CHAPTER 7

System of Particles and Rotational Motion

Syllabus

- **Centre of mass and its motion:** Rigid body, Properties of rigid body; Centre of mass-centre of mass of two particles, centre of mass of N-particles, centre of mass of homogeneous bodies; Motion of centre of mass, velocity of centre of mass, acceleration of centre of mass; Centre of mass and toppling stability.
- **Angular velocity and its relationship with linear velocity:** Angular displacement, angular velocity, relationship between linear and angular velocity, angular acceleration, equations of rotational kinematics.
- **Torque and angular momentum:** Torque, angular momentum, relationship between torque and angular momentum; Law of conservation of angular momentum.
- **Equilibrium of a rigid body:** Translatory equilibrium, rotatory equilibrium; State of equilibrium; Principle of moments; Centre of gravity.
- **Moment of inertia:** Moment of inertia of various objects; Relationship between angular momentum and moment of inertia; Perpendicular and parallel axis theorem.
- **Kinetic energy, work and power of rotational motion:** Introduction, Analogy between translatory and rotatory motion.
- **Rolling motion:** Kinetic energy of rolling body.

MIND MAP

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

- Rigid Body (Body with a perfectly definite, unchanging shape and size)
- Centre of Mass (Point where whole mass of system is supposed to be concentrated)
- Terms Related to Rotational Motion
- Torque (Measure of force that can cause an object to rotate about an axis) $\tau = \vec{r} \times \vec{p}$
- Angular Momentum (Rate of change of angular momentum of a body is equal to external torque acting upon the body) $\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$
- Moment of Inertia $I = \sum m_i r_i^2$
  - Theorem of perpendicular axes $I_z = I_x + I_y$
  - Theorem of Parallel axes $I = I_{cm} + MR^2$
- Kinetic Energy, Work and Power
  - KE, $E_k = \frac{1}{2} I\omega^2$
  - Work, $W = \tau \Delta\theta$
  - Power, $P = \tau . \omega$
- Rolling Motion (Motion that is a combination of rotational and translational motion)

Mind Map 1: System of Particles and Rotational Motion

RIGID BODY
(Body with a perfectly definite and unchanging shape and size)

- Rotatory Equilibrium
  (If a body possesses no angular acceleration)
  $\sum \vec{\tau}_{ext} = \sum \vec{r}_i \times \vec{F}_{iext} = 0$
- Translatory Equilibrium
  (If centre of mass of a body possesses no linear acceleration)
  $\sum \vec{F}_{ext} = 0$

Mind Map 2: Rigid Body

CENTRE OF MASS
(Point where whole mass of system is supposed to be concentrated)

- Position of Centre of Mass of System
  $$r_{cm} = \frac{\sum_{i=1}^{n} m_i r_i}{\sum m_i}$$
- Velocity of Centre of Mass of System
  $$v_{cm} = \frac{\sum_{i=1}^{n} m_i v_i}{\sum m_i}$$
- Acceleration of Centre of Mass of System
  $$a_{cm} = \frac{\sum_{i=1}^{n} m_i a_i}{\sum m_i}$$

Mind Map 3: Centre of Mass

TERMS RELATED TO ROTATIONAL MOTION

- Angular Displacement
  (The angle through which a point or line has been rotated)
  $S = r.\theta$
- Angular Velocity
  (Measure of how fast a body is changing its angle) $\omega = \frac{d\theta}{dt}$
- Angular Acceleration
  (Measure of how fast a body is changing its angular velocity)
  $$\alpha = \frac{d\omega}{dt} = \frac{d^2(\theta)}{dt^2}$$

Mind Map 4: Terms Related to Rotational Motion

RECAP

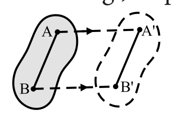
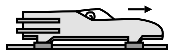
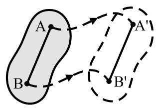
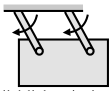
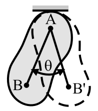
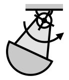
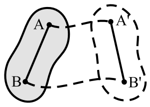
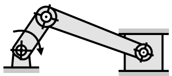
Centre of Mass and its Motion

Rigid Body

- It is an idealization of a body that does not deform or change its shape. Formally, it is defined as a collection of particles with the property that the distance between particles remains unchanged during motion of the body.
- Positions of different particles with respect to each other remains same even under the application of force.
- Has definite shape and size as distance between different pairs of particles does not change on applying force on it.
- No body is truly rigid as real bodies deform under action of force applied on them. e.g.; Copper rod, stone etc.

Properties of Rigid Body Motion

- Arbitrary rigid body motion have one of three categories:
  - **Translational rectilinear and curvilinear:** Motion in which every line in the body remains parallel to its original position. The motion of the body is completely specified by the motion of any point in the body. All points of the body have the same velocity and same acceleration.
  - **Rotation about a fixed axis:** All particles move in circular paths about the axis of rotation. The motion of the body is completely determined by the angular velocity of the rotation.
  - **General plane motion:** Any plane motion that is neither a pure rotational nor a translational, falls into this class.
- The motion of a rigid body which is not pivoted or fixed in some way is either a pure translational or a combination of translational and rotational. The motion of a rigid body which is pivoted or fixed in some way is rotational.

(a) Rectilinear translation		 Roket test sled
(b) Carvilinear translation		 Parallel-link swinging plate
(c) Fixed axis translation		 Compound pendulum
(d) General plane motion		 Connecting rod in a reciprocating engine

Centre of Mass

- It is the point where whole mass of the particle system is concentrated.
- In a uniform gravitational field, the centre of mass and the centre of gravity of a system are coincident.
- For simple rigid objects with uniform density, the centre of mass is located at the centroid. For example, the centre of mass of a uniform disc shape would be at its centre. Sometimes the centre of mass doesn't fall anywhere on the object. The centre of mass of a ring for example is located at its centre, where there isn't any material.

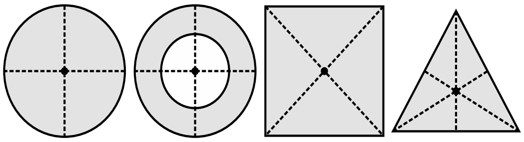


Figure: Centre of mass for some simple geometric shapes (dots)

Centre of Mass for two Particles

- For a system of two particles of masses m_1 and m_2 having their position vectors as $\vec{r}_1$ and $\vec{r}_2$ respectively, with respect to origin of the coordinate system, the position vector $\vec{R}_{CM}$ of the centre of mass is given by

$$\vec{R}_{\text{CM}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

If $\quad m_1 = m_2 = m$ (say),

then $\quad \vec{R}_{\text{CM}} = \dfrac{\vec{r}_1 + \vec{r}_2}{2}$

- For a system of two particles with equal masses, CM is the point that lies exactly in the middle of both.

Centre of Mass for *N*-Particles

- For a system of *N*-particles of masses $m_1, m_2, m_3 .... m_N$ having their position vectors $r_1, r_2, r_3, ... r_N$ respectively, then the centre of mass $\vec{R}_{COM}$ for a system of *N*-particles,

$$\vec{R}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_N\vec{r}_N}{m_1 + m_2 + \ldots + m_N} = \frac{\sum_{i=1}^{N} m_i\vec{r}_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i\vec{r}_i}{M}$$

- The coordinates of centre of mass are given by

$$X_{COM} = \frac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i x_i}{M}; Y_{COM} = \frac{\sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i y_i}{M}; Z_{COM} = \frac{\sum_{i=1}^{N} m_i z_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i z_i}{M}$$

Centre of Mass of Homogeneous Bodies

- **Homogeneous bodies** have uniformly distributed mass around body, e.g.; spheres, rings, etc. They have regular shape. We can assume that the CM for these regular bodies lies at their geometric centres.

Motion of Centre of Mass

- Now extend the concept of the centre of mass to velocity and acceleration, and thus give the tools to describe the motion of a system of particles.

