5. Electrostatic Potential and Capacitance

Very Short Answer Type Questions

Question 1. Can there be electric potential at a point with zero electric intensity? Give an example.
Answer: Yes. At the mid point of line joining two similar charges, electric field is zero but potential will exist. Ex : Inside a charged hallow spherical shell field is zero but potential is not zero.

Question 2. Can there be electric intensity at a point with zero electric potential? Give an example.
Answer: Yes. At the midpoint or on the equatorial line of an electric dipole potential is zero but eletric field is not zero.

Question 3. What are meant by equipotential surfaces?
Answer: An equipotential surface is a surface with constant value of potential at all points on the surface.

Question 4. Why is the electric field always at right angles to the equipotential surface? Explain.
Answer: If the electric field is not normal to the equipotential surface, then the work done in moving a charge from one point to the other will not be zero, which is a contradiction, thus the field is normal to equipotential surface.

Question 5. Three capacitors of capacitances 1 μF, 2 μF, and 3 μF are Connected in parallel.
a) What is the ratio of charges?
b) What is the ratio of potential differences?
Answer: Charge $q = CV$
a) $\Rightarrow q \propto C \Rightarrow q_1 : q_2 : q_3 = C_1 : C_2 : C_3 = 1 : 2 : 3$
b) In parallel combination potential across combination is
Constant : $V_1 : V_2 : V_3 = 1 : 1 : 1$

Question 6. Three capacitors of capacitances 1 μF, 2 μF, and 3 μF are connected in series
a) What is the ratio of charges?
b) What is the ratio of potential differences?
Answer: q = CV, in series combination charge q' is constant on each capacitor.
a) $\Rightarrow q_1 : q_2 : q_3 = 1 : 1 : 1$
b) $V \propto 1C \Rightarrow V_1 : V_2 : V_3 : 11{:}12{:}13 = 6 : 3 : 2$

Question 7.
What happens to the capacitance of a parallel plate capacitor if the area of its plates is doubled?
Answer:
The capacity of a parallel plate capacitor

is, $C = \frac{\varepsilon_0 A}{d}$

But $C \propto A$,

$$\therefore \frac{C}{C'} = \frac{A}{A'} = \frac{A}{2A} \Rightarrow \boxed{C' = 2C}$$

when area is doubled (A' = 2A) then capacity is also doubled.

Question 8. The dielectric strength of air is 3×10^6 Vm^{-1} at certain pressure. A parallel plate capacitor with air in between the plates has a plate separation of 1 cm. Can you charge the capacitor to 3×10^6 V?
Answer: No ; The dielectric strength of air means the max. electric field that the medium will with-stand. Given $E_{max} = 3 \times 10^6 Vm^{-1}$,

$$V = 3 \times 10^6 \text{ V. } d = 10^{-2}\text{m}$$

$$V = Ed \Rightarrow E = \frac{V}{d} = \frac{3 \times 10^6}{10^{-2}} = 3 \times 10^8 \text{ V / m}$$

$E > E_{max}$, which is not possible.

Short Answer Questions

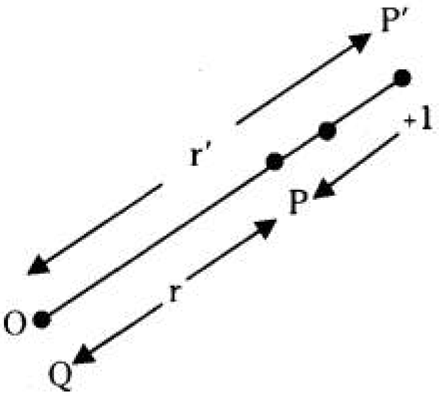
Question 1. Derive an expression for the electric potential due to a point charge. [Mar. '16; TS Mar. '16]
Answer: Consider a point charge Q at 'O' and a unit positive charge is

placed at ‘P’, distance r’. The force acting on it is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r')^2} \text{ along } \overrightarrow{OP'}$$

Let dW be the work done in moving this test charge through dr’ towards ‘O’.

$$\therefore dW = \bar{F}\cdot\bar{d}r' = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{(r')^2}\Delta r' \quad (\because \theta = 180^\circ)$$



The total work done in bringing this test charge from r’ = ∞ to r’ = r is

$$W = \int_{\infty}^{r} dW = -\frac{Q}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{1}{(r')^2}dr' = \frac{-Q}{4\pi\varepsilon_0}\left[\frac{-1}{r'}\right]_{\infty}^{r}$$

$$W = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

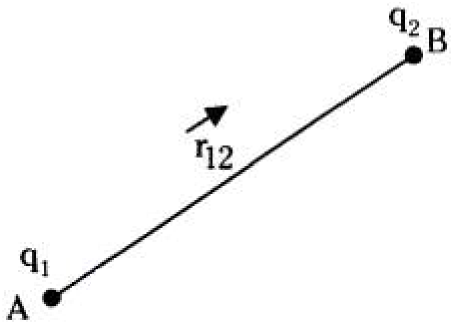
By the definition of potential this work- done is the electrostatic potential at that point.

$\therefore V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r}$

Question 2. Derive an expression for the electrostatic potential energy of a system of two point charges and find its relation with electric potential of a charge.

