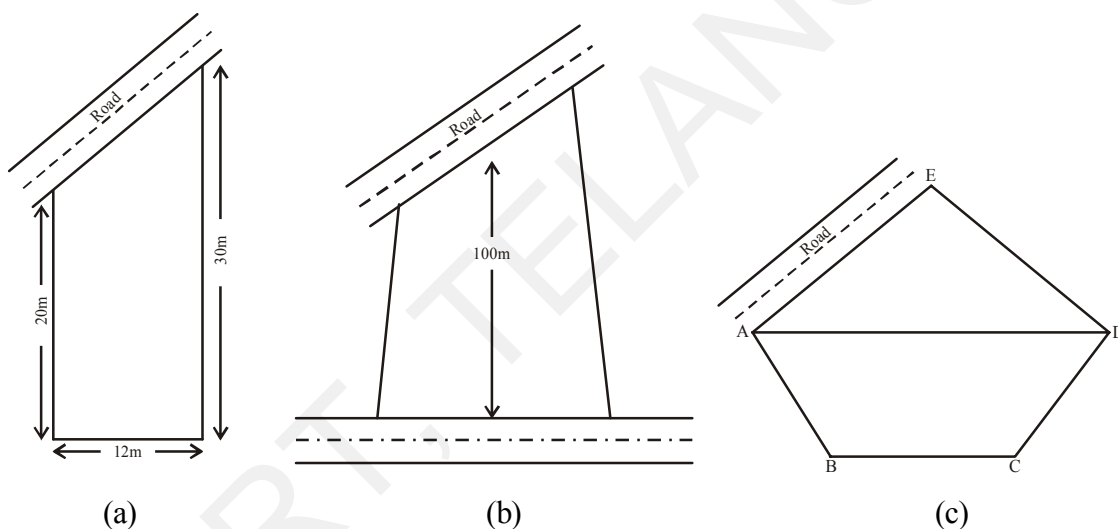# Area of Plane Figures

9.0   Introduction

Devarsh wants to purchase a plot to construct a house for himself. Some of the shapes of the plots visited by him are shown below.
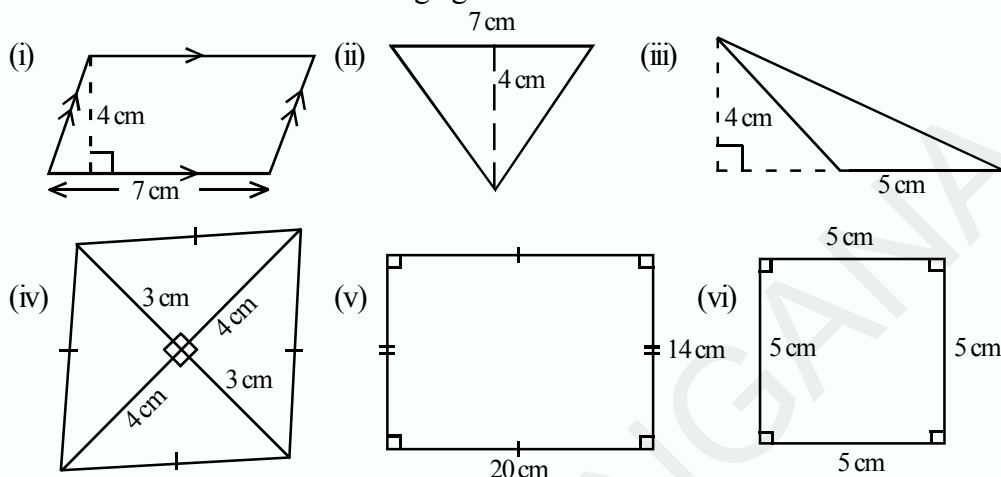

| (a) | (b) | (c) |

Plot (a) is in the shape of a trapezium, Plot (b) is in the shape of a quadrilateral and plot (c) is in the shape of a pentagon . He wants to calculate the area of such figures to construct his house in the field.

We have learnt how to find the area of a rectangle, square, parallelogram, triangle and rhombus. In this chapter we will learn how to find the area of a trapezium, quadrilateral, circle and a sector. First let us review what we have learnt about the area of a rectangle, square, parallelogram and rhombus.

Do This

1. Find the area of the following figures:

(i)
7 cm
4 cm
7 cm

(ii)
7 cm
4 cm

(iii)
4 cm
5 cm

(iv)
3 cm
4 cm
4 cm
3 cm

(v)
14 cm
20 cm

(vi)
5 cm
5 cm   5 cm
5 cm

2. The measurements of some plane figures are given in the table below. However, they are incomplete. Find the missing information

Figure		Measurements	Formula for area	Area of the given figure
	Square	Side of the square is 15 cm	A = side×side	....
	Rectangle	Length = 20 cm Breadth = ...	A = $l \times b$	280cm^2
	Triangle	Base = 5 cm Height = ...	A = ...	60cm^2
	Parallelogram	Height = 7.6cm Base = ...	A = b × h	38cm^2
	Rhombus	d_1 = 4 cm d_2 = 3 cm	....	....

9.1 Area of a Trapezium

Kumar owns a plot near the main road as in the figure below. Unlike some other rectanglar plots in his neighbourhood, the plot has only a pair of parallel sides. So, it is nearly a trapezium in shape. Can you find out its area?

(i)   (ii)
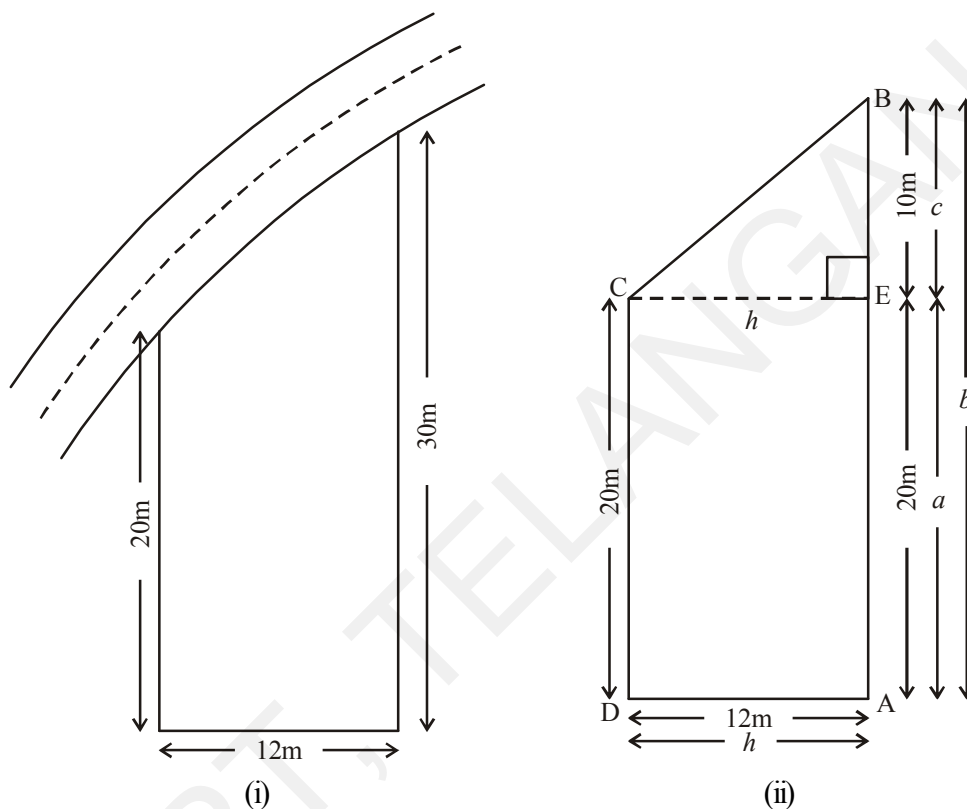
Let us name the vertices of this plot as shown in figure (i). By drawing CE ⊥ AB, we can divide it into two parts, one of rectangular shape and the other of triangular shape (which is right angled), as shown in figure (ii).

Area of $\triangle$ ECB = $\frac{1}{2}$ h $\times$ c   = $\frac{1}{2}$ $\times$ 12 $\times$ 10 = 60 m^2

Area of rectangle ADCE   =   AE $\times$ AD = 20 $\times$ 12 = 240 m^2

Area of trapezium ABCD   =   Area of $\triangle$ ECB + Area of rectangle ADCE

                                      =   60 + 240 = 300 m^2

Thus we can find the area if the trapezium ABCD by combining the two areas i.e. is rectangle ADCE and triangle ECB.

∴ Area of ABCD = Area of ADCE + Area of ECB

$$= (h \times a) + \frac{1}{2}(h \times c)$$

$$= h(a + \frac{1}{2}c)$$

$$= h\left(\frac{2a+c}{2}\right)$$

$$= h\left(\frac{2a+c}{2}\right) = \frac{h}{2}(a+a+c)$$

$$= \frac{1}{2}h(a+b) \ (\because c+a=b)$$

$$= \frac{1}{2} \text{height (sum of parallel sides)}$$

$$\boxed{\begin{array}{l} \overline{AD} = \overline{EC} = h \\ \overline{AE} = a, \ \overline{AB} = b = a+c \end{array}}$$

By substituting the values of h, b and a in the above expression

Area of trapezium ABCD $= \frac{1}{2}h(a+b)$

$$\boxed{\begin{array}{l} \text{Where } h = 12 \\ a = 20 \\ b = 30 \end{array}}$$

$$= \frac{1}{2} \times 12 \times (30+20) = 300 \text{ m}^2$$

Example1:   Here is a figure of a playground. Find the area of the playground.
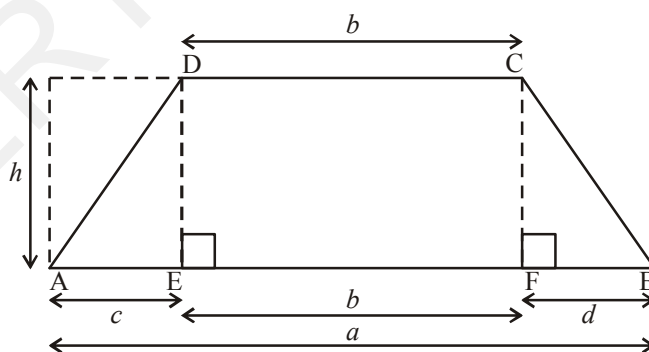
Solution: Here we can not divide the figure into one rectangle and one triangle. Instead ,we may divide it into a rectangle and two triangles conveniently. Draw DE ⊥ AB and CF ⊥ AB . So that trapezium ABCD is divided into three parts. One is rectangle DEFC and the other two are triangles; ΔADE and ΔCFB .

