CHAPTER 4

First Law of Thermodynamics

LEVEL 1

Q. 1: In different experiments an ideal gas is expanded through

(i) Isothermal
(ii) Adiabatic
(iii) Isobaric process

In which of the processes mentioned the internal energy of the gas may decrease?

Q. 2: Which of the words-out of reversible, irreversible, adiabatic, isothermal, isochoric and isobaric-will you choose to describe the following processes–

(i) A bullet stops in a target [system is bullet plus target].
(ii) A gas, enclosed in a metallic cylinder provided with a piston, is slowly expanded [System is gas]. There is friction between piston and cylinder.
(iii) A piece of hot stone (which has coefficient of thermal expansion equal to zero) is dipped into cold water [System is stone].

Q. 3: Calculate the amount of heat required in calorie to change 1 g of ice at $-10°C$ to steam at $120°C$. The entire process is carried out at atmospheric pressure. Specific heat of ice and water are 0.5 cal $g^{-1}°C^{-1}$ and 1.0 cal $g^{-1}°C^{-1}$ respectively. Latent heat of fusion of ice and vaporization of water are 80 cal g^{-1} and 540 cal g^{-1} respectively. Assume steam to be an ideal gas with its molecules having 6 degrees of freedom. Gas constant $R = 2$ cal $mol^{-1} K^{-1}$.

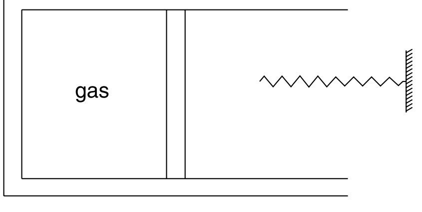
Q. 4: A mixture of 1 mole helium and 1 mole nitrogen is enclosed in a vessel of constant volume at 300 K. Find the quantity of heat absorbed by the mixture if the root mean square speed of its molecules get doubled. Give your answer in terms of universal gas constant R.

Q. 5: The average number of degree of freedom per molecule for a gas is 7. A sample of the gas perform 30 J of work when it expands at constant pressure. Find the heat absorbed by the gas in the process.

Q. 6: An ideal gas is made to undergo a process $T = T_0 e^{\alpha V}$ where T_0 and α are constants. Find the molar specific heat capacity of the gas in the process if its molar specific heat capacity at constant volume is C_v. Express your answer as a function of volume (V).

Q. 7: An ideal diatomic gas undergoes a process in which the pressure is proportional to the volume. Calculate the molar specific heat capacity of the gas for the process.

Q. 8: (i) A horizontal cylinder is fitted with a smooth movable piston. The cylinder contains an ideal gas. The gas is heated slowly so that the piston gradually moves out. After moving out for some distance the piston encounters an ideal spring and compresses it while moving out. Draw $P - V$ diagram for the entire process.



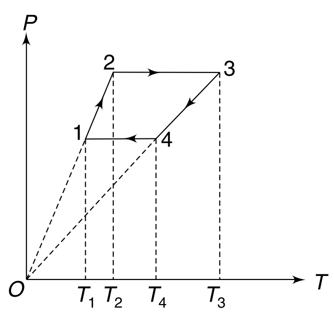
(ii) One mole of an ideal gas is expanded isothermally at temperature T_0 to double its volume from V_0 to $2V_0$. Draw a graph showing the variation of volume (V) of the gas versus the amount of heat (Q) added to it.

Q. 9: A spherical balloon contains air at pressure P_1 and is placed in vacuum. It has an initial diameter of D_1. The balloon is heated until its diameter becomes $D_2 = 2D_1$. It is known that pressure in the balloon is proportional to its diameter. Calculate the work done by the gas in expansion.

Q. 10: (i) An adiabatic cylinder contains an ideal gas. It is fitted with a freely movable insulating piston. In one experiment the piston is pulled out very fast to double the volume of the gas. In another experiment starting from same initial state, the piston is pulled out very slowly to double the volume of the gas. At the end of which experiment the final pressure of the gas will be higher?

(ii) An ideal gas is contained in a cylinder fitted with a movable piston. In an experiment '*A*' the gas is allowed to perform a work $W(>0)$ on the surrounding during an isobaric process and thereafter the pressure of the gas is reduced isochorically to half the initial value. At the end of the experiment the temperature of the gas is T_A. In a different experiment '*B*' the pressure of the gas is reduced to half in an isochoric process and then the gas performs a work W on the surrounding during an isobaric process. At the end of the experiment the gas temperature is T_B. Which is higher, T_A or T_B?

Q. 11: n moles of an ideal gas is taken through a four step cyclic process as shown in the diagram. Calculate work done by the gas in a cycle in terms of temperatures T_1, T_2, T_3 and T_4.



Q. 12: Two thermally insulated vessels are filled with an ideal gas. The pressure, volume and temperature in the two vessels are P_1, V_1, T_1 and P_2, V_2, T_2 respectively. Now, the two vessels are connected using a short insulating tube.

(a) Find final temperature of the gas.

(b) Express the final pressure of the gas in terms of P_1, V_1, P_2 and V_2 only.

Q. 13: Two tanks are connected by a valve. One tank contains 2 kg of an ideal gas at 77°C and 0.7 atm pressure. The other tank has 8kg of same gas at 27°C and 1.2 atm pressure. The valve is opened and the gases are allowed to mix. The final equilibrium temperature was found to be 42°C.

(a) Find the equilibrium pressure in both tanks.

(b) How much heat was transferred from surrounding to the tanks during the mixing process. Given: C_v for the gas is 0.745 $\text{KJ kg}^{-1}\text{K}^{-1}$.

Q. 14: The ratio of specific heats (C_p and C_v) for an ideal gas is γ. Volume of one mole sample of the gas is varied according to the law $V = \dfrac{a}{T^2}$ where T is temperature and a is a constant. Find the heat absorbed by the gas if its temperature changes by ΔT.

Q. 15: A sample of an ideal gas has diatomic molecules at a temperature at which effective degree of freedom is 5. Under the action of a suitable radiation the molecules split into two atoms. The ratio of the number of dissociated molecules to the total number of molecules is α. Plot the ratio of molar specific heats $\gamma\left(=\dfrac{C_p}{C_v}\right)$ as a function of α.

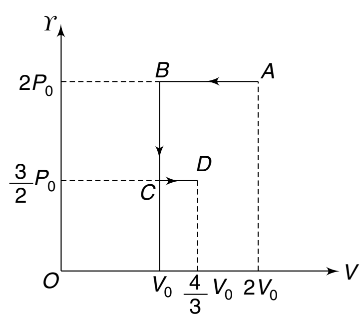
Q. 16: An ideal gas undergoes a series of processes represented by $A \to B \to C \to D$ on the P-V diagram. Answer the following questions.

(a) Is the internal energy of the gas at B and D equal?

(b) Find work done by the gas in the process $A \to B \to C \to D$.

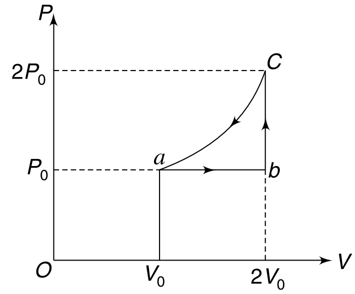
(c) Is it right to say that point B and D lie on an isotherm?

(d) Find the ratio of internal energy of the gas in state A to that in state D.



Q. 17: 30 people gather in a 10 m × 5 m × 3 m room for a confidential meeting. The room is completely sealed off and insulated. Calculate the rise in temperature of the room in half an hour. Assume that average energy thrown off by the body of a person is 2500 kcal/day, density of air is 1.2 kg/m^3 and specific heat specify capacity of air at constant volume is 0.24 $\text{kcal kg}^{-1}\,°\text{C}^{-1}$. Neglect volume occupied by human bodies.

Q. 18: One mole of an ideal monoatomic gas is taken through a cycle *a-b-c-a* as shown in figure. Find the difference in maximum and minimum value of internal energy of the gas during the cycle.

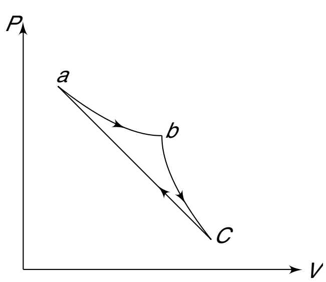


Q. 19: An ideal mono atomic gas A is supplied heat so as to expand without changing its temperature. In another process, starting with the same state, it is supplied heat at constant pressure. In both the cases a graph of work done by the gas (W) is plotted versus heat added (Q) to the gas. The ratio of slope of the graphs obtained in first and second process is η_1. The same ratio obtained for an ideal diatomic gas in η_2. Find the ratio $\frac{\eta_1}{\eta_2}$.

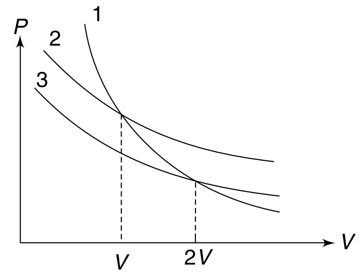
Q. 20: For an ideal gas the ratio of specific heats is $\frac{C_p}{C_v} = \gamma$. The gas undergoes a polytropic process PV^n = a constant. Find the values of n for which the temperature of the gas increases when it rejects heat to the surrounding.

Q. 21: For an ideal gas the slope of V-T graph during an adiabatic process is $\frac{dV}{dT} = -m$ at a point where volume and temperature are V_0 and T_0. Find the value of C_p of the gas. It is given that m is a positive number.

Q. 22: A gas undergoes a cyclic process a-b-c-a which is as shown in the PV diagram. The process a-b is isothermal, b-c is adiabatic and c-a is a straight line on P-V diagram. Work done in process ab and bc is 5 J and 4 J respectively. Calculate the efficiency of the cycle, if the area enclosed by the diagram abca in the figure is 3 J.

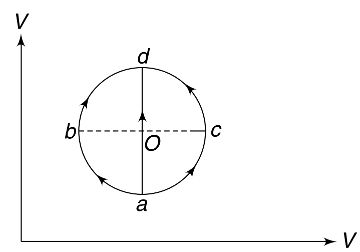


Q. 23: In the shown figure curve 1 represents an adiabat for n moles of an ideal mono atomic gas. Curve 2 and 3 are two isotherms for the same sample of the gas. Calculate the ratio of work done by the gas in doubling its volume from V to 2V along the isotherms 2 and 3.

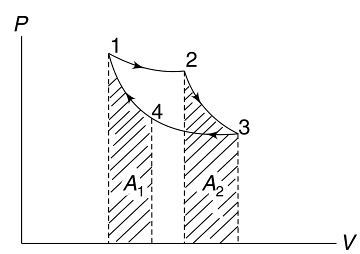


Q. 24: A mass less piston divides a thermally insulated cylinder into two parts having volumes V = 2.5 litre and 3 V = 7.5 litre. 0.1 mole of an ideal gas is confined into the part with volume V at a pressure of $P = 10^5$ N/m^2. The other part of the cylinder is empty. The piston is now released and the gas expands to occupy the entire volume of the cylinder. Now the piston is pressed back to its initial position. Find final temperature of the gas. $\left[\text{Take } R = \frac{25}{3} \text{ J mol}^{-1} \text{ K}^{-1} \text{ and } \gamma = 1.5 \text{ for the gas}\right]$

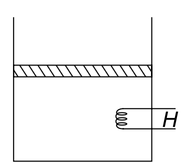
Q. 25: An ideal gas is taken from its initial state a to its find state d in three different quasi static processes marked as $a - b - d$, $a - o - d$ and $a - c - d$. Rank the net heat absorbed by the gas in the three processes. The diagram shown is a circle with centre at o.



Q. 26: The figure shows a Carnot cycle for an ideal gas on a P-V diagram. Which of the areas A_1 or A_2 is larger?



Q. 27: Air is contained in a vertical piston – cylinder assembly fitted with an electric heater. The piston has a mass of 50 kg and cross sectional area of 0.1 m^2. Mass of the air inside the cylinder is 0.3 kg. The heater is switched on and the volume of the air slowly increases by 0.045 m^3. It was found that the internal energy of the air increased by 32.2 kJ/kg

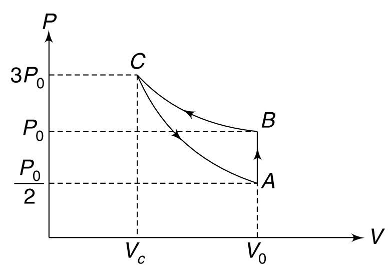


and that of the piston increased by 0.06 kJ/kg. Assume that the container walls and outer surface of the piston are well insulated and there is no friction. The atmospheric pressure is 100 kPa. Determine the heat transferred by the heater to the system consisting of air and the piston.

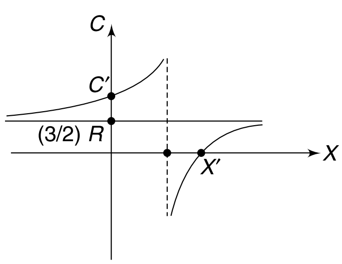
Q. 28: One mole of an ideal mono atomic gas is at a temperature T_0 = 1000 K and its pressure is P_0. The gas is adiabatically cooled so that its pressure becomes $\frac{2}{3}P_0$. Thereafter, the gas is cooled at constant volume to reduce its pressure to $\frac{P_0}{3}$. Calculate the total heat absorbed by the gas during the process.

Take $R = \frac{25}{3}$ J mol^{-1} K^{-1} and $\left(\frac{2}{3}\right)^{2/5} = 0.85$

Q. 29: One mole of an ideal gas is carried through a thermodynamics cycle as shown in the figure. The cycle consists of an isochoric, an isothermal and an adiabatic process. Find the adiabatic exponent of the gas.

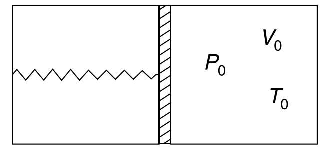


Q. 30: One mole of an ideal monatomic gas is taken along the process in which PV^x = constant. The graph shown represents the variation of molar heat capacity of such a gas with respect to 'x'. Find the values of C' and x' indicated in the diagram.



LEVEL 2

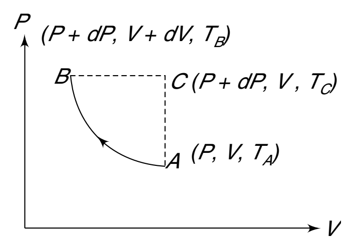
Q. 31: A container has a tight fitting movable piston which can slide without friction. The compartment containing spring has vacuum and to the left of the piston there is diatomic gas. If vacuum is created in the right compartment also the piston touches the right wall and the spring is relaxed. Find the heat capacity of the system neglecting the heat capacities of the material of spring, container and the piston. Express your answer in terms of P_0, V_0 and T_0.



Q. 32: One mole of a gas is in state $A[P, V, T_A]$. A small adiabatic process causes the state of the gas to change to $B[P + dP, V + dV, T_B]$. The changes dV & dP are infinitesimally small and dV is negative. An alternative process takes the gas from state A to B via $A \rightarrow C \rightarrow B$. $A \rightarrow C$ is isochoric and $C \rightarrow B$ is isobaric path. State at C is $[P + dP, V, T_C]$.

(a) Rank the temperatures T_A, T_B and T_C from highest to lowest.

(b) Find γ of the gas in terms of T_A, T_B and T_C.



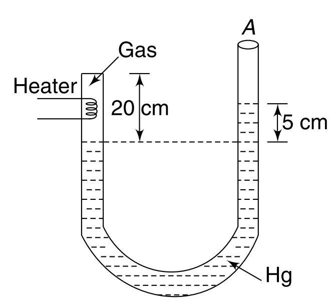
Q. 33: A metal cylinder of density d, cross sectional area A and height h is standing on a horizontal surface. Coefficient of linear expansion of the metal is α°C^{-1} and its specific heat capacity is S. Calculate the rise in temperature of the cylinder if a heat ΔQ is supplied to it. Assume no atmosphere.

Q. 34: n moles of an ideal mono atomic gas is initially at pressure $32P_0$ and volume V_0. Its volume is doubled by an isobaric process. After this the gas is adiabatically expanded so as to make its volume $16V_0$. Now the gas is isobarically expanded. Finally, the gas is made to return to its initial state by an isothermal process.

(a) Represent the process on a P–V diagram.

(b) Calculate work done by the gas in one cycle.

Q. 35: One end of an insulating U tube is sealed using insulating material. A mono atomic gas at temperature 300 K occupies 20 cm length of the tube as shown. The level of mercury on two sides of the tube differ by 5 cm. The other end of the tube is open to atmosphere.

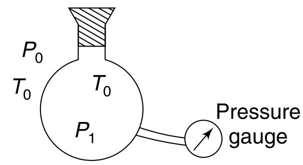


Area of cross section of the tube is uniform and is equal to 0.01 m^2. The gas in the tube is heated by an electric heater so as to raise its temperature to 562.5 K. Assume that no heat is conducted to mercury by the gas.

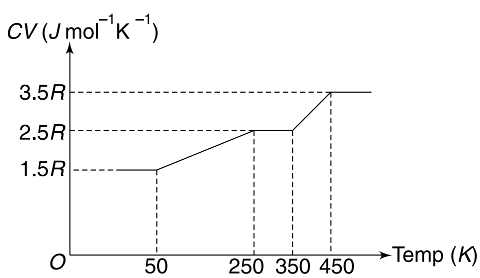
(a) Find the final length of the gas column.

(b) Find the amount of heat supplied by the heater to the gas.

Q. 36: Air is filled inside a jar which has a pressure gauge connected to it. The temperature of the air inside the jar is same as outside temperature ($=T_0$) but pressure (P_1) is slightly larger than the atmospheric pressure (P_0). The stopcock is quickly opened and quickly closed, so that the pressure inside the jar becomes equal to the atmospheric pressure P_0. The jar is now allowed to slowly warm up to its original temperature T_0. At this time the pressure of the air inside is P_2 ($P_0 < P_2 < P_1$). Assume air to be an ideal gas. Calculate the ratio of specific heats ($=\gamma$) for the air, in terms of P_0, P_1 and P_2.



Q. 37: The molar specific heat capacity at constant volume (C_V) for an ideal gas changes with temperature as shown in the graph. Find the amount of heat supplied at constant pressure in raising the temperature of one mole of the gas from 200 K to 400 K.