Answer: Consider a system of two charges q_1 and q_2 with position vectors $\overline{r_1}$ and $\overline{r_2}$ relative to origin at points A & B respectively. The electrostatic potential due to q_1 at B is $V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_2}$

The work done in bringing q2 from infinity to the point B is $W = q_2 V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$



As the electrostatic force is a conservative force, this work done will be stored as energy in the system.
Hence this work done is called electrostatic potential energy 'U' of the system of the charges q_1 and q_2.

$$\therefore U(q_1 q_2) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

Its relation with the potential can be written as $U(q_1, q_2) = V_B q_2 = V_A q_1$

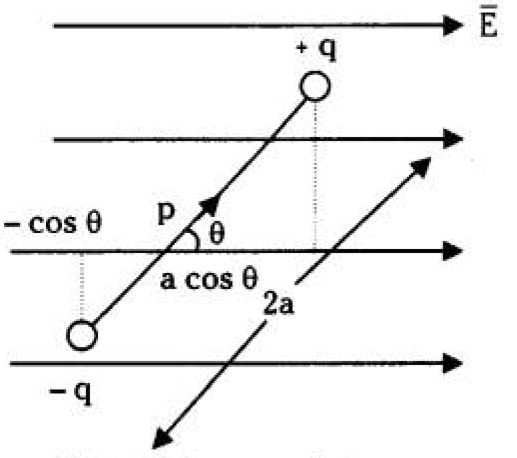
$$\text{here } V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{12}}, V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{21}}.$$

Question 3. Derive an expression for the potential energy of an electric dipole placed in a uniform electric field.
Answer: Consider a dipole with charges + q and – q placed in a uniform electric field E as shown.

Forces are $F_1 = qE$ and $F_2 = -qE$ but dipole experiences torque given by

$\bar{\tau} = \bar{p} \times \bar{E}$

$\tau = pE \sin\theta$



Potential energy between two points charges

Let ‘dW’ be the small amount of work done in rotating the dipole through d0 without any angular acceleration.
dW = τdθ
The toal work done to deflect from θ_1 to θ_2 is

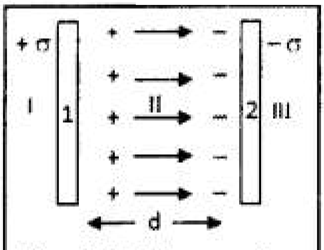
$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta = -pE[\cos\theta]_{\theta_1}^{\theta_2}$$

$W = pE [\cos \theta_1 - \cos \theta_2]$
This work done is stored as potential energy in the dipole.
If the dipole is intially parallel to $\bar{E}$ and now turned through an angle θ, the work- done is
W = pE [cosθ – cosθ] = pE [1 – cosθ]
Then the potential energy of the dipole in this displaced position is $U = U_0 + W = - pE + pE [1 - \cos \theta] = - pE \cos \theta = - \bar{E}.\bar{E}$

Question 4. Derive an expression for the capacitance of a parallel plate capacitor. [AP Mar. 18, 16, May 17. 16; TS Mar. 18. May 18. 16]
Answer: A parallel plate capacitor consists of two plane conducting plates of area A separated by a small distance d’. Let the medium between the plates is vacuum.

Let the surface charge densities of the plates (1) and (2) be + σ and – σ. The electric fields in regions I and III will be

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$



Parallel plate capacitor

In the inner region i.e., II the fields due to these plates will add up and given by

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}\left(\because \sigma = \frac{Q}{A}\right)$$

as ‘d’ is the separation between the plates, the potential difference between the plates given by

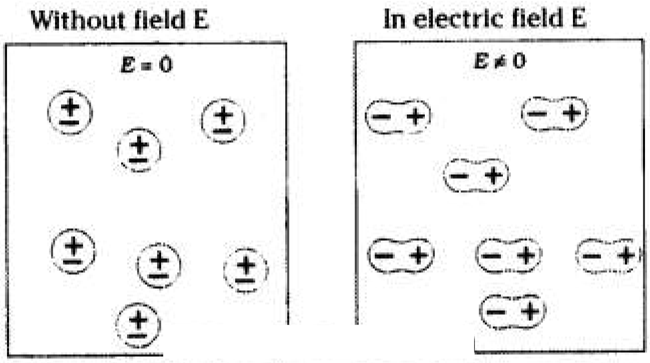
$$V = Ed = \frac{Q}{\varepsilon_0 A} d \Rightarrow \frac{V}{Q} = \frac{d}{\varepsilon_0 A}$$

The capacitance of the parallel plate capacitor given by $C = \frac{Q}{V} \Rightarrow C = \frac{\varepsilon_0 A}{d}$

$$\therefore \boxed{C = \frac{\varepsilon_0 A}{d}}$$

Question 5. Explain the behaviour of dielectrics in an external field. [AP Mar. ’19]
Answer: All dielectrics are two types 1) Non-polar dielectrics 2) Polar dielectrics.
a) For Non-polar dielectrics the centre of all positive and negative charges

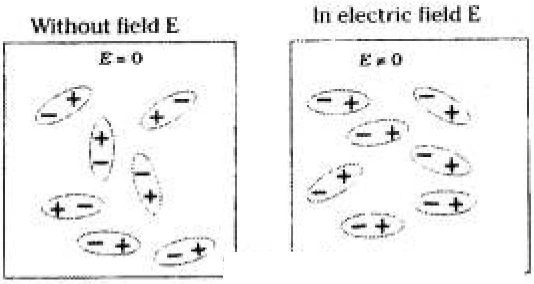
will coincide. Ex: O_2 molecule. So net dipole moment of these molecules is zero.



(a) Non-polar molecules

When these molecules are placed in an external electric field the positive and negative charges will be displaced in opposite directions. So they will develop induced dipolemoment. Total dipolemoment of these substances is the sum of all such dipolements and dielectrics are said to be polarised.

b) In case of polarised dielectrics in a molecule the centres of all positive charges and negative charges are separate. So they will develop some resultant dipolemoment. Under the absence of external electric field these dipole moments are random and resultant dipolemoment is zero.



(b) Polar molecules

When they are placed in external electric field these dipoles are arranged in an order and the dipolemoments are polarised.

The dipole moment per unit volume ($\vec{p}$) is called polarisation.

$\vec{p} = \chi \vec{E}$

where χ is called electric susceptibility.

Long Answer Questions

Question 1. Define electric potential. Derive an expression for the electric potential due to an electric dipole and hence the electric potential at a point (a) the axial line of electric dipole (b) on the equatorial line of electric dipole.

Answer: Electric potential :

Work done to bring the unit positive charge from infinity to the point in the electric field is called potential.

Potential due to point charge q at a distance r (V) = $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Electric dipole :

Two equal and opposite charges (q, -q) separated by a distance (say 2a) is called electric dipole.