Area of trapezium ABCD = Area of triangle ADE + Area of Rectangle DEFC + Area of triangle CFB

$$= (\frac{1}{2} \times h \times c) + (b \times h) + (\frac{1}{2} \times h \times d)$$

$$= h\left[\frac{1}{2}c + b + \frac{1}{2}d\right]$$

$$= h\left[\frac{c + 2b + d}{2}\right]$$

$$= h\left[\frac{c + b + d + b}{2}\right]$$

$$= h\left[\frac{a + b}{2}\right] \quad (\because c + b + d = a)$$

So, we can write the formula for the area of a trapezium

$$= \text{height} \times \left[\frac{\text{sum of parallel sides}}{2}\right]$$

$$= \frac{1}{2} \times \text{distance between two parallel sides} \times (\text{sum of parallel sides})$$

Activity

1. Draw a trapezium WXYZ on a piece of graph paper as shown in the figure and cut it out as shown in Fig. (i)
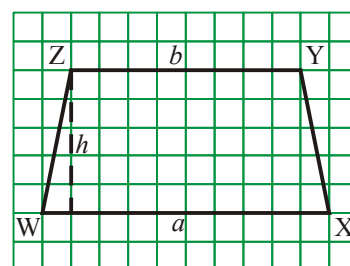
Fig. (i)

2. Find the Mid point of XY by folding its side XY and name it 'A' as shown in Fig.(ii)
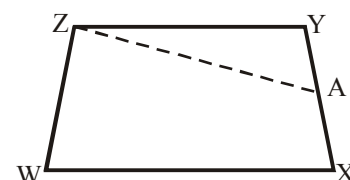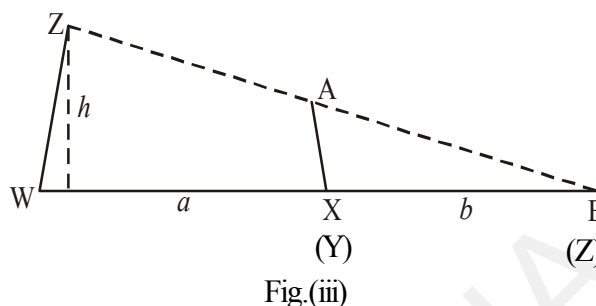

Fig. (ii)

3. Draw line AZ.

4. Cut trapezium WXAZ into two pieces by cutting along ZA. Place △ ZYA as shown in the fig. (iii) where AY placed on AX in such a way that 'Y' coincides with 'X'. We get △ WZB.

What is the length of the base of the larger triangle? Write an expression for the area of this triangle fig. (iii)

Fig.(iii)

5. The area of this triangle WZB and the area of the trapezium WXAZ are the same (How?)
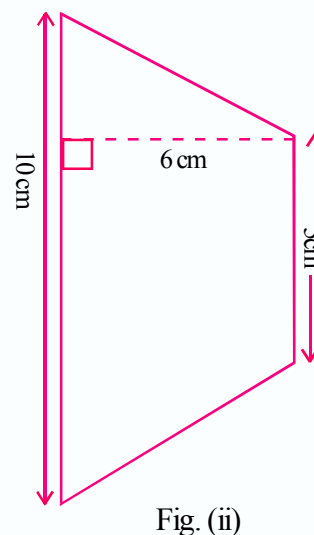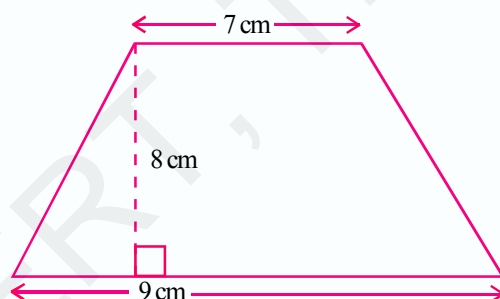
Area of trapezium WXAZ = Area of triangle WZB

$$= \frac{1}{2} \times \text{height} \times \text{base} = \frac{1}{2} \times h \times (a + b)$$

**Note :** Check the area by counting the unit squares of graph.

Do This

1. Find the area of the following trapezium.

7 cm

8 cm

9 cm

10 cm

6 cm

5 cm

Fig. (i)   Fig. (ii)

2. Area of a trapezium is 16cm^2. Length of one parallel side is 5 cm and distance between two parallel sides is 4 cm. Find the length of the other parallel side? Try to draw this trapezium on a graph paper and check the area.

3. ABCD is a parallelogram whose area is 100 sq. cm. P is any point inside the parellelogram (see fig.) find the area of ar△APB + ar△CPD.
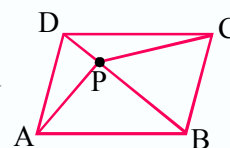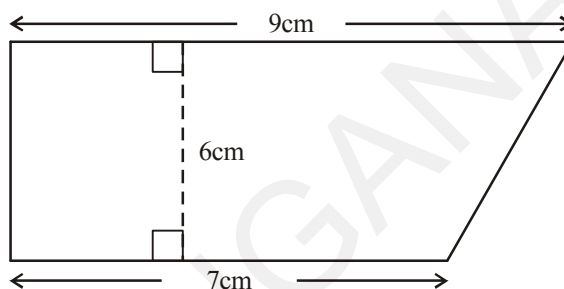
Solved examples

Example 2: The parallel sides of trapezium are 9cm and 7cm long and the distance between them is 6cm. Find the area of the trapezium.

Solution: Parallel sides of the trapezium are 9 cm and 7 cm, the sum of the lengths of parallel sides $(9 + 7)$ cm $= 16$cm

Distance between them $= 6$cm

Area of the trapezium $= \dfrac{1}{2}$ (sum of the lengths of parallel sides) $\times$ (distance between them)

$$= (\dfrac{1}{2} \times 16 \times 6) \text{ cm}^2$$

$$= 48 \text{cm}^2$$

Example 3: Area of a trapezium is 480cm^2. Length of one of the parallel sides is 24cm and the distance between the parallel sides is 8cm. Find the length of the other parallel side.

Solution : One of the parallel sides $=$ 24cm

Let the length of the other parallel sides be 'x' cm

Also, area of the trapezium $=$ 480 cm^2

Distance between the parallel sides $=$ 8 cm

$\therefore$ Area of a trapezium $= \dfrac{1}{2} \times (a + b) \times h$

$\therefore$ 480 $= \dfrac{1}{2} \times (24 + x) \times 8$

$\Rightarrow$ 480 $= 96 + 4x$

$\Rightarrow$ $480 - 96 = 4x$

$\Rightarrow$ $4x = 384$

$\Rightarrow$ $x = \dfrac{384}{4} = 96$

$\therefore$ The length of the other parallel side $= 96$ cm

Example 4: The ratio of the lengths of the parallel sides of a trapezium is 4:1. The distance between them is 10cm. If the area of the trapezium is 500 cm^2. Find the lengths of the parallel sides.

Solution: Area of the trapezium = 500cm^2

Distance between the parallel sides of the trapezium = 10 cm

Ratio of the lengths of the parallel sides of the trapezium = 4 : 1

Let the lengths of the parallel sides of the trapezium be 4x cm and x cm.

Area of the trapezium = $\frac{1}{2}$ (a + b) × h

$\Rightarrow$ 500 = $\frac{1}{2}$ (x + 4x) × 10

$\Rightarrow$ 500 = (x + 4x) 5

$\Rightarrow$ 500 = 25x

$\Rightarrow$ x = $\frac{500}{25}$ = 20

$\therefore$ One parallel side = 20cm

$\therefore$ The other parallel side = 4x = 4 × 20 = 80cm ($\because$ parallel sides are in 4:1)

Example 5: In the given figure, ABED is a parallelogram in which AB = DE = 10 cm and the area of Δ BEC is 72cm^2. If CE = 16cm, find the area of the trapezium ABCD.
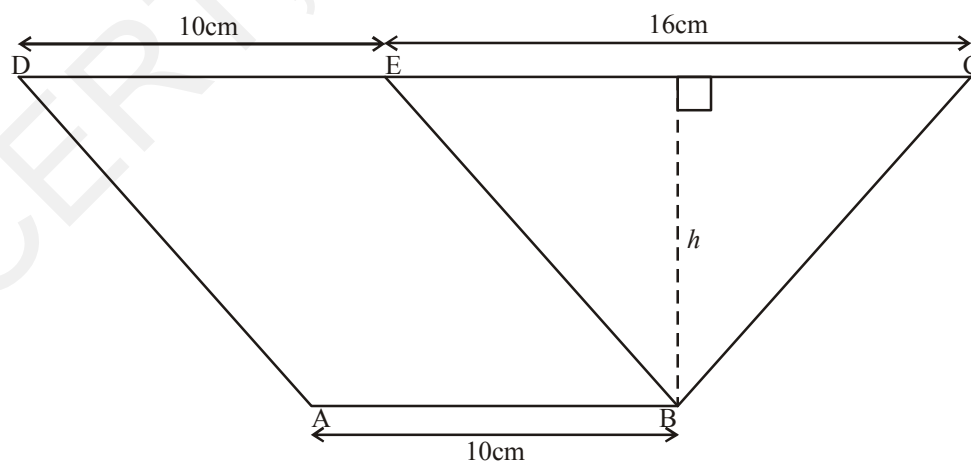
Solution: Area of Δ BEC = $\frac{1}{2}$ × Base × altitude

$$72 = \frac{1}{2} \times 16 \times h$$

$$h = \frac{72 \times 2}{16} = 9 \text{ cm}$$

In trapezium ABCD

AB   = 10 cm

DC   = DE + EC ($\because$ DE = AB)