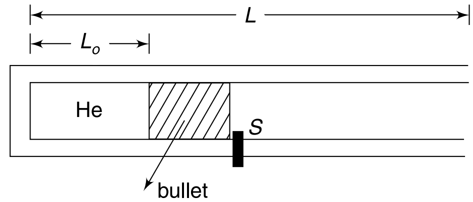


Q. 38: A sample of oxygen is heated in a process for which the molar specific heat capacity is $2R$. During the process the temperature becomes $(32)^{1/3}$ times of the original temperature. How does the volume of the gas change?

Q. 39: A gas-gun has a cylindrical bore made in an insulating material. Length of the bore is L. A small bullet having mass m just fits inside the bore and can move frictionlessly inside it. Initially n moles of helium gas is filled in the bore to a length L_0. The bullet does not allow the gas to leak and the bullet itself is kept at rest by a stopper S. The gas is at temperature T_0. The gun fires if the stopper S is removed suddenly. Neglect atmospheric pressure in your calculations [Think that the gun is in space].

(a) Calculate the speed with which the bullet is ejected from the gun.

(b) Find the maximum possible speed that can be imparted to the bullet by using n moles of helium at temperature T_0.



Q. 40: During a process carried on an ideal gas it was found that $\eta(<1)$ times the heat supplied to the gas is equal to increase in internal energy of the gas. Write the process equation in terms of pressure (P) and volume (V) of the gas. It is given that ratio of specific heats for the gas is $\frac{C_P}{C_V} = \gamma$.

Q. 41: An ideal gas is taken through a thermodynamic cycle $ABCDA$. In state A pressure and volume are P_0 and V_0 respectively. During the process $A \rightarrow B$, work done by the gas is zero and its temperature increases two fold. During the process $B \rightarrow C$, internal energy of the gas remains constant but work done by it is $W_{BC} = -P_0V_0 \ln 2$. In the process $C \rightarrow D$, the temperature decreases by 50% while the volume does not change. In the process $D \rightarrow A$ the temperature of the gas does not change.

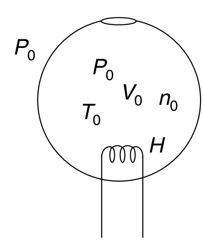
(a) Draw pressure – volume (P–V) and pressure – density ($P-\rho$) graph for the cyclic process.

(b) Calculate work done on the gas during the cycle.

Q. 42 (i) A conducting piston divides a closed thermally insulated cylinder into two equal parts. The piston can slide inside the cylinder without friction. The two parts of the cylinder contain equal number of moles of an ideal gas at temperature T_0. An external agent slowly moves the piston so as to increase the volume of one part and decrease that of the other. Write the gas temperature as a function of ratio (β) of the volumes of the larger and the smaller parts. The adiabatic exponent of the gas is γ.

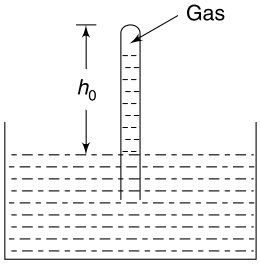
(ii) In a closed container of volume V there is a mixture of oxygen and helium gases. The total mass of gas in the container is m gram. When Q amount of heat is added to the gas mixture its temperature rises by ΔT. Calculate change in pressure of the gas.

Q. 43: A spherical container made of non conducting wall has a small orifice in it. Initially air is filled in it at atmospheric pressure (P_0) and atmospheric temperature (T_0). Using a small heater, heat is slowly supplied to the air inside the container at a constant rate of H J/s. Assuming air to be an



ideal diatomic gas find its temperature as a function of time inside the container.

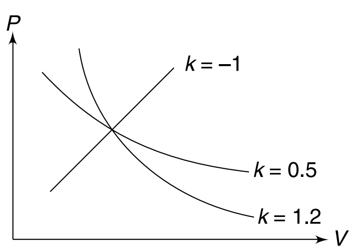
Q. 44: A glass tube is inverted and dipped in mercury as shown. One mole of an ideal monoatomic gas is trapped in the tube and the tube is held so that length of the tube above the mercury level is always h_0 meter. The atmospheric pressure is equal to h_0 meter of mercury. The mercury vapour pressure, heat capacity of mercury, tube and the container are negligible. How much heat must be supplied to the gas inside the tube so as to increase its temperature by ΔT?



Q. 45: Figure shows P versus V graph for various processes performed by an ideal gas. All the processes are polytropic following the process equation PV^k = constant.

(i) Find the value of k for which the molar specific heat of the gas for the process is $\dfrac{C_P + C_V}{2}$. Does any of the graph given in figure represent this process?

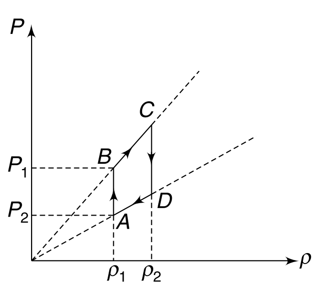
(ii) Find the value of k for which the molar specific heat of the gas is $C_V + C_P$. Assume that gas is mono atomic. Draw approximately the P versus V graph for this process in the graph given above.



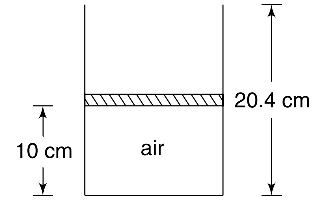
Q. 46: 'n' moles of an ideal gas having molar mass M is made to undergo a cyclic process $ABCDA$. The cycle has been represented on a pressure (P) density (ρ) diagram.

(a) Draw the corresponding $P - V$ diagram

(b) Calculate the work done by the gas in the cycle.



Q. 47: In the arrangement shown in figure, the piston can smoothly move inside the cylinder. The mass of the piston is m = 100 g and its cross sectional area is A = 10 cm^2. The length of air column at temperature of T = 27°C is 10 cm. Overall length of the cylinder is 20.4 cm. The container is turned upside down and the length of the air column in equilibrium was found to be l at 27°C.



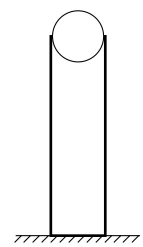
Take $R = \dfrac{25}{3}\,\text{J mol}^{-1}\,\text{K}^{-1}$ and assume air to be diatomic gas.

g = 10 m/s^2, atmospheric pressure is $1.01 \times 10^5\ \text{Nm}^{-2}$

(a) Find l

(b) If the air in the container is supplied heat in upside down position, the piston slowly begins to move down, and ultimately it gets ejected out of the cylinder. Calculate the amount of heat that the air must absorb for the piston to come out.

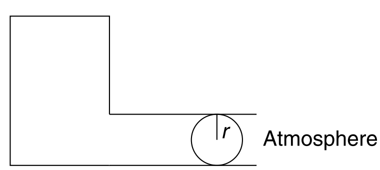
Q. 48: There is a long vertical tube of radius r containing air at atmospheric pressure. A steel ball is held at the mouth of the tube and dropped. The ball has radius r and it just fits inside the tube. The tube wall is perfectly smooth and no air can leak from the tube as the ball falls inside it. The ball falls through half the length of the tube before coming to rest. Assume that wall of the tube is perfectly conducting and temperature of the air inside the tube remains constant. Density of steel is d and atmospheric pressure is P_0 and take $L >> r$. Take air to be an ideal gas.



(a) Find the radius (r) of the tube.

(b) At what depth from the top of the tube the ball will be in equilibrium?

Q. 49: A ball of radius r fits tightly inside a tube attached to a container. There is no friction between the tube wall and the ball. Volume of air inside the container is V_0 when the ball is in equilibrium. Density of the material of the ball is d and atmospheric pressure is P_0. If the ball is displaced a little from its equilibrium position and released, find time period of its oscillation. Assume that temperature in the container remains constant and that air is an ideal gas.

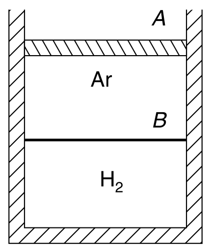


Q. 50: An ideal mono atomic gas is at temperature T_0. The pressure and volume are quasi-statically doubled such that the process traces a straight line on the PV diagram.

(a) Calculate the heat absorbed by the gas in the process if number of moles of the gas in the sample is n.

(b) Calculate the average molar specific heat capacity of the gas for the process.

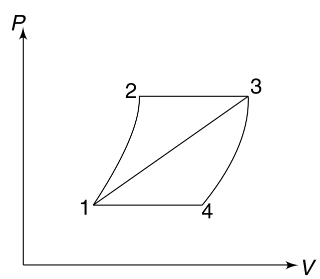
Q. 51: A cylinder contains equal volumes of Ar and H_2, separated by a freely movable piston B. Piston A can also move without friction. Volume of each gas in equilibrium is V_0. All walls of the container including piston A are non conducting.



(i) Piston A is pushed down slowly till the volume occupied by argon becomes $\frac{V_0}{4}$. Find the volume of H_2. Assume that piston B is also non conducting.

(ii) Now assume that piston B is conducting and assume that each gas has n moles. The external agent performs work W_0 in slowly pushing down the piston A. Find rise in temperature of each gas.

Q. 52: An ideal gas is taken through cycle 1231 (see figure) and the efficiency of the cycle was found to be 25%. When the same gas goes through the cycle 1341 the efficiency is 10%. Find the efficiency of the cycle 12341.



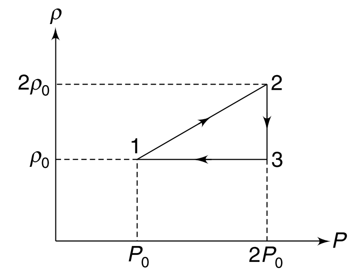
Q. 53: A heat engine is based on a gaseous cycle comprising of four processes viz. isothermal expansion at temperature (T_1), isochoric cooling to temperature T_2, isothermal compression (at T_2) and isochoric heating back to temperature T_1. The engine has been designed so as to completely use the heat rejected during isochoric cooling, in the isochoric heating process. Calculate the efficiency of this reversible cycle. Show the process on a PV graph.

Q.54: An ideal gas with a known γ, completes a cycle consisting of two isotherms and two isobars. The isothermal processes are executed at temperatures T and T' ($< T$) and isobaric processes are completed at pressures P_0 and eP_0 [e = base of natural logarithm]. Find the efficiency of the cycle.

Q. 55: One mole of a mono atomic gas of molar mass M undergoes a cyclic process as shown in the figure. Here ρ is density and P is pressure of the gas.

(a) Calculate the heat rejected by the gas in one complete cycle.

(b) Find the efficiency of the cycle.



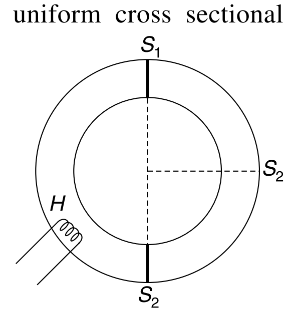
Q. 56: Helum gas is used as working substance in an engine working on a thermodynamic cycle $A - B - C - D - A$. Process AB is isobaric, BC is adiabatic compression. During process CD, pressure is increased keeping the volume constant and DA is an isothermal process. The gas has maximum volume at A and the ratio of maximum to minimum volume during the entire cycle is $8\sqrt{2}$. Also, the ratio of maximum to minimum absolute temperature is 4.

(a) Represent the cycle on a $P - V$ diagram.

(b) Calculate efficiency of the cycle in percentage if it is used as an engine.

[Take ln 2 = 0.693]

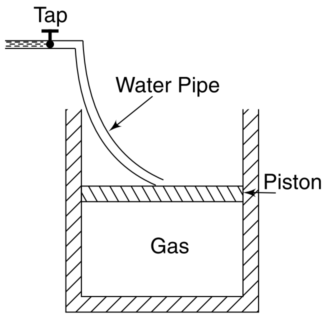
Q. 57: A ring shaped tube has uniform cross sectional area and its entire volume is $2V_0$. The tube is well insulated from the surrounding. Inside the tube there is an adiabatic fixed wall S_1 and another movable adiabatic partition S_2. Initially, the movable partition is diametrically opposite to S_1 and the two halves of the tube have equal amount of an ideal gas ($\gamma = 1.5$) at same pressure P_0. Now, a heater H is switched on which supplies heat slowly to one of the chambers. Heater is kept on till the partition S_2 moves through the quarter of the circle. At this position the heater is switched off and the partition S_2 remains in equilibrium. Neglect any friction as well as heat loss to the surrounding through the walls of the tube. Find the heat supplied by the heater to the gas.

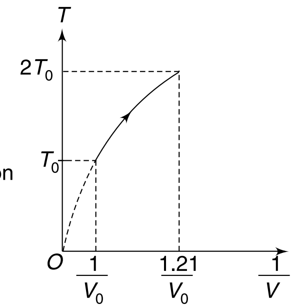


Q. 58: A cylindrical container has insulating wall and an insulating piston which can freely move up and down without any friction. It contains a mixture of ideal gases. Originally the gas is at atmospheric pressure P_0 and temperature (T_0). A tap positioned above the container is opened and it supplies water at a constant rate of $\frac{dm}{dt}$ = 0.25 kg/s. The water collects above the piston in the container and the gas compresses. The tap is kept open till the temperature of the gas is doubled. During the process the T vs $\frac{1}{V}$ graph for the gas was recorded and found to be a parabola with its vertex at origin as shown in the graph. Area of piston $A = 1.515 \times 10^{-3}$ m^2 and atmospheric pressure = 10^5 N/m^2

(a) Find the ratio of V_{rms} and speed of sound in the gaseous mixture.

(b) For how much time the tap was kept open?





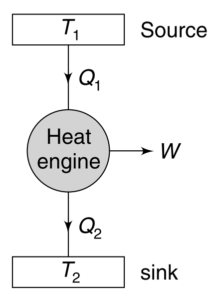
Q. 59: On a hot summer day the temperature inside is house is T_0 and the outside temperature is $T_0 + \Delta T$. How does the energy consumed by the air conditioner depend on T_0 and ΔT? Assume that the air conditioner operates ideally at its maximum coefficient of performance.

Q. 60: A room air conditioner is a Carnot cycle based heat engine run is reverse. An amount of heat Q_2 is absorbed from the room at a temperature T_2 into coils having a working gas (these gases are not good for environment!). The gas is compressed adiabatically to the outside temperature T_1. Then the gas is compressed isothermally in the unit outside the room, giving off an amount of heat Q_1. The gas expands adiabatically back to the temperature T_2 and the cycle is repeated. The electric motor electric consumes power P.

(i) Find the maximum rate at which heat can be removed from the room.

(i) Heat flows into the room at a constant rate of $k\Delta T$ where k is a constant and ΔT is temperature difference between the outside and inside of the room. Find the smallest possible room temperature in terms of T_1, k and P.

Q. 61: A Carnot cycle based ideal heat engine operates between two tanks each having same mass m of water. The source tank has an initial temperature of T_1 = 361 K and the sink tank has an initial temperature of T_2 = 289 K. Assume that the two tanks are isolated from the surrounding and exchange heat with the engine only. Specific heat of water is s.



(a) Find the final common temperature of the two tanks.

(b) Find the total work that the engine will be able to deliver by the time the two tanks reach common temperature.

Q. 62: A cylindrical container of height $3L$ and cross sectional area A is fitted with a smooth movable piston of negligible weight. It contains an ideal diatomic gas. Under normal atmospheric pressure P_0 the piston stays in equilibrium at a height L above the base of the container. The gas chamber is provided with a heater and a copper coil through which a cold liquid can be circulated to extract heat from the gas. Volume occupied by the heater and the liquid coil is negligible. Following set of operations are performed to take the gas through a cyclic process.

(1) Heater is switched on. At the same time a tap above the cylinder is opened. Water fills slowly in the container above the piston and it is observed that the piston does not move. Water is allowed to fill the container so that the height of water column becomes L. Now the tap is closed.

(2) The heater is kept on and the piston slowly moves up. Heater is switched off at the time water is at brink of overflowing.

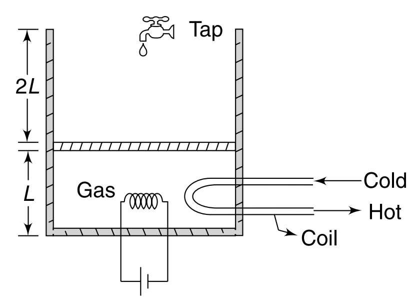
(3) Now the cold liquid is allowed to pass through the coil. The liquid extracts heat from the gas. Water is removed from the container so as to keep the position of piston fixed. Entire water is removed and the gas is brought back to atmospheric pressure.

(4) The circulation of cold liquid is continued and the piston slowly falls down to the original height L above the base of the container. Circulation of liquid is stopped.

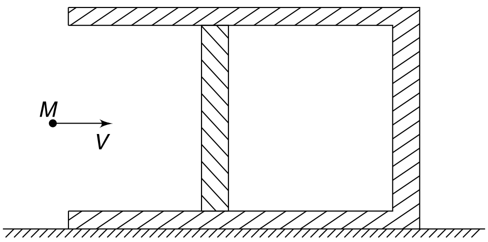
Assume that the container is made of adiabatic wall and density of water is ρ. Force on piston due to impact of falling water may be neglected.

(a) Draw the entire cycle on a P–V graph.

(b) Find the amount of heat supplied by the heater and the amount of heat extracted by the cold liquid from the gas during the complete cycle.



Q. 63: An adiabatic cylindrical chamber with a frictionless movable piston has been placed on a smooth horizontal surface as shown. One mole an ideal monotonic gas is enclosed inside the chamber. Mass of the piston is M and mass of the remaining chamber including the gas is 4 M. The gas is at atmospheric pressure and temperature. A particle of mass M moving horizontally with speed v, strikes the piston elastically. Find the change in temperature of the gas when the compression is maximum.



Q. 64: (a) A polytropic process for an ideal gas is represented by PV^x = constant, where $x \neq 1$. Show that molar specific heat capacity for such a process is given by $C = C_v + \frac{R}{1-x}$.

(b) An amount Q of heat is added to a mono atomic ideal gas in a process in which the gas performs a work $\frac{Q}{2}$ on its surrounding. Show that the process is polytropic and find the molar heat capacity of the gas in the process.

