \int \sqrt{\frac{2P}{m}} \cdot t^{1/2}dt$$

$$\Rightarrow \qquad s = C \cdot t^{3/2}$$

where, C is a constant $= \dfrac{2}{3}\sqrt{\dfrac{2P}{m}}$.

Hence, displacement (s)-time (t) graph is correctly represented in option (a).

Round II

1. Given, $\qquad s = \dfrac{t^3}{3}$

$$\therefore \qquad ds = t^2\,dt$$

$$\Rightarrow \qquad a = \frac{d^2s}{dt^2} = \frac{d^2}{dt^2}\left[\frac{t^3}{3}\right] = 2t \text{ m/s}^2$$

Now work done by the force,

$$W = \int_0^2 F \cdot ds = \int_0^2 m \cdot a\,ds$$

$$\int_0^2 3 \times 2t \times t^2\,dt = \int_0^2 6t^3\,dt = \frac{3}{2}[t^4]_0^2 = 24 \text{ J}$$

2. When a man of mass m climbs up the staircase of height L, work done by the gravitational force on the man is $-mgL$ and work done by muscular force is mgL. If we ignore air resistance and friction, then the work done by all forces on man is equal to $-mgL + mgL =$ zero. Further, force from a step does not do work because the point of application of force does not move while the force exists.

3. Here, the displacement of an object is given by

$$x = (3t^2 + 5) \text{ m}$$

Therefore, velocity $(v) = \dfrac{dx}{dt} = \dfrac{d(3t^2 + 5)}{dt}$

or $\qquad v = 6t$ m/s $\qquad$ ...(i)

The work done in moving the object from $t = 0$ to $t = 5$ s,

$$W = \int_{x_0}^{x_5} F \cdot dx \qquad \text{...(ii)}$$

The force acting on this object is given by

$$F = ma = m \times \frac{dv}{dt}$$

$$= m \times \frac{d(6t)}{dt} \quad \text{[using Eq. (i)]}$$

$$F = m \times 6 = 6\,m = 12\text{ N}$$

Also, $\quad x_0 = 3t^2 + 5 = 3 \times (0)^2 + 5 = 5\text{ m}$

and at $t = 5$ s,

$$x_5 = 3 \times (5)^2 + 5 = 80\text{ m}$$

Put the values in Eq. (ii), we get

$$W = 12 \times \int_{x_0}^{x_5} dx = 12\,[80 - 5]$$

$$W = 12 \times 75 = 900\text{ J}$$

Alternate Solution

To using work-kinetic energy theorem,

$$W = \Delta\text{KE} = \frac{1}{2} m(v_f^2 - v_i^2)$$

$$= \frac{1}{2} m \times (30^2 - 0^2)$$

$$= \frac{1}{2} \times 2 \times 900 = 900\text{ J}$$

4. For $0 \le x \le 15$ m, $\ F = 200$ N ...(i)

For 15 m $< x \le 30$ m, force F is linearly decreasing from 200 N to 100 N.

So, using two-point form of straight line, we have

$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$(F - F_1) = \left(\frac{F_2 - F_1}{x_2 - x_1}\right)(x - x_1)$$

Here, $x_1 = 15$ m, $F_1 = 200$ N

$x_2 = 30$ m, $F_2 = 100$ N

So, $$(F - 200) = \left(\frac{100 - 200}{30 - 15}\right)(x - 15)$$

$$F - 200 = \frac{-100}{15}(x - 15)$$

$$F = 200 - \frac{20}{3}(x - 15)$$

$$F = 200 - \frac{20}{3}x + 100$$

$$F = \left(300 - \frac{20}{3}x\right)\text{ N} \quad \text{...(ii)}$$

Therefore, $$F = \begin{cases} 200\text{ N}; & 0 \le x \le 15\text{ m} \\ \left(300 - \frac{20}{3}x\right)\text{ N}; & 15\text{ m} < x \le 30\text{ m} \end{cases}$$