     = 10 cm + 16 cm = 26 cm

$\therefore$ Area of the trapezium ABCD

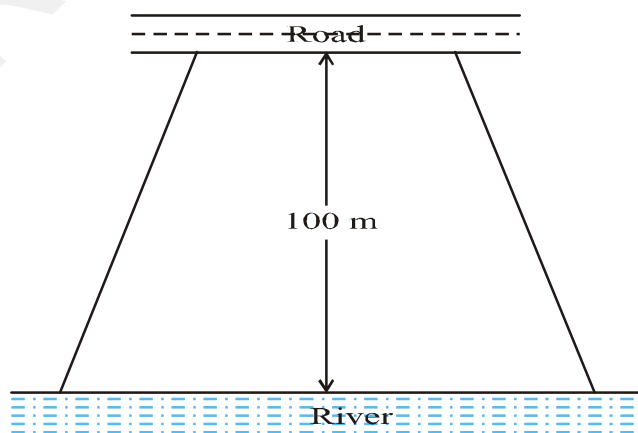$$= \frac{1}{2} \times (a + b) \times h$$

$$= \frac{1}{2}(AB + DC) h$$

$$= \frac{1}{2}(10 + 26) \times 9 \text{ cm}^2$$

$$= 18 \times 9 \text{ cm}^2$$

$$= 162 \text{ cm}^2$$

Example 6: Mohan wants to buy a field on a river-side. A plot of field as shown in the adjacent figure is available for sale. The length of the river side is double the length of the road side and are parallel.

The area of this field is 10,500m^2 and the distance between the river and road is 100 m. Find the length of the side of the plot along the river.

Solution: Let the length of the side of the field along the road be x m.

Then, length of its side along the river $= 2x$ m.

Distance between them $= 100$ m.

Area of the field $= \dfrac{1}{2}(a + b) \times h$

$\Rightarrow \qquad 10,500 = \dfrac{1}{2}(x + 2x) \times 100$

$\Rightarrow \qquad 10,500 = 3x \times 50$

$\Rightarrow \qquad x = \dfrac{10,500}{3 \times 50} = 70$

$\therefore$ Length of the plot on river side $= 2x = 2 \times 70$

$= 140$ m

9.2   Area of a Quadrilateral

A quadrilateral can be split into two triangles by drawing one of its diagonals. This 'Triangulation' helps us to find the area of a quadrilateral.

Mahesh split the quadrilateral ABCD into two triangles by drawing the diagonal AC.

We know that the area of a triangle can be found using two measurements, base of the triangle and vertical height of the triangle, which is the distance from its base to its apex (point at the top of a triangle), measured at right angles to the base.

Mahesh has drawn two perpendicular lines to AC from B and D; named their lengths as h_1 and h_2 respectively.
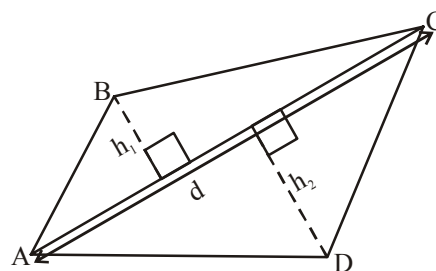
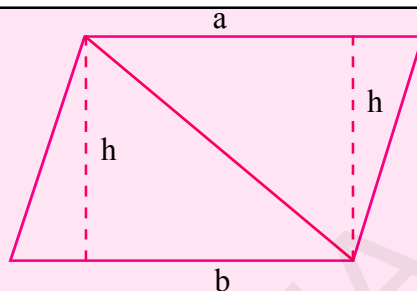Area of the quadrilateral ABCD $=$ (area of $\triangle ABC$) $+$ (area of $\triangle ADC$)

$= \left(\dfrac{1}{2} \times AC \times h_1\right) + \left(\dfrac{1}{2}AC \times h_2\right)$

$= \dfrac{1}{2} AC[h_1 + h_2]$

Area of quadrilateral ABCD $= \dfrac{1}{2} d(h_1 + h_2),$ where 'd' denotes the length of the diagonal AC.

Try These

We know that parallelogram is also a quadrilateral. Let us split such a quadrilateral into two triangles. Find their areas and subsequently that of the parallelogram. Does this process in tune with the formula that you already know?

Area of a quadrilateral = $\frac{1}{2}$ × Length of a diagonal × Sum of the lengths of the perpendiculars drawn from the remaining two vertices on the diagonal.
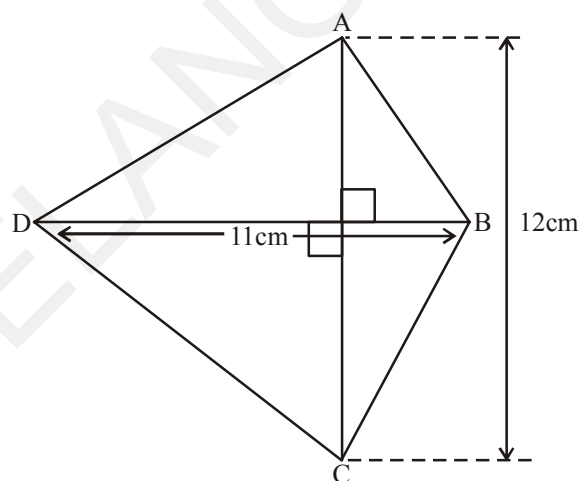
Example 7: Find the area of quadrilateral ABCD

Fig. (i)

Solution: Area of quadrilateral ABCD

$= \frac{1}{2} d(h_1 + h_2)$

Sum of the lengths of perpendiculars from the remaining two vertices on the diagonal AC = $(h_1 + h_2)$

$h_1 + h_2 = 12$ cm.
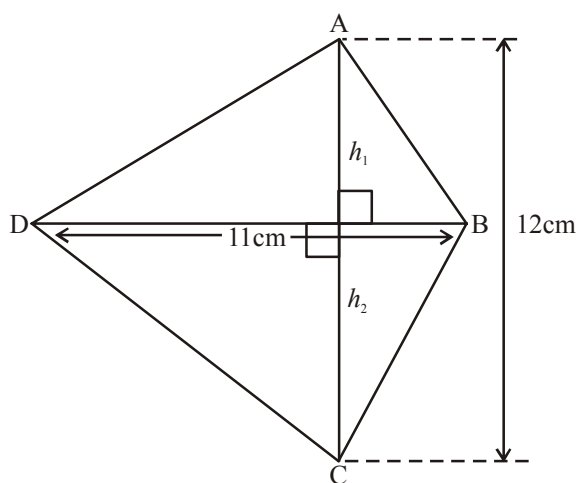
Fig. (ii)

Length of the diagonal (BD) = 11 cm.

$\therefore$ Area of quadrilateral $= \dfrac{1}{2} d(h_1 + h_2) = \dfrac{1}{2} \times 12 \times 11 = 6 \times 11 = 66 \text{ cm}^2$.

9.3   Area of Rhombus

We can use the same method of splitting into triangles (which we called triangulation) to find a formula for the area of rhombus.

In the figure ABCD is a rhombus. We know that the diagonals of a rhombus are perpendicular bisectors of each other.

$\therefore$ OA = OC,   OB = OD

And $\angle$BOA = $\angle$COB = $\angle$DOC = $\angle$AOD = 90^0
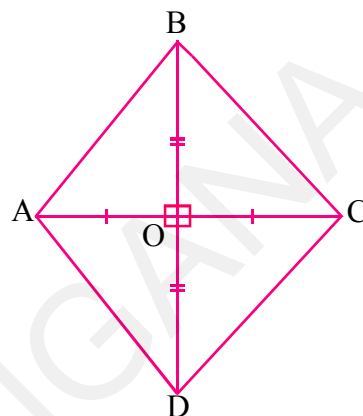
Area of rhombus ABCD = area of Δ ABC + area of Δ ADC

$$= \dfrac{1}{2} \times AC \times OB + \dfrac{1}{2} \times AC \times OD$$

$$= \dfrac{1}{2} \times AC \, (OB+OD)$$

$$= \dfrac{1}{2} \times AC \times BD \qquad (\because OB + OD = BD)$$

Thus area of a rhombus $= \dfrac{1}{2} \times d_1 d_2$, where d_1, d_2 are its diagonals.

In words we say, area of a rhombus is half the product of its diagonals.

Example 8:   Find the area of a rhombus whose diagonals are of length 10 cm and 8.2 cm.

Solution: Area of the rhombus $= \dfrac{1}{2} \times d_1 d_2$ where d_1, d_2 are lengths of diagonals

$$= \dfrac{1}{2} \times 10 \times 8.2 \text{ cm}^2$$

$$= 41 \text{cm}^2$$

9.4 Surveying the field

A surveyor has noted the measurements of a field in his field book in metres as shown below. Find the area of that field.

