PROBABILITY

1. INTRODUCTION

1.1 Factorial notation :

If $n \in N$, then the product $1 \times 2 \times 3 \times ......\times n$ is defined as factorial n which is denoted by $n!$ or $\lfloor n$

i.e., $n! = 1 \times 2 \times 3 \times ......\times n$

We also define $0! = 1$

NOTES :

$n! = n(n-1)!$

1.2 Permutation :

If n objects are given and we have to arrange $r(r \leq n)$ out of them such that the order in which we are arranging the objects is important, then such an arrangement is called permutation of n objects taking r at a time. This is denoted by

$$^nP_r = \frac{n!}{(n-r)!}$$

1.3 Combination :

If n objects are given and we have to choose $r(r \leq n)$ out of them such that the order in which we are choosing the objects is not important, then such a choice is called combination of n objects taking r at a time. This is denoted by

$$^nC_r = \frac{n!}{r!(n-r)!}$$

1.4 Fundamental Principle of Counting :

If an event can occur in 'm' different ways following which another event can occur in 'n' different ways following which another event can occur in 'p' different ways then the total number of ways of simultaneous occurence of all these events in a definite order is $m \times n \times p$.

(where the events are independent of each other)

2. DEFINITIONS AND TYPES OF EVENTS

2.1 Random experiment :

If an act or an experiment has more than one possible results which are known in advance and it is not possible to predict which one is going to occur, then such an experiment is called a **random experiment.**

The following are some random experiments :

(i) Tossing of a coin

(ii) Throwing a six-faced die

(iii) Drawing a card from a well-shuffled pack of cards

(iv) Ten horses run a race

(v) Two persons are selected out of 10 persons to form a committee.

(vi) A ball is drawn from a bag containing 7 balls.

2.2 Outcome :

The result of a random experiment is called an outcome.

2.3 Sample space :

The set of all possible outcomes of a random experiment is called a **sample space** and its elements are called **sample points.** A sample space is usually denoted by S.

Illustrations ;

(i) When a fair coin is tossed, then either head or tail will turn up.

Hence $S = \{H,T\}$ S contains 2 sample points.

(ii) When a six-faced die is thrown, then only one of 1,2,3,4,5,6 will turn up.

Hence $S = \{1,2,3,4,5,6\}$. S contains 6 sample points.

(iii) Suppose a bag contains 7 balls.

Consider the sample points.

(a) The experiment is : one ball is drawn. We can draw one ball out of the 7 balls in $^7C_1 = 7$ ways.

$\therefore$ The sample space for this experiment contains 7 sample points.

(b) The experiment is : two balls are drawn. We can draw 2 balls out of the 7 balls in ${}^7C_2 = \dfrac{7\times 6}{1\times 2} = 21$ ways

$\therefore$ the sample space for this experiment contains 21 sample points.

(c) The experiment is : three balls are drawn.

The sample space for this experiment contains

$${}^7C_3 = \frac{7\times 6\times 5}{1\times 2\times 3} = 35 \text{ sample points}$$

2.4 Event :

Any subset of a sample space is called an event.

Example :

In a single throw of a die, the event of getting a prime number is $E \equiv \{2,3,5\}$ The sample space

$S \equiv \{1,2,3,4,5,6\}$

$\therefore\ E \subseteq S \Rightarrow E$ is an event

2.5 Complementary event :

Let A be an event in a sample space S. Then A is a subset of S We can hence think of the complement of A in S, i..e., S-A. This is also a subset of S and hence an event in S. This event is called the complementary event of A and is denoted by $\overline{A}$ or A'

Now, suppose S contains n sample points, A contains m sample points, Then A' will contain

$n - m$ sample points.

2.6 Impossible event :

Let S be a sample space. Since $\phi \subseteq S$, So ϕ is an event, called an impossible event.

NOTES :

The event E and E' are such that only one of them can occur in a trial and at least one of them must occur.

2.7 Union of events :

If A and B are two events of the sample space S then $A \cup B$ *or* $A + B$ is the event that either A or B (or both) take place.

2.8 Intersection of events :

If A and B are two events of the sample space S then $A \cap B$ or A .B is the event that both A and B take place.

2.9 Mutually Exclusive events :

Two events A and B of the sample space S are said to be mutually exclusive if they cannot occur simultaneously. In such case $A \cap B$ is a null set.

2.10 Exhaustive events :

Two events A and B of the sample space S are said to be exhaustive if $A \cup B = S$ i.e. $A \cup B$ contains all sample points.

2.11 Probability of an event :

Let A be an event in a sample space S. Then the probability of the event A denoted by P(A) is defined as,

$$P(A) = \frac{\text{number of sample points in A}}{\text{number of sample points in S}} = \frac{n(A)}{n(S)}$$

Theorem :

If E is an event of a sample space S, prove that $0 \le P(E) \le 1$ and $P(E') = 1 - P(E)$, where E' is the complementary event of E.

Proof :

Suppose the sample space S contains n sample points and the event E contains m sample points. Then $P(E) = \dfrac{m}{n}$

Now, $0 \le m \le n \ \therefore 0 \le \dfrac{m}{n} \le 1$

$\therefore\ 0 \le P(E) \le 1$

Further E' contains $n - m$ sample points,

$\therefore\ P(E') = 1 - P(E)$

2.12 Equally likely event :

The events are said to be equally likely if none of them is expected to occur in preference to the other.

Ex :- When a die is thrown, then all the side faces are equally likely to come.

2.13 Odds in favour of A : It is defined as $P(A):P(\bar{A})$

Odds against A : It is defined as $P(\bar{A}):P(A)$

If $P(A):P(\bar{A}) = x:y$ then

$$P(A)=\frac{x}{x+y} \text{ and } P(\bar{A})=\frac{y}{x+y}$$

3. EXPERIMENT NO 1 : TOSSING COINS

Tossing one coin :

Let S be the sample space $S \equiv \{H,T\}$, $n(S)=2$

Tossing two coin :

Let S be the sample space $S \equiv \{HH, HT, TH, TT\} \Rightarrow n(S)=4$

Tossing three coin :

Let S be the sample space

$S \equiv \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\Rightarrow n(S)=8$

NOTES :

1. No head means all tails.
2. At least one head means one head or two heads or three heads
3. At most two heads means two heads or one head or no head (all tail).

4. EXPERIMENT NO 2 : THROWING DIE / DICE

One six faced die is thrown :

$S \equiv \{1,2,3,4,5,6\} \Rightarrow n(S)=6$

Two dice are thrown :

$$S \equiv \begin{Bmatrix} (1,1), & (2,1), & (3,1), & (4,1), & (5,1), & (6,1) \\ (1,2), & (2,2), & (3,2), & (4,2), & (5,2), & (6,2) \\ (1,3), & (2,3), & (3,3), & (4,3), & (5,3), & (6,3) \\ (1,4), & (2,4), & (3,4), & (4,4), & (5,4), & (6,4) \\ (1,5), & (2,5), & (3,5), & (4,5), & (5,5), & (6,5) \\ (1,6), & (2,6), & (3,6), & (4,6), & (5,6), & (6,6) \end{Bmatrix}$$

$\Rightarrow n(S)=36$

NOTES :

If one coin is tossed n times or n coins are tossed once the sample space consists of same number of sample points.

i.e. $n(S)=(2)^n$

Prime numbers are 2, 3, 5, 7, 11, ...

1 card is picked $n(S) = {}^{52}C_1 = 52$

2 cards are drawn. $n(S) = {}^{52}C_2 = 1326$

5. EXPERIMENT NO 3: PACK OF CARDS

5.1 There are 4 suits (spade, heart, diamond and club) each having 13 cards.

5.2 There are two colours red (heart and diamond) and black (spade and club) each having 26 cards.

5.3 Jack, Queen and King are face cards. Therefore face cards are 12 in pack of cards. Face card is also called a picture card.

5.4 There are four aces. Ace is not a picture card.

5.5 Face cards and Ace cards are known Coloured Cards.

Theorem 1

If E is an event of sample space S then $0 \le P(E) \le 1$

Proof :

As $E \subseteq S$

$\therefore \quad 0 \le n(E) \le n(S)$

$$\therefore \quad \frac{0}{n(S)} \le \frac{n(E)}{n(S)} \le \frac{n(S)}{n(S)} \quad (\because n(S) \ne 0.$$

$\therefore \quad 0 \le P(E) \le 1$

NOTES :

$P(E) = 0$ if and only if E is an impossible event and

$P(E) = 1$ if and only if E is a certain event

Theorem 2

If E is an event of sample space S and E' is the event that E does not happen then

$P(E') = 1 - P(E)$

Proof :

E' is the event that E does not happen.

$\because$ E and E' are complements of each other.

$\therefore$ $n(E) + n(E') = n(S)$ Dividing by $n(S)$, ($\because n(S) \neq 0$.)

$$\therefore \quad \frac{n(E)}{n(S)} + \frac{n(E')}{n(S)} = \frac{n(S)}{n(S)}$$

$\therefore$ $P(E) + P(E') = 1$

$\therefore$ $P(E') = 1 - P(E)$

Theorem 3

(i) If A and B are two events of sample sapce S, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(ii) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

Theorem 4

Show that if A and B are independent events defined on S then

(i) A & B′ (ii) A′ & B (iii) A′ & B′ are

independent events where A and B are mutually exclusive events of A′ and B′ respectively.

Ans. Since A and B are known to be independent events.

We have $P(A \cap B) = P(A) . P(B)$

(i) $P(A \cap B') = P(A) - P(A \cap B) = P(A) - P(A). P(B)$

$= P(A)(1 - P(B))$

$= P(A) . P(B')$

$\therefore$ A and B′ are independent events

(ii) $P(A' \cap B) = P(B) - P(A \cap B) = P(B) - P(A) . P(B)$

$= P(B)(1 - P(A)) = p(B) . P(A')$

$\therefore$ A′ and B are independent events

(iii) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [P(A) + P(B) - P(A) . (B)]$

$= 1 - PA - P(B) + P(A) . P(B)$

$= [1 - P(A)] - P(B)[1 - P(A)]$

$= [1 - P(A)][1 - P(B)] = P(A') . P(B')$

$\therefore$ A′ and B′ are independent events.

6. IMPORTANT RESULTS

1. A and B are mutually exclusive if $P(A \cap B) = 0$.
2. They are independent if $P(A \cap B) = P(A) . P(B)$
3. Two independent events with non-zero probabilities cannot be mutually exclusive.
4. If A, B and C are independent events with non-zero probabilities then $P(A \cap B \cap B) = P(A) . P(B) . P(C)$
5. If $A_1, A_2, A_3, \ldots A_n$ are independent events with non-zero probabilities, then

   $P(A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_n) = P(A_1) . P(A_2) \ldots P(A_n)$
6. P [exactly one of A or B occurs]

   $= P(A \cap B') + P(A' \cap B)$

   $= P(A \cup B) - P(A \cap B)$

   $= P(A) + P(B) - 2P(A \cap B)$
7. $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$
8. $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$
9. $P(A) = P(A \cap B) + P(A \cap B')$
10. $P(B) = P(A \cap B) + P(A' \cap B)$
11. $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

    $- P(A \cap B) - P(B \cap C) - P(C \cap A)$

    $+ P(A \cap B \cap C)$
12. $P(A \cup B \cup C) = P(A) + P(B) + P(C)$ if A, B and C are mutually exclusive.
13. If A and B are mutually exclusive and exhaustive events then $P(A) + P(B) = 1$
14. If A, B and C are mutually exclusive and exhaustive events then

    $P(A) + P(B) + P(C) = 1$

7. CONDITIONAL PROBABILITY

Let A and B be two events associated with a random experiment. Then, the probability of occurrence of event A under the condition that B has already occurred and $P(B) \neq 0$, is called the conditional probability of event A and it is denoted by P (A/B). Thus, we have

P (A/B) = Probability of occurence of A given that B has already occurred.

$$P(A/B) = \frac{\text{Number of elementary events favourable } A \cap B}{\text{Number of elementary events favourable to B}}$$

$$\Rightarrow \quad P(A/B) = \frac{n(A \cap B)}{n(B)}$$

$$\Rightarrow \quad P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Theorem

If A and B are two events associated with a random experiment, then

$P(A \cap B) = P(A)\,P(B/A)$, if $P(A) \neq 0$

NOTES :

1. $0 \leq P(A/B) \leq 1$
2. $P(A/A) = 1$

8. INDEPENDENT EVENTS

Definition :

Events are said to be independent, if the occurrence or non-occurrence of one does not affect the probability of the occurrence or non-occurrence of the other.

Theorem 1 :

If A and B are independent events associated with a random experiment, then $P(A \cap B) = P(A)\,P(B)$.

Theorem 2 :

If $A_1, A_2, \ldots, A_n$ are independent events associated with a random experiment, then

$P(A_1 \cap A_2 \cap A_3 \ldots \cap A_n) = P(A_1)\,P(A_2) \ldots P(A_n)$

Theorem 3 :

If A and B are independent events associated with a random experiment, then

(i) $\bar{A}$ and B are independent events

(ii) A and $\bar{B}$ are independent events

(iii) $\bar{A}$ and $\bar{B}$ are also independent events.

9. MULTIPLICATION THEOREM (OR) PRODUCT THEOREM (OR) THEROREM OF COMPOUND PROBABILITY

If A and B are two events in a sample space S such that $P(A) \neq 0$ & $P(B) \neq 0$, then the probability of simultaneous occurence of the two events A and B is given by

$P(A \cap B) = P(A) \,.\, P(B/A) = P(B) \,.\, P(A/B)$.

* For three events A, B and C $P(A \cap B \cap C)$

$= P(A) \,.\, P(B/A) \,.\, P(C/A \cap B)$

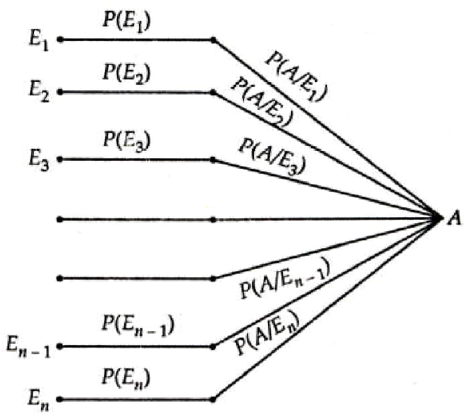
* For four events A, B, C and D $P(A \cap B \cap C \cap D)$

$= P(A) \,.\, P(B/A) \,.\, P(C/A \cap B) \,.\, P(D/A \cap B \cap C)$

10. THE LAW OF TOTAL PROBABILITY

Theorem : (Law of Total Probability) Let S be the sample space and let $E_1, E_2, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or ... or E_n, then

$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + \ldots + P(E_n)\,P(A/E_n)$

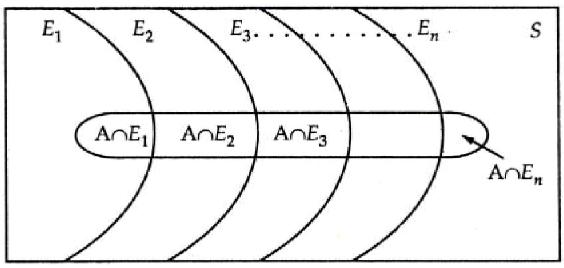


11. STATEMENT OF BAYES' THEOREM

If $B_1, B_2, B_3, \ldots B_n$ are mutually exclusive and exhaustive events & if A is an event consequent to these B_i's then for each i, where i = 1, 2, 3...., n

$$P\left(B_i/A\right) = \frac{P(B_i)\,P\left(A/B_i\right)}{P(B_1)\,P\left(A/B_1\right) + P(B_2)\,P\left(A/B_2\right) \ldots.. + P(B_n)\,P\left(A/B_n\right)}$$

$$P\left(B_i/A\right) = \frac{P(B_i)\,P\left(A/B_i\right)}{\sum_{i=1}^{n} P(B_i)\,P\left(A/B_i\right)}$$



12. RANDOM VARIABLE AND PROBABILITY DISTRIBUTION

12.1 Random Variable : Let S be a sample space associated with a random experiment. Then a real valued function X: S→R is called a random function or random variable.

12.2 Discrete Random Variable : A real valued function defined on discrete sample space S is called a discrete random variable.

12.3 Continuous Random Variable : A random variable X defined on continuous sample space S, which can take all real values in an interval (a, b) is called a continuous random variable.

12.4 Discrete Random Variable : If a discrete variable X can assume values $X_1, X_2, X_3, \dots X_n$ with respective probabilities $P(X_1), P(X_2), P(X_3) \dots P(X_n)$ such that $P(X_i) \geq 0$, $\forall$ i and $\Sigma P(X_i) = 1$, then X is said to be a discrete random variable.

12.5 Probability Mass Functions : If X is a discrete random variable which can assume values X_i; i = 1, 2, 3, ... With respective probabilities P_i; i = 1, 2, 3.... such that

$\sum_{i=1}^{\infty} P_i = 1$, then $P(X = x_i) = P_i$; i = 1, 2,3,.... is called probability mass function of a discrete random variable X. (OR) If any function P(X=x) gives the probabilities of various values of a discrete random variable X in its range, then that function is called probability mass function.

12.6 Probability Distribution : The set of ordered pairs $\{x_i, P(x_i)\}$ is called the probability distribution of a discrete random variable X.

13. MEANS AND VARIANCE

If $\{x_i, P(x_i)\}$ is the probability distribution of a discrete random variable X, then its:

13.1 Mean or Average : $(\bar{x} \text{ or } \mu)$: Expected value of x or mathematical expectation of x:E(x) or First moment about origin: $\mu_1'(0)$ is defined as $\bar{x} = \mu = \mu_1'(0) = \sum x_i P(x_i)$.

13.2 Variance (σ^2) or second moment about mean or 2nd central moment = μ_2.

$$\sigma^2 = \mu_2 = E\{x - E(x)\}^2$$

$$= E(x^2) - \{E(x)\}^2$$

$$= \mu_2^1(0) - \mu_1^1(0)^2$$

$$= \Sigma x_i^2 P(x_i) - \{\Sigma x_i P(x_i)\}^2$$

14. BINOMIAL DISTRIBUTION

14.1 Bernoulli Trials : Trials of a random experiment are called Bernoulli trials if they satisfy the following conditions :

(i) They are finite in number.