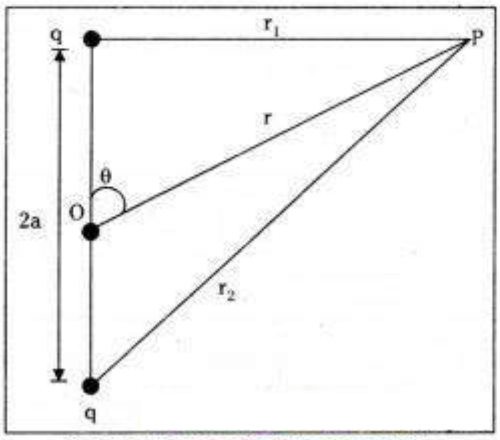
Potential due to an electric dipole :

Consider an electric dipole with charge q, – q.

Let separation between them is 2a. Let P is a point at a distance r' from centre of dipole.

Join qp, -qp and OP. Let the line OP makes an angle 'θ' with the dipole axis (q, – q).

Total potential at P is V = $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right)$



Electric dipole potential at any point

From geometry $r^2_1 = r^2 + a^2 - 2a\, r \cos\theta$ and

$r^2_2 = r^2 + a^2 + 2a\, r \cos\theta$

These equations are rearranged as

$$r_1^2 = r^2\left(1 - \frac{2a\cos\theta}{r} + \frac{a^2}{r^2}\right) \text{ and}$$

$$r_2^2 = r^2\left(1 + \frac{2a\cos\theta}{r} + \frac{a^2}{r^2}\right)$$

$\because$ $a << r$ the term a^2/r^2 can be neglected.

$$\therefore r_1^2 = r^2\left(1 - \frac{2a\cos\theta}{r}\right)$$

$$\Rightarrow r_1 = r\left(1 - \frac{2a\cos\theta}{r}\right)^{1/2}$$

$$\text{and } r_2^2 = r^2\left(1 + \frac{2a\cos\theta}{r}\right)$$

$$\Rightarrow r_2 = r\left(1 + \frac{2a\cos\theta}{r}\right)^{1/2}$$

Potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{1}{r}\left[\frac{q}{1 - a\frac{\cos\theta}{r}} - \frac{-q}{1 + \frac{a\cos\theta}{r}}\right]$$

(OR)

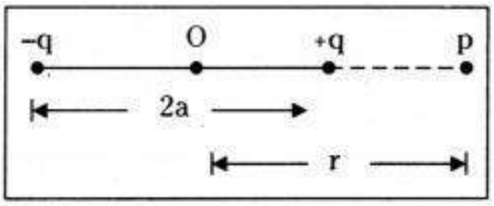
$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}\left[\frac{2a\cos\theta}{r}\right] = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\cdot 2a\cdot\cos\theta}{r^2}$$

But 2aq = p. dipole moment

$$\therefore V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\bar{p}\cdot\bar{r}}{r^2} \text{ where } \bar{r} \text{ is}$$

a unit vector along OP.

Potential at any point on axis of dipole :
Consider a dipole of charges q, – q with separation '2a'. At point p potential V is given by $V = V_1 + V_2$



Potential on the axis of dipole

where $V_1 = \frac{q}{4\pi\varepsilon_0}\frac{1}{(r^2 - a^2)}$ and

$$V_2 = \frac{-q}{4\pi\varepsilon_0(r^2 + a^2)}$$

$$\therefore\ V_{axial} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2 - a^2} - \frac{1}{r^2 + a^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\ .\ 4ar$$

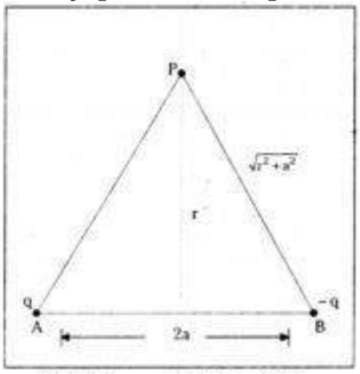
But $2qa = \overline{p}$

$$\therefore\ V_{axial} = \frac{1}{4\pi\varepsilon_0} = \frac{2p \cdot r}{[r^2 - a^2]^2}$$

where $r >> a$ then $r^2 - a^2 \simeq r^2$

$$\therefore\ V_{axial} \simeq \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

Potential at any point on equatorial line of dipole :
Let P is any point on equatorial line of a dipole at a distance r.



Potential on equitorial line of diple

Potential at P is $V_{eq} = V_1 + V_2$

where $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{(r^2+a^2)^{1/2}}$ and

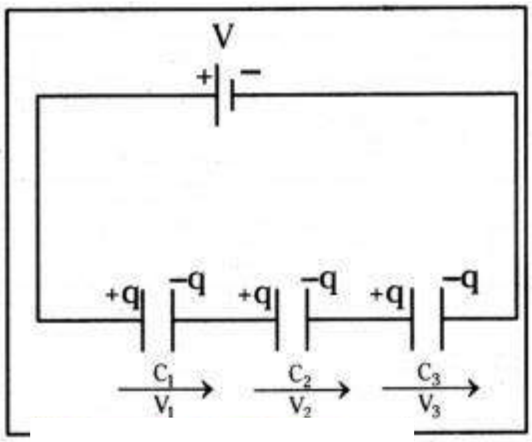
$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r^2+a^2)^{1/2}}$$

$$\therefore V_{eq} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r^2+a^2)^{1/2}} - \frac{1}{(r^2+a^2)^{1/2}}\right] = 0$$

∴ Potential on equatorial line of dipole is zero.

Question 2. Explain series and parallel combination of capacitors. Derive the formula for equivalent capacitance in each combination. [TS Mar. 19, 17, 15; AP May 16, June 15, Mar. 15]
Answer: Capacitors in series :
If number of capacitors are connected in such a way that the charge on the plates of every one of them is same, then the capacitors are said to be “connected in series”.

Explanation :
Let three capacitors C_1, C_2 and C_3 are connected in series as shown.



