Chapter 12

Algebraic Expression

Introduction to Algebraic Expressions

In class 7 teacher wants to teach algebraic expression to the students. So, she gave the following example:

Teacher: Suppose Ram and Raghav are siblings. Their mother gives them some marbles. Suppose Ram has two marbles more than Raghav, but the exact number of marbles they both have is unknown. Only we know that Raghav has two more marbles than Ram.

If you have to express this mathematically how will you do it?

Student: We don't know the exact number of marbles Ram has. Hence, we assume that Ram has x number of marbles. Now, we know that Raghav has two more marbles than Ram.

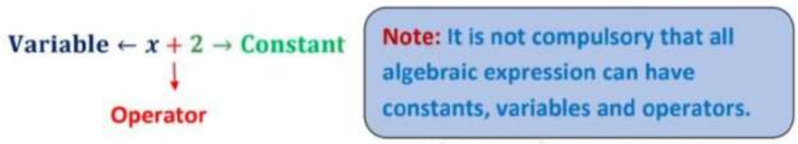
So, it can be expressed as, $x + 2$.

Teacher: Good! But what is $(x + 2)$?
This $(x + 2)$ is the algebraic expression.

So, an algebraic expression is defined as

"The combination of numbers, variables, constants and operators like plus, minus signs"

The above example is the best example of algebraic expression in which



Now, let us see what are Constants, Variables, and Operators.

(i) Constant:
A symbol having a fixed numerical value is called a constant.
Example:1, 2,$\pi = 3.14$ etc....are the examples of constants.

(ii) Variable:
A symbol which may be assigned different values is called a variable. Alphabets like x, y, z, a, b, c, r, etc.... are used to denote variables.
Example: Consider the two expressions

(i) $x - 5 = 0$;
$x = 0 + 5$;
$\therefore x = 5$

(ii) $2x - 4 = 0$;
$2x = 4$;
$\therefore x = 2$

In the first expression, the value of x is 5 and in the second expression, the value of x is 2. So, we say that depending upon the situation the value of the variable changes.

Operators: Operators includes all the operation like addition, subtraction, multiplication, division, equal to, etc..

Example: Identify the following are algebraic expressions or not?
(i) $5x - 3$ (ii) $5x$ (iii) $2x^2 + 2$

(i) $5x - 3$ in which
5 and 3 (constants)

x (Variable)
− (Operator)
Hence, it is an algebraic expression.

(ii) 5x
It is written as $5 \times x$
So, here we have
5– constant
x – Variable
× − Operator
Hence, it is an algebraic expression.

(iii) $2x^2 + 2$
Here we have
2– constant

x – Variable
+ − Operator
Hence, it is an algebraic expression.

Terms of an algebraic expression:

As the algebraic expression contain many small parts and that parts of an algebraic expression which are formed separately first and then added that each part is called a term.

"Various parts of an expression which are separated by the sign of addition or subtraction are called terms."

Example: Consider $5x^2 + 9$ is an algebraic expression.

In this expression, there are two terms.
First-term = $5x^2$
Second-term = 9
When we added these two terms together then we get an above algebraic expression.

Example: Consider 5x is an algebraic expression.
In this expression, there is only one term i.e., 5x.

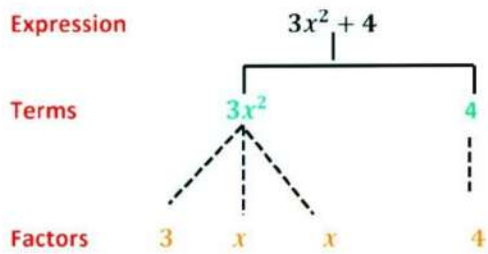
Example: Consider $2x - 2y + 2$ is an algebraic expression.
In this expression, there are three terms.
First-term = 2x

Second-term = − 2y
Third term = 2
When we added these three terms together then we get an above algebraic expression.

Factors of a Term

Factors of a Term

As we know the algebraic expression is made up of terms. Similarly, each term is also made up of constants and variables and these constants and variables are called factors of the term.
Example:



The above algebraic expression consists of two terms $3x^2$ and 4. In which the first term $3x^2$ is made up of 3, x, x.what is this 3, x, x? It is the factors of term $3x^2$ and they cannot be factorized further. Now, the second term is 4 and the factor of the second term is only 4.