LEVEL 3

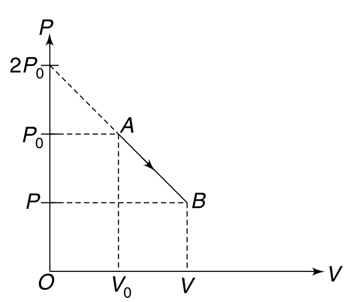
Q. 65: An adiabatic cylinder of cross section A is fitted with a mass less conducting piston of thickness d and thermal conductivity K. Initially, a monatomic gas at temperature T_0 and pressure P_0 occupies a volume V_0 in the cylinder. The atmospheric pressure is P_0 and the atmospheric temperature is $T_1 (> T_0)$. Find

(a) the temperature of the gas as a function of time

(b) the height raised by the piston as a function of time

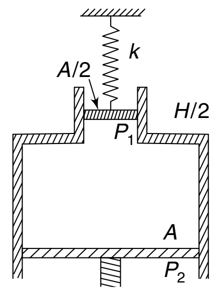
Neglect friction and heat capacities of the container and the piston.

Q. 66: One mole of an ideal gas is expanded from the state $A(P_0, V_0)$ to final state B having volume V. The process follows a path represented by a straight line on the P–V diagram (see figure). Up to what volume (V) the gas shall be expanded so that final temperature is half the maximum temperature during the process.



Q. 67: An ideal gas, in initial state $1(P_1, V_1, T_1)$ is cooled to a state $3(P_3, V_3, T_3)$ by a process which can be represented by a straight one on the P–V graph. The same gas in a different initial state $2(P_2, V_2, T_1)$ is cooled to same final state $3(P_3, V_3, T_3)$ by a process which can also be represented by a straight line on the same P–V graph. Q_1 and Q_2 are heat rejected by the gas in the two processes. Which is larger Q_1 or Q_2. It is given that $P_1 > P_2$.

Q. 68: In ideal gas is enclosed in an adiabatic container having cross section $A = 27\,\text{cm}^2$ for a part of it and $\frac{A}{2}$ for remaining part. Pistons P_1 and P_2 can move freely without friction along the inner wall. In the position shown in the figure the spring attached to the piston P_1 is relaxed. Sprig constant the spring is $K = 3700$ N/m. Piston P_2 is pushed up gradually through a distance $\frac{H}{2}$ and its was observed that the piston P_1 goes up by $\frac{3H}{32}$. Take $\gamma = 1.5$, mass of piston $P_1 = 13.5$ kg and atmospheric pressure $P_0 = 1 \times 10^5$ N/m^2



(a) Find H.

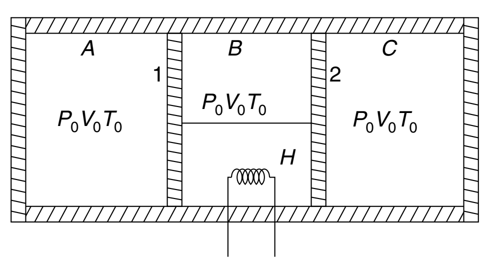
(b) Find final temperature of the gas if its initial temperature is 300 K.

Q. 69: An insulated cylinder is divided into three parts A, B and C. Pistons 1 and 2 are connected by a rigid rod and can slide without friction inside the cylinder. Piston 1 is perfectly conducting while piston 2 is perfectly insulating. Equal quantity of an ideal gas is filled in three compartments and the state of gas in every part is same $(P_0 V_0 T_0)$. Adiabatic exponent of the gas is $\gamma = 1.5$. The compartment B is slowly given heat through a heater H such that the final volume of gas in part C becomes $\frac{4V_0}{9}$

(a) Calculate the heat supplied by the heater.

(b) Calculate the amount of heat flow through piston 1.

(c) If heater were in compartment A, instead of B how would your answers to (a) and (b) change?

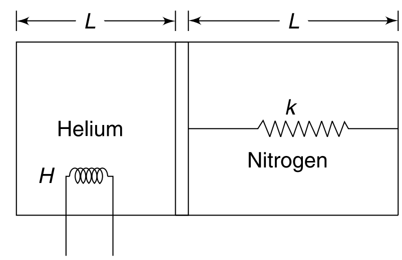


Q. 70: An adiabatic cylinder has length $2L$ and cross sectional area A. A freely moving non conducting piston of negligible thickness divides the cylinder into two equal parts. The piston is connected to the right face of the cylinder with an ideal spring of force constant k. The right chamber contains 28 g nitrogen in which one third of the molecules are dissociated into atoms. The left chamber container 4 g helium. With piston in equilibrium and spring relaxed the pressure in both chamber is P_0. The helium chamber is slowly given heat using an electric heater (H), till the piston moves to right by a distance $\frac{3L}{4}$. Neglect the volume occupied by the spring and the heating coil. Also neglect heat capacity of the spring.

(a) Find the ratio of C_P and C_V for nitrogen gas in right chamber.

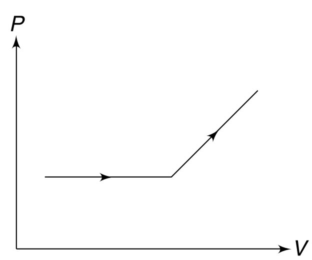
(b) Calculate change in temperature of helium.

(c) Calculate heat supplied by the heater.

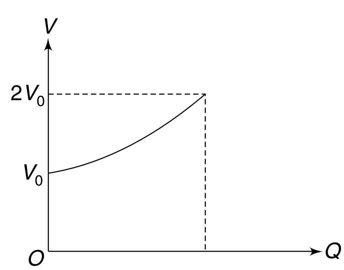


ANSWERS

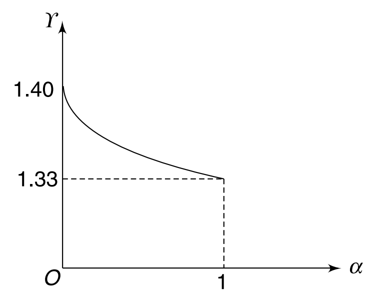
1. Adiabatic
2. (i) irreversible adiabatic
   (ii) isothermal
   (iii) irreversible isochoric
3. 733.8 cal
4. 3600R
5. 135 J
6. $C = C_v + \frac{R}{\alpha V}$
7. $3R$
8. (i)



(ii)



9. $W = \frac{15\pi}{4} P_1 D_1^3$
10. (i) Experiment 1 (ii) T_B
11. $W = nR[T_1 + T_3 - T_2 - T_4]$
12. (a) $\frac{T_1 T_2(P_1V_1 + P_2V_2)}{(P_1V_1T_2 + P_2V_2T_1)}$ (b) $\frac{P_1V_1 + P_1V_2}{V_1 + V_2}$
13. (a) 1.05 atm (b) 37.25 KJ
14. $\Delta Q = R\left(\frac{3 - 2\gamma}{\gamma - 1}\right) \Delta T$
15.



16. (a) Yes (b) $-\frac{3}{2} P_0V_0$
    (c) Yes (d) 2 : 1
17. 36°C
18. $\frac{9}{2} P_0V_0$
19. $\frac{5}{7}$
20. $1 < n < \gamma$
21. $\left(\frac{mT_0}{V_0} + 1\right) R$
22. 0.6
23. $\frac{W_1}{W_2} = 2^{2/3}$
24. 600 K
25. $\Delta Q_{abd} > \Delta Q_{aod} > \Delta Q_{acd}$
26. $A_1 = A_2$

27. 17.38 kJ

28. 5312.5 J

29. $\gamma = \dfrac{\ell n6}{\ell n3}$

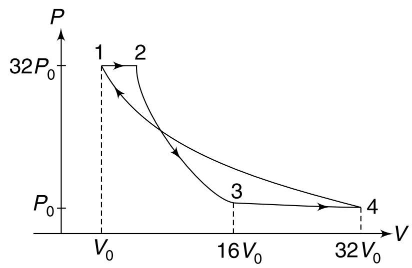
30. $C' = \dfrac{5}{2} R \quad x' = \dfrac{5}{3}$

31. $C = \dfrac{3P_0V_0}{T_0}$

32. (a) $T_C > T_B > T_A$ (b) $\gamma = \dfrac{T_C - T_A}{T_C - T_B}$

33. $\dfrac{2\Delta Q}{A \cdot d \cdot h(2s + \alpha \cdot g \cdot h)}$

34. (a)



(b) $W = 8\ P_0V_0$

35. (a) 30 cm (b) 406 J

36. $\gamma = \dfrac{\ln(P_0/P_1)}{\ln(P_2/P_1)}$

37. 706.25 R

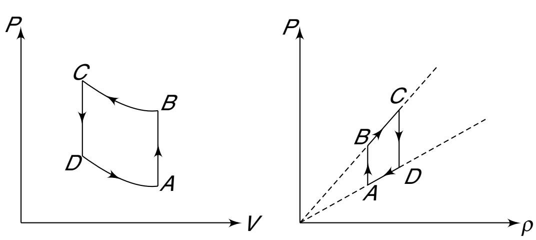
38. Volume doubles

39. (a) $u = \sqrt{\dfrac{3nRT_0}{m}\left[1 - \left(\dfrac{L_0}{L}\right)^{2/3}\right]}$

(b) $u_{\max} = \sqrt{\dfrac{3nRT_0}{m}}$

40. $PV^{\left[1 - \frac{\eta(\gamma-1)}{(1-\eta)}\right]}$

41. (a)



(b) $W = \dfrac{1}{2}P_0V_0\ln 2$

42. (i) $T = T_0\left[\dfrac{4\beta}{(1+\beta)^2}\right]^{\frac{1-\gamma}{2}}$

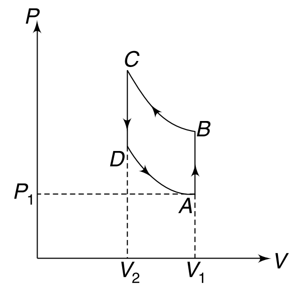
(ii) $\dfrac{28Q - mR\Delta T}{38\ \text{V}}$

43. $T = T_0 e^{\frac{2Ht}{7n_0T_0R}}$

44. $2R\Delta T$

45. (i) $k = -1$ (ii) $k = 0.6$

46. (a)



(b) $W = -\dfrac{nM}{\rho_1}(P_2 - P_1)\ln\left(\dfrac{\rho_2}{\rho_1}\right)$

47. (a) 10.2 cm (b) 35.7 J

48. (a) $r = \dfrac{3}{2}\ln(2)\dfrac{P_0}{dg}$ (b) $L\left[1 - \dfrac{1}{2\ln(2)}\right]$

49. $T = 4\sqrt{\dfrac{\pi V_0 d}{3P_0 r}}$

50. (a) $\Delta Q = 6nRT_0$ (b) $2R$

51. (i) $\dfrac{V_0}{(2)^{\frac{25}{21}}}$ (ii) $\dfrac{W_0}{4nR}$

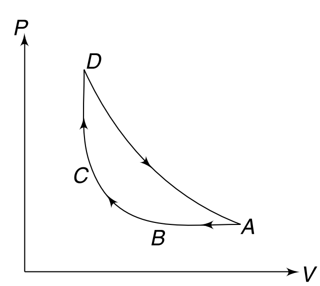
52. 32.5%

53. $\eta = \dfrac{T_1 - T_2}{T_1}$

54. $\dfrac{(T - T')(\gamma - 1)}{(2\gamma - 1)T - \gamma T'}$

55. (a) $\dfrac{P_0M}{\rho_0}\left(\dfrac{3}{2} + \ln 2\right)$ (b) $\dfrac{2}{5}(1 - \ln 2)$

56. (a)



(b) 41%

57. $4(2\sqrt{2} - 1)\ P_0V_0$

58. (a) $\sqrt{2}$ (b) 20 second

59. Energy consumed $\propto \dfrac{(\Delta T)^2}{T_0}$

60. (i) $P\left(\dfrac{T_2}{T_1 - T_2}\right)$

(ii) $T_1 - \dfrac{P}{k}\left[\sqrt{1 + \dfrac{4kT_1}{P}} - 1\right]$

61. (a) $T_0 = \sqrt{T_1 T_2} = 323$ K

(b) $W = ms(T_1 + T_2 - 2\sqrt{T_1 T_2}) = 4$ ms

62 (b) $\Delta Q_H = \dfrac{1}{2}(7P_0 + 12\rho gh)\ AL;$

$\Delta Q_L = \dfrac{1}{2}(7P_0 + 10\rho gL)\ AL$

63. $\dfrac{4Mv^2}{15R}$

64. $PV^{1/3} =$ constant; $C = 3R$

65. (a) $T = T_1 - (T_1 - T_0)\ e^{-\beta t}$

(b) $\Delta h = \dfrac{V_0(T_1 - T_0)}{AT_0}\ [1 - e^{-\beta t}]$ where $\beta = \dfrac{2T_0 KA}{5P_0 V_0 d}$

66. $V = \dfrac{\sqrt{2} + 1}{\sqrt{2}}\ V_0$

67. $Q_2 > Q_1$

68. (a) $\dfrac{16}{15}$ m (b) 400 K

69. (a) $18\ P_0V_0$ (b) $\dfrac{19}{2}\ P_0V_0$

(c) Answer to (a) does not change. Answer to (b) is $\dfrac{17}{2}\ P_0V_0$

70. (a) $\gamma = \dfrac{3}{2}$ (b) $\dfrac{1}{R}\left[\dfrac{15kL^2}{16} + 9P_0AL\right]$

(c) $\dfrac{27}{16}\ kL^2 + \dfrac{35}{2}\ P_0AL$

SOLUTIONS

1. Work done by gas is positive during all processes as the gas expands.

In an isothermal process there in no change in internal energy of the gas. In an isobaric process the temperature will increase as PV increases with expansion.

In adiabatic expansion the work done by the gas is at the expense of its internal energy.

3. Molar specific heat of steam at constant pressure will be

$$C_p = 4R = 8 \text{ cal mol}^{-1}\text{K}^{-1}$$

$$= \frac{8}{18} \text{ cal g}^{-1}\text{K}^{-1} = 0.44 \text{ cal g}^{-1}\text{K}^{-1}$$

$\therefore$ Q = Heat needed to heat the ice from –10°C to 0°C + Heat needed to melt the ice + Heat needed to heat water from 0°C to 100°C + Heat needed to convert water into steam + Heat needed to heat the steam from 100°C to 120°C.

$$= 1 \times 0.5 \times 10 + 1 \times 80 + 1 \times 1 \times 100 + 1 \times 540 + 1 \times 0.44 \times 20$$

$$= 5 + 80 + 100 + 540 + 8.8 = 733.8 \text{ cal.}$$

4. To double the rms speed, the temperature must be made 4 times.

$$\therefore \quad T_1 = 300 \text{ K}, \quad T_2 = 4 \times 300 = 1200 \text{ K}$$

$$\therefore \quad \Delta Q = n_1 C_{v_1}\ \Delta T + n_2 C_{v_2}\ \Delta T$$

$$= 1 \times \frac{3}{2} R \times 900 + 1 \times \frac{5}{2} R \times 900 = 3600\ R$$

5.

$$\Delta U = nC_v\ \Delta T = n\frac{f}{2} R\Delta T = n\frac{7}{2} R\Delta T$$

$$W = P\Delta V = nR\Delta T = \frac{2\Delta U}{7}$$

$$\therefore \quad \Delta U = \frac{30 \times 7}{2} = 105 \text{ J}$$

$$\therefore \quad \Delta Q = \Delta U + W = 105 + 30 = 135 \text{ J}$$

6.

$$T = T_0\ e^{\alpha V}$$

$$\Rightarrow \quad \ln T = \ln T_0 + \alpha V$$

$$\therefore \quad \frac{dT}{T} = \alpha dV \quad ...(1)$$

First law of thermodynamics–

$$dQ = dU + dW$$

$$nCdT = nC_v dT + PdV = nC_v dT + \left(\frac{nRT}{V}\right)\left(\frac{dT}{\alpha T}\right)$$

$$\therefore \quad C = C_v + \frac{R}{\alpha V}$$

7.

$$\Delta W = \int_{V_1}^{V_2} PdV$$

$$\because \quad P = KV$$

$$\therefore \quad \Delta W = K\int_{V_1}^{V_2} VdV = \frac{K}{2}(V_2^2 - V_1^2)$$

$$= \frac{(KV_2)(V_2) - (KV_1)(V_1)}{2} = \frac{P_2V_2 - P_1V_1}{2}$$

$$= \frac{nRT_2 - nRT_1}{2} = nR\frac{\Delta T}{2}$$

$$\Delta U = nC_v\Delta T = \frac{5}{2}nR\Delta T$$

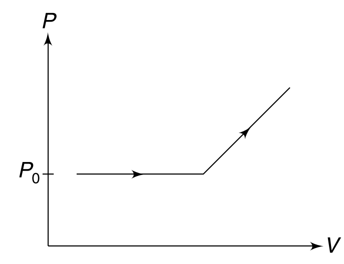
First law of thermodynamics

$$\Delta Q = \Delta U + W$$

$$nC\Delta T = \frac{5}{2}nR\Delta T + nR\frac{\Delta T}{2}$$

$$C = 3R$$

8. (i) Till the piston hits the spring, the gas pressure remains constant at atmospheric pressure (P_0). After that the gas pressure increases linearly with change in volume since spring force can be written as $kx = k\frac{\Delta V}{A}$ where ΔV = change in volume and A is area of cross sectional area of the piston.

