26

Three Dimensional Geometry

TOPIC 1 **Direction Ratios & Direction cosines of a Line, Angle between two lines in terms of dc's and dr's, Condition of Parallelism & Perpendicularity of two Lines, Projection of a Point on a Line, Projection of a Line Segment Joining two Points**

1. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to: **[April 10, 2019 (I)]**

(a) 1 (b) 2

(c) –1 (d) –2

2. The vertices B and C of a “ABC lie on the line, $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A (1, –1, 2), is: **[April 09, 2019 (II)]**

(a) $5\sqrt{17}$ (b) $2\sqrt{34}$

(c) 6 (d) $\sqrt{34}$

3. If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4) and Q(8, 0, 10), then distance of R from the origin is : **[April 08, 2019 (II)]**

(a) $2\sqrt{14}$ (b) $2\sqrt{21}$

(c) 6 (d) $\sqrt{53}$

4. A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(–1, 1, 2) and O(0, 0, 0). The angle between the faces OPQ and PQR is: **[Jan. 12, 2019 (I)]**

(a) $\cos^{-1}\left(\frac{17}{31}\right)$ (b) $\cos^{-1}\left(\frac{19}{35}\right)$

(c) $\cos^{-1}\left(\frac{9}{35}\right)$ (d) $\cos^{-1}\left(\frac{7}{31}\right)$

5. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, $x + y + z = 7$ is: **[2018]**

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\sqrt{\frac{2}{3}}$ (d) $\frac{2}{\sqrt{3}}$

6. An angle between the lines whose direction cosines are given by the equations, $l + 3m + 5n = 0$ and $5lm - 2mn + 6nl = 0$, is **[Online April 15, 2018]**

(a) $\cos^{-1}\left(\frac{1}{8}\right)$ (b) $\cos^{-1}\left(\frac{1}{6}\right)$

(c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{1}{4}\right)$

7. ABC is triangle in a plane with vertices A (2, 3, 5), B (–1, 3, 2) and C (λ, 5, μ). If the median through A is equally inclined to the coordinate axes, then the value of $(\lambda^3 + \mu^3 + 5)$ is : **[Online April 10, 2016]**

(a) 1130 (b) 1348

(c) 1077 (d) 676

8. The angle between the lines whose direction cosines satisfy the equations $l+m+n=0$ and $l^2+m^2+n^2$ is **[2014]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

9. Let A (2, 3, 5), B (– 1, 3, 2) and C (λ, 5, μ) be the vertices of a ΔABC. If the median through A is equally inclined to the coordinate axes, then: **[Online April 11, 2014]**

(a) $5\lambda - 8\mu = 0$ (b) $8\lambda - 5\mu = 0$

(c) $10\lambda - 7\mu = 0$ (d) $7\lambda - 10\mu = 0$

10. A line in the 3-dimensional space makes an angle θ $\left(0 < \theta \leq \frac{\pi}{2}\right)$ with both the x and y axes. Then the set of all values of θ is the interval: **[Online April 9, 2014]**

(a) $\left(0, \frac{\pi}{4}\right]$ (b) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

(c) $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$ (d) $\left(\frac{\pi}{3},\frac{\pi}{2}\right]$

11. Let ABC be a triangle with vertices at points A (2, 3, 5), B (–1, 3, 2) and C (λ, 5, μ) in three dimensional space. If the median through A is equally inclined with the axes, then (λ, μ) is equal to : **[Online April 25, 2013]**

(a) (10, 7) (b) (7, 5)
(c) (7, 10) (d) (5, 7)

12. If the projections of a line segment on the x, y and z-axes in 3-dimensional space are 2, 3 and 6 respectively, then the length of the line segment is : **[Online April 23, 2013]**

(a) 12 (b) 7
(c) 9 (d) 6

13. The acute angle between two lines such that the direction cosines l, m, n, of each of them satisfy the equations $l+m+n=0$ and $l^2+m^2-n^2=0$ is : **[Online April 22, 2013]**

(a) 15° (b) 30°
(c) 60° (d) 45°

14. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals **[2010]**

(a) 45° (b) 60°
(c) 75° (d) 30°

15. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are : **[2009]**

(a) $\frac{6}{5},\frac{-3}{5},\frac{2}{5}$ (b) $\frac{6}{7},\frac{-3}{7},\frac{2}{7}$

(c) $\frac{-6}{7},\frac{-3}{7},\frac{2}{7}$ (d) $6,-3,2$

16. If a line makes an angle of $\pi/4$ with the positive directions of each of x- axis and y- axis, then the angle that the line makes with the positive direction of the z-axis is **[2007]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

17. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta=3\sin^2\theta$, then $\cos^2\theta$ equals **[2004]**

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

TOPIC 2 Equation of a Straight Line in Cartesian and Vector Form, Angle Between two Lines, Condition for Coplanarity of two Lines Perpendicular Distance of a Point from a Line, Shortest Distance between two Skew Lines, Distance Between two Parallel Lines.

18. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1,1,2). Then the equation of the line through this centroid and perpendicular to the plane P is: **[Sep. 06, 2020 (II)]**

(a) $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-2}{1}$ (b) $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-2}{2}$

(c) $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$ (d) $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$

19. If (a, b, c) is the image of the point (1, 2, –3) in the line, $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}$, then $a+b+c$ is equals to: **[Sep. 05, 2020 (I)]**

(a) 2 (b) – 1
(c) 3 (d) 1

20. The lines $\vec{r}=(\hat{i}-\hat{j})+l(2\hat{i}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j})+m(\hat{i}+\hat{j}-\hat{k})$ **[Sep. 03, 2020 (I)]**

(a) do not intersect for any values of l and m
(b) intersect for all values of l and m
(c) intersect when $l=2$ and $m=\frac{1}{2}$
(d) intersect when $l=1$ and $m=2$

21. The shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is: **[Jan. 08, 2020 (I)]**

(a) $2\sqrt{30}$ (b) $\frac{7}{2}\sqrt{30}$
(c) $3\sqrt{30}$ (d) 3

22. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through (α, 7, 1) is , then α is equal to _________. **[NA Jan. 07, 2020 (II)]**

23. A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane $x+y+z=3$ such that the foot of the perpendicular Q also lies on the plane $x-y+z=3$. Then the co-ordinates of Q are : **[April 10, 2019 (II)]**

(a) $(1, 0, 2)$ (b) $(2, 0, 1)$
(c) $(-1, 0, 4)$ (d) $(4, 0, -1)$

24. The length of the perpendicular from the point $(2, -1, 4)$ on the straight line, $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is : **[April 08, 2019 (I)]**

(a) greater than 3 but less than 4
(b) less than 2
(c) greater than 2 but less than 3
(d) greater than 4

25. Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy- plane has coordinates : **[Jan. 11, 2019 (II)]**

(a) $(2, -4, -7)$ (b) $(2, 4, 7)$
(c) $(2, -4, 7)$ (d) $(-2, 4, 7)$

26. If the lines $x = ay + b$, $z = cy + d$ and $x = a'z + b'$, $y = c'z + d'$ are perpendicular, then: **[Jan. 09, 2019 (II)]**

(a) $ab' + bc' + 1 = 0$
(b) $cc' + a + a' = 0$
(c) $bb' + cc' + 1 = 0$
(d) $aa' + c + c' = 0$

27. If the angle between the lines,

$\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{5-x}{-2}=\frac{7y-14}{P}=\frac{z-3}{4}$ is

$\cos^{-1}\left(\frac{2}{3}\right)$, then P is equal to **[Online April 16, 2018]**

(a) $-\frac{7}{4}$ (b) $\frac{2}{7}$
(c) $-\frac{4}{7}$ (d) $\frac{7}{2}$

28. The number of distinct real values of λ for which the lines

$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{\lambda^2}$ and $\frac{x-3}{1}=\frac{y-2}{\lambda^2}=\frac{z-1}{2}$ are

coplanar is : **[Online April 10, 2016]**

(a) 2 (b) 4
(c) 3 (d) 1

29. The shortest distance between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and

$\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$ lies in the interval :

[Online April 9, 2016]

(a) $(3, 4]$ (b) $(2, 3]$
(c) $[1, 2)$ (d) $[0, 1)$

30. Equation of the line of the shortest distance between the lines $\frac{x}{1}=\frac{y}{-1}=\frac{z}{1}$ and $\frac{x-1}{0}=\frac{y+1}{-2}=\frac{z}{1}$ is:

[Online April 19, 2014]

(a) $\frac{x}{1}=\frac{y}{-1}=\frac{z}{-2}$ (b) $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{-2}$
(c) $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{1}$ (d) $\frac{x}{-2}=\frac{y}{1}=\frac{z}{2}$

31. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and

$\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then k can have

[2013]

(a) any value (b) exactly one value
(c) exactly two values (d) exactly three values

32. If two lines L_1 and L_2 in space, are defined by

$L_1=\left\{x=\sqrt{\lambda}y+\left(\sqrt{\lambda}-1\right),\ z=\left(\sqrt{\lambda}-1\right)y+\sqrt{\lambda}\right\}$ and

$L_2=\left\{x=\sqrt{\mu}y+\left(1-\sqrt{\mu}\right),\ z=\left(1-\sqrt{\mu}\right)y+\sqrt{\mu}\right\}$

then L_1 is perpendicular to L_2, for all non-negative reals λ and μ, such that : **[Online April 23, 2013]**

(a) $\sqrt{\lambda}+\sqrt{\mu}=1$ (b) $\lambda \neq \mu$
(c) $\lambda+\mu=0$ (d) $\lambda=\mu$

33. If the lines $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z+1}{3}$ and $\frac{x+2}{2}=\frac{y-k}{3}=\frac{z}{4}$ are coplanar, then the value of k is : **[Online April 9, 2013]**

(a) $\frac{11}{2}$ (b) $-\frac{11}{2}$
(c) $\frac{9}{2}$ (d) $-\frac{9}{2}$

34. If the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and

$\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to: **[2012]**

(a) -1 (b) $\frac{2}{9}$
(c) $\frac{9}{2}$ (d) 0

35. The distance of the point $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line that passes through the point $2\hat{i}+3\hat{j}-4\hat{k}$ and is parallel to the vector $6\hat{i}+3\hat{j}-4\hat{k}$ is

[Online May 26, 2012]

(a) 9 (b) 8
(c) 7 (d) 10

36. **Statement 1:** The shortest distance between the lines $\frac{x}{2}=\frac{y}{-1}=\frac{z}{2}$ and $\frac{x-1}{4}=\frac{y-1}{-2}=\frac{z-1}{4}$ is $\sqrt{2}$.
Statement 2: The shortest distance between two parallel lines is the perpendicular distance from any point on one of the lines to the other line. **[Online May 19, 2012]**
(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true, , Statement 2 is not a correct explanation for Statement 1

37. The coordinates of the foot of perpendicular from the point (1, 0, 0) to the line
$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ are **[Online May 12, 2012]**
(a) $(2,-3,8)$ (b) $(1,-1,-10)$
(c) $(5,-8,-4)$ (d) $(3,-4,-2)$

38. The length of the perpendicular drawn from the point $(3,-1,11)$ to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is : **[2011RS]**
(a) $\sqrt{29}$ (b) $\sqrt{33}$
(c) $\sqrt{53}$ (d) $\sqrt{66}$

39. **Statement-1:** The point $A(1, 0, 7)$) is the mirror image of the point $B(1, 6, 3)$ in the line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$
Statement-2: The line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$. **[2011]**
(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

40. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is **[2010]**
(a) $\sqrt{17}$ (b) $\frac{17}{\sqrt{15}}$
(c) $\frac{23}{\sqrt{17}}$ (d) $\frac{23}{\sqrt{15}}$

41. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to **[2008]**
(a) –5 (b) 5
(c) 2 (d) –2

42. If non zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is **[2005]**
(a) $(-1,2)$ (b) $(-1,-2)$
(c) $(1,-2)$ (d) $\left(1,-\frac{1}{2}\right)$

43. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is **[2005]**
(a) 0° (b) 90°
(c) 45° (d) 30°

44. If the straight lines **[2004]**
$x=1+s, y=-3-\lambda s, z=1+\lambda s$
and $x=\frac{t}{2}, y=1+t, z=2-t$, with parameters s and t respectively, are co-planar, then λ equals.
(a) 0 (b) –1
(c) $-\frac{1}{2}$ (d) –2

45. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x=y+a=z$ and $x+a=2y=2z$. The co-ordinates of each of the points of intersection are given by **[2004]**
(a) $(2a,3a,3a),(2a,a,a)$ (b) $(3a,2a,3a),(a,a,a)$
(c) $(3a,2a,3a),(a,a,2a)$ (d) $(3a,3a,3a),(a,a,a)$

46. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ **[2003]**
and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if
(a) k = 3 or –2 (b) k = 0 or –1
(c) k = 1 or –1 (d) k = 0 or –3.

47. The two lines $x=ay+b, z=cy+d$ and $x=a'y+b', z=c'y+d'$ will be perpendicular, if and only if **[2003]**
(a) $aa'+cc'+1=0$
(b) $aa'+bb'+cc'+1=0$
(c) $aa'+bb'+cc'=0$
(d) $(a+a')(b+b')+(c+c')=0$.

TOPIC 3 Equation of a Plane in Different Forms, Equation of a Plane Passing Through the Intersection of two Given Planes, Plane Containing two Lines, Angle Between two Planes, Angle Between a Plane and a Line, Distance Between two Parallel Planes, Position of Point and Line wrt a Plane, Projection of a Line on a Plane

48. The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and $x+y+z+1=0, 2x-y+z+3=0$ is : **[Sep. 06, 2020 (I)]**

(a) 1 (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

49. If for some $\alpha \in \mathbf{R}$, the lines $L_1 : \frac{x+1}{2}=\frac{y-2}{-1}=\frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha}=\frac{y+1}{5-\alpha}=\frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point : **[Sep. 05, 2020 (II)]**

(a) $(10, 2, 2)$ (b) $(2, -10, -2)$

(c) $(10, -2, -2)$ (d) $(-2, 10, 2)$

50. If the equation of a plane P, passing through the intersection of the planes, $x+4y-z+7=0$ and $3x+y+5z=8$ is $ax+by+6z=15$ for some $a, b \in \mathbf{R}$, then the distance of the point (3, 2, –1) from the plane P is ___________. **[Sep. 04, 2020 (I)]**

51. The distance of the point (1, –2, 3) from the plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is : **[NA Sep. 04, 2020 (II)]**

(a) $\frac{7}{5}$ (b) 1

(c) $\frac{1}{7}$ (d) 7

52. The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, –2, 3) and (1,1, 0) lies on the plane : **[Sep. 03, 2020 (I)]**

(a) $2x+y-z=1$ (b) $x-y-2z=1$

(c) $x-2y+z=1$ (d) $x+2y-z=1$

53. The plane which bisects the line joining the points (4, – 2, 3) and (2, 4, – 1) at right angles also passes through the point: **[Sep. 03, 2020 (II)]**

(a) (4, 0, 1) (b) (0, –1, 1)

(c) (4, 0, –1) (d) (0, 1, –1)

54. Let a plane P contain two lines

$\vec{r}=\hat{i}+\lambda(\hat{i}+\hat{j}), \lambda \in \mathbf{R}$ and $\vec{r}=-\hat{j}+\mu(\hat{j}-\hat{k}), \mu \in \mathbf{R}$.

If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point $M(1, 0, 1)$ to P, then $3(\alpha+\beta+\gamma)$ equals ____________. **[NA Sep. 03, 2020 (II)]**

55. The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, $2x=3y, z=1$ also through the point : **[Sep. 02, 2020 (I)]**

(a) (0, 6, –2) (b) (–2, 0, 1)

(c) (0, –6, 2) (d) (2, 0, –1)

56. A plane passing through the point (3, 1, 1) contains two lines whose direction ratios are 1, –2, 2 and 2, 3, –1 respectively. If this plane also passes through the point $(\alpha, -3, 5)$, then α is equal to : **[Sep. 02, 2020 (II)]**

(a) 5 (b) –10

(c) 10 (d) –5

57. If for some α and β in $\mathbf{R}$, the intersection of the following three planes

$x+4y-2z=1$

$x+7y-5z=\beta$

$x+5y+\alpha z=5$

is a line in R^3, then $\alpha+\beta$ is equal to: **[Jan. 9, 2020 (I)]**

(a) 0 (b) 10

(c) 2 (d) –10

58. If the distance between the plane, $23x-10y-2z+48=0$ and the plane containing the lines

$\frac{x+1}{2}=\frac{y-3}{4}=\frac{z+1}{3}$

and $\frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda}(\lambda \in R)$

is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ______.