Velocity of Centre of Mass

- Taking a simple time derivative of our expression for R_{CM}

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_N\vec{v}_N}{m_1 + m_2 + \ldots + m_N} = \frac{\sum_{i=1}^{N} m_i\vec{v}_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i\vec{v}_i}{M}$$

Acceleration of Centre of Mass

- Differentiating again to generate an expression for acceleration.

$$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots + m_N\vec{a}_N}{m_1 + m_2 + \ldots + m_N} = \frac{\sum_{i=1}^{N} m_i\vec{a}_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i\vec{a}_i}{M}$$

- Hence, the total mass of the system times the acceleration of its centre of mass is equal to vector sum of all the forces acting on the group of particles.
- If total external force acting on the system is zero, then the total linear momentum of the system is conserved. Also, when the total external force acting on the system is zero, the velocity of centre of mass remains constant.

Centre of Mass and Toppling Stability

- One useful application of the centre of mass is determining the maximum angle that an object can be tilted before it will topple over.
- Figure shows a cross section of a truck. The truck has been poorly loaded with many heavy items, loaded on the left-hand side. The centre of mass is shown by a dot. A dark line extends down from the centre of mass, representing the force of gravity. Gravity acts on all the weight of the truck through this line.
- If truck is tipped at angle θ_t then all the weight of the truck will be supported by the left-most edge of the left wheel.

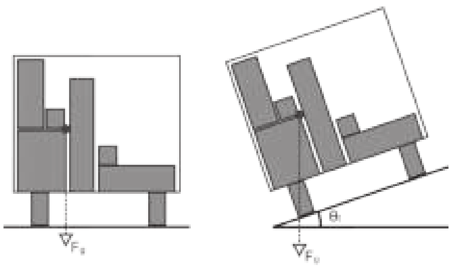


Figure: Topple limit of a poorly loaded truck.

- If the angle is increased further, then truck will get topple over because a vertical line from its centre of gravity falls outside its base. Then this angle θ_t is the topple limit.

Angular Velocity and its Relationship with Linear Velocity

Angular Displacement

- It is 'the angle in radians (degrees, revolutions) through which point or line has been rotated in specified sense about specified axis'. It is the angle of the movement of a body in a circular path.

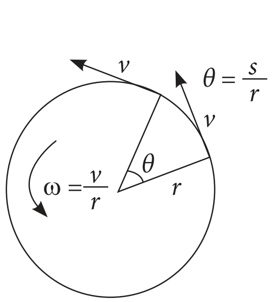


Figure: A body is moving in circular direction

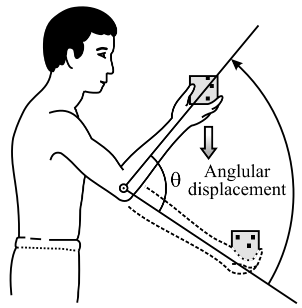


Figure: Example of angular displacement

- If object moves θ angle on circular path of radius r. Then linear displacement is related to angular displacement as

$$S = r.\theta \quad ...(i)$$

The equation (i) is the angular displacement equation.

Angular Velocity

- Like the linear motion, circular/rotational motion has too all the equivalent quantities.
- Measure of how fast a body is changing its angle. It is measured in radians per second. The angular velocity is represented by ω and hence it is defined as the time rate of change of angular displacement and is given by

$$\omega = \frac{\theta}{t} \text{ or, } \omega = \frac{d\theta}{dt} \quad ...(ii)$$

- It is directed along axis of rotation and is vector quantity. Its SI unit is rad/s and dimensional formula is $[M^0L^0T^{-1}]$.

Relationship between Linear Velocity and Angular Velocity

- Let us consider a body P moving along the circumference of a circle of radius r with linear velocity v and angular velocity ω. Let it move from P to Q in time dt and $d\theta$ be the angle swept by the radius vector.
- Let PQ = ds, be the arc length covered by the particle moving along the circle, then the angular displacement $d\theta$ is expressed as $d\theta = dS/r$. But $dS = vdt$.

$$dS = vdt$$

$$\therefore \quad d\theta = \frac{vdt}{r}; \frac{d\theta}{dt} = \frac{v}{r}$$

$$\text{Angular velocity, } \quad \omega = \frac{v}{r}$$

$$\text{or} \quad v = \omega r$$

In vector notation, $\vec{v} = \vec{\omega} \times \vec{r}$

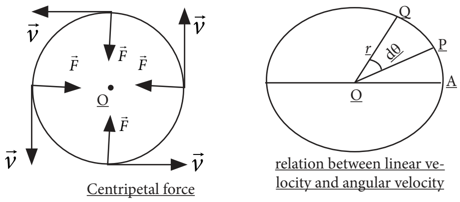


Figure: Relationship between linear velocity and angular velocity

- For given angular velocity ω, linear velocity v of particle is directly proportional to distance from centre of circular path.

Angular Acceleration

- Measure of how fast a body is changing its angular velocity. The angular acceleration is measured in rad per sec^2.
- Represented by α and hence it is defined as the time rate of change of angular velocity and it is given by

$$\alpha = \frac{\omega}{t} = \frac{\theta}{t^2}; \alpha = \frac{d\omega}{dt} = \frac{d^2(\theta)}{dt^2}$$

- Angular acceleration is a vector quantity and its dimensional formula is $[M^0L^0T^{-2}]$.

Equations of Rotational Kinematic

- The kinematics equations for rotational motion at constant angular acceleration are,
  - Angular velocity after a time t second: $\omega_f = \omega_i + \alpha t$
  - Angular displacement after t second: $\theta_f = \theta_i + \omega_i t + 1/2\alpha t^2$
  - Angular velocity after a certain rotation: $\omega^2_f = \omega^2_i + 2\alpha(\theta_f - \theta_i)$
  - Angle traversed in n^{th} second: $\theta_{nth} = \omega_i + \alpha/2(2n-1)$

Torque and Angular Momentum

Torque

- It is measure of force that can cause an object to rotate about an axis.
- It is the rotational analogue of force. It is also termed as the moment of force and denoted by τ.
- It is a vector quantity. The direction of the torque vector depends on the direction of the force on the axis.
- The SI unit of torque is Newton-metre (Nm).

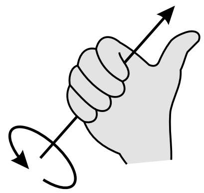


Figure: Direction of the torque vector found with the right-hand rule

Mathematical Expression of the Torque

- Magnitude of torque vector τ for torque produced by given force F is $\tau = F \cdot r \sin\theta$.
- Direction of torque vector is found by convention using right-hand grip rule. If hand is curled around axis of rotation with fingers pointing in direction of the force, then torque vector points in the direction of the thumb.

Angular Momentum

- Torque and angular momentum are closely related to each other. Angular momentum is the rotational analogue of linear momentum p and is denoted by L.
- It is a vector product. Angular momentum of the particle is

$$\vec{L} = \vec{r} \times \vec{p}; \text{ In magnitude, } L = rp\sin\theta$$

- Angular momentum is a vector quantity. Its SI unit is kgm^2s^{-1} and its dimensional formula is $[ML^2T^{-1}]$.

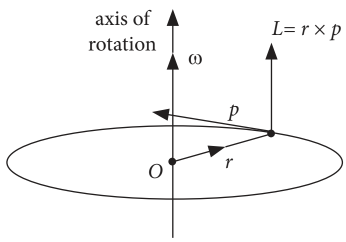


Figure: Angular Momentum

Angular Momentum for an Object Rotating about a Fixed Axis

- Consider an object rotating about a fixed axis, as shown in the figure.
- If a particle P in the body that rotates about the axis as shown above, the expression of total angular momentum for this system can be given by:

$$L = \sum_{i=1}^{N} r_i \times P_i$$

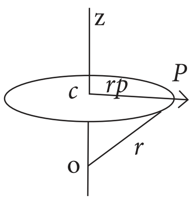


Figure: Angular Momentum for an object rotating about a fixed axis

Relationship between Torque and Angular Momentum

- Rate of change of angular momentum of a body is equal to the external torque acting upon the body.

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

Law of Conservation of Angular Momentum

- When the net external torque acting on a an object or system about a given axis is zero, the total angular momentum of the system about that axis remains constant. Mathematically,

$$\vec{\tau}_{ext} = 0$$

Then, $$\frac{d\vec{L}}{dt} = 0$$

or $$\vec{L} = \text{ Constant}$$

- For systems that consist of many rigid bodies and/or particles, the total angular momentum about any axis is the sum of the individual angular momenta. The conservation of angular momentum also applies to such systems. In the absence of external forces acting on the system, the total angular momentum of the system remains constant.