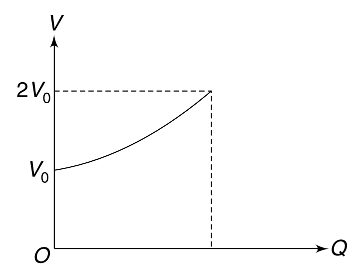


(ii) For isothermal process

$$Q = W = nRT_0 \ln\left(\frac{V}{V_0}\right)$$

$$\Rightarrow \quad \frac{Q}{nRT_0} + \ln V_0 = \ln V$$

$$\Rightarrow \quad V = e^{\left(\frac{Q}{nRT_0} + \ln V_0\right)}$$



9.

$$V = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \Rightarrow D = \left(\frac{6V}{\pi}\right)^{1/3} \quad ...(1)$$

Given $\quad P = KD$

$$\Rightarrow \quad P = K\left(\frac{6V}{\pi}\right)^{1/3} \quad [K = a \text{ Constant}]$$

This is the process equation.

$$\therefore \quad W = \int_{V_1}^{V_2} PdV$$

$$= K\left(\frac{6}{\pi}\right)^{1/3}\int_{V_1}^{V_2} V^{1/3}\, dV$$

$$= K\left(\frac{6}{\pi}\right)^{1/3}\left(\frac{3}{4}\right)\left[V_2^{4/3} - V_1^{4/3}\right]$$

$$= K\left(\frac{6}{\pi}\right)^{1/3}\left(\frac{3}{4}\right)V_1^{4/3}\left[\left(\frac{V_2}{V_1}\right)^{4/3} - 1\right]$$

From initial state $P_1 = K\left(\frac{6}{\pi}\right)^{1/3}\cdot V_1^{1/3}$

$\Rightarrow$ $K = P_1\left(\frac{6V_1}{\pi}\right)^{-1/3}$

$\therefore$ $W = P_1\left(\frac{6V_1}{\pi}\right)^{-1/3}\left(\frac{3}{4}\right)V_1^{4/3}\left[\left(\frac{V_2}{V_1}\right)^{4/3} - 1\right]$

$$= \frac{3}{4}P_1V_1\left[\left(\frac{V_2}{V_1}\right)^{4/3} - 1\right]$$

Put $V_1 = \frac{4}{3}\pi\left(\frac{D_1}{2}\right)^3$ and $V_2 = \frac{4}{3}\pi\left(\frac{D_2}{2}\right)^3$

$$W = \frac{\pi}{4}P_1D_1^3\left[\left(\frac{D_2}{D_1}\right)^4 - 1\right]$$

$$= \frac{\pi}{4}P_1D_1^3\,[2^4 - 1] = \frac{15\pi}{4}P_1D_1^3$$

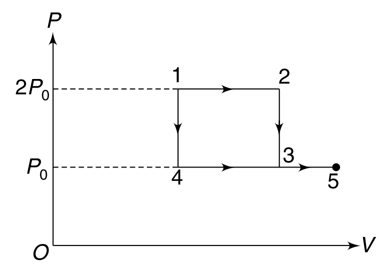
10. (i) When the piston is pulled out very fast, it is like free expansion of the gas. The gas temperature does not change. Therefore, PV = a constant and at the end the pressure becomes half the original value.

In second experiment the gas performs work. Therefore, its internal energy falls (notice that $\Delta Q = 0$). It means final temperature of the gas is less than its initial temperature.

$\therefore$ $P_1V_1 > P_2V_2$

$\therefore$ If $V_2 = 2V_1$, it means final pressure will be less then the original value.

(ii) The $P - V$ diagram for two processes is as shown in the figure.



Experiment A is represented by 1-2-3

Experiment B is represented by 1-4-5

Since $P_5V_5 > P_3V_3$

$\therefore$ $T_5 > T_3$

$\Rightarrow$ $T_B > T_A$

Notice that $W_{12} = W_{45}$

$\Rightarrow$ $2P_0(\Delta V_{12}) = P_0(\Delta V_{45})$

$\Rightarrow$ $2\cdot\Delta V_{12} = \Delta V_{45}$

11. Process 1 – 2 and 3 – 4 are isochoric process.

$\therefore$ $W_{12} = W_{34} = 0$

2 – 3 and 4 – 1 are isobaric processes.

$\therefore$ $W_{23} = P_2\Delta V = P_2V_3 - P_2V_2$

$$= nRT_3 - nRT_2 = nR(T_3 - T_2)$$

$$W_{41} = P_1\Delta V = P_1V_1 - P_1V_4$$

$$= nR(T_1 - T_4) = -nR(T_4 - T_1)$$

$\therefore$ $W = nR[T_3 - T_2 - T_4 + T_1]$

12. (a)

$$\Delta Q = 0,\ \Delta W = 0$$

$$\therefore \quad \Delta U = 0$$

$$U_{\text{initial}} = n_1 C_V T_1 + n_2 C_V T_2$$

$$= \frac{C_v}{R}[n_1 R T_1 + n_2 R T_2]$$

$$U_{\text{final}} = (n_1 + n_2)\ C_v T = \frac{C_v}{R}(n_1 + n_2)\ RT$$

$$\because \quad U_{\text{final}} = U_{\text{initial}}$$

$$\therefore \quad (n_1 + n_2)\ T = n_1 T_1 + n_2 T_2$$

$$T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2} \qquad \text{...(1)}$$

$$\frac{\frac{P_1 V_1}{R} + \frac{P_2 V_2}{R}}{\frac{P_1 V_1}{R T_1} + \frac{P_2 V_2}{R T_2}} = \frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{(P_1 V_1 T_2 + P_2 V_2 T_1)}$$

(b)

$$P = \frac{(n_1 + n_2)\ RT}{V_1 + V_2}$$

Using (1)

$$P = \frac{R(n_1 T_1 + n_2 T_2)}{V_1 + V_2} = \frac{P_1 V_1 + P_2 V_2}{V_1 + V_2}$$

13. (a)

$$V_1 = \frac{n_1 R T_1}{P_1};\ V_2 = \frac{n_2 R T_2}{P_2}$$

After mixing

$$P = \frac{(n_1 + n_2) R T_0}{V_1 + V_2}$$

$[T_0 = \text{final temperature} = 42 + 273 = 315 \text{ K}]$

$$= \frac{(n_1 + n_2) R T_0}{\frac{n_1 R T_1}{P_1} + \frac{n_2 R T_2}{P_2}}$$

$$= \frac{\left(\frac{m_1}{M} + \frac{m_2}{M}\right) R T_0}{\frac{m_1 R T_1}{M P_1} + \frac{m_2 R T_2}{M P_2}} = \frac{(m_1 + m_2) T_0}{\frac{m_1 T_1}{P_1} + \frac{m_2 T_2}{P_2}}$$

$$= \frac{(2 \text{ kg} + 8 \text{ kg})\ (315 \text{ K})}{\frac{(2 \text{ kg})(350 \text{ K})}{0.7 \text{ atm}} + \frac{(8 \text{ kg})(300 \text{ K})}{1.2 \text{ atm}}} = 1.05 \text{ atm}$$

(b)

$$W = 0$$

$$\therefore \quad \Delta Q = \Delta U = U_f - U_i$$

$$= (m_1 + m_2) C_V T_0 - m_1 C_V T_1 - m_2 C_V T_2$$

$$= (10 \text{ kg})\left(0.745 \frac{\text{kJ}}{\text{kgK}}\right)(315 \text{ K}) - (2 \text{ kg})\left(0.745 \frac{\text{kJ}}{\text{kgK}}\right)(350 \text{ K})$$

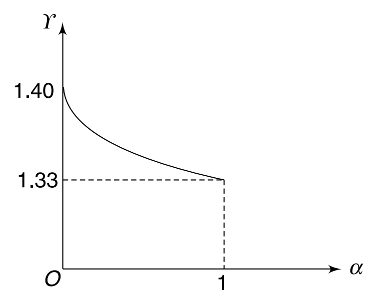
$$- (8 \text{ kg})\left(0.745 \frac{\text{kJ}}{\text{kgK}}\right)(300 \text{ K})$$

$$= 37.25 \text{ kJ}$$

14.
$$VT^2 = a$$
$$\therefore \quad T^2dV + V(2TdT) = 0$$
$$\therefore \quad TdV = -2VdT$$
$$\therefore \quad dV = -2V\frac{dT}{T}$$
$$PdV = -2PV\frac{dT}{T} = -nRT\frac{dT}{T} = -2nRdT$$

First law of thermodynamics for an infinitesimal change will be
$$dQ = dU + dW$$
$$= nC_VdT + PdV$$
$$= n\frac{R}{\gamma-1}dT + (-2nR\,dT)$$
$$= R\left(\frac{3-2\gamma}{\gamma-1}\right)dT \qquad [\because \; n = 1]$$
$$\therefore \quad \Delta Q = R\left(\frac{3-2\gamma}{\gamma-1}\right)\Delta T$$

15. Consider 1 mole sample of the diatomic gas. Due to radiation α mole molecules split into 2α mole of atoms and $1-\alpha$ mole remain as diatomic gas.



C_v for the mixture will be
$$C_v = \frac{2\alpha\cdot\frac{3}{2}R + (1-\alpha)\frac{5}{2}R}{1} = \frac{(\alpha+5)}{2}R$$
$$C_p = C_v + R = \left(\frac{\alpha+7}{2}\right)R$$
$$\therefore \quad \gamma = \frac{C_p}{C_V} = \frac{\alpha+7}{\alpha+5} = 1 + \frac{2}{\alpha+5}$$

Graph is as shown

16. (a) Product PV is same at B and D. It means that temperature is same at these two points.

(b) $W = (2P_0)V_0 - \frac{3}{2}P_0\left(\frac{V_0}{3}\right) = -\frac{3}{2}P_0V_0$

(c) Yes, reason being what has been said in part (a).

(d) Internal energy is proportional to temperature, which is proportional to product of pressure and volume. Hence the answer is 2.

17. Work done by air in the room $W = 0$
$$\therefore \quad \Delta U = Q = 30 \times 2500 = 75000 \text{ kcal/day}$$
$$= \frac{75000}{24 \times 60} \text{ kcal/min} = 52 \text{ kcal/min}$$

Volume of air in the room $\quad V = 10 \times 5 \times 3 = 150 \text{ m}^3$

$\therefore$ Mass of air in the room $\quad m = 1.20 \times 150 = 180$ kg
$$\therefore \quad mc\Delta T = 52 \times 30 \text{ Kcal}$$
$$\Delta T = \frac{52 \times 30}{180 \times 0.24} = 36°\text{C (!)}$$

18. Let temperature in state 'a' be T_0. Temperature at 'b' will be $2T_0$ [$\because P_BV_B = 2P_AV_A$] and temperature at C is $4T_0$ [$\because P_CV_C = 4P_AV_A$]

The product PV is least at 'a' and is maximum at 'c'.

$$\therefore \quad U_{min} = U_a \quad \text{and} \quad U_{max} = U_c$$

$$U_{max} - U_{min} = U_c - U_a = nC_V(T_c - T_a)$$

$$= 1 \times \frac{3R}{2} \times 3T_0 = \frac{9}{2} RT_0$$

$$= \frac{9}{2} P_0V_0 \qquad [\because \; P_0V_0 = 1 \cdot R \cdot T_0]$$

19. First law $\quad Q = W + \Delta U$

For isothermal process $\quad \Delta U = 0$

$\therefore \quad Q = W$

Graph of W vs Q is a straight line with slope $m_1 = 1$

For isobaric process:

$$\frac{Q}{\Delta U} = \frac{nC_p \Delta T}{nC_v \Delta T} = \gamma = \frac{5}{3}$$

$$\therefore \quad \frac{Q}{Q - W} = \frac{5}{3}$$

$$\Rightarrow \quad W = \frac{2}{5} Q$$

Slope of W vs Q graph is $m_2 = \frac{2}{5}$

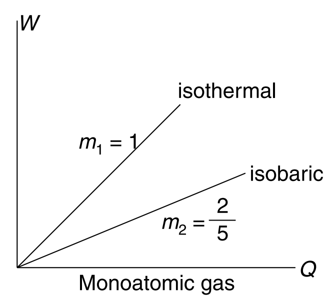
$$\therefore \quad \eta_1 = \frac{m_1}{m_2} = \frac{5}{2}$$

Similarly, for Diatomic gas

$$m_1 = 1 \quad \text{and} \quad m_2 = \frac{2}{7}$$

$$\therefore \quad \eta_2 = \frac{m_1}{m_2} = \frac{7}{2}$$

$$\therefore \quad \frac{\eta_1}{\eta_2} = \frac{5/2}{7/2} = \frac{5}{7}$$



20. Temperature of the gas increases when it rejects heat to the surrounding. This means that specific heat capacity of the gas for the given polytropic process must be negative.

For one mole gas–

$$W = \int PdV = \frac{R(T_1 - T_2)}{n - 1} \quad \text{[prove yourself by integration]}$$

$$\Delta U = C_v \Delta T = \frac{R(T_2 - T_1)}{\gamma - 1}$$

$$\therefore \quad \Delta Q = \Delta U + W$$

$$C\Delta T = \frac{R}{\gamma - 1}(\Delta T) - \frac{R}{n - 1}(\Delta T)$$

$$\therefore \quad C = \frac{R}{\gamma - 1} - \frac{R}{n - 1}$$

$$\therefore \quad C = \frac{R(n - \gamma)}{(n - 1)(\gamma - 1)}$$

γ is always > 1

$\therefore \quad C$ is negative if $(n - \gamma)$ is negative and $(n - 1)$ is positive

$$\Rightarrow \quad 1 < n < \gamma$$

21. $TV^{\gamma-1}$ = Constant

$$\ell n\, T + (\gamma - 1)\, \ell n V = \text{Const}$$

$$\frac{1}{T} + \frac{\gamma - 1}{V}\frac{dV}{dT} = 0$$

$$\Rightarrow \quad \frac{dV}{dT} = -\frac{V}{T(\gamma - 1)}$$

Given $\left(\frac{dV}{dT}\right)_{V_0, T_0} = -m$

$$\therefore \quad \frac{V_0}{T_0(\gamma - 1)} = m$$

$$\Rightarrow \quad \frac{1}{\gamma - 1} = \frac{mT_0}{V_0}$$

$$C_v = \frac{R}{\gamma - 1} = \frac{mRT_0}{V_0}$$

$$C_p = C_v + R = \left(\frac{mT_0}{V_0} + 1\right) R$$

22.

$$W_{ab} = 5 \text{ J}$$

$$\Delta U_{ab} = 0 \quad \text{(isothermal)}$$

$$Q_{ab} = 5 \text{ J} \quad \text{(First law)}$$

$$W_{bc} = 4 \text{ J}$$

$$Q_{bc} = 0 \quad \text{(Adiabatic)}$$

$$\Delta U_{bc} = -4 \text{ J} \quad \text{(Adiabatic)}$$

$$\Delta U_{bc} = -4 \text{ J} \quad \text{(First law)}$$

$$\because \quad \Delta U_{ab} + \Delta U_{bc} + \Delta U_{ca} = 0$$

$$\therefore \quad \Delta U_{ca} = 4 \text{ J}$$

Also, $W_{ab} + W_{bc} + W_{ca} = 3$ J (Area inside cycle)

$$5 + 4 + W_{ca} = 3$$

$$W_{ca} = -6 \text{ J}$$

First law for process *ca* $\quad Q_{ca} = 4 - 6 = -2$ J

Efficiency of cycle $\quad \eta = \frac{W_{\text{cycle}}}{Q_{ab}} = \frac{3}{5} = 0.6$

23. If temperature along two isotherms are T_1 & T_2 then,

$$T_1 V^{\gamma - 1} = T_2 (2V)^{\gamma - 1} \quad [\because \; A \text{ and } B \text{ lie on an adiabat}]$$

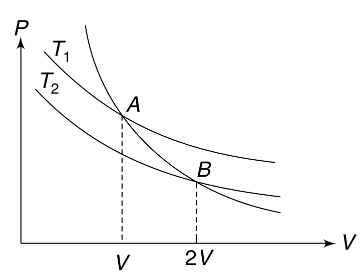
Put $\gamma = \frac{5}{3}$ to get $\frac{T_1}{T_2} = 2^{2/3}$

Work done in doubling the volume:

Along 1st isotherm $\quad W_1 = nRT_1\, \ell n\left(\frac{2V}{V}\right)$

Along 2nd isotherm $\quad W_2 = nRT_2\, \ell n\left(\frac{2V}{V}\right)$

$$\therefore \quad \frac{W_1}{W_2} = \frac{T_1}{T_2} = 2^{2/3}$$



24. **Expansion:**

$$P_1V_1 = PV$$

$$P_1(4V) = PV \Rightarrow P_1 = \frac{P}{4} \qquad ...(1)$$

During expansion, no work is done and

$$\Delta Q = 0 \qquad \therefore \quad \Delta U = 0$$

∴ Temperature does not change.

Initial temperature $$T = \frac{PV}{nR} = \frac{10^5 \times 2.5 \times 10^{-3}}{0.1 \times \frac{25}{3}} = 300 \text{ K}$$

Compression:

$$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$$

$$T_2V^{0.5} = 300 \times (4V)^{0.5}$$

$$T_2 = 600 \text{ K}$$

25. In all three processes ΔU is same.

$$W_{abd} = \text{area } (abdoa)$$

$$W_{aod} = 0$$

$$W_{acd} = -\text{area}(acdoa)$$

$$\therefore \quad W_{abd} > W_{aod} > W_{acd}$$

$$\therefore \quad \Delta Q_{abd} > \Delta Q_{aod} > \Delta Q_{acd}$$

26. 2 – 3 and 4 – 1 are adiabatic processes and 1 – 2 and 3 – 4 are isotherms (Why?)

$$\therefore \quad T_1 = T_2; \quad T_3 = T_4$$

Work done in an adiabatic process depends on change in temperature. Change in temperature in both 2 – 3 and 4 – 1 is same.

$$\therefore \quad A_1 = A_2$$

27.

$$\Delta U = 32.2 \times 0.3 + 0.06 \times 50 = 12.66 \text{ kJ}$$

Rise in gravitational potential energy of the piston

$$\Delta U_g = \text{mgh} = 50 \times 9.8 \times \frac{0.045}{0.1} = 0.221 \text{ kJ}$$

Work done by (air + piston) system against the atmospheric pressure is

$$W = P_0\, \Delta V = (100 \text{ kPa}) \times (0.045 \text{ m}^3) = 4.5 \text{ kJ}$$

$$\therefore \quad \Delta Q = W + \Delta U + \Delta U_g$$

$$= 4.5 + 12.66 + 0.221 = 17.38 \text{ kJ}$$

28. For the adiabatic process we have

$$P_2^{1-\gamma}\, T_2^{\gamma} = P_1^{1-\gamma} T_1^{\gamma}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}}$$

$$\Rightarrow \quad T_2 = 1000\left(\frac{3}{2}\right)^{\frac{3}{5}-1} = 1000\left(\frac{2}{3}\right)^{2/5} = 1000 \times 0.85 = 850 \text{ K}$$

For the isochoric process

$$\frac{P_3}{T_3} = \frac{P_2}{T_2} \Rightarrow T_3 = \left(\frac{P_3}{P_2}\right) T_2$$

$$\Rightarrow \quad T_3 = \left(\frac{P_0/3}{2P_0/3}\right) \times 850$$

$$\Rightarrow \qquad T_3 = \frac{850}{2} = 425 \text{ K}$$

Heat is lost in 2nd process only.

$$\text{Heat lost} = nCv\,\Delta T = 1 \times \frac{3}{2} R \times (850 - 425)$$

$$= \frac{3}{2} \times \frac{25}{3} \times 425 = 5312.5 \text{ J}$$

29. Since BC is isothermal process $(3P_0)V_C = P_0V_0 \quad \Rightarrow \quad V_C = \frac{V_0}{3}$

For adiabatic process CA $\qquad (3P_0)\left(\frac{V_0}{3}\right)^{\gamma} = \left(\frac{P_0}{2}\right)(V_0^{\gamma})$

$$\Rightarrow \qquad \gamma = \frac{\ell n 6}{\ell n 3}$$

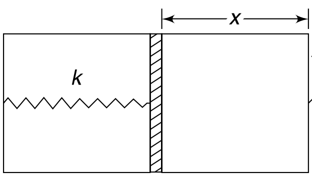
30. For $\qquad PV^x = \text{constant}$

$$C = \frac{R}{\gamma - 1} - \frac{R}{x-1} \qquad \left(\gamma = \frac{5}{3} \text{ for monatomic gas}\right)$$

$$C' \text{ corresponds to } x = 0 \quad \Rightarrow \quad C' = \frac{R}{\frac{5}{3} - 1} + R = \frac{5}{2}R$$

$$x' \text{ corresponds to } C = 0 \quad \Rightarrow \quad x' = \gamma = \frac{5}{3}$$

31. Assume that the system is given dQ amount of heat. This is used in increasing the internal energy of the gas and spring potential energy.



