DPP - Daily Practice Problems

Date : ______ Start Time : ______ End Time : ______

PHYSICS

CP14

SYLLABUS : Waves

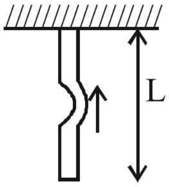
Max. Marks : 120 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 30 MCQs. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

1. When a wave travel in a medium, the particle displacement is given by the equation $y = a \sin 2\pi (bt - cx)$ where a, b and c are constants. The maximum particle velocity will be twice the wave velocity if

(a) $c = \frac{1}{\pi a}$ (b) $c = \pi a$ (c) $b = ac$ (d) $b = \frac{1}{ac}$

2. A thick uniform rope of length L is hanging from a rigid support. A transverse wave of wavelength λ_0 is set up at the middle of rope as shown in figure. The wavelength of the wave as it reaches to the topmost point is



(a) $2\lambda_0$ (b) $\sqrt{2}\lambda_0$ (c) $\frac{\lambda_0}{\sqrt{2}}$ (d) λ_0

3. An engine approaches a hill with a constant speed. When it is at a distance of 0.9 km, it blows a whistle whose echo is heard by the driver after 5 seconds. If the speed of sound in air is 330 m/s, then the speed of the engine is :

(a) 32 m/s (b) 27.5 m/s
(c) 60 m/s (d) 30 m/s

4. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively?

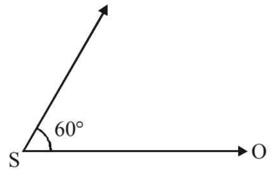
(a) 188.5 Hz (b) 178.2 Hz
(c) 200.5 Hz (d) 770 Hz

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

5. The fundamental frequency of a sonometer wire of length ℓ is f_0. A bridge is now introduced at a distance of $\Delta\ell$ from the centre of the wire ($\Delta\ell << \ell$). The number of beats heard if both sides of the bridges are set into vibration in their fundamental modes are –

(a) $\frac{8f_0\Delta\ell}{\ell}$ (b) $\frac{f_0\Delta\ell}{\ell}$ (c) $\frac{2f_0\Delta\ell}{\ell}$ (d) $\frac{4f_0\Delta\ell}{\ell}$

6. A source of sound S emitting waves of frequency 100 Hz and an observor O are located at some distance from each other. The source is moving with a speed of 19.4 ms^{-1} at an angle of 60° with the source observer line as shown in the figure. The observor is at rest. The apparent frequency observed by the observer is (velocity of sound in air is 330 ms^{-1})



(a) 103 Hz (b) 106 Hz (c) 97 Hz (d) 100 Hz

7. Two identical piano wires kept under the same tension T have a fundamental frequency of 600 Hz. The fractional increase in the tension of one of the wires which will lead to occurrence of 6 beats/s when both the wires oscillate together would be

(a) 0.02 (b) 0.03 (c) 0.04 (d) 0.01

8. Two sound sources emitting sound each of wavelength λ are fixed at a given distance apart. A listener moves with a velocity u along the line joining the two sources. The number of beats heard by him per second is

(a) $\frac{u}{2\lambda}$ (b) $\frac{2u}{\lambda}$ (c) $\frac{u}{\lambda}$ (d) $\frac{u}{3\lambda}$

9. A star, which is emitting radiation at a wavelength of 5000 Å, is approaching the earth with a velocity of 1.50×10^6 m/s. The change in wavelength of the radiation as received on the earth is

(a) 0.25 Å (b) 2.5 Å (c) 25 Å (d) 250 Å

10. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is

(a) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (b) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$

(c) $300\left(\frac{2\rho}{2\rho-1}\right)$ (d) $300\left(\frac{2\rho-1}{2\rho}\right)$

11. The transverse displacement y (x, t) of a wave on a string is given by $y(x,t) = e^{-\left(ax^2+bt^2+2\sqrt{ab}xt\right)}$.