Equilibrium of a Rigid Body

- Accordingly, the equilibrium is classified into following two categories:
  - **Translatory equilibrium:** When centre of mass of body possesses no linear acceleration in an inertial frame of reference. Basic condition is that the vector sum of all the external forces acting on the body should be zero, then body at rest will remain at rest. This is static equilibrium. A body moving with uniform velocity, along a straight line, will keep on doing so. This is dynamic translatory equilibrium.
  - **Rotatory equilibrium:** Concept is equivalent to Newton's 1st law for rotational system. Object at rest, remains at rest or object rotating, continues to rotate with constant angular velocity unless acted on by external torque. Body is in rotatory equilibrium if it possesses no angular acceleration about any axis in inertial frame. To be in rotatory equilibrium vector sum of all the external torques acting on the body is zero.
- For a body to be in equilibrium it must satisfy both the conditions stated above simultaneously, i.e.,
  - The vector sum of all the external forces acting on the body should vanish.
  - The vector sum of all the external torque acting on the body should vanish.

State of Equilibrium

- Equilibrium can be classified into three categories:
  - **Stable equilibrium:** On being slightly disturbed, it tends to come back to its original position.
  - **Unstable equilibrium:** On being slightly disturbed, it shows no tendency to come back to its original position and moves away from it.
  - **Neutral equilibrium:** On being slightly displaced, it remains in the new position.

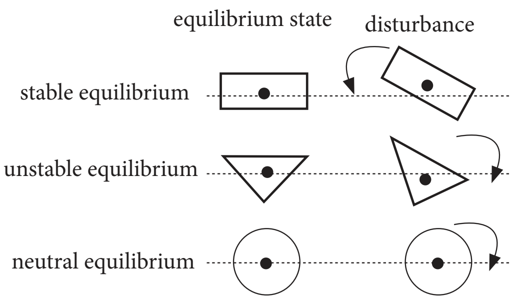


Figure: States of Equilibrium

Principle of Moments

- A body will be in rotational equilibrium if algebraic sum of the moments of all forces acting on the body, about a fixed point is zero.

Centre of Gravity

- The centre of gravity of a body is that point where the total gravitational torque on the body is zero.

Moment of Inertia

- It is sum of products of masses of different particles constituting body and square of their distances from axis of rotation. It depends upon following factors: Mass of body and Distribution of mass about the axis of rotation.
- It does not depend upon state of motion of rotating body. It is same for body at rest, rotating slowly or rotating fast about given axis. The general definition of moment of inertia, also called rotational inertia, for a rigid body is $I = \Sigma\, m_i\, r_i^2$ and it is measured in S.I. units of kilogram-metres^2.

Moments of Inertia of Various Objects

Table: Moments of inertia of various objects

S. No.	Shape of Regular body		Axis of rotation	Moment of inertia
1	Thin rod of mass M and length L	(a)	Centre of rod and perpendicular to length	$\frac{ML^2}{12}$
		(b)	One end and perpendicular to length	$\frac{ML^2}{3}$
2	Circular ring of mass M and radius R	(a)	Through centre, perpendicular to the plane of the ring	MR^2
		(b)	Any diameter	$\frac{1}{2}MR^2$
		(c)	Any tangent in the plane of the ring	$\frac{3}{2}MR^2$
		(d)	Any tangent perpendicular to the plane of the ring	$2MR^2$

3	Circular disc of mass M and radius R	(a)	Through centre, perpendicular to the plane of the disc	$\frac{1}{2}MR^2$
		(b)	Any diameter	$\frac{1}{4}MR^2$
		(c)	Tangent in the plane of the disc	$\frac{5}{4}MR^2$
		(d)	Tangent perpendicular to the plane of the disc	$\frac{3}{2}MR^2$
4	Flat annular disc of mass M and inner and outer radii R_1 and R_2	(a)	Through centre and perpendicular to the plane	$\frac{M}{2}\left(R_1^2+R_2^2\right)$
		(b)	About its diameter	$\frac{M}{4}\left(R_1^2+R_2^2\right)$
		(c)	About tangential axis lying in its plane	$\frac{M}{4}\left(5R_1^2+R_2^2\right)$
		(d)	About tangential axis perpendicular to the plane	$\frac{M}{2}\left(3R_1^2+R_2^2\right)$
5	Solid sphere of mass M and radius R	(a)	Any diameter	$\frac{2}{5}MR^2$
		(b)	Any tangent	$\frac{7}{5}MR^2$
6	Hollow sphere of mass M and radius R	(a)	Diameter	$\frac{2}{3}MR^2$
		(b)	Tangent	$\frac{5}{3}MR^2$
7	Cylinder of mass M, radius R and length L	(a)	Own axis	$\frac{1}{2}MR^2$
		(b)	Through centre perpendicular to length	$M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$
		(c)	Through end face and perpendicular to length	$M\left(\frac{R^2}{4}+\frac{L^2}{3}\right)$
8	Rectangular lamina of mass M, length L and breadth B	(a)	Length of lamina and in its plane	$\frac{MB^2}{3}$
		(b)	Breadth of lamina and in its plane	$\frac{ML^2}{3}$
		(c)	Centre of mass of lamina and perpendicular to its plane	$M\left(\frac{L^2+B^2}{12}\right)$
		(d)	Centre of length and perpendicular to its plane	$M\left(\frac{L^2}{12}+\frac{B^2}{3}\right)$
		(e)	Centre of breadth and perpendicular to its plane	$M\left(\frac{L^2}{3}+\frac{B^2}{12}\right)$

9.	Rectangular block of mass M, length L, breadth B and height H	(a) Through centre of block and parallel to	
		1. Length	$M\left(\frac{B^2+H^2}{12}\right)$
		2. Breadth	$M\left(\frac{H^2+L^2}{12}\right)$
		3. Height	$M\left(\frac{L^2+B^2}{12}\right)$
		(b) Through end face and parallel to	
		1. Length	$M\left(\frac{H^2}{3}+\frac{B^2}{12}\right)$
		2. Breadth	$M\left(\frac{L^2}{3}+\frac{H^2}{12}\right)$
		3. Height	$M\left(\frac{B^2}{3}+\frac{L^2}{12}\right)$

- **Radius of gyration** is perpendicular distance from axis of rotation to point where total mass of body is concentrated, so that about axis may remain same. Gyration is distribution of components of object.
- It is denoted by K. If M is mass of the body and I is moment of inertia of the body then,

$$I = MK^2$$

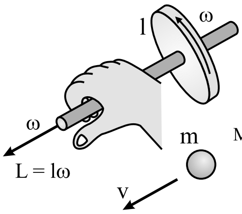
$$\therefore K = \sqrt{\frac{I}{M}}$$

Relation between Angular Momentum and Moment of Inertia

- The angular momentum of a rigid object is also defined as the product of the moment of inertia and the angular velocity.
- It is analogous to linear momentum and is subject to the fundamental constraints of the conservation of angular momentum principle, if there is no external torque on the object. Angular momentum is vector quantity, derivable from expression for angular momentum of particle.

Relation between Angular and Linear Momentum

- Angular momentum and linear momentum are examples of parallels between linear and rotational motion. They have the same form and are subject to constraints of conservation laws, conservation of momentum and angular momentum.



Anglular momentum	=	Moment of inertia	X	Angular velocity
L	=	I	X	ω
Linear Momentum	=	Mass	X	Velocity
p	=	m	X	v

The X implies simple multiplication here.