(ii) They are independent of each other.

(iii) Each trial has exactly two outcomes : success or failure.

(iv) The probability of success or failure remains same in each trial.

(v) The probability of success is p and failure is q such that p + q = 1

(vi) The probability of r successes in n trails in any order is given by ${}^nC_r p^r q^{n-r}$.

14.2 Binomial Distribution : Let X denote the random variable which associates every outcome to the number of successes in it. Then, X assumes values 0, 1, 2, ..., n such that

$P(X = r) = {}^nC_r p^r q^{n-r}$, r = 0, 1, 2, ..., n.

The probability distribution of the random variable X is therefore given by

X :	0	1	2	...	r	...	n
P(X) :	${}^nC_0 p^0 q^{n-0}$	${}^nC_1 p^1 q^{n-1}$	${}^nC_2 p^2 q^{n-2}$	...	${}^nC_r p^r q^{n-r}$	...	${}^nC_n p^n q^{n-n}$

14.3 Mean & Variance :

Mean = np

Variance = npq

SOLVED EXAMPLES

Example – 1

5 letters are to be posted in 5 post boxes. If any number of letters can be posted in 5 post boxes, what is the probability that each box contains only one letter?

Sol. Since any number of letters can be posted in all 5 post boxes, each letter can be posted in 5 different ways.

$\therefore$ $n(S) = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$

Let $A \equiv$ the event that each box contains only one letter.

The first letter can be posted in 5 post boxes in 5 different ways. Since each box contains only one letter, the second letter can be posted in the remaining 4 post boxes in 4 different ways.

Similarly, the third letter can be posted in 3 different ways, the fourth letter can be posted in 2 different ways and the fifth letter can be posted in 1 way.

$\therefore$ $n(A) = 5 \times 4 \times 3 \times 2 \times 1 = 5!$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{5!}{5^5}$$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{5 \times 5^4} = \frac{24}{625}$$

Example – 2

A doctor is called to see a sick child. The doctor has prior information that 80% of sick children in that area have the flu, while the other 20% are sick with measles. Assume that there is no other disease in that area. A well-known symptom of measles is a rash. From the past records it is known that, chances of having rashes given that sick child is suffering from measles is 0.95. However, occasionally children with flu also develop rash, whose chances are 0.08. Upon examining the child, the doctor finds a rash. What is the probability that the child has measles?

Sol. Let A $\equiv$ event that the child is sick with flu

B $\equiv$ event that the child is sick with measles

C $\equiv$ event that the child has rash

$$\therefore \quad P(A) = 80\% = \frac{80}{100} = \frac{4}{5}$$

$$P(B) = 20\% = \frac{20}{100} = \frac{1}{5}$$

Since the chances of having rashes, if the child is suffering from measles is 0.95 and the chances of having rashes, if the child has flu is 0.08,

$$P(C/B) = 0.95 = \frac{95}{100}$$

and $P(C/A) = 0.08 = \dfrac{8}{100}$

By Baye's Theorem, probability that the child has measles provided he has the rashes is given by

$$P(B/C) = \frac{P(B).P(C/B)}{P(A).P(C/A) + P(B).P(C/B)}$$

$$= \frac{\left(\frac{1}{5}\right)\left(\frac{95}{100}\right)}{\left(\frac{4}{5}\right)\left(\frac{8}{100}\right) + \left(\frac{1}{5}\right)\left(\frac{95}{100}\right)}$$

$$= \frac{95}{32+95} = \frac{95}{127}$$

$= 0.748$

Example – 3

Suppose you have a large barrel containing a number of plastic eggs. Some eggs contain pearls, the rest contain nothing. Some eggs are painted blue, the rest are painted red. Suppose that 40% of the eggs are painted blue, $\dfrac{5}{13}$ of the eggs painted blue contain pearls and 20% of the red eggs are empty. What is the probability that an egg containing pearl is painted blue ?

Sol. Let event A = An egg is painted blue.

event B = An egg is painted red.

The barrel contains egg with blue paint as 40% and red paint as 60%.

$\therefore \quad P(A) = 40\% = \frac{40}{100} = \frac{2}{5}$

$\therefore \quad P(B) = 60\% = \frac{60}{100} = \frac{3}{5}$

Let event C = an egg selected contains a pearl.

Then C/A = A blue painted egg contains pearl. given that P (C/A) = 5/13.

P(C/B) = A red painted egg contains pearl. given that 20% of red eggs are empty.

i.e. 80% of red eggs contain pearls.

$\therefore \quad P(C/B) = 80\% = \frac{80}{100} = \frac{4}{5}$

$\therefore$ Required probability, that an egg containing pearl is painted blue is

$$P(A/C) = \frac{P(A).P(C/A)}{P(A).P(C/A) + P(B).P(C/B)}$$

$$= \frac{\frac{2}{5} \times \frac{5}{13}}{\frac{2}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{4}{5}}$$

$$= \frac{\frac{2}{13}}{\frac{206}{13 \times 25}} = \frac{50}{206}$$

$$= \frac{25}{103} = 0.243$$

Example – 4

The mean and variance of a binomial distribution are 4 and 4/3 respectively, find P (X ≥ 1).

Sol. Let X be a binomial variate with parameters n and p. Then,

Mean = np and Variance = npq

$\Rightarrow \quad np = 4$ and $npq = \frac{4}{3}$

$[\because \text{Mean} = 4, \text{Var}(X) = \frac{4}{3} \text{(Given)}]$

$\Rightarrow \quad \frac{npq}{np} = \frac{\frac{4}{3}}{4} \Rightarrow q = \frac{1}{3} \Rightarrow p = 1 - \frac{1}{3} = \frac{2}{3}$

$[p = 1 - q]$

Putting p = 2/3 in np = 4, we get

$$n \times \frac{2}{3} = 4 \Rightarrow n = 6$$

Thus, we have

$$n = 6, p = \frac{2}{3} \text{ and } q = \frac{1}{3}$$

$\therefore \quad P(X = r) = {}^nC_r p^r q^{n-r} \Rightarrow P(X = r) = {}^6C_r \left(\frac{2}{3}\right)^r \left(\frac{1}{3}\right)^{6-r}$,

$r = 0, 1, 2, ..., 6$

Now, $P(X \geq 1) = 1 - P(X < 1)$

$\Rightarrow \quad P(X \geq 1) = 1 - P(X = 0)$

$\Rightarrow \quad P(X \geq 1) = 1 - {}^6C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^6 = 1 - \left(\frac{1}{3}\right)^6 = 1 - \frac{1}{729} = \frac{728}{729}$

Example – 5

Events A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$. Then set of possible values of x are in the interval

(a) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (b) $\left[\frac{1}{3}, \frac{13}{3}\right]$

(c) [0, 1] (d) $\left[\frac{1}{3}, \frac{1}{2}\right]$

Ans. (d)

Sol. A, B, C are mutually exclusive

$\therefore \; P(A) + P(B) + P(C) \leq 1 \quad$ ..(1)

$0 \leq P(A), P(B), P(C) \leq 1 \quad$ ...(ii)

Now on solving (i) and (ii), we get

$$\frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \leq 1$$

$12x + 4 + 3 - 3x + 6 - 13 \times \leq 12$

$13 - 3 \times \leq 12$

$$\Rightarrow \boxed{x \geq \frac{1}{3}}$$

Also, $P(C) \geq 0$

$\Rightarrow 1 - 2x \geq 0$

$2x \leq 1$

$\Rightarrow x \leq \frac{1}{2}$

$\Rightarrow \frac{1}{3} \leq x \leq \frac{1}{2}$.

Example – 6

Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse, is

(a) 3/5 (b) 1/5

(c) 2/5 (d) 4/5

Ans. (c)

Sol. Let two horses selected are A and B.

No. of horses = 5

$\therefore$ Probability that A can't win the race

Where $p(A) = \frac{1}{5}$ and $P(B) = \frac{1}{4}$

$= p(\bar{A}).p(\bar{B}) = \frac{4}{5} \times \frac{3}{4}$

Probability that 'A' must win the race = $1 - P(\bar{A})P(\bar{B})$

$= 1 - \frac{12}{20} = \frac{2}{5}$.

Example – 7

The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then P (X = 1) is

(a) 1/16 (b) 1/8

(c) 1/4 (d) 1/32

Ans. (d)

Sol. Given mean np = 4, npq = 2

$\Rightarrow \frac{npq}{np} = \frac{2}{4} \Rightarrow q = \frac{1}{2} \quad \therefore q = p = \frac{1}{2}$ and $n = 8$

Now $P(X = r) = {}^8C_r p^r \left(\frac{1}{2}\right)^{8-r}$

(Use $P(X = r) = {}^nC_r p^r q^{n-r}$)

$\therefore P(X = 1) = {}^8C_1 \left(\frac{1}{2}\right)^8 = \frac{8}{16 \times 16} = \frac{1}{32}$

Example – 8

The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

(a) 128/256 (b) 219/256

(c) 37/256 (d) 28/256

Ans. (d)

Sol. Given np = 4 and npq = 2

$q = \frac{npq}{np} = \frac{2}{4} = \frac{1}{2} \Rightarrow p = 1 - \frac{1}{2} = \frac{1}{2}$

Now npq = 2 $\quad \therefore$ n = 8

$\therefore$ Binomial Distribution is given by

$P(X = r) = {}^8C_r p^r q^{n-r}$

$\therefore P(X = r = 2) = {}^8C_2 \left(\frac{1}{2}\right)^8 = \frac{28}{256}$.

Example – 9

A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice, is

(a) 8/729 (b) 8/243

(c) 1/729 (d) 8/9

Ans. (b)

Sol. Possibility of getting 9 are (5, 4), (4, 5), (6, 3), (3, 6)

Probability of getting score 9 in a single throw

$= p = \frac{4}{36} = \frac{1}{9}$

so, probability of not getting 9 in a single throw

$= 1 - \frac{1}{9} = \frac{8}{9}$

Required probability = probability of getting score 9

exactly twice $= {}^3C_2 \left(\frac{1}{9}\right)^2 \times \left(\frac{8}{9}\right) = \frac{8}{243}$.

Example – 10

A die is thrown. Let A be the event that the number obtained is greater than 3, Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) 0 (d) 1

Ans. (d)

Sol. $A = \{4, 5, 6\}$

Also $B = \{1, 2, 3, 4\}$

We have $A \cup B = \{1, 2, 3, 4, 5, 6\} = S$

Where S is the sample space of the experiment of throwing a die. $P(S) = 1$, for it is a sure event.

Hence $P(A \cup B) = 1$

Example – 11

A set S contains 7 elements. A non-empty subset A of S and an element x of S are chosen at random. Then the probability that $x \in A$ is

(a) $\frac{1}{2}$ (b) $\frac{64}{127}$

(c) $\frac{63}{128}$ (d) $\frac{31}{128}$

Ans. (b)

Sol. $S = \{a, b, c, d, e, f, g\}$

A can be selected in $(2^7 - 1)$ ways

Hence, the number of non-empty subsets of S can be chosen in

$^7C_1 + {}^7C_2 + {}^7C_3 + ... + {}^7C_7 = 128 - 1$ ways

$= 127$

Now, any x if not included in A can happen in $2^6 - 1$ ways

$\therefore$ Number of sets which include x are

$= (2^7 - 1) - (2^6 - 1)$

$= 2^6(2 - 1) = 64$ ways

Hence, probability that $x \in A$ is

$$\frac{64}{2^7 - 1} = \frac{64}{127}.$$

Example – 12

One hundred identical coins, each with probability p, of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is

(a) 1/2 (b) 49/101

(c) 50/101 (d) 51/101

Ans. (d)

Sol. Lex X be the number of coins showing heads. Let X be a binomial variate with parameters $n = 100$ and p.

Since, $P(X = 50) = P(X = 51)$

$$\Rightarrow {}^{100}C_{50}\, p^{50}(1-p)^{50} = {}^{100}C_{51}(p)^{51}(1-p)^{49}$$

$$\Rightarrow \frac{(100)!}{(50!)(50!)} \cdot \frac{(51!) \times (49!)}{100!} = \frac{p}{1-p}$$

$$\Rightarrow \frac{p}{1-p} = \frac{51}{50} \Rightarrow p = \frac{51}{101}$$

Example – 13

Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral, equals

(a) 1/2 (b) 1/5

(c) 1/10 (d) 1/20

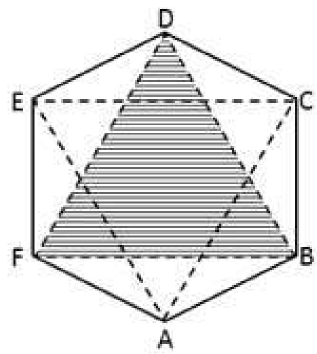
Ans. (c)

Sol. Three vertices out of 6 can be chosen in $^6C_3$ ways.

So, total ways $= {}^6C_3 = 20$

Only two equilateral triangles can be formed

ΔAEC and ΔBFD



$\therefore$ Favourable ways = 2

So, required probability $= \frac{2}{20} = \frac{1}{10}$

Example – 14

If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5, equals

(a) $\frac{1}{4}$ (b) $\frac{1}{7}$

(c) $\frac{1}{8}$ (d) $\frac{1}{49}$

Ans. (a)

Sol. $7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401, \ldots$

There fore, for 7^r, $r \in N$ then number ends at unit

Place 7, 9, 3, 1, 7, ...

$\therefore$ $7^m + 7^n$ will be divisible by 5 if it end at 5 or 0.

But it cannot end at 5.

Also for end at 0.

For this *m* ad *n* should be as follows

	m	*n*
1	$4r$	$4r - 2$
2	$4r - 1$	$4r - 3$
3	$4r - 2$	$4r$
4	$4r - 3$	$4r - 1$

For any given value of *m*, there will be 25 values of *n*.

Hence, the probability of the required event is

$$= \frac{100 \times 25}{100 \times 100} = \frac{1}{4}$$

Example – 15

If M and N are any two events, then the probability that exactly one of them occurs is

(a) $P(M) + P(N) - 2P(M \cap N)$

(b) $P(M) + P(N) - P(\overline{M \cup N})$

(c) $P(\bar{M}) + P(\bar{N}) - 2P(\bar{M} \cap \bar{N})$

(d) $P(M \cap \bar{N}) - P(\bar{M} \cap N)$

Ans. (a,c)

Sol. P(exactly one of M, N occurs)

$$= P\{(M \cap \bar{N}) \cup (\bar{M} \cap N)\} = P(M \cap \bar{N}) + P(\bar{M} \cap N)$$

$$= P(M) - P(M \cap N) + P(N) - P(M \cap N)$$

$$= P(M) + P(N) - 2P(M \cap N)$$

Also, P (exactly one of them occurs)

$$= \{1 - P(\bar{M} \cap \bar{N})\}\{1 - P(\bar{M} \cup \bar{N})\}$$

$$= P(\bar{M} \cup \bar{N}) - P(\bar{M} \cap \bar{N}) = P(\bar{M}) + P(\bar{N}) - 2P(\bar{M} \cap \bar{N})$$

Hence, (a) and (c) are correct answers.

Example – 16

For two given events A and B, $P(A \cap B)$ is

(a) not less than $P(A) + P(B) - 1$

(b) not greater than $P(A) + P(B)$

(c) equal to $P(A) + P(B) - P(A \cup B)$

(d) equal to $P(A) + P(B) + P(A \cup B)$

Ans. (a,b,c)

Sol. We know that,

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

Also, $P(A \cup B) \le 1$

$\therefore P(A \cap B)_{min}$, when $P(A \cup B)_{max} = 1$

$\Rightarrow P(A \cap B) \ge P(A) + P(B) - 1$

$\therefore$ Option (a) is true.

Again, $P(A \cup B) \ge 0$

$\therefore P(A \cap B)_{max}$, when $P(A \cup B)_{min} = 0$

$\Rightarrow P(A \cap B) \le P(A) + P(B)$

$\therefore$ Option (b) is true.

Also, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$, Thus, (c) is also correct.

Hence, (a), (b),(c) are correct options.

Example – 17

If E and F are independent events such that $0 < P(E) < 1$ and $0 < P(F) < 1$, then

(a) E and F are mutually exclusive

(b) E and F^c (the complement of the event F) are independent

(c) E^c and F^c are independent

(d) $P(E/F) + P(E^c/F) = 1$

Ans. (b,c,d)

Sol. Since, E and F are independent events. Therefore, $P(E \cap F) = P(E) \cdot P(F) \ne 0$, so E and F are not mutually exclusive events.

Now, $P(E\cap \overline{F}) = P(E) - P(E\cap F) = P(E) - P(E).P(F)$

$= P(E)[1 - P(F)] = P(E).P(\overline{F})$

and, $P(\overline{E}\cap\overline{F}) = P(\overline{E\cup F}) = 1 - P(E\cup F)$

$= 1 - \left[1 - P(\overline{E}).P(\overline{F})\right]$

[$\because$ E and F are independent]

$= P(\overline{E}).P(\overline{F})$

So, E and $\overline{F}$ as well as $\overline{E}$ and $\overline{F}$ are independent events.

Now, $P(E/F) + P(\overline{E}/F) = \dfrac{P(E\cap F) + P(\overline{E}\cap F)}{P(F)}$

$= \dfrac{P(F)}{P(F)} = 1$

Example – 18

For any two events A and B in a sample space

(a) $P\left(\dfrac{A}{B}\right) \geq \dfrac{P(A)+P(B)-1}{P(B)}$, $P(B) \neq 0$ is always true

(b) $P(A\cap\overline{B}) = P(A) - P(A\cap B)$ does not hold

(c) $P(A\cup B) = 1 - P(\overline{A})P(\overline{B})$, if A and B are independent

(d) $P(A\cup B) = 1 - P(\overline{A})P(\overline{B})$, if A and B are disjoint

Ans. (a,c)

Sol. We know that,

$P\left(\dfrac{A}{B}\right) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{P(A)+P(B)-P(A\cup B)}{P(B)}$

Since, $P(A\cup B) \leq 1$

$\Rightarrow -P(A\cup B) \geq -1$

$\Rightarrow P(A)+P(B)-P(A\cup B) \geq P(A)+P(B)-1$

$\Rightarrow \dfrac{P(A)+P(B)-P(A\cup B)}{P(B)} \geq \dfrac{P(A)+P(B)-1}{P(B)}$

$\Rightarrow P\left(\dfrac{A}{B}\right) \geq \dfrac{P(A)+P(B)-1}{P(B)}$

Hence, option (a) is correct.

