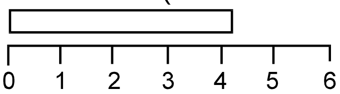
1. ERRORS IN MEASUREMENT AND SIGNIFICANT FIGURES

To get some overview of error and significant figures, lets consider the example given below.
Suppose we have to measure the length of a rod. How can we!

(a) Lets use a cm. scale: (a scale on which only cm. marks are there)



We will measure length = 4 cm.

Although the length will be a bit more than 4 , but we cannot say its length to be 4.1 cm or 4.2 cm., as the scale can measure upto cms only , not closer than that.

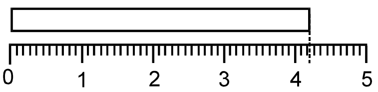
* It (this scale) can measure upto cms accuracy only.
* so we'll say that its ***least count**** is 1 cm.

To get a closer measurement,
We have to use a more minute scale, that is mm scale

*
Least count : Smallest quantity an instument can measure

- mm. scale L.C. = 1 mm
- Vernier L.C. =0.1 mm
- Screw gauge L.C. = 0.01 mm
- stop watch L.C. = 0.1 sec
- Temp thermometer L.C. = 1°C

(b) Lets use an mm scale : (a scale on which mm. marks are there)



We will measure length ℓ = 4.2 cm., which is a more closer measurement. Here also if we observe closely, we'll find that the length is a bit more than 4.2, but we cannot say its length to be 4.21, or 4.22, or 4.20 as this scale can measure upto 0.1 cms (1 mm) only, not closer than that.

* It (this scale) can measure upto 0.1 cm accuracy
Its ***least count*** is 0.1 cm.

Max ***uncertainty*** in ℓ can be = 0.1cm

Max possible error in ℓ can be = 0.1cm

Measurement of length = 4.2 cm. has two ***significant figures*** ; 4 and 2, in which 4 is absolutely correct, and 2 is reasonably correct (Doubtful) because uncertainty of 0.1 cm is there.

To get more closer measurement

(c) We can use Vernier callipers : (which can measure more closely , upto 0.01 cm)
Then we'll measure length ℓ = 4.23 cm which is more closer measurement.

* It can measure upto 0.01 cm accuracy

Least count = 0.01 cm   Max ***uncertainty*** in ℓ can be = 0.01cm

Max possible error in ℓ can be = 0.01cm

Measurement of length = 4.23 cm. has three ***significant figures*** ; 4 , 2 and 3, in which 4 and 2 are absolutely correct , and 3 is reasonable correct (Doubtful) because uncertainty of 0.01 cm is there.

To get further more closer measurement :

(d) We can use Screw Gauge : (which can measure more closely, upto 0.001 cm)
we'll measure length ℓ = 4.234 cm.

* ***Max possible uncertainty (error)*** in ℓ can be = 0.001 cm

* length = 4.234 cm. has four ***significant figures*** ; 4 , 2 , 3 and 4.

4	2	3	4
absolutely correct	absolutely correct	absolutely correct	Reasonably correct

To get further more closer measurement

(e) We can Use microscope :
we'll measure length ℓ = 4.2342 cm.

* ***Max possible uncertainty (error)*** in ℓ can be = 0.0001cm

* length = 4.2342cm. has five ***significant figures*** ; 4, 2, 3, 4 and 2

SIGNIFICANT FIGURES

From the above example, we can conclude that, in a measured quantity,

Significant figures are = Figures which are absolutely correct + The first uncertain figure

Common rules of counting significant figures :

Rule 1 :

All non-zero digits are significant

e.i. 123.56 has five S.F.

Rule 2 :

All zeros occurring between two non-zeros digits are significant (obviously)

e.i. 1230.05 has six S.F.