Now, work done during the complete movement of the box,

$$W = \int_0^{30} F\,dx = \int_0^{15} 200\,dx + \int_{15}^{30}\left(300 - \frac{20x}{3}\right)dx$$

$$= 200\,[x]_0^{15} + \left[300x - \frac{20}{3}\frac{x^2}{2}\right]_{15}^{30}$$

$$= 200\,[15 - 0] + \left[300x - \frac{10}{3}x^2\right]_{15}^{30}$$

$$= 200 \times 15 + \left[\left\{300(30) - \frac{10}{3}(30)^2\right\} - \left\{300\,(15) - \frac{10}{3}(15)^2\right\}\right]$$

$$= 3000 + [\{9000 - 3000\} - \{4500 - 750\}]$$

$$= 3000 + [6000 - 3750]$$

$$= 3000 + 2250 = 5250\text{ J}$$

5. As, $E_1 = \frac{1}{2}mv^2$

$$E_2 = \frac{1}{2}m\,(v+1)^2$$

$$\frac{E_2 - E_1}{E_1} = \frac{\frac{1}{2}m\,[(v+1)^2 - v^2]}{\frac{1}{2}mv^2} = \frac{44}{100}$$

On solving, we get $v = 5\text{ ms}^{-1}$.

6. Given, $m = 0.5$ kg and $u = 20$ m/s

Initial kinetic energy, $(K_i) = \frac{1}{2}mu^2$

$$= \frac{1}{2} \times 0.5 \times 20 \times 20 = 100\text{ J}$$

After deflection it moves with 5% of K_i

$\therefore \quad K_f = \frac{5}{100} \times K_i \Rightarrow \frac{5}{100} \times 100$

$\Rightarrow \quad K_f = 5$ J

Now, let the final speed be v m/s, then

$$K_f = 5 = \frac{1}{2}mv^2$$

$\Rightarrow \quad v^2 = 20$

$\Rightarrow \quad v = \sqrt{20} = 4.47$ m/s

7. $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$

$$x = \sqrt{\frac{2mv^2}{k}}$$

8. From work-energy theorem,

$$\Delta\text{KE} = W_{\text{net}}$$

or $\quad K_f - K_i = \int P\,dt$

or $\quad \frac{1}{2}mv^2 = \int_0^2 \left(\frac{3}{2}t^2\right)dt$

$$v^2 = \left[\frac{t^3}{2}\right]_0^2$$

$$v = 2\text{ ms}^{-1}$$

9. As slope of problem graph is positive and constant upto certain distance and then it becomes zero.

So, from $F = \frac{-dU}{dx}$, upto distance F = constant (negative) and become zero suddenly.

10. Here, $m_1 = 20$ kg

$m_2 = 0.1$ kg

where, v_1 = velocity of recoil of gun

and v_2 = velocity of bullet.

As, $m_1 v_1 = m_2 v_2$ ($\because$ momentum is conserved)

$$v_1 = \frac{m_2}{m_1} v_2 = \frac{0.1}{20} v_2 = \frac{v_2}{200}$$

Recoil energy of gun $= \frac{1}{2} m_1 v_1^2 = \frac{1}{2} \times 20 \left(\frac{v_2}{200}\right)^2$

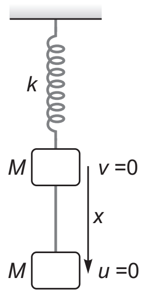
$$804 = \frac{10\, v_2^2}{4 \times 10^4} = \frac{v_2^2}{4 \times 10^3}$$

$$v_2 = \sqrt{804 \times 4 \times 10^3} \text{ ms}^{-1}$$

11. KE lost is $\frac{3}{4}$th, therefore KE left is $\frac{1}{4}$th. Hence, velocity of particle reduces from v_0 to $\frac{v_0}{2} = v_0 - \mu g\, t_0$

or $$\mu = \frac{v_0}{2 g t_0}$$

12. Let x be the maximum extension of the spring as shown in figure. From conservation of mechanical energy, decreases in gravitational potential energy = increase in elastic potential energy