The choice (b) holds only for disjoint i.e. $P(A\cap B) = 0$

Finally, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= P(A) + P(B) - P(A).P(B)$,

if A, B are independent

$= 1 - \{1 - P(A)\}\{1 - P(B)\} = 1 - P(\overline{A}).P(\overline{B})$

Hence, option (c) is correct, but option (d) is not correct.

Example – 19

Let E and F be two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happen is 1/2. Then,

(a) $P(E) = 1/3$, $P(F) = 1/4$

(b) $P(E) = 1/2$, $P(F) = 1/6$

(c) $P(E) = 1/6$, $P(F) = 1/2$

(d) $P(E) = 1/4$, $P(F) = 1/3$

Ans. (a,d)

Sol. Both E and F happen $\Rightarrow P(E\cap F) = \dfrac{1}{12}$

and neither E nor F happens $\Rightarrow P(\overline{E}\cap\overline{F}) = \dfrac{1}{2}$

But for independent events, we have

$P(E\cap F) = P(E)P(F) = \dfrac{1}{12}$ ...(i)

and $P(\overline{E}\cap\overline{F}) = P(\overline{E})P(\overline{F})$

$= \{1 - P(E)\}.\{1 - P(F)\}$

$= 1 - P(E) - P(F) + P(E)P(F)$

$\Rightarrow \dfrac{1}{2} = 1 - \{P(E) + P(F)\} + \dfrac{1}{12}$

$\Rightarrow P(E) + P(F) = 1 - \dfrac{1}{2} + \dfrac{1}{12} = \dfrac{7}{12}$ ...(ii)

On solving Eqs. (i) and (ii), we get

either $P(E) = \dfrac{1}{4}$ and $P(F) = \dfrac{1}{3}$.

or, $P(E) = \dfrac{1}{3}$ and $P(F) = \dfrac{1}{4}$

Example – 20

The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the students has a 75% chance of passing in at least one, a 50% chance of passing in at least two, and a 40% chance of passing in exactly two. Which of the following relations are true?

(a) $p+m+c=\frac{19}{20}$ (b) $p+m+c=\frac{27}{20}$

(c) $pmc=\frac{1}{10}$ (d) $pmc=\frac{1}{4}$

Ans. (b,c)

Sol. Let A, B and C respectively denote the events that the student passes in Maths, Physics and Chemistry.

It is given.

$P(A)=m, P(B)=p$ and $P(C)=c$ and

P (passing atleast in one subject) = $P(A\cup B\cup C) = 0.75$

$\Rightarrow 1-P(A'\cap B'\cap C') = 0.75$

$\because [P(A)=1-P(\overline{A})]$

and $[P(\overline{A\cup B\cup C}) = P(A'\cap B'\cap C')]$

$\Rightarrow 1-P(A').P(B').P(C')=0.75$

($\because$ A, B and C are independent events, therefore A', B' and C' are independent events)

$\Rightarrow 0.75 = 1-(1-m)(1-p)(1-c)$

$\Rightarrow 0.25 = (1-m)(1-p)(1-c)$ ...(i)

Also, P(passing exactly in two subjects) = 0.4

$\Rightarrow P(A\cap B\cap\overline{C}\cup A\cap\overline{B}\cap C\cup\overline{A}\cap B\cap C) = 0.4$

$\Rightarrow P(A\cap B\cap\overline{C})+P(A\cap\overline{B}\cap C)+P(\overline{A}\cap B\cap C) = 0.4$

$\Rightarrow P(A)\ P(B)\ P(\overline{C})+P(A)\ P(\overline{B})\ P(C)$

$+P(\overline{A})\ P(B)\ P(C) = 0.4$

$\Rightarrow pm(1-c)+p(1-m)c+(1-p)mc=0.4$

$\Rightarrow pm-pmc+pc-pmc+mc-pmc=0.4$ ....(iii)

Again, P(passing atleast in two subjects) = 0.5

$\Rightarrow P(A\cap B\cap\overline{C})+P(A\cap\overline{B}\cap C)+P(\overline{A}\cap B\cap C)$

$+P(A\cap B\cap C)=0.5$

$\Rightarrow pm(1-c)+pc(1-m)+cm(1-p)+pcm=0.5$

$\Rightarrow pm-pcm+pc-pcm+cm-pcm+pcm=0.5$

$\Rightarrow (pm+pc+mc)-2pcm=0.5$ ...(ii)

From Eq. (ii),

$pm+pc+mc-3pcm=0.4$ ...(iv)

From Eq.(i),

$0.25=1-(m+p+c)+(pm+pc+cm)-pcm$ ...(v)

On solving Eqs. (iii), (iv) and (v), we get

$p+m+c=1.35=\frac{27}{20}$

Therefore, option (b) is correct.

Also, from Eqs, (ii) and (iii), we get $pmc=\frac{1}{10}$

Hence, option (c) is correct.

Example – 21

If p & q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} with replacement. Determine the probability that the roots of the equation $x^2+px+q=0$ are real.

Ans. (31/50)

Sol. The required probability = 1- (probability of the event that the roots of $x^2+px+q=0$ are non-real).

The roots of $x^2+px+q=0$ will be non-real if and only if $p^2-4q<0$, i.e. if $p^2<4q$

The possible values of p and q can be possible according to the following table.

Value of q	Value of p	Number of pairs of p,q
1	1	1
2	1, 2	2
3	1, 2, 3	3
4	1, 2, 3	3
5	1, 2, 3, 4	4
6	1, 2, 3, 4	4
7	1, 2, 3, 4, 5	5
8	1, 2, 3, 4, 5	5
9	1, 2, 3, 4, 5	5
10	1, 2, 3, 4, 5, 6	6

Therefore, the number of possible pairs = 38

Also, the total number of possible pairs is $10\times10=100$

$\therefore$ The required probability $= =1-\frac{38}{100}=1-0.38=0.62$

Example – 22

There is 30% chance that it rains on any particular day. What is the probability that there is at least one rainy day within a period of 7 – days ? Given that there is at least one rainy day, what is the probability that there are at least two rainy days ?

Ans. $1-(0.7)^7$, $\dfrac{\left[1-(0.7)^7-{}^7C_1(0.3)(0.7)^6\right]}{1-(0.7)^7}$

Sol. Probability of rain on any particular day = 0.3

Probability of no rain on any particular day

$= 1 - 0.3 = 0.7$

Probability of no rain for 7 days = $(0.7)^7$

probability for atleast one rainy day = $1 - (0.7)^7$

Now let A = there are atleast two rainy days

B = there is atleast one rainy day.

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{\left[1-(0.7)^7-{}^7C_1(0.3)(0.7)^6\right]}{1-(0.7)^7}$$

Example – 23

3 players A, B & C toss a coin cyclically in that order (that is A, B, C, A, B, C, A, B, ...) till a head shows. Let p be the probability that the coin shows a head. Let α, β & γ be respectively the probabilities that A, B and C gets the first head . Prove that

$\beta = (1-p)\alpha$. Determine α, β & γ (in terms of p).

Ans. $\alpha=\dfrac{p}{1-(1-p)^3},\ \beta=\dfrac{(1-p)\,p}{1-(1-p)^3},\ \gamma=\dfrac{(1-p)^2\,p}{1-(1-p)^3}$

Sol. Let $q = 1 - p$ = probability of getting the tail. We have

α = probability of A getting the head on tossing firstly

$= P(H_1 \text{ or } T_1T_2T_3H_4 \text{ or } T_1T_2T_3T_4T_5T_6H_7 \text{ or } \ldots)$

$= P(H) + P(H)P(T)^3 + P(H)P(T)^6 + \ldots$

$$=\frac{P(H)}{1-P(T)^3}=\frac{p}{1-q^3}$$

Also,

β = probability of B getting the head on tossing secondly

$= P(T_1H_2 \text{ or } T_1T_2T_3H_4H_5 \text{ or } T_1T_2T_3T_4T_5T_6T_7H_8 \text{ or } \ldots)$

$= P(H)[P(T)+P(H)P(T)^4 + P(H)P(T)^7 + \ldots]$

$= P(T)[P(H)+P(H)P(T)^3 + P(H)P(T)^6 + \ldots]$

$$= q\alpha = (1-p)\alpha = \frac{p(1-p)}{1-q^3}$$

Again, we have

$\alpha + \beta + \gamma = 1$

$$\Rightarrow \gamma = 1-(\alpha+\beta) = 1-\frac{p+p(1-p)}{1-q^3}$$

$$=1-\frac{p+p(1-p)}{1-(1-p)^3}$$

$$=\frac{1-(1-p)^3-p-p(1-p)}{1-(1-p)^3}$$

$$\gamma=\frac{1-(1-p)^3-2p+p^2}{1-(1-p)^3}=\frac{p-2p^2+p^3}{1-(1-p)^3}$$

Also, $\alpha=\dfrac{p}{1-(1-p)^3},\ \beta=\dfrac{p(1-p)}{1-(1-p)^3}$

Example – 24

Eight players P_1, P_2, P_3, ...P_8 play a knock-out tournament. It is known that whenever the players P_i and P_j play, the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final.

Ans. (4/35)

Sol. The number of ways in which $P_1, P_2, \ldots P_8$ can be paired in four pairs

$$=\frac{1}{4!}[({}^8C_2)({}^6C_2)({}^4C_2)({}^2C_2)]$$

$$=\frac{1}{4!}\times\frac{8!}{2!6!}\times\frac{6!}{2!4!}\times\frac{4!}{2!2!}\times 1$$

$$=\frac{1}{4!}\times\frac{8\times7}{2!\times1}\times\frac{6\times5}{2!\times1}\times\frac{4\times3}{2!\times1}=\frac{8\times7\times6\times5}{2.2.2.2}=105$$

Now, atleast two players certainly reach the second round between P_1, P_2 and P_3 and P_4 can reach in final if exactly two players play against each other between P_1, P_2, P_3 and remaining player will play against one of the remaining three from $P_5 \ldots P_8$.

This can be possible in

$^{3}C_2 \times {}^{4}C_1 \times {}^{3}C_1 = 3.4.3 = 36$ ways

$\therefore$ Probability that P_4 and exactly one of P_5 ...P_8 reach second round

$= \frac{36}{105} = \frac{12}{35}$

If P_1, P_i, P_4 and P_j, where $i = 2$ or 3 and $j = 5$ or 6 or 7 reach the second round, then they

can be paired in 2 pairs in $\frac{1}{2!}({}^{4}C_2)({}^{2}C_2) = 3$ ways.

But P_4 will reach the final, if P_1 plays

Against P_i and P_4 plays against P_j,

Hence, the probability that P_4 will reach the final round

from the second $= \frac{1}{3}$

$\therefore$ Probability that P_4 will reach the final is $\frac{12}{35} \times \frac{1}{3} = \frac{4}{35}$

Example – 25

Four cards are drawn from a pack of 52 playing cards. Find the probability (correct upto two places of decimals) of drawing exactly one pair.

Ans. (0.3)

Sol. Total number of outcomes = $^{52}C_4$

Now there are exactly 13 group of cards each group containing card having equal number. Choose any one group and in that group out of 4 cards choose any two cards which will make exactly one pair. Now rest of 12 groups select one card from each group. Hence

Favourable outcomes $= {}^{13}C_1 \times {}^{4}C_2 \times {}^{12}C_2 \times {}^{4}C_1 \times {}^{4}C_1$

Required probability

$$= \frac{{}^{13}C_1 \times {}^{4}C_2 \times {}^{12}C_2 \times {}^{4}C_1 \times {}^{4}C_1}{{}^{52}C_4} = 0.3$$

Example – 26

A box contains 2 black, 4 white and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept beside the first. This process is repeated till all the balls are drawn from the box. Find the probability that the balls drawn are in the sequence of 2 black, 4 white and 3 red.

Ans. $\frac{1}{1260}$

Sol. Since, the drawn balls are in the sequence black, black, white, white, white, white, red, red and red.

Let the corresponding probabilities be $p_1, p_2, \ldots p_9$

Then, $p_1 = \frac{2}{9}, p_2 = \frac{1}{8}, p_3 = \frac{4}{7}, p_4 = \frac{3}{6}, p_5 = \frac{2}{5},$

$p_6 = \frac{1}{4}, p_7 = \frac{3}{3}, p_8 = \frac{2}{2}, p_9 = 1$

$\therefore$ Required probability

$$p_1 \cdot p_2 \cdot p_3 \cdots p_9 = \left(\frac{2}{9}\right)\left(\frac{1}{8}\right)\left(\frac{4}{7}\right)\left(\frac{3}{6}\right)\left(\frac{2}{5}\right)\left(\frac{1}{4}\right)\left(\frac{3}{3}\right)\left(\frac{2}{2}\right)(1) = \frac{1}{1260}$$

Example – 27

An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2, and 0.1 respectively. What is the probability that the gun hits the plane ?

Ans. (0.6976)

Sol. Let $P(H_1) = 0.4, P(H_2) = 0.3, P(H_3) = 0.2, P(H_4) = 0.1$

P (gun hits the plane)

$= 1 - P(\overline{H_1}).P(\overline{H_2}).P(\overline{H_3}).P(\overline{H_4})$

$= 1 - (0.6)(0.7)(0.8)(0.9) = 1 - 0.3024 = 0.6976$

Example – 28

Cards are drawn one by one at random from a well shuffled full pack of 52 playing cards until 2 aces are obtained for the first time. If N is the number of cards required to be drawn, then show that

$$P_r\{N=n\}=\frac{(n-1)(52-n)(51-n)}{50\times 49\times 17\times 13}$$

where $2<n\leq 50$.

Sol. P(N^{th} draw gives 2^{nd} ace)

$=P\{1$ ace and $(n–2)$ other cards are drawn in $(N–1)$draws$\}$

$\times P\{$Nth draw is 2nd ace$\}$

$$=\frac{4_{C_1}\times 48_{C_{n-2}}\times 3_{C_1}}{52_{C_{n-1}}\times(53-n)}$$

$$=\frac{4.(48)!.(n-1)!(53-n)!}{(52)!.(n-2)!(50-n)!}.\frac{3}{(53-n)}$$

$$=\frac{4(n-1)(52-n)(51-n).3}{52.51.50.49}$$

$$=\frac{(n-1)(52-n)(51-n)}{50.49.17.13}$$

Example – 29

A, B, C are events such that

$P_r(A)=0.3, P_r(B)=0.4, P_r(C)=0.8,$

$P_r(AB)=0.08, P_r(AC)=0.28, P_r(ABC)=0.09.$

If $P_r(A\cup B\cup C)\geq 0.75$, then show that $P_r(BC)$ lies in the interval $0.23\leq x\leq 0.48$.

Sol. We know that,

$P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-(C\cap A)$

$+P(A\cap B\cap C)=P(A\cup B\cup C)$

$\Rightarrow 0.3+0.4+0.8-\{0.08+0.28+P(BC)\}+0.09$

$=P(A\cup B\cup C)$

$\Rightarrow 1.23-P(BC)=P(A\cup B\cup C)$

where, $0.75\leq P(A\cup B\cup C)\leq 1$

$\Rightarrow 0.75\leq 1.23-P(BC)\leq 1$

$\Rightarrow -0.48\leq -P(BC)\leq -0.23$

$\Rightarrow 0.23\leq P(BC)\leq 0.48$

$\Rightarrow 0.23\leq x\leq 0.48$

Example – 30

In a certain city only two newspapers A and B are published, it is known that 25% of the city population reads A and 20% reads B, while 8% reads both A and B. It is also known that 30% of those who read A but not B look into advertisements and 40% of those who read B but not A look into advertisements while 50% of those who read both A and B look into advertisements. What is the percentage of the population reads an advertisement ?

Ans. (13.9%)

Sol. Let P(A) and P(B) denote respectively the percentage of city population that reads newspapers A and B. Then,

$$P(A)=\frac{25}{100}=\frac{1}{4}, P(B)=\frac{20}{100}=\frac{1}{5},$$

$$P(A\cap B)=\frac{8}{100}=\frac{2}{25},$$

$$P(A\cap B)=P(A)-P(A\cap B)=\frac{1}{4}-\frac{2}{25}=\frac{17}{100},$$

$$P(A\cap B)=P(B)-P(A\cap B)=\frac{1}{5}-\frac{2}{25}=\frac{3}{25}$$

Let P(C) be the probability that the population who reads advertisements.

$\therefore P(C)=30\%$ of $P(A\cap\overline{B})+40\%$ of $P(\overline{A}\cap B)$

$+50\%$ of $P(A\cap B)$

[since, $A\cap\overline{B}$, $\overline{A}\cap B$ and $A\cap B$ are all mutually exclusive]

$$\Rightarrow P(C)=\frac{3}{10}\times\frac{17}{100}+\frac{2}{5}\times\frac{3}{25}+\frac{1}{2}\times\frac{2}{25}=\frac{139}{1000}=13.9\%$$

Example – 31

In a multiple-choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks the correct answers. The candidate decides to tick the answers at random. If he is allowed upto three chances to answer the questions, find the probability that he will get marks in the question.

Ans. $\frac{1}{5}$

Sol. The total number of ways to answer the question

$= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1 = 15$

P(getting marks) = P(correct answer in I chance)

+ P(correct answer in II chance)

+ P(correct answer in III chance)

$$= \frac{1}{15} + \left(\frac{14}{15}.\frac{1}{14}\right) + \left(\frac{14}{15}.\frac{13}{14}.\frac{1}{13}\right) = \frac{3}{15} = \frac{1}{5}$$

Example – 32

A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one by one without replacement and are tested till all defective articles are found. What is the probability that the testing procedure ends at the twelfth testing ?