If the spring compresses further by dx when heat dQ is supplied, change in PE can be calculated as

$$U = \frac{1}{2}kx^2$$

$$\Rightarrow \qquad \frac{dU}{dx} = kx \quad \Rightarrow \quad dU = kxdx$$

$$\therefore \qquad dQ = nC_v dT + kxdx$$

$$CdT = \frac{5}{2}nRdT + kxdx \qquad \text{...(1)}$$

But $\qquad kx = PA \qquad$ ...(2)

and $\qquad PV = nRT$

$$\therefore \qquad PAx = nRT$$

$$\Rightarrow \qquad PA = \frac{nRT}{x}$$

Put in (2) $\qquad kx = \frac{nRT}{x}$

$$\therefore \qquad kx^2 = nRT$$

$$\therefore \qquad 2kxdx = nRdT$$

Putting in (1) $\qquad CdT = \frac{5}{2}nRdT + \frac{1}{2}nRdT$

$$\Rightarrow \qquad C = 3nR = 3 \cdot \frac{P_0V_0}{T_0}$$

32. $A \rightarrow B$ (Adiabatic)

$$dW = PdV$$

$\because \quad dQ = 0$

$\therefore \quad dU = -dW = -PdV$

$A \to C$ (Isochoric)

$$dW = 0$$

$$dU_1 = dQ = C_V(T_C - T_A)$$

$C \to B$ (isobaric)

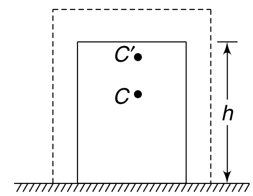
$$dU_2 = dQ - dW = -C_p(T_C - T_B) - PdV$$

$\because \quad dU = dU_1 + dU_2$

$\therefore \quad -PdV = C_V(T_C - T_A) - C_p(T_C - T_B) - PdV$

$\therefore \quad \dfrac{C_p}{C_V} = \dfrac{T_C - T_A}{T_C - T_B}$

33. Let the temperature rise by $\Delta\theta$



Increase in height of the centre of the cylinder is

$$\Delta h_{cm} = \alpha \frac{h}{2} \Delta\theta$$

$\therefore$ Rise in gravitational potential energy of the cylinder is

$$\Delta U = mg\,\Delta h_{\text{cm}}$$

$$= Ahdg\,\alpha \frac{h}{2}\Delta\theta = \frac{1}{2}\alpha Ad\cdot g\cdot h^2\Delta\theta$$

This is actually work done (W) by the expanding cylinder against gravity.
From first law of thermodynamics

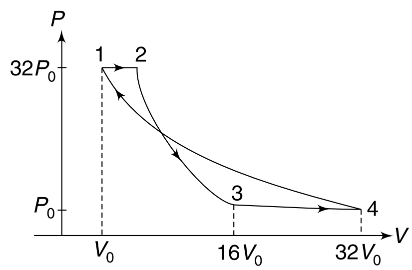
$$\Delta Q = W + \text{change in internal energy}$$

$$\Delta Q = \frac{1}{2}\alpha Adgh^2\,\Delta\theta + mS\Delta\theta$$

$$= \frac{1}{2}\alpha Ad\cdot g\cdot h^2\Delta\theta + A\cdot d\cdot h\cdot S\cdot\Delta\theta$$

$\therefore \quad \Delta\theta = \dfrac{2\Delta Q}{A\cdot d\cdot h\cdot(2S + \alpha\cdot g\cdot h)}$

34. (a)



(b) $\quad W_{12} = 32P_0V_0$

2 – 3 is an adiabatic process

$$P_3V_3^{\gamma} = P_2V_2^{\gamma}$$

$$P_3(16V_0)^{5/3} = 32P_0(2V_0)^{5/3}$$

$\Rightarrow \quad P_3 = P_0$

$$W_{23} = \frac{P_2V_2 - P_3V_3}{\gamma - 1} = \frac{(32P_0)\ (2V_0) - P_0(16V_0)}{\frac{5}{3} - 1}$$

$$= 72\ P_0V_0$$

Since final process is isothermal

$\therefore$ $P_4V_4 = P_1V_1$

$\Rightarrow$ $P_0 \cdot V_4 = 32\ P_0 \cdot V_0 \Rightarrow V_4 = 32V_0$

$\therefore$ $W_{34} = P_0(32V_0 - 16V_0) = 16P_0V_0$

And $W_{41} = P_4V_4 \ell n \dfrac{V_1}{V_4} = 32P_0V_0 \ell n\left(\dfrac{1}{32}\right) = -160P_0V_0\ell n2$

$\therefore$ Total work done is

$$W = 32P_0V_0 + 72P_0V_0 + 16P_0V_0 - 160\ \ell n 2P_0V_0$$

$$= 120P_0V_0 - 160 \times 0.7P_0V_0$$

$$= 8P_0V_0$$

35. Initial pressure of gas $P_1 = 75 + 5 = 80$ cm of Hg.

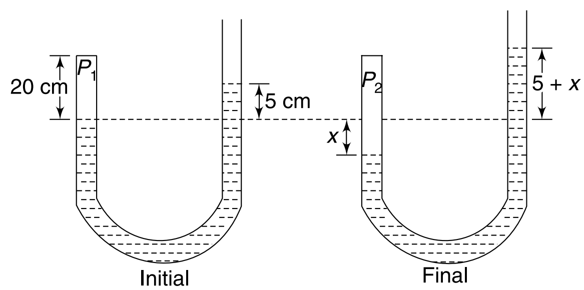
Let its final pressure be P_2

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

$$\frac{(75 + 5 + 2x)[A(20 + x)]}{562.5} = \frac{(80)\cdot(A\cdot 20)}{300}$$

$\Rightarrow$ $3(80 + 2x)\ (20 + x) = 9000 \Rightarrow 2x^2 + 120x + 1600 = 3000$

$\Rightarrow$ $x^2 + 60x - 700 = 0$



Solving this quadratic equation gives $x = 10$ cm

Work done by gas = Work done against atmospheric pressure in pushing the Hg column by x + Work done against gravity (i.e., rise in gravitational P.E. of Hg)

$\therefore$ $W = P_{atm}Ax + mg(5 + x)$

Where m = mass of Hg in column of length x.

$\therefore$ $W = 10^5 \times 0.01 \times 0.1 + 13.6 \times 10^3 \times 0.01 \times 0.1 \times 10 \times 0.15$

$$= 100 + 20.4 = 120.4 \text{ J}$$

Change in internal energy of the gas

$$\Delta U = nC_V\ \Delta T = \left(\frac{P_1V_1}{RT_1}\right)\left(\frac{3}{2}R\right)(T_2 - T_1)$$

$$= \frac{0.8 \times 13.6 \times 10^3 \times 10 \times 0.01 \times 0.2}{300} \times \frac{3}{2} \times 262.5 = 285.6 \text{ J}$$

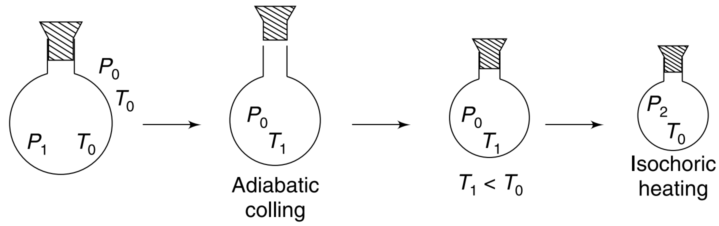
$\therefore$ Heat supplied by heater

$$Q = W + \Delta U = 120.4 + 285.6 = 406 \text{ J}$$

36. The expansion of air on opening the stopcock is sudden. The process is close to adiabatic.

$$P_1^{-(\gamma-1)}T_0^{\gamma} = P_0^{-(\gamma-1)}T_1^{\gamma}$$

$$\Rightarrow \qquad \left(\frac{P_1}{P_0}\right)^{-(\gamma-1)} = \left(\frac{T_1}{T_0}\right)^{\gamma} \qquad ...(1)$$



The slightly colder air inside the jar picks up heat from the surrounding and warms up to temperature T_0. The process is isochoric.

$$\therefore \qquad \frac{P_2}{T_0} = \frac{P_0}{T_1}$$

$$\therefore \qquad \left(\frac{T_1}{T_0}\right) = \frac{P_0}{P_2} \qquad ...(2)$$

From (1) and (2)
$$\left(\frac{P_1}{P_0}\right)^{-(\gamma-1)} = \left(\frac{P_0}{P_2}\right)^{\gamma}$$

$$\left(\frac{P_0}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{P_0}{P_2}$$

$$\therefore \qquad \left(1 - \frac{1}{\gamma}\right) \ln\left(\frac{P_0}{P_1}\right) = \ln\left(\frac{P_0}{P_2}\right)$$

$$\Rightarrow \qquad \ln\left(\frac{P_0}{P_1}\right) - \ln\left(\frac{P_0}{P_2}\right) = \frac{1}{\gamma} \ln \frac{P_0}{P_1} \text{ simplifying gives-}$$

$$\therefore \qquad \gamma = \frac{\ln\left(\frac{P_0}{P_1}\right)}{\ln\left(\frac{P_2}{P_1}\right)}$$

37.
$$C_p = C_V + R$$

$$\Delta Q = \int_{T_1}^{T_2} nC_p\, dT = 1 \cdot \int_{200}^{400} (C_V + R)\, dT$$

$$\Delta Q = \int_{200}^{400} C_V dT + R \times 200 \qquad ...(1)$$

The integral can be obtained by area under the graph.

At $\qquad T = 200$ K, $C_V = 2.25\ R$

and $\qquad T = 400$ K, $C_V = 3.0\ R$

$$\therefore \qquad \int_{200}^{400} C_V\, dT = \frac{1}{2} \times 50 \times (2.25\ R + 2.5\ R) + 100 \times 2.5\ R + \frac{1}{2} \times 50 \times (2.5\ R + 3\ R)$$

$$= (118.75 + 250 + 137.5)\text{R} = 506.25\ \text{R}$$

$\therefore$ From (1) $\qquad \Delta Q = 706.25\ R$

38. Consider an infinitesimal change in the state of the gas.

Let dQ = heat added dT = change in temperature C = molar heat capacity

$$dQ = nCdT \Rightarrow dQ = 2nRdT \quad ...(i)$$

Work done by the gas $dW = PdV$

Since $PV = nRT$

$$\Rightarrow \quad dW = \frac{nRT}{V}dV \quad ...(ii)$$

Change in internal energy

$$dU = nC_V dT = \frac{7}{5}nRdT \quad \left[\because \ C_V = \frac{7}{5}R\right] \quad ...(iii)$$

First law of thermodynamics says

$$dQ = dU + dW$$

$$2nRdT = \frac{7}{5}nRdT + nRT\frac{dV}{V}$$

$$\Rightarrow \quad \frac{3}{5}dT = T\frac{dV}{V} \Rightarrow \frac{3}{5}\frac{dT}{T} = \frac{dV}{V}$$

$$\Rightarrow \quad \frac{3}{5}\int\frac{dT}{T} = \int\frac{dV}{V}$$

$$\Rightarrow \quad \frac{3}{5}\ln T = \ln V + \ln k \quad [k = a \quad \text{constant}]$$

$$\Rightarrow \quad \ln T^{3/5} = \ln(kV)$$

$$\Rightarrow \quad T^{3/5} = kV$$

$$\Rightarrow \quad V \propto T^{3/5}$$

$\therefore$ If temperature is made $(32)^{1/3}$ times,

The volume becomes $(32)^{1/5} = 2$ times.

39. (a) The gas expands adiabatically.

Its volume changes from $V_0 = AL_0$ to V = AL as the bullet moves out of the gun.

$$\therefore \quad T_0V_0^{\gamma-1} = TV^{\gamma-1}$$

Where T = temperature of gas as the bullet moves out

$$\therefore \quad T_0(AL_0)^{\gamma-1} = T(AL)^{\gamma-1}$$

$$\Rightarrow \quad T = T_0\left(\frac{L_0}{L}\right)^{\gamma-1} \quad ...(i)$$

Kinetic energy gained by the bullet is equal to work done by the gas in adiabatic expansion.

$$\therefore \quad \frac{1}{2}mu^2 = \frac{nR(T_0 - T)}{\gamma - 1}$$

$$\therefore \quad u^2 = \frac{2nRT_0}{m(\gamma-1)}\left[1 - \left(\frac{L_0}{L}\right)^{\gamma-1}\right]$$

For helium $\gamma = \frac{5}{3}$

$$\therefore \quad u^2 = \frac{3nRT_0}{m}\left[1 - \left(\frac{L_0}{L}\right)^{2/3}\right]$$

$$u = \sqrt{\frac{3nRT_0}{m}\left[1 - \left(\frac{L_0}{L}\right)^{2/3}\right]}$$

(b) Speed will be maximum when $\frac{L_0}{L} << 1$

$$u_{max} = \sqrt{\frac{3nRT_0}{m}}$$

In this case ($L >> L_0$) the entire internal energy of the gas gets converted into kinetic energy of the bullet. Therefore, we may also write

$$\frac{1}{2} mu_{max}^2 = nC_V T_0$$

$$\Rightarrow \quad \frac{1}{2} mu_{max}^2 = n\frac{3R}{2} T_0$$

$$\Rightarrow \quad u_{max} = \sqrt{\frac{3nRT_0}{m}}$$

40. $$dU = \eta \cdot dQ$$

$$\therefore \quad dW = dQ - dU = \left(\frac{1}{\eta} - 1\right) dU$$

$$PdV = \left(\frac{1-\eta}{\eta}\right) nC_V dT \qquad \text{...(i)}$$

Ideal gas equation $PV = nRT$

$$\Rightarrow \quad PdV + VdP = nRdT$$

$$\Rightarrow \quad PdV + VdP = \frac{\eta R}{(1-\eta) C_V} PdV \qquad \text{[using (i)]}$$

$$\Rightarrow \quad PdV\left[1 - \frac{\eta R}{(1-\eta) C_V}\right] + VdP = 0$$

$$\Rightarrow \quad \left[1 - \frac{\eta R}{(1-\eta) C_V}\right]\frac{dV}{V} + \frac{dP}{P} = 0$$

$$\Rightarrow \quad \left[1 - \frac{\eta R}{(1-\eta) C_V}\right]\int \frac{dV}{V} + \int \frac{dP}{P} = \text{a constant}$$

$$\Rightarrow \quad \left[1 - \frac{\eta R}{(1-\eta) C_V}\right]\ln V + \ln P = \text{constant}$$

$$\Rightarrow \quad PV^{\left[1 - \frac{\eta R}{(1-\eta) C_V}\right]} = \text{constant} \quad \text{Put } C_V = \frac{R}{\gamma - 1}$$

$$\Rightarrow \quad PV^{\left(1 - \frac{(\gamma-1)\eta}{(1-\eta)}\right)} = \text{a constant}$$

41. $A \rightarrow B$

$$W = 0$$

$$\Rightarrow \quad \Delta V = 0 \quad \Rightarrow \quad \Delta\rho = 0 \text{ (density and volume constant)}$$

$$\therefore \quad P \propto T$$

$\therefore$ Temperature changes from T_0 to $2T_0$ and pressure changes from P_0 to $2P_0$

$B \rightarrow C$

$$\therefore \quad \Delta U = 0 \quad \Rightarrow \quad T = \text{constant at } 2T_0$$

Process is isothermal

$\therefore \qquad W = 2P_0 \cdot V_0 \ln \dfrac{V_C}{V_B}$

$\Rightarrow \qquad -P_0V_0 \ln 2 = 2P_0V_0 \ln \dfrac{V_C}{V_0}$

$\Rightarrow \qquad \left(\dfrac{V_C}{V_0}\right)^2 = \dfrac{1}{2} \quad \Rightarrow \quad V_C = \dfrac{V_0}{\sqrt{2}}$

$\therefore \qquad P_C = \dfrac{2P_0V_0}{V_C} = 2\sqrt{2}\,P_0$

$\Rightarrow \qquad 2\sqrt{2}P_0V_C = 2P_0V_0 \quad \Rightarrow \quad V_C = \dfrac{V_0}{\sqrt{2}}$

$C \rightarrow D$

Constant volume process

$\therefore \qquad P \propto T$

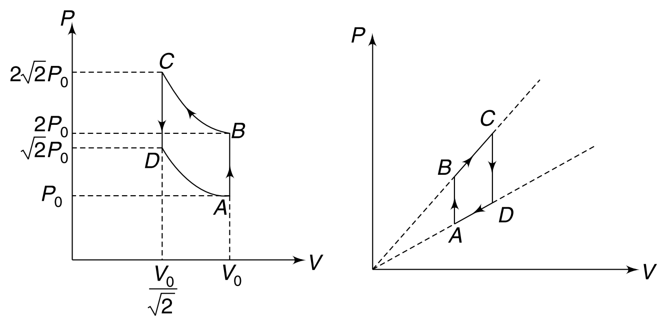
Temperature becomes half ($2T_0 \rightarrow T_0$) and pressure also becomes half $= \dfrac{P_C}{2} = \sqrt{2}P_0$

$D \rightarrow A$

Isothermal process $\left(\text{Volume changes from } \dfrac{V_0}{\sqrt{2}} \text{ to } V_0\right)$

$$W = (\sqrt{2}\ P_0)\left(\frac{V_0}{\sqrt{2}}\right) \ln\sqrt{2} = \frac{1}{2}\,P_0V_0\ln 2$$

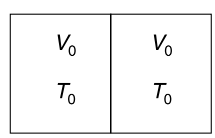
(a)

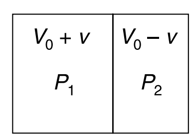


(c) Work done by the gas

$$W = -P_0V_0\ln 2 + \frac{1}{2}\,P_0V_0\ \ln 2 = -\frac{1}{2}\,P_0V_0\ln 2$$

$\therefore$ Work done on the gas $\qquad W = \dfrac{1}{2}\,P_0V_0\ln 2$





42. (i) Piston is conducting. It means temperature in both compartments is always same.