Rule 3 :

ℓ = 3.5 cm	ℓ = 3.50 cm	ℓ = 3.500 cm
$\downarrow$	$\downarrow$ closer!	$\downarrow$ more closer!
0.1 cm accuracy	0.01 cm accuracy	0.001 cm accuracy
$\downarrow$	$\downarrow$	$\downarrow$
ℓ lies between(3.4 – 3.6)	ℓ lies between (3.49 – 3.51)	ℓ lies between (3.499 – 3.501)

So trailing zeroes after decimal place are significant (Shows the further accuracy)

3.5 cm	3.50 cm	3.500 cm
↓	↓	↓ more closer!
Two S.F	Three S.F	Four S.F

Once a measurement is done, significant figures will be decided according to closeness of measurement. Now if we want to display the measurement in some different units, the S.F. shouldn't change (S.F. depends only on accuracy of measurement)

<u>Number of S.F. is always conserved, change of units cannot change S.F.</u>

Suppose measurement was done using mm scale, and we get ℓ = 85 mm (Two S. F.)

If we want to display it in other units.

85 mm ⟶ 8.5 cm ⟶ 0.085 m

85 mm ↓ 85000 μm = 8.5×10^4 μm

0.085 m ↓ 0.000085 km = 8.5×10^{-5} km

All should have two S.F.

The following rules support the conservation of S.F.

Rule 4:

From the previous example, we have seen that,

0.000085 km ⟶ also should has two S.F.; 8 and 5, So leading Zeros are not significant.

000 ↓ Not significant

<u>In the number less than one, all zeros after decimal point and to the left of first non-zero digit are insignificant (arises only due to change of unit)</u>

0.000305 has three S.F.

⇒ 3.05×10^{-4} has three S.F.

Rule 5 :

From the previous example, we have also seen that

85000 μm ⟶ should also has two S.F., 8 and 5. So the trailing zeros are also not significant.

000 ↓ Not significant

<u>The terminal or trailing zeros in a number without a decimal point are not significant. (Also arises only due to change of unit)</u>

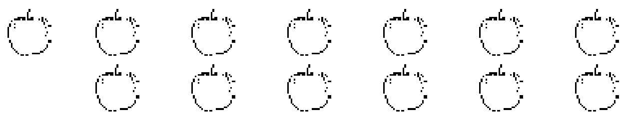
154 m = 15400 cm = 15400 mm

= 154×10^9 nm

All has only three S.F. all trailing zeros are insignificant

Rule 6 :

There are certain measurement, which are exact i.e.



Number of apples are = 12 (exactly) = 12.000000........... ∞

This type of measurement is infinitely accurate so, it has ∞ S.F.

* Numbers of students in class = 125 (exact)

* Speed of light in the vacuum = 299,792,458 m/s (exact)

Solved Examples

Example 1. Count total number of S.F. in 3.0800
Solution : S.F. = Five , as trailing zeros after decimal place are significant.

Example 2. Count total number of S.F. in 0.00418
Solution : S.F. = Three, as leading zeros are not significant.

Example 3. Count total number of S.F. in 3500

Solution : S.F. = Two, the trailing zeros are not significant.

Example 4. Count total number of S.F. in 300.00

Solution : S.F. = Five, trailing zeros after decimal point are significant.

Example 5. Count total number of S.F. in 5.003020

Solution : S.F. = Seven, the trailing zeros after decimal place are significant.

Example 6. Count total number of S.F. in 6.020×10^{23}

Solution : S.F. = Four ; 6, 0, 2, 0 ; remaining 23 zeros are not significant.

Example 7. Count total number of S.F. in 1.60×10^{-19}

Solution : S.F. = Three ; 1, 6, 0 ; remaining 19 zeros are not significant.

Solve the following questions

1. Find significant figures in the following observations -
(i) 0.007 gm (ii) 2.64×10^{24} kg (iii) 0.2370 gm/cm^3 (iv) 6.320 J/K
(v) 6.032 N/m^2 (vi) 0.0006032 K^{-1}

Sol.

2. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are
(A) 5, 1, 2 (B) 5, 1, 5 (C) 5, 5, 2 (D) 4, 4, 2

Sol.

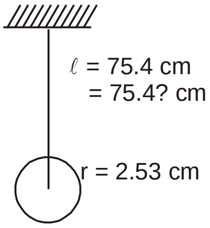
OPERATIONS ACCORDING TO SIGNIFICANT FIGURES

Now lets see how to do arithmetic operations ie. addition, subtraction, multiplication and division according to significant figures

(a) Addition $\longleftrightarrow$ subtraction

For this, lets consider the example given below.