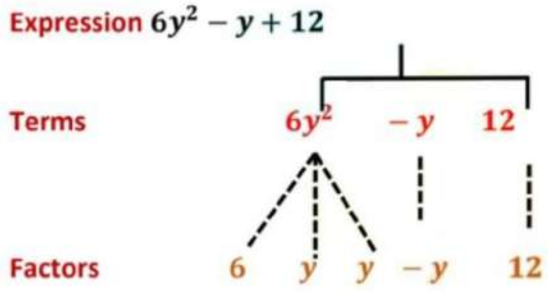
Now, when we see the factors we see certain factors are numbers and certain factors are variable. So, numerals or numbers or constants are called numeric factors, and the variables are called algebraic factors. In the given example 6 and 12 are numeric factors and y is the algebraic factor

Coefficients

In the above section, we see there are two types of factors
(1) Numeric factors
(2) Algebraic factors

"That numerical factor is said to be the numerical coefficient or simply the coefficient of the term."

Example:



The above example consists of three terms, $6y^2$, $-y$ and 12, in which

Factors of the first term $6y^2 = 6, y, y$
The factor of the second term $-y = -y$
The factor of the third term $12 = 12$
Now, when we see all the factors, we see
6 and 12 are numeric factors and y is an algebraic factor.
Hence, in this example, the coefficient of $6y^2$ is 6 and the coefficient of $-y$ is $-$ 1.

Example: $1x$ is written as x .
Whereas, $-1x$ is written as $-x$.

Note: When the coefficient of a term is +1, it is usually omitted. But when the coefficient is –1 then it is indicated by the minus sign.

Example: Write the coefficient of x^2 in the following example.

(i) $17 - 2x + 7x^2$
The coefficient of x^2 in this expression is 7.

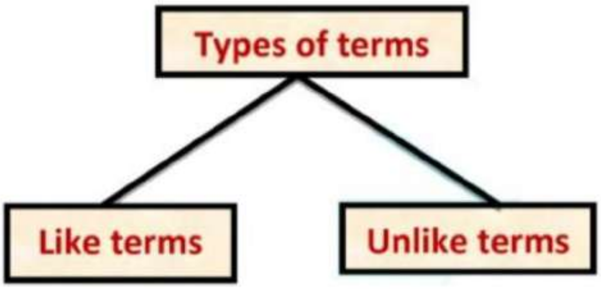
Example: Write the coefficient of the given terms
(i) – x (ii) $3x^2$

(i) – x
In this example, the coefficient of x is -1
(ii) $3x^2$
In this example, the coefficient of x^2 is 3

Example: Write the coefficient of the given terms
(i) 7xy and (ii) $-5x^2y^2$

(i) In the first example, the coefficient of xy is 7
(ii) In the second example, the coefficient of x^2y^2 is -5

Like and Unlike Terms
There are two types of terms.



(i) Like terms: Like terms are terms that have the same variables and powers. The coefficients do not need to match.
Example:

$5x, 2x, -3x, \quad 9x$
All are like terms, because they all have same variable x and they all have same power 1.

(ii) Unlike terms: Unlike terms are two or more terms that do not have the same variables or powers.
Example:

Example	Variables	Powers
$5x$	x	1
$2y$	y	1
$3x^2 + 5y^2 + 3$	x and y	2

These are not like terms, because they all have different variables and their powers are also different.

Example: Group the like terms from the following:
$-2xy^2$, $3yx^2$, x^2, $5xy^2$, 9y, $11x^2$,

$-150x$, $-11yx$, $2x^2 y$, $-5x^2$, y, xy, 3x

Like terms are:
(i) – $2xy^2$, $5xy^2$ (ii) $3yx^2$, $2x^2 y$ (iii)x^2, $-11x^2$, $-5x^2$