[NA Jan. 9, 2020 (II)]

59. The mirror image of the point (1, 2, 3) in a plane is $\left(-\frac{7}{3},-\frac{4}{3},-\frac{1}{3}\right)$. Which of the following points lies on this plane? **[Jan. 8, 2020 (II)]**

(a) $(1, 1, 1)$ (b) $(1, -1, 1)$
(c) $(-1, -1, 1)$ (d) $(-1, -1, -1)$

60. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is: **[Jan. 7, 2020 (I)]**

(a) $(6, 5, 2)$ (b) $(6, 5, -2)$
(c) $(4, 3, 2)$ (d) $(3, 4, -2)$

61. If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane $2x+3y-z+13=0$ at a point P and the plane $3x+y+4z=16$ at a point Q, then PQ is equal to: **[April 12, 2019 (I)]**

(a) 14 (b) $\sqrt{14}$
(c) $2\sqrt{7}$ (d) $2\sqrt{14}$

62. A plane which bisects the angle between the two given planes $2x-y+2z-4=0$ and $x+2y+2z-2=0$, passes through the point : **[April 12, 2019 (II)]**

(a) $(1, -4, 1)$ (b) $(1, 4, -1)$
(c) $(2, 4, 1)$ (d) $(2, -4, 1)$

63. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines
$\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=(\hat{i}+\hat{j})+\mu(-\hat{i}+\hat{j}-2\hat{k})$ is : **[April 12, 2019 (II)]**

(a) 3 (b) $\frac{1}{3}$
(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

64. If Q $(0, -1, -3)$ is the image of the point P in the plane $3x-y+4z=2$ and R is the point $(3, -1, -2)$, then the area (in sq. units) of ΔPQR is : **[April 10, 2019 (I)]**

(a) $2\sqrt{13}$ (b) $\frac{\sqrt{91}}{4}$
(c) $\frac{\sqrt{91}}{2}$ (d) $\frac{\sqrt{65}}{2}$

65. If the plane $2x-y+2z+3=0$ has the distances $\frac{1}{2}$ and $\frac{2}{3}$ units from the planes $4x-2y+4z+\lambda=0$ and $2x-y+2z+\mu=0$, respectively, then the maximum value of $\lambda+\mu$ is equal to : **[April 10, 2019 (II)]**

(a) 9 (b) 15
(c) 5 (d) 13

66. If the line, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane, $x+2y+3z=15$ at a point P, then the distance of P from the origin is: **[April 09 2019I]**

(a) $\sqrt{5}/2$ (b) $2\sqrt{5}$
(c) 9/2 (d) 7/2

67. A plane passing through the points $(0, -1, 0)$ and $(0, 0, 1)$ and making an angle $\frac{\pi}{4}$ with the plane $y-z+5=0$, also passes through the point: **[April 09 2019I]**

(a) $(-\sqrt{2}, 1, -4)$ (b) $(\sqrt{2}, -1, 4)$
(c) $(-\sqrt{2}, -1, -4)$ (d) $(\sqrt{2}, 1, 4)$

68. Let P be the plane, which contains the line of intersection of the planes, $x+y+z-6=0$ and $2x+3y+z+5=0$ and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to: **[April 09, 2019 (II)]**

(a) $17/\sqrt{5}$ (b) $63\sqrt{5}$
(c) $205\sqrt{5}$ (d) $11/\sqrt{5}$

69. The equation of a plane containing the line of intersection of the planes $2x-y-4=0$ and $y+2z-4=0$ and passing through the point (1, 1, 0) is : **[April 08 2019 I]**

(a) $x-3y-2z=-2$ (b) $2x-z=2$
(c) $x-y-z=0$ (d) $x+3y+z=4$

70. The vector equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$ is : **[April 08, 2019 (II)]**

(a) $\vec{r}\times(\hat{i}-\hat{k})+2=0$ (b) $\vec{r}.(\hat{i}-\hat{k})-2=0$
(c) $\vec{r}\times(\hat{i}+\hat{k})+2=0$ (d) $\vec{r}.(\hat{i}-\hat{k})+2=0$

71. The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is : **[Jan. 12, 2019 (I)]**

(a) $11\sqrt{6}$ (b) $11/\sqrt{6}$
(c) 11 (d) $6\sqrt{11}$

72. If an angle between the line, $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and the plane, $x-2y-kx=3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is **[Jan. 12, 2019 (II)]**

(a) $\sqrt{\frac{5}{3}}$ (b) $\sqrt{\frac{3}{5}}$

(c) $-\frac{3}{5}$ (d) $-\frac{5}{3}$

73. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point- $(-1, -1, 1)$. Then S is equal to : **[Jan. 12, 2019 (II)]**

(a) $\{\sqrt{3}\}$ (b) $\{\sqrt{3}, -\sqrt{3}\}$

(c) $\{1, -1\}$ (d) $\{3, -3\}$

74. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points? **[Jan. 11, 2019 (I)]**

(a) $(2, 2, 0)$ (b) $(-2, 2, 2)$

(c) $(0, -2, 2)$ (d) $(2, 0, -2)$

75. The direction ratios of normal to the plane through the points $(0, -1, 0)$ and $(0, 0, 1)$ and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$ are : **[Jan. 11, 2019 (I)]**

(a) $2, -1, 1$ (b) $2, \sqrt{2}, -\sqrt{2}$

(c) $\sqrt{2}, 1, -1$ (d) $2\sqrt{3}, 1, -1$

76. If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points $(3, 4, 2)$ and $(7, 0, 6)$ and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to : **[Jan. 11, 2019 (II)]**

(a) 12 (b) 7

(c) 5 (d) 17

77. The plane which bisects the line segment joining the points $(-3, -3, 4)$ and $(3, 7, 6)$ at right angles, passes through which one of the following points? **[Jan. 10, 2019 (II)]**

(a) $(-2, 3, 5)$ (b) $(4, -1, 7)$

(c) $(2, 1, 3)$ (d) $(4, 1, -2)$

78. On which of the following lines lies the point of inter-section of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, $x + y + z = 2$? **[Jan. 10, 2019 (II)]**

(a) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

(b) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

(c) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

(d) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

79. The system of linear equations

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$ **[Jan 09 2019I]**

(a) is inconsistent when $a = 4$

(b) has a unique solution for $|a| = \sqrt{3}$

(c) has infinitely many solutions for $a = 4$

(d) is inconsistent when $|a| = \sqrt{3}$

80. The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is: **[Jan 09 2019I]**

(a) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

(b) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(c) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(d) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

81. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point: **[Jan 09 2019I]**

(a) $(-3, 0, -1)$ (b) $(-3, 1, 1)$

(c) $(3, 3, -1)$ (d) $(3, 2, 1)$

82. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is: **[Jan. 09, 2019 (II)]**

(a) $x - 2y + z = 0$

(b) $3x + 2y - 3z = 0$

(c) $x + 2y - 2z = 0$

(d) $5x + 2y - 4z = 0$

83. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is : **[2018]**

(a) $\frac{1}{3\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{4\sqrt{2}}$

84. The sum of the intercepts on the coordinate axes of the plane passing through the point $(-2, -2, 2)$ and containing the line joining the points $(1, -1, 2)$ and $(1, 1, 1)$ is **[Online April 16, 2018]**

(a) 12 (b) -8
(c) -4 (d) 4

85. A variable plane passes through a fixed point $(3, 2, 1)$ and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to $yz-plane$ through A, a second plane is drawn parallel $zx-plane$ through B and a third plane is drawn parallel to $xy-plane$ through C. Then the locus of the point of intersection of these three planes, is **[Online April 15, 2018]**

(a) $x+y+z=6$ (b) $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$
(c) $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$ (d) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$

86. An angle between the plane, $x + y + z = 5$ and the line of intersection of the planes, $3x + 4y + z - 1 = 0$ and $5x + 8y + 2z + 14 = 0$, is **[Online April 15, 2018]**

(a) $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (b) $\cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$
(c) $\sin^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (d) $\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

87. A plane bisects the line segment joining the points $(1, 2, 3)$ and $(-3, 4, 5)$ at right angles. Then this plane also passes through the point. **[Online April 15, 2018]**

(a) $(-3, 2, 1)$ (b) $(3, 2, 1)$
(c) $(1, 2, -3)$ (d) $(-1, 2, 3)$

88. If the image of the point P(1, –2, 3) in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to: **[2017]**

(a) $6\sqrt{5}$ (b) $3\sqrt{5}$
(c) $2\sqrt{42}$ (d) $\sqrt{42}$

89. The distance of the point $(1, 3, -7)$ from the plane passing through the point $(1, -1, -1)$, having normal perpendicular to both the lines **[2017]**

$\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$, is:

(a) $\frac{10}{\sqrt{74}}$ (b) $\frac{20}{\sqrt{74}}$
(c) $\frac{10}{\sqrt{83}}$ (d) $\frac{5}{\sqrt{83}}$

90. If $x = a$, $y = b$, $z = c$ is a solution of the system of linear equations **[Online April 9, 2017]**

$x + 8y + 7z = 0$
$9x + 2y + 3z = 0$
$x + y + z = 0$

such that the point (a, b, c) lies on the plane $x + 2y + z = 6$, then $2a + b + c$ equals :

(a) –1 (b) 0
(c) 1 (d) 2

91. If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ΔABC is : **[Online April 9, 2017]**

(a) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$
(b) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=3$
(c) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{9}$
(d) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$

92. If the line, $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+\lambda}{-2}$ lies in the plane, $2x - 4y + 3z = 2$, then the shortest distance between this line and the line, $\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}$ is: **[Online April 9, 2017]**

(a) 2 (b) 1
(c) 0 (d) 3

93. The coordinates of the foot of the perpendicular from the point $(1, -2, 1)$ on the plane containing the lines,

$\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and

$\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$, is : **[Online April 8, 2017]**

(a) $(2, -4, 2)$ (b) $(-1, 2, -1)$
(c) $(0, 0, 0)$ (d) $(1, 1, 1)$

94. The line of intersection of the planes

$\vec{r}.(3\hat{i}-\hat{j}+\hat{k})=1$ and

$\vec{r}.(\hat{i}+4\hat{j}-2\hat{k})=2$, is: **[Online April 8, 2017]**

(a) $\frac{x-\frac{4}{7}}{-2}=\frac{y}{7}=\frac{z-\frac{5}{7}}{13}$
(b) $\frac{x-\frac{4}{7}}{2}=\frac{y}{-7}=\frac{z+\frac{5}{7}}{13}$
(c) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{-13}$
(d) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{7}=\frac{z}{-13}$

95. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $lx+my-z=9$, then l^2+m^2 is equal to : **[2016]**

(a) 5 (b) 2
(c) 26 (d) 18

96. The distance of the point (1, –5, 9) from the plane $x-y+z=5$ measured along the line $x=y=z$ is : **[2016]**

(a) $\frac{10}{\sqrt{3}}$ (b) $\frac{20}{3}$
(c) $3\sqrt{10}$ (d) $10\sqrt{3}$

97. The distance of the point (1, –2, 4) from the plane passing through the point (1, 2, 2) and perpendicular to the planes $x-y+2z=3$ and $2x-2y+z+12=0$, is : **[Online April 9, 2016]**

(a) 2 (b) $\sqrt{2}$
(c) $2\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

98. The equation of the plane containing the line $2x-5y+z=3$; $x+y+4z=5$, and parallel to the plane, $x+3y+6z=1$, is: **[2015]**

(a) $x+3y+6z=7$ (b) $2x+6y+12z=-13$
(c) $2x+6y+12z=13$ (d) $x+3y+6z=-7$

99. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=16$, is **[2015]**

(a) $3\sqrt{21}$ (b) 13
(c) $2\sqrt{14}$ (d) 8

100. The shortest distance between the z–axis and the line $x+y+2z-3=0=2x+3y+4z-4$, is **[Online April 11, 2015]**

(a) 1 (b) 2
(c) 4 (d) 3

101. A plane containing the point (3, 2, 0) and the line $\frac{x-1}{1}=\frac{y-2}{5}=\frac{z-3}{4}$ also contains the point : **[Online April 11, 2015]**

(a) (0, 3, 1) (b) (0, 7, –10)
(c) (0, –3, 1) (d) 0, 7, 10

102. If the points (1, 1, λ) and (–3, 0, 1) are equidistant from the plane, $3x+4y-12z+13=0$, then λ satisfies the equation : **[Online April 10, 2015]**

(a) $3x^2+10x-13=0$ (b) $3x^2-10x+21=0$
(c) $3x^2-10x+7=0$ (d) $3x^2+10x-7=0$

103. If the shortest distance between the lines $\frac{x-1}{\alpha}=\frac{y+1}{-1}=\frac{z}{1}, (\alpha\neq-1)$ and $x+y+z+1=0$ $=2x-y+z+3$ is $\frac{1}{\sqrt{3}}$, then a value α is : **[Online April 10, 2015]**

(a) $-\frac{16}{19}$ (b) $-\frac{19}{16}$
(c) $\frac{32}{19}$ (d) $\frac{19}{32}$

104. The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x-y+z+3=0$ is the line: **[2014]**

(a) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$
(b) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$
(c) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$
(d) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

105. If the angle between the line $2(x+1)=y=z+4$ and the plane $2x-y+\sqrt{\lambda}\,z+4=0$ is $\frac{\pi}{6}$, then the value of λ is: **[Online April 19, 2014]**

(a) $\frac{135}{7}$ (b) $\frac{45}{11}$
(c) $\frac{45}{7}$ (d) $\frac{135}{11}$

106. If the distance between planes, $4x-2y-4z+1=0$ and $4x-2y-4z+d=0$ is 7, then d is: **[Online April 12, 2014]**

(a) 41 or –42 (b) 42 or –43
(c) –41 or 43 (d) –42 or 44

107. A symmetrical form of the line of intersection of the planes $x=ay+b$ and $z=cy+d$ is **[Online April 12, 2014]**

(a) $\frac{x-b}{a}=\frac{y-1}{1}=\frac{z-d}{c}$
(b) $\frac{x-b-a}{a}=\frac{y-1}{1}=\frac{z-d-c}{c}$
(c) $\frac{x-a}{b}=\frac{y-0}{1}=\frac{z-c}{d}$
(d) $\frac{x-b-a}{b}=\frac{y-1}{0}=\frac{z-d-c}{d}$

108. The plane containing the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and parallel to the line $\frac{x}{1}=\frac{y}{1}=\frac{z}{4}$ passes through the point: **[Online April 11, 2014]**

(a) $(1,-2,5)$ (b) $(1,0,5)$
(c) $(0,3,-5)$ (d) $(-1,-3,0)$

109. Equation of the plane which passes through the point of intersection of lines

$\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and

$\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

and has the largest distance from the origin is: **[Online April 9, 2014]**

(a) $7x+2y+4z=54$ (b) $3x+4y+5z=49$
(c) $4x+3y+5z=50$ (d) $5x+4y+3z=57$

110. Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is **[2013]**

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$
(c) $\frac{7}{2}$ (d) $\frac{9}{2}$

111. The equation of a plane through the line of intersection of the planes $x+2y=3, y-2z+1=0$, and perpendicular to the first plane is : **[Online April 25, 2013]**

(a) $2x-y-10z=9$ (b) $2x-y+7z=11$
(c) $2x-y+10z=11$ (d) $2x-y-9z=10$

112. Let Q be the foot of perpendicular from the origin to the plane $4x-3y+z+13=0$ and R be a point $(-1,-6)$ on the plane. Then length QR is : **[Online April 22, 2013]**

(a) $\sqrt{14}$ (b) $\sqrt{\frac{19}{2}}$
(c) $3\sqrt{\frac{7}{2}}$ (d) $\frac{3}{\sqrt{2}}$

113. A vector $\vec{n}$ is inclined to x-axis at 45°, to y-axis at 60° and at an acute angle to z-axis. If $\vec{n}$ is a normal to a plane passing through the point $(\sqrt{2},-1,1)$ then the equation of the plane is : **[Online April 9, 2013]**

(a) $4\sqrt{2}x+7y+z-2$ (b) $2x+y+2z=2\sqrt{2}+1$
(c) $3\sqrt{2}x-4y-3z=7$ (d) $\sqrt{2}x-y-z=2$

114. A equation of a plane parallel to the plane $x-2y+2z-5=0$ and at a unit distance from the origin is : **[2012]**

(a) $x-2y+2z-3=0$ (b) $x-2y+2z+1=0$
(c) $x-2y+2z-1=0$ (d) $x-2y+2z+5=0$

115. The equation of a plane containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and the point $(0, 7, -7)$ is **[Online May 26, 2012]**

(a) $x+y+z=0$
(b) $x+2y+z=21$
(c) $3x-2y+5z+35=0$
(d) $3x+2y+5z+21=0$

116. Consider the following planes

$P: x+y-2z+7=0$
$Q: x+y+2z+2=0$
$R: 3x+3y-6z-11=0$ **[Online May 26, 2012]**

(a) P and R are perpendicular
(b) Q and R are perpendicular
(c) P and Q are parallel
(d) P and R are parallel

117. If the three planes $x = 5$, $2x - 5ay + 3z - 2 = 0$ and $3bx+y-3z=0$ contain a common line, then(a, b) is equal to **[Online May 19, 2012]**

(a) $\left(\frac{8}{15},-\frac{1}{5}\right)$ (b) $\left(\frac{1}{5},-\frac{8}{15}\right)$
(c) $\left(-\frac{8}{15},\frac{1}{5}\right)$ (d) $\left(-\frac{1}{5},\frac{8}{15}\right)$

118. A line with positive direction cosines passes through the point P (2, – 1, 2) and makes equal angles with the coordinate axes. If the line meets the plane $2x+y+z=9$ at point Q, then the length PQ equals **[Online May 7, 2012]**

(a) $\sqrt{2}$ (b) 2
(c) $\sqrt{3}$ (d) 1

119. The values of a for which the two points (1, a, 1) and (– 3, 0, a) lie on the opposite sides of the plane $3x + 4y - 12z + 13 = 0$, satisfy **[Online May 7, 2012]**

(a) $0<a<\frac{1}{3}$ (b) $-1<a<0$
(c) $a<-1$ or $a<\frac{1}{3}$ (d) $a=0$

120. The distance of the point $(1, -5, 9)$ from the plane $x-y+z=5$ measured along a straight $x=y=z$ is **[2011RS]**

(a) $10\sqrt{3}$ (b) $5\sqrt{3}$
(c) $3\sqrt{10}$ (d) $3\sqrt{5}$

121. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals **[2011]**

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$

(c) $\frac{5}{3}$ (d) $\frac{2}{3}$

122. **Statement -1 :** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$.
Statement -2: The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4). **[2010]**
(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

123. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals **[2009]**
(a) $(-6, 7)$ (b) $(5, -15)$
(c) $(-5, 5)$ (d) $(6, -17)$

124. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then **[2008]**
(a) $a = 2, b = 8$ (b) $a = 4, b = 6$
(c) $a = 6, b = 4$ (d) $a = 8, b = 2$

125. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then cos α equals **[2007]**

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$.