Series combination of capacitors

The charge q on the plates of the capacitors is same, let, V_1, V_2 and V_3 be the potential differences across C_1, C_2 and C_3 respectively. Let V be the p.d across the combination

then $V = V_1 + V_2 + V_3$

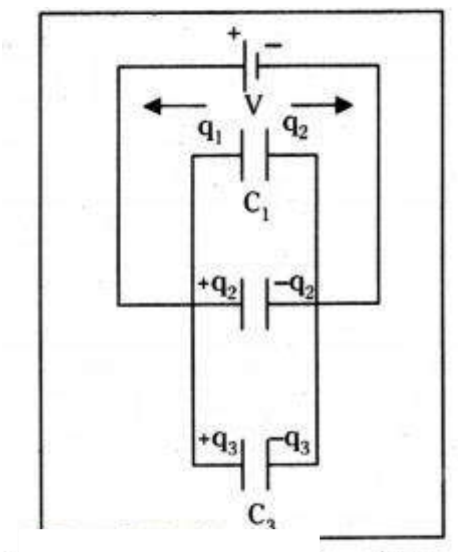
here $V_1 = \frac{q}{C_1}$, $V_2 = \frac{q}{C_2}$, $V_3 = \frac{q}{C_3}$

$$\therefore V = \frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \Rightarrow \boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

The reciprocal of equivalent capacity = the sum of reciprocal values of individual capacities of the combination.

Capacitors in parallel :

If a number of capacitors connected in such a way that the p. d between the plates of every one of them is same, then the capacitors are said to be “connected in parallel”.



Parallel combination of capacitors

Explanation :
Let the capacitors of capacities C_1, C_2 and C_3 connected in parallel as shown.

The potential difference across each condenser is same and is equal to V.
Let q_1 and q_2 and q_3 be the charges on the plates of the capacitors.
$\therefore q = q_1 + q_2 + q_3$. here $q_1 = C_1V$, $q_2 = C_2V$, $q_3 = C_3V$
If C is equivalent capacity of the combination, the $C = pV \Rightarrow q = CV$
$\therefore CV = C_1V + C_2V + C_3V$
$\therefore C = C_1 + C_2 + C_3$
The equivalent capacity of the parallel combination = the sum of the capacities of the capacitors.

Question 3. Derive an expression for the energy stored in a capacitor. What is the energy stored when the space between the plates is filled with a dielectric.
a) With charging battery disconnected?
b) With charging battery connected in the circuit?
Answer: Let 'q' be the charge on the plates of a capacitor and V be the potential difference between plates. The work done dW in charging the capacitor with an additional charge dq.
$dW = Vdq$
$\Rightarrow dW = qCdq$ ($\because q = CV$)
The total work done in charging the plates it to a charge Q = W

$$W = \int_0^Q dW = \int_0^Q V dr = \int_0^Q \frac{q}{C} dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$= \frac{Q^2}{2C} = \frac{1}{2}CV^2 \quad (Q = CV) \Rightarrow W = \frac{QV}{2}$$

This work done in charging the capacitor stored as electrostatic potential energy.
Effect of dielectric :
a) When charging battery is disconnected:
The charge on the plates remain constant, but capacity increases.

$$Q' = Q, \quad C' = KC \quad \text{and} \quad U = \frac{Q^2}{2C}$$

$$\therefore U' = \frac{(Q')^2}{2C} = \frac{Q^2}{2KC} = \frac{U}{K}$$

∴ The energy stored will get reduced to 1K th of initial value.
b) When charging battery remain connected the p.d across the capacitor remains same, but the capacity and the charge increases.

$$C' = KC \text{ and } V' = V$$

$$\text{But } U = \frac{1}{2} CV^2$$

$$\Rightarrow U' = \frac{1}{2}C'(V')^2$$

$$= \frac{1}{2}KCV^2 = KU$$

∴ The energy stored will increases by K times initial value.

Problems

Question 1. An elementary particle of mass in’ and charge + e initially at a very large distance is projected with velocity ‘v’ at a much more massive particle of charge + Ze at rest. The closest possible distance of approach of the incident particle is

Answer: At closest approach kinetic energy of charged particle = electrostatic potential between them.

Given : Mass of particle = m; velocity = V
∴ KE = 1/2 mv^2 → (1)
Charge of particle q_1 = e ; Charge on massive particle q_2 = Ze
∴ Electrostatic potential

$$V = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{d} \rightarrow (2)$$

From equations (1) and (2)

$$\frac{1}{2} mv^2 = \frac{1}{4\pi\varepsilon_0} \frac{e \cdot Ze}{d}$$

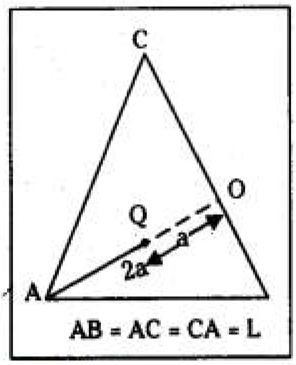
∴ Closest approach $d = \dfrac{Ze^2}{2\pi\varepsilon_0 mv^2}$

Question 2. In a hydrogen atom the electron and proton are at a distance of 0.5 Å. The dipole moment of the system is
Answer: Distance between electron and proton = 0.5Å = 0.5×10^{-10} m
Charge on proton = Charge on electron = 1.6×10^{-19} C
Dipole moment = charge × separation between charges
∴ Dipolemoment p = $1.6 \times 10^{-19} \times 0.5 \times 10^{-10} = 0.8 \times 10^{-29} = 8 \times 10^{-30}$ Cm

Question 3. There is a uniform electric field in the XOY plane represented by $(40\hat{i}+30\hat{j})$ Vm^{-1}. If the electric potential at the origin is 200 V, the electric potential at the point with co-ordinates (2m, lm) is
Answer: Intensity of electric field E = $40\hat{i}+30\hat{j}$
Position of the given point = 2 m, 1 m
i.e., 2m along $\hat{i}$ and lm along $\hat{j}$. Potential
= $\vec{E}.\vec{r}$ = 40 × 2 + 30 × 1 = 80 + 30 = 110V
Potential at origin = 200V.
Potential difference at point p = 200 – 110 = 90V.