Ans. $\frac{99}{1900}$

Sol. The testing procedure may terminate at the twelfth testing in two mutually exclusive ways.

I : When lot contains 2 defective articles.

II : When lot contains 3 defective articles.

Let A = testing procedure ends at twelfth testing

A_1 = lot contains 2 defective articles

A_2 = lot contains 3 defective articles

$\therefore$ Required probability = $P(A_1)$. $P(A/A_1)$ + $P(A_2)$. $P(A/A_2)$

Here, $P(A/A_1)$ = probability that first 11 draws contains 10 non-defective and one-defective and twelfth draw contains a defective article.

$$= \frac{{}^{18}C_{10} \times {}^2C_1}{{}^{20}C_{11}} \times \frac{1}{9} \quad ...(i)$$

$P(A/A_2)$ = probability that first 11 draws contains

9 non-defective and 2-defective articles and twelfth draw

contains defective $= \frac{{}^{17}C_9 \times {}^3C_2}{{}^{20}C_{11}} \times \frac{1}{9}$ ...(ii)

$\therefore$ Requiredprobability $= (0.4)P(A/A_1) + 0.6P(A/A_2)$

$$= \frac{0.4 \times {}^{18}C_{10} \times {}^2C_1}{{}^{20}C_{11}} \times \frac{1}{9} + \frac{0.6 \times {}^{17}C_9 \times {}^3C_2}{{}^{20}C_{11}} \times \frac{1}{9} = \frac{99}{1900}$$

Example – 33

A man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.

Ans. $({}^{11}C_6(0.24)^5)$

Sol. The man will be one step away from the starting point, if (i) either he is one step ahead or (ii) one step behind the starting point

The man will be one step ahead at the end of eleven steps, if he moves six steps forward and five steps backward. The probability of this event is ${}^{11}C_6\ (0.4)^6\ (0.6)^5$.

The man will be one step behind at the end of eleven steps, if he moves six steps backward and five steps forward. The probability of this event is ${}^{11}C_6\ (0.6)^6\ (0.4)^5$.

$\therefore$ Required probability $= {}^{11}C_6(0.4)^6(0.6)^5$

$+ {}^{11}C_6(0.6)^6(0.4)^5 = {}^{11}C_6(0.24)^5$

Example – 34

An urn contains 2 white and 2 blacks balls. A ball is drawn at random. If it is white it is not replaced into the urn. Otherwise it is replaced along with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black.

Ans. $\frac{23}{30}$

Sol. Let B_i = ith ball drawn is black.

W_i = ith ball drawn is white, where $i = 1, 2$

and A = third ball drawn is black.

We observe that the black ball can be drawn in the third draw in one of the following mutually exclusive ways.

i) Both first and second balls drawn are white and third ball drawn is black.

i.e. $(W_1 \cap W_2) \cap A$

ii) Both first and second balls are black and third ball drawn is black.

i.e. $(B_1 \cap B_2) \cap A$

iii) The first ball drawn is white, the second ball drawn is black and the third ball drawn is black

i.e. $(W_1 \cap B_2) \cap A$

iv) The first ball drawn is black, the second ball drawn is white and the third ball drawn is black

i.e. $(B_1 \cap W_2) \cap A$

$\therefore P(A) = P[\{(W_1 \cap W_2) \cap A\} \cup \{(B_1 \cap B_2) \cap A\}$

$\cup \{(W_1 \cap B_2) \cap A\} \cup \{(B_1 \cap W_2) \cap A\}$

$= P[\{(W_1 \cap W_2) \cap A\} + P\{(B_1 \cap B_2) \cap A\}$

$+ P\{(W_1 \cap B_2) \cap A\} + P\{(B_1 \cap W_2) \cap A\}$

$= P\{(W_1 \cap W_2)\}.\ P(A/(W_1 \cap W_2)) + P(B_1 \cap B_2)$

$\therefore P(A/(B_1 \cap B_2)) + P(W_1 \cap B_2).\ P(A/(W_1 \cap B_2))$

$+ P(B_1 \cap W_2).\ P(A/(B_1 \cap W_2))$

$$= \left(\frac{2}{4} \times \frac{1}{3}\right) \times 1 + \left(\frac{2}{4} \times \frac{3}{5}\right) \times \frac{4}{6} + \left(\frac{2}{4} \times \frac{2}{3}\right) \times \frac{3}{4} + \left(\frac{2}{4} \times \frac{2}{5}\right) \times \frac{3}{4}$$

$$= \frac{1}{6} + \frac{1}{5} + \frac{1}{4} + \frac{3}{20} = \frac{23}{30}$$

Example – 35

A box contains 2 fifty paise coins, 5 twenty five paise coins and a certain fixed number n (≥ 2) of ten and five paise coins. Five coins are taken out of the box at random. Find the probability that the total value of these 5 coins is less than one rupee and fifty paise.

Ans. $1 - \dfrac{10\,(n+2)}{{}^{n+7}C_5}$

Sol. There are $(n + 7)$ coins in the box out of which five coin can be taken out in ${}^{n+7}C_5$ ways.

The total value of 5 coins can be equal to or more than one rupee and fifty paise in the following ways.

i) When one 50 paise coin and four 25 paise coins are chosen.

ii) When two 50 paise coins and three 25 paise coins are chosen.

iii) When two 50 paise coins, 2 twenty five paise coins and one from n coins of ten and five paise

$\therefore$ The total number of ways of selecting the coins

so that the total value of the coins is not less than one rupee and fifty paise is

$({}^2C_1 . {}^5C_4 . {}^nC_0) + ({}^2C_2 . {}^5C_3 . {}^nC_0) + ({}^2C_2 . {}^5C_2 . {}^nC_1) = 10 + 10 + 10n = 10\,(n + 2)$

So, the number of ways of selecting five coins, so that the total value of the coins is less than one rupee and fifty paise is ${}^{n+7}C_5 - 10\,(n + 2)$

$$\therefore \text{Required probability } = \frac{{}^{n+7}C_5 - 10(n+2)}{{}^{n+7}C_5}$$

$$= 1 - \frac{10(n+2)}{{}^{n+7}C_5}$$

Example – 36

Suppose the probability for A to win a game against B is 0.4. If A has an option of playing either a "best of 3 games" or a "best of 5 games" match against B, which option should choose so that the probability of his winning the match is higher ? (No game ends in a draw).

Ans. (best of 3 games)

Sol. **Case I:** When A plays 3 games against B.

In this case, we have $n = 3, p = 0.4$ and $q = 0.6$

Let X denote the number of wins. Then,

$P(X = r) = {}^nC_r p^r q^{n-r} = {}^3C_r (0.4)^r (0.6)^{3-r};\ r = 0, 1, 2, 3$

$\therefore P_1$ = probability of winning the best of 3 games

$= P(X \geq 2)$

$= P(X = 2) + P(X = 3)$

$= {}^3C_2 (0.4)^2 (0.6)^1 + {}^3C_3 (0.4)^3 (0.6)^0$

$= 0.288 + 0.064 = 0.352$

Case II: When A plays 5 games against B.

In this case, we have $n = 5, p = 0.4$ and $q = 0.6$

Let X denote the number of wins in 5 games.

Then,

$P(X = r) = {}^5C_r (0.4)^r (0.6)^{5-r};$ where $r = 0, 1, 2, \ldots, 5$

$\therefore P_2$ = probability of winning the best of 5 games

$= P(X \geq 3)$

$= P(X = 3) + P(X = 4) + P(X = 5)$

$= {}^5C_3 (0.4)^3 (0.6)^2 + {}^5C_4 (0.4)^4 (0.6) + {}^5C_5 (0.4)^5 (0.6)^0$

$= 0.2304 + 0.0768 + 0.1024 = 0.31744$

Clearly, $P_1 > P_2$. Therefore, first option i.e. 'best of 3 games' has higher probility of winning the match.

Example – 37

An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4, ..., 12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8 ?

Ans. $\frac{193}{792}$

Sol. Let, E_1 = the event that the noted number is 7

E_2 = the event that the noted number is 8

H = getting head on coin

T = getting tail on coin

$\therefore$ By law of total probability,

$P(E_1) = P(H).\ P(E_1/H) + P(T).\ P(E_1/T)$

and $P(E_2) = P(H).\ P(E_2/H) + P(T).\ P(E_2/T)$

Where, $P(H) = 1/2 = P(T)$

$P(E_1/H)$ = probability of getting a sum of 7 on two dice.

Here, favourable cases are

$\{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}$.

$$\therefore P(E_1/H) = \frac{6}{36} = \frac{1}{6}$$

Also, $P(E_1/T)$ = probability of getting 7 numbered card out of 11 cards

$$= \frac{1}{11}$$

$P(E_2/H)$ = probability of getting a sum of 8 on two dice

Here, favourable cases are

$\{(2, 6), (6, 2), (4, 4), (5, 3), (3, 5)\}$

$$\therefore P(E_2/H) = \frac{5}{36}$$

$P(E_2/T)$ = probability of getting ‘8’ numbered card out of 11 cards

$= 1/11$

$$\therefore P(E_1) = \left(\frac{1}{2} \times \frac{1}{6}\right) + \left(\frac{1}{2} \times \frac{1}{11}\right) = \frac{1}{12} + \frac{1}{22} = \frac{17}{132}$$

$$\text{and } P(E_2) = \left(\frac{1}{2} \times \frac{5}{36}\right) + \left(\frac{1}{2} \times \frac{1}{11}\right) = \frac{1}{2}\left(\frac{91}{396}\right) = \frac{91}{792}$$

Now, E_1 and E_2 are mutually exclusive events.

Therefore,

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) = \frac{17}{132} + \frac{91}{792} = \frac{193}{792}$$

Example – 38

In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back ? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats ? Now, if all the seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on adjacent seats ?

Ans. $\frac{14!}{2!}, 2 \times 2 \times 3! \times {}^{11}P_9, \frac{1}{91}$

Sol. We have 14 seats in two vans and there are 9 boys and 3 girls. The number of ways of arranging 12 people on 14 seats without restriction is

$${}^{14}P_{12} = \frac{14!}{2} = 7(13!)$$

Now, the number of ways of choosing back seats is 2. and the number of ways of arranging 3 girls on adjacent seats is 2(3!) and the number of ways of arranging 9 boys on the remaining 11 seats is ${}^{11}P_9$ ways.

Therefore, the required number of ways.

$$= 2.(2.3!).{}^{11}P_9 = \frac{4.3!11!}{2!} = 12!$$

Hence, the probability of the required event

$$= \frac{12!}{7.13!} = \frac{1}{91}$$

Example – 39

Sixteen players $S_1, S_2, ..., S_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength.

(a) Find the probability that the players S_1 is among the eight winners.

(b) Find the probability that exactly one of the two players S_1 and S_2 is among the eight winners.

Ans. $(a)\frac{1}{2}(b)\frac{8}{15}$

Sol. i) Probability of S_1 to be among the eight winners

= (Probability of S_1 being a pair) × (Probability of S_1 winning in the group)

$= 1\times\frac{1}{2}=\frac{1}{2}$ [since, S_1 is definitely in a group)

ii) If S_1 and S_2 are in the same pair, then exactly one wins.

If S_1 and S_2 are in two pairs separately, then exactly one of S_1 and S_2 will be among the eight winners. If S_1 wins and S_2 loses or S_1 loses and S_2 wins.

Now, the probability of S_1, S_2 being in the same pair and one wins

= (Probability of S_1, S_2 being in the same pair)

× (Probability of any one winning in the pair)

and the probability of S_1, S_2 being the same pair $= \frac{n(E)}{n(S)}$

where, $n(E)$ = the number of ways in which 16 persons can be divided in 8 pairs.

$\therefore n(E) = \frac{(14)!}{(2!)^7.7!}$ and $n(S) = \frac{(16)!}{(2!)^8.\,8!}$

$\therefore$ Probability of S_1, S_2 being in the same pair

$$= \frac{(14)!.(2!)^8.\,8!}{(2!)^7.7!.\,(16)!} = \frac{1}{15}$$

The probability of any one wining in the pairs of S_1, S_2 = P (certain event) = 1

$\therefore$ The pairs of S_1, S_2 being in two pairs separately and S_1 wins, S_2 loses + The probability of S_1, S_2 being in two pairs separately and S_1 loses, S_2 wins.

$$= \left[1 - \frac{\frac{(14)!}{(2!)^7.7!}}{\frac{(16)!}{(2!)^8.\,8!}}\right]\times\frac{1}{2}\times\frac{1}{2} + \left[1 - \frac{\frac{(14)!}{(2!)^7.7!}}{\frac{(16)!}{(2!)^8.\,8!}}\right]\times\frac{1}{2}\times\frac{1}{2}$$

$$= \frac{1}{2}\times\frac{14\times(14)!}{15\times(14)!} = \frac{7}{15}$$

$\therefore$ Required probability $= \frac{1}{15}+\frac{7}{15} = \frac{8}{15}$

Example – 40

A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of the determinant chosen is positive, is....

Ans. $\frac{3}{16}$

Sol. Since, determinat is of order 2 × 2 and each element is 0 or 1 only.

$\therefore n(S) = 2^4 = 16$

and the determinat is positive are $\begin{vmatrix}1 & 0\\0 & 1\end{vmatrix}, \begin{vmatrix}1 & 1\\0 & 1\end{vmatrix}, \begin{vmatrix}1 & 0\\1 & 1\end{vmatrix}$

$\therefore n(E) = 3$

Thus, the required probability $= \frac{3}{16}$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Classical Probability

1. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house, is

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$

(c) $\frac{7}{9}$ (d) $\frac{8}{9}$

2. Fifteen coupons are numbered 1, 2, ..., 15, respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is

(a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$

(c) $\left(\frac{3}{5}\right)^7$ (d) None of these

3. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently, equals

(a) $\frac{1}{2}$ (b) $\frac{7}{15}$

(c) $\frac{2}{15}$ (d) $\frac{1}{3}$

4. An Urn contains 'm' white and 'n' black balls. All the balls except for one ball, are drawn from it. The probability that the last ball remaining in the Urn is white, is

(a) $\frac{m}{m+n}$ (b) $\frac{n}{m+n}$

(c) $\frac{1}{(m+n)!}$ (d) $\frac{mn}{(m+n)!}$

5. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral equals

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$

(c) $\frac{1}{10}$ (d) $\frac{1}{20}$

6. $x_1, x_2, x_3, ... x_{50}$ are fifty numbers such that $x_r < x_{r+1}$ for $r = 1, 2, 3, ..., 49$. Five numbers out of these are picked up at random. The probability that the five numbers have x_{20} as the middle number is

(a) $\frac{{}^{20}C_2 \times {}^{30}C_2}{{}^{50}C_2}$ (b) $\frac{{}^{30}C_2 \times {}^{19}C_2}{{}^{50}C_5}$

(c) $\frac{{}^{19}C_2 \times {}^{31}C_3}{{}^{50}C_5}$ (d) none the these

7. The probability that the birthdays of six different people will fall in exactly two calendar months is

(a) $\frac{1}{6}$ (b) ${}^{12}C_2 \times \frac{2^6}{12^6}$

(c) ${}^{12}C_2 \times \frac{2^6-1}{12^6}$ (d) $\frac{341}{12^5}$

8. A, B, C are three events for which $P(A) = 0.6$, $P(B) = 0.4$, $P(C) = 0.5$, $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$ and $P(A \cap B \cap C) = 0.2$. If $P(A \cup B \cup C) \geq 0.85$ then the interval of values of $P(B \cap C)$ is

(a) [0.2, 0.35] (b) [0.55, 0.7]

(c) [0.2, 0.55] (d) none of these

9. A and B are two events. Odds against A are 2 : 1. Odds in favour of $A \cup B$ are 3 : 1. If $x \leq P(B) \leq y$, then the ordered pair (x, y) is

(a) $\left(\frac{5}{12}, \frac{3}{4}\right)$ (b) $\left(\frac{2}{3}, \frac{3}{4}\right)$

(c) $\left(\frac{1}{3}, \frac{3}{4}\right)$ (d) none of these

10. If $P(B)=\frac{3}{5}, P(A/B)=\frac{1}{2}$ and $P(A\cup B)=\frac{4}{5}$, then $P((A\cup B)')+P(A'\cup B)=$

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{2}$ (d) 1

Multiplication Theorem & Independent Events

11. A problem in mathematics is given to three student A, B, C and their respective probability of solving the problem is 1/2, 1/3 and 1/4. Probability that the problem is solved, is

(a) 3/4 (b) 1/2

(c) 2/3 (d) 1/3

12. The probability that A speaks truth is 4/5, while this probability for B is 3/4. The probability that they contradict each other when asked to speak on a fact, is

(a) 7/20 (b) 1/5

(c) 3/20 (d) 4/5

13. If A and B are independent events such that $0<P(A)<1$ and $0<P(B)<1$, then which of the following is not correct ?

(a) A and B are mutually exclusive

(b) A and B' are independent

(c) A' and B are independent

(d) A' and B' are independent

14. Let A and B be two events such that

$P(\overline{A\cup B})=\frac{1}{6}, P(A\cap B)=\frac{1}{4}$ and

$P(\bar{A})=\frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are :

(a) independent and equally likely

(b) mutually exclusive and independent

(c) equally likely but not independent

(d) independent but not equally likely

15. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black balls will be drawn, is

(a) $\frac{13}{32}$ (b) $\frac{1}{4}$

(c) $\frac{1}{32}$ (d) $\frac{3}{16}$

16. Three persons A, B and C fire at a target in turn, starting with A. Their probabilities of hitting the target are 0.4, 0.3 and 0.2 respectively. The probability of two hits is

(a) 0.024 (b) 0.188

(c) 0.336 (d) 0.452

17. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is

(a) $\frac{2}{21}$ (b) $\frac{3}{28}$

(c) $\frac{1}{28}$ (d) $\frac{167}{168}$

18. The probability that certain electronic component fails when first used is 0.10. If it does not fail immediately, the probability that it lasts for one year is 0.99. The probability that a new component will last for one year is