126. The image of the point (–1, 3, 4) in the plane $x - 2y = 0$ is **[2006]**

(a) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$ (b) $(15, 11, 4)$

(c) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$ (d) None of these

127. The distance between the line
$\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(i - j + 4k)$ and the plane
$\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is **[2005]**

(a) $\frac{10}{9}$ (b) $\frac{10}{3\sqrt{3}}$

(c) $\frac{3}{10}$ (d) $\frac{10}{3}$

128. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}\, z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$ then the value of λ is **[2005]**

(a) $\frac{5}{3}$ (b) $\frac{-3}{5}$

(c) $\frac{3}{4}$ (d) $\frac{-4}{3}$

129. Distance between two parallel planes
$2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is **[2004]**

(a) $\frac{9}{2}$ (b) $\frac{5}{2}$

(c) $\frac{7}{2}$ (d) $\frac{3}{2}$

130. Two system of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a', b', c' from the origin then **[2003]**

(a) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

(b) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} = 0$

(c) $\frac{1}{a^2} + \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

(d) $\frac{1}{a^2} - \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$.

131. The d.r. of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle $\pi/4$ with plane $x + y = 3$ are **[2002]**

(a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$

(c) $1, 1, 2$ (d) $\sqrt{2}, 1, 1$

132. A plane which passes through the point (3, 2, 0) and the line $\frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4}$ is **[2002]**

(a) $x - y + z = 1$ (b) $x + y + z = 5$
(c) $x + 2y - z = 1$ (d) $2x - y + z = 5$

TOPIC 4 Sphere and Miscellaneous Problems on Sphere

133. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the cooordinates of the other end of the diameter are **[2007]**

(a) $(4, 3, 5)$ (b) $(4, 3, -3)$
(c) $(4, 9, -3)$ (d) $(4, -3, 3)$.

134. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius **[2005]**

(a) 3 (b) 1
(c) 2 (d) $\sqrt{2}$

135. If the plane 2ax – 3ay + 4az + 6 = 0 passes through the midpoint of the line joining the centres of the spheres

$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and

$x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals **[2005]**

(a) -1 (b) 1
(c) -2 (d) 2

136. The intersection of the spheres

$x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and

$x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane **[2004]**

(a) $2x - y - z = 1$ (b) $x - 2y - z = 1$
(c) $x - y - 2z = 1$ (d) $x - y - z = 1$

137. The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is **[2003]**

(a) 4 (b) 1
(c) 2 (d) 3

138. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is **[2003]**

(a) 39 (b) 26
(c) $11\frac{4}{13}$ (d) 13.

Hints & Solutions

1. **(c)** Given, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}=p$ (let) and point P $(\beta, 0, \beta)$

Any point on line A = $(p, 1, -p-1)$

Now, DR of AP a” $<p-\beta, 1-0, -p-1-\beta>$

Which is perpendicular to line.

$\therefore (p-\beta)\,1+0.1-1\,(-p-1-\beta)=0$

$\Rightarrow p-\beta+p+1+\beta=0 \Rightarrow p=\frac{-1}{2}$

$\therefore$ Point $A\left(\frac{-1}{2}, 1-\frac{1}{2}\right)$

Given that distance AP = $\sqrt{\frac{3}{2}} \Rightarrow AP^2=\frac{3}{2}$

$$\Rightarrow \left(\beta+\frac{1}{2}\right)^2+1+\left(\beta+\frac{1}{2}\right)^2=\frac{3}{2} \text{ or } 2\left(\beta+\frac{1}{2}\right)^2=\frac{1}{2}$$

$$\Rightarrow \left(\beta+\frac{1}{2}\right)^2=\frac{1}{4} \Rightarrow \beta=0,-1,(\beta\neq 0)$$

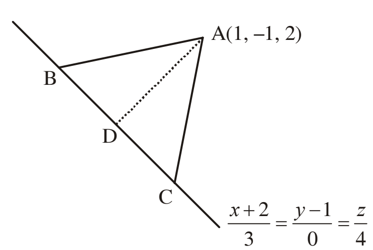
$\therefore \beta=-1$

2. **(d)** Let a point D on BC = $(3\lambda-2, 1, 4\lambda)$

$\overrightarrow{AD}=(3\lambda-3)\hat{i}+2\hat{j}+(4\lambda-2)\hat{k}$

$\because \overrightarrow{AD}\perp\overrightarrow{BC}$, $\therefore \overrightarrow{AD}.\overrightarrow{BC}=0$

$$\Rightarrow (3\lambda-3)+3+2(0)+(4\lambda-2)4=0 \Rightarrow \lambda=\frac{17}{25}$$



Hence, $D=\left(\frac{1}{25}, 1, \frac{68}{25}\right)$

$$|\overrightarrow{AD}|=\sqrt{\left(\frac{1}{25}-1\right)^2+(2)^2+\left(\frac{68}{25}-2\right)^2}$$

$$=\sqrt{\frac{(24)^2+4(25)^2+(18)^2}{25}}=\sqrt{\frac{3400}{25}}=\frac{2\sqrt{34}}{5}$$

Area of triangle $=\frac{1}{2}\times|\overline{BC}|\times|\overline{AD}|$

$$=\frac{1}{2}\times 5\times\frac{2\sqrt{34}}{5}=\sqrt{34} \qquad [\mapsto BC=5]$$

3. **(a)** Here, P, Q, R are collinear

$\therefore \overrightarrow{PR}=\lambda\overrightarrow{PQ}$

$2\hat{i}+(y+3)\hat{j}+(z-4)\hat{k}=\lambda[6\hat{i}+3\hat{j}+6\hat{k}]$

$\Rightarrow 6\lambda=2,\ y+3=3\lambda,\ z-4=6\lambda$

$\Rightarrow \lambda=\frac{1}{3}, y=-2, z=6$

$\therefore$ Point $R\,(4, -2, 6)$

Now, OR = $\sqrt{(4)^2+(-2)^2+(6)^2}=\sqrt{56}=2\sqrt{14}$

4. **(b)** Let $\vec{v}_1$ and $\vec{v}_2$ be the vectors perpendicular to the plane OPQ and PQR respectively.

$$\vec{v}_1=\overrightarrow{PQ}\times\overrightarrow{OQ}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 1\\ 2 & 1 & 3\end{vmatrix}$$

$=5\hat{i}-\hat{j}-3\hat{k}$

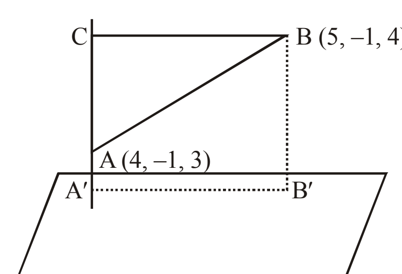
$$\vec{v}_2=\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ -2 & -1 & 1\end{vmatrix}$$

$=\hat{i}-5\hat{j}-3\hat{k}$

$$\because \quad \cos\theta=\frac{\vec{v}_1\cdot\vec{v}_2}{|\vec{v}_1||\vec{v}_2|}=\frac{5+5+9}{25+1+9}=\frac{19}{35}$$

$$\therefore \quad \theta=\cos^{-1}\left(\frac{19}{35}\right)$$

5. **(c)**



$$AC = \overrightarrow{AB}.\widehat{AC} = \left(\hat{i}+\hat{k}\right).\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Now, $A'B' = BC = \sqrt{AB^2 - AC^2} = \sqrt{2-\frac{4}{3}} = \sqrt{\frac{2}{3}}$

$\therefore$ Length of projection $= \sqrt{\frac{2}{3}}$

6. **(b)** Given

$l + 3m + 5n = 0$ (1)

and $5lm - 2mn + 6nl = 0$ (2)

From eq. (1) we have

$l = -3m - 5n$

Put the value of l in eq. (2), we get;

$5(-3m-5n)m - 2mn + 6n(-3m-5n) = 0$

$\Rightarrow 15m^2 + 45mn + 30n^2 = 0$

$\Rightarrow m^2 + 3mn + 2n^2 = 0$

$\Rightarrow m^2 + 2mn + mn + 2n^2 = 0$

$\Rightarrow (m+n)(m+2n) = 0$

$\therefore m = -n$ or $m = -2n$

For $m = -n$, $l = -2n$

And for $m = -2n$, $l = n$

$\therefore (l, m, n) = (-2n, -n, n)$ Or $(l, m, n) = (n, -2n, n)$

$\Rightarrow (l, m, n) = (-2, -1, 1)$ Or $(l, m, n) = (1, -2, 1)$

Therefore, angle between the lines is given as:

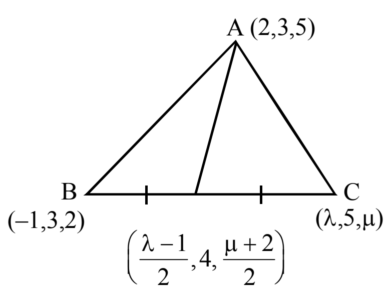
$$\cos(\theta) = \frac{(-2)(1) + (-1).(-2) + (1)(1)}{\sqrt{6}.\sqrt{6}}$$

$$\Rightarrow \cos(\theta) = \frac{1}{6} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{6}\right)$$

7. **(b)** DR's of AD are $\frac{\lambda-1}{2} - 2, 4-3, \frac{\mu+2}{2} - 5$

i.e. $\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$

$\because$ This median is making equal angles with coordinate axes, therefore,



$\frac{\lambda-5}{2} = 1 = \frac{\mu-8}{2}$

$\Rightarrow \lambda = 7$ & $\mu = 10$

$\therefore \lambda^3 + \mu^3 = 5 = 1348$

8. **(c)** Given, $l + m + n = 0$ and $l^2 = m^2 + n^2$

Now, $(-m-n)^2 = m^2 + n^2$

$\Rightarrow mn = 0 \Rightarrow m = 0$ or $n = 0$

If $m = 0$ then $l = -n$

We know $l^2 + m^2 + n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

i.e. $(l_1, m_1, n_1) = \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

If $n = 0$ then $l = -m$

$l^2 + m^2 + n^2 = 1 \Rightarrow 2m^2 = 1$

$\Rightarrow m = \pm\frac{1}{\sqrt{2}}$

Let $m = \frac{1}{\sqrt{2}}$

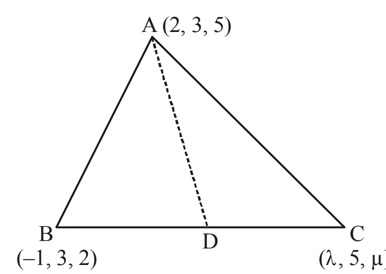
$\Rightarrow l = -\frac{1}{\sqrt{2}}$ and $n = 0$

$(l_2, m_2, n_2) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

9. **(c)** If D be the mid-point of BC, then

$D = \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$



Direction ratios of AD are $\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$

Since median AD is equally inclined with coordinate axes, therefore direction ratios of AD will be equal, i.e,

$$\frac{\left(\frac{\lambda-5}{2}\right)^2}{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2} = \frac{1}{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2}$$

$$= \frac{\left(\frac{\mu-8}{2}\right)^2}{\left(\frac{\lambda-5}{2}\right)^2 + 1 + \left(\frac{\mu-8}{2}\right)^2}$$

$\Rightarrow \left(\frac{\lambda-5}{2}\right)^2 = 1 = \left(\frac{\mu-8}{2}\right)^2$

$\Rightarrow \lambda = 7, 3$ and $\mu = 10, 6$

If $\lambda = 7$ and $\mu = 10$

Then $\frac{\lambda}{\mu} = \frac{7}{10} \Rightarrow 10\lambda - 7\mu = 0$

$\Rightarrow \lambda = 7$ and $\mu = 10$

10. **(c)** It makes θ with x and y-axes.

$l = \cos\theta$, $m = \cos\theta$, $n = \cos(\pi - 2\theta)$

we have $l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\theta + \cos^2\theta + \cos^2(\pi - 2\theta) = 1$

$\Rightarrow 2\cos^2\theta + (-\cos2\theta)^2 = 1$

$\Rightarrow 2\cos^2\theta - 1 + \cos^2 2\theta = 0$

$\Rightarrow \cos2\theta - [1 + \cos2\theta] = 0$

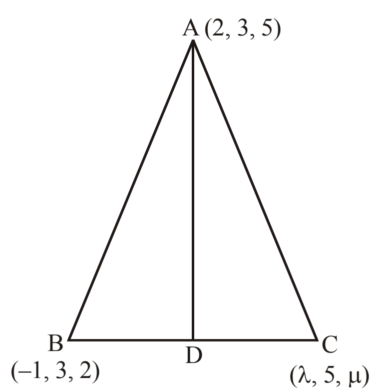
$\Rightarrow \cos 2\theta = 0$ or $\cos2\theta = -1$

$\Rightarrow 2\theta = \pi/2$ or $2\theta = \pi$

$\Rightarrow \theta = \pi/4$ or $\theta = \frac{\pi}{2}$

$\Rightarrow \theta = \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

11. **(c)** Since AD is the median



$\therefore\ D = \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$

Now, dR's of AD is

$a = \left(\frac{\lambda-1}{2} - 2\right) = \frac{\lambda-5}{2}$

$b = 4 - 3 = 1,\ c = \frac{\mu+2}{2} - 5 = \frac{\mu-8}{2}$

Also, a, b, c are dR's

$\therefore\ a = kl, b = km, c = kn$ where $l = m = n$ and $l^2 + m^2 + n^2 = 1$

$\Rightarrow l = m = n = \frac{1}{\sqrt{3}}$

Now, $a = 1, b = 1$ and $c = 1$

12. **(b)** Length of the line segment

$= \sqrt{(2)^2 + (3)^2 + (6)^2} = 7$

13. **(c)** Let l_1, m_1, n_1 and l_2, m_2, n_2 be the d.c of line 1 and 2 respectively, then as given

$l_1 + m_1 + n_1 = 0$

and $l_2 + m_2 + n_2 = 0$

and $l_1^2 + m_1^2 - n_1^2 = 0$ and

$l_2^2 + m_2^2 - n_2^2 = 0$

$(\because\ l + m + n = 0$ and $l^2 + m^2 - n^2 = 0)$

Angle between lines, θ is

$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ ...(1)

As given $l^2 + m^2 = n^2$ and $l + m = -n$

$\Rightarrow (-n)^2 - 2lm = n^2 \Rightarrow 2lm = 0$ or $lm = 0$

So $l_1 m_1 = 0, l_2 m_2 = 0$

If $l_1 = 0, m_1 \neq 0$ then $l_1 m_2 = 0$

If $m_1 = 0, l_1 \neq 0$ then $l_2 m_1 = 0$

If $l_2 = 0, m_2 \neq 0$ then $l_2 m_1 = 0$

If $m_2 = 0, l_2 \neq 0$ then $l_1 m_2 = 0$

Also $l_1 l_2 = 0$ and $m_1 m_2 = 0$

$l^2 + m^2 - n^2 = l^2 + m^2 + n^2 - 2n^2 = 0$

$\Rightarrow 1 - 2n^2 = 0 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

$\therefore\ n_1 = \pm\frac{1}{\sqrt{2}}, n_2 = \pm\frac{1}{\sqrt{2}}$

$\therefore\ \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$ (acute angle)

14. **(b)** As per question, direction cosines of the line :

$\ell = \cos 45° = \frac{1}{\sqrt{2}},\ m = \cos 120° = \frac{-1}{2},\ n = \cos\theta$

where θ is the angle, which line makes with positive z-axis.