Question 4. An equilateral triangle has a side length L. A charge + q is kept at the centroid of the triangle. P is a point on the perimeter of the triangle. The ratio of the minimum and maximum possible electric potentials for the point P is
Answer: Length of side = L. Charge at centroid = + q
Let ‘O’ be the centroid.

Centroid will divide the angle bisector in the ratio 2 : 1.
∴ Maximum distance from centroid is say '2a' then minimum distance from centroid is 'a'.



Potential max. = $V_{max} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$ (on angular bisector at a')

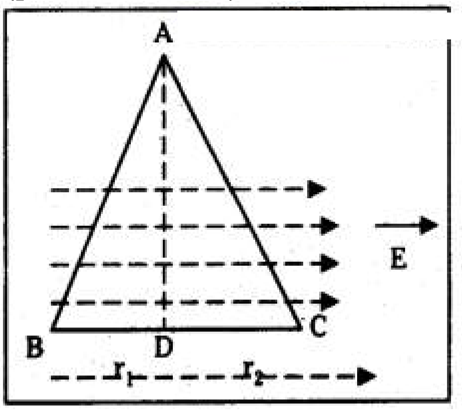
Potential minimum = $V_{min} = \frac{1}{4\pi\varepsilon_0}\frac{q}{2a}$ (at vertex)

∴ Ratio of $V_{min} : V_{max} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{2a}\right] : \frac{q}{4\pi\varepsilon_0}\frac{1}{a}$

Ratio of minimum potential to maximum potential is 1 : 2.

Question 5. ABC is an equilateral triangle of side 2m. There is a uniform electric field of intensity 100 V/m in the plane of the triangle and parallel to BC as shown. If the electric potential at A is 200 V, then the electric potentials at B and C are respectively
Answer: Side of triangle L = 2m
Intensity of electric field = 100 V/m

(parallel to BC)

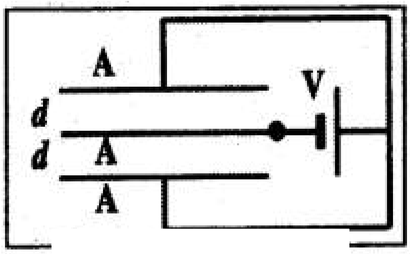


Potential at A = 200V.
Let D is the mid point of BC.
Now AD is an equipotential line with a potential of 200V.
Potential at B = 200 + E. r where r_1 = BD = 1m.
∴ Potential at B = 200 + 100 × 1 = 300 V
∴ Potential at C = Pot. at D – E. r_2 where
r_2 = 1m
V_D = 200 – 100 × 1 = 100v

Note : By definition potential is the work done against the field.

Question 6. An electric dipole of moment p is called in a uniform electric field E, with p parallel to E. It is then rotated by an angle q. The work done is
Answer: Electric dipolemoment = p;
Intensity of electric field = E
p and E are parallel ⇒ θ = 0;
Angle rotated = q
Work done by external force to rotate the dipole without acceleration
W = PE (cos θ_1 – cos θ_2)
Here θ_1 = 0 and θ_2 = q
∴ Work done = PE (cos 0 – cos q) = pE (1 – cos q)

Question 7. Three identical metal plates each of area 'A' are arranged parallel to each other, 'd' is the distance between the plates as shown. A battery of V volts is connected as shown. The charge stored in the system of plates is



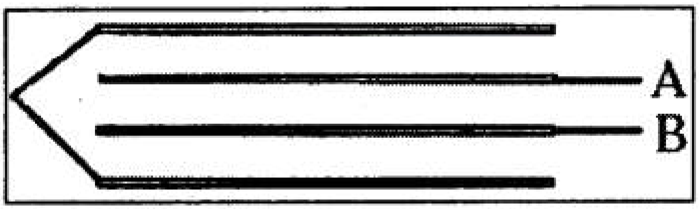
Answer: Area of plates = A, Separation between the plates = d.
Potential supplied by battery = V.
From given arrangement it is a parallel combination of two identical capacitors.

Capacity of a capacitor $= \frac{\varepsilon_0 A}{d}$.

Charge stored $Q = CV = \frac{\varepsilon_0 A}{d} \cdot V$

For two similar capacitors in parallel charge stored $q = \frac{2\varepsilon_0 A}{d} V$.

Question 8. Four identical metal plates each of area A are separated mutually by a distance d and are connected as shown. Find the capacity of the system between the terminals A and B.

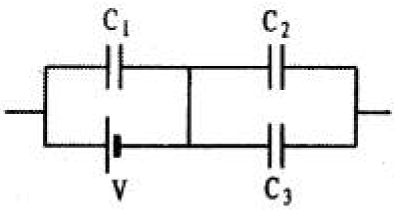


Answer: Area of each plate = A
Total separation between the plates = d
Separation between two adjacent plates = $d_1 = d/3$
Capacity of each capacitor $C_1 = \varepsilon_0 A d_1 = \varepsilon_0 A d/3 = 3\varepsilon_0 A d$
For two similar capacitors in series the resultant

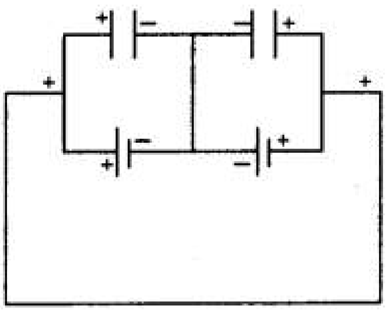
$$\text{capacity } C = \frac{C_1 C_2}{C_1 + C_2} = C/2$$

$$\therefore C_R = \frac{1}{2}\frac{3\varepsilon_0 A}{d} = \frac{3\varepsilon_0 A}{2d}$$

Question 9. In the circuit shown the battery of'V' volts has no internal resistance. All three con-densers are equal in capacity. Find the condenser that carries more charge.