Theorems of Perpendicular and Parallel Axes

- **Parallel axes theorem:** The moment of inertia I of a body about any axis is equal to the moment of inertia I_{CM} about a parallel axis through the centre of gravity of the body plus MR^2, where M is the mass of the body and R is the distance between the two axes.

$$I = I_{CM} + MR^2$$

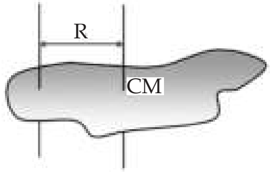


Figure: Parallel axes theorem

- **Perpendicular axes theorem:** For any plane body (a rectangular sheet of metal) the moment of inertia about any axis perpendicular to the plane is equal to the sum of the moments of inertia about any two perpendicular axes in the plane of the body which intersect the first axis in the plane. This theorem is useful when considering a body which is of regular form (symmetrical) about two out of the three axes. If the moment of inertia about these axes is known, then that about the third axis may be calculated.

$$I_z = I_x + I_y$$

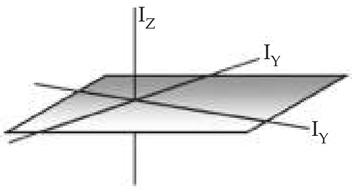


Figure: Perpendicular axes theorem

Kinetic Energy, Work and Power of Rotational Motion

- A rigid body rotating with uniform angular speed ω about a fixed axis possesses kinetic energy of rotation. Its value may be calculated by summing up the individual kinetic energies of all the particles of which the body is composed.
- Particle of mass m_i at distance r_i from axis of rotation has kinetic energy given by $1/2\ m_i v_i^2$, where v_i is the speed of the particle. There will be a similar term for each particle making up the body, so for the total kinetic energy E_k.

$$E_k = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \cdots + \frac{1}{2}m_nv_n^2 = \sum \frac{1}{2}m_iv_i^2$$

- Each particle of a rigid body rotates with uniform angular speed ω. Let us express the instantaneous linear speed of each particle in terms of the common angular speed. Remembering that $v = \omega r$, we substitute for v in the above equation to find

$$E_k = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2;\omega^2 + \cdots + \frac{1}{2}m_nr_n^2\omega^2 = \frac{1}{2}\omega^2\left(m_1r_1^2 + m_2r_2^2 + \cdots + m_nr_n^2\right)$$

Replace, $I = m_1r_1^2 + m_2r_2^2 + \cdots + m_nr_n^2 = \sum m_ir_i^2$

so that the kinetic energy of the rotating body may be written as $E_k = \frac{1}{2}I\omega^2$

- We know that when we apply force on any object in direction of the displacement of the object, work is said to be done.
- Similarly force applied to the rotational body does work on it and this work done can be expressed in terms of moment of force (torque) and angular displacement θ.
- A force F acts on the wheel of radius R pivoted at point O so that it can rotate through point O. This force F rotates the wheel through angle $d\theta$ and $d\theta$ is small enough to regard force to be constant during corresponding time interval dt. Work done by this force is

$$dW = FdS$$

but $\quad dS = Rd\theta$

So, $\quad dW = FRd\theta$

- Now FR is the torque τ due to force F so we have

$$dW = \tau\, d\theta \quad \text{....(i)}$$

- If the torque is constant while angle changes from θ_1 to θ_2 then

$$W = \tau(\theta_2 - \theta_1) = \tau\Delta\theta \quad \text{...(ii)}$$

- Thus, work done by the constant torque is equal to the product of the torque and angular displacement.
- We know that rate of doing work is the power input of torque so

$$P = dW/dt = \tau(d\theta/dt) = \tau\omega;\ P = \tau.\omega$$

Analogy between Translatory and Rotatory Motion

- Each physical concept used to analyse rotational motion has its translational concomitant. Every law of physics governing rotational motion has a translational equivalent.

Table: Analogy between translatory and rotatory motion

Rotational motion about a fixed axis	Translatory motion
Mass, m	Moment of inertia, I
Angular speed, $\omega = d\theta/dt$	Translational speed, $v = dx/dt$
Angular acceleration, $\alpha = d\omega/dt$	Translational acceleration, $a = dv/dt$
Net torque, $\Sigma\tau = I\alpha$	Net force, $\Sigma F = ma$
$\left.\begin{array}{l}\text{If}\\ \alpha = \text{constant}\end{array}\right\} \left\{\begin{array}{l}\omega_f = \omega_i + \alpha t\\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2\\ \omega_f^2 = \omega_i^2 + 2\alpha\left(\theta_f - \theta_i\right)\end{array}\right.$	$\left.\begin{array}{l}\text{If}\\ a = \text{constant}\end{array}\right\} \left\{\begin{array}{l}v_f = v_i + at\\ x_f = x_i + v_i t + \frac{1}{2}at^2\\ v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)\end{array}\right.$

Work, $W=\int_{\theta_i}^{\theta_f}\tau d\theta$	Work, $W=\int_{x_i}^{x_j}F_x dx$
Rotational kinetic energy, $K_R=\frac{1}{2}I\omega^2$	Kinetic energy, $K=\frac{1}{2}mv^2$
Power, $P=\tau\omega$	Power, $P=Fv$
Angular momentum, $L=I\omega$ $L=\sqrt{2IE}$	Linear momentum, $p=mv$
Net torque, $\Sigma\tau=dL/dt$	Net force, $\Sigma F=dp/dt$

Rolling Motion

- A motion that is a combination of rotational and translational motion, e.g.; a wheel rolling downs the road.

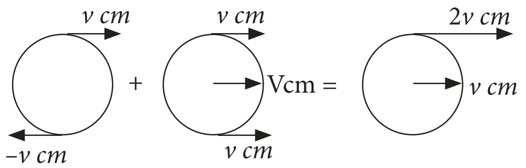


Figure: Motion of wheel is sum of rotational and translational motion

pure translation

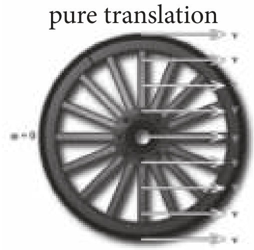


Figure: Wheel of radius R in pure translational motion

- Rolling without slipping is combination of translation and rotation where the point of contact is instantaneously at rest.
- When object experiences pure translational motion, all of its points move with same velocity as CM; that is in the same direction and with same speed. $v(r)=v_{\text{centre of mass}}$
- The object will also move in a straight line in the absence of a net external force.
- When an object experiences pure rotational motion about its centre of mass, all its points move at right angles to the radius in a plane perpendicular to the axis of rotation with a speed proportional to the distance from the axis of rotation.

  $v(r)=r\omega$

pure rotational

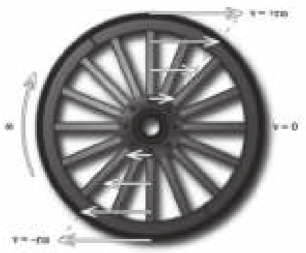


Figure: Wheel of radius R in pure rotational motion

- Thus, points on opposite sides of the axis move in opposite directions, points on the axis do not move at all since $r=0$ there $v_{\text{centre of mass}}=0$ and points on the outer edge move at the maximum speed.
- When an object experiences rolling motion the point of the object in contact with the surface is instantaneously at rest, $v_{\text{point of contact}}=0$ and is the instantaneous axis of rotation. Thus, the centre of mass of the object moves with speed $v_{\text{centre of mass}}=R\omega$ and the point farthest from the point of contact moves with twice that speed,

  $$v_{\text{opposite the point of contact}}=2v_{cm}=2R\omega$$

Rolling without slipping

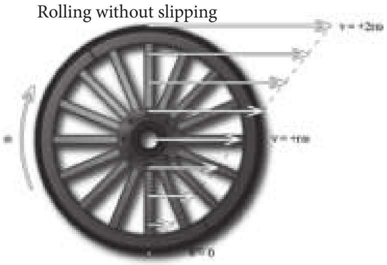


Figure: Wheel of Radius R in Rolling Without Slipping

Kinetic Energy of a Rolling Body

- If an object is rolling without slipping, then its kinetic energy can be expressed as the sum of the translational kinetic energy of its centre of mass plus the rotational kinetic energy about the centre of mass.

  Kinetic energy of a rolling body = translational kinetic energy (K_r) + rotational kinetic energy (K_R)

  $$=\frac{1}{2}Mv^2+\frac{1}{2}I\omega^2=\frac{1}{2}Mv^2\left[1+\frac{K^2}{R^2}\right]$$ (where K is the radius of gyration)

- The angular velocity is of course related to the linear velocity of the centre of mass, so the energy can be expressed in terms of either of them as the problem dictates, such as in the rolling of an object down an incline.