We know that, $\ell^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$

$\cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2} = \cos\frac{\pi}{2}$ (θ being acute)

$\Rightarrow \theta = \frac{\pi}{3}$

15. **(b)** Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be the initial and final points of the vector whose projections on the three coordinate axes are $6, -3, 2$ then

$x_2 - x_1, = 6;\ y_2 - y_1 = -3;\ z_2 - z_1 = 2$

So that direction ratios of $\overrightarrow{PQ}$ are $6, -3, 2$

$\therefore$ Direction cosines of $\overrightarrow{PQ}$ are

$$\frac{6}{\sqrt{6^2+(-3)^2+2^2}}, \frac{-3}{\sqrt{6^2+(-3)^2+2^2}},$$

$$\frac{2}{\sqrt{6^2+(-3)^2+2^2}} = \frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$$

16. **(b)** Let the line makes an angle θ with the positive direction of z-axis. Given that lines makes angle $\frac{\pi}{4}$ with x-axis and y-axis.

$\therefore\ l = \cos\frac{\pi}{4}, m = \cos\frac{\pi}{4}, n = \cos\theta$

We know that, $l^2 + m^2 + n^2 = 1$

$\therefore\ \cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} + \cos^2\theta = 1$

$\Rightarrow\ \frac{1}{2} + \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow\ \cos^2\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

Hence, angle with positive direction of the z-axis is $\frac{\pi}{2}$.

17. **(c)** As per question the direction cosines of the line are $\cos\theta, \cos\beta, \cos\theta$

$\therefore\ \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

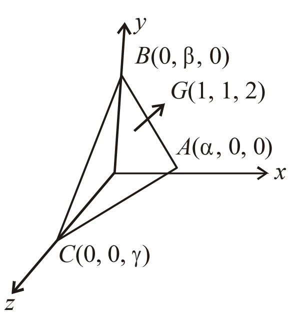
$\because\ 2\cos^2\theta = 1 - \cos^2\theta$

$\Rightarrow\ 2\cos^2\theta = \sin^2\beta = 3\sin^2\theta$ (given)

$\Rightarrow\ 2\cos^2\theta = 3 - 3\cos^2\theta$

$\therefore\ \cos^2\theta = \frac{3}{5}$

18. **(c)**



$\therefore\ \alpha = 3, \beta = 3$ and $\gamma = 6$ as G is centroid.

$\therefore$ The equation of plane is

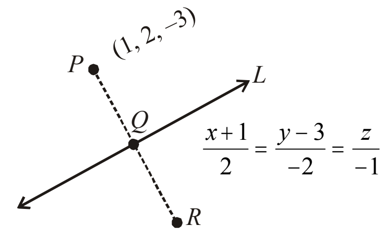
$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$$

$$\Rightarrow \frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1 \Rightarrow 2x + 2y + z = 6$$

$\therefore$ The required line is, $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

19. **(a)** $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$

Any point on line $= Q(2\lambda - 1, -2\lambda + 3, -\lambda)$



$\therefore$ D.r. of $PQ = [2\lambda - 2, -2\lambda + 1, -\lambda + 3]$

D.r. of given line $= [2, -2, -1]$

$\because$ PQ is perpendicular to line L

$\therefore 2(2\lambda - 2) - 2(-2\lambda + 1) - 1(-\lambda + 3) = 0$

$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$

$\Rightarrow 9\lambda - 9 = 0 \Rightarrow \lambda = 1$

$\because$ Q is mid point of $PR = Q = (1, 1, -1)$

$\therefore$ Coordinate of image $R = (1, 0, 1) = (a, b, c)$

$\therefore a + b + c = 2.$

20. **(a)** $L_1 \equiv \vec{r} = (\hat{i} - \hat{j}) + \ell(2\hat{i} + \hat{k})$

$L_2 \equiv \vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

Equating coeff. of $\hat{i}$, $\hat{j}$ and $\hat{k}$ of L_1 and L_2

$2\ell + 1 = m + 2$ ...(i)

$-1 = -1 + m \Rightarrow m = 0$ ...(ii)

$\ell = -m$ ...(iii)

$\Rightarrow m = \ell = 0$ which is not satisfy eqn. (i) hence lines do not intersect for any value of ℓ and m.

21. **(c)** $\overrightarrow{AB} = 6\hat{i} + 15\hat{j} + 3\hat{k}$

$\vec{p} = \hat{i} + 4\hat{j} + 22\hat{k}$

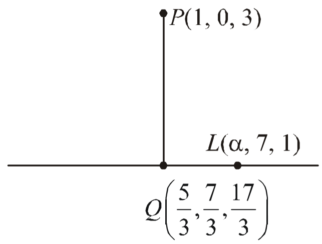
$\vec{q} = \hat{i} + \hat{j} + 7\hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} i & j & k \\ 1 & 4 & 22 \\ 1 & 1 & 7 \end{vmatrix} = 6\hat{i} + 15\hat{j} - 3\hat{k}$$

Shortest distance between the lines is

$$= \frac{|\overrightarrow{AB}.(\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|} = \frac{|36 + 225 + 9|}{\sqrt{36 + 225 + 9}} = 3\sqrt{30}$$

22. **(4)** Since, PQ is perpendicular to L

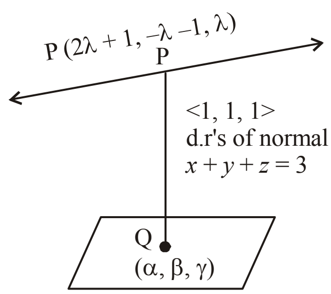


$$\therefore \quad \left(1-\frac{5}{3}\right)\left(\alpha-\frac{5}{3}\right)+\left(\frac{-7}{3}\right)\left(7-\frac{7}{3}\right)$$
$$+\left(3-\frac{17}{3}\right)\left(1-\frac{17}{3}\right)=0$$

$$\Rightarrow \quad \frac{-2\alpha}{3}+\frac{10}{9}-\frac{98}{9}+\frac{112}{9}=0$$

$$\Rightarrow \quad \frac{2\alpha}{3}=\frac{24}{9} \quad \Rightarrow \quad \alpha=4$$

23. **(b)**



Let co-ordinates of Q be (α, β, γ), then

$\alpha+\beta+\gamma=3$ ...(i)

$\alpha-\beta+\gamma=3$ ...(ii)

$\Rightarrow \alpha+\gamma=3$ and $\beta=0$

Equating direction ratio's of PQ, we get

$$\frac{\alpha-2\lambda-1}{1}=\frac{\lambda+1}{1}=\frac{\gamma-\lambda}{1}$$

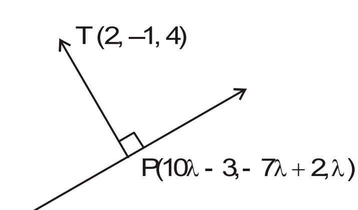
$\Rightarrow \alpha=3\lambda+2, \gamma=2\lambda+1$

Substituting the values of α and γ in equation (i), we get

$\Rightarrow 5\lambda+3=3 \Rightarrow \lambda=0$

Hence, point is Q (2, 0, 1)

24. **(a)** Let P be the foot of perpendicular from point T (2, – 1, 4) on the given line. So P can be assumed as P $(10\lambda-3, -7\lambda+2, \lambda)$



DR's of $TP \propto$ to $10\lambda-5, -7\lambda+3, \lambda-4$

$\because$ TP and given line are perpendicular, so

$10(10\lambda-5)-7(-7\lambda+3)+1(\lambda-4)=0$

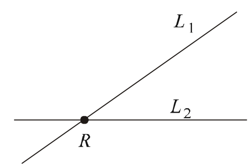
$$\Rightarrow \quad \lambda=\frac{1}{2}$$

$$\Rightarrow TP=\sqrt{(10\lambda-5)^2+(-7\lambda+3)^2+(\lambda-4)^2}$$

$$=\sqrt{0+\frac{1}{4}+\frac{49}{4}}=\sqrt{12.5}=3.54$$

Hence, the length of perpendicular is greater than 3 but less than 4.

25. **(a)** Let the coordinate of P with respect to line



$$\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}=\lambda$$

$$\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}=\mu$$

$L_1=(\lambda+3, 3\lambda-1, -\lambda+6)$

and coordinate of P w.r.t.

line $L_2=(7\mu-5, -6\mu+2, 4\mu+3)$

$\therefore \quad \lambda-7\mu=-8, 3\lambda+6\mu=3, \lambda+4\mu=3$

From above equation : $\lambda=-1, \mu=1$

$\therefore$ Coordinate of point of intersection $R=(2, -4, 7)$.

Image of R w.r.t. xy plane $=(2, -4, -7)$.

26. **(d)** First line is: $x=ay+b, z=cy+d$

$$\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$$

and another line is: $x=a'z+b', y=c'z+d'$

$$\Rightarrow \quad \frac{x-b'}{a'}=\frac{y-d'}{c'}=\frac{z}{1}$$

$\because$ Both lines are perpendicular to each other

$\therefore \quad aa'+c'+c=0$

27. **(d)** Let θ be the angle between the two lines

Here direction cosines of $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ are 2, 2, 1

Also second line can be written as:

$$\frac{x-5}{2}=\frac{y-2}{\frac{P}{7}}=\frac{z-3}{4}$$

$\therefore$ its direction cosines are $2, \frac{P}{7}, 4$

Also, $\cos\theta = \frac{2}{3}$ (Given)

$\because \quad \cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

$\Rightarrow \frac{2}{3} = \left|\frac{(2\times 2)+\left(2\times\frac{P}{7}\right)+(1\times 4)}{\sqrt{2^2+2^2+1^2}\sqrt{2^2+\frac{P^2}{49}+4^2}}\right|$

$= \frac{4+\frac{2P}{7}+4}{3\times\sqrt{2^2+\frac{P^2}{49}+4^2}}$

$\Rightarrow \left(4+\frac{P}{7}\right)^2 = 20+\frac{P^2}{49} \Rightarrow 16+\frac{8P}{7}+\frac{P^2}{49} = 20+\frac{P^2}{49}$

$\Rightarrow \frac{8P}{7} = 4 \Rightarrow P = \frac{7}{2}$

28. **(c)** Lines are coplanar

$$\begin{vmatrix} 3-1 & 2-2 & 1-(-3) \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2 & 0 & 4 \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix} = 0$$

$\Rightarrow 2(4-\lambda^4)+4(\lambda^2-2)=0$

$\Rightarrow 4-\lambda^4+2\lambda^2-4=0 \Rightarrow \lambda^2(\lambda^2-2)=0$

$\Rightarrow \lambda = 0, \sqrt{2}, -\sqrt{2}$

29. **(b)** Shortest distance between two lines

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$$\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}\right|$$

$\therefore$ The shortest distance between given lines are

$$\left|\frac{\begin{vmatrix} -2 & 4 & 5 \\ 2 & 2 & 1 \\ -1 & 8 & 4 \end{vmatrix}}{\sqrt{(8-8)^2+(-1-8)^2+(16+2)^2}}\right|$$

$= \left|\frac{0-36+90}{\sqrt{405}}\right| = \frac{54}{20.1} = 2.68$

30. **(b)** Let equation of the required line be

$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ ...(i)

Given two lines

$\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ ...(ii)

and $\frac{x-1}{0} = \frac{y+1}{0} = \frac{z}{1}$ ...(iii)

Since the line (i) is perpendicular to both the lines (ii) and (iii), therefore

$a - b + c = 0$ ...(iv)

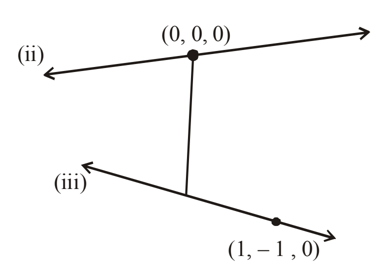
$-2b + c = 0$ ...(v)

From (iv) and (v) $c = 2b$ and $a + b = 0$, which are not satisfy by options (c) and (d). Hence options (c) and (d) are rejected.

Thus point (x_1, y_1, z_1) on the required line will be either (0, 0, 0) or (1, –1, 0).

Now foot of the perpendicular from point (0, 0, 0) to the line (iii)

$= (1, -2r - 1, r)$



The direction ratios of the line joining the points (0, 0, 0) and $(1, -2r - 1, r)$ are $1, -2r-1, r$

Since sum of the x and y-coordinate of direction ratio of the required line is 0.

$\therefore\ 1 - 2r - 1 = 0, \Rightarrow r = 0$

Hence direction ratio are 1, – 1, 0

But the z-direction ratio of the required line is twice the y-direction ratio of the required line

i.e. 0 = 2 (–1), which is not true.

Hence the shortest line does not pass through the point (0, 0, 0). Therefore option (a) is also rejected.

31. **(c)** Given lines will be coplanar

$$\text{If}\begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow\ -1(1+2k)-(1+k^2)+1(2-k)=0$

$\Rightarrow\ k = 0, -3$

32. **(d)** For L_1,

$$x = \sqrt{\lambda}y + (\sqrt{\lambda}-1) \Rightarrow y = \frac{x-(\sqrt{\lambda}-1)}{\sqrt{\lambda}} \quad \text{...(i)}$$

$$z = (\sqrt{\lambda}-1)y + \sqrt{\lambda} \Rightarrow y = \frac{z-\sqrt{\lambda}}{\sqrt{\lambda}-1} \quad \text{...(ii)}$$

From (i) and (ii)

$$\frac{x-(\sqrt{\lambda}-1)}{\sqrt{\lambda}} = \frac{y-0}{1} = \frac{z-\sqrt{\lambda}}{\sqrt{\lambda}-1} \quad \text{...(A)}$$

The equation (A) is the equation of line L_1.

Similarly equation of line L_2 is

$$\frac{x-(1-\sqrt{\mu})}{\sqrt{\mu}} = \frac{y-0}{1} = \frac{z-\sqrt{\mu}}{1-\sqrt{\mu}} \quad \text{...(B)}$$

Since $L_1 \perp L_2$, therefore

$\sqrt{\lambda}\sqrt{\mu} + 1\times 1 + (\sqrt{\lambda}-1)(1-\sqrt{\mu}) = 0$

$\Rightarrow \sqrt{\lambda} + \sqrt{\mu} = 0 \Rightarrow \sqrt{\lambda} = -\sqrt{\mu}$

$\Rightarrow \lambda = \mu$

33. **(a)** Two given planes are coplanar, if

$$\begin{vmatrix} -2-(-1) & k-1 & 0-(-1) \\ 2 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -1 & k-1 & 1 \\ 2 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$\Rightarrow (-1)(4-9)-(k-1)(8-6)+6-2=0$

$\Rightarrow k = \frac{11}{2}$

34. **(c)** Given lines are $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$

and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$

$\therefore\ a_1 = \hat{i} - \hat{j} + \hat{k},\ \vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$a_2 = 3\hat{i} + k\hat{j},\ \vec{b}_2 = \hat{i} + 2\hat{j} + \hat{k}$

Given lies are intersect if

$$\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1||\vec{b}_2|} = 0$$

$\Rightarrow\ (\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2) = 0$

$$\Rightarrow \begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow\ 2(3-8)-(k+1)(2-4)-1(4-3)=0$

$\Rightarrow\ 2(-5)-(k+1)(-2)-1(1)=0$

$\Rightarrow\ -10+2k+2-1=0 \Rightarrow k = \frac{9}{2}$

35. **(c)** Point is (–1, 2, 6)

Line passes through the point (2, 3, –4) parallel to vector whose direction ratios is 6, 3, – 4.

Equation is $\frac{x-2}{6} = \frac{y-3}{3} = \frac{z+4}{-4} = \lambda$

Any point on this line is given by $x = 6\lambda + 2$, $y = 3\lambda + 3$, $z = -4\lambda - 4$

Now, d.Rs of line passing through (–1, 2, 6) and $\perp$ to this line is

$\{(x+1), (y-2), (z-6)\}$

So, $6(x+1) + 3(y-2) - 4(z-6) = 0$

$\Rightarrow 6x + 3y - 4z + 24 = 0$

Now, $6(6\lambda+2) + 3(3\lambda+3) + 4(4\lambda+4) + 24 = 0$

$\Rightarrow 61\lambda + 61 = 0 \Rightarrow \lambda = -1$

So, $x = -4$, $y = 0$, $z = 0$

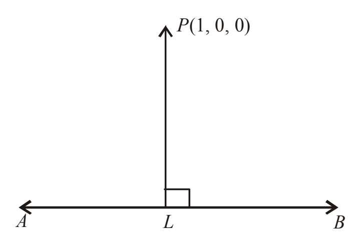
Now, distance between (–1, 2, 6) and (– 4, 0, 0) is $\sqrt{9+4+36} = \sqrt{49} = 7$

36. **(c)** On solving we will get shortest distance $\neq \sqrt{2}$

37. **(d)** Let the equation of AB is

$$\frac{x-1}{2} = \frac{y-(-1)}{-3} = \frac{z-(-10)}{8} = k$$

Let L be the foot of the perpendicular drawn from $P(1, 0, 0)$.



$\therefore\ L = (2k+1, -3k-1, 8k-10)$.

Now, direction ratio of $PL = (2k, -3k-1, 8k-10)$ and direction ratio of $AB = (2, -3, 8)$

Since, PL is perpendicular to AB

$\therefore \quad 2(2k)-3(-3k-1)+8(8k-10)=0$

Now, $k=\dfrac{2(1-1)+(-3)(0+1)+8(0+10)}{(2)^2+(-3)^2+(8)^2}$

$=\dfrac{0-3+80}{4+9+64}=\dfrac{77}{77}=1$

$\therefore$ Required co-ordinate $=L=(2+1,-3-1,8-10)$
$=(3,-4,-2)$.