Answer: In the diagram the battery polarities are as show.



So all capacitors are connected in parallel. Since capacity 'C' is same and potential of battery V is same charge will exist on all the three capacitors.

Question 10. Two capacitors A and B of capacities C and 2C are connected in parallel and the combination is connected to a battery of volts. After the charging is over, the bat-tery is removed. Now a dielectric slab of K = 2 is inserted between the plates of A so as to fill file space completely. The energy lost by the system during the sharing of charges is
Answer: Capacity $C_1 = C$; Capacity $C_2 = 2C$
Let the capacitors are charged to the potential V.
Charge on capacitor $Q_1 = CV$
Charge on capacitor $Q_2 = 2CV$

Total charge $Q = Q_1 + Q_2 = 3CV \rightarrow (1)$
Energy stored $U_1 = 12CV^2 + 12(2C) V^2$
$CV22 + CV^2 = 32CV^2 \rightarrow (2)$
Now dielectric is introduced in C_1
New capacity $= KC_1$
Where $K_1 = 2$ and $C_1 = C \therefore C_N = 2C$.
Cherge is not changed because battery is disconnected from circuit.

$$\therefore \text{New potential } V_1 = \frac{Q}{C} = \frac{\text{total charge}}{\text{total capacity}}$$

$$= \frac{3CV}{C_N + C_2} = \frac{3CV}{4C} \Rightarrow V_1 = \frac{3V}{4}$$

$$\text{New energy stored} = \frac{1}{2}C_1V_1^2$$

$$= \frac{1}{2}4C \cdot \left(\frac{3V}{4}\right)^2 = \frac{1}{2}\frac{9V^2C}{4} = \frac{9}{8}CV^2$$

$$\text{Loss of energy} = \frac{3}{2}CV^2 - \frac{9}{8}CV^2$$

$$= \frac{12CV^2 - 9CV^2}{8} = \frac{3}{8}CV^2$$

Question 11. A condenser of certain capacity is charged to a potential V and stores some energy. A second condenser of twice the capacity is to stored half the energy of the first, find to what potential one must be charged?
Answer: For 1st capacitor
Let capacity of capacitor $= x$; Potential $= V$
Energy stored $U_1 = 12CV^2 = 12 \times V^2{}_1 \rightarrow (1)$
For 2nd capacitor capacity $C_2 = 2x$
Energy stored $= 12U_1 = 12.12 \times V^2{}_1 \rightarrow (2)$
But for 2nd capacitor energy stored

$$U_2 = \frac{1}{2}(2x)V_2^2 \rightarrow (2)$$

From eq. (2) & (3)

$$\cancel{\frac{1}{2}}2x V_2^2 = \cancel{\frac{1}{2}} \cdot \frac{1}{2} x V_1^2$$

$$\Rightarrow V_2^2 = \frac{V_1^2}{4} \text{ OR } V_2 = \frac{V_1}{2}$$

$\therefore$ Potential on New Capacitor $= \frac{V_1}{2}$

Intext Question and Answers

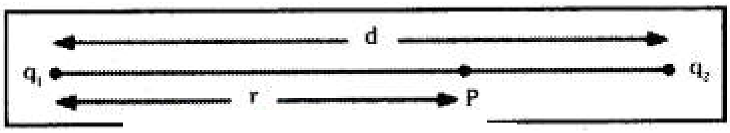
Question 1. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Answer: There are two charges,

$q_1 = 5 \times 10^{-8}C$, $q_2 = -3 \times 10^{-8}C$

Distance between the two charges, d = 16 cm = 0.16 m

Consider a point P on the line joining the two charges, as shown in the given figure.



Let the electric potential (V) at point P be zero. It is at a distance r from q_1

Potential at point P is the sum of potentials caused by charges q_1 and

q_2 respectively.

$$\therefore\ V = \frac{q_1}{4\pi\epsilon_0} + \frac{q_2}{4\pi\epsilon_0(d-r)} \text{ when } v = 0 \text{ then}$$

$$\frac{q_1}{4\pi\epsilon_0 r} = -\frac{q_2}{4\pi\epsilon_0(d-r)}$$

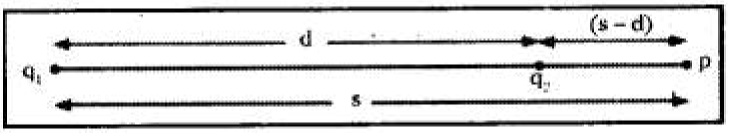
$$\therefore \frac{q_1}{r} = \frac{-q_2}{d-r} \quad \text{(OR)} \quad \frac{5\times10^{-8}}{r} = \frac{(-3\times10^{-8})}{(0.16-r)}$$

$$\therefore \frac{0.16}{r} - 1 = \frac{3}{5} \Rightarrow \frac{0.16}{r} = \frac{8}{5};$$

$$\therefore r = 0.1\text{m} = 10\text{cm}$$

Therefore, the potential is zero at a distance of 10 cm from the positive charge between the charges.

Suppose point P is outside the system of two charges at a distance s from the negative charge, where potential is zero, as shown in the following figure.



For this arrangement, potential is given by,

$$V = \frac{q_1}{4\pi\epsilon_0 s} + \frac{q_2}{4\pi\epsilon_0(s-d)}.$$

For V = 0 we get

$$\frac{q_1}{4\pi\epsilon_0 s} = -\frac{q_2}{4\pi\epsilon_0(s-d)} \Rightarrow \frac{q_1}{s} = \frac{-q_2}{s-d}$$

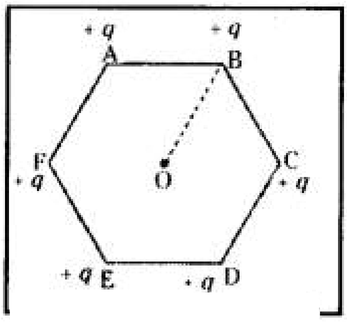
$$\frac{5\times10^{-8}}{s} = -\frac{(-3\times10^{-8})}{(s-0.16)} \Rightarrow 1 - \frac{0.16}{s} = \frac{3}{5}$$

$$\Rightarrow \frac{0.16}{s} = \frac{2}{5}$$

$$\therefore s = 0.4 \text{ m} = 40 \text{ cm}$$

Therefore, the potential is zero at a distance of 40 cm from the positive charge outside the system of charges.