- The moment of inertia used must be the moment of inertia about the centre of mass. If it is known about some other axis, then the parallel axis theorem may be used to obtain the needed moment of inertia.
- When a body rolls down an inclined plane of inclination θ without slipping its velocity at the bottom of incline is given by

$$v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$$

- When a body rolls down on an inclined plane without slipping, its acceleration down the inclined plane is given by

$$a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$$

- When a body rolls down on an inclined plane without slipping, time taken by body to reach bottom is given by

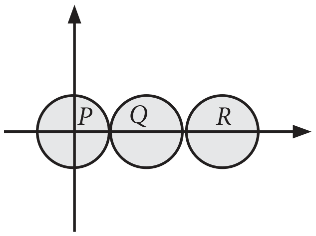
$$t = \sqrt{\frac{2l\left(1+\frac{K^2}{R^2}\right)}{g\sin\theta}}$$

PRACTICE TIME

Centre of Mass and its Motion

1. The centre of mass of a body:
(a) lies always outside the body
(b) may lie within, outside or on the surface of the body
(c) lies always inside the body
(d) lies always on the surface of the body

2. Three identical spheres, each of mass 1 kg are kept as shown in figure, touching each other, with their centres on a straight line. If their centres are marked *P*, *Q*, and *R* respectively, the distance of centre of mass of the system from *P* is:

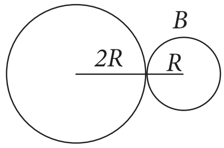


(a) $\frac{PQ+PR+QR}{3}$ (b) $\frac{PQ+PR}{3}$
(c) $\frac{PQ+QR}{3}$ (d) $\frac{PR+QR}{3}$

3. Two identical particles are located at $\vec{x}$ and $\vec{y}$ with reference to the origin of three-dimensional coordinate system. The position vector of centre of mass of the system is given by:
(a) $\vec{x}-\vec{y}$ (b) $\frac{\vec{x}+\vec{y}}{2}$
(c) $(\vec{x}-\vec{y})$ (d) $\frac{\vec{x}-\vec{y}}{2}$

4. The centre of mass of three bodies each of mass 1 kg located at the points (0, 0), (3, 0), and (0, 4) in the *XY*-plane is:
(a) $\left(\frac{4}{3},1\right)$ (b) $\left(\frac{1}{3},\frac{2}{3}\right)$
(c) $\left(\frac{1}{2},\frac{1}{2}\right)$ (d) $\left(1,\frac{4}{3}\right)$

5. Two spheres *A* and *B* of masses *m* and 2 *m* and radii 2 *R* and *R* respectively are placed in contact as shown. The CM of the system lies:



(a) inside *A*
(b) inside *B*
(c) at the point of contact
(d) None of these

6. The motion of a rigid body which is not pivoted or fixed in some way is either a pure ___*A*___ or a combination of translation and rotation. The motion of a rigid body which is pivoted or fixed in some way is ___*B*___ Here, *A* and B refer to:
(a) rotation and translation
(b) translation and rotation
(c) translation and the combination of rotation and translation
(d) None of the above

7. In rotation of a rigid body about a fixed axis, every ___A___ of the body moves in a ___B___, which lies in a plane ___C___ to the axis and has its centre on the axis. Here, A, B, and C refer to:
(a) particle, perpendicular, and circle
(b) circle, particle, and perpendicular
(c) particle, circle, and perpendicular
(d) particle, perpendicular, and perpendicular

8. Consider the following statements and choose for correct option.

I. Position vector of centre of mass of two particles of equal mass is equal to the position vector of either particle.

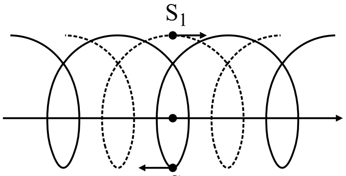
II. Centre of mass is always at the mid-point of the line joining two particles.

III. Centre of mass of a body can lie where there is no mass.

(a) I and II (b) II only

(c) III only (d) I, II, and III

9. The motion of binary stars, S_1, and S_2 is the combination of ___*X*___ and ___*Y*___ Here, *X* and *Y* refer to:



(a) motion of the CM and motion about the CM

(b) motion about the CM and motion of one star

(c) position of the CM and motion of the CM

(d) motion about CM and position of one star

10. Three masses are placed on the x-axis: 300 g at origin, 500 g at x = 40 cm and 400 g at x = 70 cm. The distance of the centre of mass from the origin is:

(a) 40 cm (b) 45 cm

(c) 50 cm (d) 30 cm

11. A body *A* of mass *M* while falling vertically downwards under gravity breaks into two parts; a body *B* of mass *M*/3 and a body *C* of mass 2/3 *M*. The centre of mass of bodies *B* and *C* taken together shifts compared to that of body *A*:

(a) does not shift

(b) depends on height of breaking

(c) towards body *B*

(d) towards body *C*

12. A system consists of three particles, each of mass m and located at (1, 1), (2, 2), and (3, 3). The coordinates of the centre of mass are:

(a) (1, 1) (b) (2, 2)

(c) (3, 3) (d) (6, 6)

13. Position vector of centre of mass of two particles system is given by:

(a) $\vec{R}=\dfrac{m_1\vec{r}_1-m_2\vec{r}_2}{m_1+m_2}$ (b) $\vec{R}=\dfrac{m_1\vec{r}_1\cdot m_2\vec{r}_2}{\vec{r}_1+\vec{r}_2}$

(c) $\vec{R}=\dfrac{m_1\vec{r}_1+m_2\vec{r}_2}{\vec{r}_1+\vec{r}_2}$ (d) $\vec{R}=\dfrac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2}$

14. The position of centre of mass of a system of particles does not depend upon the:

(a) mass of particles

(b) symmetry of the body

(c) position of the particles

(d) relative distance between the particles

15. The mass per unit length of a non-uniform rod of length L varies $m=\lambda x$ where λ is constant. The centre of mass of the rod will be at:

(a) $\frac{2}{3}L$ (b) $\frac{3}{2}L$

(c) $\frac{1}{2}L$ (d) $\frac{4}{3}L$

16. The motion of the centre of mass depends on:

(a) total external forces (b) total internal forces

(c) Sum of (a) and (b) (d) None of these

Angular Velocity and its Relationship with Linear Velocity

17. A pulley fixed to the ceiling carries a string with blocks of mass m and 3 m attached to its ends. The masses of string and pulley are negligible. When the system is released, its centre of mass moves with what acceleration?

(a) 0 (b) $-\dfrac{g}{4}$

(c) $\dfrac{g}{2}$ (d) $-\dfrac{g}{2}$

18. In rotatory motion, linear velocities of all the particles of the body are:

(a) same (b) different

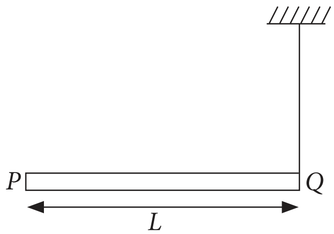
(c) zero (d) Cannot say

19. A wheel of moment of inertia 2.5 kg m^2 has an initial angular velocity of 40 rad/s. A constant torque of 10 N m acts on the wheel. The time during which the wheel is accelerated to 60 rad/s^2 is:

(a) 4.4 s (b) 6 s

(c) 5 s (d) 2.5 s

20. A rod PQ of mass *M* and length *L* is hinged at end *P*. The rod kept horizontal by a massless string tied to point *Q* as shown in the figure. When string is cut, the initial angular acceleration of the rod is:



(a) $\dfrac{3g}{2L}$ (b) $\dfrac{g}{L}$

(c) $\dfrac{2g}{L}$ (d) $\dfrac{2g}{3L}$

21. A solid cylinder of mass 50 kg and radius 0.5 m is, free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of 2 revolution s^{-2} is:

(a) 25 N (b) 50 N
(c) 78.5 N (d) 157 N

22. A wheel rotates with a constant acceleration of 2.0 rad/s^2. If the wheel starts from rest, the number of revolutions it makes in first 10 seconds is:

(a) 8 (b) 16
(c) 24 (d) 32

23. A rigid body rotates about a fixed axis with variable angular velocity equal to $(a - bt)$ at time t where a and b are constants. The angle through which it rotates before it comes to rest is:

(a) $\frac{a^2}{b}$ (b) $\frac{a^2}{2b}$ (c) $\frac{a^2}{4b}$ (d) $\frac{a^2}{2b^2}$

24. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of disc is:

(a) $\frac{I_2\omega}{I_1+I_2}$ (b) ω

(c) $\frac{I_1\omega}{I_1+I_2}$ (d) $\frac{(I_1+I_2)\omega}{I_1}$

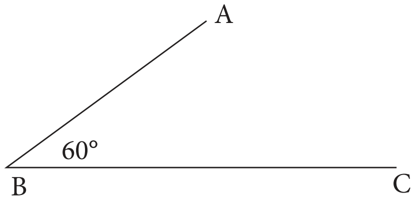
25. An athlete throws a discus from rest to a final angular velocity of 15 rad/s in 0.270 s before releasing it. During acceleration, discus moves a circular arc of radius 0.810 m. Acceleration of discus before it is released is:

(a) 45 m/s^2 (b) 182 m/s^2
(c) 187 m/s^2 (d) 192 m/s^2

26. Two bodies of masses 2 kg and 4 kg are moving with velocities 2 m/s and 10 m/s respectively along same direction. Then the velocity of their centre of mass will be:

(a) 8.1 m/s (b) 7.3 m/s
(c) 6.4 m/s (d) 5.3 m/s

27. In the figure shown, ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\frac{BC}{AB}$ is close to:



(a) 1.85 (b) 1.5
(c) 1.37 (d) 3

28. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The centre of mass of the new disc is α/R from the centre of the bigger disc. The value of α is:

(a) 1/4 (b) 1/3
(c) 1/2 (d) 1/6

29. Which of the following is incorrect?

(a) $\vec{v} = \vec{\omega} \times \vec{r}$ (b) $\vec{a} = \vec{v} \times \vec{r}$

(c) $a = \frac{d\omega}{dt}$ (d) None of these

30. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute (rpm) to ensure proper mixing is close to (take the radius of the drum to be 1.25 m and its axle to be horizontal):

(a) 27 (b) 0.4
(c) 1.3 (d) 8

31. Angular velocity of each particle of a rotating rigid body about axis of rotation is:

(a) same
(b) different
(c) depends on relative position
(d) None of these

32. A wheel rotates with a constant acceleration of 4.0 rad/s^2. If the wheel starts from rest, the number of revolutions it makes in the first 10 seconds will be approximately:

(a) 8 (b) 31
(c) 24 (d) 32

33. A cord is wound over the rim of a flywheel of mass 20 kg and radius 25 cm. A mass 2.5 kg attached to the cord can fall under gravity. The angular acceleration of the flywheel is:

(a) 25 rad/s^2 (b) 20 rad/s^2
(c) 10 rad/s^2 (d) 5 rad/s^2

34. Initial angular velocity of a circular disc of mass M is ω_1. Then, two small spheres of m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$

(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$

35. A rope of negligible mass is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder, if the rope is pulled with a force of 30 N? Assume that there is no slipping:

(a) 10 rad/s^2 (b) 15 rad/s^2
(c) 20 rad/s^2 (d) 25 rad/s^2

36. When the disc rotates with uniform angular velocity. Which of the following is not true?
(a) The sense of rotation remains same.
(b) The orientation of the axis of rotation remains same.
(c) The speed of rotation is non-zero and remains same.
(d) The angular acceleration is non-zero and remains same.

37. A flywheel of moment of inertia 3×10^2 kgm^2 is rotating with uniform angular speed of 4.6 rad/s. If a torque of 6.9×10^2 N m retards the wheel, then the time in which the wheel comes to rest is:
(a) 1.5 s (b) 2 s
(c) 0.5 s (d) 1 s

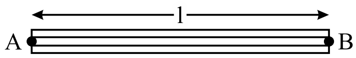
38. A thin circular ring of mass m and radius r is rotating about its axis with a constant angular momentum, ω. Four objects each of mass, m are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be:
(a) $\left(\dfrac{M\omega}{M+4m}\right)$ (b) $\dfrac{(M+4m)\omega}{M}$
(c) $\dfrac{(M-4m)\omega}{M+4m}$ (d) $\left(\dfrac{M\omega}{4m}\right)$

39. If the frequency of the rotating platform is v and the distance of a boy from the centre is r, what is the area swept out per second by the line connecting the boy to the centre?
(a) $\pi r v$ (b) $2\pi r v$
(c) $\pi r^2 v$ (d) $2\pi r^2 v$

40. A thin and circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. If another disc of same dimensions but of mass M/4 is placed gently on the first disc coaxially, then the new angular velocity of the system is:
(a) 5ω/4 (b) 2ω/3
(c) 4ω/5 (d) 3ω/2

41. A uniform rod of length l and mass m is free to rotate in a vertical plane about A. the rod initially in horizontal position is released. The initial angular acceleration of rod is:

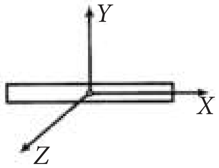
[***Hint:*** *Moment of inertia of rod about A is* $\dfrac{ml^2}{3}$.]



(a) $\dfrac{3g}{2l}$ (b) $\dfrac{2l}{3g}$
(c) $\dfrac{3g}{2l^2}$ (d) $mg\dfrac{l}{2}$

42. When a ceiling fan is switched on, it makes 10 revolutions in the first 3 s. Assuming a uniform angular acceleration, how many rotations it will make in the next 3 s?
(a) 10 (b) 20
(c) 30 (d) 40

43. A uniform rod of mass M and length L is free to rotate in XY-plane, i.e., about y-axis. If a force of $F = (3\,i + 2\,j + 6\,k)$ N is acting on (L/2, 0, 0) in the situation as shown in figure. The angular acceleration of rod is: (Take, M = 6 kg, L = 4 m)



(a) $-\dfrac{3}{2}\hat{i}+\dfrac{1}{2}\hat{k}$ (b) $-\dfrac{3}{2}\hat{j}$
(c) $\dfrac{1}{2}\hat{k}$ (d) $4\hat{j}$

44. A tube of length L is filled completely with an incompressible liquid of mass m and closed at both ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity, ω. The force exerted by the liquid at the other end is:
(a) $M\omega^2 L/2$ (b) $M\omega^2 L$
(c) $M\omega^2 L/4$ (d) $M\omega^2 L^2/2$

45. If 2 kg mass is rotating on a circular path of radius 0.8 m with angular velocity of 44 rad/sec. If radius of the path becomes 1 m, then what will be the value of angular velocity?
(a) 28.16 rad/sec (b) 19.28 rad/sec
(c) 8.12 rad/sec (d) 35.26 rad/sec

Torque and Angular Momentum

46. Angular momentum of a body is defined as the product of:
(a) mass and angular velocity
(b) centripetal force and radius
(c) linear velocity and angular velocity
(d) moment of inertia and angular velocity

47. The drive shaft of an automobile rotates at 3600 rpm and transmits 80 HP up from the engine to the rear wheels. The torque developed by the engine is:
(a) 16.58 Nm (b) 0.022 Nm
(c) 158.31 Nm (d) 141.6 Nm

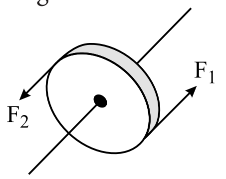
48. A particle of mass 1 kg is moving along the line $y = x + 2$ (here x and y are in metres) with speed 2 m/s. The magnitude of angular momentum of particle about origin is:

(a) 4 kg m^2s^{-1} (b) $2\sqrt{2}$ kg m^2s^{-1}
(c) $4\sqrt{2}$ kg m^2s^{-1} (d) 2 kg m^2s^{-1}

49. In an orbital motion, the angular momentum vector is:
(a) along the radius vector
(b) parallel to the linear momentum
(c) in the orbital plane
(d) perpendicular to the orbital plane

50. A rigid horizontal smooth rod AB of mass 0.75 kg and length 40 cm can rotate freely about a fixed vertical axis through its mid-point O. Two rings each of mass 1 kg initially at rest are placed at a distance of 10 cm from O on either side of the rod. The rod is set in rotation with an angular velocity of 30 radian per sec and when the rings reach the ends of the rod, the angular velocity in rad/sec is:
(a) 5 (b) 10
(c) 15 (d) 20