$\therefore$ $$Mgx = \frac{1}{2} kx^2$$

$\therefore$ $$x = \frac{2\, Mg}{k}$$

13. Let v be the velocity with which the bullet will emerge. Now, change in kinetic energy = work done

For first case, $\frac{1}{2} m\, (100)^2 - \frac{1}{2} m \times 0 = F$ ...(i)

For second case, $\frac{1}{2} m (100)^2 - \frac{1}{2} m v^2 = F \times 0.5$ ...(ii)

Dividing Eq. (ii) by Eq.(i), we get

$$\frac{(100)^2 - (v)^2}{(100)^2} = \frac{0.5}{1} = \frac{1}{2}$$

or $$v = \frac{100}{\sqrt{2}} = 50\sqrt{2} \text{ ms}^{-1}$$

14. Initial velocity of particle, $v_i = 20 \text{ ms}^{-1}$

Final velocity of the particle, $v_f = 0$

According to work-energy theorem,

$$W_{\text{net}} = \Delta\text{KE} = K_f - K_i$$
$$= \frac{1}{2} m (v_f^2 - v_i^2) = \frac{1}{2} \times 2 (0^2 - 20^2)$$
$$= -400 \text{ J}$$

15. If W_1 = work done by applied force

W_2 = work done against friction then applying work-energy theorem

$$W_1 - W_2 = \text{PE} + \text{KE (at the top)}$$
$$F \times s - W_2 = mgh + \frac{1}{2} \times mv^2$$
$$100 \times 12 - W_2 = 50 \times 10 \times 2 + \frac{1}{2} \times 50 \times 2^2$$
$$1200 - W_2 = 1100$$
$$W_2 = 100 \text{ J}$$

16. Velocity at B when dropped from A

where, $AC = s$

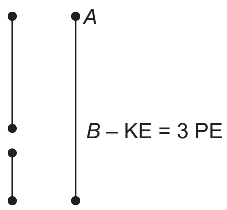
$v^2 = u^2 + 2g\,(s - x)$ ...(i)

$v^2 = 2g(s - x)$ ...(ii)

Potential energy at $B = mgx$

$\therefore$ kinetic energy = 3 × potential energy

$$\frac{1}{2} m \times 2g(s - x) = 3 \times mgx$$



or $(s - x) = 3x$

or $s = 4x$

or $x = \frac{s}{4}$

From Eq. (i),

$$v^2 = 2g\,(s - x)$$
$$= 2g\left(s - \frac{s}{4}\right)$$
$$= \frac{2g \times 3s}{4} = \frac{3gs}{2}$$

$\therefore$ $x = \frac{s}{4}$ and $v = \sqrt{\frac{3gs}{2}}$

17. Let mass of boy be m, therefore mass of man $= 2m$.

As, KE of man $= \frac{1}{2}$ KE of boy

$\therefore$ $$\frac{1}{2} (2m)\, u^2 = \frac{1}{2} \times \frac{1}{2} m\, u'^2$$

$$u^2 = \frac{u'^2}{4},\ u = \frac{u'}{2}$$

When man speeds up to 1 m/s

KE of man = KE of boy

$$\frac{1}{2} (2m)\, (u + 1)^2 = \frac{1}{2} m u'^2 = \frac{1}{2} m\, (2\,u)^2$$
$$(u + 1)^2 = 2\,u^2$$
$$u + 1 = \sqrt{2}\, u$$
$$u = \frac{1}{\sqrt{2} - 1} = \frac{\sqrt{2} + 1}{(\sqrt{2} - 1)\,(\sqrt{2} + 1)}$$
$$u = (\sqrt{2} + 1) \text{ ms}^{-1}$$
$$u' = 2\,u = 2\,(\sqrt{2} + 1) \text{ ms}^{-1}$$