This represents a:

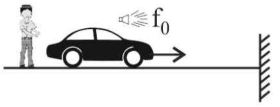
(a) wave moving in –x direction with speed $\sqrt{\frac{b}{a}}$

(b) standing wave of frequency $\sqrt{b}$

(c) standing wave of frequency $\frac{1}{\sqrt{b}}$

(d) wave moving in + x direction with speed $\sqrt{\frac{a}{b}}$

12. In the figure shown the wave speed is v. The velocity of car is v_0. The beat frequency for the observer will be



(a) $\frac{2f_0vv_0}{v^2+v_0^2}$ (b) $\frac{2f_0v^2}{v^2-v_0^2}$

(c) $\frac{2f_0vv_0}{v^2-v_0^2}$ (d) $\frac{f_0vv_0}{v^2-v_0^2}$

13. A source of sound A emitting waves of frequency 1800 Hz is falling towards ground with a terminal speed v. The observer B on the ground directly beneath the source receives waves of frequency 2150 Hz. The source A receives waves, reflected from ground of frequency nearly: (Speed of sound = 343 m/s)

(a) 2150 Hz (b) 2500 Hz (c) 1800 Hz (d) 2400 Hz

RESPONSE GRID	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	

14. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s.

(a) 12 (b) 8 (c) 6 (d) 4

15. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02(\text{m}) \sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right].$$

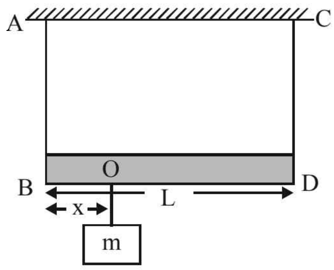
The tension in the string is

(a) 4.0 N (b) 12.5 N (c) 0.5 N (d) 6.25 N

16. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of

(a) 100 (b) 1000 (c) 10000 (d) 10

17. A massless rod of length L is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to 'x'. Further it is observed that the frequency of 1st harmonic in AB is equal to 2nd harmonic frequency in CD. 'x' is



(a) $\frac{L}{5}$

(b) $\frac{4L}{5}$

(c) $\frac{3L}{4}$

(d) $\frac{L}{4}$

18. An organ pipe P_1, closed at one end vibrating in its first harmonic and another pipe P_2, open at both ends vibrating in its third harmonic, are in resonance with a given tuning fork. The ratio of the lengths of P_1 and P_2 is :

(a) $\frac{8}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

19. Two points are located at a distance of 10 m and 15 m from the source of oscillation. The period of oscillation is 0.05 sec and the velocity of the wave is 300 m/sec. What is the phase difference between the oscillations of two points?

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) π (d) $\frac{\pi}{6}$

20. A uniform rope of length L and mass m_1 hangs vertically from a rigid support. A block of mass m_2 is attached to the free end of the rope. A transverse pulse of wavelength λ_1 is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope is λ_2 the ratio λ_2/λ_1 is

(a) $\sqrt{\frac{m_1}{m_2}}$ (b) $\sqrt{\frac{m_1+m_2}{m_2}}$

(c) $\sqrt{\frac{m_2}{m_1}}$ (d) $\sqrt{\frac{m_1+m_2}{m_1}}$

21. If n_1, n_2 and n_3 are the fundamental frequencies of three segments into which a string is divided, then the original fundamental frequency n of the string is given by

(a) $n = n_1 + n_2 + n_3$

(b) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

(c) $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}} + \frac{1}{\sqrt{n_3}}$

(d) $\sqrt{n} = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3}$

22. The vibrations of a string of length 60 cm fixed at both the ends are represented by the equation $y = 2\sin\left(\frac{4\pi x}{15}\right)\cos(96\pi t)$ where x and y are in cm. The maximum number of loops that can be formed in it is

(a) 4 (b) 16 (c) 5 (d) 15

RESPONSE GRID					
	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ
	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	

23. The frequency of whistle of an engine appears to be $(4/5)^{th}$ of initial frequency when it crosses a stationary observer. If the velocity of sound is 330 m/s, then the speed of engine will be

(a) 30 m/s (b) 36.6 m/s (c) 40 m/s (d) 330 m/s

24. A sonometer wire supports a 4 kg load and vibrates in fundamental mode with a tuning fork of frequency 416 Hz. The length of the wire between the bridges is now doubled. In order to maintain fundamental mode, the load should be changed to

(a) 1 kg (b) 2 kg (c) 4 kg (d) 16 kg

25. The wavelength of two waves are 50 and 51 cm respectively. If the temperature of the room is 20°C then what will be the number of beats produced per second by these waves, when the speed of sound at 0°C is 332 m/s?