51. A couple consisting of two forces F_1 and F_2 each equal to 5 N is acting at the rim of a disc of mass 2 kg and radius ½ m for 5 sec. Initially, the disc is at rest, the final angular momentum of the disc is:



(a) 15 (b) 20
(c) 25 (d) 30

52. A particle is moving along a straight line parallel to x-axis with constant velocity. Its angular momentum about the origin:
(a) decreases with time (b) increases with time
(c) remains constant (d) is zero

53. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected?
(a) Moment of inertia
(b) Angular momentum
(c) Angular velocity
(d) Rotational kinetic energy

54. The radius vector and linear momentum are respectively given by vectors $2\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-3\hat{j}+\hat{k}$. Then the angular momentum is:
(a) $2i-4\hat{k}$ (b) $4\hat{i}-8\hat{k}$
(c) $2\hat{i}-4\hat{j}+2k$ (d) $4i-8j$

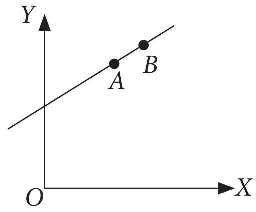
55. A mass is whirled in a circular path with constant angular velocity and its angular momentum is L. If the string is now halved keeping the angular velocity the same, the angular momentum is:
(a) $L/4$ (b) $L/2$
(c) L (d) $2L$

56. The position of a particle is given $\hat{j}$ by $\vec{r}=\hat{i}+2\hat{j}-\hat{k}$ and its linear momentum is given by $\vec{p}=3\hat{i}+4\hat{j}-2\hat{k}$. Then its angular momentum, about the origin is perpendicular to:
(a) yz-plane (b) z-axis
(c) y-axis (d) x-axis

57. If r denotes the distance between the Sun and the Earth, then the angular momentum of the Earth around the Sun is proportional to:
(a) r^3/r (b) r
(c) $\sqrt{r}$ (d) r^2

58. If I is the moment of inertia and E is the kinetic energy of rotation of a body, then its angular momentum will be:
(a) $\sqrt{(EI)}$ (b) $2EI$
(c) E/I (d) $\sqrt{(2EI)}$

59. A particle of mass m in the XY-plane with a velocity v along the straight line AB. If the angular momentum of the particle with respect to origin O is L_A when it is at B, then:

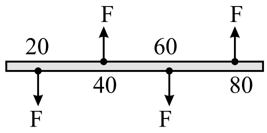


(a) $L_A > L_B$
(b) $L_A = L_B$
(c) The relationship between L_A and L_B depends upon the slope of the line AB
(d) $L_A < L_B$

60. Total angular momentum of a rotating body remains constant, if the net torque acting on the body is:
(a) zero (b) maximum
(c) minimum (d) unity

Equilibrium of a Rigid Body

61. Four equal and parallel forces are acting on a rod of length 100 cm, as shown in figure, at distances of 20 cm, 40 cm, 60 cm, and 80 cm respectively from one end of the rod. Under the influence of these forces, the rod: (neglecting its weight)

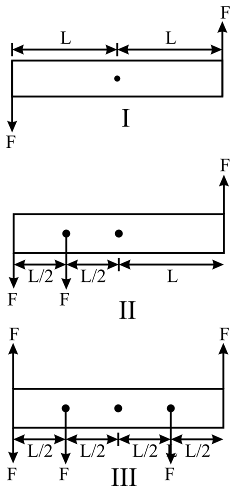


(a) experiences no torque
(b) experiences torque
(c) experiences a linear motion
(d) experiences torque and also a linear motion

62. A uniform horizontal metre scale of mass m is suspended by two vertical strings attached to its two ends. A body of mass 2 m is placed on the 75 cm mark. The tensions in the two strings are in the ratio is:
(a) 1:2 (b) 1:3
(c) 2:3 (d) 3:4

63. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?
(a) 56 g (b) 66 g
(c) 76 g (d) 86 g

64. A rigid rod of length 2 L is acted upon by some forces. All forces labelled F have the same magnitude. Which cases have a non-zero net torque acting on the rod about its centre?



(a) I and II only
(b) II and III only
(c) I and III only
(d) The net torque is zero in all cases

Moment of Inertia

65. The moment of inertia of a ___A___ body about an axis ___B___ to its plane is equal to the sum of its moments of inertia about two ___C___ axes concurrent with perpendicular axis and lying in the plane of the body. Here, A, B, and C refer to:
(a) three dimensional, perpendicular, and perpendicular
(b) planar, perpendicular, and parallel
(c) planar, perpendicular, and perpendicular
(d) three dimensional, parallel, and perpendicular

66. A thin rod of length l and mass m is bent at mid-point O at angle of 60°. The moment of inertia of the rod about an axis passing through O and perpendicular to the plane of the rod will be:
(a) $\frac{ml^2}{3}$ (b) $\frac{ml^2}{6}$
(c) $\frac{ml^2}{8}$ (d) $\frac{ml^2}{12}$

67. Three-point masses, each of mass, m are placed at the corner of an equilateral triangle of side, l. Then the moment of inertia of this system about an axis along one side of the triangle is:
(a) $3ml^2$ (b) ml^2
(c) $\frac{3}{4}ml^2$ (d) $\frac{3}{2}ml^2$

68. The moment of inertia of a uniform circular disc of radius, R and mass, M about an axis passing from the edge of the disc and normal to the disc is:
(a) MR^2 (b) $\frac{1}{2}MR^2$
(c) $\frac{3}{2}MR^2$ (d) $\frac{7}{2}MR^2$

69. Moment of inertia of a hollow cylinder of mass M and radius r about its own axis is:
(a) $\frac{2}{3}Mr^2$ (b) $\frac{2}{5}Mr^2$
(c) $\frac{1}{3}Mr^2$ (d) Mr^2

70. A constant torque of 3.14 N m is exerted on a pivoted wheel. If the angular acceleration of the wheel is 4π rad/s^2, then the moment of inertia of the wheel is:
(a) 0.25 kg m^2 (b) 2.5 kg m^2
(c) 4.5 kg m^2 (d) 25 kg m^2

71. Which of the following has the highest moment of inertia when each of them has the same mass and the same outer radius:
(a) a ring about its axis, perpendicular to the plane of the ring.
(b) a disc about its axis, perpendicular to the plane of the disc.
(c) a solid sphere about one of its diameters.
(d) a spherical shell about one of its diameters.

72. The moment of inertia of a thin uniform rod of mass M and length l about an axis perpendicular to the rod through its centre is I. The moment of inertia of the rod through its end point is:
(a) $\frac{I}{4}$ (b) $\frac{I}{2}$
(c) $2I$ (d) $4I$

73. The correct relation between moment of inertia I, radius of gyration K and mass M of the body is:

(a) $K = I^2 M$ (b) $K = IM^2$

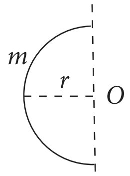
(c) $K = \sqrt{\frac{M}{I}}$ (d) $K = \sqrt{\frac{I}{M}}$

74. The radius of gyration of a uniform rod of length L about an axis passing through its centre of mass is:

(a) $\frac{L}{\sqrt{12}}$ (b) $\frac{L^2}{\sqrt{12}}$

(c) $\frac{L}{\sqrt{3}}$ (d) $\frac{L}{\sqrt{2}}$

75. A thin wire of length l and mass m is bent in the form of a semicircle as shown in the figure. Its moment of inertia about an axis joining its free ends will be:



(a) ml^2 (b) 0

(c) ml^2/π^2 (d) $ml^2/2\pi^2$

76. If two circular discs A and B are of same mass but of radii r and $2r$ respectively, then the moment of inertia of A is:

(a) the same as that of B.
(b) twice that of B.
(c) four times that of B.
(d) ¼ that of B.