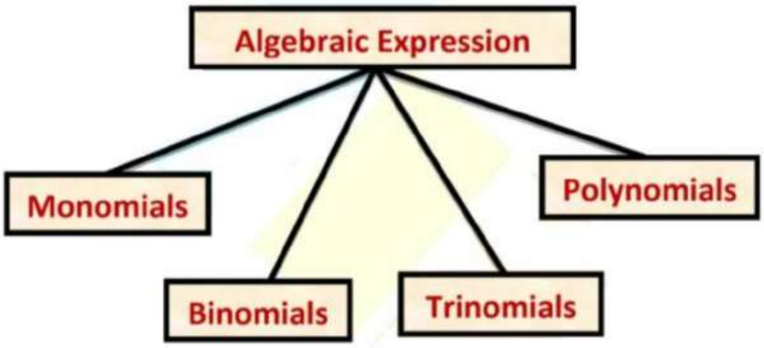
(iv) 9y, y (v) −150x, 3x (vi) −11yx, xy

Types of Algebraic Expressions

Types of Algebraic Expressions

Monomials, Binomials, Trinomials, and Polynomials

Based on the number of terms, an algebraic expression can be classified as:



1) Monomial:
An algebraic expression that contains only one term is called a monomial. Example: 2x, 3, 4t, 9pq, $2c^2$ etc... each of these expressions contains only one term hence is called a monomial.

2) Binomial:
An algebraic expression that contains two unlike terms is called a binomial.

Example: (i) $6x + 2x^2$

The given expression is a binomial since it contains two unlike terms, i.e., 6x and $2x^2$ which are separated by a plus sign.

(ii) $6x - 2x^2$

The given expression is a binomial since it contains two unlike terms, 6x and $2x^2$ which are separated by a minus sign.

Note: If the terms, 6x and 2x2 are separated by a multiplication sign ($6x \times 2x^2$) then it is not binomial because when we multiply two unlike terms then a monomial is formed.

$$6x \times 2x^2 = (6 \times 2)(x \times x^2) = 12x^3$$

3) Trinomial:
An algebraic expression that contains three unlike terms is called a trinomial.
Example: (i) $x + x^2 - 3x^3$ is trinomial since it contains three unlike terms namely, x, x^2 and $-3x^3$.

(ii) $7y^3 + 8y^2 + 9y$ is also a trinomial since it contains three unlike terms namely, 9y, $8y^2$ and $7y^3$.

4) Polynomial:
An algebraic expression that contains one or more terms is called a polynomial. A polynomial can be monomial, binomial or trinomial
Example: $x + x^2 - 3x^3$, $2a^2$, $7y^3 + 8y^2$

these are examples of a polynomial.

Classify into monomials, binomials, trinomial and polynomial.
(i)$3 \times a + a$ (ii)$7a^2 + 8b + 9c$ (iii) $6a^2 + 5b^2$

(iv) $3a \times 4b + 2c$

(i) $3 \times a + a$
$= 3a + a$
$= 6a$
Here, the number of the term is 1. Hence, it is a monomial.

(ii)$5a_2 + 8b + 9c$
Here, the number of terms is 3. Hence, it is a trinomial.

(iii) $6a^2 + b^2$

Here, the number of terms is 2. Hence, it is a binomial.

(iv)$3a \times 3b + 2c$

$= 9ab + 2c$

Here, the number of terms is 2. Hence, it is a binomial.

(ii)$y^2 + xy + 9 + y$
This is a polynomial.

Addition and Subtraction of Algebraic Expressions

Suppose, Ram has 1 pencil and Shyam has 2 pencils, then how many pencils do they have in total?
Here, we can easily make the addition of two numbers like 1 pencil + 2 pencils = 3 pencils.

So, they together have three pencils.
Now if I say, Ram has 1 pen and Shyam has 2 pencils. Now, can you add these two things together?

No, because we see in the first case they both have the same thing, i.e., pencils. But in the second case, they both have different things i.e., pens and pencils.

The same concept is applied for the addition and subtraction of an algebraic expression.

Addition of an algebraic expression

In addition, we can add only like terms together means those terms having the same algebraic factors. If you have unlike terms you cannot add them together. So, you can leave them as it is.