(a) 24 (b) 14 (c) 10 (d) 25

26. In a resonance tube with tuning fork of frequency 512 Hz, first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7 cm. The maximum possible error in the speed of sound is

(a) 51.2 cm/s (b) 102.4 cm/s
(c) 204.8 cm/s (d) 153.6 cm/s

27. Each of the two strings of length 51.6 cm and 49.1 cm are tensioned separately by 20 N force. Mass per unit length of both the strings is same and equal to 1 g/m. When both the strings vibrate simultaneously the number of beats is

(a) 7 (b) 8 (c) 3 (d) 5

28. The fundamental frequency of a closed organ pipe of length 20 cm is equal to the second overtone of an organ pipe open at both the ends. The length of organ pipe open at both the ends is

(a) 100 cm (b) 120 cm (c) 140 cm (d) 80 cm

29. In a standing wave experiment, a 1.2 kg horizontal rope is fixed in place at its two ends (x = 0 and x = 2.0 m) and made to oscillate up and down in the fundamental mode, at frequency 5.0 Hz. At t = 0, the point at x = 1.0 m has zero displacement and is moving upward in the positive direction of y axis with a transverse velocity 3.14 m/s.

Tension in the rope is

(a) 60 N (b) 100 N (c) 120 N (d) 240 N

30. The equation of a travelling wave is $y = 60\cos(180t - 6x)$ where y is in μm, *t* in second and x in metres. The ratio of maximum particle velocity to velocity of wave propagation is

(a) 3.6×10^{-2} (b) 3.6×10^{-4}
(c) 3.6×10^{-6} (d) 3.6×10^{-11}

RESPONSE GRID	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ
	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ		

DAILY PRACTICE PROBLEM DPP CHAPTERWISE CP14 - PHYSICS			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	45	Qualifying Score	60
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

DAILY PRACTICE PROBLEMS

PHYSICS SOLUTIONS

DPP/CP14

1. (a) Equation of the harmonic progressive wave given by : $y = a\sin 2\pi(bt - cx)$.

Here $\upsilon = b$

$k = \frac{2\pi}{\lambda} = 2\pi c$ take, $\frac{1}{\lambda} = c$

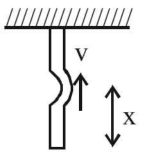
$\therefore$ Velocity of the wave $= \upsilon\lambda = b\frac{1}{c} = \frac{b}{c}$

$\frac{dy}{dt} = a2\pi b\cos 2\pi(bt - cx) = a\omega\cos(\omega t - kx)$

Maximum particle velocity $= a\omega = a2\pi b = 2\pi ab$

given this is $2\times\frac{b}{c}$ i.e. $2\pi a = \frac{2}{c}$ or $c = \frac{1}{\pi a}$

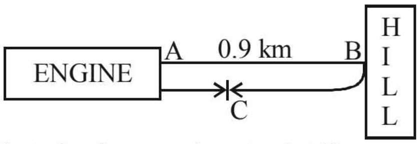
2. (b) Speed of pulse at a distance x from bottom, $v = \sqrt{gx}$.



While traveling from mid point to the top, frequency remains unchanged.

$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2} \Rightarrow \frac{\sqrt{g(L/2)}}{\lambda_0} = \frac{\sqrt{gL}}{\lambda_2} \Rightarrow \lambda_2 = \sqrt{2}\lambda_0$

3. (d)



Let after 5 sec engine at point C

$t = \frac{AB}{330} + \frac{BC}{330}$

$5 = \frac{0.9\times 1000}{330} + \frac{BC}{330}$

$\therefore BC = 750\,m$

Distance travelled by engine in 5 sec

$= 900\,m - 750\,m = 150\,m$

Therefore velocity of engine

$= \frac{150\,m}{5\,sec} = 30$ m/s

4. (b) Fundamental frequency,

$f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{A\rho}}$

$\left[\because v = \sqrt{\frac{T}{\mu}} \text{ and } \mu = \frac{m}{\ell}\right]$