(a) 0.891 (b) 0.692

(c) 0.92 (d) none of these

19. The probability that an event A happens in one trial of an experiment is 0.4. Three independent trials of the experiments are performed. The probability that the event A happens at least once is

(a) 0.936 (b) 0.784

(c) 0.904 (d) None of these

20. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting points 0, 1 and 2 are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points, is

(a) 0.8750 (b) 0.0875

(c) 0.0625 (d) 0.0250

21. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5, is

(a) 16/81 (b) 1/81

(c) 80/81 (d) 65/81

22. A coin is tossed n times. The probability of getting at least one head is greater than that of getting at least two tails by $\frac{5}{32}$. Then n is

(a) 5 (b) 10

(c) 15 (d) none of these

23. An unbiased cubic die marked with 1, 2, 2, 3, 3, 3 is rolled 3 times. The probability of getting a total score of 4 or 6 is

(a) $\frac{16}{216}$ (b) $\frac{50}{216}$

(c) $\frac{60}{216}$ (d) none

24. A Urn contains 'm' white and 'n' black balls. Balls are drawn one by one till all the balls are drawn. Probability that the second drawn ball is white, is

(a) $\frac{m}{m+n}$ (b) $\frac{n(m+n-1)}{(m+n)(m+n-1)}$

(c) $\frac{m(m-1)}{(m+n)(m+n-1)}$ (d) $\frac{mn}{(m+n)(m+n-1)}$

Conditional Probability

25. The probabilities of four cricketers A, B, C and D scoring more than 50 runs in a match are $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ and $\frac{1}{10}$. It is known that exactly two of the players scored more than 50 runs in a particular match. The probability that players were A and B is

(a) $\frac{27}{65}$ (b) $\frac{5}{6}$

(c) $\frac{1}{6}$ (d) none of these

26. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has atleast one girl is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{4}{7}$

27. Let $A = \{2, 3, 4, ..., 20, 21\}$. A number is chosen at random from the set A and it is found to be a prime number. The probability that it is more than 10 is

(a) $\frac{9}{10}$ (b) $\frac{1}{10}$

(c) $\frac{1}{2}$ (d) none of these

28. Three distinguishable balls are distributed in three cells. The probability that all three occupy the same cell, given that atleast two of them are in the same cell, is

(a) $\frac{1}{7}$ (b) $\frac{1}{9}$

(c) $\frac{1}{6}$ (d) none of these

29. In a certain town, 40% of the people have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. If a person selected at random from the town, has brown hair, the probability that he also has brown eyes, is

(a) $\frac{1}{5}$ (b) $\frac{3}{8}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

30. A pair of numbers is picked up randomly (without replacement) from the set $\{1, 2, 3, 5, 7, 11, 12, 13, 17, 19\}$. The probability that the number 11 was picked given that the sum of the numbers was even, is nearly :

(a) 0.1 (b) 0.125

(c) 0.24 (d) 0.18

31. For a biased die the probabilities for the diffferent faces to turn up are given below :

Faces :	1	2	3	4	5	6
Probabilities :	0.10	0.32	0.21	0.15	0.05	0.17

The die is tossed & you are told that either face one or face two has turned up. Then the probability that it is face one is :

(a) 1/6 (b) 1/10

(c) 5/49 (d) 5/21

32. The probability that an automobile will be stolen and found within one week is 0.0006. The probability that an automobile will be stolen is 0.0015. The probability that a stolen automobile will be found in one week is

(a) 0.3 (b) 0.4

(c) 0.5 (d) 0.6

Total Probability Law and Baye's Theorem

33. A letter is known to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that it came from LONDON is

(a) $\frac{5}{17}$ (b) $\frac{12}{17}$

(c) $\frac{17}{30}$ (d) $\frac{3}{5}$

34. For k = 1, 2,3 the box B_k contains k red balls and (k + 1) white balls. Let $P(B_1) = \frac{1}{2}$, $P(B_2) = \frac{1}{3}$ and $P(B_3) = \frac{1}{6}$. A box is selected at random and a ball is drawn from it. If a red ball is drawn, then the probability that it has come from box B_2, is

(a) $\frac{35}{78}$ (b) $\frac{14}{39}$

(c) $\frac{10}{13}$ (d) $\frac{12}{13}$

35. In an entrance test there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing, is

(a) $\frac{37}{40}$ (b) $\frac{1}{37}$

(c) $\frac{36}{37}$ (d) $\frac{1}{9}$

36. Two coins are available, one fair and the other two headed. Choose a coin and toss it once assume that the unbiased coin is chosen with probability $\frac{3}{4}$. Given that the outcome is head, the probability that the two-headed coin was chosen is

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$

(c) $\frac{1}{5}$ (d) $\frac{2}{7}$

37. A man is known to speak truth 3 out of 4 times. He throws a dice and reports that it is six. The probability that it is actually six is

(a) $\frac{3}{8}$ (b) $\frac{1}{5}$

(c) $\frac{3}{5}$ (d) none of these

38. One bag contains 5 white and 4 black balls. Another bag contains 7 white and 9 black balls. A ball is transferred from the first bag to the second and then a ball is drawn from second. The probability that the ball is white, is

(a) 8/17 (b) 40/153

(c) 5/9 (d) 4/9

39. Three groups A, B, C are competing for positions on the Board of Directors of a company. The probabilities of their winning are 0.5, 0.3, 0.2 respectively. If the group A wins, the probability of introducing a new product is 0.7 and the corresponding probabilities for group B and C are 0.6 and 0.5 respectively. The probability that the new product will be introduced, is

(a) 0.18 (b) 0.35

(c) 0.10 (d) 0.63

40. A survey of people in a given region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was 10 times the probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the region is 0.006, what is the probability of death due to lung cancer given that a person is a smoker

(a) 1/140 (b) 1/70

(c) 3/140 (d) 1/10

41. A, B and C are contesting the election for the post of secretary of a club which does not allow ladies to become members. The probabilities of A, B and C winning the election are $\frac{1}{3}, \frac{2}{9}$ and $\frac{4}{9}$ respectively. The probabilities of introducing the clause of admitting lady members to the club by A, B and C are 0.6, 0.7 and 0.5 respectively. The probability that ladies will be taken as members in the club after the election is

(a) $\frac{26}{45}$ (b) $\frac{5}{9}$

(c) $\frac{19}{45}$ (d) none of these

42. A certain player, say X, is know to win with probability 0.3 if the track is fast and 0.4 if the track is slow. For Monday, there is a 0.7 probability of a fast-track and 0.3 probability of slow track. The probability that player X will win on Monday, is

(a) 0.22 (b) 0.11

(c) 0.33 (d) none of these

Bernoulli trials and binomial distribution

43. A coin is tossed 7 times. Each time a man calls head. The probability that he wins the toss on more occasions is

(a) $\frac{1}{4}$ (b) $\frac{5}{8}$

(c) $\frac{1}{2}$ (d) none of these

44. If X and Y are the independent random variables B $\left(5, \frac{1}{2}\right)$ and B $\left(7, \frac{1}{2}\right)$, then $P(X+Y \geq 1) =$

(a) $\frac{4095}{4096}$ (b) $\frac{309}{4096}$

(c) $\frac{4032}{4096}$ (d) none of these

45. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then $P(X=1)$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{32}$

(c) $\frac{1}{16}$ (d) $\frac{1}{8}$

46. 6 ordinary dice are rolled. The probability that at least half of them will show at least 3 is

(a) $41 \times \frac{2^4}{3^6}$ (b) $\frac{2^4}{3^6}$

(c) $20 \times \frac{2^4}{3^6}$ (d) none of these

47. The probability of guessing correctly atleast 8 out of 10 answers on a true-false type examination is

(a) $\frac{7}{64}$ (b) $\frac{7}{128}$

(c) $\frac{45}{1024}$ (d) $\frac{7}{41}$

48. Suppose that a random variable X follows Binomial distribution with parameters n and p, where $0 < p < 1$. If $P(X=r)/P(X=n-r)$ is independent of n and r, then p is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{5}$ (d) $\frac{1}{7}$

49. How many times must a man toss a fair coin so that the probability of having atleast one head is more than 90%?

(a) 2 (b) 3

(c) 4 (d) 5

50. One hundred identical coins, each with probability, p, of showing up heads are tosses once. If $0 < p < 1$ and the probability of heads showing on fifty coins is equal to that of heads showing on 51 coins, then the value of p is:

(a) 1/2 (b) 49/101
(c) 50/101 (d) 51/101

Misc Examples-Probability

51. An unbiased die is tossed until a number greater than 4 appears. The probability that an even number of tosses is needed is

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$

(c) $\frac{1}{5}$ (d) $\frac{2}{3}$

52. Ram and Shyam throw with one dice for a prize of Rs 88 which is to be won by the player who throws 1 first. If Ram starts, then mathematical expectation for Shyam is

(a) Rs 32 (b) Rs 40
(c) Rs 48 (d) none of these

53. A square is inscribed in a circle. If p_1 is the probability that a randomly chosen point of the circle lies within the square and p_2 is the probability that the point lies outside the square then

(a) $p_1 = p_2$

(b) $p_1 > p_2$ and $p_1^2 - p_2^2 < \frac{1}{3}$

(c) $p_1 < p_2$

(d) none of these

54. A pair of dice is rolled again and again till a total of 5 or a total of 7 is obtained. The chance that a total of 5 comes before a total of 7 is

(a) $\frac{2}{5}$ (b) $\frac{3}{7}$

(c) $\frac{3}{13}$ (d) none of these

55. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that value of the determinant chosen is positive is

(a) $\frac{16}{81}$ (b) $\frac{7}{16}$

(c) $\frac{3}{16}$ (d) none of these

56. If the integers m and n belongs to set of first hundred natural numbers then the probability that a number of the form $7^m + 7^n$ is divisible by 5 is

(a) $\frac{1}{5}$ (b) $\frac{1}{7}$

(c) $\frac{1}{4}$ (d) $\frac{1}{49}$

57. The number 'a' is randomly selected from the set $\{0, 1, 2, 3, ... 98, 99\}$. The number 'b' is selected from the same set. Probability that the number $3^a + 7^b$ has a digit equal to 8 at the units place, is

(a) $\frac{1}{16}$ (b) $\frac{2}{16}$

(c) $\frac{4}{16}$ (d) $\frac{3}{16}$

58. Four tickets marked 00, 01, 10 and 11 respectively are placed in bag. A ticket is drawn at random five times, being replaced each time. The probability that the sum of the numbers on the ticket is 15, is

(a) $\frac{3}{1024}$ (b) $\frac{5}{1024}$

(c) $\frac{7}{1024}$ (d) none of these

59. 2n boys are randomly divided into two subgroups containing n boys each. The probability that the two tallest boys are in different groups is

(a) $\frac{n}{2n-1}$ (b) $\frac{n-1}{2n-1}$

(c) $\frac{2n-1}{4n^2}$ (d) none of these

60. 5 girls and 10 boys sit at random in a row having 15 chairs numbered as 1 to 15. Find the probability that end seats are occupied by the girls and between any two girls odd number of boys sit, is

(a) $\frac{20 \times 10! \times 5!}{15!}$ (b) $\frac{20 \times 10!}{15!}$

(c) $\frac{20 \times 5!}{15!}$ (d) none of these

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. If A and B are any two events such that $P(A)=\frac{2}{5}$ and $P(A\cap B)=\frac{3}{20}$, then the conditional, probability, $P(A|(A'\cup B'))$, where A' denotes the complement of A, is equal to : **(2016/Online Set–1)**

(a) $\frac{1}{4}$ (b) $\frac{5}{17}$

(c) $\frac{8}{17}$ (d) $\frac{11}{20}$

2. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is : **(2016/Online Set–2)**

(a) $\frac{240}{729}$ (b) $\frac{192}{729}$

(c) $\frac{256}{729}$ (d) $\frac{496}{729}$

3. For three events A, B and C, P(Exactly one of A or B occurs)

= P(Exactly one of B or C occurs)

= P(Exactly one of C or A occurs) $=\frac{1}{4}$ and P (All the three events occur simultaneously) $=\frac{1}{16}$.

Then the probability that at least one of the events occurs, is: **(2017)**

(a) $\frac{7}{32}$ (b) $\frac{7}{16}$

(c) $\frac{7}{64}$ (d) $\frac{3}{16}$

4. If two different numbers are taken from the set $\{0, 1, 2, 3, \ldots\ldots, 10\}$; then the probability that their sum as well as absolute difference are both multiple of 4, **(2017)**

(a) $\frac{6}{55}$ (b) $\frac{12}{55}$

(c) $\frac{14}{45}$ (d) $\frac{7}{55}$

5. Three persons P, Q and R independently try to hit a target. If the probabilities of their hitting the target are $\frac{3}{4}, \frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is : **(2017)**

(a) $\frac{21}{64}$ (b) $\frac{9}{64}$

(c) $\frac{15}{64}$ (d) $\frac{39}{64}$

6. An unbiased coin is tossed eight times. The probability of obtaining at least one head and at least one tail is : **(2017)**

(a) $\frac{255}{256}$ (b) $\frac{127}{128}$

(c) $\frac{63}{64}$ (d) $\frac{1}{2}$

7. From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one woman. Then the probability for these committees to have more women than men, is **(2017)**

(a) $\frac{21}{220}$ (b) $\frac{3}{11}$

(c) $\frac{1}{11}$ (d) $\frac{2}{23}$

8. Let E and F be two independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then a value of $\frac{P(E)}{P(F)}$ is: **(2017)**

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$

(c) $\frac{1}{3}$ (d) $\frac{5}{12}$

9. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is: **(2018)**

(a) $\frac{3}{4}$ (b) $\frac{3}{10}$

(c) $\frac{2}{5}$ (d) $\frac{1}{5}$

10. A box 'A' contains 2 white, 3 red and 2 black balls. Another box 'B' contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box 'B' is : **(2018/Online Set–1)**

(a) $\frac{9}{16}$ (b) $\frac{7}{16}$

(c) $\frac{9}{32}$ (d) $\frac{7}{8}$

11. A player X has a biased coin whose probability of showing heads is p and a player Y has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If X starts the game, and the probability of winning the game by both the players is equal, then the value of 'p' is: **(2018/Online Set–2)**

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$

(c) $\frac{2}{5}$ (d) $\frac{1}{4}$

12. Two different families A and B are blessed with equal number of children. There are 3 tickets to be distributed amongst the children of these families so that no child gets more than one ticket. If the probability that all the tickets go to the children of the family B is $\frac{1}{12}$, then the number of children in each family is : **(2018/Online Set–3)**

(a) 3 (b) 4

(c) 5 (d) 6

13. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is: **(2019-04-08/Shift-2)**

(a) 5 (b) 3

(c) 4 (d) 2

14. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is: **(2019-04-09/Shift-1)**

(a) $\frac{25}{192}$ (b) $\frac{7}{32}$

(c) $\frac{1}{192}$ (d) $\frac{25}{32}$

15. Minimum number of times a fair coinmust be tossed so that the probability of getting at least one head is more than 99% is: **(2019-04-10/Shift-2)**

(a) 5 (b) 6

(c) 8 (d) 7

16. If three of the six vertices of a regular hexagon are chosen at random, then theprobability that the triangle formed with these chosen vertices is equilateral is : **(2019-04-12/Shift-1)**

(a) $\frac{1}{10}$ (b) $\frac{1}{5}$

(c) $\frac{3}{10}$ (d) $\frac{3}{20}$

17. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X \le 2) = \frac{k}{2^{16}}$, then k is equal to ______. **(2019-04-12/Shift-1)**

18. For an initial screening of an admission test, a candidate is given fifty problemsto solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable tosolve less than two problems is : **(2019-04-12/Shift-2)**

(a) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$ (b) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$

(c) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$ (d) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$

19. A person throws two fair dice. He winsRs. 15 for throwing a doublet (same numbers on the two dice), wins Rs. 12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcomeon the throw. Then the expected gain/loss (in Rs.) of the person is _____. **(2019-04-12/Shift-2)**

(a) $\frac{1}{2}$ gain (b) $\frac{1}{4}$ loss

(c) $\frac{1}{2}$ loss (d) 2 gain

20. Two cards are drawn successively with replacement from a well-shuflled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals: **(2019-01-09/Shift-1)**

(a) 49/169 (b) 52/169

(c) 24/169 (d) 25/169

21. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn . If the drawn ball is green, then a red ball is added to the urn and if the drawnball is red, then a green ball is added to the urn; the original ball is not returnedto the urn. Now, a second ball is drawnat random from it. The probability thatthe second ball is red is: **(2019-01-09/Shift-2)**

(a) $\frac{21}{49}$ (b) $\frac{27}{49}$

(c) $\frac{26}{49}$ (d) $\frac{32}{49}$

22. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rol l ed and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of ninecards num bered 1,2,3, …, 9 is randomly picked and the number on the card is noted. The probability that thenoted number is either 7 or 8 is: **(2019-01-10/Shift-1)**

(a) $\frac{13}{36}$ (b) $\frac{15}{72}$

(c) $\frac{19}{72}$ (d) $\frac{19}{36}$

23. If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is **(2019-01-10/Shift-2)**

(a) 3 (b) 6

(c) 5 (d) 4

24. Two integers are selected at randomfrom the set $\{1, 2, ..., 11\}$. Given thatthe sum of selected numbers is even,the conditional probability that both the numbers are even is: **(2019-01-11/Shift-1)**

(a) $\frac{7}{10}$ (b) $\frac{1}{2}$

(c) $\frac{2}{5}$ (d) $\frac{3}{5}$

25. A bag contains 30 white balls and 10red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn,the $\left(\frac{\text{mean of X}}{\text{standard deviation of X}}\right)$ is equal to: **(2019-01-11/Shift-2)**

(a) 4 (b) $4\sqrt{3}$

(c) $3\sqrt{2}$ (d) $\frac{4\sqrt{3}}{3}$

26 Let S = {l, 2, ... , 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Than the probability that a randomly chosen subset of S is "nice" is : **(2019-01-11/Shift-2)**

(a) $\frac{7}{2^{20}}$ (b) $\frac{5}{2^{20}}$

(c) $\frac{4}{2^{20}}$ (d) $\frac{6}{2^{20}}$

27. In a game, a man wins Rs100 if he gets 5 or 6 on a throw of a fair die and loses Rs 50 forgetting any other number on the die. If hedecides to throw the die either till he gets a fiveor a six or to a maximum of three throws, then his expected gain/loss (in rupees) is:

(2019-01-12/Shift-2)

(a) $\frac{400}{9}$ loss (b) 0

(c) $\frac{400}{3}$ gain (d) $\frac{400}{3}$ loss

28. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the studentSelected has opted neither for NCC norfor NSS is : **(2019-01-12/Shift-2)**

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{5}{6}$

29. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawnfrom it. The number on the card isfound to be a non-prime number. The probability that the card was drawn from Box I is :

(2020-09-02/Shift-1)

(a) $\frac{4}{17}$ (b) $\frac{8}{17}$

(c) $\frac{2}{5}$ (d) $\frac{2}{3}$

30. Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwiseindependent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$. Then is equalto : **(2020-09-02/Shift-2)**

(a) $P(E_3^C) - P(E_2^C)$ (b) $P(E_3) - P(E_2^C)$

(c) $P(E_3^C) - P(E_2)$ (d) $P(E_2^C) + P(E_3)$

31. A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is : **(2020-09-03/Shift-1)**

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{9}$

32. The probability that a randomly chosen 5-digit number is made from exactly two digits is :

(2020-09-03/Shift-2)

(a) $\frac{134}{10^4}$ (b) $\frac{121}{10^4}$

(c) $\frac{135}{10^4}$ (d) $\frac{150}{10^4}$

33. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is…. **(2020-09-04/Shift-1)**

34. In a game two players A and B take turns in throwing a pair of fair dice starting withplayer A and total of scores on the two dice, in each throw is noted. A wins the game ifhe throws a total of 6 before B throws a total of 7 and B wins the game if he throws atotal of 7 before A throws a total of six. The game stops as soon as either of the playerswins. The probability of A winning the game is:

(2020-09-04/Shift-2)

(a) $\frac{5}{31}$ (b) $\frac{31}{61}$

(c) $\frac{30}{61}$ (d) $\frac{5}{6}$

35. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice shown up a three or a five, is ....