	To S	
	200	
25 to T ←	160	
	110	→ 60 to R
25 to L ←	70	
	30	→ 40 to Q
	From P	

The data gives the following information

1. The field is in the shape of a hexagon whose vertices are P, Q, R, S, T and L.

2. PS is taken as diagonal

3. Vertices Q and R on one side of the diagonal and the vertices T and L are on another side.

4. The perpendicular drawn from Q to A is 40 m. Similarly other perpendiculars.

5. In the field book the measurements are real and recorded from bottom to top.

6. The field is divided into 2 triangles, 2 trapeziums.

We can find the following measurements from the above figure

$AC = PC - PA$

$= 110 - 30 = 80$ m

$CS = PS - PC$

$= 200 - 110 = 90$ m

$DS = PS - PD$

$= 200 - 160 = 40$ m

$BD = PD - PB$

$= 160 - 70 = 90$ m
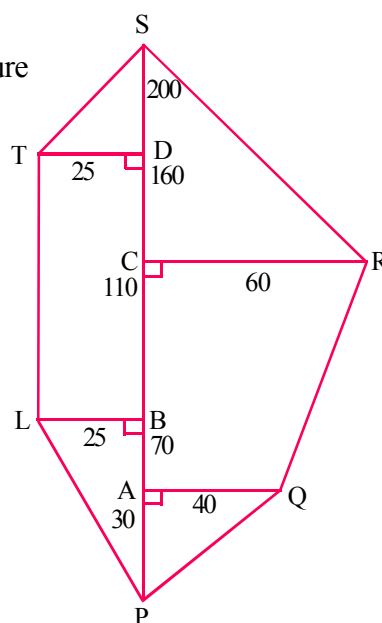
Area of $\triangle$ APQ $= \dfrac{1}{2} \times b \times h$

$= \dfrac{1}{2} \times 30 \times 40 = 600$ Sq.m.

Area of trapezium AQRC $= \dfrac{1}{2} \times h(a + b)$

$= \dfrac{1}{2} \times AC \, (AQ + CR)$

$= \dfrac{1}{2} \times 80 \times (40 + 60)$

$= \dfrac{1}{2} \times 80 \times 100$

$= 4000$ Sq. m.

Area of $\triangle$ CRS $= \dfrac{1}{2} \times CR \times CS = \dfrac{1}{2} \times 60 \times 90 = 2700$ Sq.m.

Area of trapezium PLTS $= \dfrac{1}{2} \times h(a + b)$

$= \dfrac{1}{2} \times LB \, (TL + SP)$

$= \dfrac{1}{2} \times 25(90 + 200) \qquad (\because TL = BD = 90)$

$= \dfrac{1}{2} \times 25 \times 290$

$= 3625$ Sq.m.

Area of the field $= 600 + 4000 + 2700 + 3625$

$= 10,925$ Sq. m.

Do This

The following details are noted in metres in the field book of a surveyor. Find the area of the fields.

(i)

	To D	
	140	
50 to E ←	80	
	50	→ 50 to C
	30	→ 30 to B
	From A	

(ii)

	To C	
	160	
30 to D ←	130	
	90	→ 60 to B
40 to E ←	60	
	From A	

Think and Discuss:

A parallelogram is divided into two congruent triangles by drawing a diagonal across it. Can we divide a trapezium into two congruent triangles?

Try These

Find the area of following quadrilaterals.
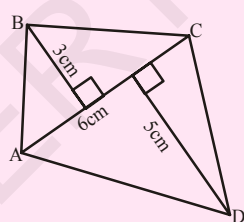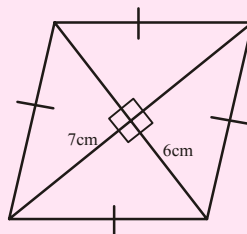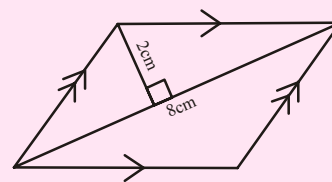
(i)   (ii)   (iii)

9.5 Area of a Polygon

The area of a polygon may be obtained by dividing the polygon into a number of simple shapes (triangles, rectangles etc.) Then the areas of each of them can be calculated and added up to get the required area.

Observe the following pentagon in the given figure:
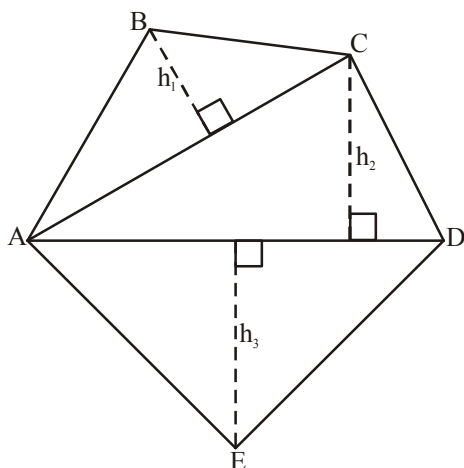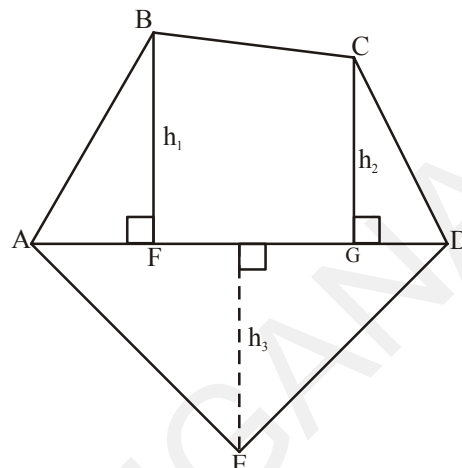
Fig. (i)   Fig. (ii)

Fig.(i) : By drawing two diagonals AC and AD the pentagon ABCDE is divided into three parts. So, area ABCDE = area of ΔABC + area of ΔACD + area of ΔAED

Fig.(ii) : By drawing one diagonal AD and two perpendiculars BF and CG on it, pentagon ABCDE is divided into four parts. So, area of ABCDE = area of right angled ΔBFA + area of trapezium BFGC + area of right angled ΔDGC + area of ΔDEA. Why is this so? (Identify the parallel sides of trapezium BFGC).

Try These

(i)   Divide the following polygon into parts (triangles and trapezium) to find out its area.

FI is a diagonal of polygon EFGHI   NQ is a diagonal of polygon MNOPQR
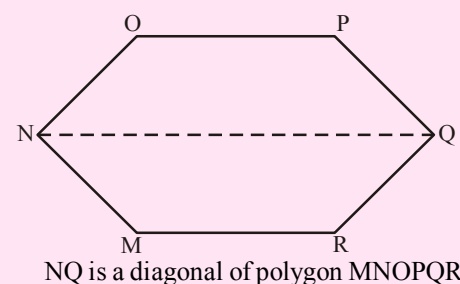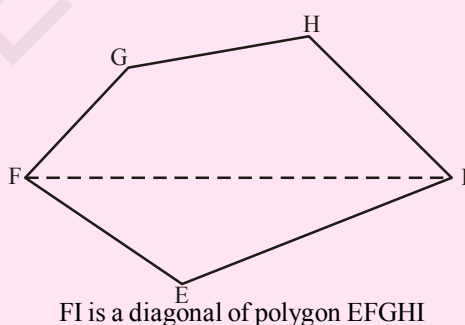
(ii) Polygon ABCDE is divided into parts as shown in the figure. Find the area if AD = 8cm, AH = 6 cm, AF = 3cm and perpendicular, BF = 2cm, CH = 3cm and EG = 2.5cm
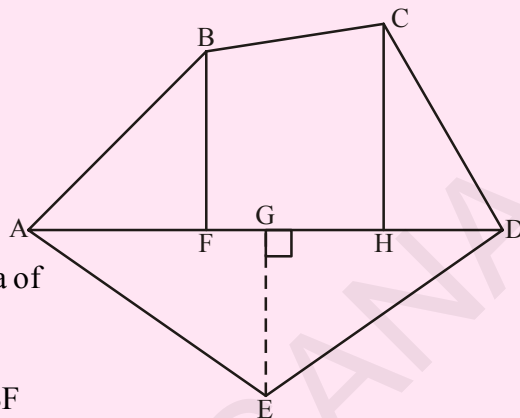
Area of polygon ABCDE = area of

ΔAFB + _____

Area of $\triangle AFB = \dfrac{1}{2} \times AF \times BF$

$\qquad = \dfrac{1}{2} \times 3 \times 2 = $ _____

Area of trapezium FBCH $\qquad = FH \times \dfrac{(BF + CH)}{2}$

$\qquad\qquad = 3 \times \dfrac{(2+3)}{2} \; [\because \; FH = AH - AF]$

Area of $\triangle CHD \qquad = \dfrac{1}{2} \times HD \times CH = $ _____

Area of $\triangle ADE \qquad = \dfrac{1}{2} \times AD \times GE = $ _____

So, the area of polygon ABCDE = ….

(iii) Find the area of polygon MNOPQR if MP = 9 cm, MD = 7 cm, MC = 6 cm, MB = 4 cm, MA = 2 cm

NA, OD, QC and RB are perpendiculars to diagonal MP
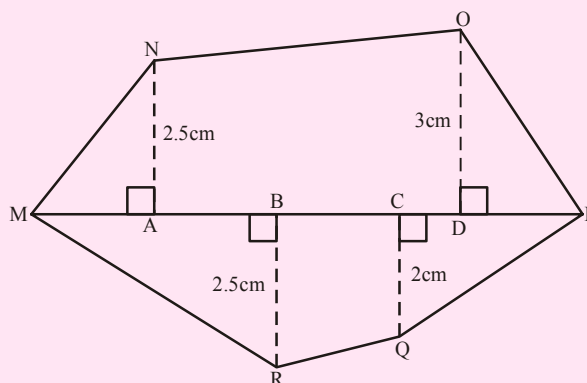
Example 9: Find the area of the field shown along side. All dimension are in metres.