Also, $Y = \frac{T\ell}{A\Delta\ell} \Rightarrow \frac{T}{A} = \frac{Y\Delta\ell}{\ell}$

$\Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{\gamma\Delta\ell}{\ell\rho}}$ ....(i)

$\ell = 1.5$ m, $\frac{\Delta\ell}{\ell} = 0.01$,

$\rho = 7.7\times 10^3$ kg/m^3 (given)

$\gamma = 2.2\times 10^{11}$ N/m^2 (given)

Putting the value of ℓ, $\frac{\Delta\ell}{\ell}$, ρ and γ in eq^n. (i) we get,

$f = \sqrt{\frac{2}{7}}\times\frac{10^3}{3}$ or $f \approx 178.2$ Hz

5. (a) $f_0 = \frac{v}{2\ell}$

Now beat frequency $= f_1 - f_2$

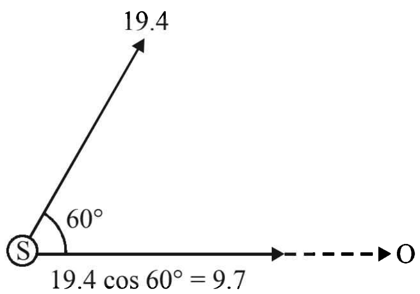
$= \frac{v}{2\left(\frac{\ell}{2} - \Delta\ell\right)} - \frac{v}{2\left(\frac{\ell}{2} + \Delta\ell\right)} = \frac{v}{2}\left[\frac{1}{\frac{\ell}{2} - \Delta\ell} - \frac{1}{\frac{\ell}{2} + \Delta\ell}\right]$

$= (f_0\ell)\left[\frac{2}{\ell - 2\Delta\ell} - \frac{2}{\ell + 2\Delta\ell}\right]$

$= 2f_0\ell\left[\frac{\ell + 2\Delta\ell - \ell + 2\Delta\ell}{\ell^2 - 4(\Delta\ell)^2}\right] \approx 2f_0\ell\left(\frac{4\Delta\ell}{\ell^2}\right) \approx \frac{8f_0\Delta\ell}{\ell}$

6. (a) Here, original frequency of sound, $f_0 = 100$ Hz

Speed of source $V_s = 19.4\cos 60° = 9.7$



From Doppler's formula

$f = f_0\left(\frac{V - V_0}{V - V_s}\right)$

$f = 100\left(\frac{V - 0}{V - (+9.7)}\right)$

$f = 100\frac{V}{V\left(1 - \frac{9.7}{V}\right)}$

$f = 100\left(1 + \frac{9.7}{330}\right) = 103$Hz

Apparent frequency f = 103 Hz

7. **(a)** For fundamental mode,

$$f=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

Taking logarithm on both sides, we get

$$\log f=\log\left(\frac{1}{2\ell}\right)+\log\left(\sqrt{\frac{T}{\mu}}\right)$$

$$=\log\left(\frac{1}{2\ell}\right)+\frac{1}{2}\log\left(\frac{T}{\mu}\right)$$

or $\log f=\log\left(\frac{1}{2\ell}\right)+\frac{1}{2}[\log T-\log\mu]$

Differentiating both sides, we get

$$\frac{df}{f}=\frac{1}{2}\frac{dT}{T} \quad \text{(as } \ell \text{ and } \mu \text{ are constants)}$$

$$\Rightarrow \frac{dT}{T}=2\times\frac{df}{f}$$

Here df = 6

f = 600 Hz

$$\therefore \frac{dT}{T}=\frac{2\times 6}{600}=0.02$$

8. **(b)** Frequency received by listener from the rear source,

$$n'=\frac{v-u}{v}\times n=\frac{v-u}{v}\times\frac{v}{\lambda}=\frac{v-u}{\lambda}$$

Frequency received by listener from the front source,

$$n''=\frac{v+u}{v}\times\frac{v}{\lambda}=\frac{v+u}{\lambda}$$

No. of beats = $n''-n'$

$$=\frac{v+u}{\lambda}-\frac{v-u}{\lambda}=\frac{v+u-v+u}{\lambda}=\frac{2u}{\lambda}$$

9. **(c)** Given : Wavelength (λ) = 5000 Å

velocity of star (v) = 1.5×10^6 m/s.