In a simple pendulum, length of the thread is measured (from mm scale) as 75.4 cm. and the radius of the bob is measured (from vernier) as 2.53 cm.



Find $\ell_{eq} = \ell + r$

ℓ is known upto 0.1 cm (first decimal place) only. We don't know what is at the next decimal place. So we can write ℓ = 75.4 cm = 75.4? cm and the radius r = 2.53 cm .

If we add ℓ and r, we don't know which number will be added with 3. So we have to leave that position.

ℓ_{eq} = 75.4? + 2.53 = 77.9? cm = 77.9 cm

Rules for Addition $\longleftrightarrow$ subtraction : (based on the previous example)

* First do the addition/subtraction in normal manner.
* Then round off all quantities to the decimal place of least accurate quantity.

i.e.

423.5 + 20.23 + 10.15

$$\begin{array}{r} 423.5 \\ +\ 20.23 \\ +\ 10.15 \\ \hline 453.8\textcircled{8} \end{array} \xrightarrow[\text{decimal place}]{\text{Round off to one}} 453.9$$

486.2 – 35.18

$$\begin{array}{r} 486.2 \\ -\ 35.18 \\ \hline 451.0\textcircled{2} \end{array} \xrightarrow[\text{decimal place}]{\text{Round off to one}} 451.0$$

Rules for Multiply $\longleftrightarrow$ Division

Suppose we have to multiply 2.11 × 1.2 = 2.11 ? × 1.2 ?

```
     2 . 11 ?
     x  1.2 ?
     ?   ? ? ?
   4  2  2 ? x
 2 1 1  ? x x
 2.5  ?  ? ? ?   = 2.5
```

So answer will come in least significant figures out of the two numbers.

- ✪ Multiply divide in normal manner.
- ✪ Round off the answer to the weakest link (number having least S.F.)

312.65 × 26.4 = 8253.960

5 S.F. 3 S.F

$$8253.960 \xrightarrow{\text{round off to three S.F.}} 8250$$

Rules of Rounding off

✪ If removable digit is less than 5 (50%) ; drop it.

$\underline{\mathbf{47.8}}33 \xrightarrow[\text{till one decimal place}]{\text{Round off}} 47.8$

✪ If removable digit is greater than 5(50%), increase the last digit by 1.

$\underline{\mathbf{47.8}}62 \xrightarrow[\text{till one decimal place}]{\text{Round off}} 47.9$

✪ If removable number is exactly 5(50%)

- If last number is even drop 5: $\underline{20.6}5 \rightarrow 20.6$
- If last number is odd, increase the last digit by 1: $\underline{20.7}5 \rightarrow 20.8$

Solved Example

Example 8. A cube has a side $\ell = 1.2 \times 10^{-2}$ m. Calculate its volume

Solution : $\ell = 1.2 \times 10^{-2}$

$V = \ell^3 = \underset{\text{Two S.F.}}{(1.2 \times 10^{-2})}\ \underset{\text{Two S.F.}}{(1.2 \times 10^{-2})}\ \underset{\text{Two S.F.}}{(1.2 \times 10^{-2})}$

$= 1.728 \times 10^{-6}\ m^3$

Round off to 2 S.F.

$= 1.7 \times 10^{-6}\ m^3$ **Ans.**

Example 9. In ohm's law exp., reading of voltmeter across the resistor is 12.5 V and reading of current i = 0.20 Amp. Estimate the resistance in correct S.F.