18. $\frac{1}{2}mv^2 = a(t)$

or $\quad v \propto t^{1/2}$

$$a = \frac{dv}{dt} \propto t^{-1/2}$$

or $\quad F \propto \frac{1}{\sqrt{t}}$ and $F \propto \frac{1}{v}$

19. Here $K = \frac{1}{2}mv^2 = as^2$

$\therefore \quad mv^2 = 2as^2$

Differentiating w.r.t. time t, we get

$$2mv\frac{dv}{dt} = 4as\frac{ds}{dt} = 4\,asv \Rightarrow m\frac{dv}{dt} = 2as$$

This is tangential force, $F_t = 2as$

Centripetal force, $F_c = \frac{mv^2}{R} = \frac{2as^2}{R}$

$\therefore$ Force acting on the particle,

$$F = \sqrt{F_t^2 + F_c^2} = \sqrt{(2as)^2 + \left(\frac{2as^2}{R}\right)^2} = 2as\sqrt{1 + s^2/R^2}$$

20. From $F = -\frac{dU}{dx}$

$$dU = -Fdx$$

$$U = \int_0^{U(x)} dU = -\int_0^x Fdx = -\int_0^x kx\,dx$$

$$U = \frac{kx^2}{2}$$

As $U(0) = 0, \propto x^2$ and U is negative.

21. According to work-energy theorem,

$W =$ change in kinetic energy

$$Fs\cos\theta = \frac{1}{2}mv^2 - \frac{1}{2}mv^2$$

Substituting the given values, we get

$$20 \times 4 \times \cos\theta = 40 - 0$$

$$\cos\theta = \frac{40}{80} = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

22. Given, $\quad \mathbf{F} = 3x^2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}, \mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{k}}$

Let, $\quad d\mathbf{r} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}$

Work done, $W = \int \mathbf{F}\,d\mathbf{r} = \int_{(23)}^{(3,\,0)} (3x^2\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\,(dx\,\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$

$$= \int_{(2,\,3)}^{(3,\,0)} (3x^2\,dx + 4\,dy) = (x^3 + 4\,y)_{(2,\,3)}^{(3,\,0)}$$

$$= 3^3 + 4 \times 0 - (2^3 + 4 \times 3)$$

$$= 27 + 0 - 20 = +7\text{ J}$$

23. $\int F\,dt = \Delta p$

$$\Rightarrow \quad \frac{1}{2} \times 4 \times 3 - \frac{1}{2} \times 1.5 \times 2 = p_f - 0$$

$$\Rightarrow \quad p_f = 6 - 1.5 = \frac{9}{2}$$

$$\text{KE} = \frac{p^2}{2m} = \frac{81}{4 \times 2 \times 2}$$

$$\text{KE} = 5.06\text{ J}$$

24. From given information $a = -kx$, where a is acceleration, x is displacement and k is a proportionality constant.

$$\frac{vdv}{dx} = -kx \Rightarrow v\,dv = -kx\,dx$$

Let for any displacement from 0 to x, the velocity changes from v_0 to v.

$$\Rightarrow \quad \int_{v_0}^{v} v\;dv = -\int_0^x kx\,dx$$

$$\Rightarrow \quad \left[\frac{v^2}{2}\right]_{v_0}^{v} = -k\left[\frac{x^2}{2}\right]_0^x$$

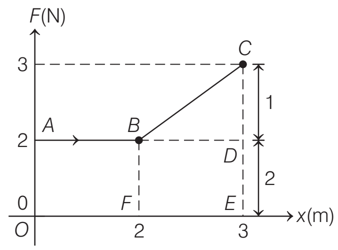
$$\Rightarrow \quad \frac{v^2 - v_0^2}{2} = -\frac{kx^2}{2}$$

$$\Rightarrow \quad m\left(\frac{v^2 - v_0^2}{2}\right) = -\frac{mkx^2}{2}$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = -\frac{mkx^2}{2}$$

$\Rightarrow \quad \Delta K \propto x^2 \qquad$ [ΔK is loss in KE]

25. Area under force-displacement graph gives the value of work done.



$\therefore$ Work done on the particle

$=$ Area under the curve ABC

$W =$ Area of square $ABFO +$ Area of ΔBCD

$+$ Area of rectangle $BDEF$

$$= 2 \times 2 + \frac{1}{2} \times 1 \times 1 + 2 \times 1 = 6.5\text{ J}$$

Now, from work-energy theorem,

$$\Delta W = K_f - K_i$$

$\Rightarrow \quad K_f = \Delta W = 6.5\text{ J} \qquad [\because K_i = 0]$

26. According to the work-energy theorem,

Net work done = Change in the kinetic energy

Here, net work done = work done due to external force (W_{ext}) + work done due to the spring (W_{spr}).