Solution: Area of ABCDE = Area of Δ ABH + area of trap BCFH + area of Δ CDF + + Area of Δ AED

Now, area of Δ ABH

$$= \frac{1}{2} \times AH \times HB$$

$$= \frac{1}{2} \times 25 \times 25$$

$$= \frac{625}{2} \, m^2 = 312.5 \, m^2$$
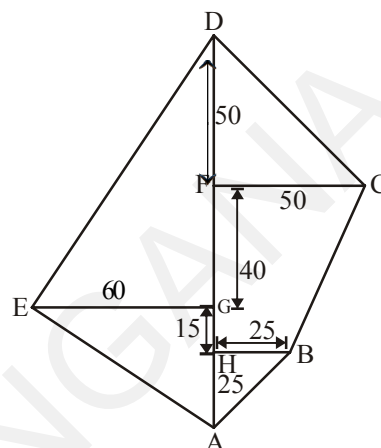
Area of trap BCFH $\quad = \frac{1}{2} \times (HB + FC) \times HF$

$$= \frac{1}{2}(25 + 50) \times 55 m^2$$

$$= \frac{75 \times 55}{2} \, m^2 = 2062.5 \, m^2$$

Area of Δ CDF $\quad = \frac{1}{2} \times FC \times DF$

$$= \frac{1}{2} \times 50 \times 50 \, m^2 = 1250 \, m^2$$

Area of Δ AED $\quad = \frac{1}{2} \times AD \times EG$

$$= \frac{1}{2} \times 130 \times 60$$

$$= 3900 \, m^2$$

Thus, area of ABCDE $= 312.5 \, m^2 + 2062.5 \, m^2 + 1250 \, m^2 + 3900 \, m^2$

$$= 7525 m^2$$

Example 10: There is a hexagon MNOPQR of each side 5 cm and symmetric about NQ. Suresh and Rushika divided it into different ways. Find the area of this hexagon using both ways.
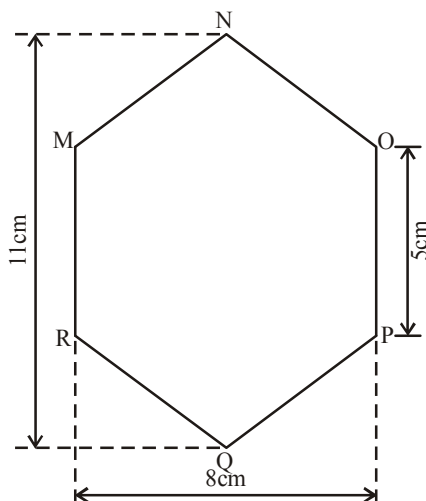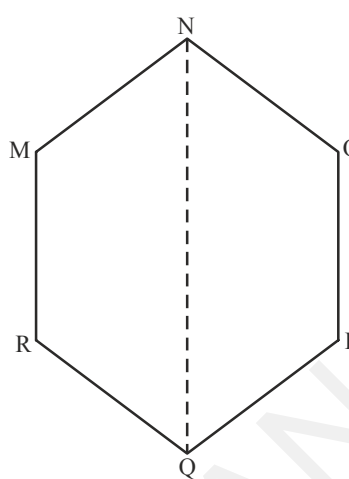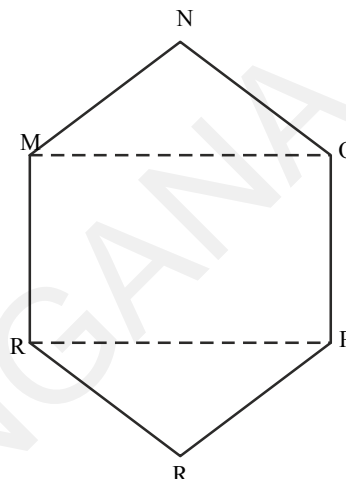

Suresh's Method   **Rushika's Method**

Fig. (i)   Fig. (ii)

Solution: Method adopted by Suresh

Since it is a regular hexagon. So, NQ divides the hexagon into two congruent trapeziums. You can verify it by paper folding.

Now area of trapezium MNQR

$$= 4 \times \frac{11 + 5}{2}$$

$$= 2 \times 16 = 32 \text{ cm}^2$$

So the area of hexagon MNOPQR $= 2 \times 32 = 64 \text{ cm}^2$

Method adopted by Rushika

$\triangle$ MNO and $\triangle$ RPQ are congruent triangles with altitude 3 cm (fig.4). You can verify this by cutting off these two triangles and placing them on one another.

Area of $\triangle$MNO $= \frac{1}{2} \times 8 \times 3 = 12 \text{ cm}^2$

$=$ Area of $\triangle$RPQ

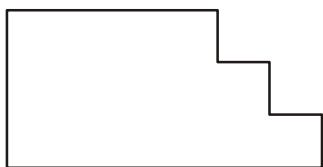Area of rectangle MOPR $= 8 \times 5 = 40 \text{ cm}^2$

Now, area of hexagon MNOPQR $= 40 + 12 + 12 = 64 \text{ cm}^2$.
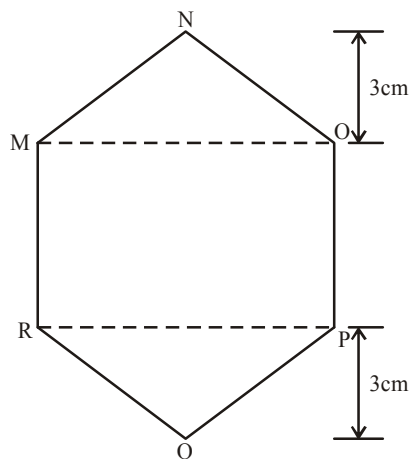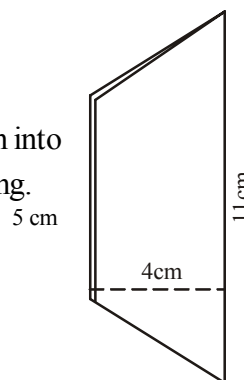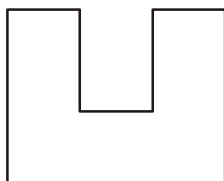
Exercise - 9.1

1. Divide the given shapes as instructed
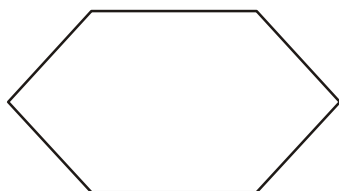
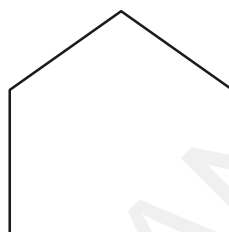   (i) into 3 rectangles   (ii) into 3 rectangles   (iii) into 2 trapezium
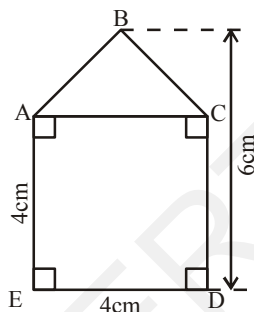
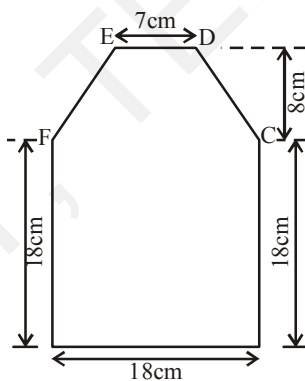   (iv) 2 triangles and a rectangle   (v) into 3 triangles
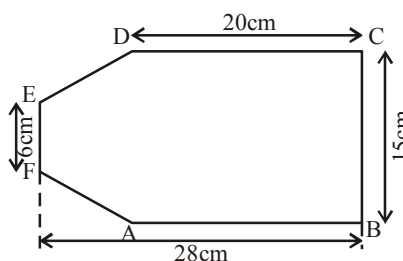
2. Find the area enclosed by each of the following figures
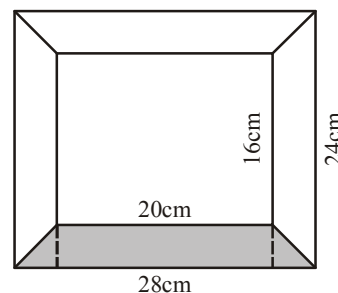
   (i)   (ii)   (iii)

3. Calculate the area of a quadrilateral ABCD when length of the diagonal AC = 10 cm and the lengths of perpendiculars from B and D on AC be 5 cm and 6 cm respectively.

4. Diagram of the adjacent picture frame has outer dimensions 28 cm × 24 cm and inner dimensions 20 cm × 16 cm. Find the area of shaded part of frame, if width of each section is the same.

5. Find the area of each of the following fields. All dimensions are in metres.
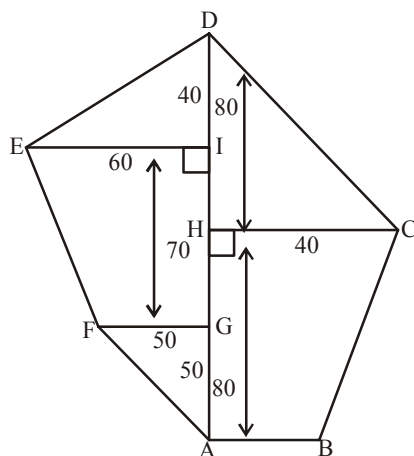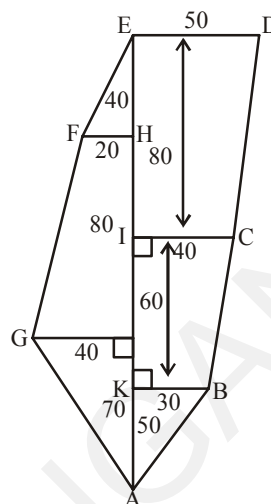


(i)   (ii)

6. The ratio of the length of the parallel sides of a trapezium is 5:3 and the distance between them is 16cm. If the area of the trapezium is 960 cm^2, find the length of the parallel sides.

7. The floor of a building consists of around 3000 tiles which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of flooring if each tile costs rupees 20 per m^2.