Let volume of two parts be $(V_0 + v)$ and $(V_0 - v)$ at an instant.

$$\beta = \frac{V_0 + v}{V_0 - v} \quad \Rightarrow \quad V = \left(\frac{\beta - 1}{\beta + 1}\right)V_0 \qquad \ldots\text{(i)}$$

If T is temperature of the two parts then

$$P_1 = \frac{nRT}{V_0 + v};\ P_2 = \frac{nRT}{V_0 - v}$$

Work done by external agent in further changing the volume of two parts by dv is

$$dW_{ext} = F_{ext} \cdot dv = (P_2 - P_1)\ dv$$

$$= nRT\left(\frac{1}{V_0 - v} - \frac{1}{V_0 + v}\right) dv \qquad \ldots\text{(ii)}$$

For the entire gas as system, $dQ = 0$

$$\therefore \quad dW_{ext} = dU$$

$$\therefore \quad nRT\left(\frac{2v}{V_0^2 - v^2}\right) dv = 2 \cdot nC_V dT$$

$$\Rightarrow \quad R\int_0^V \frac{vdv}{V_0^2 - v^2} = \frac{R}{\gamma - 1}\int_{T_0}^{T} \frac{dT}{T}$$

$$\Rightarrow \quad -\frac{(\gamma - 1)}{2}\left[\ln\left[V_0^2 - v^2\right]\right]_0^v = [\ln T]_{T_0}^T$$

$$\Rightarrow \quad -\left(\frac{\gamma - 1}{2}\right)\ln\left[\frac{V_0^2 - v^2}{V_0^2}\right] = \ln\left(\frac{T}{T_0}\right)$$

$$\Rightarrow \quad \left[1 - \frac{v^2}{V_0^2}\right]^{-\frac{\gamma - 1}{2}} = \frac{T}{T_0}$$

$$\Rightarrow \quad T = T_0\left[1 - \left(\frac{\beta - 1}{\beta + 1}\right)^2\right]^{\frac{1-\gamma}{2}} = T_0\left[\frac{4\beta}{(\beta + 1)^2}\right]^{\frac{1-\gamma}{2}}$$

(ii) Let n_1 and n_2 be number of moles of oxygen and helium respectively

$$32n_1 + 4n_2 = m \quad \text{...(i)}$$

Since volume of gas does not change, therefore $W = 0$

$$Q = \Delta U = (n_1 C_{V O_2} + C_{V He})\, \Delta T$$

$$\Rightarrow \quad Q = \left(\frac{5}{2}n_1 R + \frac{3}{2}n_2 R\right)\Delta T$$

$$\Rightarrow \quad 5n_1 + 3n_2 = \frac{2Q}{R\Delta T} \quad \text{...(ii)}$$

Solving (i) and (ii) gives

$$n_1 = \frac{1}{76}\left[3m - \frac{8Q}{R\Delta T}\right] = \frac{3m}{76} - \frac{2Q}{19R\Delta T}$$

And

$$n_2 = -\frac{5m}{76} + \frac{16Q}{19R\Delta T}$$

$\therefore$ Total number of moles of gas

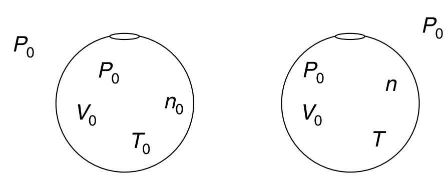
$$n = n_1 + n_2 = \frac{14Q}{19R\Delta T} - \frac{m}{38}$$

$\therefore$ change in pressure is given by

$$\Delta P = \frac{nR\Delta T}{V} = \frac{14Q}{19V} - \frac{mR\Delta T}{38V}$$

$$= \frac{28Q - mR\Delta T}{38V}$$

43. Let number of moles of air inside the container at time t be n, and temperature be T. It is pressure and volume that is not changing. In time dt, heat supplied is



$$dQ = H dt$$

$$\therefore \quad H dt = nC_P dT \;\Rightarrow\; H dt = n \cdot \frac{7}{2} R dT$$

But

$$nT = n_0 T_0 = \frac{P_0 V_0}{R}$$

$$\therefore \quad n = \frac{n_0 T_0}{T}$$

$$\therefore \quad \frac{7}{2}\frac{n_0 T_0}{T} R\,dT = H\,dt$$

$$\therefore \quad \frac{7n_0T_0R}{2H}\int_{T_0}^{T}\frac{dT}{T} = \int_0^t dt$$

$$\therefore \quad \ln\frac{T}{T_0} = \frac{2Ht}{7n_0T_0R}$$

$$\therefore \quad T = T_0 e^{\frac{2Ht}{7n_0T_0R}}$$

44. Let P_1 = initial pressure of the gas inside the tube

$$P_0 = P_1 + \rho g h_1 \qquad [h_1 \text{ height of Hg column}]$$

$$\rho g h_0 = P_1 + \rho g h_1$$

$$\therefore \quad P_1 = \rho g (h_0 - h_1)$$

Temperature of the gas is given by

$$T_1 = \frac{P_1 V_1}{nR} = \frac{\rho g (h_0 - h_1) A (h_0 - h_1)}{R} \qquad [A = \text{area of cross section}]$$

$$\therefore \quad T_1 = \frac{(h_0 - h_1)^2 \rho g A}{R}$$

After increasing the temperature, let the height of Hg column be h_2.

$$T_2 = \frac{(h_0 - h_2)^2 \rho g A}{R}$$

$$\therefore \quad \Delta T = T_2 - T_1 = \frac{\rho g A}{R}\left[(h_0 - h_2)^2 - (h_0 - h_1)^2\right] \qquad \text{...(i)}$$

Let's calculate the work done by the gas in pushing the *Hg* column.

$$W = \int P\,dV = -\int_{h_1}^{h_2} \rho g (h_0 - h)\, A\,dh \qquad [-ve \text{ sign as } dh \text{ is negative}]$$

$$= \rho g A\left[\frac{(h_0 - h)^2}{2}\right]_{h_1}^{h_2}$$

$$= \frac{\rho g A}{2}\left[(h_0 - h_2)^2 - (h_0 - h_1)^2\right] = \frac{R\Delta T}{2} \qquad [\text{using (i)}]$$

First law of thermodynamics

$$\Delta Q = nC_V \Delta T + W$$

$$= \frac{3}{2}R\Delta T + \frac{R\Delta T}{2} = 2R\Delta T$$

45. PV^k = constant

For small changes in P, V and T, we can write–

$$PkV^{k-1}\Delta V + \Delta P V^k = 0 \qquad \text{...(i)}$$

For an ideal gas

$$PV = nRT \qquad \text{...(ii)}$$

$$\therefore \quad P\Delta V + V\Delta P = nR\Delta T \qquad \text{...(iii)}$$

From (i) $$(V\Delta P)V^{k-1} = -kPV^{k-1}\Delta V$$

$$\therefore \quad V\Delta P = -kP\Delta V$$

Putting in (iii) $\quad P\Delta V - kP\Delta V = nR\Delta T$

$$\Rightarrow \quad P\Delta V(1-k) = nR\Delta T \quad \text{... (iv)}$$

First law of thermodynamics–

$$\Delta Q = \Delta U + W$$

$$nC\Delta T = nC_V\Delta T + P\Delta V$$

$$nC\Delta T = nC_V\Delta T + \frac{nR\Delta T}{1-k} \quad \text{[using (iv)]}$$

$$\therefore \quad C = C_V + \frac{R}{1-k} \quad \text{... (v)}$$

(i) $\quad C = \dfrac{C_P + C_V}{2} = \dfrac{C_V + R + C_V}{2} = C_V + \dfrac{R}{2}$

$$\therefore \quad k = -1$$

(ii) $\quad C = C_P + C_V$

For mono atomic gas $\quad C_P = \dfrac{5R}{2}$

$\therefore$ from (v) $\quad \dfrac{R}{1-k} = \dfrac{5R}{2}$

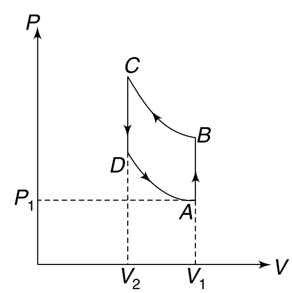
$$\Rightarrow \quad 2 = 5 - 5k \Rightarrow k = 0.6$$

46. (a) AB and CD are isochoric.

BC and DA are isothermal processes because $P \propto \rho$ means T is constant as shown below-

$$PV = nRT \Rightarrow PV = \frac{m}{M}RT$$

$$\Rightarrow \quad P = \frac{m}{V}\frac{RT}{M} \Rightarrow P = \rho\frac{RT}{M}$$



$P \propto \rho$ means T is constant.

(b) $\quad V_1 = \dfrac{nM}{\rho_1}$ and $V_2 = \dfrac{nM}{\rho_2}$

$$W_{AB} = W_{CD} = 0$$

$$W_{BC} = P_BV_B\ln\left(\frac{V_C}{V_B}\right) = P_2V_1\ln\left(\frac{V_2}{V_1}\right) = P_2\frac{nM}{\rho_1}\ln\left(\frac{\rho_1}{\rho_2}\right)$$

$$W_{DA} = P_AV_A\ln\left(\frac{V_A}{V_D}\right) = P_1V_1\ln\left(\frac{V_1}{V_2}\right) = P_1\frac{nM}{\rho_1}\ln\left(\frac{\rho_2}{\rho_1}\right)$$

$$\therefore \quad W = \frac{nM}{\rho_1}\left[-P_2\ln\left(\frac{\rho_2}{\rho_1}\right) + P_1\ln\left(\frac{\rho_2}{\rho_1}\right)\right]$$

$$= -\frac{nM}{\rho_1}\ln\left(\frac{\rho_2}{\rho_1}\right)(P_2 - P_1)$$

47. (a) In upside down position

$$P_2A = P_0A - mg$$

$$\therefore \quad P_2 = P_0 - \frac{mg}{A}$$

$$= 1.01 \times 10^5 - \frac{0.1 \times 10}{10 \times 10^{-4}} = 1.0 \times 10^5 \text{ N/m}^2$$

In original piston

$$P_1A = P_0A + mg$$

$$\Rightarrow \quad P_1 = 1.02 \times 10^5 \text{ N/m}^2$$

Since, temperature remained constant

$$P_2V_2 = P_1V_1$$

$$\therefore \quad V_2 = 1.02 \times 100 = 102\, cc$$

$$\therefore \quad \text{length of air column} \quad l_2 = \frac{V_2}{A} = \frac{102}{10} = 10.2 \text{ cm}$$

(b) As the piston reaches the mouth of the cylinder, length of air column becomes 20.4 cm, i.e., volume of air doubles. The piston moves slowly, it means pressure remains constant.

$$\therefore \quad W = P_2 \Delta V = 1.0 \times 10^5 \times (10.2 \times 10 \times 10^{-6})$$

$$= 10.2 \text{ J}$$

Keeping pressure constant, the volume has been doubled. It means temperature has doubled.

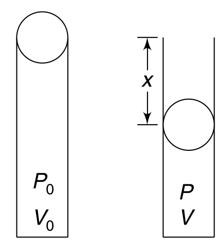
$$\therefore \quad \Delta U = nC_V\Delta T = n \cdot \frac{5}{2} R \times 300$$

$$= (nR \times 300) \times \frac{5}{2} = P_1V_1 \times \frac{5}{2}$$

$$= 1.02 \times 10^5 \times 100 \times 10^{-6} \times \frac{5}{2} = 25.5 \text{ J}$$

$$\therefore \quad \Delta Q = 25.5 + 10.2 = 35.7 \text{ J}$$

48. $V_0 = AL;\ V = A(L - x) \qquad [A = \pi r^2]$



For isothermal compression of air

$$PV = P_0V_0$$

$$PA[L - x] = P_0AL \quad \Rightarrow \quad P = P_0\left(\frac{L}{L - x}\right)$$

(a) Work done by the gas on the ball $= -P_0V_0 \ln\left(\frac{L}{L - x}\right)$

Work done by the gravity on the ball $= mgx$

If the ball comes to rest

$$mgx - P_0V_0 \ln\left(\frac{L}{L - x}\right) = 0$$

Given $\quad x = \frac{L}{2}$

$$mg\,\frac{L}{2} = P_0AL \ln\left(\frac{L}{L - \frac{L}{2}}\right)$$

$$\therefore \quad \frac{4}{3}\pi r^3 \cdot d \cdot g \cdot \frac{1}{2} = P_0 \pi r^2 \ln(2)$$

$$\therefore \quad r = \frac{3}{2}\,\frac{P_0 \ln(2)}{d \cdot g \cdot}$$

(b) In equilibrium, $\quad P\pi r^2 = mg$

$$P_0\left(\frac{L}{L - x}\right) \pi r^2 = \frac{4}{3}\pi r^3 \cdot d \cdot g$$

$$L - x = \frac{3P_0L}{4rdg} = \frac{3}{4}L\cdot\frac{2}{3\ln(2)} = \frac{L}{2\ln(2)}$$

$$x = L\left[1 - \frac{1}{2\ln(2)}\right]$$

49. Ball is in equilibrium means pressure inside the container is equal to the atmospheric pressure (P_0).

If the ball is displaced a little; volume of air inside the container changes and hence, the pressure changes. This creates a restoring force which causes oscillations.

Assume that the ball is displaced by a small distance x towards right. Change in pressure can be calculated as follows.

$$PV = \text{constant}$$

$$\ln P + \ln V = \text{constant}$$

$$\frac{1}{P}\frac{\Delta P}{\Delta V} + \frac{1}{V} = 0$$

$\therefore$ $$\Delta P = -\frac{P\Delta V}{V}$$

[Negative sign means pressure drops when volume increases and vice-versa]

$$\Delta P = -\frac{P_0}{V_0}\pi r^2\cdot x$$

$\therefore$ Restoring force towards left = (outside atmospheric pressure – Inside gas pressure) × A

$$= \Delta P\cdot \pi r^2$$

$\therefore$ $$\frac{4}{3}\pi r^3\cdot d\left(\frac{d^2x}{dt^2}\right) = -\frac{P_0}{V_0}\pi r^2\cdot x\cdot \pi r^2$$

$$\frac{d^2x}{dt^2} = -\frac{3P_0\pi r}{4V_0 d}\cdot x$$

$\therefore$ Motion is SHM.

$$\omega = \sqrt{\frac{3\pi P_0 r}{4V_0 d}}$$

$$T = 4\pi\sqrt{\frac{V_0 d}{3\pi P_0\cdot r}}$$

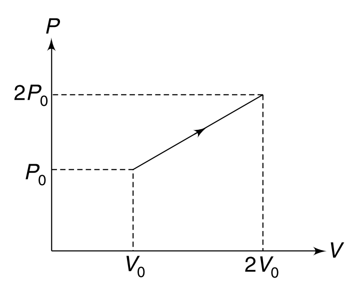
$$T = 4\sqrt{\frac{\pi V_0 d}{3P_0 r}}$$

50. (a) $$P_0V_0 = nRT_0$$

$$W = \text{area under the graph} = \frac{1}{2}V_0[P_0 + 2P_0]$$

$$= \frac{3}{2}P_0V_0 = \frac{3}{2}nRT_0$$

And $$\Delta U = nC_V\Delta T = n\cdot\frac{3}{2}R(T_f - T_i)$$



Final temperature will be 4 times the initial temperature since $P_fV_f = 4P_iV_i$

$\therefore$ $$\Delta U = \frac{3}{2}nR(4T_0 - T_0) = \frac{9}{2}nRT_0$$

$\therefore$ $$\Delta Q = W + \Delta U = \frac{3}{2}nRT_0 + \frac{9}{2}nRT_0 = 6nRT_0$$

(b)
$$nC\Delta T = \Delta Q$$
$$\therefore \quad nC(3T_0) = 6nRT_0$$
$$\therefore \quad C = 2R$$

51. (i) The process is adiabatic for each gas.

For argon: $\gamma = \frac{5}{3}$

$$\therefore \quad P_{Ar}\left(\frac{V_0}{2}\right)^{\frac{5}{3}} = P_0 V_0^{5/3}$$

$$\Rightarrow \quad P_{\text{Ar}} = 2^{\frac{5}{3}} P_0 \quad ...(i)$$

For H_2: $\gamma = \frac{7}{5}$

$$P_{H_2} V_{H_2}^{7/5} = P_0 V_0^{7/5} \quad ...(ii)$$

But $P_{Ar} = P_{H_2}$ (process is slow and pressure is same in both chambers at all instants)

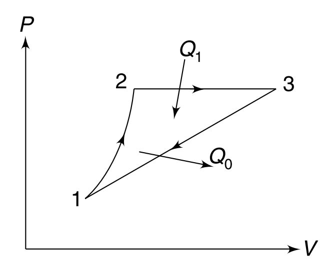
$\therefore$ from (1) and (2) $\quad V_{H_2} = \dfrac{V_0}{(2)^{\frac{25}{21}}}$

(ii) There is exchange of heat between the gases but no exchange of heat from surrounding.

For both gases as system

$$\Delta Q = 0$$
$$\therefore \quad W + \Delta U = 0$$
$$\therefore \quad \Delta U = -W$$
$$nC_{V_{Ar}}\Delta T + nC_{V_{H_2}}\Delta T = W_0 \quad \text{[Work done by gas } W = -W_0]$$
$$n\cdot\frac{3}{2}R\Delta T + n\cdot\frac{5}{2}R\Delta T = W_0$$
$$4nR\Delta T = W_0$$
$$\therefore \quad \Delta T = \frac{W_0}{4nR}$$

52. Cycle 1–2–3–1



From 1 $\rightarrow$ 2, work done is $+ve$ ($\because$ volume increases) and ΔU is also positive ($\because$ temperature increased).