Solution : $R = \frac{V}{i} = \frac{12.5 \rightarrow 3\text{ SF}}{0.20 \rightarrow 2\text{ SF}} = 62.5\ \Omega \xrightarrow[\text{to 2 S.F.}]{\text{round off}} 62\ \Omega$

Example 10. Using screw gauge radius of wire was found to be 2.50 mm. The length of wire found by mm. scale is 50.0 cm. If mass of wire was measured as 25 gm, the density of the wire in correct S.F. will be
(use π = 3.14 exactly)

Solution : $\rho = \frac{m}{\pi r^2 \ell} = \frac{\overset{\text{(two S.F.)}}{25}}{\pi \underset{\text{three S.F}}{(0.250)^2} \underset{\text{three S.F.}}{(50.0)}} = \underline{2.5}465 \xrightarrow[\text{S.F.}]{\text{two}} 2.5\ \text{gm/cm}^3$

Solve the following questions

3. The mass of a ball is 1.76 kg. The mass of 25 such balls is

(A) 0.44×10^3 kg (B) 44.0 kg (C) 44 kg (D) 44.00 kg

Sol.

4. The edge of a cube is a = 1.2×10^{-2} m. Then its volume will be recorded as :

(A) 1.72×10^{-6} m^3 (B) 1.728×10^{-6} m^3

(C) 1.7×10^{-6} m^3 (D) 1.73×10^{-6} m^3

Sol.

5. Round off the following numbers within three significant figures -

(i) 0.03927 kg (ii) 4.085×10^8 sec (iii) 5.2354 m (iv) 4.735×10^{-6} kg

Sol.

2. PERMISSIBLE ERROR

Error in measurement due to the limitation (least count) of the instrument, is called permissible error.

From mm scale → we can measure upto 1 mm accuracy (least count = 1mm). From this we will get measurement like ℓ = 34 mm

↓

Max uncertainty can be 1 mm.

Max permissible error ($\Delta\ell$) = 1 mm.

But if from any other instrument, we get ℓ = 34.5 mm then max permissible error ($\Delta\ell$) = 0.1 mm and if from a more accurate instrument, we get ℓ = 34.527 mm then max permissible error ($\Delta\ell$) = 0.001 mm = place value of last number

Max permissible error in a measured quantity is = least count of the measuring instrument and if nothing is given about least count then Max permissible error = place value of the last number

MAX.PERMISSIBLE ERROR IN RESULT DUE TO ERROR IN EACH EASURABLE QUANTITY :

Let Result f(x, y) contains two measurable quantity x and y

Let error in x is = $\pm \Delta x$ i.e. $x \in (x - \Delta x, x + \Delta x)$

error in y is = $\pm \Delta y$ i.e. $y \in (y - \Delta y, y + \Delta y)$

Case : (I) If f(x, y) = x + y

df = dx + dy

error in f = $\Delta f = \pm \Delta x \pm \Delta y$

max possible error in f = $(\Delta f)_{max}$ = max of $(\pm \Delta x \pm \Delta y)$

$(\Delta f)_{max} = \Delta x + \Delta y$

Case : (II) If $f = x - y$

$df = dx - dy$

$(\Delta f) = \pm \Delta x \mp \Delta y$

max possible error in f = $(\Delta f)_{max}$ = max of $(\pm \Delta x \mp \Delta y)$

$\Rightarrow \quad (\Delta f)_{max} = \Delta x + \Delta y$

For getting maximum permissible error , sign should be adjusted, so that errors get added up to give maximum effect

i.e. $f = 2x - 3y - z$

$(\Delta f)_{max} = 2\Delta x + 3\Delta y + \Delta z$

Case (III) If $f(x, y, z) = (\text{constant})\, x^a y^b z^c$

to scatter all the terms, Lets take log on both sides

$\ell n\, f = \ell n\,(\text{constant}) + a\,\ell n\, x + b\,\ell n\, y + c\,\ell n\, z$

$\downarrow$ Differentiating both sides

$$\frac{df}{f} = 0 + a\frac{dx}{x} + b\frac{dy}{y} + c\frac{dz}{z}$$

$$\frac{\Delta f}{f} = \pm a\frac{\Delta x}{x} \pm b\frac{\Delta y}{y} \pm c\frac{\Delta z}{z}$$

$$\left(\frac{\Delta f}{f}\right)_{max} = \text{max of } \left(\pm a\frac{\Delta x}{x} \pm b\frac{\Delta y}{y} \pm c\frac{\Delta z}{z}\right)$$