38. **(c)** Any point on line $\dfrac{x}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}=\alpha$ is

$(2\alpha, 3\alpha+2, 4\alpha+3)$

$\Rightarrow$ Direction ratio of the $\perp$ line is

$2\alpha-3, 3\alpha+3, 4\alpha-8$. and

Direction ratio of the given line are 2, 3, 4

$\Rightarrow \quad 2(2\alpha-3)+3(3\alpha+3)+4(4\alpha-8)=0$

$\Rightarrow \quad 29\alpha-29=0$

$\Rightarrow \quad \alpha=1$

$\Rightarrow$ Foot of $\perp$ is (2, 5, 7)

$\Rightarrow$ Length $\perp$ is $\sqrt{1^2+6^2+4^2}=\sqrt{53}$

39. **(a)** The direction ratio of the line segment AB is 0, 6, –4 and the direction ratio of the given line is 1, 2, 3.
Clearly $1\times 0+2\times 6+3\times(-4)=0$
So, the given line is perpendicular to line AB.
Also, the mid point of A and B is (1, 3, 5) which satisfy the given line.
So, the image of B in the given line is A statement-1 and 2 both true but 2 is not correct explanation. of 1.

40. **(c)** Slope of line L $=-\dfrac{b}{5}$

Slope of line K $=-\dfrac{3}{c}$

Line L is parallel to line k.

$\Rightarrow \dfrac{b}{5}=\dfrac{3}{c} \Rightarrow bc=15$

(13, 32) is a point on L.

$\therefore \dfrac{13}{5}+\dfrac{32}{b}=1 \Rightarrow \dfrac{32}{b}=-\dfrac{8}{5}$

$\Rightarrow b=-20 \Rightarrow c=-\dfrac{3}{4}$

Equation of K :

$y-4x=3 \Rightarrow 4x-y+3=0$

Distance between L and K $=\dfrac{|52-32+3|}{\sqrt{17}}=\dfrac{23}{\sqrt{17}}$

41. **(a)** When the two lines intersect then shortest distance between them is zero i.e.

$\dfrac{(\vec{a}_2-\vec{a}_1)\cdot\vec{b}_1\times\vec{b}_2}{|\vec{b}_1\times\vec{b}_2|}=0$

$\Rightarrow \quad (\vec{a}_2-\vec{a}_1)\cdot\vec{b}_1\times\vec{b}_2=0$

where $\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}_1=k\hat{i}+2\hat{j}+3\hat{k}$

$\vec{a}_2=2\hat{i}+3\hat{j}+\hat{k}$, $\hat{b}_2=3\hat{i}+k\hat{j}+2\hat{k}$

$\Rightarrow \begin{vmatrix}1 & 1 & -2\\ k & 2 & 3\\ 3 & k & 2\end{vmatrix}=0$

$\Rightarrow \quad 1(4-3k)-1(2k-9)-2(k^2-6)=0$

$\Rightarrow \quad -2k^2-5k+25=0 \Rightarrow k=-5$ or $\dfrac{5}{2}$

$\because k$ is an integer, therefore $k=-5$

42. **(c)** a, b, c are in H.P. $\Rightarrow \dfrac{1}{a},\dfrac{1}{b},\dfrac{1}{c}$ are in A.P.

$\Rightarrow \dfrac{2}{b}=\dfrac{1}{a}+\dfrac{1}{c} \Rightarrow \dfrac{1}{a}-\dfrac{2}{b}+\dfrac{1}{c}=0$

$\therefore \quad \dfrac{x}{a}+\dfrac{y}{a}+\dfrac{1}{c}=0$ passes through (1, –2)

43. **(b)** The given lines are $2x=3y=-z$

or $\quad \dfrac{x}{3}=\dfrac{y}{2}=\dfrac{z}{-6}$ [Dividing by 6]

and $6x=-y=-4z$

or $\quad \dfrac{x}{2}=\dfrac{y}{-12}=\dfrac{z}{-3}$ [Dividing by 12]

$\therefore$ Angle between two lines is

$\cos\theta=\dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$\cos\theta=\dfrac{3.2+2.(-12)+(-6).(-3)}{\sqrt{3^2+2^2+(-6)^2}\sqrt{2^2+(-12)^2+(-3)^2}}$

$=\dfrac{6-24+18}{\sqrt{49}\sqrt{157}}=0 \Rightarrow \theta=90^\circ$

44. **(d)** The given lines are

$x-1=\dfrac{y+3}{-\lambda}=\dfrac{z-1}{\lambda}=s$ ....(1)

and $2x=y-1=\dfrac{z-2}{-1}=t$ ....(2)

The lines are coplanar, if

$\begin{vmatrix}0-1 & 1-(-3) & 2-1\\ 1 & -\lambda & \lambda\\ \dfrac{1}{2} & 1 & -1\end{vmatrix}=0$

$\Rightarrow \begin{vmatrix}-1 & 4 & 1\\ 1 & -\lambda & \lambda\\ \dfrac{1}{2} & 1 & -1\end{vmatrix}=0$

Apply $c_2 \to c_2 + c_3$; $\begin{vmatrix} -1 & 5 & 1 \\ 1 & 0 & \lambda \\ \frac{1}{2} & 0 & -1 \end{vmatrix} = 0$

$\Rightarrow -5\left(-1-\frac{\lambda}{2}\right) = 0 \Rightarrow \lambda = -2$

45. **(b)** Let a point on the line $x = y + a = z = \lambda$ is $(\lambda, \lambda - a, \lambda)$ and a point on the line

$x + a = 2y = 2z = \mu$ is $\left(\mu - a, \frac{\mu}{2}, \frac{\mu}{2}\right)$, then Direction ratio of the line joining these points are

$\lambda - \mu + a,\ \lambda - a - \frac{\mu}{2},\ \lambda - \frac{\mu}{2}$

If it respresents the required line whose $d \cdot r.$ be 2, 1, 2, then

$$\frac{\lambda - \mu + a}{2} = \frac{\lambda - a - \frac{\mu}{2}}{1} = \frac{\lambda - \frac{\mu}{2}}{2}$$

on solving we get $\lambda = 3a, \mu = 2a$

$\therefore$ The required points of intersection are

$(3a, 3a\text{-}a, 3a)$ and $\left(2a - a, \frac{2a}{2}, \frac{2a}{2}\right)$

or $(3a, 2a, 3a)$ and (a, a, a)

46. **(d)** Two planes are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 + C_1, C_3 \to C_3 + C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 1-k \\ k & k+2 & 1+k \end{vmatrix} = 0$$

$\Rightarrow\ 1[2 + 2k - (k+2)(1-k)] = 0$

$\Rightarrow\ 2 + 2k - (-k^2 - k + 2) = 0$

$k^2 + 3k = 0 \Rightarrow k(k+3) = 0$

or $k = 0$ or -3

47. **(a)** $\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}; \frac{x-b'}{a'} = \frac{y}{1} = \frac{z-d'}{c'}.$

For perpendicularity of lines,

$aa' + 1 + cc' = 0$

48. **(b)** For line of intersection of planes $x + y + z + 1 = 0$ and $2x - y + z + 3 = 0$:

$$\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = 2\hat{i} + \hat{j} - 3\hat{k}$$

Put $y = 0$, we get $x = -2$ and $z = 1$

$L_2 : \bar{r} = (-2\hat{i} + \hat{k}) + \lambda(2\hat{i} + \hat{j} - 3\hat{k})$ and

$L_1 : \bar{r} = (\hat{i} - \hat{j}) + \mu(-\hat{j} + \hat{k})$ (Given)

Now, $\bar{b}_1 \times \bar{b}_2 = -2[\hat{i} + \hat{j} + \hat{k}]$ and $\bar{a}_2 - \bar{a}_1 = -3\hat{i} + \hat{j} + \hat{k}$

$\therefore$ Shortest distance $= \frac{1}{\sqrt{3}}$

49. **(b)** Since, lince are coplanar

$$\therefore \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(-1 - 5 + \alpha) - 3(2 - \alpha) + 2(10 - 2\alpha + \alpha) = 0$

$\therefore \alpha = -4$

$\therefore$ Equation of $L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

$\therefore$ Point $(2, -10, -2)$ lies on line L_2.

50. **(3.00)**

Equation of plane P is

$(x + 4y - z + 7) + \lambda(3x + y + 5z - 8) = 0$

$\Rightarrow x(1 + 3\lambda) + y(4 + \lambda) + z(-1 + 5\lambda) + (7 - 8\lambda) = 0$

$\Rightarrow \frac{1+3\lambda}{a} = \frac{4+\lambda}{b} = \frac{5\lambda - 1}{6} = \frac{7 - 8\lambda}{-15}$

From last two ratios, $\lambda = -1$

$\Rightarrow \frac{-2}{a} = \frac{3}{b} = -1$

$\therefore a = 2, b = -3$

$\therefore$ Equation of plane is, $2x - 3y + 6z - 15 = 0$

Distance $= \frac{|6 - 6 - 6 - 15|}{7} = \frac{21}{7} = 3.$

51. **(b)** Equation of line through point $P(1, -2, 3)$ and parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$$

So, any point on line $= Q(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

Since, this point lies on plane $x - y + 2 = 5$

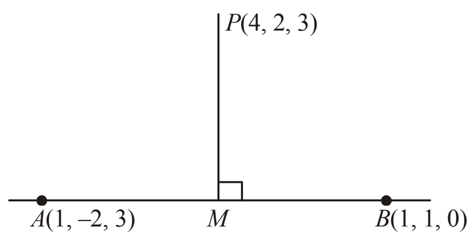
$\therefore 2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5 \Rightarrow \lambda = \frac{1}{7}$

$\therefore$ Point of intersection line and plane, $Q = \left(\frac{9}{7}, \frac{11}{7}, \frac{15}{7}\right)$

$\therefore$ Required distance PQ

$$= \sqrt{\left(\frac{9}{7}-1\right)^2 + \left(-\frac{11}{7}+2\right)^2 + \left(\frac{15}{7}-3\right)^2} = 1$$

52. **(a)** Equation of line through points $(1, -2, 3)$ and $(1, 1, 0)$ is



$$\frac{x-1}{0} = \frac{y-1}{-3} = \frac{z-0}{3-0} \ (=\lambda \text{ say})$$

$\therefore M(1, -\lambda+1, \lambda)$

Direction ratios of PM $= [-3, -\lambda-1, \lambda-3]$

$\because PM \perp AB$

$\therefore (-3)\cdot 0 + (-1-\lambda)(-1) + (\lambda-3)\cdot 1 = 0$

$\therefore \lambda = 1$

$\therefore$ Foot of perpendicular $= (1, 0, 1)$

This point satisfy the plane $2x + y - z = 1$.

53. **(c)** Direction ratios of normal to the plane are $<1, -3, 2>$.
Plane passes through $(3, 1, 1)$.
Equation of plane is,

$1(x-3) - 3(y-1) + 2(z-1) = 0$

$\Rightarrow x - 3y + 2z - 2 = 0$

54. **(5)**

Normal of plane $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$\vec{n} = -\hat{i} + \hat{j} + \hat{k}$

Direction ratios of normal to the plane $= <-1, 1, 1>$
Equation of plane

$-1(x-1) + 1(y-0) + 1(z-0) = 0$

$\Rightarrow x - y - z - 1 = 0$

If (x, y, z) is foot of perpendicular of $M(1, 0, 1)$ on the plane then

$$\Rightarrow \frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1} = \frac{-(1-0-1-1)}{3}$$

$\therefore x = \frac{4}{3}, y = -\frac{1}{3}, z = \frac{2}{3}$

$\alpha + \beta + \gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{3}$

$\therefore 3(\alpha+\beta+\gamma) = 3 \times \frac{5}{3} = 5.$

55. **(b)** Let plane passes through $(2, 1, 2)$ be

$a(x-2) + b(y-1) + (z-2) = 0$

It also passes through $(1, 2, 1)$

$\therefore -a + b - c = 0 \Rightarrow a - b + c = 0$

The given line is

$\frac{x}{3} = \frac{y}{2} = \frac{z-1}{0}$ is parallel to plane

$\therefore 3a + 2b + c(0) = 0$

$$\Rightarrow \frac{a}{0-2} = \frac{b}{3-0} = \frac{c}{2+3}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{-3} = \frac{c}{2+3}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{-3} = \frac{c}{-5}$$

$\therefore$ plane is $2x - 4 - 3y + 3 - 5z + 10 = 0$

$\Rightarrow 2x - 3y - 5z + 9 = 0$

The plane satisfies the point $(-2, 0, 1)$.

56. **(a)** $\because$ Plane contains two lines

$\therefore \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix}$

$= \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$

$= -4\hat{i} + 5\hat{j} + 7\hat{k}$

So, equation of plane is

$-4(x-3) + 5(y-1) + 7(z-1) = 0$

$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$

$\Rightarrow -4x + 5y + 7z = 0$

This also passes through $(\alpha, -3, 5)$

So, $-4\alpha - 15 + 35 = 0$

$\Rightarrow -4\alpha = -20 \Rightarrow \alpha = 5.$

57. **(b)** $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$

$\Rightarrow \quad (7\alpha+25)-(4\alpha+10)+(-20+14)=0$

$\Rightarrow \quad 3\alpha+9=0 \;\Rightarrow\; \alpha=-3$

Also, $D_z=0 \;\Rightarrow\; \begin{vmatrix}1 & 4 & 1\\ 1 & 7 & \beta\\ 1 & 5 & 5\end{vmatrix}=0$

$\Rightarrow 1(35-5\beta)-(15)+1\,(4\beta-7)=0 \;\Rightarrow\; \beta=13$

Hence, $\alpha+\beta=-3+13=10$

58. **(3)** Since, the line $\frac{x+1}{2}=\frac{y-3}{4}=\frac{z+1}{3}$ contains the point $(-1, 3, -1)$ and line $\frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda}$ contains the point $(-3, -2, 1)$.

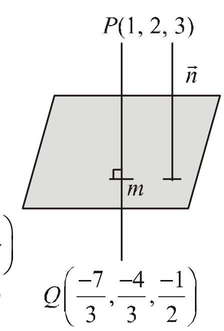
Then, the distance between the plane $23x-10y-2z+48=0$ and the plane containing the lines = perpendicular distance of plane $23x-10y-2z+48=0$ either from $(-1, 3, -1)$ or $(-3, -2, 1)$.

$$=\left|\frac{23(-1)-10(3)-2(-1)}{\sqrt{(23)^2+(10)^2+(-2)^2}}\right|=\frac{3}{\sqrt{633}}$$

It is given that distance between the planes

$$=\frac{k}{\sqrt{633}} \;\Rightarrow\; \frac{k}{\sqrt{633}}=\frac{3}{\sqrt{633}} \;\Rightarrow\; k=3$$

59. **(b)** $\vec{n}=\frac{-7}{3}-1, \frac{-4}{3}-2, \frac{-1}{3}-3$



$\vec{n}=\frac{10}{3}, \frac{10}{3}, \frac{10}{3}$

$D.r$ of normal to the plane $(1, 1, 1)$

Midpoint of P and Q is $\left(\frac{-2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

$\therefore$ Equation of required plane Q

$\vec{r}.\vec{n}=\vec{a}.\vec{n}$

$\vec{r}.(\hat{i}+\hat{j}+\hat{k})=\frac{-2}{3}+\frac{1}{3}+\frac{4}{3}$

$\therefore$ Equation of plane is $x+y+z=1$

60. **(b)** Equation of plane is $x+y-2z=3$

$\Rightarrow \quad \frac{x-2}{1}=\frac{y-1}{1}=\frac{z-6}{-2}=\frac{-2(2+1-12-3)}{6}$

$\Rightarrow \quad (x, y, z)=(6, 5, -2)$

61. **(d)** Let points $P\,(3\lambda+2, 2\lambda-1, -\lambda+1)$ and $Q\,(3\mu+2, 2\mu-1, -\mu+1)$

$\because P$ lies on $2x+3y-z+13=0$

$\therefore \quad 6\lambda+4+6\lambda-3+\lambda-1+13=0$

$\Rightarrow \quad 13\lambda=-13 \Rightarrow \lambda=-1$

Hence, $P\,(-1, -3, 2)$

Similarly, Q lies on $3x+y+4z=16$

$\therefore \quad 9\mu+6+2\mu-1-4\mu+4=16$

$\Rightarrow \quad 7\mu=7 \Rightarrow \mu=1$

Hence, Q is $(5, 1, 0)$

Now, $PQ=\sqrt{36+16+4}=\sqrt{56}=2\sqrt{14}$

62. **(d)** The equations of angle bisectors are,

$$\frac{x+2y+2z-2}{3}=\pm\frac{2x-y+2z-4}{3}$$

$\Rightarrow \quad x-3y-2=0$

or $\quad 3x+y+4z-6=0$

$(2, -4, 1)$ lies on the second plane.