We know that wavelength of the approaching star (λ') = $\lambda\frac{c-v}{c}$

or, $\frac{\lambda'}{\lambda}=\frac{c-v}{c}=1-\frac{v}{c}$

or, $\frac{v}{c}=1-\frac{\lambda'}{\lambda}=\frac{\lambda-\lambda'}{\lambda}=\frac{\Delta\lambda}{\lambda}$. Therefore,

$$\Delta\lambda=\lambda\times\frac{v}{c}=5000\times\frac{1.5\times10^6}{3\times10^8}=25\text{ Å}.$$

[where $\Delta\lambda$ = Change in the wavelength]

10. **(a)** *In air* : $T=mg=\rho Vg$

$$\therefore\ f=\frac{1}{2\ell}\sqrt{\frac{\rho Vg}{m}} \quad \text{...(i)}$$

In water : $T=mg$ – upthrust

$$=V\rho g-\frac{V}{2}\rho_\omega g=\frac{Vg}{2}(2\rho-\rho_\omega)$$

$$\therefore\ f'=\frac{1}{2\ell}\sqrt{\frac{\frac{Vg}{2}(2\rho-\rho_\omega)}{m}}=\frac{1}{2\ell}\sqrt{\frac{Vg\rho}{m}}\sqrt{\frac{(2\rho-\rho_\omega)}{2\rho}}$$

$$\frac{f'}{f}=\sqrt{\frac{2\rho-\rho_\omega}{2\rho}} \qquad f'=f\left(\frac{2\rho-\rho_\omega}{2\rho}\right)^{1/2}$$

$$=300\left[\frac{2\rho-1}{2\rho}\right]^{1/2}\text{ Hz}$$

11. **(a)** Given wave equation is $y(x,t)$

$$=e^{\left(-ax^2+bt^2+2\sqrt{ab}\,xt\right)}$$

$$=e^{-[(\sqrt{a}x)^2+(\sqrt{b}t)^2+2\sqrt{a}x.\sqrt{b}t]}$$

$$=e^{-(\sqrt{a}x+\sqrt{b}t)^2}$$

$$=e^{-\left(x+\sqrt{\frac{b}{a}}t\right)^2}$$

It is a function of type $y=f(x+vt)$

$\Rightarrow$ Speed of wave = $\sqrt{\frac{b}{a}}$

12. **(c)** $f_2=\frac{f_0v}{v+v_0}$

The wave which reaches wall f_1 is reflected.

$$f_1=\frac{f_0v}{v-v_0}$$

The reflected frequency is f_1 as the wall is at rest.

$$Beats=f_1-f_2=\frac{f_0v}{v-v_0}-\frac{f_0v}{v+v_0}=\frac{2f_0vv_0}{v^2-v_0^2}$$

13. **(b)** Given $f_A=1800\text{Hz}$

$v_t=v$

$f_B=2150$ Hz

Reflected wave frequency received by A, $f_A'=?$

Applying doppler's effect of sound,

$$f'=\frac{v_sf}{v_s-v_t}$$

here, $v_t=v_s\left(1-\frac{f_A}{f_B}\right)$

$$=343\left(1-\frac{1800}{2150}\right)$$

v_t = 55.8372 m/s

Now, for the reflected wave,

$$\therefore\ f_A'=\left(\frac{v_s+v_t}{v_s-v_t}\right)f_A$$

$$=\left(\frac{343+55.83}{343-55.83}\right)\times1800$$

= 2499.44 ≈ 2500Hz

14. **(c)** Length of pipe = 85 cm = 0.85m

Frequency of oscillations of air column in closed organ pipe is given by,

$$f=\frac{(2n-1)v}{4L}$$

$$f = \frac{(2n-1)v}{4L} \le 1250$$

$$\Rightarrow \frac{(2n-1)\times 340}{0.85\times 4} \le 1250$$

$$\Rightarrow 2n-1 \le 12.5 \approx 6$$

15. (d) $y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)}\right) - \frac{x}{0.50(m)}\right]$