8. There is a pentagonal shaped parts as shown in figure. For finding its area Jyothi and Rashida divided it in two different ways. Find the area in both ways and what do you observe?
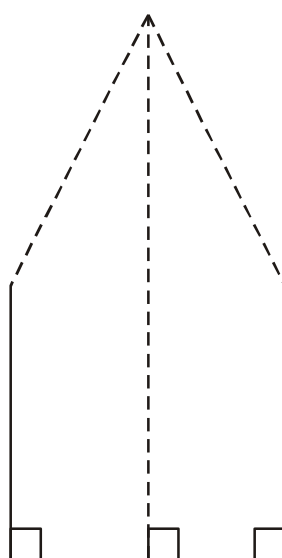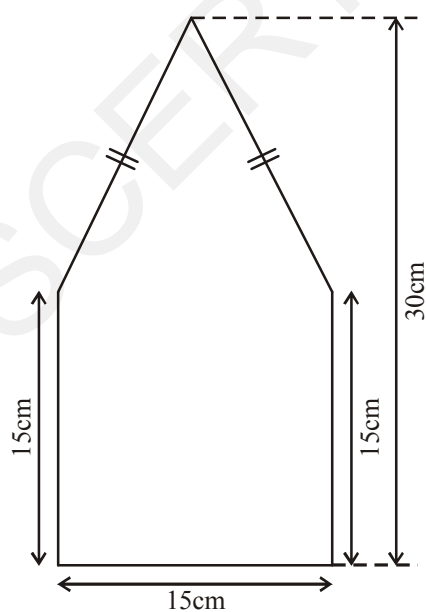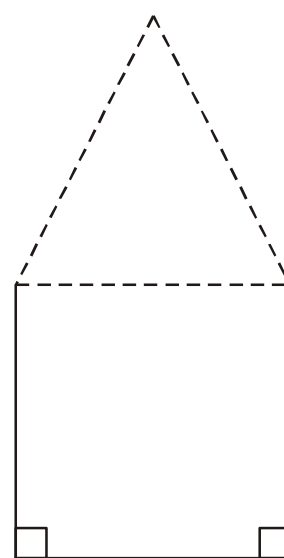
Jyoti Diagram   Rashida's Diagram

9.6   Area of circle

Let us find the area of a circle, using graph paper.

Draw a circle of radius 4 cm. on a graph paper . Find the area by counting the number of squares enclosed.

As the edges are not straight, we roughly estimate the area of circle by this method. There is another way of finding the area of a circle.
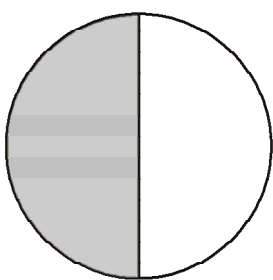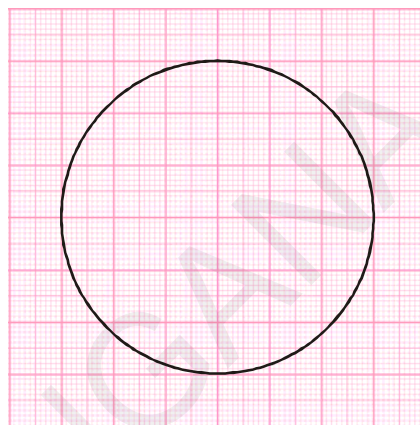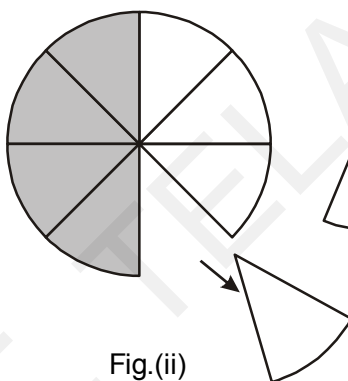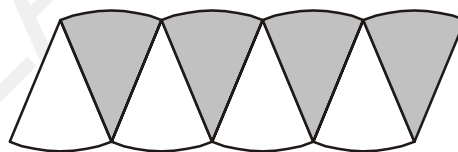

Fig.(i)   Fig.(ii)   Fig.(iii)

Draw a circle and shade one half of the circle as in (Fig.(i)), now fold the circle into eight equal parts and cut along the folds as in Fig (ii))

Arrange the separate pieces as shown in Fig. (iii), which is roughly a parallelogram. The more sectors we have, the nearer we reach an appropriate parallelogram as done above. If we divide the circle into 64 sectors, and arrange these sectors as shown in figure it forms almost a rectangle Fig(iv)

Half of the Circumference

Fig.(iv)

What is the breadth of this rectangle? The breadth of this rectangle is the radius of the circle 'r'
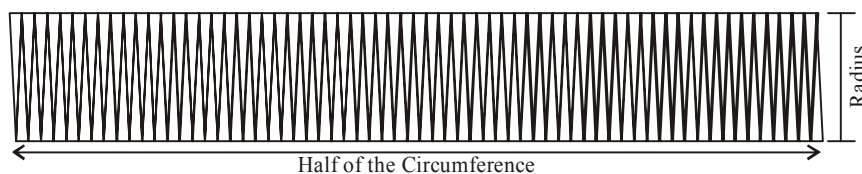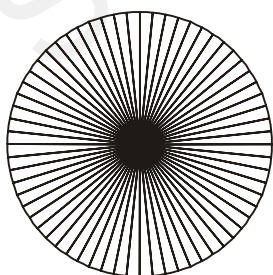
As the whole circle is divided into 64 sectors and on each side we have 32 sectors, the length of the rectangle is the length of the 32 sectors, which is half of the circumference (Fig.(iv)).

$$\text{Area of the circle} = \text{Area of rectangle thus formed}$$

$$= l \times b$$

$$= (\text{half of circumference}) \times \text{radius}$$

$$= \frac{1}{2} \times 2\pi r \times r = \pi r^2$$

So the area of the circle $= \pi r^2$

Thread activity:

The commentaries of the Talmud (A book of Jews) presents a nice approach to the formula, $A = \pi r^2$ to calculate the area of a circle. Imagine that the interior of a circle is covered by concentric circles of yarn. Cut the yarn circles along a vertical radius. Each strand is then straightened and arranged as shown in the figure below to form an isosceles triangle

(i)   (ii)

Radius

(iii)
Circumference of the circle
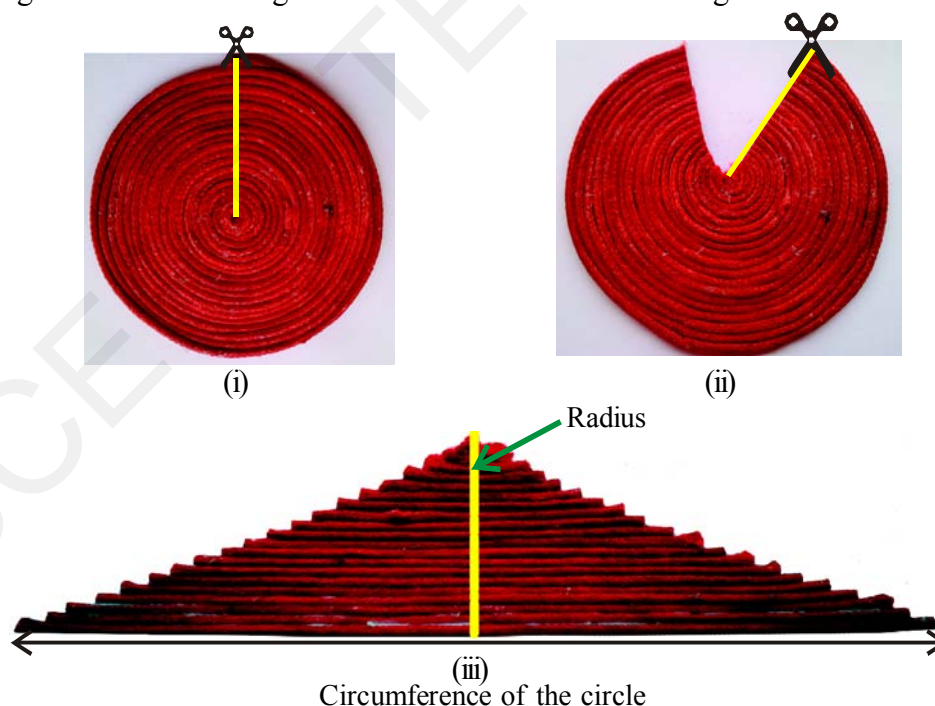
The base of the isosceles triangle is equal to the circumference of the circle and height is equal to the radius of the circle.

$$\text{The area of the triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 2\pi r \times r$$

$$= \pi r^2$$

∴ Area of a circle $= \pi r^2$ (Where r is the radius of the circle)

Try These

Draw circles having different radius on a graph paper. Find the area by counting the number of squares. Also find the area by using formula. Compare the two answers.

Example 11: A wire is bent into the form of a square of side 27.5 cm. The wire is straightened and bent into the form of a circle. What will be the radius of the circle so formed?

Solution: Length of wire = perimeter of the square

$= (27.5 \times 4)$ cm $= 110$ cm.

When the wire is bent into the form of a circle, then it represents the circumference of the circle which would be 110 cm.

Let r be the radius of this circle

Then, circumference $= 110$ cm

we have $2 \times \frac{22}{7} \times r = 110$ \qquad $|\because c = 2\pi r$

∴ $\frac{44}{7} \times r = 110$

$\Rightarrow r = \frac{110 \times 7}{44}$ cm

$= 17.5$ cm

Example 12: The circumference of a circle is 22 cm. Find its area? And also find the area of the semicircle.

Solution: Let the radius of the circle be r cm

Then circumference $= 2\pi r$

$$\therefore \quad 2\pi r \quad = 22$$

$$2 \times \frac{22}{7} \times r \quad = 22$$

$$r = 22 \times \frac{1}{2} \times \frac{7}{22} = \frac{7}{2} \text{ cm}$$

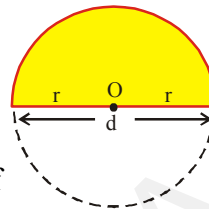$\therefore$ Radius of the circle $\quad = 3.5 \text{cm}$

Area of the circle $\pi r^2 \quad = \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \right) \text{ cm}^2$

$$= 38.5 \text{ cm}^2$$

Area of the semi circle $\quad = \frac{1}{2} \pi r^2$

$$= \frac{1}{2} \times 38.5 = 19.25 \text{ cm}^2$$

> **What is area of Semi-circle?**
>
> Imagine, if we fold the circle along its diameter, we get the area of the shaded region. Can we say area of the shaded region is 1/2 of area of the circle?
>
> Its area is 1/2 of the area of circle $= \frac{1}{2}\pi r^2$ what will be the perimeter of the semi-circle ?
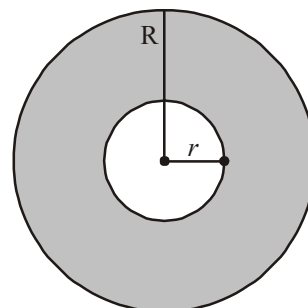
9.7 Area of a Circular Path or Area of a ring

In a park a circular path is laid as shown in the given figure. Its outer and inner circles are concentric. Let us find the area of this circular path.