Question 2. A regular hexagon of side 10 cm has a charge 5 µC at each of its vertices. Calculate the potential at the centre of the hexagon.
Answer:
The given figure shows six equal amount of charges, q, at the vertices of a regular hexagon.



Charge, $q = 5\ \mu C = 5 \times 10^{-6} C$
Side of the hexagon, $l = AB = BC = CD = DE$
$= EF = FA = 10$ cm
Distance of each vertex from centre O, d = 10 cm

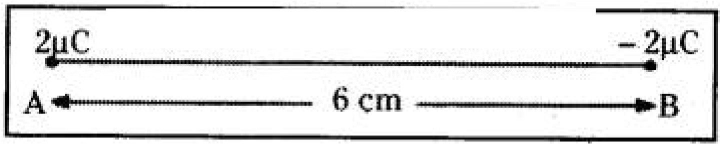
Electric potential at point O, is $V = \dfrac{6 \times q}{4\pi\varepsilon_0 d}$

$$\therefore V = \frac{6 \times 9 \times 10^9 \times 5 \times 10^{-6}}{0.1} = 2.7 \times 10^6 V$$

Therefore, the potential at the centre of the hexagon is 2.7×10^6 V.

Question 3. Two charges 2 µC and -2 µC are placed at points A and B 6 cm apart.
(a) Identify an equipotential surface of the system.

Answer: (a) The situation is represented in the given figure.



An equipotential surface is the plane on which total potential is zero everywhere. This plane is normal to line AB.

The plane is located at the mid-point of line AB because the magnitude of charges is the same.

(b) The direction of the electric field at every point on this surface is normal to the plane in the direction of AB.

Question 4. A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7}C distributed uniformly on its surface. What is the electric field
(a) Inside the sphere?
(b) Just outside the sphere?
(c) At a point 18 cm from the centre of the sphere?
Answer: (a) Radius of the spherical conductor, r = 12 cm = 0.12 m

Charge is uniformly distributed over the conductor, $q = 1.6 \times 10^{-7}$ C
Electric field inside a spherical conductor is zero. This is because if there is field inside the conductor, then charges will move to neutralize it.

(b) Electric field E just outside the conductor is given by the relation,

$$E = \frac{q}{4\pi\varepsilon_0 r^2};$$

$$\therefore\ E = \frac{1.6 \times 10^{-7} \times 9 \times 10^{-9}}{(0.12)^2} = 10^5 NC^{-1}$$

Therefore, the electric field just outside the sphere is 10^5 NC^{-1}.

(c) Electric field at a point 18 m from the centre of the sphere = E_1
Distance of the point from the centre, d = 18 cm = 0.18 m

$$E_1 = \frac{1}{4\pi\varepsilon_0 d^2}$$

$$= \frac{9\times10^9 \times 1.6\times10^{-7}}{\left(18\times10^{-2}\right)^2} = 4.4\times10^4 \text{ N/C}$$

Therefore, the electric field at a point 18 cm from the centre of the sphere is 4.4×10^4 N/C.

Question 5. A parallel plate capacitor with air between the plates has a capacitance of 8 pF (lpF = 10^{-12} F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Answer: Capacitance between the parallel plates of the capacitor, C = 8 pF
Initially, distance between the parallel plates was d and it was filled with air. Dielectric constant of air, k = 1
Capacitance, C, is given by the formula,
$C = k\varepsilon_0 Ad = \varepsilon_0 Ad$ …………. (i)
Where, A = Area of each plate; ϵ_0 = permittivity of free space
If distance between the plates is reduced to half, then new distance, d' = d2.
Dielectric constant of the substance filled in between the plates, k' = 6

Hence, capacitance of the capacitor
becomes C' = = kε0Ad=6ε0Ad2 ……….. (ii)
From eqn (i) & (ii)
C' = 2 × 6C = 12 C = 12 × 8 = 96 pF
Therefore, the capacitance between the plates is 96 pF.

Question 6. Three capacitors each of capacitance 9 pF are connected in series. [AP Mar '14]
(a) What is the total capacitance of the combination?
(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?
Answer: (a) Capacitance of each of the three capacitors, C = 9 pF
Equivalent capacitance (C') of the combination of the capacitors is given by the relation,

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$$

$$\therefore C' = 3\mu F$$

Therefore, total capacitance of the combination is 3μF.

(b) Supply voltage, V = 100 V
Potential difference (V') across each capacitor is equal to one-third of the supply voltage.
∴ V' = V3=1203 = 40V
Therefore, the potential difference across each capacitor is 40 V.
Therefore, total capacitance of the combination is pμF.
(b) Supply voltage, V = 100 V

Question 7. Three capacitors of capacitances 2μF, 3μF and 4μF are connected in parallel.
(i) What is the total capacitance of the combination?
(ii) Determine the charge on each capacitor, if the combination is connected to a 200V supply. [AP Mar. 1 7; TS May 1 7, June 15]
Answer: (a) Capacitances of the given capacitors are
C_1 = 2μF ; C_2 = 3μF ; C_3 = 4μF
For the parallel combination of the capacitors, equivalent capacitor C' is given by the algebraic sum, C' = 2 + 3 + 4 = 9
Therefore, total capacitance of the combination is pμF.