But $y = a\sin(\omega t - kx)$

$\therefore \omega = \frac{2\pi}{0.04} \Rightarrow \nu = \frac{1}{0.04} = 25\,Hz$

$k = \frac{2\pi}{0.50} \Rightarrow \lambda = 0.5\,\text{m}$

$\therefore$ velocity, $v = \nu\lambda = 25 \times 0.5$ m/s $= 12.5$ m/s

Velocity on a string is given by

$v = \sqrt{\frac{T}{\mu}} \quad \therefore T = v^2 \times \mu = (12.5)^2 \times 0.04 = 6.25\,\text{N}$

16. (a) We have, $L_1 = 10\log\left(\frac{I_1}{I_0}\right)$; $L_2 = 10\log\left(\frac{I_2}{I_0}\right)$

$\therefore L_1 - L_2 = 10\log\left(\frac{I_1}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right)$

or, $\Delta L = 10\log\left(\frac{I_1}{I_0}\times\frac{I_0}{I_2}\right)$ or, $\Delta L = 10\log\left(\frac{I_1}{I_2}\right)$

or, $20 = 10\log\left(\frac{I_1}{I_2}\right)$ or, $2 = \log\left(\frac{I_1}{I_2}\right)$

or, $\frac{I_1}{I_2} = 10^2$ or, $I_2 = \frac{I_1}{100}$.

$\Rightarrow$ Intensity decreases by a factor 100.

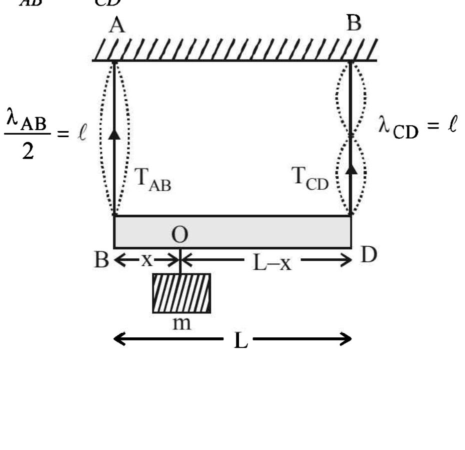
17. (a) Frequency of Ist harmonic of $AB = \frac{1}{2\ell}\sqrt{\frac{T_{AB}}{m}}$

Frequency of 2nd harmonic of $CD = \frac{1}{\ell}\sqrt{\frac{T_{CD}}{m}}$

Given that the two frequencies are equal.

$\therefore \frac{1}{2\ell}\sqrt{\frac{T_{AB}}{m}} = \frac{1}{\ell}\sqrt{\frac{T_{CD}}{m}} \Rightarrow \frac{T_{AB}}{4} = T_{CD}$

$\Rightarrow T_{AB} = 4T_{CD}$ ...(i)



For rotational equilibrium of massless rod, taking torque about point O.

$T_{AB} \times x = T_{CD}(L-x)$ ...(ii)

For translational equilibrium,

$T_{AB} + T_{CD} = mg$ ...(iii)

On solving, (i) and (iii), we get

$T_{CD} = \frac{mg}{5} \quad \therefore T_{AB} = \frac{4mg}{5}$

Substituting these values in (ii), we get

$\frac{4mg}{5}\times x = \frac{mg}{5}(L-x) \Rightarrow x = \frac{L}{5}$

18. (b) $\frac{v}{4\ell_1} = \frac{3v}{2\ell_2}$, $\quad \therefore \frac{\ell_1}{\ell_2} = \frac{1}{6}$

19. (b) Here, T = 0.05 sec, v = 300 ms^{-1}.