(2020-09-05/Shift-1)

36. In a bombing attack, there is 50% chance that a bomb will hit the target. At least two indepen dent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is________

(2020-09-05/Shift-2)

37. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is: **(2020-09-06/Shift-1)**

(a) $\frac{10}{99}$ (b) $\frac{5}{33}$

(c) $\frac{15}{101}$ (d) $\frac{5}{101}$

38. The probabilities of three events A, B and C are given by P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$,

$P(A \cap C) = 0.3, P(A \cap B \cap C) = 0.2,$

$P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where

$0.85 \le \alpha \le 0.95$, then β lies in the interval:

(2020-09-06/Shift-2)

(a) [0.36,0.40] (b) [0.25, 0.35]

(c) [0.35, 0.36] (d) [0.20, 0.25]

39. An unbiased coin is tossed 5 times. Suppose that a variable x is assigned the value k when k consecutive heads are obtained for k = 3, 4, 5, otherwise x takes the value -1. The expected value of x, is

(2020-01-07/Shift-3)

(a) $\frac{1}{8}$ (b) $\frac{3}{16}$

(c) $-\frac{1}{8}$ (d) $-\frac{3}{16}$

40. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3$ k, then k is equal to : **(2020-01-07/Shift-2)**

(a) $\frac{17}{2}$ (b) 4

(c) $\frac{17}{4}$ (d) $\frac{17}{8}$

41. Let A and B be two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{6}$. Then, which of the following is TRUE? **(2020-01-08/Shift-1)**

(a) $P(A/(A \cup B)) = \frac{1}{4}$ (b) $P(A/B') = \frac{1}{3}$

(c) $P(A/B) = \frac{2}{3}$ (d) $P(A'/B') = \frac{1}{3}$

42. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and theprobability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is **(2020-01-08/Shift-2)**

(a) 0.10 (b) 0.20

(c) 0.01 (d) 0.02

43. In a box, there are 20 cards out of which 10 are labelled as A and remaining 10 are labelled as B Cards are drawn at random, one after the other and with replacement, till a second A-card isobtained. The probability that the second A-card appears before the third B-card is:

(2020-01-09/Shift-1)

(a) 15/16 (b) 9/16

(c) 13/16 (d) 11/16

44. If 10 different balls has to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is :**(2020-01-09/Shift-2)**

(a) $\frac{965}{2^{10}}$ (b) $\frac{945}{2^{10}}$

(c) $\frac{945}{2^{11}}$ (d) $\frac{965}{2^{11}}$

45. A random variable X has the following probability distribution:

X	1	2	3	4	5
P(X)	K^2	2K	K	2K	$5K^2$

Then $P(X>2)$ is equal to **(2020-01-09/Shift-2)**

(a) $\frac{7}{12}$ (b) $\frac{23}{36}$

(c) $\frac{1}{36}$ (d) $\frac{1}{6}$

46. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is: **(2021-07-20/Shift-1)**

(a) $\frac{1}{9}$ (b) $\frac{1}{66}$

(c) $\frac{2}{11}$ (d) $\frac{1}{11}$

47. The probability of selecting integers $a \in [-5,30]$ such that $x^2+2(a+4)x-5a+64>0$, for all $x \in R$, is:

(2021-07-20/Shift-1)

(a) $\frac{1}{4}$ (b) $\frac{7}{36}$

(c) $\frac{2}{9}$ (d) $\frac{1}{6}$

48. Let A, B and C be three events such that the probability that exactly one of A and B occurs is (1–k), the probability that exactly one of B and C occurs is (1–2k), the probability that exactly one of C and A occurs is (1–k) and the probability of all A, B and C occur simultaneously is k^2, where $0<k<1$. Then the probability that at least one of A, B and C occur is ?

(2021-07-20/Shift-2)

(a) Greater than $\frac{1}{2}$

(b) Greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

(c) Exactly equal to $\frac{1}{2}$

(d) Greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

49. Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability that B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ then k lies in the set ?

(2021-07-25/Shift-1)

(a) $\{x \in R : |x-5| \le 1\}$ (b) $\{x \in R : |x-2| \le 1\}$

(c) $\{x \in R : |x-3| < 1\}$ (d) $\{x \in R : |x-1| < 1\}$

50. A student appeared in an examination consisting of 8 true – false type questions. The student guesses the answers with equal probability. The smallest value of n, so that the probability of guessing at least 'n' correct answers is less than $\frac{1}{2}$, is: **(2021-07-27/Shift-2)**

(a) 5 (b) 3

(c) 6 (d) 4

51. Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2×2 matrices. The probability that such formed matrices have all different entries and are non-singular, is:

(2021-07-22/Shift-2)

(a) $\frac{23}{81}$ (b) $\frac{22}{81}$

(c) $\frac{45}{162}$ (d) $\frac{43}{162}$

52. Let X be a random variable such that the probability function of a distribution is given by $P(X=0)=\frac{1}{2}$, $P(X=j)=\frac{1}{3^j}(j=1,2,3,...,\infty)$. Then the mean of the distribution and P(X is positive and even) respectively are: **(2021-07-25/Shift-2)**

(a) $\frac{3}{4}$ and $\frac{1}{9}$ (b) $\frac{3}{4}$ and $\frac{1}{16}$

(c) $\frac{3}{8}$ and $\frac{1}{8}$ (d) $\frac{3}{4}$ and $\frac{1}{8}$

53. A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is ____ . **(2021-07-25/Shift-2)**

54. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(x \geq 5 | x > 2)$ is: **(2021-08-26/Shift-2)**

(a) $\frac{25}{36}$ (b) $\frac{11}{36}$

(c) $\frac{125}{216}$ (d) $\frac{5}{6}$

55. When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6}-x$ and its opposite face occurs with probability $\frac{1}{6}+x$. All other faces occur with probability $\frac{1}{6}$. Note that opposite faces sum to in any die. If $0<x<\frac{1}{6}$, and the probability of obtaining total sum $=7$, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is: **(2021-08-27/Shift-1)**

(a) $\frac{1}{9}$ (b) $\frac{1}{12}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

56. Let A and B be independent events such that $P(A)=p$, $P(B)=2p$. The largest value of p, for which P (exactly one of A, B occurs) $=\frac{5}{9}$, is: **(2021-08-26/Shift-1)**

(a) $\frac{4}{9}$ (b) $\frac{2}{9}$

(c) $\frac{1}{3}$ (d) $\frac{5}{12}$

57. Each of the persons A and B independently tosses three fair coins. The probability that both of them get the same number of heads is: **(2021-08-27/Shift-2)**

(a) $\frac{5}{8}$ (b) $\frac{5}{16}$

(c) $\frac{1}{8}$ (d) 1

58. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98p is equal to ___________ ? **(2021-08-31/Shift-1)**

59. Let $S=\{1,2,3,4,5,6\}$. Then the probability that a randomly chosen onto function g from S to S satisfies $g(3)=2g(1)$ is: **(2021-08-31/Shift-2)**

(a) $\frac{1}{5}$ (b) $\frac{1}{30}$

(c) $\frac{1}{15}$ (d) $\frac{1}{10}$

60. The probability distribution of random variable X is given by :

X	1	2	3	4	5
P(X)	K	2K	2K	3K	K

Let $p=P\left(\frac{1<X<4}{X<3}\right)$, If $5p=\lambda K$, then λ is equal to___________. **(2021-08-27/Shift-2)**

61. Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to **(2021-03-16/Shift-2)**

(a) $\frac{9}{56}$ (b) $\frac{11}{27}$

(c) $\frac{3}{7}$ (d) $\frac{4}{9}$

62. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is **(2021-03-16/Shift-1)**

(a) $\frac{39}{50}$ (b) $\frac{52}{867}$

(c) $\frac{22}{425}$ (d) $\frac{3}{4}$

63. Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to : **(2021-03-17/Shift-2)**

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$

(c) $\frac{1}{18}$ (d) $\frac{1}{6}$

64. Two dices are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is : **(2021-03-17/Shift-1)**

(a) $\frac{5}{12}$ (b) $\frac{1}{2}$

(c) $\frac{17}{36}$ (d) $\frac{4}{9}$

65. Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equation $(\alpha-2\beta)\,p=\alpha\beta$ and $(\beta-3\gamma)\,p=2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then, $\frac{\text{Probability of occrrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to ………………. **(2021-03-17/Shift-1)**

66. Let in a Binomial distribution, consisting of 5 independent trails, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to : **(2021-03-18/Shift-2)**

(a) $\frac{40}{243}$ (b) $\frac{80}{243}$

(c) $\frac{32}{625}$ (d) $\frac{128}{625}$

67. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is : **(2021-02-24/Shift-2)**

(a) $\frac{65}{2^8}$ (b) $\frac{35}{2^7}$

(c) $\frac{135}{2^9}$ (d) $\frac{65}{2^7}$

68. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is **(2021-02-24/Shift-1)**

(a) $\frac{1}{32}$ (b) $\frac{5}{16}$

(c) $\frac{3}{16}$ (d) $\frac{1}{2}$

69. Let $B_i\left(i=1,2,3\right)$ be three independent events in a sample space. The probability that only B_1 occurs is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $\left(\alpha-2\beta\right)p=\alpha\beta$ and $\left(\beta-3\gamma\right)p=2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\dfrac{P\left(B_1\right)}{P\left(B_3\right)}$ is equal to ______.

(2021-02-24/Shift-1)

70. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is : **(2021-02-25/Shift-2)**

(a) $\dfrac{97}{297}$ (b) $\dfrac{122}{297}$

(c) $\dfrac{2}{9}$ (d) $\dfrac{1}{5}$

71. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is:

(2021-02-25/Shift-2)

(a) $\dfrac{14}{45}$ (b) $\dfrac{7}{45}$

(c) $\dfrac{8}{45}$ (d) $\dfrac{28}{45}$

72. The coefficients a, b and c of the quadratic equation, $ax^2+bx+c=0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is: **(2021-02-25/Shift-1)**

(a) $\dfrac{5}{216}$ (b) $\dfrac{1}{36}$

(c) $\dfrac{1}{54}$ (d) $\dfrac{1}{72}$

73. When a missile is fired from a ship, the probability that it is intercepted is $\dfrac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\dfrac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is:

(2021-02-25/Shift-1)

(a) $\dfrac{3}{4}$ (b) $\dfrac{3}{8}$

(c) $\dfrac{1}{27}$ (d) $\dfrac{1}{8}$

74. A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is **(2021-02-26/Shift-2)**

(a) $\dfrac{1}{7}$ (b) $\dfrac{6}{7}$

(c) $\dfrac{4}{7}$ (d) $\dfrac{3}{7}$

75. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is : **(2021-02-26/Shift-1)**

(a) $\dfrac{15}{2^{13}}$ (b) $\dfrac{15}{2^{8}}$

(c) $\dfrac{15}{2^{14}}$ (d) $\dfrac{15}{2^{12}}$

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

1. A purse contains 2 six sided dice. One is a normal fair die, while the other has 2 ones, 2 threes, and 2 fives. A die is picked up and rolled. Because of some secret magnetic attraction of the unfair die, there is 75% chance of picking the unfair die and a 25% chance of picking a fair die. The die is rolled and shows up the face 3. The probability that a fair die was picked up, is

(a) $\frac{1}{7}$ (b) $\frac{1}{4}$

(c) $\frac{1}{6}$ (d) $\frac{1}{24}$

2. An instrument consists of two units. Each unit must function for the instrument to operate. The reliability of the first unit is 0.9 & that of the second unit is 0.8. The instrument is tested & fails. The probability that "only the first unit failed & the second unit is sound" is :

(a) 1/7 (b) 2/7

(c) 3/7 (d) 4/7

3. A box has four dice in it. Three of them are fair dice but the fourth one has the number five on all of its faces. A die is chosen at random from the box and is rolled three times and shows up the face five on all the three occassions. The chance that the die chosen was a rigged die, is

(a) $\frac{216}{217}$ (b) $\frac{215}{219}$

(c) $\frac{216}{219}$ (d) none

4. On a Saturday night 20% of all drivers in U.S.A. are under the influence of alcohol. The probability that a driver under the influence of alcohol will have an accident is 0.001. The probability that a sober driver will have an accident is 0.0001. If a car on a saturday night smashed into a tree, the probability that the driver was under the influence of alcohol, is

(a) 3/7 (b) 4/7

(c) 5/7 (d) 6/7

5. Mr. Dupont is a professional wine taster. When given a French wine, he will identify it with probability 0.9 correctly as French, and will mistake it for a Californian wine with probability 0.1. When given a Californian wine, he will identify it with probability 0.8 correctly as Californian, and will mistake it for a French wine with probability 0.2. Suppose that Mr. Dupont is given ten unlabelled glasses of wine, three with French and seven with Californian wines. He randomly picks a glass, tries the wine, and solemnly says : "French". The probability that the wine he tasted was Californian, is nearly equal to

(a) 0.14 (b) 0.24

(c) 0.34 (d) 0.44

6. A box contains a normal coin and a double headed coin. A coin selected at random and tossed twice, fell headwise on both the occasions. The probability that the drawn coin is a double headed coin is

(a) $\frac{2}{3}$ (b) $\frac{5}{8}$

(c) $\frac{3}{4}$ (d) $\frac{4}{5}$

7. A box contains 5 red and 4 white marbles. Two marbles are drawn successively from the box without replacement and the second drawn marble drawn is found to be white. Probability that the first marble is also white is

(a) $\frac{3}{8}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

8. Events A and C are independent. If the probabilities relating A, B and C are P (A) = 1/5;

P (B) = 1/6 ; $P(A \cap C) = 1/20$; $P(B \cup C) = 3/8$ then

(a) events B and C are independent

(b) events B and C are mutually exclusive

(c) events B and C are neither independent nor mutually exclusive

(d) events B and C are equiprobable

9. Assume that the birth of a boy or girl to a couple to be equally likely, mutually exclusive, exhaustive and independent of the other children in the family. For a couple having 6 children, the probability that their "three oldest are boys" is

(a) $\frac{20}{64}$ (b) $\frac{1}{64}$

(c) $\frac{2}{64}$ (d) $\frac{8}{64}$

10. A and B are two events such that P(A) = 0.2 and $P(A \cup B) = 0.7$. If A and B are independent events then P(B) equals

(a) 2/7 (b) 7/9

(c) 5/8 (d) none of these

11. Box A contains 3 red and 2 blue marbles while box B contains 2 red and 8 blue marbles. A fair coin is tossed. If the coin turns up heads, a marble is drawn from A, if it turns up tails, a marble is drawn from bag B. The probability that a red marble is chosen, is

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{1}{2}$

12. Lot A consists of 3G and 2D articles. Lot B consists of 4G and 1D article. A new lot C is formed by taking 3 articles from A and 2 from B. The probability that an article chosen at random from C is defective, is

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$

(c) $\frac{8}{25}$ (d) none

13. A bowl has 6 red marbles and 3 green marbles. The probability that a blind folded person will draw a red marble on the second draw from the bowl without replacing the marble from the first draw, is

(a) $\frac{2}{3}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{5}{8}$

14. A number x is chosen at random from the set {1, 2, 3, 4..... 100}. Define the event: A = the chosen number x satisfies $\frac{(x-10)(x-50)}{(x-30)} \geq 0$. Then P(A) is:

(a) 0.71 (b) 0.70

(c) 0.51 (d) 0.20

15. Three identical dice are rolled. The probability that the same number will appear on each of them, is

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$

(c) $\frac{1}{18}$ (d) $\frac{3}{28}$

16. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is

(a) 8/3 (b) 3/8

(c) 4/5 (d) 5/4

17. A fair die is tossed repeatedly. A wins if it is 1 or 2 on two consecutive tosses and B wins if it is 3, 4, 5 or 6 on two consecutive tosses. The probability that A wins if the die is tossed indefinitely, is

(a) $\frac{1}{3}$ (b) $\frac{5}{21}$

(c) $\frac{1}{4}$ (d) $\frac{2}{5}$

18. A and B play a game of tennis. The situation of the game is as follows; if one scores two consecutive points after a deuce he wins; if loss of a point is followed by win of a point, it is deuce. The chance of a server to win a point is 2/3. The game is at deuce and A is serving. Probability that A will win the match is, (serves are changed after each pt)

(a) 3/5 (b) 2/5

(c) 1/2 (d) 4/5

19. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb if the first misses the target. The probability that the target is hit by the second plane, is

(a) 0.2 (b) 0.7

(c) 0.06 (d) 0.14

20. It is given that the event A and B are such that $P(A)=\frac{1}{4}, P\left(\frac{A}{B}\right)=\frac{1}{2}$ and $P\left(\frac{B}{A}\right)=\frac{2}{3}$. Then P (B) is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

21. Let A and E by any two events with positive probabilities:

Statement 1 : $P(E/A) \geq P(A/E)\,P(E)$

Statement 2 : $P(A/E) \geq P(A \cap E)$

(a) Both the statements are false

(b) Both statements are True

(c) Statement–1 is true, Statement–2 is false

(d) Statement–1 is false, Statement–2 is true

22. If A and B are two independent events such that $P(A) > 0$, and $P(B) \neq 1$, then $P(\bar{A}/\bar{B})$ is equal to

(a) $1-P(\bar{A}/B)$ (b) $1-P(A/\bar{B})$

(c) $\frac{1-P(A \cup B)}{P(B)}$ (d) $\frac{P(\bar{A})}{P(\bar{B})}$

23. Indicate the correct order sequence in respect of the following :

I. If the probability that a computer will fail during the first hour of operation is 0.01, then if we turn on 100 computers, exactly one will fail in the first hour of operation.