As, $\quad W_{\text{ext}} = F \cdot x$

and $\quad W_{\text{spr}} = \frac{-1}{2}kx^2$

$$\Rightarrow \quad \Delta\text{KE} = F \cdot x + \left(-\frac{1}{2}kx^2\right)$$

$$(\Delta\text{KE})_f - (\Delta\text{KE})_i = F \cdot x - \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}mv_{\max}^2 - \frac{1}{2}m(0)^2 = F \cdot \left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2$$

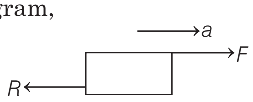
[using Eq. (i)]

$$\Rightarrow \quad \frac{1}{2}mv_{\max}^2 = \frac{F^2}{k} - \frac{F^2}{2k} = \frac{F^2}{2k}$$

or $$v_{\max}^2 = \frac{F^2}{km}$$

$$\Rightarrow \quad v_{\max} = F/\sqrt{km}$$

27. From the diagram,



$$F - R = ma \quad \text{or} \quad F = R + ma$$

Rate of doing work = Power = $F \cdot v = (R + ma) \cdot v$

28. We know that, $P = F \cdot v = F \cdot \frac{l}{t}$

As $\quad F =$ constant

$\therefore \quad l \propto t^2$

$$\therefore \quad P = F \cdot \frac{l}{t} = F \cdot \frac{t^2}{t} = F \cdot T$$

or $\quad P \propto t$

29. From $v = u + at, v_1 = 0 + at_1$ $\quad \left(\because a = \frac{v_1}{t_1}\right)$

$$F = ma = \frac{mv_1}{t_1}$$

Velocity acquired in t seconds $= at = \frac{v_1}{t_1}t$

$$\text{Power} = F \cdot v = \frac{mv_1}{t_1} \times \frac{v_1 t}{t_1} = \frac{mv_1^2 t}{t_1^2}$$

30. As, $\quad P = (mg\sin\theta + F)v$

$$= \left(1000 \times 10 \times \frac{1}{20} + 200\right) \times 20$$

$$= 14000 \text{ W} = 14\text{kW}$$

31. $\because$ Speed is constant.

$\therefore$ Work done by force = 0

$$\therefore \quad \text{Power} = \frac{\text{Work}}{\text{Time}} = 0$$

32. We know that,

$$P = F \cdot v \quad \text{...(i)}$$

But $$F = \frac{\Delta p}{\Delta t} = \frac{mv}{t} \quad [\because \Delta p = mv]$$

$$= \frac{mv}{(l/v)} \quad \left[\because l = v \cdot t \Rightarrow t = \frac{l}{v}\right]$$

$$= \frac{mv^2}{l}$$

$$\Rightarrow \quad F = \left(\frac{m}{l}\right) \cdot v^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$P = \left(\frac{m}{l}\right) \cdot v^2 \cdot v$$

$$= \left(\frac{m}{l}\right) \cdot v^3 = 100 \times 2^3 = 800 \text{ W}$$

33. Given, $P = 3t^2 - 2t + 1 = \frac{dE}{dt}$

$$\therefore \quad dE = (3t^2 - 2t + 1)dt$$

$$\Rightarrow \quad E = \int_{t=2\text{s}}^{t=4\text{s}} (3t^2 - 2t + 1)\, dt$$

$$= \left[\frac{3t^3}{3} - \frac{2t^2}{2} + t\right]_{t=2\text{s}}^{t=4\text{s}}$$