Now $\lambda = \frac{v}{\nu} = vT = (300\times 0.05)$m

or, $\lambda = 15$ m

Phase of the point at 10 m from the source

$= \frac{2\pi}{\lambda}\times x = \frac{2\pi}{15}\times 10 = \frac{4\pi}{3}$ rad

Phase of the point at 15 m from the source

$\frac{2\pi}{\lambda}\times x = \frac{2\pi}{15}\times 15 = 2\pi$ rad

$\therefore$ The phase difference between the points

$= 2\pi - \frac{4\pi}{3} = \frac{2\pi}{3}$ rad

20. (b) From figure, tension $T_1 = m_2 g$

$T_2 = (m_1 + m_2)g$

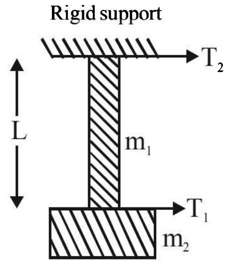
As we know

Velocity $\propto \sqrt{T}$ So,

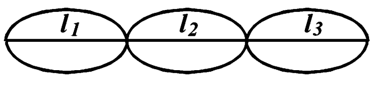
$\lambda \propto \sqrt{T}$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$

$\Rightarrow \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{m_1 + m_2}{m_2}}$



21. (b)



$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

or, $n \propto \frac{1}{l}$ or nl = constant, K

$\therefore n_1 l_1 = K,$

$n_2 l_2 = K, n_3 l_3 = K$

Also, $l = l_1 + l_2 + l_3$

or, $\frac{K}{n}=\frac{K}{n_1}+\frac{K}{n_2}+\frac{K}{n_3}$

or, $\frac{1}{n}=\frac{1}{n_1}+\frac{1}{n_2}+\frac{1}{n_3}$

22. **(b)** Let the string vibrates in p loops, wavelength of the p^{th} mode of vibration is given by

$\lambda_p=\frac{2l}{p}$

Given, $y=2\sin\left(\frac{4\pi x}{15}\right)\cos(96\pi t)$

or $y=2\left[\sin\left(\frac{4\pi x}{15}+96\pi t\right)+\sin\left(\frac{4\pi x}{15}-96\pi t\right)\right]$

Comparing it with standard equation, we get

$\upsilon=\frac{96\pi}{2\pi}=48$ Hz and $k=\frac{4\pi}{15}$

$\frac{1}{48}=\frac{2\times60}{p}\times\frac{4\pi}{15\times96\pi}$

$\Rightarrow p=16.$

23. **(b)** $n'=\frac{nv}{v-v_s}$ ....(1)

$n''=\frac{nv}{v+v_s}$ ....(2)

From (1) and (2), $\frac{n'}{n''}=\frac{v+v_s}{v-v_s}$ ....(3)

According to question, $\frac{n'}{n''}=\frac{5}{4}$

$v_s=?\ v=330$ m/s ....(4)

From eq. (3) and (4)

$\frac{5}{4}=\left[\frac{330+v_s}{330-v_s}\right]$

$9v_s=330$

$\therefore\ v_s=36.6$ m/s

24. **(d)** Load supported by sonometer wire = 4 kg

Tension in sonometer wire = 4 g

If μ = mass per unit length

then frequency $\upsilon=\frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$\Rightarrow\ 416=\frac{1}{2l}\sqrt{\frac{4g}{\mu}}$

When length is doubled, i.e., $l'=2l$

Let new load = L

As, $\upsilon'=\upsilon$

$\therefore\ \frac{1}{2l'}\sqrt{\frac{Lg}{\mu}}=\frac{1}{2l}\sqrt{\frac{4g}{\mu}}$

$\Rightarrow\ \frac{1}{4l}\sqrt{\frac{Lg}{\mu}}=\frac{1}{2l}\sqrt{\frac{4g}{\mu}}$

$\Rightarrow\ \sqrt{L}=2\times2\ \Rightarrow\ L=16$ kg

25. **(b)** $\lambda_1=50$ cm. $\lambda_2=51$ cm.

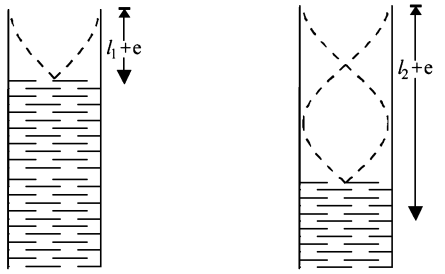
$v\propto\sqrt{T}\Rightarrow\frac{v_1}{v_2}=\sqrt{\frac{T_2}{T_1}}=\sqrt{\frac{273+20}{273}}\Rightarrow v_2=319.23.$

$\nu_1=\frac{v_2}{\lambda_1}=\frac{319.23}{0.50}=640$ Hz.