Example: Add 2x and 5x
In the example, we see that 2x and 5x are monomials and they both contain the same algebraic coefficient. Hence, both of them are like terms. Hence, we can easily add them together.

$$2x + 5x = (2 + 5)x = 7x$$

Example: Add $5x^3$ and $\frac{1}{5}x^3$

In the example, we see that $5x^3$ and $\frac{1}{5}x^3$ are monomials and they both contain the same algebraic coefficient. Hence, both of them are like terms. Hence, we can easily add them together.

$$5x^3 + \frac{1}{5}x^3 = \left(5 + \frac{1}{5}\right) x^3 = \left(\frac{25+1}{5}\right) x^3 = \frac{26}{5} x^3$$

Example: Add $(3x + 2y)$ and $(2x + y)$
Now, first we collect the like terms together from both groups.

$$= 3x + 2x + 2y + y$$
$$= (3 + 2)x + (2 + 1)y$$
$$= 5x + 3y$$

Example:
Add $4x^2 - 3x + 1$ to the sum of $3x^2 - 2x$ and $x + 7$

Sum of $3x^2 - 2x$ and $x + 7$

$$= (3x^2 - 2x) + (x + 7)$$

$$= 3x^2 - 2x + x + 7$$

$$= 3x^2 - x + 7$$

Now, the required expression

$$= 4x^2 - 3x + 1 + 3x^2 - x + 7$$

$$= 4x^2 + 3x^2 - 3x - x + 1 + 7$$

$$= 7x^2 - 4x + 8$$

Example:

Add: $(4ab - 5bc + 7ca) + (-2ab + 2bc - 3ca) + (5ab - 3bc + 4ca)$
Collecting positive and negative like terms together, we get
$4ab - 2ab + 5ab - 5bc + 2bc - 3bc + 7ca - 3ca + 4ca$

$= 9ab - 2ab - 8bc + 2bc + 11ca - 3ca$
$= 7ab - 6bc + 8ca$

Subtraction of an algebraic expression

In addition, we can add only like terms together means those terms having the same algebraic factors. If you have unlike terms you cannot add them together. So, you can leave them as it is. In a similar way, we will do subtraction of an algebraic expression.

Example: Subtract 5x and 2x
In the example, we see that 5x and 2x are monomials and they both contain the same algebraic coefficient. Hence, both of them are like terms. Hence, we can easily subtract them together.

$$5x - 2x = (5 - 2)x = 3x$$

Example: Subtract $5x^3$ and $\frac{1}{5}x^3$

In the example, we see that $5x^3$ and $\frac{1}{5}x^3$ are monomials and they both contain the same algebraic coefficient. Hence, both of them are like terms. Hence, we can easily subtract them together.

$$5x^3 - \frac{1}{5}x^3 = \left(5 - \frac{1}{5}\right)x^3 = \left(\frac{25 - 1}{5}\right)x^3 = \frac{24}{5}x^3$$

Subtract $(3x + 2y)$ and $(x + y)$

$$(3x + 2y) - (x + y)$$
$$3x + 2y - x - y$$

First we collect the like terms together

$$= 3x - x + 2y - y$$
$$= (3 - 1)x + (2 - 1)y$$
$$= 2x + y$$

Note: In order to subtract an algebraic expression from another, we change the sign (from '+' to '-' or from '-' to '+') of all the terms of the expression which is to be subtracted and then the two expressions are added.

Example:
Subtract: $(-6x + 7x^2 - 4)$ from $(2x^2 + 5x - 3)$

$$(-6x + 7x^2 - 4) - (2x^2 + 5x - 3)$$

$$-6x + 7x^2 - 4 - 2x^2 - 5x + 3$$

First we collect the like terms together

$$7x^2 - 2x^2 - 6x - 5x - 4 + 3$$

$$(7 - 2)x^2 - (6 + 5)x - 4 + 3$$

$$5x^2 - 11x - 1$$

Alternate method:
We subtract $(-6x + 7x^2 - 4)$ from $(2x^2 + 5x - 3)$ column wise.