(b) Supply voltage, $V = 100$ V
The voltage through all the three capacitors is same = $V = 100$V
Charge on a capacitor of C and potential difference V is given by the relation,
$q = CV$ ... (i)
For $C = 2\mu F$, Charge = $VC = 100 \times 2 = 200\mu C = 2 \times 10^{-4}$ C
For $C = 3\mu F$,
Charge = $VC = 100 \times 3 = 300\ \mu C = 3 \times 10^{-4}$ C
For $C = 4\mu F$,
Charge = $VC = 100 \times 4 = 200\mu C = 4 \times 10^{-4}$ C

Question 8. Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.
(a) What is the total capacitance of the combination?
(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.
Answer: (a) Capacitances of the given capacitors are
$C_1 = 2$ pF ; $C_2 = 3$ pF ; $C_3 = 4$ pF
For the parallel combination of the capacitors, equivalent capacitor C' is given by the algebraic sum, $C' = 2 + 3 + 4 = 9$
Therefore, total capacitance of the combination is 9 pF.

(b) Supply voltage, $V = 100$ V
The voltage through all the three capacitors is same = $V = 100$ V
Charge on a capacitor of capacitance C and potential difference V is given by the relation,
$q = VC$ ... (i)
For C = 2 pF, Charge = $VC = 100 \times 2 = 200$ pC = 2×10"10 C
For C = 3 pF,
Charge = $VC = 100 \times 3 = 300$ pC = 3×10^{-10} C
For C = 4 pF,
Charge = $VC = 100 \times 4 = 200$ pC = 4×10^{-10} C

Question 9. In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3}\ m^2$ and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?
Answer: Area of each plate of the parallel plate capacitor, $A = 6 \times 10^{-3}\ m^2$
Distance between the plates, $d = 3$ mm
$= 3\times 10^{-3}$ m; Supply voltage, $V = 100$ V

Capacitance C of a parallel plate capacitor is given by, $C = \varepsilon_0 Ad$
Where, ε_0 = Permittivity of free space

$$= 8.854 \times 10^{-12}\ N^{-1}\ m^{-2}\ C^{-2}$$

$$\therefore C = \frac{8.854 \times 10^{-12} \times 6}{3 \times 10^{-3}} = 17.71 \times 10^{-12}\ F$$

$$= 17.71\ pF$$

Potential V is related with the charge q and capacitance C as $V = qC$
$\therefore q = VC = 100 \times 17.71 \times 10^{-12} = 1.771 \times 10^{-9} C$.
Therefore, capacitance of the capacitor is 17.71 pF and charge on each plate is $1.771 \times 10^{-9} C$.

Question 10. Expalin what would happen if in the capacitor given in Exercise 8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,
(a) while the voltage supply remained connected.
(b) after the supply was disconnected.
Answer: (a) Dielectric constant of the mica sheet, k = 6;
Initial capacitance, $C = 1.771 \times 10^{-11}$ F
New capacitance,
$C' = kc = 6 \times 1.771 \times 10^{-11} = 106$ pF;
Supply voltage, V = 100 V
New charge,
$q' C'V = 6 \times 1.771 \times 10^{-9} = 1.06 \times 10^{-8} C$
Potential across the plates remains 100 V.

(b) Dielectric constant, k = 6 ;
Initial capacitance, $C = 1.771 \times 10^{-11}$ F
New capacitance,
$C' = kC = 6 \times 1.771 \times 10^{-11} = 106$ pF

If supply voltage is removed, then there will be no effect on the amount of charge in the plates.
$\therefore$ Charge $= 1.771 \times 10^{-9}$ C
Potential across the plates is gives by,

$$\therefore V' = \frac{q}{C'} = \frac{1.771 \times 10^{-9}}{106 \times 10^{-12}} \; 16.7 \text{ V}$$

Question 11. A 12 pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?
Answer: Capacitor of the capacitance, $C = 12$ pF $= 12 \times 10^{-12}$ F ; Potential difference, $V = 50$ V
Electrostatic energy stored in the capacitor is given by the relation,
$E = 12CV^2 = 12 \times 12 \times 10^{-12} \times (50)^2 = 1.5 \times 10^{-8}$ J
Therefore, the electrostatic energy stored in the capacitor is 1.5×10^{-8} J.

Question 12. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?
Answer: Capacitance of the capacitor, $C = 600$ pF ; Potential difference, $V =$ 200 V Electrostatic energy stored in the capacitor is given by,
$E = 12cV^2 = 12 \times (600 \times 10^{-12}) \times (200)^2 = 1.2 \times 10^{5}$ J
If supply is disconnected from the capacitor and another capacitor of capacitance $C = 600$ pF is connected to it, then equivalent capacitance (C') of the combination is given by,

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} = \frac{1}{600} + \frac{1}{600} = \frac{2}{600} = \frac{1}{300}$$

$$\therefore C' = 300 \text{ pF}$$

New electrostatic energy can be calculated as
$E' = 12 \times C' \times V^2 = 12 \times 300 \times (200)^2 = 0.6 \times 10^{-6}$J
Loss in electrostatic energy $= E - E'$
$= 1.2 \times 10^{-5} - 0.6 \times 10^{-5} = 0.6 \times 10^{-5} = 6 \times 10^{-6}$ J
Therefore, the electrostatic energy lost in the process is 6×10^{-6} J.

Question 13. In a Van de Graaff type generator, a spherical metal shell is to be a 15×10^{6} V electrode. The dielectric strength of the gas surrounding the electrode is 5×10^{7} Vm^{-1}. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)
Answer: Potential difference, $V = 15 \times 10^{6}$V ;
Dielectric strength of the surrounding gas $= 5 \times 10^{7}$ V/m

Electric field intensity, E = Dielectric strength = 5×10^7 V/m
Minimum radius of the spherical shell required for the purpose is given by,

$$r = \frac{V}{E} = \frac{15 \times 10^6}{5 \times 10^7} = 0.3\,m = 30\,cm$$

Hence, the minimum radius of the spherical shell required is 30 cm.

Question 14. A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.
Answer: According to Gauss's law, the electric field between a sphere and a shell is determined by the charge q_1 on a small sphere. Hence, the potential difference, V, between the sphere and the shell is independent of charge q_2. For positive charge q_1, potential difference V is always positive.