II. A man has ten keys only one of which fits the lock. He tries them in a door one by one discarding the one he has tried. The probability that fifth key fits the lock is 1/10.

III. Given the events A and B in a sample space. If $P(A) = 1$, then A and B are independent.

IV. When a fair six sided die is tossed on a table top, the bottom face can not be seen. The probability that the product of the numbers on the five faces that can be seen is divisible by 6 is one.

(a) FTFT (b) FTTT

(c) TFTF (d) TFFF

24. A child throws 2 fair dice. If the numbers showing are unequal, he adds them together to get his final score. On the other hand, if the numbers showing are equal, he throws 2 more dice & adds all 4 numbers showing to get his final score. The probability that his final score is 6 is:

(a) $\frac{145}{1296}$ (b) $\frac{146}{1296}$

(c) $\frac{147}{1296}$ (d) $\frac{148}{1296}$

25. A examination consists of 8 questions in each of which one of the 5 alternatives is the correct one. On the assumption that a candidate who has done no preparatory work chooses for each question any one of the five alternatives with equal probability, the probability that he gets more than one correct answer is equal to :

(a) $(0.8)^8$ (b) $3\,(0.8)^8$

(c) $1-(0.8)^8$ (d) $1-3\,(0.8)^8$

26. A number is chosen at random from the numbers 10 to 99. By seeing the number a man will laugh if product of the digits is 12. If he choose three numbers with replacement then the probability that he will laugh at least once is

(a) $1-\left(\frac{3}{5}\right)^3$ (b) $\left(\frac{43}{45}\right)^3$

(c) $1-\left(\frac{4}{25}\right)^3$ (d) $1-\left(\frac{43}{45}\right)^3$

27. A fair die is tossed eight times. Probability that on the eighth throw a third six is observed is,

(a) ${}^8C_3\,\frac{5^5}{6^8}$

(b) $\frac{{}^7C_2 . 5^5}{6^8}$

(c) $\frac{{}^7C_2 . 5^5}{6^7}$

(d) none of these

28. Two cards are drawn from a well shuffled pack of 52 playing cards one by one. If

A : the event that the second card drawn is an ace and

B : the event that the first card drawn is an ace card.

then which of the following is true?

(a) $P(A) = \frac{4}{17}$; $P(B) = \frac{1}{13}$

(b) $P(A) = \frac{1}{13}$; $P(B) = \frac{1}{13}$

(c) $P(A) = \frac{1}{13}$; $P(B) = \frac{1}{17}$

(d) $P(A) = \frac{16}{221}$; $P(B) = \frac{4}{51}$

29. There are n different gift coupons, each of which can occupy $N(N > n)$ different envelopes, with the same probability $1/N$

P_1: The probability that there will be one gift coupon in each of n definite envelopes out of N given envelopes

P_2: The probability that there will be one gift coupon in each of n arbitrary envelopes out of N given envelopes

Consider the following statements

(i) $P_1 = P_2$ (ii) $P_1 = \frac{n!}{N^n}$

(iii) $P_2 = \frac{N!}{N^n (N-n)!}$

(iv) $P_2 = \frac{n!}{N^n (N-n)!}$ (v) $P_1 = \frac{N!}{N^n}$

Now, which of the following is true

(a) Only (i) (b) (ii) and (iii)
(c) (ii) and (iv) (d) (iii) and (v)

30. A bag contains 3 R & 3 G balls and a person draws out 3 at random. He then drops 3 blue balls into the bag & again draws out 3 at random. The chance that the 3 later balls being all of different colours is

(a) 15% (b) 20%
(c) 27% (d) 40%

31. From an urn containing six balls, 3 white and 3 black ones, a person selects at random an even number of balls (all the different ways of drawing an even number of balls are considered equally probable, irrespective of their number). Then the probability that there will be the same number of black and white balls among them

(a) $\frac{4}{5}$ (b) $\frac{11}{15}$

(c) $\frac{11}{30}$ (d) $\frac{2}{5}$

32. One purse contains 6 copper coins and 1 silver coin ; a second purse contains 4 copper coins. Five coins are drawn from the first purse and put into the second, and then 2 coins are drawn from the second and put into the first. The probability that the silver coin is in the second purse is

(a) $\frac{1}{2}$ (b) $\frac{4}{9}$

(c) $\frac{5}{9}$ (d) $\frac{2}{3}$

33. If $\frac{(1+3p)}{3}, \frac{(1-p)}{4}$ & $\frac{(1-2p)}{2}$ are the probabilities of three mutually exclusive events defined on a sample space S, then the true set of all values of p is

(a) $\left[\frac{1}{3}, \frac{1}{2}\right]$ (b) $\left[\frac{1}{3}, 1\right]$

(c) $\left[\frac{1}{4}, \frac{1}{3}\right]$ (d) $\left[\frac{1}{4}, \frac{1}{2}\right]$

34. The probabilities of events, $A \cap B$, A, B & $A \cup B$ are respectively in A.P. with probability of second term equal to the common difference. Therefore the events A and B are

(a) compatible

(b) independent

(c) such that one of them must occur

(d) such that one is twice as likely as the other

Objective Questions II [One or more than one correct option]

35. A bag initially contains one red & two blue balls. An experiment consisting of selecting a ball at random, noting its colour & replacing it together with an additional ball of the same colour. If three such trials are made, then :

(a) probability that atleast one blue ball is drawn is 0.9

(b) probability that exactly one blue ball is drawn is 0.2

(c) probability that all the drawn balls are red given that all the drawn balls are of same colour is 0.2

(d) probability that atleast one red ball is drawn is 0.6.

36. If E_1 and E_2 are two events such that $P(E_1) = 1/4$, $P(E_2/E_1) = 1/2$ and $P(E_1/E_2) = 1/4$

(a) then E_1 and E_2 are independent

(b) E_1 and E_2 are exhaustive

(c) E_2 is twice as likely to occur as E_1

(d) Probabilities of the events $E_1 \cap E_2$, E_1 and E_2 are in GP.

37. Let $0 < P(A) < 1$, $0 < P(B) < 1$ & $P(A \cup B) = P(A) + P(B) - P(A).\,P(B)$, then :

(a) $P(B/A) = P(B) - P(A)$

(b) $P(A^C \cup B^C) = P(A^C) + P(B^C)$

(c) $P((A \cup B)^C) = P(A^C).\,P(B^C)$

(d) $P(A/B) = P(A)$

38. If M & N are independent events such that $0 < P(M) < 1$ & $0 < P(N) < 1$, then :

(a) M & N are mutually exclusive

(b) M & $\bar{N}$ are independent

(c) $\bar{M}$ & $\bar{N}$ are independent

(d) $P\left(M/N\right) + P\left(\bar{M}/N\right) = 1$

39. If $\bar{E}$ and $\bar{F}$ are the complementary events of E and F respectively and if $0 < P(F) < 1$, then

(a) $P(E/F) + P(\bar{E}/F) = 1$

(b) $P(E/F) + P(E/\bar{F}) = 1$

(c) $P(\bar{E}/F) + P(E/\bar{F}) = 1$

(d) $P(E/\bar{F}) + P(\bar{E}/\bar{F}) = 1$

40. Two real numbers, x & y are selected at random. Given that $0 \le x \le 1$; $0 \le y \le 1$. Let A be the event that $y^2 \le x$; B be the event that $x^2 \le y$, then :

(a) $P(A \cap B) = \dfrac{1}{3}$

(b) A & B are exhaustive events

(c) A & B are mutually exclusive

(d) A & B are independent events.

41. If A & B are two events such that $P(B) \ne 1$, B^C denotes the event complementry to B, then

(a) $P\left(A/B^C\right) = \dfrac{P(A) - P(A \cap B)}{1 - P(B)}$

(b) $P(A \cap B) \ge P(A) + P(B) - 1$

(c) $P(A) > P\left(A/B\right)$ if $P\left(A/B^C\right) > P(A)$

(d) $P\left(A/B^C\right) + P\left(A^C/B^C\right) = 1$

42. For any two events A & B defined on a sample space ,

(a) $P(A/B) \ge \dfrac{P(A) + P(B) - 1}{P(B)}$, $P(B) \ne 0$ is always true

(b) $P\left(A \cap \bar{B}\right) = P(A) - P(A \cap B)$

(c) $P(A \cup B) = 1 - P(A^c).\,P(B^c)$, if A & B are independent

(d) $P(A \cup B) = 1 - P(A^c).\,P(B^c)$, if A & B are disjoint

43. For two given events A & B, $P(A \cap B)$ is :

(a) not less than $P(A) + P(B) - 1$

(b) not greater than $P(A) + P(B)$

(c) equal to $P(A) + P(B) - P(A \cup B)$

(d) equal to $P(A) + P(B) + P(A \cup B)$

Numerical Value Type Questions

44. Two integers r and s are chosen one at a time without replacement from the numbers 1, 2, 3,... 100. Let p be the probability that $r \leq 25$ given that $s \leq 25$. Find the value of 33p.

45. A bag contains n + 1 coins. It is known that one of these coins has heads on both sides, whereas the other coins are fair. One coin is selected at random and tossed. If the probability that the toss results in heads is 7/12, find n.

46. 7 persons are stopped on the road at random and asked about their birthdays. If the probability that 3 of them are born on Wednesday, 2 on Thursday and the remaining 2 on Sunday is $\frac{K}{7^6}$, then K is equal to

Assertion & Reason

(A) If both assertion and reason are correct and reason is the correct explanation of assertion.

(B) If both assertion and reason are true but reason is not the correct explanation of assertion.

(C) If assertion is true but reason is false.

(D) If assertion is false but reason is true.

47. From an urn containing a white and b black balls, k (< a, b) are drawn and laid aside, their colour unnoted. Then another ball, that is, $(k + 1)^{th}$ ball is drawn.

Assertion : Probability that $(k + 1)^{th}$ ball drawn is white is $\frac{a}{a+b}$.

Reason : Probability that $(k + 1)^{th}$ ball drawn is black is $\frac{a}{a+b}$

(a) A (b) B

(c) C (d) D

48. Let A and B are two events such that P(A) > 0.

Assertion : If P (A) + P (B) > 1, then $P(B/A) \geq 1 - P(B')/P(A)$

Reason : If $P(A/B') \geq P(A)$, then $P(A) \geq P(A/B)$.

(a) A (b) B

(c) C (d) D

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

49. A determinant Δ is chosen at random from the set of all determinant of order two with elements 0 and 1 only.

Value of Δ	Probability
(A) 1	**(P)** 5/8
(B) 0	**(Q)** 3/16
(C) 2	**(R)** 3/8
(D) non zero	**(S)** 0

The correct matching is

(a) A–Q; B-P; C-S; D-R

(b) A–P; B-Q; C-S; D-R

(c) A–Q; B-S; C-P; D-R

(d) A–Q; B-P; C-R; D-S

50. A ten digit number N is formed by using the digits 0 to 9 exactly once. The probability that N is divisible by

(A) 4	**(P)** 1
(B) 5	**(Q)** 20/81
(C) 45	**(R)** 17/81
(D) 12	**(S)** 2/81

The correct matching is

(a) A–Q; B–R; C–R; D–Q

(b) A–R; B–Q; C–R; D–Q

(c) A–Q; B–R; C–Q; D–R

(d) A–R; B–R; C–Q; D–Q

Paragraph Type Questions

Passage

Using the following passage, solve Q.51 to Q.53

Let S and T are two events defined on a sample space with probabilities

$P(S) = 0.5, P(T) = 0.69, P(S/T) = 0.5$

51. Events S and T are:

(a) mutually exclusive

(b) independent

(c) mutually exclusive and independent

(d) neither mutually exclusive nor independent

52. The value of P(S and T)

(a) 0.3450 (b) 0.2500

(c) 0.6900 (d) 0.350

53. The value of P(S or T)

(a) 0.6900 (b) 1.19

(c) 0.8450 (d) 0

Using the following passage, solve Q.54 to Q.56

A JEE aspirant estimates that she will be successful with an 80 percent chance if she studies 10 hours per day, with a 60 percent chance if she studies 7 hours per day and with a 40 percent chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively

54. The chance she will be successful, is

(a) 0.28 (b) 0.38

(c) 0.48 (d) 0.58

55. Given that she is successful, the chance she studied for 4 hours, is

(a) $\frac{6}{12}$ (b) $\frac{7}{12}$

(c) $\frac{8}{12}$ (d) $\frac{9}{12}$

56. Given that she does not achieve success, the chance she studied for 4 hour, is

(a) $\frac{18}{26}$ (b) $\frac{19}{26}$

(c) $\frac{20}{26}$ (d) $\frac{21}{26}$

Using the following passage, solve Q.57 to Q.60

Read the passage given below carefully before attempting these questions.

A standard deck of playing cards has 52 cards. There are four suit (clubs, diamonds, hearts and spades), each of which has thirteen numbered cards (2, ..., 9, 10, Jack, Queen, King, Ace)

In a game of card, each card is worth an amount of points. Each numbered card is worth its number (e.g. a 5 is worth 5 points) ; the Jack, Queen and King are each worth 10 points; and the Ace is worth your choice of either 1 point or 11 points. The object of the game is to have more points in your set of cards than your opponent without going over 21. Any set of cards with sum greater than 21 automatically loses.

Here's how the game played. You and your opponent are each dealt two cards. Usually the first card for each player is dealt face down, and the second card for each player is dealt face up. After the initial cards are dealt, the first player has the option of asking for another card or not taking any cards. The first player can keep asking for more cards until either he or she goes over 21, in which case the player loses, or stops at some number less than or equal to 21. When the first player stops at some number less than or equal to 21, the second player then can take more cards until matching or exceeding the first player's number without going over 21, in which case the second player wins, or until going over 21, in which case the first player wins.

We are going to simplify the game a little and assume that all cards are dealt face up, so that all cards are visible. Assume your opponent is dealt cards and plays first.

57. The chance that the second card will be a heart and a Jack, is

(a) $\frac{4}{52}$ (b) $\frac{13}{52}$

(c) $\frac{17}{52}$ (d) $\frac{1}{52}$

58. The chance that the first card will be a heart or a Jack, is

(a) $\frac{13}{52}$ (b) $\frac{16}{52}$

(c) $\frac{17}{52}$ (d) none

59. Given that the first card is a Jack, the chance that it will be the heart, is

(a) $\frac{1}{13}$ (b) $\frac{4}{13}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

60. Your opponent is dealt a King and a 10, and you are dealt a Queen and a 9. Being smart, your opponent does not take any more cards and stays at 20. The chance that you will win if you are allowed to take as many cards as you need, is

(a) 0.771 (b) 0.088
(c) 0.0797 (d) 0.0907

Text

61. In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he make a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question given that he correctly answered it.

62. Six boys and six girls sit in a row at random. Find the probability that

(a) the six girls sit together

(b) the boys and girls sit alternatively

Fill in the blanks

63. For a biased die the probabilities for the different faces to turn up are given below

Face	1	2	3	4	5	6
Probability	0.1	0.32	0.21	0.15	0.05	0.17

This die is tossed and you are told that either face 1 or face 2 has turned up. Then, the probability that it is face 1, is....

64. A box contains 100 tickets numbered 1, 2, ..., 100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5 with probability....

65. If two events A and B are such that $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$ then $P[B / (A \cup B^c)]$ =....