i.e., $f = 15\, x^2\, y^{-3/2}\, z^{-5}$

$$\frac{df}{f} = 0 + 2\frac{dx}{x} - \frac{3}{2}\frac{dy}{y} - 5\frac{dz}{z}$$

$$\frac{\Delta f}{f} = \pm 2\frac{\Delta x}{x} \mp \frac{3}{2}\frac{\Delta y}{y} \mp 5\frac{\Delta z}{z}$$

$$\left(\frac{\Delta f}{f}\right)_{max} = \text{max of } \left(\pm 2\frac{\Delta x}{x} \mp \frac{3}{2}\frac{\Delta y}{y} \mp 5\frac{\Delta z}{z}\right)$$

$$\left(\frac{\Delta f}{f}\right)_{max} = 2\frac{\Delta x}{x} + \frac{3}{2}\frac{\Delta y}{y} + 5\frac{\Delta z}{z}$$

✪ ***sign should be adjusted, so that errors get added up***

Solved Examples

Example 11. In resonance tube exp. we find $\ell_1 = 25.0$ cm and $\ell_2 = 75.0$ cm. The least count of of the scale used to measure ℓ is 0.1 cm. If there is no error in frequency. What will be max permissible error in speed of sound (take $f_0 = 325$ Hz.)

Solution : $V = 2f_0\,(\ell_2 - \ell_1)$

$(dV) = 2f_0\,(d\ell_2 - d\ell_1)$

$(\Delta V)_{max}$ = max of $[2f_0(\pm \Delta\ell_2 \mp \Delta\ell_2] = 2f_0\,(\Delta\ell_2 + \Delta\ell_1)$

$\Delta\ell_1$ = least count of the scale = 0.1 cm

$\Delta\ell_2$ = least count of the scale = 0.1 cm

So max permissible error in speed of sound $(\Delta V)_{max}$ = 2(325Hz) (0.1 cm + 0.1 cm) = 1.3 m/s

Value of $V = 2f_0\,(\ell_2 - \ell_1)$ = 2(325Hz) (75.0 cm - 25.0 cm) = 325 m/s so V = (325 ± 1.3) m/s

Example 12. If measured value of resistance R = 1.05 Ω, wire diameter d = 0.60 mm, and length ℓ = 75.3 cm. If maximum error in resistance measurment is 0.01 Ω and least count of diameter and lenth measuring device are 0.01 mm and 0.1 cm respectively, then find max. permissible error in resistivity $\rho = \frac{R\left(\frac{\pi d^2}{4}\right)}{\ell}$

Solution : $\left(\frac{\Delta\rho}{\rho}\right)_{max} = \frac{\Delta R}{R} + 2\frac{\Delta d}{d} + \frac{\Delta\ell}{\ell}$

ΔR = 0.01 Ω, Δd = 0.01 mm (least count), $\Delta\ell$ = 0.1 cm (least count)

$\left(\frac{\Delta\rho}{\rho}\right)_{max} = \left(\frac{0.01\Omega}{1.05\,\Omega} + 2\frac{0.01mm}{0.60mm} + \frac{0.1cm}{75.3cm}\right) \times 100 = 4.3\ \%.$

Example 13. In ohm's law experiment, potential drop across a resistance was measured as v = 5.0 volt and current was measured as i = 2.0 amp. If least count of the voltmeter and ammeter are 0.1 V and 0.01 A respectively then find the maximum permissible error in resistance.

Solution : $R = \frac{v}{i} = v \times i^{-1} \Rightarrow \left(\frac{\Delta R}{R}\right)_{max} = \frac{\Delta v}{v} + \frac{\Delta i}{i}$

Δv = 0.1 volt (least count), Δi = 0.01 amp (least count)

$\%\left(\frac{\Delta R}{R}\right)_{max} = \left(\frac{0.1}{5.0} + \frac{0.01}{2.00}\right) \times 100\ \% = 2.5\ \%$

value of R from the observation $R = \frac{v}{i} = \frac{5.0}{2.00} = 2.5\ \Omega$