- **To subtract this column wise we arranging the terms of the given expression in descending powers of x.**
- **Then subtract column wise.**

$$\begin{array}{rrr} 7x^2 & -\ 6x & -\ 4 \\ 2x^2 & +\ 5x & -\ 3 \\ - & - & + \\ \hline 5x^2 & -\ 11x & -\ 1 \end{array}$$

Example:
Subtract: $(8 - 5x + 4x^2 + 8x^3) - (6x^3 - 7x^2 + 5x - 3)$

$$= 8 - 5x + 4x^2 + 8x^3 - 6x^3 + 7x^2 - 5x + 3$$

Now, first we collect the like terms together from both groups.

$$= 8x^3 - 6x^3 + 7x^2 + 4x^2 - 5x - 5x + 3 + 8$$

$$= (8 - 6)x^3 + (7 + 4)x^2 - (5 + 5)x + 11$$

$$= 2x^3 + 11x^2 - 10x + 11$$

Example: Subtract $(3x^2 - y + 2z + 7) - (-x^2 - 3z)$

$$3x^2 - y + 2z + 7 + x^2 + 3z$$

Now, first we collect the like terms together from both groups

$$3x^2 + x^2 - y + 2z + 3z + 7$$

$$(3 + 1)x^2 - y + (2 + 3)z + 7$$

$$4x^2 - y + 5z + 7$$

Collect like terms and simplify the expression:

$$9m^2 - 7m + 5m - 4m^2 - 5m + 10$$

Now, first we collect the like terms together from both groups

$$9m^2 - 4m^2 + 5m - 7m - 5m + 10$$
$$= (9 - 4)m^2 + (5 - 7 - 5)\ m + 10$$
$$= 5m^2 + (-2 - 5)\ m + 10$$
$$= 5m^2 + (-7)\ m + 10$$
$$= 5m^2 - 7m + 1$$

Example: What should be added to $(xy - 2yz + 4zx)$ to get $(4xy - 3zx + 4yz + 6)$?
Let a be the required expression.

$$\therefore a + (xy - 2yz + 4zx) = (4xy - 3zx + 4yz + 6)$$
$$\therefore a = (4xy - 3zx + 4yz + 6) - (xy - 2yz + 4zx)$$

Now, we open up the brackets, the sign of each term is changed.

$$a = 4xy - 3zx + 4yz + 6 - xy + 2yz - 4zx$$

Rearranging the terms to put like terms together

$$a = 4xy - xy - 3zx - 5zx + 4yz + 2yz + 6$$

$$a = (4 - 1)xy - (3 + 5)zx + (4 + 2)yz + 6$$

$$\therefore a = 3xy - 8zx + 6yz + 6$$

Finding Value of an Expression

Finding the value of an Expression

We know the algebraic expression contains both variables and constants. So, how can we find the value of an algebraic expression? We can find the value of expression only if we know the value of the variables.

Example: We have $3a + 5$
Here, a is the variable.
Now, if $a = 2$, then
$3 \times 2 + 5$
$= 6 + 5$
$= 11$
Hence, the value of this algebraic expression is 11

> **Note:** The value of an algebraic expressions changes as the value of variable change.

Now, if $a = 3$, then
$3 \times 3 + 5$
$= 9 + 5$
$= 14$
Hence, the value of this algebraic expression is 14.

Find the value of the algebraic expression $x^2 + y^2 - xy$ if $x = 2$ and $y = 3$.

Sol:
$x^2 + y^2 - xy$

$= x \times x + y \times y - x \times y$
$= 2 \times 2 + 3 \times 3 - 2 \times 3$
$= 4 + 9 - 6$

$= 13 - 6$
$= 7$
Hence, the value of this algebraic expression is 7.
Find the value of algebraic expression $100 - 10x^2$ if $x = 2$

$$100 - 10x^2$$

Put $x = 2$
$= 100 - (10 \times 2^2)$
$= 100 - (10 \times 4)$
$= 100 - 40$
$= 60$
Hence, the value of this algebraic expression is 60.