63. **(c)** The equation of plane containing two given lines is,

$$\begin{vmatrix}x-1 & y-1 & z\\ 1 & 2 & -1\\ -1 & 1 & -2\end{vmatrix}=0$$

On expanding, we get $x-y-z=0$

Now, the length of perpendicular from $(2, 1, 4)$ to this plane

$$=\left|\frac{2-1-4}{\sqrt{1^2+1^2+1^2}}\right|=\sqrt{3}$$

64. **(c)** Image of Q $(0, -1, -3)$ in plane is,

$$\frac{(x-0)}{3}=\frac{(y+1)}{-1}=\frac{z+3}{+4}=\frac{-2(1-12-2)}{9+1+16}=1$$

$\Rightarrow x=3, y=-2, z=1$

$\Rightarrow$ P $(3, -2, 1)$, Q $(0, -1, -3)$, R $(3, -1, -2)$

$\therefore$ Area of ΔPQR is

$$\frac{1}{2}|\vec{QP}\times\vec{QR}|=\frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & k\\ 3 & -1 & 4\\ 3 & 0 & 1\end{matrix}\right\|$$

$$=\frac{1}{2}|\{\hat{i}(-1)-\hat{j}(3-12)+\hat{k}(3)\}|$$

$$=\frac{1}{2}\sqrt{(1+81+9)}=\frac{\sqrt{91}}{2}$$

65. **(d)** Let, $P_1: 2x-y+2z+3=0$

$P_2: 2x-y+2z+\frac{\lambda}{2}=0$

$P_3: 2x-y+2z+\mu=0$

Given, distance between P_1 and P_2 is $\frac{1}{3}$

$$\frac{1}{3}=\frac{\left|3-\frac{\lambda}{2}\right|}{\sqrt{9}} \Rightarrow \left|3-\frac{\lambda}{2}\right|=1 \Rightarrow \lambda_{max}=8$$

And distance between P_1 and P_3 is $\frac{2}{3}$

$$\frac{2}{3}=\frac{|\mu-3|}{\sqrt{9}} \Rightarrow \mu_{max}=5$$

$\Rightarrow (\lambda+\mu)_{max}=13$

66. **(c)** Let point on line be P $(2k+1, 3k-1, 4k+2)$
Since, point P lies on the plane $x+2y+3z=15$
$\therefore 2k+1+6k-2+12k+6=15$

$$\Rightarrow k=\frac{1}{2}$$

$$\therefore P \equiv \left(2, \frac{1}{2}, 4\right)$$

Then the distance of the point P from the origin is

$$OP=\sqrt{4+\frac{1}{4}+16}=\frac{9}{2}$$

67. **(d)** Let the required plane passing through the points $(0, -1, 0)$ and $(0, 0, 1)$ be $\frac{x}{\lambda}+\frac{y}{-1}+\frac{z}{1}=1$ and the given plane is $y-z+5=0$

$$\therefore \cos\frac{\pi}{4}=\frac{-1-1}{\sqrt{\left(\frac{1}{\lambda^2}+1+1\right)}\sqrt{2}}$$

$$\Rightarrow \lambda^2=\frac{1}{2} \Rightarrow \frac{1}{\lambda}=\pm\sqrt{2}$$

Then, the equation of plane is

$\pm\sqrt{2}x-y+z=1$

Then the point $(\sqrt{2},1,4)$ satisfies the equation of plane

68. **(d)** Let the plane be
$P \equiv (2x+3y+z+5)+\lambda(x+y+z-6)=0$
$\because$ above plane is perpendicular to xy plane.

$$\therefore \left((2+\lambda)\hat{i}+(3+\lambda)\hat{j}+(1+\lambda)\hat{k}\right).\hat{k}=0 \Rightarrow \lambda=-1$$

Hence, the equation of the plane is,
$P \equiv x+2y+11=0$
Distance of the plane P from $(0, 0, 256)$

$$\left|\frac{0+0+11}{\sqrt{5}}\right|=\frac{11}{\sqrt{5}}$$

69. **(c)** Let the equation of required plane be;
$(2x-y-4)+\lambda(y+2z-4)=0$

$\because$ This plane passes through the point $(1, 1, 0)$ then $(2-1-4)+\lambda(1+0-4)=0$
$\Rightarrow \lambda=-1$
Then, equation of required plane is,
$(2x-y-4)-(y+2z-4)=0$
$\Rightarrow \quad 2x-2y-2z=0 \Rightarrow x-y-z=0$

70. **(d)** Equation of the plane passing through the line of intersection of $x+y+z=1$ and $2x+3y+4z=5$ is
$(2x+3y+4z-5)+\lambda(x+y+z-1)=0$
$\Rightarrow (2+\lambda)x+(3+\lambda)y+(4+\lambda)z+(-5-\lambda)=0 \quad ...(i)$
$\because$ plane (i) is perpendicular to the plane $x-y+z=0$
$\therefore (2+\lambda)(1)+(3+\lambda)(-1)+(4+\lambda)(1)=0$
$2+\lambda-3-\lambda+4+\lambda=0 \Rightarrow \lambda=-3$
Hence, equation of required plane is
$-x+z-2=0$ or $x-z+2=0$

$$\Rightarrow \vec{r}.(\hat{i}-\hat{k})+2=0$$

71. **(b)** $\because$ plane containing both lines.

$$\therefore \quad \text{D.R. of plane}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 5 & 7\\ 1 & 4 & 7\end{vmatrix}=7\hat{i}-14\hat{j}+7\hat{k}$$

Now, equation of plane is,
$7(x-1)-14(y-4)+7(z+4)=0$
$\Rightarrow \quad x-1-2y+8+z+4=0$
$\Rightarrow \quad x-2y+z+11=0$
Hence, distance from $(0, 0, 0)$ to the plane,

$$=\frac{11}{\sqrt{1+4+1}}=\frac{11}{\sqrt{6}}$$

72. **(a)** Let angle between line and plane is θ, then

$$\sin\theta=\left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}|\cdot|\vec{n}|}\right|$$

$$=\left|\frac{(2\hat{i}+\hat{j}-2\hat{k})\cdot(\hat{i}-2\hat{j}-K\hat{k})}{\sqrt{9}\cdot\sqrt{1+4+K^2}}\right|$$

$$=\left|\frac{2-2\_2K}{3\sqrt{5+K^2}}\right|=\frac{2|K|}{3\sqrt{4+K^2}}$$

Since, $\cos\theta=\frac{2\sqrt{2}}{3} \Rightarrow \sin\theta=\frac{1}{3}$

Then, $\frac{2|K|}{3\sqrt{5+K^2}}=\frac{1}{3} \Rightarrow 4K^2=5+K^2$

$$3K^2=5 \Rightarrow K=\pm\sqrt{\frac{5}{3}}$$

73. **(b)** Let $A(-\lambda^2, 1, 1)$, $B(1, -\lambda^2, 1)$, $C(1, 1, -\lambda^2)$, $D(-1, -1, 1)$ lie on same plane, then

$$\begin{vmatrix} 1-\lambda^2 & 2 & 0 \\ 2 & 1-\lambda^2 & 0 \\ 2 & 2 & -\lambda^2-1 \end{vmatrix} = 0$$

$\Rightarrow$ $(\lambda^2+1)((1-\lambda^2)^2-4)=0$

$\Rightarrow$ $(3-\lambda^2)(\lambda^2+1)=0 \Rightarrow \lambda^2=3$

$\lambda = \pm\sqrt{3}$

Hence, $S = \{-\sqrt{3}, \sqrt{3}\}$

74. **(d)** Let normal to the required plane is $\vec{n}$

$\Rightarrow$ $\vec{n}$ is perpendicular to both vector $2\hat{i}-\hat{j}+3\hat{k}$ and $2\hat{i}+3\hat{j}-3\hat{k}$.

$$\Rightarrow \quad \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 2 & 3 & -1 \end{vmatrix} = -8\hat{i}+8\hat{j}+8\hat{k}$$

$\Rightarrow$ Equation of the required plane is

$\Rightarrow$ $(x-3)(-8)+(y+2)\times 8+(z-1)\times 8=0$

$\Rightarrow$ $(x-3)(-1)+(y+2)\times 1+(z-1)\times 1=0$

$\Rightarrow$ $x-3-y-2-z+1=0$

$\because$ $x-y-z=4$ passes through $(2, 0, -2)$

$\therefore$ plane contains $(2, 0, -2)$.

75. **(b, c)** Let the d.r's of the normal be $\langle a, b, c\rangle$

Equation of the plane is

$a(x-0)+b(y+1)+c(z-0)=0$

$\because$ It passes through $(0, 0, 1)$

$\therefore$ $b+c=0$

Also $\dfrac{0\cdot a+b-c}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{2}} = \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow$ $b-c=\sqrt{a^2+b^2+c^2}$

And $b+c=0$

$\Rightarrow$ $b=\pm\dfrac{1}{\sqrt{2}}a.$

$\therefore$ The d.r's are $\sqrt{2}, 1, -1$ or $2, \sqrt{2}, -\sqrt{2}$

76. **(b)** Let the normal to the required plane is $\vec{n}$, then

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -4 & 4 \\ 2 & -5 & 0 \end{vmatrix} = 20\hat{i}+8\hat{j}-12$$

$\therefore$ Equation of the plane

$(x-3)\times 20+(y-4)\times 8+(z-2)\times(-12)=0$

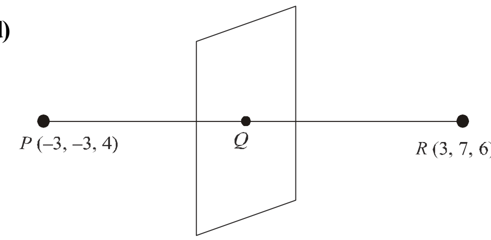
$5x-15+2y-8-3z+6=0$

$5x+2y-3z-17=0$ ...(1)

Since, equation of plane (1) passes through $(2, \alpha, \beta)$, then

$10+2\alpha-3\beta-17=0 \Rightarrow 2\alpha-3\beta=7$

77. **(d)**



Q (0, 2, 5)

Since, direction ratios of normal to the plane is $\vec{n}=6\hat{i}+10\hat{j}+2\hat{k}$

Then, equation of the plane is

$(x-0)6+(y-2)10+(z-5)2=0$

$3x+5y-10+z-5=0$

$3x+5y+z=15$ ...(1)

Since, plane (1) satisfies the point $(4, 1, -2)$

Hence, required point is $(4, 1, -2)$

78. **(c)** Let any point on the line $\dfrac{x-4}{2}=\dfrac{y-5}{2}=\dfrac{z-3}{1}$ be $A(2\lambda+4, 2\lambda+5, \lambda+3)$ which lies on the plane $x+y+z=2$

$\Rightarrow$ $2\lambda+4+2\lambda+5+\lambda+3=2$

$\Rightarrow$ $5\lambda=-10 \Rightarrow \lambda=-2$

Then, the point of intersection is $(0, 1, 1)$

which lies on the line $\dfrac{x-1}{1}=\dfrac{y-3}{2}=\dfrac{z+4}{-5}$

79. **(d)** Since the system of linear equations are

$x+y+z=2$ ...(1)

$2x+3y+2z=5$ ...(2)

$2x+3y+(a^2-1)z=a+1$ ...(3)

$$\text{Now, } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2-1 \end{vmatrix}$$

(Applying $R_3 \to R_3 - R_2$)

$$\Rightarrow \quad \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 0 & a^2-3 \end{vmatrix}$$

$= a^2-3$

When, $\Delta=0 \Rightarrow a^2-3=0 \Rightarrow |a|=\sqrt{3}$

If $a^2 = 3$, then plane represented by eqn (2) and eqn (3) are parallel.

Hence, the given system of equation is inconsistent.

80. **(c)** Let any point on the intersecting line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1} = \lambda \text{ (say)}$$

is $(-3\lambda - 1, 2\lambda + 3, -\lambda + 2)$

Since, the above point lies on a line which passes through the point $(-4, 3, 1)$

Then, direction ratio of the required line

$= \langle -3\lambda - 1 + 4, 2\lambda + 3 - 3, -\lambda + 2 - 1 \rangle$

or $\langle -3\lambda + 3, 2\lambda, -\lambda + 1 \rangle$

Since, line is parallel to the plane

$x + 2y - z - 5 = 0$

Then, perpendicular vector to the line is $\hat{i} + 2\hat{j} - \hat{k}$

Now $(-3\lambda + 3)(1) + (2\lambda)(2) + (-\lambda+1)(-1) = 0$

$\Rightarrow \lambda = -1$

Now direction ratio of the required line $= \langle 6, -2, 2 \rangle$ or $\langle 3, -1, 1 \rangle$

Hence required equation of the line is

$$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

81. **(d)** Since, equation of plane through intersection of planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is

$(2x + 3y - z + 4) + \lambda(x + y + z - 1) = 0$

$(2 + \lambda)x + (3 + \lambda)y + (-1 + \lambda)z + (4 - \lambda) = 0$ ...(1)

But, the above plane is parallel to y-axis then

$(2 + \lambda) \times 0 + (3 + \lambda) \times 1 + (-1 + \lambda) \times 0 = 0$

$\Rightarrow \lambda = -3$

Hence, the equation of required plane is

$-x - 4z + 7 = 0$

$\Rightarrow x + 4z - 7 = 0$

Therefore, (3, 2, 1) the passes through the point.

82. **(a)** Let the direction ratios of the plane containing lines

$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is $\langle a, b, c \rangle$

$\therefore$ $3a + 4b + 2c = 0$

$4a + 2b + 3c = 0$

$$\therefore \quad \frac{a}{12-4} = \frac{b}{8-9} = \frac{c}{6-16}$$

$$\frac{a}{8} = \frac{b}{-1} = \frac{c}{-10}$$

$\therefore$ Direction ratio of plane $= \langle -8, 1, 10 \rangle$.

Let the direction ratio of required plane is $\langle l, m, n \rangle$

Then $-8l + m + 10n = 0$ ...(1)

and $2l + 3m + 4n = 0$ ...(2)

From (1) and (2),

$$\frac{l}{-26} = \frac{m}{52} = \frac{n}{-26}$$

$\therefore$ D.R.s are $\langle 1, -2, 1 \rangle$

$\therefore$ Equation of plane: $x - 2y + z = 0$

83. **(a)** Equation of plane passing through the line of intersection of first two planes is:

$(2x - 2y + 3z - 2) + \lambda(x - y + z + 1) = 0$

or $x(\lambda + 2) - y(2 + \lambda) + z(\lambda + 3) + (\lambda - 2) = 0$ ...(i)

is having infinite number of solution with

$x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$, then

$$\begin{vmatrix} (\lambda+2) & -(\lambda+2) & (\lambda+3) \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow \lambda = 5$

Now put $\lambda = 5$ in (i), we get

$7x - 7y + 8z + 3 = 0$

Now perpendicular distance from (0, 0, 0) to the place containing L_1 and $L_2 = \frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$

84. **(c)** Equation of plane passing through three given points is:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+2 & y+2 & z-2 \\ 1+2 & -1+2 & 2-2 \\ 1+2 & 1+2 & 1-2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+2 & y+2 & z-2 \\ 3 & 1 & 0 \\ 3 & 3 & -1 \end{vmatrix} = 0$$

$\Rightarrow -x + 3y + 6z - 8 = 0$

$$\Rightarrow \frac{x}{8} - \frac{3y}{8} - \frac{6z}{8} + \frac{8}{8} = 0$$

$$\Rightarrow \frac{x}{8} - \frac{y}{\frac{8}{3}} - \frac{z}{\frac{8}{6}} = -1$$

$$\Rightarrow \frac{x}{-8} + \frac{y}{\frac{8}{3}} + \frac{z}{\frac{8}{6}} = 1$$

$\therefore$ Sum of intercepts $= -8+\frac{8}{3}+\frac{8}{6}=-4$

85. **(c)** If a, b, c are the intercepts of the variable plane on the x, y, z axes respectively, then the equation of the plane is

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

And the point of intersection of the planes parallel to the xy, yz and zx planes is (a, b, c).

As the point (3, 2, 1) lies on the variable plane, so

$\frac{3}{a}+\frac{2}{b}+\frac{1}{c}=1$

Therefore, the required locus is $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

86. **(d)** Normal to $3x+4y+z=1$ is $3\hat{i}+4\hat{j}+\hat{k}$.

Normal to $5x+8y+2z=-14$ is $5\hat{i}+8\hat{j}+2\hat{k}$

The line of intersection of the planes is perpendicular to both normals, so, direction ratios of the intersection line are directly proportional to the cross product of the normal vectors.