Similarly, for 2 $\rightarrow$ 3, W as well as ΔU are positive.

In both the processes 1 $\rightarrow$ 2 and 2 $\rightarrow$ 3 the gas absorbs heat.

Let the total heat absorbed in the process 1 $\rightarrow$ 2 $\rightarrow$ 3 be Q_1.

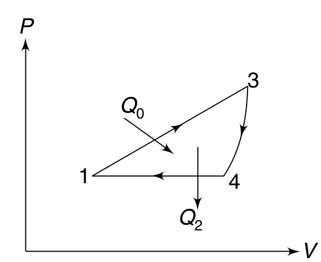
Similarly, one can argue that the gas rejects heat (say Q_0) to the surrounding in the process 3 $\rightarrow$ 1. Let work done in the cycle be W_1.

Using first law of thermodynamics for complete cycle–

$$\Delta Q = \Delta U + W \quad [\Delta U_{\text{cycle}} = 0]$$
$$Q_1 - Q_0 = W_1 \quad ...(i)$$

And efficiency $\quad \eta = \dfrac{W_1}{Q_1}$

$$\frac{1}{4} = \frac{W_1}{Q_1} \quad ...(ii)$$



Cycle 1–3–4–1

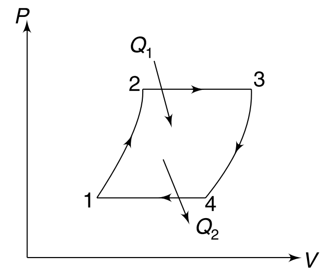
Q_0 = heat gained by the gas in process 1 $\to$ 3.

Q_2 = heat rejected by the gas in process 3 $\to$ 4 $\to$ 1

W_2 = work done in cycle.

$$\eta = \frac{W_2}{Q_0}$$

$$\frac{1}{10} = \frac{W_2}{Q_1 - W_1} \quad \text{[using (i)]}$$



$$\Rightarrow \quad Q_1 - W_1 = 10W_2$$

$$\Rightarrow \quad Q_1 = W_1 + 10W_2 \quad \text{...(iii)}$$

Using (i) and (iii)

$$4W_1 = W_1 + 10W_2$$

$$3W_1 = 10W_2 \quad \text{...(iv)}$$

Efficiency of cycle 1 – 2 – 3 – 4 – 1

$$\eta = \frac{W_1 + W_2}{Q_1} = \frac{W_1 + W_2}{W_1 + 10W_2}$$

$$= \frac{1 + \frac{W_2}{W_1}}{1 + 10\frac{W_2}{W_1}} = \frac{1 + \frac{3}{10}}{1 + 3} \quad \text{[using 4]}$$

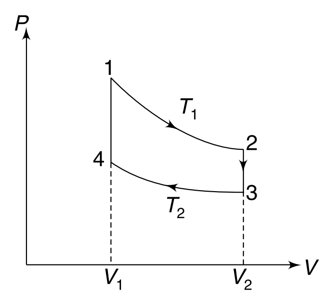
$$= \frac{13}{40}$$

In percentage: $\eta = \frac{13}{40} \times 100 = 32.5\%$

53. $$W_{cycle} = W_{12} + W_{34} = nRT_1 \ln\frac{V_2}{V_1} + nRT_2 \ln\frac{V_1}{V_2}$$

$$= \left(nR \ln\frac{V_2}{V_1}\right)(T_1 - T_2)$$

$$Q_{12} = W_{12}$$

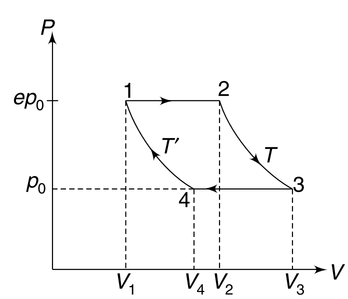


Heat rejected during 3-4 is a wastage.

$\therefore$ Efficiency $$\eta = \frac{\left(nR \ln\frac{V_2}{V_1}\right)(T_1 - T_2)}{\left(nR \ln\frac{V_2}{V_1}\right)(T_1)}$$

$\therefore$ $$\eta = \frac{T_1 - T_2}{T_1}$$

54. Isobaric process 1-2



$$W_{12} = eP_0(V_2 - V_1) = nR(T - T')$$

$$\Delta U_{12} = nC_V\Delta T = \frac{nR}{\gamma - 1}(T - T')$$

$\therefore$ $$\Delta Q_{12} = nR(T - T')\left[1 + \frac{1}{\gamma - 1}\right] = \frac{nR\gamma}{\gamma - 1}(T - T')$$

Isothermal process 2-3:

$$W_{23} = nRT\ln\left(\frac{P_2}{P_3}\right) = nRT\ln e = nRT$$

$$\Delta Q_{23} = W_{23} = nRT$$

Isobaric process 3-4:

$$W_{34} = P_0(V_4 - V_3) = P_1V_1 - P_2V_2 = -W_{12}$$

$$\Delta U < 0$$

Heat is rejected in the process.

Isothermal process 4-1:

$$W_{41} = nRT'\ln\left(\frac{P}{eP_0}\right) = -nRT'$$

Total work done in cycle = $nRT - nRT' = nR(T - T')$

Total heat absorbed in cycle = $\frac{nR\gamma}{\gamma - 1}(T - T') + nRT$

$$= \frac{nR}{\gamma - 1}[\gamma T - \gamma T' + \gamma T - T]$$

$$= \frac{nR[(2\gamma - 1)T - \gamma T']}{\gamma - 1}$$

∴ Efficiency $\eta = \frac{\text{Total work done}}{\text{total heat absorbed}} = \frac{(T - T')(\gamma - 1)}{(2\gamma - 1)T - \gamma T'}$

55. The line 1-2 passes through origin if extended. It means $\rho \propto P$. The process must be isothermal.

$$\Delta W_{12} = nRT\ln\frac{P_1}{P_2} = P_1V_1\ln\frac{1}{2} = P_0\frac{M}{\rho_0}\ln\left(\frac{1}{2}\right) = -\frac{P_0M}{\rho_0}\ln 2$$

$$\Delta U_{12} = 0$$

∴ $$\Delta Q_{12} = -\frac{P_0M}{\rho_0}\ln 2$$

$$\text{Heat rejected} = \frac{P_0M}{\rho_0}\ln 2$$

Process 2-3 is isobaric

$$\Delta W_{23} = 2P_0(V_3 - V_2) = 2P_0\left(\frac{M}{\rho_3} - \frac{M}{\rho_2}\right)$$

$$= 2P_0M\left(\frac{1}{\rho_0} - \frac{1}{2\rho_0}\right) = \frac{P_0M}{\rho_0}$$

$$\Delta U_{23} = nC_V\Delta T = 1 \times \frac{3}{2}R[T_3 - T_2]$$

$$= \frac{3}{2}[P_3V_3 - P_2V_2] = \frac{3}{2}\cdot 2P_0\left(\frac{M}{\rho_0} - \frac{M}{2\rho_0}\right) = \frac{3}{2}\frac{P_0M}{\rho_0}$$

$$\Delta Q_{23} = \Delta W_{23} + \Delta U_{23} = \frac{5}{2}\frac{P_0M}{\rho_0}$$

Process 3-1 is isochoric.

$$\Delta W_{31} = 0$$

$$\Delta U_{31} = nC_V\Delta T = n\cdot\frac{3}{2}R[T_1 - T_3]$$

$$= \frac{3}{2}[P_1V_1 - P_3V_3] = \frac{3}{2}\left[\frac{P_0M}{\rho_0} - \frac{2P_0M}{\rho_0}\right]$$

$$= -\frac{3}{2}\frac{P_0M}{\rho_0}$$

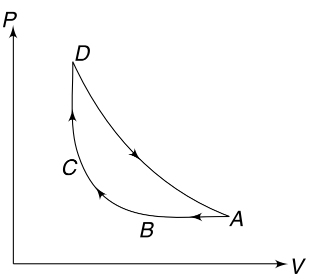
$$\Delta Q_1 = -\frac{3}{2}\frac{P_0M}{\rho_0}$$

(a) $$\Delta Q_{\text{rejected}} = |\Delta Q_{12}| + |\Delta Q_{31}| = \frac{P_0M}{\rho_0}\left(\frac{3}{2} + \ln 2\right)$$

(b) $$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{\frac{P_0M}{\rho_0} - \frac{P_0M}{\rho_0}\ln 2}{\frac{5}{2}\frac{P_0M}{\rho_0}}$$

$$= \frac{1 - \ln 2}{\frac{5}{2}} = \frac{2}{5}(1 - \ln 2)$$

56. (a)



(b) Let $V_C = V_D = V_0$ (minimum volume)

$V_A = 8\sqrt{2}V_0$ (maximum volume)

Obviously T is maximum at A and D ($\because T \propto PV$) and minimum at B. In adiabatic compression temperature is increased. Hence $T_C > T_B$.

Let $T_B = T_0$; then $T_A = T_D = 4T_0$

For process AB $V \propto T$

$$\therefore \quad V_B = T_B\left(\frac{V_A}{T_A}\right) = T_0\left(\frac{8\sqrt{2}V_0}{4T_0}\right) = 2\sqrt{2}V_0$$

Process BC is adiabatic

$$\therefore \quad T_C = T_B\left(\frac{V_B}{V_C}\right)^{\gamma - 1} = T_0\left(\frac{2\sqrt{2}\,V_0}{V_0}\right)^{\frac{5}{3} - 1} = 2T_0 \qquad \left[\gamma = \frac{5}{3} \text{ for He}\right]$$

$$\therefore \quad T_A = 4T_0,\ T_B = T_0,\ T_C = 2T_0 \text{ and } T_D = 4T_0$$

Now in process AB (isobaric)-

$$\Delta W_{AB} = \Delta Q_{AB} - \Delta U_{AB} = nC_P\Delta T - nC_V\Delta T = nR\Delta T$$

$$= nR(T_B - T_A) = -3nRT_0$$

Also ΔQ_{AB} is $-ve$

In process BC (adiabatic) $\Delta Q_{BC} = 0$

$$\Delta W_{BC} = -\Delta U_{BC} = -nC_V\Delta T = n\left(\frac{3}{2}R\right)(T_B - T_C) = -\frac{3}{2}nRT_0$$

For CD (isochoric); $\Delta W_{CD} = 0$

$$\Delta Q_{CD} = \Delta U_{CD} = nCV(T_D - T_C) = n\left(\frac{3}{2}R\right)(2T_0) = 3nRT_0$$

For DA (isothermal): $\Delta U_{DA} = 0$

$$\Delta Q_{DA} = \Delta W_{DA} = nRT_D \ln \frac{V_A}{V_B} = nR(4T_0)\ln(8\sqrt{2})$$

$$= 14nRT_0 \ln 2$$

$\therefore$ Total work done; $W = -3nRT_0 - \frac{3}{2}nRT_0 + 14nRT_0 \ln 2$

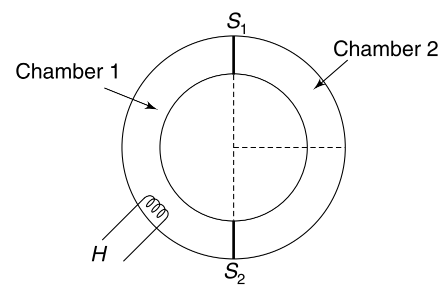
$$= nRT_0\left[14\ln 2 - \frac{9}{2}\right] = 5.202\ nRT_0$$

Heat absorbed $Q = 3nRT_0 + 14nRT_0 \ln(2)$

$$= 12.702\ nRT_0$$

$\therefore$ $$\eta = \frac{W}{Q} \times 100 = \frac{5.202}{12.702} \times 100 = 41\%$$

57. The process in chamber 2 is adiabatic.
Let final pressure in chamber 2 be P_2.



$$P_2\left(\frac{V_0}{2}\right)^\gamma = P_0V_0^\gamma$$

$\Rightarrow$ $$P_2 = (2)^{3/2}P_0 \qquad \left[\because\ \gamma = \frac{3}{2}\right]$$

$$= 2\sqrt{2}\,P_0$$

In equilibrium, pressure in both chambers must be same, hence final pressure in chamber 1 is

$$P_1 = P_2 = 2\sqrt{2}\,P_0$$

Work done on the gas in chamber 2 by gas in chamber 1 is

$$W = \frac{P_2V_2 - P_0V_0}{\gamma - 1} = \frac{2\sqrt{2}\cdot P_0\frac{V_0}{2} - P_0V_0}{\frac{3}{2} - 1}$$

$$W = 2(\sqrt{2} - 1)P_0V_0$$

Change in internal energy of the gas in chamber 1

$$\Delta U = nC_V\Delta T = \frac{nR\Delta T}{\gamma - 1} = 2nR\Delta T$$

$$= 2[P_1V_1 - P_0V_0] = 2\left[2\sqrt{2}P_0\cdot\frac{3}{2}V_0 - P_0V_0\right]$$

$$= 2(3\sqrt{2} - 1)\ P_0V_0$$

$\therefore$ $$\Delta Q = \Delta U + W = 2(3\sqrt{2} - 1)\ P_0V_0 + 2(2\sqrt{2} - 1)\ P_0V_0$$

$$= 2P_0V_0[3\sqrt{2} - 1 + \sqrt{2} - 1] = 4P_0V_0[2\sqrt{2} - 1]$$

58. (a) Equation of a parabola passing through origin will be of the form $y^2 = kx$

$\Rightarrow$ $$T^2 = \frac{k}{V} \quad \Rightarrow \quad T^2V = k$$

$\Rightarrow$ $$TV^{1/2} = k$$

Because the process is adiabatic, $TV^{\gamma - 1}$ = Const.

$\therefore$ $$\gamma - 1 = \frac{1}{2} \quad \Rightarrow \quad \gamma = \frac{3}{2}$$

$$\frac{V_{rms}}{V_{sound}} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{\gamma RT}{M}}} = \sqrt{\frac{3}{\gamma}} = \sqrt{2}$$

(b) During the process volume changed from V_0 to $\frac{V_0}{1.21}$

$$\therefore \quad P_0V_0^{\gamma} = P\left(\frac{V_0}{1.21}\right)^{\gamma}$$

$$\Rightarrow \quad P = P_0(1.21)^{3/2} = P_0(1.1)^3 = 1.33P_0$$

Increase in pressure

$$\Delta P = 0.03P_0 = 0.33 \times 10^5 \text{ N/m}^2$$

∴ Weight of water over piston

$$mg = \Delta PA = 0.33 \times 10^5 \times 1.515 \times 10^{-3}$$

$$= 0.5 \times 10^2 = 50 \text{ N}$$

$$\therefore \quad m = 5 \text{ kg}$$

∴ Time required $\quad t = \frac{5}{0.25} = 20$ sec

59. The rate at which energy flows into the house

$$\frac{dQ}{dt} \propto \Delta T \quad \Rightarrow \quad \frac{dQ}{dt} = k\Delta T$$

k is a constant that depends on geometry and material of the walls of the room. To maintain the temperature the air conditioner must remove heat at the same rate.

If β is COP, then $\frac{dQ/dt}{dW/dt} = \beta$

$$\Rightarrow \quad \frac{dW}{dt} = \frac{1}{\beta}\frac{dQ}{dt} = \frac{k}{\beta}\Delta T$$

Now $\quad \beta = \frac{T_0}{\Delta T}$

$$\therefore \quad \frac{dW}{dt} = \frac{k(\Delta T)^2}{T_0}$$

60. (i) $\quad \frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

And $\quad Q_1 = Q_2 + W$

$$\Rightarrow \quad Q_2 = Q_1 - W = Q_2\left(\frac{T_1}{T_2}\right) - W$$

$$\Rightarrow \quad W = Q_2\left(\frac{T_1}{T_2} - 1\right) \quad \Rightarrow \quad Q_2 = W\left(\frac{T_2}{T_1 - T_2}\right)$$

$$\frac{dQ_2}{dt} = \frac{dW}{dt}\left(\frac{T_2}{T_1 - T_2}\right)$$

$$\therefore \quad \frac{dQ_2}{dt} = P\left(\frac{T_2}{T_1 - T_2}\right) \qquad \text{...(i)}$$

(ii) From $\quad k\Delta T = P\,\frac{T_2}{\Delta T}$

$$\Rightarrow \quad k\Delta T^2 = P(T_1 - \Delta T) \Rightarrow k\Delta T^2 + P\Delta T - PT_1 = 0$$

$$\therefore \quad \Delta T = \frac{-P \pm \sqrt{P^2 + 4PkT_1}}{2k}$$

Only +ve sign is meaningful

$$\therefore \quad \Delta T = \frac{P}{k}\left[\sqrt{1 + \frac{4kT_1}{P}} - 1\right]$$

$$\therefore \quad T_2 = T_1 - \Delta T = T_1 - \frac{P}{k}\left[\sqrt{1 + \frac{4kT_1}{P}} - 1\right]$$

61. Let instantaneous temperature of the two tanks be θ_1 and θ_2 respectively. A small amount of heat (dQ_1) is extracted from the source and its temperature falls by $(-d\theta_1)$. In the same time a heat (dQ_2) is rejected into the sink and its temperature increases by $d\theta_2$.

(a) We know that–

$$\frac{dQ_1}{\theta_1} = \frac{dQ_2}{\theta_2}$$

$$\Rightarrow \quad -\frac{msd\theta_1}{\theta_1} = \frac{msd\theta_2}{\theta_2} \Rightarrow -\int_{T_1}^{T_0} \frac{d\theta_1}{\theta_1} = \int_{T_2}^{T_0} \frac{d\theta_2}{\theta_2}$$

$$\Rightarrow \quad \ln\frac{T_1}{T_0} = \ln\frac{T_0}{T_2} \Rightarrow T_0 = \sqrt{T_1T_2} = \sqrt{361 \times 289}$$

$$= 323K$$

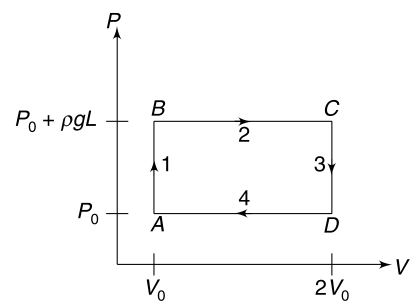
(b)
$$W = \Delta Q_1 - \Delta Q_2$$
$$= ms(T_1 - T_0) - ms(T_0 - T_2)$$
$$= ms(T_1 + T_2 - 2T_0) = 4ms$$

62. (a) Process 1 is a constant volume process. Pressure is increased from P_0 to $P_0 + \rho gL$

Process 2 is constant pressure process. Volume of gas in increased from $V_0(=AL)$ to $2V_0(=2AL)$

Process 3 is again a constant volume process in which pressure reduced from $P_0 + \rho gL$ to P_0.