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

1. If $P(B)=\frac{3}{4}$, $P(A\cap B\cap \bar{C})=\frac{1}{3}$ and $P(\bar{A}\cap B\cap \bar{C})=\frac{1}{3}$, then $P(B\cap C)$ is **(2002)**

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{15}$ (d) $\frac{1}{9}$

2. Two numbers are selected randomly from the set $S=\{1,2,3,4,5,6\}$ without replacement one by one. The probability that minimum of the two numbers is less than 4, is **(2003)**

(a) 1/15 (b) 14/15

(c) 1/5 (d) 4/5

3. If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3, is **(2004)**

(a) $\frac{4}{55}$ (b) $\frac{4}{35}$

(c) $\frac{4}{33}$ (d) $\frac{4}{1155}$

4. A fair die is rolled. The probability that the first time 1 occurs at the even throw, is **(2005)**

(a) 1/6 (b) 5/11

(c) 6/11 (d) 5/36

5. One Indian and four American men and their wives are to be seated randomly around a circular table. Then, the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife, is **(2007)**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{5}$ (d) $\frac{1}{5}$

6. Let E^c denotes the complement of an event E. Let E, F, G be pairwise independent events with $P(G) > 0$ and $P(E\cap F\cap G)=0$. Then, $P(E^c\cap F^c\,|\,G)$ equals **(2007)**

(a) $P(E^c)+P(F^c)$ (b) $P(E^c)-P(F^c)$

(c) $P(E^c)-P(F)$ (d) $P(E)-P(F^c)$

7. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is **(2008)**

(a) 2, 4, or 8 (b) 3, 6 or 9

(c) 4 or 8 (d) 5 or 10

8. Let ω be a complex cube root of unity with $\omega\neq 1$. A fair die is thrown three times, If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1}+\omega^{r_2}+\omega^{r_3}=0$ is **(2010)**

(a) 1/18 (b) 1/9

(c) 2/9 (d) 1/36

9. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the singal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal green is **(2010)**

(a) $\frac{3}{5}$ (b) $\frac{6}{7}$

(c) $\frac{20}{23}$ (d) $\frac{9}{20}$

10. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is **(2014)**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

11. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that P (computer turns out to be defective given that it is produced in plant T_1) = 10P (computer turns out to be defective given that it is produced in plant T_2), where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plants T_2 is **(2016)**

(a) $\frac{36}{73}$ (b) $\frac{47}{79}$

(c) $\frac{78}{93}$ (d) $\frac{75}{83}$

12. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation $x+y+z=10$. Then the probability that z is even, is **(2017)**

(a) $\frac{1}{2}$ (b) $\frac{36}{55}$

(c) $\frac{6}{11}$ (d) $\frac{5}{11}$

13. Let C_1 and C_2 be two biased coins such that the probabilities of getting head in a single toss are $\frac{2}{3}$ and $\frac{1}{3}$, respectively. Suppose α is the number of heads that appear when C_1 is tossed twice, independently, and suppose β is the number of heads that appear when C_2 is tossed twice, independently, Then probability that the roots of the quadratic polynomial $x^2 - \alpha x + \beta$ are real and equal, is **(2020)**

(a) $\frac{40}{81}$ (b) $\frac{20}{81}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

14. Consider three sets $E_1 = \{1,2,3\}, F_1 = \{1,3,4\}$ and $G_1 = \{2,3,4,5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement from the set G_2 and let S_3 denote the set of these chosen elements.

Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1,2\}$. Then the value of p is **(2021)**

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$

(c) $\frac{1}{2}$ (d) $\frac{2}{5}$

Objective Questions II [One or more than one correct option]

15. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occuring is $\frac{2}{25}$. If P (T) denotes the probability of occurence of the event T, then **(2011)**

(a) $P(E)=\frac{4}{5}, P(F)=\frac{3}{5}$ (b) $P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$

(c) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$ (d) $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

16. Let X and Y be two events such that $P(X)=\frac{1}{3}$, $P(X|Y)=\frac{1}{2}$ and $P(Y|X)=\frac{2}{5}$. Then **(2017)**

(a) $P(Y)=\frac{4}{15}$ (b) $P(X'|Y)=\frac{1}{2}$

(c) $P(X \cup Y)=\frac{2}{5}$ (d) $P(X \cap Y)=\frac{1}{5}$

17. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls. B_2 contains 3 red and 5 green balls and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities 3/10, 3/10 and 4/10 respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct ? **(2019)**

(a) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(b) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{5}{13}$

(c) Probability that the chosen ball is green equals $\frac{39}{80}$

(d) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{3}{10}$

18. Let E, F and G be three events having probabilities $P(E)=\frac{1}{8}, P(F)=\frac{1}{6}$ and $P(G)=\frac{1}{4}$, and $P(E\cap F\cap G)=\frac{1}{10}$. For any event H, if H^c denotes its complement, then which of the following statements is(are) TRUE? **(2021)**

(a) $P(E\cap F\cap G^c)\le\frac{1}{40}$

(b) $P(E^c\cap F\cap G)\le\frac{1}{15}$

(c) $P(E\cup F\cup G)\le\frac{13}{24}$

(d) $P(E^c\cap F^c\cap G^c)\le\frac{5}{12}$

Numerical Value Type Questions

19. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is. **(2015)**

20. Let S be the sample space of all 3×3 matrices with entries from the set {0,1}. Let the events E_1 and E_2 is given by

$$E_1=\{A\in S:\det A=0\}$$

$$E_2=\{A\in S:\text{Sum of Entries of A is 7}\}$$

If a matrix is chosen at random from S, then the conditional probability $P(E_1|E_2)$ equals _____ **(2019)**

21. Let |X| denote the number of elements in a set X. Let S = {1,2,3,4,5,6} be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A,B) such that $1\le|B|<|A|$ equals **(2019)**

22. The probability that a missile hits a target successfully is 0.75. In order to destroy the target completely, at least three successful hits are required. Then the minimum number of missiles that have to be fired so that the probability of completely destroying the target is NOT less than 0.95, is _____ . **(2020)**

23. Two fair dice, each with faces numbered 1,2,3,4,5 and 6, are rolled together and the sum of the numbers on the faces is observed. This process is repeated till the sum is either a prime number or a perfect square. Suppose the sum turns out to be a perfect square before it turns out to be a prime number. If P is the probability that this perfect square is an odd number, then the value of 14 P is **(2020)**

24. A number is chosen at random from the set {1, 2, 3......, 2000}. Let be the probability that the chosen number is a multiple of 3 or a multiple of 7. Then the value of 500p is ______. **(2021)**

Assertion & Reason

(A) If both assertion and reason are correct and reason is the correct explanation of assertion.

(B) If both assertion and reason are true but reason is not the correct explanation of assertion.

(C) If assertion is true but reason is false.

(D) If assertion is false but reason is true.

25. Let $H_1, H_2,......, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, i = 1, 2,...., n. Let E be any other event with $0 < P(E) < 1$. **(2007)**

Assertion : $P(H_i/E) > P(E/H_i) \cdot P(H_i)$ for $i = 1, 2, ..., n$

Reason : $\sum_{i=1}^{n} P(H_i) = 1$

(a) A (b) B

(c) C (d) D

26. Consider the system of equations

$ax + by = 0, cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$.

Assertion : The probability that the system of equations has a unique solution, is 3/8.

Reason : The probability that the system of equations has a solution, is 1. **(2008)**

(a) A (b) B

(c) C (d) D

Comprehension Type

Passage – 1

Using the following passage, solve Q.27 to Q.29

There are n urns each containing $n + 1$ balls such that the i^{th} urn contains i white balls and $(n + 1 - i)$ red balls. Let u_i be the event of selecting i^{th} urn, $i = 1, 2, 3, ..., n$ and w denotes the event of getting a white ball.

(2006)

27. If $P(u_i) \propto i$ where $i = 1, 2, 3,....., n$ then $\lim\limits_{n \to \infty} P(w)$ is equal to

(a) 1 (b) 2/3

(c) 3/4 (d) 1/4

28. If $P(u_i) = c$, where c is a constant then $P(u_n/w)$ is equal to

(a) $\dfrac{2}{n+1}$ (b) $\dfrac{1}{n+1}$

(c) $\dfrac{n}{n+1}$ (d) $\dfrac{1}{2}$

29. If n is even and E denotes the event of choosing even numbered urn $\left(P(u_i) = \dfrac{1}{n}\right)$, then the value of $P(w/E)$, is

(a) $\dfrac{n+2}{2n+1}$ (b) $\dfrac{n+2}{(n+1)}$

(c) $\dfrac{n}{n+1}$ (d) $\dfrac{1}{n+1}$

Passage – 2

Using the following passage, solve Q.30 to Q.32

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required. **(2009)**

30. The probability that $X = 3$ equals

(a) $\dfrac{25}{216}$ (b) $\dfrac{25}{36}$

(c) $\dfrac{5}{36}$ (d) $\dfrac{125}{216}$

31. The probability that $X \geq 3$ equals

(a) $\dfrac{125}{216}$ (b) $\dfrac{25}{36}$

(c) $\dfrac{5}{36}$ (d) $\dfrac{25}{216}$

32. The conditional probability that $X \geq 6$ given $X > 3$ equals

(a) $\dfrac{125}{216}$ (b) $\dfrac{25}{216}$

(c) $\dfrac{5}{36}$ (d) $\dfrac{25}{36}$

Passage – 3

Using the following passage, solve Q.33 and Q.34

Box 1 contains three cards bearing numbers 1,2,3 ; box 2 contains five cards bearing numbers 1,2,3,4,5 and box 3 contains seven cards bearing numbers 1,2,3,4,5,6,7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, $i = 1,2,3$. **(2014)**

33. The probability that $x_1 + x_2 + x_3$ is odd, is

(a) $\dfrac{29}{105}$ (b) $\dfrac{53}{105}$

(c) $\dfrac{57}{105}$ (d) $\dfrac{1}{2}$

34. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(a) $\frac{9}{105}$ (b) $\frac{10}{105}$

(c) $\frac{11}{105}$ (d) $\frac{7}{105}$

Passage – 4

Using the following passage, solve Q.35 and Q.36

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n4 be the number of red and black balls, respectively, in box II. **(2015)**

35. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is (are).

(a) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$

(b) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$

(c) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$

(d) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

36. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are)

(a) $n_1 = 4$ and $n_2 = 6$

(b) $n_1 = 2$ and $n_2 = 3$

(c) $n_1 = 10$ and $n_2 = 20$

(d) $n_1 = 3$ and $n_2 = 6$

Passage – 5

Using the following passage, solve Q.37 and Q.38

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2, respectively, after two games. **(2016)**

37. $P(X > Y)$ is

(a) $\frac{1}{4}$ (b) $\frac{5}{12}$

(c) $\frac{1}{2}$ (d) $\frac{7}{12}$

38. $P(X = Y)$ is

(a) $\frac{11}{36}$ (b) $\frac{1}{3}$

(c) $\frac{13}{36}$ (d) $\frac{1}{2}$

Passage – 6

Using the following passage, solve Q.39 and Q.40

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, $i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats. **(2018)**

39. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1, and none of the remaining students gets the seat previously allotted to him/her is

(a) $\frac{3}{40}$ (b) $\frac{1}{8}$

(c) $\frac{7}{40}$ (d) $\frac{1}{5}$

40. For i = 1, 2, 3, 4, let T_i denote the event that the students S_i and S_{i+1} do not sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(a) $\frac{1}{15}$ (b) $\frac{1}{10}$

(c) $\frac{7}{60}$ (d) $\frac{1}{5}$

Passage – 7

Using the following passage, solve Q.41 and Q.42

41. Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, ..., 100\}$. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40.

The value of $\frac{625}{4} p_1$ is ___. **(2021)**

42. Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, ..., 100\}$. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40.

The value of $\frac{125}{4} p_2$ is ____. **(2021)**

Text

43. A coin has probability 'p' of showing head when tossed. It is tossed 'n' times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that, $p_1 = 1$, $p_2 = 1 - p^2$ & $p_n = (1-p)\,p_{n-1} + p\,(1-p)\,p_{n-2}$, for all $n \geq 3$. **(2000)**

44. A and B are two independent events. The probability that both occur simultaneously is 1/6 and the probability that neither occurs is 1/3. Find the probabilities of occurance of the events A and B separately. **(2000)**

45. Two cards are drawn at random from a pack of playing cards. Find the probability that one card is a heart and the other is an ace. **(2001)**

46. (a) An urn contains 'm' white and 'n' black balls. A ball is drawn at random and is put back into the urn along with K additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white.

(b) An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6, only three numbers appear in the list.

(2001)

47. A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? **(2002)**

48. (a) A person takes three tests in succession. The probability of his passing the first test is p, that of his passing each successive test is p or p/2 according as he passes or fails in the preceding one. He gets selected provided he passes at least two tests. Determine the probability that the person is selected.

(b) In a combat, A targets B, and both B and C target A. The probabilities of A, B, C hitting their targets are 2/3, 1/2 and 1/3 respectively. They shoot simultaneously and A is hit. Find the probability that B hits his target whereas C does not. **(2003)**

49. (a) If A and B are independent events, prove that

$P(A \cup B) \cdot P(A' \cap B') \le P(C)$, where C is an event defined that exactly one of A or B occurs.

(b) A bag contains 12 red balls and 6 white balls. Six balls are drawn one by one without replacement of which atleast 4 balls are white. Find the probability that in the next two draws exactly one white ball is drawn (leave the answer in terms of ${}^nC_r$). **(2004)**

50. A person goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car.

(2005)

Answer Key

CHAPTER -9 | PROBABILITY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (a)	**2.** (d)	**3.** (b)	**4.** (a)
5. (c)	**6.** (b)	**7.** (d)	**8.** (a)
9. (a)	**10.** (d)	**11.** (a)	**12.** (a)
13. (a)	**14.** (d)	**15.** (a)	**16.** (b)
17. (b)	**18.** (a)	**19.** (b)	**20.** (b)
21. (a)	**22.** (a)	**23.** (b)	**24.** (a)
25. (a)	**26.** (d)	**27.** (c)	**28.** (a)
29. (b)	**30.** (c)	**31.** (d)	**32.** (b)
33. (b)	**34.** (b)	**35.** (b)	**36.** (b)
37. (a)	**38.** (d)	**39.** (d)	**40.** (c)
41. (a)	**42.** (c)	**43.** (c)	**44.** (a)
45. (b)	**46.** (a)	**47.** (b)	**48.** (a)
49. (c)	**50.** (d)	**51.** (b)	**52.** (b)
53. (b)	**54.** (a)	**55.** (c)	**56.** (c)
57. (d)	**58.** (b)	**59.** (a)	**60.** (a)

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. (b)	**2.** (c)	**3.** (b)	**4.** (a)
5. (a)	**6.** (b)	**7.** (c)	**8.** (a)
9. (c)	**10.** (b)	**11.** (b)	**12.** (c)
13. (c)	**14.** (d)	**15.** (d)	**16.** (a)
17. (137)	**18.** (c)	**19.** (c)	**20.** (d)
21. (d)	**22.** (c)	**23.** (c)	**24.** (c)
25. (b)	**26.** (b)	**27.** (b)	**28.** (a)
29. (b)	**30.** (c)	**31.** (d)	**32.** (c)
33. (3)	**34.** (c)	**35.** (11)	**36.** (11)
37. (b)	**38.** (b)	**39.** (a)	**40.** (d)
41. (b)	**42.** (a)	**43.** (d)	**44.** (b)
45. (b)	**46.** (d)	**47.** (c)	**48.** (a)
49. (c)	**50.** (a)	**51.** (d)	**52.** (d)
53. (4)	**54.** (a)	**55.** (c)	**56.** (d)
57. (b)	**58.** (28)	**59.** (d)	**60.** (30)
61. (d)	**62.** (a)	**63.** (b)	**64.** (c)
65. (6)	**66.** (c)	**67.** (c)	**68.** (d)
69. (6)	**70.** (a)	**71.** (d)	**72.** (a)
73. (d)	**74.** (d)	**75.** (a)	

ANSWER KEY

CHAPTER -9 | PROBABILITY

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (a)	**2.** (b)	**3.** (c)	**4.** (c)
5. (c)	**6.** (d)	**7.** (a)	**8.** (a)
9. (d)	**10.** (c)	**11.** (b)	**12.** (c)
13. (a)	**14.** (a)	**15.** (b)	**16.** (d)
17. (b)	**18.** (c)	**19.** (d)	**20.** (c)
21. (b)	**22.** (b)	**23.** (b)	**24.** (d)
25. (d)	**26.** (d)	**27.** (b)	**28.** (b)
29. (b)	**30.** (c)	**31.** (b)	**32.** (c)
33. (a)	**34.** (d)	**35.** (a,b,c,d)	**36.** (a,c,d)
37. (c,d)	**38.** (b,c,d)	**39.** (a,d)	**40.** (a,b)
41. (a,b,c,d)	**42.** (a,b,c)	**43.** (a,b,c)	**44.** (8)
45. (5)	**46.** (30)	**47.** (c)	**48.** (b)
49. (a)	**50.** (a)	**51.** (b)	**52.** (a)
53. (c)	**54.** (c)	**55.** (b)	**56.** (d)
57. (d)	**58.** (b)	**59.** (c)	**60.** (d)
61. $\frac{24}{29}$	**62.** (a) $\frac{1}{132}$ (b) $\frac{1}{462}$		**63.** $\frac{5}{21}$
64. $\frac{1}{9}$	**65.** $\frac{1}{4}$		

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (a)	**2.** (d)	**3.** (d)	**4.** (b)
5. (c)	**6.** (c)	**7.** (d)	**8.** (c)
9. (c)	**10.** (a)	**11.** (c)	**12.** (c)
13. (b)	**14.** (a)	**15.** (a,d)	**16.** (a,b)
17. (a,c)	**18.** (a,b,c)	**19.** (8)	**20.** (0.50)
21. (422)	**22.** (6)	**23.** (8)	**24.** (214)
25. (d)	**26.** (b)	**27.** (b)	**28.** (a)
29. (b)	**30.** (a)	**31.** (b)	**32.** (d)
33. (b)	**34.** (c)	**35.** (a,b)	**36.** (c,d)
37. (b)	**38.** (c)	**39.** (a)	**40.** (c)
41. (76.25)	**42.** (24.50)	**44.** $\frac{1}{2} \& \frac{1}{3}$ or $\frac{1}{3} \& \frac{1}{2}$	

45. $\frac{1}{26}$

46. (a) $\frac{m}{m+n}$ (b) $\frac{{}^6C_3\,(3^n - 3.2^n - 3)}{6^n}$

47. $\frac{9m}{m+8N}$

48. (a) $p^2(2-p)$ (b) $1/2$

49. $\dfrac{{}^{12}C_2\,{}^6C_4\,{}^{10}C_1\,{}^2C_1 + {}^{12}C_1\,{}^6C_5\,{}^{11}C_1\,{}^1C_1}{{}^{12}C_2\left({}^{12}C_2\,{}^6C_4 + {}^{12}C_1\,{}^6C_5 + {}^{12}C_0\,{}^6C_6\right)}$

50. $\frac{1}{7}$