The Area of the circular path is the difference of Area of outer circle and inner circle.

If we say the radius of outer circle is 'R' and inner circle is 'r' then

Area of the circular path = Area of outer circle – Area of inner circle

$$= \pi R^2 - \pi r^2$$

$$= \pi (R^2 - r^2)$$

> Hence
>
> Area of the circular path or Area of a ring $= \pi (R^2 - r^2)$ or $\pi (R + r)(R - r)$

Where R, r are radii of outer circle and inner circle respectively

Example:13 Observe the adjacent figure. It shows two circles with the same centre. The radius of the larger circle is 10cm and the radius of the smaller circle is 4cm.

Find (i) the area of the larger circle

(ii) The area of the smaller circle

(iii) The shaded area between the two circles. (take $\pi = 3.14$)
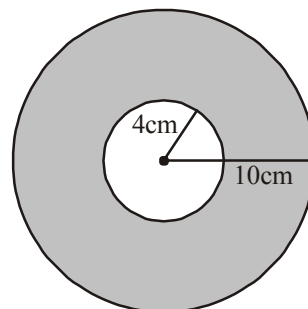
Solution: (i)   Radius of the larger circle = 10 cm

So, area of the larger circle = πR^2

$= 3.14 \times 10 \times 10 = 314 \text{ cm}^2$

(ii)   Radius of the smaller circle = 4 cm

So, area of the smaller circle = πr^2

$= 3.14 \times 4 \times 4 = 50.24 \text{ cm}^2$

(iii)   Area of the shaded region = Area of the larger circle – Area of the smaller circle

$= (314 - 50.24) \text{ cm}^2$

$= 263.76 \text{ cm}^2.$

Example 14:   Calculate the area of shaded part of the figure given below
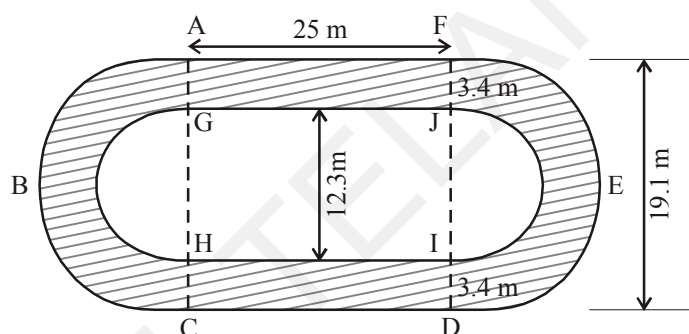
Solution: Shaded Area = Area of rectangle AGJF + Area of rectangle HCDI + Area of semi circular ring ABCHG + Area of semicircular ring DEFJI

Area of rectangle AGJF = $25 \times 3.4 = 85 \text{ m}^2.$

Area of rectangle HCDI = $25 \times 3.4 = 85 \text{ m}^2.$

Area of a semi circular ring ABCHG = $\frac{\pi}{2}[(R^2 - r^2)] = \frac{22}{7} \times \frac{1}{2}[(9.55)^2 - (6.15)^2]$

Area of a semi circular ring DEFJI = $\frac{\pi}{2}[(R^2 - r^2)] = \frac{22}{7} \times \frac{1}{2}[(9.55)^2 - (6.15)^2]$

$= (25 \times 3.4) + (25 \times 3.4) + \frac{1}{2}\pi[(9.55)^2 - (6.15)^2] + \frac{1}{2}\pi[(9.55)^2 - (6.15)^2]$

$= [85 + 85 + \frac{22}{7} \times 15.7 \times 3.4]\text{m}^2$

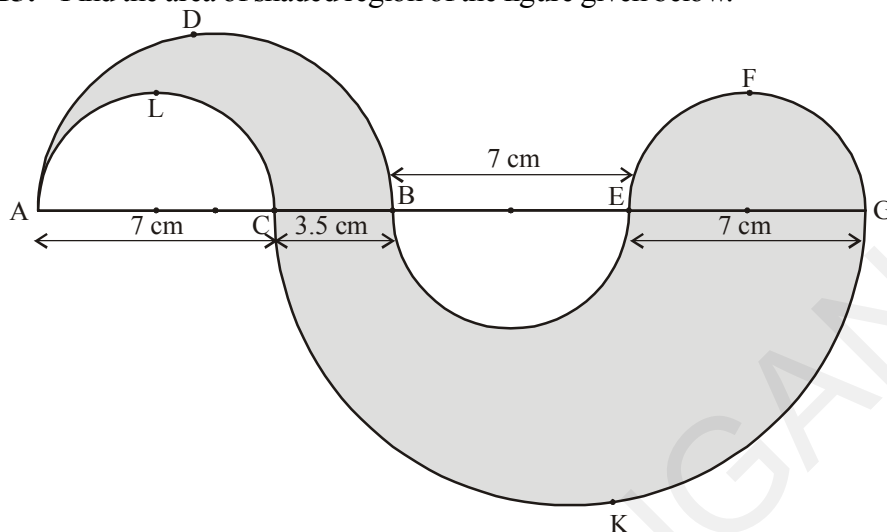$= (170 + 167.77)\text{m}^2$

$= 337.77\text{m}^2$

$R = \frac{19.1}{2} = 9.55$

$r = \frac{12.3}{2} = 6.15$

Example 15: Find the area of shaded region of the figure given below.

Solution: Shaded area = Area ADBCLA + Area EFGE + Area BEGKCB

$$= \frac{1}{2} \times \pi \left[\left(\frac{10.5}{2} \right)^2 - \left(\frac{7}{2} \right)^2 \right] + \frac{1}{2} \pi \left(\frac{7}{2} \right)^2 + \frac{1}{2} \pi \left[\left(\frac{17.5}{2} \right)^2 - \left(\frac{7}{2} \right)^2 \right] \text{cm}^2$$

$$= \left(\frac{1}{2} \times \frac{22}{7} \times \frac{35}{4} \times \frac{7}{4} \right) + \left(\frac{1}{2} \times \frac{22}{7} \times \frac{49}{4} \right) + \left(\frac{1}{2} \times \frac{22}{7} \times \frac{21}{4} \times \frac{49}{4} \right) \text{cm}^2$$

$$= \left(\frac{385}{16} + \frac{77}{4} + \frac{1617}{16} \right) \text{cm}^2$$

$$= \left(\frac{2310}{16} \right) \text{cm}^2$$

$$= 144.375 \text{ cm}^2$$

9.8 Length of the arc

Observe the following circles and complete the table

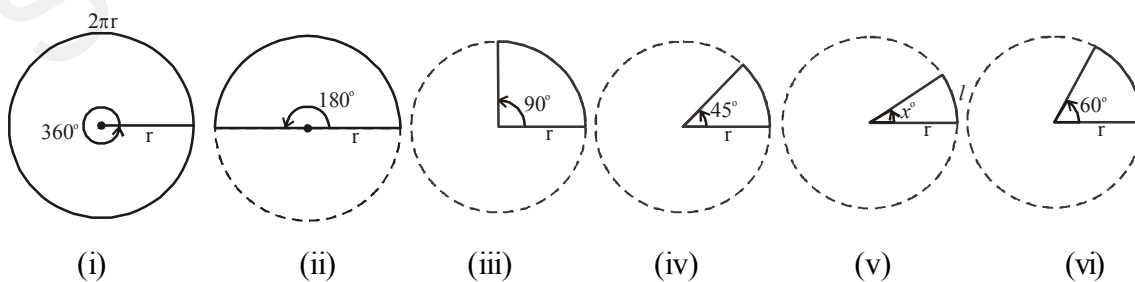| (i) | (ii) | (iii) | (iv) | (v) | (vi) |

| Fig	Angle	Length of Arc	Relation between angle and length of arc
(i)	360^0	$2\pi r$	$\dfrac{360^0}{360^0} \times 2\pi r = 2\pi r$
(ii)	180^0	πr	$\dfrac{180^0}{360^0} \times 2\pi r = \pi r$
(iii)	90^0	$\dfrac{\pi r}{2}$	_____
(iv)	45^0	$\dfrac{\pi r}{4}$	_____
(v)	x^0	l	$\dfrac{x^0}{360^0} \times 2\pi r = l$
(vi)	60^0	$\dfrac{\pi r}{3}$	_____

From the above observations, the length of the arc of a sector (l) is $\dfrac{x^\circ}{360^\circ} \times 2\pi r$ where 'r' is the radius of the circle and 'x' is the angle subtended by the arc of the sector at the centre.