What should be the value of x if $9p^2 + p + 3x = 47$ when $p = 2$

When $p = 2$;

$$9p^2 + p + 3x = 47$$
$$9(2)^2 + 2 + 3x = 47$$
$$9(2 \times 2) + 2 + 3x = 47$$
$$9 \times 4 + 2 + 3x = 47$$
$$36 + 2 + 3x = 47$$
$$38 + 3x = 47$$
$$38 - 38 + 3x = 47 - 38$$
$$3x = 47 - 38$$
$$3x = 9$$
$$x = 3$$

Hence, the value of x is 3

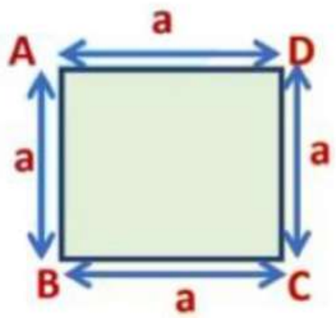
Formulae and Rules

Using Algebraic Expressions – Formulae and Rules

In daily life, we need to find out certain values like the area of the square field or the perimeter of the rectangular plot, the perimeter of the square, and so on... for that we need formulas.

Example: If you have a square plot and you are thinking to fence that plot, for that you need to find the perimeter of the plot. How we find that?

We know that all the sides in a square are of equal length and the perimeter is nothing but the sum of the length of all the sides.

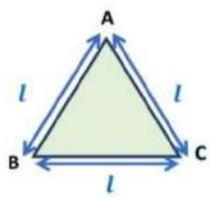


In the figure, the length of each side of a square plot is a.
So, perimeter of a square = Side + Side + Side + Side
Perimeter of a square = a + a + a + a
Here, the length of the side which we added 4 times.
Hence, we can write it as
The Perimeter of a square = $4 \times a$

What is this $4 \times a$?
It is the algebraic expression because it contains constant, operator, and variable.

But why we are using such formulas?
Because such formulas make our calculations easier

Similarly, we can find the perimeter of each closed figure
The perimeter of the equilateral triangle:



The perimeter of the equilateral triangle = 3 × length of its side
Here we denote the length of the side of the equilateral triangle by l. Hence,
The perimeter of the equilateral triangle = 3l

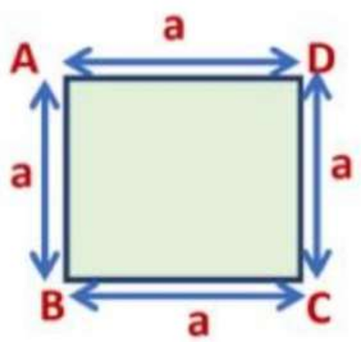
The perimeter of the regular pentagon:

The perimeter of the regular pentagon = 5 × length of its side
Here, we denote the length of the side of a regular pentagon by l,
Hence,
The perimeter of the regular pentagon = 5l

Area formulas

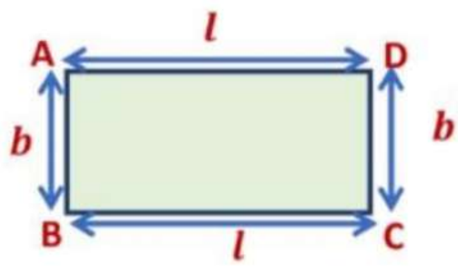
The area is the amount of region occupied by the figure.

Area of the square:
In the adjacent figure, the side of the square is denoted by a hence,
Area of the square = a × a = a^2



Area of the rectangle:

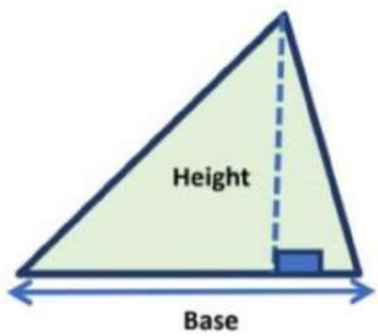
Area of the rectangle = length × breadth In the adjacent figure, the length of a rectangle is denoted by land breadth is denoted by b,
Hence, area of the rectangle = l × b = lb.



Area of the triangle:

Area of the triangle = $\frac{1}{2}$ × Base × Height
In the adjacent figure, bis the base and h is the height of a triangle

Hence, the area of the triangle = $\frac{bXh}{2} = \frac{bh}{2}$



Example: Find the perimeter of a square whose side is 3 cm.
We know that,
The perimeter of a square = 3×length of the side of the square
The perimeter of a square = 3×5
The perimeter of a square = 15 cm

Example: Find the area of a square whose side is 2 cm.