$$= [(4^3 - 2^3) - (4^2 - 2^2) + (4 - 2)]$$

or $$E = 56 - 12 + 2 = 46 \text{ J}$$

34. Work output of engine $= mgh = 100 \times 10 \times 10 = 10^4$ J

$$\text{Efficiency } (\eta) = \frac{\text{output}}{\text{input}} = \frac{10^4}{60} \times 100 = \frac{10^5}{6} \text{ J}$$

$$\therefore \quad \text{Power} = \frac{\text{input energy}}{\text{time}} = \frac{10^5/6}{5}$$

$$= \frac{10^5}{30} = 3.3 \text{ kW}$$

35. Power given to turbine $= \frac{mgh}{t}$

$$P_{\text{in}} = \left(\frac{m}{t}\right)gh = P_{\text{in}} = 15 \times 10 \times 60$$

$$P_{\text{in}} = 9000 \text{ W}$$

$$\Rightarrow \quad P_{\text{in}} = 9 \text{ kW}$$

As efficiency of turbine is 90%, therefore power generate = 90% of 9 kW

$$\Rightarrow \quad P_{\text{out}} = 9 \times \frac{90}{100}$$

$$\Rightarrow \quad P_{\text{out}} = 8.1 \text{ kW}$$

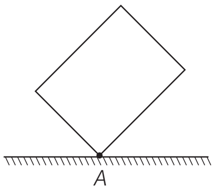
36. Work done by bowling machine = Initial kinetic energy of ball = Final potential energy of ball.

$\Rightarrow$ Force × displacement $= mgh$

$$\Rightarrow \quad F\,(0.2) = (0.15)\,(10)\,(20)$$

$$F = 150 \text{ N}$$

37. To slide the block, the boy has to overcome the force of friction $\mu\, mg = 300$ N



$$\therefore \quad W_1 = 300 \times 2 = 600 \text{ J}$$

To displace the block by tipping about corner A, the change in the height of centre of gravity is

$$h = \sqrt{2} - 1 = 1.414 - 1 = 0.414$$

$$\therefore \quad W_2 = mgh = 414 = 207 \times 2 \quad \text{(least)}$$

$$\therefore \quad n = 2$$

38. Here, $\frac{1}{2}mv^2 = b\,t^4$ or $mv^2 = 2bt^4$

The centripetal force on the body,

$$F_c = \frac{mv^2}{R} = \frac{2bt^4}{R}$$

and $$a_t = \frac{dv}{dt} = \sqrt{\frac{2b}{m}}\,\frac{d(t)^2}{dt} = \sqrt{\frac{8b}{m}}\,t$$

$\therefore$ Tangential force,

$$F_t = ma_t = m\sqrt{\frac{8b}{m}}\,t$$

The force on the particle,

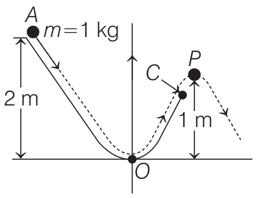
$$F = \sqrt{\left(\frac{2bt^4}{R}\right)^2 + 8mbt^2}$$

$$= \sqrt{4+8} = \sqrt{12} = 2\sqrt{3}\ \text{N}$$

$\therefore \qquad 2\sqrt{3} = (2x)^{1/2}$

$\Rightarrow \qquad 4 \times 3 = 2x \quad \Rightarrow \quad x = 6$

39. Following is the situation given



Since, the given path AOC is frictionless.
So, it starting from point A, the particle during the path AOC will attain maximum height at P.
Energy conservation at A and P gives
PE at A = (PE + KE) at P