Therefore the direction ratios of the line is $-\hat{j}+4\hat{k}$

Hence the angle between the plane $x+y+z+5=0$ and the intersection line is $\sin^{-1}\left(\frac{-1+4}{\sqrt{17}\sqrt{3}}\right)=\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

87. **(a)** Since the plane bisects the line joining the points (1, 2, 3) and (–3, 4, 5) then the plane passes through the midpoint of the line which is :

$\left(\frac{1-3}{2},\frac{2+4}{2},\frac{5+3}{2}\right)\equiv\left(\frac{-2}{2},\frac{6}{2},\frac{8}{2}\right)\equiv(-1,3,4).$

As plane cuts the line segment at right angle, so the direction cosines of the normal of the plane are $(-3-1, 4-2, 5-3)=(-4, 2, 2)$

So the equation of the plane is : $-4x+2y+2z=\lambda$

As plane passes through $(-1, 3, 4)$ so

$-4(-1)+2(3)+2(4)=\lambda\Rightarrow\lambda=18$

Therefore, equation of plane is : $-4x+2y+2z=18$

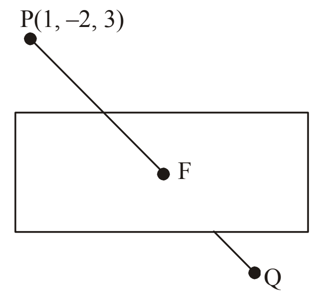
Now, only $(-3, 2, 1)$ satisfies the given plane as

$-4(-3)+2(2)+2(1)=18$

88. **(c)** Equation of line PQ is

$\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}$

Let F be $(\lambda+1, 4\lambda-2, 5\lambda+3)$



Since F lies on the plane

$\therefore\quad 2(\lambda+1)+3(4\lambda-2)-4(5\lambda+3)+22=0$

$2\lambda+2+12\lambda-6-20\lambda-12+22=0$

$\Rightarrow\quad -6\lambda+6=0\Rightarrow\lambda=1$

$\therefore\quad$ F is (2, 2, 8)

$PQ=2\,PF=2\sqrt{1^2+4^2+5^2}=2\sqrt{42}$

89. **(c)** Let the plane be

$a(x-1)+b(y+1)+c(z+1)=0$

Normal vector

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 3\\ 2 & -1 & -1\end{vmatrix}=5\hat{i}+7\hat{j}+3\hat{k}$$

So plane is $5(x-1)+7(y+1)+3(z+1)=0$

$\Rightarrow\quad 5x+7y+3z+5=0$

Distance of point (1, 3, –7) from the plane is

$\frac{5+21-21+5}{\sqrt{25+49+9}}=\frac{10}{\sqrt{83}}$

90. **(c)** $x+8y+7z=0$

$9x+2y+3z=0$

$x+y+z=0$

$x=\lambda \quad y=6\lambda \quad z=-7\lambda$

$x=\lambda \quad y=6\lambda \quad z=-7\lambda$

Now, $\lambda+12\lambda-7\lambda=6$	$\therefore$	$2\lambda+6\lambda-7\lambda$
$6\lambda=6$		$=\lambda$
$\lambda=1$		$=1$

91. **(a)** Suppose centroid be (h, k, ℓ)

$\therefore\quad$ x – intp $=3h$, y – intp $=3k$, z – intp $=3\ell$

Equation $\frac{x}{3h}+\frac{y}{3k}+\frac{z}{3\ell}=1$

$\therefore\quad$ Distance from (0, 0, 0)

$$\left|\frac{-1}{\sqrt{\frac{1}{9h^2}+\frac{1}{9k^2}+\frac{1}{9\ell^2}}}\right|=3$$

$\Rightarrow \boxed{\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1}$

92. **(c)** Point $(3, -2, -\lambda)$ on p line $2x-4y+3z-2=0$
$=6+8-3\lambda-2=0=3\lambda=12$

$\boxed{\lambda=4}$

Now,

$\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+4}{-2}=k_1$ ...(i)

$\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}=k_2$ ...(ii)

Point on equation (i) P $(k_1+3, -k_1-2, -2k_1-4)$
Point on equation (ii) Q $(12k_2+1, 9k_2, 4k_2)$
$k_1+3=12k_2+1 \mid -k_1-2=9k_2 \mid -2k_1-4=4k_2$
$k_2=0$
$k_1=-2$
p (1, 0, 0) lie on equation of a line 1
gives shortest distance = 0

93. **(c)** $\vec{n}=\vec{n_1}\times\vec{n_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 6 & 7 & 8\\ 3 & 5 & 7\end{vmatrix}=(9,-18,9)=(1,-2,1)$

$\therefore$ Equation of plane is
$1(x+1)-2(y-1)+(z-3)=0$
$\Rightarrow x-2y+z=0$
foot to z

$\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-1}{1}=-\frac{[1+4+1]}{6}$

$\boxed{x=0, y=0, z=0}$

94. **(c)** $\vec{n}=\vec{n_1}\times\vec{n_2}$

$\Rightarrow \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 1\\ 1 & 4 & -2\end{vmatrix}=\hat{i}(-2)-\hat{j}(-7)+\hat{k}(13)$

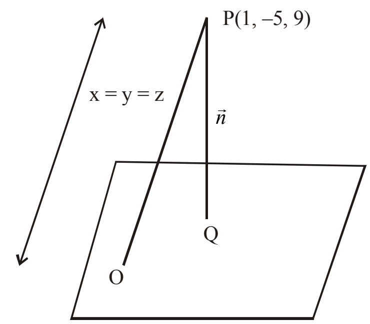
$\Rightarrow \vec{n}=-2\hat{i}+7\hat{j}+13\hat{k}$

Now,
$3x-y+z=1$
$x+4y-2z=2$
but $z=0$ & solving the given
$x=6/13$ & $y=5/13$
$\therefore$ required equation of a line is

$\frac{x-6/13}{2}=\frac{y-5/13}{-7}=\frac{z}{-13}$

95. **(b)** Line lies in the plane $\Rightarrow (3, -2, -4)$ lie in the plane
$\Rightarrow 3\ell-2m+4=9$ or $3\ell-2m=5$ ....(1)
Also, ℓ, m, -1 are dr's of line perpendicular to plane and 2, -1, 3 are dr's of line lying in the plane
$\Rightarrow 2\ell-m-3=0$ or $2\ell-m=3$ ....(2)
Solving (1) and (2) we get $\ell=1$ and $m=-1$
$\Rightarrow \ell^2+m^2=2.$

96. **(d)**



Eq^n of PO : $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda$
$\Rightarrow x=\lambda+1;\ y=\lambda-5; z=\lambda+9.$
Putting these in eq^n of plane :
$\lambda+1-\lambda+5+\lambda+9=5$
$\Rightarrow \lambda=-10$
$\Rightarrow$ O is $(-9, -15, -1)$
$\Rightarrow$ distance OP $=10\sqrt{3}$

97. **(c)** Let equation of plane be
$a(x-1)+b(y-2)+c(z-2)=0$ .....(1)
(1) is perpendicular to given planes then
$a-b+2c=0$
$2a-2b+c=0$
Solving above equation $c=0$ and $a=b$
equation of plane (1) can be
$x+y-3=0$
distance from $(1, -2, 4)$ will be

$D=\frac{|1-2-3|}{\sqrt{1+1}}=\frac{4}{\sqrt{2}}=2\sqrt{2}$

98. **(a)** Equation of the plane containing the lines
$2x-5y+z=3$ and $x+y+4z=5$ is
$2x-5y+z-3+\lambda(x+y+4z-5)=0$
$\Rightarrow (2+\lambda)x+(-5+\lambda)y+(1+4\lambda)z+(-3-5\lambda)=0$ ...(i)
Since the plane (i) parallel to the given plane
$x+3y+6z=1$

$\therefore \frac{2+\lambda}{1}=\frac{-5+\lambda}{3}=\frac{1+4\lambda}{6}$

$\Rightarrow \lambda=-\frac{11}{2}$

Hence equation of the required plane is

$$\left(2-\frac{11}{2}\right)x+\left(-5-\frac{11}{2}\right)y+\left(1-\frac{44}{2}\right)z+\left(-3+\frac{55}{2}\right)=0$$

$\Rightarrow (4-11)x+(-10-11)y+(2-44)z+(-6+55)=0$

$\Rightarrow -7x-21y-42z+49=0$

$\Rightarrow x+3y+6z-7=0$

$\Rightarrow x+3y+6z=7$

99. **(b)** General point on given line $\equiv P(3r+2, 4r-1, 12r+2)$

Point P must satisfy equation of plane

$(3r+2)-(4r-1)+(12r+2)=16$

$11r+5=16$

$r=1$

$P(3\times 1+2, 4\times 1-1, 12\times 1+2)=P(5, 3, 14)$

distance between P and (1, 0, 2)

$D=\sqrt{(5-1)^2+3^2+(14-2)^2}=13$

100. **(b)** The equation of any plane passing through given line is

$(x+y+2z-3)+\lambda(2x+3y+4z-4)=0$

$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(2+4\lambda)z-(3+4\lambda)=0$

If this plane is parallel to z–axis then normal to the plane will be perpendicular to z–axis.

$\therefore (1+2\lambda)(0)+(1+3\lambda)(0)+(2+4\lambda)(1)=0$

$\lambda=-\frac{1}{2}$

Thus, Required plane is

$(x+y+2z-3)-\frac{1}{2}(2x+3y+4z-4)=0 \Rightarrow y+2=0$

$\therefore$ S.D $=\frac{2}{\sqrt{(1)^2}}=2$

101. **(c)** Equation of the plane containing the given line

$\frac{x-1}{1}=\frac{y-2}{5}=\frac{z-3}{4}$ is

$A(x-1)+B(y-2)+C(z-3)=0$ ...(i)

where $A+5B+4C=0$ ...(ii)

Since the point (3, 2, 0) contains in the plane (i), therefore

$2A+0.B-3C=0$ ...(iii)

From equations (ii) and (iii),

$\frac{A}{-15-0}=\frac{B}{6+3}=\frac{C}{0-10}=k$ (let)

$\Rightarrow A=-15k, B=9k$ and $C=-10k$

Putting the value of A, B and C in equation (i), we get

$-15(x-1)+9(y-2)-10(z-3)=0$...(iv)

Now the coordinates of the point (0, –3, 1) satisfy the equation of the plane (iv) as

$-15(0-1)+9(-3-2)-10(1-3)$

$=15-45+20=0$

Hence the point (0, –3, 1) contains in the plane.

102. **(c)** $|3+4-12\lambda+13|=|-9+0-12+13|$

$\Rightarrow |-12\lambda+20|=|8| \Rightarrow |3\lambda-5|=2$

$\Rightarrow 9\lambda^2+25-30\lambda=4 \Rightarrow 9\lambda^2-30\lambda+21=0$

$\Rightarrow 3\lambda^2-10\lambda+7=0$

103. **(c)** Plane passing through $x+y+z+1=0$ and $2x-y+z+3=0$ is $x+y+z+1+\lambda(2x-y+z+3)=0$

$\Rightarrow (2\lambda+1)x+(1-\lambda)y+(1+\lambda)z+3\lambda+1=0$

Parallel to the given line if

$\alpha(2\lambda+1)-1(1-\lambda)+1(1+\lambda)=0$

$\Rightarrow \alpha=\frac{-2\lambda}{2\lambda+1}$ ...(i)

Also, $\left|\frac{2\lambda+1-(1-\lambda)+0+3\lambda+1}{\sqrt{(2\lambda+1)^2+(1-\lambda)^2+(1+\lambda)^2}}\right|=\frac{1}{\sqrt{3}}$

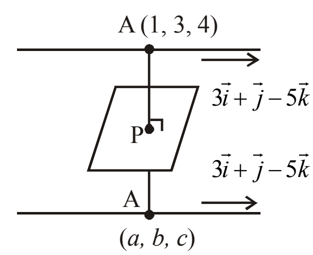
$\Rightarrow \lambda=0, \frac{-32}{102}; \alpha=0$ or $\alpha=\frac{32}{19}$

104. **(c)** $\frac{a-1}{2}=\frac{b-3}{-1}=\frac{c-4}{1}=\lambda$(let)

$\Rightarrow a=2\lambda+1$

$b=3-\lambda$

$c=4+\lambda$



$P=\left(\frac{a+1}{2},\frac{b+3}{2},\frac{c+4}{2}\right)=\left(\lambda+1,\frac{6-\lambda}{2},\frac{\lambda+8}{2}\right)$

$\therefore\ 2(\lambda+1)-\frac{6-\lambda}{2}+\frac{\lambda+8}{2}+3=0$

$3\lambda+6=0 \Rightarrow \lambda=-2$

$a=-3, b=5, c=2$

Required line is $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

105. **(c)** Given equation of line can be written as

$\frac{x+1}{1}=\frac{y}{2}=\frac{z+4}{2}$

Eqn of plane is $2x-y+\sqrt{\lambda}z+4=0$

Since, angle between the line and the plane is $\frac{\pi}{6}$ therefore

$\sin\frac{\pi}{6}=\frac{2(1)+2(-1)+2(\sqrt{\lambda})}{\sqrt{1+4+4}\sqrt{4+1+\lambda}}$

$$\frac{1}{2}=\frac{2-2+2\sqrt{\lambda}}{\sqrt{9}\sqrt{5+\lambda}}$$

$$\Rightarrow \frac{\sqrt{\lambda}}{\sqrt{5+\lambda}}=\frac{3}{4}\Rightarrow\frac{\lambda}{5+\lambda}=\frac{9}{16}$$

$$\Rightarrow 7\lambda = 45 \Rightarrow \lambda=\frac{45}{7}$$

106. (c) Given planes are

$4x-2y-4z+1=0$

and $4x-2y-4z+d=0$

They are parallel.

Distance between them is $\pm 7=\frac{d-1}{\sqrt{16+4+16}}$

$\Rightarrow \frac{d-1}{6}=\pm 7 \Rightarrow d=42+1$

or $-42+1$ i.e. $d=-41$ or 43.

107. (b) Given two planes :

$x-ay-b=0$ and $cy-z+d=0$

Let, l, m, n be the direction ratio of the required line. Since the required line is perpendicular to normal of both the plane, therefore $l-am=0$ and $cm-n=0$

$\Rightarrow l-am+0.n=0$ and $0.l+cm-n=0$

$\therefore \frac{l}{a-0}=\frac{m}{0+1}=\frac{n}{c-0}$

Hence, $d.R$ of the required line are a, 1, c.

Hence, options (c) and (d) are rejected.

Now, the point $(a+b, 1, c+d)$ satisfy the equation of the two given planes.

$\therefore$ Option (b) is correct.

108. (b) Equation of the plane containing the line

$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ is

$a(x-1)+b(y-2)+c(z-3)=0$ .... (i)

where $a.1+b.2+c.3=0$

i.e., $a+2b+3c=0$ .... (ii)

Since the plane (i) parallel to the line

$\frac{x}{1}=\frac{y}{1}=\frac{z}{4}$

$\therefore a.1+b.1+c.4=0$

i.e., $a+b+4c=0$ ...... (iii)

From (ii) and (iii),

$\frac{a}{8-3}=\frac{b}{3-4}=\frac{c}{1-2}=k$ (let)

$\therefore a=5k,\ b=-k,\ c=-k$

On putting the value of a, b and c in equation (i),

$5(x-1)-(y-2)-(z-3)=0$

$\Rightarrow 5x-y-z=0$ .... (iv)

when $x=1$, $y=0$ and $z=5$; then

L.H.S. of equation (iv) $=5x-y-2$

$=5\times 1-0-5=0$

= R.H.S. of equation (iv)

Hence coordinates of the point (1, 0, 5) satisfy the equation plane represented by equations (iv),

Therefore the plane passes through the point (1,0,5)

109. (c) Given equation of lines are

$\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ ...(1)

and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ ...(2)

Any point on line (1) is P $(3\lambda+1, \lambda+2, 2\lambda+3)$ and on line (2) is Q $(\mu+3, 2\mu+1, 3\mu+2)$.

On solving $3\lambda+1=\mu+3$ and $\lambda+2=2\mu+1$

we get $\lambda=1$, $\mu=1$

$\therefore$ Point of intersection of two lines is R (4, 3, 5)

So, equation of plane $\perp$ to OR where O is (0, 0, 0) and passing through R is

$4x+3y+5z=50$

110. (c) $2x+y+2z-8=0$ ....(Plane 1)

$2x+y+2z+\frac{5}{2}=0$ ....(Plane 2)

Distance between Plane 1 and 2

$$=\left|\frac{-8-\frac{5}{2}}{\sqrt{2^2+1^2+2^2}}\right|=\left|\frac{-21}{6}\right|=\frac{7}{2}$$

111. (c) Equation of a plane through the line of intersection of the planes

$x+2y=3, y-2z+1=0$ is

$(x+2y-3)+\lambda(y-2z+1)=0$

$\Rightarrow x+(2+\lambda)y-2\lambda(z)-3+\lambda=0$ (i)

Now, plane (i) is $\perp$ to $x+2y=3$

$\therefore$ Their dot product is zero

i.e. $1+2(2+\lambda)=0 \Rightarrow \lambda=-\frac{5}{2}$

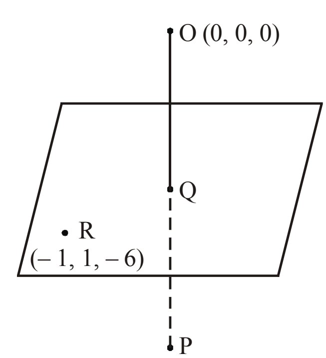
Thus, required plane is

$$x+\left(2-\frac{5}{2}\right)y-2\times\frac{-5}{2}(z)-3-\frac{5}{2}=0$$

$$\Rightarrow x-\frac{y}{2}+5z-\frac{11}{2}=0$$

$$\Rightarrow 2x-y+10z-11=0$$

112. (c) Let P be the image of O in the given plane.