So we can write R = (2.5 ± 2.5%) Ω

Example 14. To find the value of 'g' using simple pendulum. T = 2.00 sec; ℓ = 1.00 m was measured. Estimate maximum permissible error in 'g'. Also find the value of 'g'. (Use π^2 = 10)

Solution : $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow g = \frac{4\pi^2\ell}{T^2}$

$\left(\frac{\Delta g}{g}\right)_{max} = \frac{\Delta\ell}{\ell} + 2\frac{\Delta T}{T} = \left(\frac{0.01}{1.00} + 2\frac{0.01}{2.00}\right) \times 100\ \%. \quad = 2\ \%$

value of $g = \frac{4\pi^2\ell}{T^2} = \frac{4\times10\times1.00}{(2.00)^2} = 10.0\ m/s^2$

$\left(\frac{\Delta g}{g}\right)_{max} = 2/100$ so $\frac{\Delta g_{max}}{10.0} = \frac{2}{100}$ so $(\Delta g)_{max} = 0.2$ = max error in 'g'

So 'g' = (10.0 ± 0.2) m/s^2

Solve the following questions :

6. The external and internal diameters of a hollow cylinder are measured to be (4.23 ± 0.01) cm and (3.89 ± 0.01) cm. The thickness of the wall of the cylinder is

(A) (0.34 ± 0.02) cm (B) (0.17 ± 0.02) cm (C) (0.17 ± 0.01) cm (D) (0.34 ± 0.01) cm

Sol.

7. The length of a rectangular plate is measured by a meter scale and is found to be 10.0 cm. Its width is measured by vernier callipers as 1.00 cm. The least count of the meter scale and vernier callipers are 0.1 cm and 0.01 cm respectively (Obviously). Maximum permissible error in area measurement is -

(A) $\pm$ 0.2 cm^2 (B) $\pm$ 0.1 cm^2 (C) $\pm$ 0.3 cm^2 (D) Zero

Sol.

8. In the previous question, minimum possible error in area measurement can be -

(A) $\pm$ 0.02 cm^2 (B) $\pm$ 0.01 cm^2 (C) $\pm$ 0.03 cm^2 (D) Zero

Sol.

9. For a cubical block, error in measurement of sides is $\pm$ 1% and error in measurement of mass is $\pm$ 2%, then maximum possible error in density is -

(A) 1% (B) 5% (C) 3% (D) 7%

Sol.

10. To estimate 'g' (from $g = 4\pi^2 \frac{L}{T^2}$), error in measurement of L is $\pm$ 2% and error in measurement of T is $\pm$ 3%. The error in estimated 'g' will be -

(A) $\pm$ 8% (B) $\pm$ 6% (C) $\pm$ 3% (D) $\pm$ 5%

Sol.

11. The least count of a stop watch is 0.2 second. The time of 20 oscillations of a pendulum is measured to be 25 seconds. The percentage error in the time period is

(A) 16% (B) 0.8 % (C) 1.8 % (D) 8 %

Sol.

12. The dimensions of a rectangular block measured with a vernier callipers having least count of 0.1 mm is 5 mm × 10 mm × 5 mm. The maximum percentage error in measurement of volume of the block is
(A) 5 % (B) 10 % (C) 15 % (D) 20 %

Sol.

13. An experiment measures quantities x, y, z and then t is calculated from the data as $t = \frac{xy^2}{z^3}$.
If percentage errors in x, y and z are respectively 1%, 3%, 2%, then percentage error in t is :
(A) 10 % (B) 4 % (C) 7 % (D*) 13 %

Sol.

14. A wire has a mass (0.3 ± 0.003)g, radius (0.5 ± 0.005)mm and length (6 ± 0.06)cm. The maximum percentage error in the measurement of its density is :
(A) 1 (B) 2 (C) 3 (D*) 4

Sol.

3. OTHER TYPES OF ERRORS :

1. Error due to external Causes :
These are the errors which arise due to reasons beyond the control of the experimentalist, e.g., change in room temperature, atmospheric pressure etc. A suitable correction can, however, be applied for these errors if the factors affecting the result are also recorded.