We know that,
Area of a square = Side × Side
Area of a square = 2 cm ×2 cm
∵ Square is a closed figure in which all the sides are equal
Area of a square = 4 cm^2

Example: Find the perimeter of a rectangle of sides is 5 cm and 2cm.
We know that,
Perimeter of a rectangle = 2×(length + breadth)

Perimeter of a rectangle = $2\times(5+2)$
Perimeter of a rectangle = $2(7) = 14$ cm

Example: The area of a triangle is 24 cm^2. If the height is 2 cm, what is its base?

We have,

Area of triangle = $\frac{1}{2}\times$ Base $\times$ Height

Base = $\frac{2X area}{Height}$
Here,
Height= 2 cm and area = 24cm^2

The base of a triangle = $\frac{2X24}{2}$
Base of a triangle = 24cm

Rules for Number Patterns

(i) If a natural number is denoted by n, its successor is (n + 1). We can check this for any natural number.
Example: if n = 5, then
Its successor = (n + 1) = 5+ 1= 6, which is known.

2. If a natural number is denoted by n, 2n is an even number and (2n + 1) an odd number.
Let us check it for any number
Example: If n = 5;
$2n = 2\times n = 2\times 5 = 10$
Which is an even number
Now,
$2n + 1 = 2\times 5 + 1 = 10 + 1 = 11$
Which is an odd number

Number Patterns
Take line segments of equal length such as matchsticks, pen, pencil, chalk, or pieces of straws and cut into smaller pieces of equal length. Join them in patterns.
In the following pattern, we take 4 pens and joint them together to form one square.

Now to form two such square or pattern, we joint three more pens.
So, a total of 7 pens are used.

Now, to form three such square or pattern, we joint three more pens. So, a total of 10 pens are used.

What do you see in the above picture? In the above picture, we see repetitions of the shape.
Here, to make one shape or square you need 4 pens, for two squares you need 7 pens, for three squares you need 10 pens and so on.

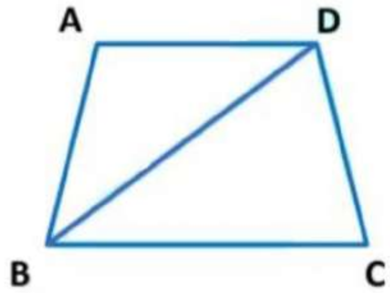
In general, if n is the number of shapes, then the number of segments required to form n shapes is given by $(3n + 1)$

Example: Suppose you want to find how many pens are required for making 10 such square.
So, we simply put the value of n in the formula i.e.,$(3n + 1)$
Here, n – number of squares you want

$(3n + 1) = 3 \times 10 + 1 = 30 + 1 = 31$
So, there are 31 pens required for making 10 squares.

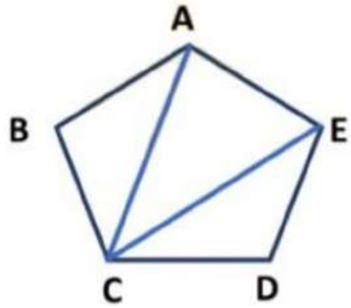
Patterns in Geometry

What is the number of diagonals we can draw from one vertex of a quadrilateral?



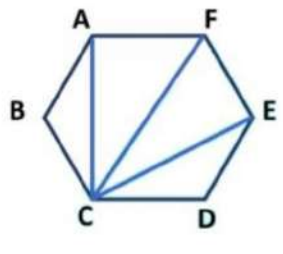
From one vertex of a quadrilateral, we can draw only one diagonal.
That is four sides and one diagonal.

What is the number of diagonals we can draw from one vertex of a pentagon?



From one vertex of a pentagon, we can draw two diagonals that are five sides and two diagonals.

What is the number of diagonals we can draw from one vertex of a hexagon?



From one vertex of a hexagon, we can draw three diagonals that are six sides and three diagonals.

So, in the above three diagrams, we see the pattern, because when the side of the polygon is increased by one then the number of diagonals is also increased by one.

So, the general rule to find out the number diagonals we can draw from one vertex of a polygon of n side is $(n - 3)$

Here, n: sides of the polygon.