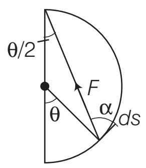
$\Rightarrow \qquad U_A = U_P + K_P$

$$mgh_A = mgh_P + K_P$$

$$1 \times 10 \times 2 = 1 \times 10 \times 1 + K_P$$

$\therefore \qquad K_P = 10\ \text{J}$

40. Here, $\frac{\pi}{2} - \alpha + \frac{\pi}{2} - \alpha = \theta$



or $\qquad \pi - 2\alpha = \theta \quad$ or $\quad 2\alpha = \pi - \theta$

or $\qquad \alpha = \frac{\pi}{2} - \frac{\theta}{2}$

$$dW = Fds\cos\alpha = F(r\,d\theta)\,\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

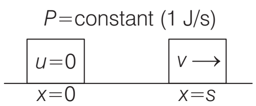
$$= Fr\sin\frac{\theta}{2}\,d\theta$$

Total work done, $W = \int dW = Fr\int_0^{\pi}\sin\frac{\theta}{2}\,d\theta$

$$= 2Fr\left[-\cos\frac{\theta}{2}\right]_0^{\pi} = 2\,Fr\,[0+1]$$

$$= 2Fr = 2 \times 10 \times 0.1 = 2\ \text{J}$$

41. Let s be the required distance.



From work-energy theorem,
Work = Change in kinetic energy

$\Rightarrow \qquad$ Power $\times$ Time $= \Delta K$

i.e. $\qquad Pt = \Delta K \;\Rightarrow\; Pt = \frac{1}{2}mv^2 \qquad$ ...(i)

Given, $\qquad P = 1\ \text{Js}^{-1},\ t = 9\ \text{s},\ m = 2\ \text{kg}$

Substituting all the given values in Eq. (i), we get

$$1 \times 9 = \frac{1}{2}(2)\,v^2$$

$$v^2 = 9 \;\Rightarrow\; v = 3\ \text{m/s} \qquad (\text{at } t = 9\ \text{s})$$

As, $\qquad Fv = P \;\Rightarrow\; (ma)v = P \qquad [\because F = ma]$

$\Rightarrow \qquad m\left[\frac{dv}{dt}\right]v = P$

$\Rightarrow \qquad m\left[\frac{ds}{dt}\frac{dv}{ds}\right]v = P$

$\Rightarrow \qquad m\left[v\frac{dv}{ds}\right]v = P$

$\Rightarrow \qquad 2v^2dv = ds \qquad \{\because P = 1\ \text{J/s and } m = 2\ \text{kg}\}$

Integrating both sides, we get

$$\int_0^3 2\,v^2dv = \int_0^s ds \;\Rightarrow\; \frac{2}{3}[v^3]_0^3 = s$$

$$\frac{2}{3}[27 - 0] = s \;\Rightarrow\; s = 18\ \text{m}$$

Hence, after 9 s, the body has moved a distance of 18 m.

42. Let's say the compression in the spring by y,
so, by work-energy theorem, we have

$\Rightarrow \qquad \frac{1}{2}mv^2 = \frac{1}{2}ky^2$

$\Rightarrow \qquad y = \sqrt{\frac{m}{k}}\cdot v \;\Rightarrow\; y = \sqrt{\frac{4}{100}} \times 10$

$\Rightarrow \qquad y = 2\ \text{m}$

Final length of spring $= 8 - 2 = 6$ m

43. Let for bouncing of the block A, the elongation in the spring is x_1.

$$kx_1 > m_A\,g \quad \text{or} \quad kx_1 > 100$$

$\therefore \qquad x_1 > \frac{100}{150} \quad$ or $\quad x_1 > \frac{2}{3}$

$\therefore \qquad x_{1\,\min} = \frac{2}{3}$

For $x_{1\,\min}$, the velocity of block B will be zero.
According to conservation principle of mechanical energy,

$$U_i + T_i = U_f + T_f$$

$$\frac{1}{2}kx_0^2 + \frac{1}{2}mv_{0\,\min}^2 = mg(x_0 + x_{1\,\min}) + \frac{1}{2}kx_{1\,\min}^2$$

Here, $\qquad x_0 = \frac{mg}{k}$

By putting the value, we get

$$v_{0\,\min} = \frac{20}{\sqrt{15}}\ \text{m/s}$$

$\therefore \qquad n = 5$