Equation of the plane, $4x - 3y + z + 13 = 0$

OP is normal to the plane, therefore direction ratio of OP are proportional to 4, – 3, 1

Since OP passes through (0, 0, 0) and has direction ratio proportional to 4, –3, 1. Therefore equation of OP is

$$\frac{x-0}{4} = \frac{y-0}{-3} = \frac{z-0}{1} = r \text{ (let)}$$

$\therefore x = 4r, y = -3r, z = r$

Let the coordinate of P be $(4r, -3r, r)$

Since Q be the mid point of OP

$$\therefore Q = \left(2r, -\frac{3}{2}r, \frac{r}{2}\right)$$

Since Q lies in the given plane

$4x - 3y + z + 13 = 0$

$$\therefore 8r + \frac{9}{2}r + \frac{r}{2} + 13 = 0$$

$$\Rightarrow r = \frac{-13}{8 + \frac{9}{2} + \frac{1}{2}} = \frac{-26}{26} = -1$$

$$\therefore Q = \left(-2, \frac{3}{2}, -\frac{1}{2}\right)$$

$$QR = \sqrt{(-1+2)^2 + \left(1 - \frac{3}{2}\right)^2 + \left(-6 + \frac{1}{2}\right)^2}$$

$$= \sqrt{1 + \frac{1}{4} + \frac{121}{4}} = 3\sqrt{\frac{7}{2}}$$

113. **(b)** Direction cosines of $\vec{n}$ are $\frac{1}{2}, \frac{1}{4}, \frac{1}{2}$.

Equation of the plane,

$$\frac{1}{2}(x - \sqrt{2}) + \frac{1}{4}(y+1) + \frac{1}{2}(z-1) = 0$$

$$\Rightarrow 2(x - \sqrt{2}) + (y+1) + 2(z-1) = 0$$

$$\Rightarrow 2x + y + 2z = 2\sqrt{2} - 1 + 2$$

$$\Rightarrow 2x + y + 2z = 2\sqrt{2} + 1$$

114. **(a)** Given that, equation of a plane is

$x - 2y + 2z - 5 = 0$

So, Equation of parallel plane is

$x - 2y + 2z + d = 0$

Now, it is given that distance from origin to the parallel plane is 1.

$$\therefore \left|\frac{d}{\sqrt{1^2 + 2^2 + 2^2}}\right| = 1 \Rightarrow d = \pm 3$$

So equation of required plane

$x - 2y + 2z \pm 3 = 0$

115. **(a)** The equation of the plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ is a $(x + 1) + b\,(y - 3) + c\,(z + 2) = 0$ where

$-3a + 2b + c = 0$ ...(A)

This passes through (0, 7, – 7)

$\therefore a\,(0 + 1) + b\,(7 - 3) + c\,(-7 + 2) = 0$

$\Rightarrow a + 4b - 5c = 0$ ...(B)

On solving equation (A) and (B) we get

$a = 1, b = 1, c = 1$

$\therefore$ Required plane is

$x + 1 + y - 3 + z + 2 = 0$

$\Rightarrow x + y + z = 0$

116. **(d)** Given planes are

$P : x + y - 2z + 7 = 0$

$Q : x + y + 2z + 2 = 0$

and $R : 3x + 3y - 6z - 11 = 0$

Consider Plane P and R.

Here $a_1 = 1, b_1 = 1, c_1 = -2$

and $a_2 = 3, b_2 = 3, c_2 = -6$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{3}$

therefore P and R are parallel.

117. **(b)** Let the direction ratios of the common line be l, m and n.

$\therefore l \times 1 + m \times 0 + n \times 0 = 0 \Rightarrow l = 0$ ....(1)

$2l - 5ma + 3n = 0 \Rightarrow 5ma - 3n = 0$ ....(2)

$3lb + m - 3n = 0 \Rightarrow m - 3n = 0$ ....(3)

Subtracting (3) from (1), we get

$m(5a - 1) = 0$

Now, value of m can not be zero because if $m = 0$ then $n = 0$

$\Rightarrow l = m = n = 0$ which is not possible.

Hence, $5a - 1 = 0 \Rightarrow a = \frac{1}{5}$

Thus, option (b) is correct.

118. **(c)** Point P is $(2, -1, 2)$

Let this line meet at $Q\,(h, k, w)$

Direction ratio of this line is

$(h - 2, k + 1, w - 2)$

Since, dc_s are equal & dr_s are also equal,

So, $h - 2 = k + 1 + w - 2$

$\Rightarrow k = h - 3$ and $w = h$

This line meets the plane

$2x + y + z = 9$ at Q, so,

$2h + k + w = 9$ or $2h + h - 3 + h = 9$

$\Rightarrow 4h - 3 = 9 \Rightarrow h = 3$

and $k = 0$ and $w = 3$

Distance

$$PQ = \sqrt{(3-2)^2 + (0-(-1))^2 + (3-2)^2}$$

$= \sqrt{1^2+1^2+1^2} = \sqrt{3}$

119. (d) Given equation of plane is

$3x+4y-12z+13=0$

$(1, a, 1)$ and $(-3, 0, a)$ satisfy the equation of plane.

$\therefore$ We have

$3+4(a)-12+13=0$ and $\quad 3(-3)-12(a)+13=0$

$\Rightarrow\ 4+4a=0$ and $4-12a=0$

$\Rightarrow\ a=-1$ and $a=\frac{1}{3}$

Since, $(1, a, 1)$ and $(-3, 0, a)$ lie on the opposite sides of the plane $\therefore a=0$

120. (a) Equation of line through $P(1,-5,9)$ and parallel to the line $x=y=z$ is

$\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda(say)$

$Q=(x=1+\lambda, y=-5+\lambda, z=9+\lambda)$

Since Q lies on plane $x-y+z=5$

$\therefore\ 1+\lambda+5-\lambda+9+\lambda=5$

$\Rightarrow\ \lambda=-10$

$\therefore\ Q=(-9,-15,-1)$

$\therefore\ PQ=\sqrt{(1+9)^2+(15-5)^2+(9+1)^2}$

$=\sqrt{300}=10\sqrt{3}$

121. (d) Let θ be the angle between the given line and plane, then

$$\sin\theta=\frac{1\times1+2\times2+\lambda\times3}{\sqrt{1^2+2^2+\lambda^2}.\sqrt{1^2+2^2+3^2}}=\frac{5+3\lambda}{\sqrt{14}.\sqrt{5+\lambda^2}}$$

$$\Rightarrow\ \cos\theta=\sqrt{1-\frac{(5+3\lambda)^2}{14(5+\lambda^2)}}$$

$$\Rightarrow\ \sqrt{\frac{5}{14}}=\sqrt{1-\frac{(5+3\lambda)^2}{14(5+\lambda^2)}}$$

Squaring both sides, we get

$$\frac{5}{14}=\frac{5\lambda^2-30\lambda+45}{14(5+\lambda^2)}$$

$\Rightarrow\ \lambda=\frac{2}{3}$

122. (a) $A(3, 1, 6); B=(1, 3, 4)$

Putting coordinate of mid-point of $AB=(2, 2, 5)$ in plane $x-y+z=5$ then $2-2+5=5$, satisfy

So, mid-point of $AB=(2, 2, 5)$ lies on the plane.

d.r's of $AB=(2,-2,2)$

d.r's of normal to plane $=(1,-1,1)$.

Direction ratio of AB and normal to the plane are proportional therefore,

AB is perpendicular to the normal of plane

$\therefore$ A is image of B

Statement-1 is correct.

Statement-2 is also correct but it is not correct explanation.

123. (a) Given that, the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$

$\therefore$ Pt $(2, 1, -2)$ lies on the plane

i.e. $2+3+2\alpha+\beta=0$

or $\quad 2\alpha+\beta+5=0 \qquad$ ....(i)

Also normal to plane will be perpendicular to line,

$\therefore\ 3\times1-5\times3+2\times(-\alpha)=0$

$\Rightarrow\ \alpha=-6$

From equation (i) then, $\beta=7$

$\therefore (\alpha, \beta)=(-6, 7)$

124. (c) Equation of line through $(5, 1, a)$ and $(3, b, 1)$ is $\frac{x-5}{-2}=\frac{y-1}{b-1}=\frac{z-a}{1-a}=\lambda$

$x=-2\lambda+5$

$y=(b-1)\lambda+1$

$z=(1-a)\lambda+a$

$\therefore$ Any point on this line is a

$[-2\lambda+5, (b-1)\lambda+1, (1-a)\lambda+a]$

Given that it crosses yz plane $\therefore -2\lambda+5=0$

$\lambda=\frac{5}{2}$

$$\therefore \left(0, (b-1)\frac{5}{2}+1, (1-a)\frac{5}{2}+a\right)=\left(0, \frac{17}{2}, \frac{-13}{2}\right)$$

$\Rightarrow\ (b-1)\frac{5}{2}+1=\frac{17}{2}$

and $(1-a)\frac{5}{2}+a=-\frac{13}{2}$

$\Rightarrow\ b=4$ and $a=6$

125. (c) Let the direction cosines of line L be l, m, n. Since line L lies on both planes.

$\therefore\ 2l+3m+n=0 \qquad$ ....(i)

and $l+3m+2n=0 \qquad$ ....(ii)

on solving equation (i) and (ii), we get

$\frac{l}{6-3}=\frac{m}{1-4}=\frac{n}{6-3} \Rightarrow \frac{l}{3}=\frac{m}{-3}=\frac{n}{3}$

Now $\frac{l}{3}=\frac{m}{-3}=\frac{n}{3}=\frac{\sqrt{l^2+m^2+n^2}}{\sqrt{3^2+(-3)^2+3^2}}$

$\because\ l^2+m^2+n^2=1$

$\therefore\ \frac{l}{3}=\frac{m}{-3}=\frac{n}{3}=\frac{1}{\sqrt{27}}$

$\Rightarrow\ l=\frac{3}{\sqrt{27}}=\frac{1}{\sqrt{3}}, m=-\frac{1}{\sqrt{3}}, n=\frac{1}{\sqrt{3}}$

Line L, makes an angle α with +ve x-axis

$\therefore\ l=\cos\alpha \Rightarrow \cos\alpha=\frac{1}{\sqrt{3}}$

126. (d) Let (α, β, γ) be the image, then mid point of (α, β, γ) and $(-1, 3, 4)$ must lie on $x - 2y = 0$

$$\therefore \frac{\alpha - 1}{2} - 2\left(\frac{\beta + 3}{2}\right) = 0$$

$$\therefore \alpha - 1 - 2\beta - 6 = 0 \Rightarrow \alpha - 2\beta = 7 \quad \text{....(1)}$$

Also line joining (α, β, γ) and $(-1, 3, 4)$ should be parallel to the normal of the plane $x - 2y = 0$

$$\therefore \frac{\alpha+1}{1} = \frac{\beta-3}{-2} = \frac{\gamma-4}{0} = \lambda$$

$$\Rightarrow \alpha = \lambda - 1, \beta = -2\lambda + 3, \gamma = 4 \quad \text{....(2)}$$

From (1) and (2)

$$\alpha = \frac{9}{5}, \ \beta = -\frac{13}{5}, \gamma = 4$$

None of the option matches.

127. (b) The given line is $\vec{r} = 2\vec{i} - 2\vec{j} + 3\vec{k} + \lambda(\vec{i} - \vec{j} + 4\vec{k})$

and the plane is $\vec{r} \cdot (\vec{i} + 5\vec{j} + \vec{k}) = 5$

$\Rightarrow x + 5y + z = 5$

$$\text{Required distance} = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

$$= \left|\frac{2 - 10 + 3 - 5}{\sqrt{1 + 25 + 1}}\right| = \frac{10}{3\sqrt{3}}$$

128. (a) Let θ is the angle between line and plane then

$$\sin\theta = \frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$$

$$= \frac{(\hat{i} + 2\hat{j} + 2\hat{k}).(2\hat{i} - \hat{j} + \sqrt{\lambda}\hat{k})}{\sqrt{1+4+4}\sqrt{4+1+\lambda}} = \frac{2 - 2 + 2\sqrt{\lambda}}{3 \times \sqrt{5+\lambda}}$$

$$\Rightarrow \sin\theta = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}} = \frac{1}{3} \Rightarrow 4\lambda = 5 + \lambda$$

$$\Rightarrow \lambda = \frac{5}{3}.$$

129. (c) The planes are $2x + y + 2z - 8 = 0$ ...(1)

and $4x + 2y + 4z + 5 = 0$

or $2x + y + 2z + \frac{5}{2} = 0$ ...(2)

Since, both planes are parallel

$\therefore$ Distance between (1) and (2)

$$= \left|\frac{\frac{5}{2} + 8}{\sqrt{2^2 + 1^2 + 2^2}}\right| = \left|\frac{21}{2\sqrt{9}}\right| = \frac{7}{2}$$

130. (a) Equation of planes in intercept form be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

& $\frac{x}{a'} + \frac{y}{b'} + \frac{z}{c'} = 1$

($\perp r$ distance on plane from origin is same.)

$$\left|\frac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}\right| = \left|\frac{-1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}}\right|$$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$$

131. (b) Equation of plane through (1, 0, 0) is

$a(x - 1) + by + cz = 0$ ...(i)

It is also passes through (0, 1, 0).

$\therefore -a + b = 0 \Rightarrow b = a;$

$$\cos 45° = \frac{a + a}{\sqrt{2(2a^2 + c^2)}}$$

$$\Rightarrow 2a = \sqrt{2a^2 + c^2} \Rightarrow 2a^2 = c^2 \Rightarrow c = \sqrt{2}a.$$

So d.r of normal are a, a $\sqrt{2}a$ i.e. 1, 1, $\sqrt{2}$.

132. (a) Since the point (3, 2, 0) lies on the given line

$$\frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4}$$

$\therefore$ There can be infinite many planes passing through this line. We observed that only option (a) is satisfied by the coordinates of both the points (3, 2, 0) and (4, 7, 4)

$\therefore x - y + z = 1$ is the required plane.

133. (c) We know that centre of sphere

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

is $(-u, -v, -w)$

Given that, $x^2+y^2+z^2-6x-12y-2z+20=0$

$\therefore$ Centre $\equiv$ (3, 6, 1)

Coordinates of one end of diameter of the sphere are (2, 3, 5).

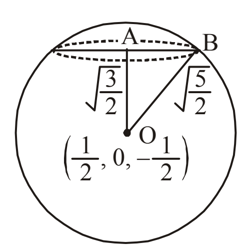
Let the coordinates of the other end of diameter are (α, β, γ)

$$\therefore \frac{\alpha+2}{2} = 3, \frac{\beta+3}{2} = 6, \frac{\gamma+5}{2} = 1$$

$\Rightarrow \alpha = 4, \beta = 9$ and $\gamma = -3$

$\therefore$ Coordinate of other end of diameter are (4, 9, –3)

134. (b)



Centre of sphere $= \left(\frac{1}{2}, 0, -\frac{1}{2}\right)$ and radius of sphere

$$= \sqrt{\frac{1}{4} + \frac{1}{4} + 2} = \sqrt{\frac{5}{2}}$$

Perpendicular distance OA of centre from $x+2y-z=4$ is given by

$$\frac{\left|\frac{1}{2}+\frac{1}{2}-4\right|}{\sqrt{6}}=\sqrt{\frac{3}{2}}$$

$\therefore$ radius of circle

$$AB=\sqrt{OB^2-OA^2}=\sqrt{\frac{5}{2}-\frac{3}{2}}=1.$$

135. **(c)** Plane $2ax-3ay+4az+6=0$ passes through the mid point of the line joining the centres of spheres

$x^2+y^2+z^2+6x-8y-2z=13$ and

$x^2+y^2+z^2-10x+4y-2z=8$

respectively centre of spheres are $c_1(-3, 4, 1)$ and $c_2(5,-2, 1)$. Mid point of c_1c_2 is $(1, 1, 1)$.

Satisfying this in the equation of plane, we get

$2a-3a+4a+6=0$

$\Rightarrow\ a=-2.$

136. **(a)** Given that, the equations of spheres are

$S_1: x^2+y^2+z^2+7x-2y-z-13=0$ and

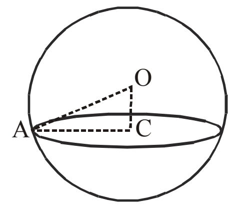
$S_2: x^2+y^2+z^2-3x+3y+4z-8=0$

We know that eqn. of intersection plane be

$S_1-S_2=0 \Rightarrow 10x-5y-5z-5=0$

$\Rightarrow\ 2x-y-z=1$

137. **(d)**



Centre of sphere $=(-1, 1, 2)$

Radius of sphere $\sqrt{1+1+4+19}=5$

Perpendicular distance from centre to the plane

$$OC=d=\left|\frac{-1+2+4+7}{\sqrt{1+4+4}}\right|=\frac{12}{3}=4.$$

In right, ΔAOC

$AC^2=AO^2-OC^2=5^2-4^2=9$

$\Rightarrow\ AC=3$

138. **(d)** Centre of sphere be $(-2, 1, 3)$ and radius 13

We know that,

Shortest distance = perpendicular distance between the plane and sphere = distance of plane from centre of sphere – radius

$$=\left|\frac{-2\times12+4\times1+3\times3-327}{\sqrt{144+9+16}}\right|-13$$

$=26-13=13$