$\nu_2=\frac{v_2}{\lambda_2}=\frac{319.23}{51\times10^{-2}}=625.94=626$ Hz.

No. of beats $=\nu_2-\nu_1=14$ Hz

26. **(c)**



For first resonance **For second resonance**

$\ell_1+e=\frac{\lambda}{4}$ $\ell_2+e=\frac{3\lambda}{4}$

But $v=\nu\lambda$

$\therefore\ v=\nu\frac{4}{3}(\ell_2+e)\ \Rightarrow\ \ell_2+e=\frac{3v}{4\nu}$ ...(i)

$\therefore\ v=\nu\,4(\ell_1+e)\ \Rightarrow\ \ell_1+e=\frac{v}{4\nu}$ ...(ii)

Subtracting (i) and (ii),

$v=2\nu(\ell_2-\ell_1)\ \therefore\ \Delta v=2\nu(\Delta\ell_2+\Delta\ell_1)$

$=2\times512\times(0.1+0.1)$ cm/s $=204.8$ cm/s

27. **(a)** The frequency of vibration of a string is given by,

$f=\frac{1}{2l}\sqrt{\frac{T}{m}}$ where m is mass per unit length.

$f_1=\frac{1}{2l_1}\sqrt{\frac{T}{m}},\ f_2=\frac{1}{2l_2}\sqrt{\frac{T}{m}},$

$f_2-f_1=\frac{1}{2}\sqrt{\frac{T}{m}}\frac{(l_1-l_2)}{l_1l_2}$

$\sqrt{\frac{T}{m}}=\sqrt{\frac{20}{10^{-3}}}=\sqrt{2}\times10^2=1.414\times100$

$=141.4$

$\frac{l_1-l_2}{l_1l_2}=\frac{(51.6-49.1)\times10^2}{51.6\times49.1}$

$=\frac{2.5\times10^2}{50\times50}=\frac{1}{10}$

$\therefore\ f_2-f_1=\frac{1}{2}\times141.4\times\frac{1}{10}=7$ beats

28. **(b)** Fundamental frequency of closed organ pipe

$V_c=\frac{V}{4l_c}$

Fundamental frequency of open organ pipe

$$V_0 = \frac{V}{2l_0}$$

Second overtone frequency of open organ pipe $= \frac{3V}{2l_0}$

From question,

$$\frac{V}{4l_c} = \frac{3V}{2l_0}$$

$\Rightarrow \quad l_0 = 6l_c = 6 \times 20 = 120$ cm

29. (d) $\mu = \frac{1.2}{2} = 0.6 \text{ kg/m}$

$f = 5$ Hz, $\lambda = 2\ell = 4$m

$v = n\lambda = 5 \times 4 = 20$ m/s

Using $v = \sqrt{\frac{T}{\mu}} \Rightarrow T = 20^2 \times 0.6 = 240 \text{ N}$

$$\left(\frac{\partial y}{\partial t}\right)_{max} = 3.14 \text{ m/s}$$

$(2A)\omega = 3.14$

Amplitude $2A = \frac{3.14}{2 \times (3.14) \times 5} = 0.1\text{m}$

Equation of standing wave is

$y = (0.1) \sin(\pi/2) x \sin(10\pi) t$

30. (b) $y = 60 \cos(180t - 6x)$ ....(1)

$\omega = 180, k = 6 \Rightarrow \frac{2\pi}{\lambda} = 6$

$$v = \frac{\omega}{k} = \frac{2\pi}{T} \times \frac{\lambda}{2\pi} = \frac{180}{6} = 30 \text{ m/s}$$

Differentiating (1) w.r.t. t,

$$v = \frac{dy}{dt} = -60 \times 180 \sin(180t - 6x)$$

$v_{max} = 60 \times 180 \ \mu\text{m/s}$

$= 10800 \ \mu\text{m/s} = 0.0108 \text{ m/s}$

$$\frac{v_{max}}{v} = \frac{0.0108}{30} = 3.6 \times 10^{-4}$$