2. Instrumental errors :
Every instrument, however cautiously or manufactured, possesses imperfection to some extent. As a result of this imperfection, the measurements with the instrument cannot be free from errors. Errors, however small, do occur owing to the inherent manufacturing defects in the measuring instruments are called instrumental errors. These errors are of constant magnitude and suitable corrections can be applied for these errors. e.i., Zero errors in vernier callipers, and screw gauge, backlash errors etc

3. Personal or chance error :
Two observers using the same experiment set up, do not obtain exactly the same result. Even the observations of a single experimentalist differ when it is repeated several times by him or her. Such errors always occur inspire of the best and honest efforts on the part of the experimentalist and are known as personal errors. These errors are also called chance errors as they depend upon chance. The effect of the chance error on the result can be considerably reduced by taking a large number of observations and then taking their mean. How to take mean, is described in next point.

4. **Errors in averaging :**

Suppose to measure some quantity, we take several observations, a_1, a_2, a_3…. a_n. To find the absolute error in each measurement and percentage error, we have to follow these steps

(a) First of all mean of all the observations is calculated : $a_{mean} = (a_1 + a_2 + a_3 + \ldots + a_n) / n$. The mean of these values is taken as the best possible value of the quantity under the given conditions of measurements.

(b) Absolute Error :

The magnitude of the difference between the best possible or mean value of the quantity and the individual measurement value is called the absolute error of the measurement. The absolute error in an individual measured value is:

$$\Delta a_n = | a_{mean} - a_n |$$

The arithmetic mean of all the absolute errors is taken as the final or mean absolute error.

$$\Delta a_{mean} = (|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots\ldots\ldots + |\Delta a_n|)/n$$

$$\Delta a_{mean} = \left(\sum_{i=1}^{n} | \Delta a_i |\right) \Big/ n$$

we can say $a_{mean} - \Delta a_{mean} \le a \le a_{mean} + \Delta a_{mean}$

(c) **Relative and Percentage Error**

Relative error is the ratio of the mean absolute error and arithmetic mean.

$$\text{Relative error} = \frac{\Delta a_{mean}}{a_{mean}}$$

When the relative error is expressed in percent, it is called the percentage error.

Thus, Percentage error = $\frac{\Delta a_{mean}}{a_{mean}} \times 100\%]$

Solved Example

Example 15. In some observations, value of 'g' are coming as

9.81, 9.80, 9.82, 9.79, 9.78, 9.84, 9.79, 9.78, 9.79 and 9.80 m/s^2.

Calculate absolute errors and percentage error in g.

Solution :

S.N.	Value of g	Absolute error $\Delta g = \|g_i - \bar{g}\|$
1	9.81	0.01
2	9.80	0.00
3	9.82	0.02
4	9.79	0.01
5	9.78	0.02
6	9.84	0.04
7	9.79	0.01
8	9.78	0.02
9	9.79	0.01
10	9.80	0.00
	$g_{mean} = 9.80$	$\Delta g_{mean} = \frac{\sum \Delta g_i}{10} = \frac{0.14}{10} = 0.014$

percentage error $= \frac{\Delta g_{mean}}{g_{mean}} \times 100 = \frac{0.014}{9.80} \times 100\ \% = 0.14\ \%$ so 'g' = (9.80 ± 0.014) m/s^2

Solve the following question :

15. Using screw gauge, the observation of the diameter of a wire are 1.324, 1.326, 1.334, 1.336 cm respectively. Find the average diameter, the mean error, the relative error and % error.

ANSWER KEY

1. (i) 1 (ii) 3 (iii) 4 (iv) 4 (v) 4 (vi) 4 **2.** (A) **3.** (B) **4.** (C)

5. (i) 0.0393 kg (ii) 4.08 x 10^8 sec (iii) 5.24 m (iv) 4.74 × 10^{-6} kg **6.** (C) **7.** (A)

8. (D) **9.** (B) **10.** (A) **11.** (B) **12.** (A) **13.** (D) **14.** (D)

15. $\bar{D} = 1.330\,\text{cm}$, $\overline{\Delta D} = 0.005\,\text{cm}$, Relative error = ± 0.004 %, error = 0.4%