If the length of arc of a sector is l

$$\frac{2\pi r}{l} = \frac{360^0}{x^0}$$

Then

$$l = \frac{x^\circ}{360^\circ} \times 2\pi r$$
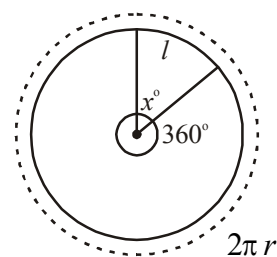
9.9 Area of Sector

We know that part of a circle bounded by two radii and an arc is called sector.
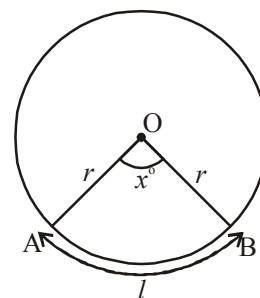
The Area of a circle with radius r $= \pi r^2$

Angle subtended by the arc of the sector at centre of the circle is x^0

Area of a sector and its angle are in direct proportion

$\therefore$ Area of sector : Area of circle $= x^0 : 360^0$

The area of sector OAB $= \dfrac{x^\circ}{360^\circ} \times$ Area of the circle

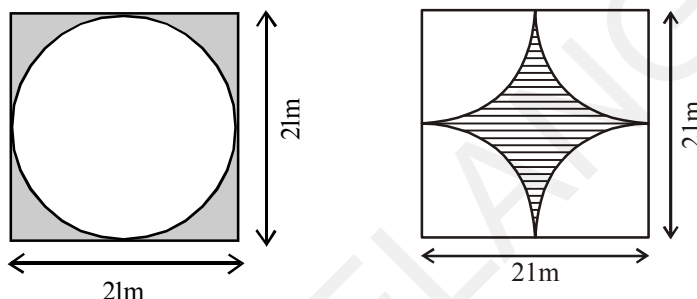Hence   Area of sector OAB = $\dfrac{x^o}{360^o} \times \pi r^2$ $[\pi r^2 = \pi r \times \dfrac{2r}{2}]$

$$= \dfrac{x^o}{360^o} \times 2\pi r \times \dfrac{r}{2}$$

$$= l \times \dfrac{r}{2}$$

$$A = \dfrac{lr}{2} \ (l \text{ is length of the arc})$$

Example 13:   Find the area of shaded region in each of the following figures.

Solution: **(i)**   Area of the shaded region

={Area of the square with side 21m} – {Area of the circle with diameter 21m}

If the diameter of the circle is 21m

Then the radius of the circle = $\dfrac{21}{2}$ =10.5m

Area of the shaded region = $(21 \times 21) - \left(\dfrac{22}{7} \times \dfrac{21}{2} \times \dfrac{21}{2} \right) m^2$

$$= 441 - 346.5$$

$$= 94.5 \ m^2$$

(ii) Area of shaded region = {Area of the square with side 21 m} –

{4×Area of the sector}

$$= (21 \times 21) - \left(4 \times \dfrac{90^o}{360^o} \times \dfrac{22}{7} \times \dfrac{21}{2} \times \dfrac{21}{2} \right) \ m^2$$
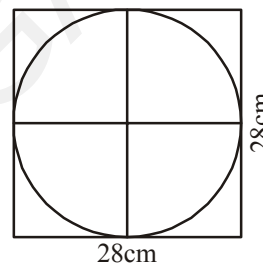
(if diameter is 21m, then radius is $\dfrac{21}{2}$ m)

$$= (21 \times 21) - \left(4 \times \frac{1}{4} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \right)$$

$$= (441 - 346.5) \, \text{m}^2$$
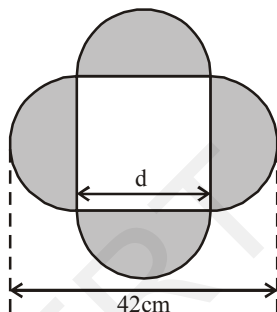
$$= 94.5 \, \text{m}^2$$

Exercise - 9.2

1. A rectangular acrylic sheet is 36 cm by 25 cm. From it, 56 circular buttons, each of diameter 3.5 cm have been cut out. Find the area of the remaining sheet.

2. Find the area of a circle inscribed in a square of side 28 cm.

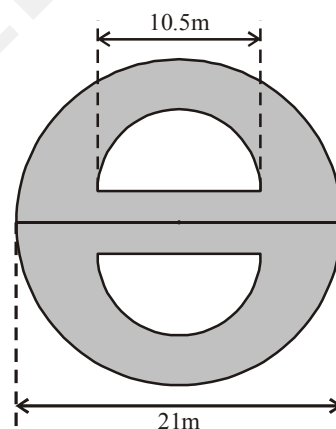   [**Hint.** Diameter of the circle is equal to the side of the square]

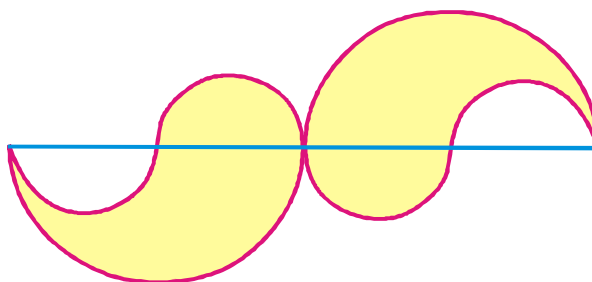3. Find the area of the shaded region in each of the following figures.
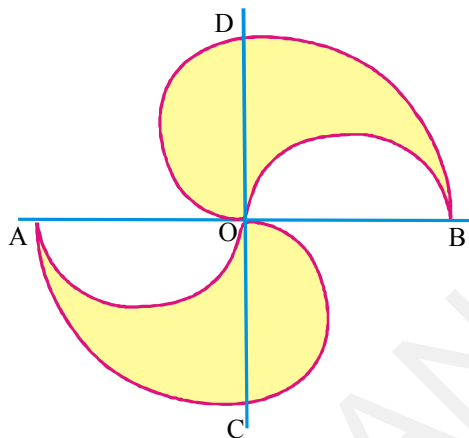
[Hint: $\frac{d}{2} + d + \frac{d}{2} = 42$]

d = 21

∴ side of the square 21 cm

4. The adjacent figure consists of four small semi-circles of equal radii and two big semi-circles of equal radii (each 42 cm). Find the area of the shaded region

5. The adjacent figure consists of four half circles and two quarter circles. If OA = OB = OC = OD = 14 cm. Find the area of the shaded region.

6. In adjacent figure A, B, C and D are centres of equal circles which touch externally in pairs and ABCD is a square of side 7 cm. Find the area of the shaded region.
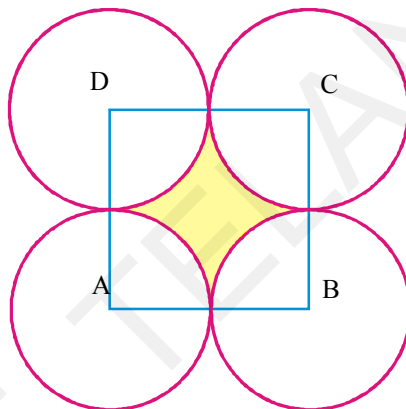
7. The area of an equilateral triangle is $49\sqrt{3}$ cm². Taking each angular point as centre, a circle is described with radius equal to half the length of the side of the triangle as shown in the figure. Find the area of the portion in the triangle not included in the circles.
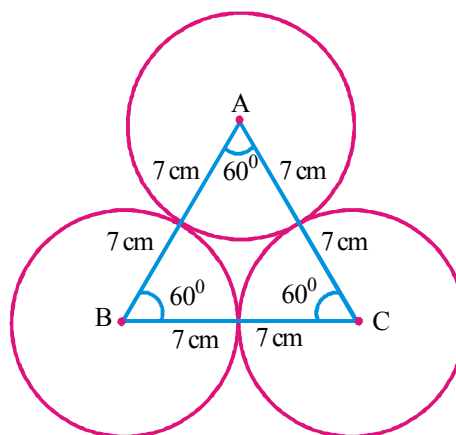
8. (i) Four equal circles, each of radius 'a' touch one another. Find the area between them.

(ii) Four equal circles are described about the four corners of a square so that each circle touches two of the others. Find the area of the space enclosed between the circumferences of the circles, each side of the square measuring 24 cm.

9. From a piece of cardbord, in the shape of a trapezium ABCD, and AB||CD and $\angle BCD = 90^0$, quarter circle is removed. Given AB = BC = 3.5 cm and DE = 2 cm. Calcualte the area of the remaining piece of the cardboard. (Take π to be $\dfrac{22}{7}$)
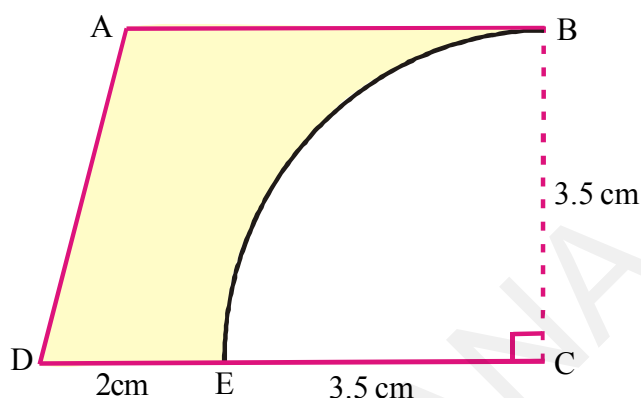
10. A horse is placed for grazing inside a rectangular field 70 m by 52 m and is tethered to one corner by a rope 21 m long. How much area can it graze?
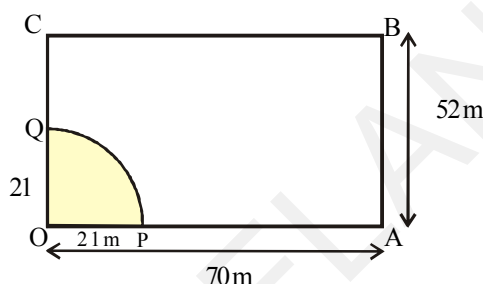
What we have discussed

Area of a trapezium =

$\dfrac{1}{2}$ (Sum of the lengths of parallel sides) × (Distance between them)

• Area of a quadrilateral = $\dfrac{1}{2}$ × length of a diagonal × Sum of the lengths of the perpendiculars from the remaining two vertices on the diagonal)

• Area of a rhombus = Half of the product of diagonals.

• Area of a circle = πr^2 where 'r' is the radius of the circle.

• Area of a circular path (or) Area of a Ring = $\pi(R^2 - r^2)$ or $\pi(R + r)(R-r)$ where R, r are radii of outer circle and inner circle respectively.

• Area of a sector = $\dfrac{x^0}{360^0} \times \pi r^2$ where x° is the angle subtended by the arc of the sector at the center of the circle and r is radius of the circle.

$A = \dfrac{lr}{2}$