Process 4 is again an isobaric process in which the gas is brought back to original volume. The $P - V$ graph is as shown.



(b) Heater supplies heat in the process 1 and 2

$$\Delta Q_H = \Delta Q_1 + \Delta Q_2 = nC_v\Delta T_1 + nC_p\Delta T_2$$
$$= \frac{5}{2}nR(T_B - T_A) + \frac{7}{2}nR(T_C - T_B)$$
$$= \frac{5}{2}[(P_0 + \rho gL)V_0 - P_0V_0] + \frac{7}{2}[(P_0 + \rho gL)\, 2V_0 - (P_0 - \rho gL)\, V_0]$$
$$= \frac{5}{2}\rho gLV_0 + \frac{7}{2}(P_0 + \rho gL)\, V_0$$
$$= \frac{1}{2}(7P_0 + 12\rho gL)V_0$$

In the entire cycle $\quad dU = 0$

First law of thermodynamics for entire cycle

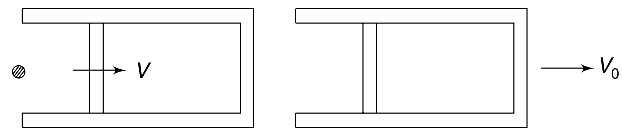
$$\Delta Q = \Delta U + W$$

$$\Delta Q = 0 + \rho g L V_0 \quad [W = \text{area under } P - V \text{ graph}]$$

$\therefore$ Heat extracted by cold liquid

$$\Delta Q_L = \Delta Q_H - \Delta Q = \frac{1}{2}(7P_0 + 10\rho g L)V_0 = \frac{1}{2}(7P_0 + 10\rho g L)AL$$

63. Immediately after collision, the particle comes to rest and the piston starts moving with velocity v, because the collision is elastic and two colliding objects have equal mass.



The compression is maximum when the piston stops moving relative to the cylinder.

Let v_0 = velocity of entire system at the point of maximum compression

The first figure shows situation immediately after collision and the second figure shows situation at maximum compression.

Momentum conservation gives–

$$5Mv_0 = Mv \Rightarrow v_0 = \frac{v}{5}$$

Energy conservation gives–

$$nC_v\Delta T = \frac{1}{2}Mv^2 - \frac{1}{2}(5M)\left(\frac{V}{5}\right)^2$$

$$1 \times \frac{3}{2}R\Delta T = \frac{2}{5}Mv^2 \quad \left[\text{for monatomic gas } C_v = \frac{3}{2}R\right]$$

$$\Delta T = \frac{4Mv^2}{15R}$$

64. (a)
$$C = \frac{\Delta Q}{\Delta T} \quad (\text{for } n = 1)$$

$\therefore$
$$C = \frac{\Delta U}{\Delta T} + \frac{\Delta W}{\Delta T} = C_v + \frac{\Delta W}{\Delta T}$$

Let's find ΔW in the process $PV^x = k$ as the state of gas changes from (P_1, V_1, T_1) to (P_2, V_2, T_2)

$$\Delta W = \int_{V_1}^{V_2} Pdv = k\int_{V_1}^{V_2} V^{-x}dv = \left[\frac{kV^{-x+1}}{-x+1}\right]_{V_1}^{V_2}$$

$$= \frac{kV_2^{-x+1} - kV_1^{-x+1}}{-x+1}$$

$$= \frac{P_2V_2^xV_2^{-x+1} - P_1V_1^xV_1^{-x+1}}{1-x} = \frac{P_2V_2 - P_1V_1}{1-x} \quad [\because P_1V_1^x = P_2V_2^x = k]$$

$\therefore$
$$\Delta W = \frac{RT_2 - RT_1}{1-x} = \frac{R\Delta T}{1-x}$$

$$\frac{\Delta W}{\Delta T} = \frac{R}{1-x}$$

$\therefore$
$$C = C_v + \frac{R}{1-x}$$

(b)
$$Q = \Delta U + \Delta W$$

$\Rightarrow$
$$Q = \Delta U + \frac{Q}{2} \Rightarrow \Delta U = \frac{Q}{2}$$

$$\therefore \quad \Delta U = \Delta W \Rightarrow nC_v dT = PdV$$

$$n\frac{3R}{2}dT = PdV \quad [\because \; PV = nRT \; \therefore \; Pdv + VdP = nRdT]$$

$$\Rightarrow \quad \frac{3}{2}[PdV + VdP] = PdV$$

$$\Rightarrow \quad PdV = -3VdP \Rightarrow \int \frac{dV}{V} = -3\int \frac{dP}{P}$$

$$\Rightarrow \quad \ln V = -3\ln P + \ln K$$

$$\Rightarrow \quad \ln P^3V = \ln K$$

$$\Rightarrow \quad P^3V = \text{constant}$$

As shown in part (a) $\quad C = C_V + \dfrac{R}{1-x}$

$$\therefore \quad C = \frac{3}{2}R + \frac{R}{1-\frac{1}{3}} = 3R$$

Alternatively, we can simply say

$$dQ = dU + dW$$

$$\Rightarrow \quad dQ = 2dU \qquad [\because \; dU = dW]$$

$$\Rightarrow \quad nCdT = 2nC_V dT \Rightarrow C = 2C_V$$

$$= 2\cdot\frac{3}{2}R = 3R$$

65. (a) Rate of teat flow through the piston is $\dfrac{dQ}{dt} = \dfrac{KA(T_1 - T)}{d}$

Where T is the instantaneous temperature of the gas.

The gas receives this heat at a constant pressure. Hence,

$$nC_p\frac{dT}{dt} = \frac{dQ}{dt}$$

$$\Rightarrow \quad nC_p\frac{dT}{dt} = \frac{KA(T_1 - T)}{d}$$

$$\Rightarrow \quad \frac{P_0V_0}{RT_0}\cdot\frac{5}{2}R\frac{dT}{dt} = \frac{KA}{d}(T_1 - T)$$

$$\Rightarrow \quad \frac{5P_0V_0d}{2T_0KA}\int_{T_0}^{T}\frac{dT}{T_1 - T} = \int_0^t dt$$

$$\Rightarrow \quad \ln\left(\frac{T_1 - T}{T_1 - T_0}\right) = -\frac{2T_0KA}{5P_0V_0d}t$$

$$\Rightarrow \quad T_1 - T = (T_1 - T_0)e^{\left(\frac{-2T_0KAt}{5P_0V_0d}\right)}$$

$$\therefore \quad T = T_1 - (T_1 - T_0)e^{\left(\frac{-2T_0KAt}{5P_0V_0d}\right)} \qquad \text{...(i)}$$

(b) Let the height at time t be h and initial height be $h_0\left(=\dfrac{V_0}{A}\right)$

$$\frac{PV}{RT} = \frac{P_0V_0}{RT_0}$$

$$\therefore \quad P_0Ah = P_0V_0\left(\frac{T}{T_0}\right)$$

$$\Rightarrow \quad h = \frac{V_0}{A}\left(\frac{T}{T_0}\right)$$

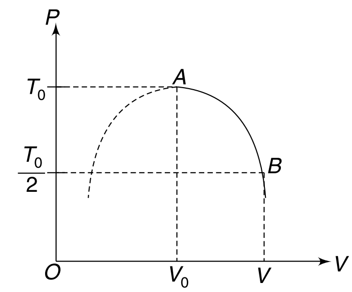
$\therefore$ Height raised $\qquad \Delta h = h - h_0 = h - \dfrac{V_0}{A}$

$$= \frac{V_0}{A}\left[\frac{T}{T_0} - 1\right] = \frac{V_0}{AT_0}(T - T_0)$$

$$= \frac{V_0}{AT_0}\left[(T_1 - T_0) - (T_1 - T_0)e^{\frac{-2T_0KAt}{5P_0V_0d}}\right]$$

$$= \frac{V_0(T_1 - T_0)}{AT_0}\left[1 - e^{\frac{-2T_0KAt}{5P_0V_0d}}\right]$$

66. Equation of the given straight line is the process equation.



$$P = -\left(\frac{P_0}{V_0}\right)V + 2P_0 \qquad \text{...(i)}$$

But for an ideal gas $\qquad PV = nRT$

$$\therefore \qquad \left[-\frac{P_0}{V_0}V + 2P_0\right]V = nRT$$

$$\Rightarrow \qquad T = -\frac{P_0}{V_0R}V^2 + \frac{2P_0}{R}V \quad [\text{As } n = 1] \qquad \text{...(ii)}$$

This is equation of a parabola.

The vertex of parabola corresponds to $\dfrac{dT}{dV} = 0$

$$-\frac{2P_0}{V_0R}V + \frac{2P_0}{R} = 0 \quad \Rightarrow \quad V = V_0$$

Temperature in this state is

$$T_0 = \frac{P_0V_0}{nR}$$

$$\because \qquad n = 1 \quad \therefore \quad T_0 = \frac{P_0V_0}{R}$$

This is the maximum temperature during the process. If final temperature at B is $\dfrac{T_0}{2} = \dfrac{P_0V_0}{2R}$ then volume can be obtained using (ii)

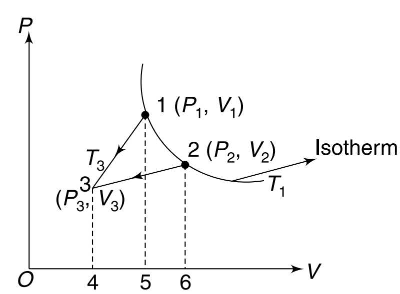
$$\frac{P_0V_0}{2R} = -\frac{P_0}{RV_0}V^2 + \frac{2P_0}{R}V$$

$$\Rightarrow \qquad \frac{V_0}{2} = -\frac{V^2}{V_0} + 2V$$

$$\Rightarrow \qquad 2V^2 - 4V_0V + V_0^2 = 0$$

$$\therefore \qquad V = \left(\frac{\sqrt{2} + 1}{\sqrt{2}}\right)V_0$$

67. State 1 and 2 lie on an isotherm



$$\therefore \qquad \Delta U_{1-3} = \Delta U_{2-3}$$

Hence, more heat will be rejected in the process in which work done on the gas is higher.

$$W_{1-3}^{\text{on gas}} = \text{area}(1345)$$

$$= \frac{1}{2} \times (V_1 - V_3)(P_1 + P_3)$$

$$= \frac{1}{2}[P_1V_1 + P_3V_1 - P_1V_3 - P_3V_3]$$

$$W_{2-3}^{\text{on gas}} = \text{area (2346)}$$

$$= \frac{1}{2} \times (V_2 - V_3)(P_2 + P_3)$$

$$= \frac{1}{2}[V_2P_2 + V_2P_3 - V_3P_2 - V_3P_3]$$

$$W_{1-3}^{\text{on gas}} - W_{2-3}^{\text{on gas}} = \frac{1}{2}[P_3(V_1 - V_2) + V_3(P_2 - P_1)]$$

But $V_1 - V_2 < 0$ and $P_2 - P_1 < 0$

$\therefore$ $W_{1-3}^{\text{on gas}} - W_{2-3}^{\text{on gas}} < 0$

$\therefore$ $W_{1-3}^{\text{on gas}} < W_{2-3}^{\text{on gas}}$

$\therefore$ $Q_1 < Q_2$

68. Initial volume

$$V_1 = AH + \frac{A}{2}\frac{H}{2} = \frac{5AH}{4}$$

Find volume $V_2 = A\frac{H}{2} + \frac{A}{2}\left[\frac{H}{2} - \frac{3H}{32}\right] = \frac{45AH}{64}$

Process is adiabatic

$\therefore$ $P_2V_2^{\gamma} = P_1V_1^{\gamma}$

$$P_2\left(\frac{45AH}{64}\right)^{3/2} = P_1\left(\frac{5AH}{4}\right)^{3/2}$$

$$P_2\left(\frac{9}{16}\right)^{3/2} = P_1 \Rightarrow P_2 = \frac{64}{27}P_1 \quad \text{...(i)}$$

But $P_1\frac{A}{2} = Mg + P_0\frac{A}{2}$ ...(ii)

[P_0 = atmospheric pressure, M = mass of piston P_1]

And $P_2\frac{A}{2} = Mg + \frac{P_0A}{2} + k\frac{3H}{32}$ ...(ii)

$\Rightarrow$ $\frac{64}{27}P_1\frac{A}{2} = Mg + \frac{P_0A}{2} + k\frac{3H}{32}$

$\Rightarrow$ $\frac{64}{27}\left[Mg + \frac{P_0A}{2}\right] = \left(Mg + P_0\frac{A}{2}\right) + \frac{3kH}{32}$

$\Rightarrow$ $\frac{37}{27}\left[Mg + \frac{P_0A}{2}\right] = \frac{3kH}{32}$

$\therefore$ $H = \frac{37}{27} \times \frac{32}{3} \times \frac{1}{3700}\left[\frac{27}{2} \times 10 + \frac{10^5 \times 27 \times 10^{-4}}{2}\right] = \frac{16}{15}\,m$

$$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$$

$$T_2\left[\frac{45}{64}AH\right]^{1/2} = T_1\left[\frac{5}{4}AH\right]^{1/2}$$

$\Rightarrow$ $T_2 = \frac{4}{3} \times T_1$

$\Rightarrow$ $T_2 = 400\text{ K}$

69. Gas in A and B are always at equal temperature. Process for gas in C is adiabatic.

$$\therefore \quad P_0V_0^{3/2} = P_C\left(\frac{4V_0}{9}\right)^{3/2} \quad \therefore \quad P_C = \frac{27}{8}P_0$$

For mechanical equilibrium $\quad P_A = P_C$

$$\therefore \quad P_A = P_C = \frac{27}{8}P_0$$

Again for C: $\quad T_C\left(\frac{4V_0}{9}\right)^{3/2-1} = T_0V_0^{\frac{3}{2}-1} \Rightarrow T_C = \frac{3}{2}T_0$

For A: $\quad P_0V_0 = nRT_0$

And $\quad P_A\left[V_0 + \frac{5V_0}{9}\right] = nRT_A$

$$\left[\because \text{ volume change} = V_0 - \frac{4V_0}{9} = \frac{5V_0}{9}\right]$$

$$\therefore \quad \frac{27}{8}P_0\frac{14}{9}V_0 = nRT_A$$

$$\therefore \quad T_A = \frac{21}{4}T_0$$

Also $\quad T_A = T_B = \frac{21}{4}T_0$

(a) Work done on gas in C = work done by gas in A.

$$W_A = -\left(\frac{P_0V_0 - P_CV_C}{\gamma - 1}\right) = -\frac{P_0V_0 - \frac{27}{8}P_0\frac{4V_0}{9}}{0.5} = P_0V_0$$

Work done by gas in B is zero.

Change in internal energy of $A + B$ is

$$\Delta U_{A+B} = 2nC_V\Delta T = 2n\frac{R}{\gamma - 1}\left(\frac{21}{4} - 1\right)T_0$$

$$= \frac{2nR}{0.5} \times \frac{17}{4}T_0 = 17nRT_0 = 17P_0V_0$$

$$Q_{\text{Heater}} = \Delta U_{A+B} + W_{\text{by}A} = 18P_0V_0$$

(b) Heat flow through piston 1

$$H = \Delta U_A + W_{\text{by}A}$$

$$= \frac{17}{2}P_0V_0 + P_0V_0 = \frac{19}{2}P_0V_0$$

(c) Answer to (a) does not change answer to (b) becomes

$$H = \Delta U_B = \frac{17}{2}P_0V_0$$

70. The right chamber has 1 mole N_2. Out of this $\frac{1}{3}$ mole N_2 dissociates into atoms. Therefore, the chamber has a mixture of $\frac{2}{3}$ mole of diatomic gas and $\frac{2}{3}$ mole of mono atomic gas

$$\therefore \quad C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2} \quad \left[n_1 = n_2 = \frac{2}{3}\right]$$

$$= \frac{C_{V_1} + C_{V_2}}{2} = \frac{\frac{3}{2}R + \frac{5}{2}R}{2} = 2R$$

$$C_P = C_V + R = 3R$$

$$\therefore \quad \gamma = \frac{C_P}{C_V} = \frac{3}{2}$$

For the adiabatic compression in this part $PV^\gamma = P_0V_0^\gamma$

$$P\left(\frac{AL}{4}\right)^\gamma = P_0(AL)^\gamma \quad \Rightarrow \quad P = (4)^{\frac{3}{2}}P_0 = 8P_0$$

Work done by nitrogen during the process

$$W_{Ad} = \frac{P_0V_0 - PV}{\gamma - 1} = \frac{P_0V_0 - 8P_0\dfrac{V_0}{4}}{\dfrac{3}{2} - 1}$$

$$= -4P_0V_0 = -4P_0AL$$

Work done on nitrogen by the gas in the other chamber is

$$W = 4P_0AL$$

Let the final pressure of He chamber be P_1. For equilibrium

$$P_1A = k\left(\frac{3L}{4}\right) + PA$$

$$\therefore \quad P_1 = \frac{3}{4}\frac{kL}{A} + 8P_0$$

Change in temperature of He is

$$\Delta T = \frac{P_1V_1 - P_0V_0}{n \cdot R} \quad [n = 1]$$

$$= \frac{\frac{3}{4}\frac{kL}{A}\frac{5}{4}AL + 8P_0\frac{5}{4}AL - P_0AL}{R}$$

$$= \frac{1}{R}\left[\frac{15}{16}kL^2 + 9P_0AL\right]$$

$\therefore$ Change in internal energy of He

$$\Delta U = nC_V\Delta T = \frac{3}{2}\left[\frac{15}{16}kL^2 + 9P_0AL\right]$$

$$= \frac{45}{32}kL^2 + \frac{27}{2}P_0AL$$

$$\therefore \quad \Delta Q = \Delta U + \text{work to compress the spring} + \text{work on nitrogen}$$

$$\therefore \quad \Delta Q = \frac{45}{32}kL^2 + \frac{27}{2}P_0AL + \frac{1}{2}k\left(\frac{3L}{4}\right)^2 + 4P_0AL$$

$$= \frac{27}{16}kL^2 + \frac{35}{2}P_0AL$$