77. For a given mass and size, moment of inertia of a solid disc is:

(a) more than that of a ring
(b) less than that of a ring
(c) equal to that of a ring
(d) depend on the material of ring and disc

Kinetic Energy, Work and Power of Rotational Motion

78. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad/s. The radius of the cylinder is 0.25 m. The kinetic energy associated with the rotation of the cylinder is:

(a) 3025 J (b) 3225 J
(c) 3250 J (d) 3125 J

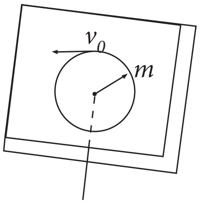
79. A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system becomes doubled. The kinetic energy of the system now is:

(a) $2K$ (b) $K/2$
(c) $K/4$ (d) $4K$

80. A circular disc rolls down an inclined plane. The ratio of rotational kinetic energy to total kinetic energy is:

(a) 1/2 (b) 1/3
(c) 2/3 (d) 3/4

81. A mass m moves in a circle on a smooth horizontal plane with velocity v_0 at a radius R_0. The mass is attached to a string which passes through a smooth hold in the plane as shown in figure:



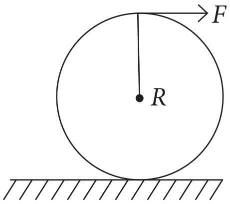
The tension in the string is increased gradually and finally m moves in a circle of radius $\frac{R_0}{2}$. The final value of the kinetic energy is:

(a) $\frac{1}{4}mv_0^2$ (b) $2mv_0^2$

(c) $\frac{1}{2}mv_0^2$ (d) mv_0^2

Rolling Motion

82. A tangential force F acts at the top of a thin spherical shell of mass m and radius R. The acceleration of the shell, if it rolls without slipping is:



(a) $\frac{5}{6}\frac{F}{m}$ (b) $\frac{6}{5}\frac{F}{m}$

(c) $\frac{7}{2}\frac{m}{F}$ (d) $\frac{2}{7}\frac{m}{F}$

83. A solid sphere, disc and solid cylinder all of the same mass and made up of same material are allowed to roll down (from rest) on inclined plane, then:

(a) solid sphere reaches the bottom first
(b) solid sphere reaches the bottom late
(c) disc will reach the bottom first
(d) all of them reach the bottom at the same time

84. An inclined plane makes an angle 30° with the horizontal. A solid sphere rolling down this inclined plane from rest without slipping has a linear acceleration equal to:

(a) g/3 (b) 2g/3
(c) 5g/7 (d) 5g/14

85. A uniform solid cylindrical roller of mass m is being pulled on a horizontal surface with force F parallel to the surface and applied at its centre. If the acceleration of the cylinder is a and it is rolling without slipping, then the value of F is:
(a) ma (b) $5/3\ ma$
(c) $3/2\ ma$ (d) $2\ ma$

86. A body rolls down an inclined plane. If its kinetic energy of rotation is 40% of its kinetic energy of translation motion, then the body is:
(a) hollow cylinder (b) ring
(c) solid disc (d) solid sphere

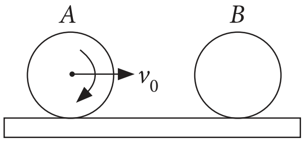
87. A solid cylinder of mass 2 kg and radius 0.1 m rolls down an inclined plane of height 3 m without slipping. Its rotational kinetic energy when it reaches the foot of the plane would be:
(a) 22.7 J (b) 19.6 J
(c) 10.2 J (d) 9.8 J

88. The acceleration of a disc (mass m and radius R) rolling down an incline of angle θ without slipping is:
(a) $2/3\ g \sin \theta$ (b) $5/7\ g \sin \theta$
(c) $1/2\ g \sin \theta$ (d) $7/5\ g \sin \theta$

89. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be:
(a) $\dfrac{K^2}{R^2}$ (b) $\dfrac{K^2}{K^2+R^2}$
(c) $\dfrac{R^2}{K^2+R^2}$ (d) $\dfrac{K^2+R^2}{R^2}$

90. A sphere rolls down on an inclined plane of inclination 6. What is the acceleration as the sphere reaches the bottom?
(a) $\dfrac{5}{7} g \sin\theta$ (b) $\dfrac{3}{5} g \sin\theta$
(c) $\dfrac{2}{7} g \sin\theta$ (d) $\dfrac{2}{5} g \sin\theta$

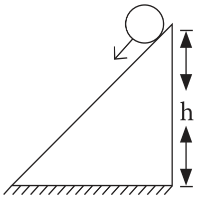
91. A hollow smooth uniform sphere A of mass m rolls without sliding on a smooth horizontal surface. It collides head on elastically with another stationary smooth solid sphere B of the same mass m and same radius. The ratio of kinetic energy of B to that of A just after the collision is:



(a) 1:1 (b) 2:3
(c) 3:2 (d) None of these

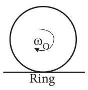
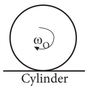
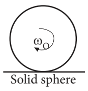
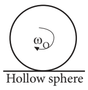
92. A solid sphere, disc, and solid cylinder all of the same mass and made of the same material are allowed to roll down (from rest) on an inclined plane, then
(a) solid sphere reaches the bottom first.
(b) solid sphere reaches the bottom last.
(c) disc will reach the bottom first.
(d) all reach the bottom at the same time.

93. A solid cylinder of mass m and radius R rolls down inclined plane without slipping. The speed of its CM when it reaches the bottom is:



(a) $\sqrt{2gh}$ (b) $\sqrt{4gh/3}$
(c) $\sqrt{3/4gh}$ (d) $\sqrt{4gh}$

94. Match the following:

Column I		Column II	
A		(i)	Rotational work done by the friction is negative till pure rolling begins
B		(ii)	Translational work done is positive till pure rolling starts
C		(iii)	When pure rolling begins velocity of centre of mass is minimum
D		(iv)	Takes maximum time for pure rolling to begin

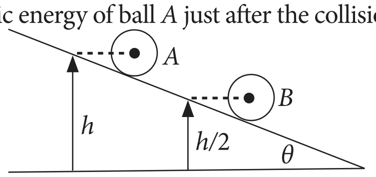
	A	B	C	D
(a)	(i), (ii)	(i)	(i), (ii)	(i), (iii)
(b)	(ii)	(iii)	(iv)	(i)
(c)	(i), (ii), (iv)	(i), (ii)	(i), (ii), (iii)	(i), (ii)
(d)	(i), (ii)	(i), (iii)	(i), (ii), (iii)	(iv)

95. The ratio of the acceleration for a solid sphere (mass m and radius R) rolling down an incline of angle θ without slipping and slipping down the incline without rolling is:
(a) 5 : 7 (b) 2 : 3
(c) 2 : 5 (d) 7 : 5

96. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $3\ v^2/4\ g$ with respect to the initial position. The object is:

(a) ring (b) solid sphere
(c) hollow sphere (d) disc

97. Two identical uniform solid spherical ball A and B of mass m each are placed on the fixed wedge as shown in figure. Ball B is kept at rest and it is released just before two ball collides. Ball A rolls down without slipping on inclined plane and collide elastically with ball B. The kinetic energy of ball A just after the collision with B is:

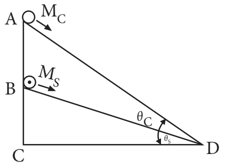


(a) $\frac{mgh}{7}$ (b) $\frac{mgh}{2}$

(c) $\frac{2mgh}{5}$ (d) $\frac{7mgh}{5}$

98. A cylinder of mass M_c and sphere of mass M_s are placed at points A and B of two inclines, respectively (see figure). If they roll on the incline without sipping such that their accelerations are the same, then the ratio

$\frac{\sin\theta_c}{\sin\theta_s}$ is:



(a) $\sqrt{\frac{8}{7}}$ (b) $\sqrt{\frac{15}{14}}$

(c) $\frac{8}{7}$ (d) $\frac{15}{14}$

HIGH-ORDER THINKING SKILL

Centre of Mass and its Motion

1. Three identical metal balls, each of radius r are placed, touching each other on a horizontal surface. When the centres of the three balls are joined, an equilateral triangle is formed. The centre of mass of the system is located at:
(a) the centre of one of the balls
(b) the line joining the centre of any two balls
(c) the point of intersection of the medians
(d) the horizontal surface

2. The centre of mass of a right circular cone of height h, radius R and constant density ρ is at:

(a) $\left(0,0,\frac{h}{4}\right)$ (b) $\left(0,0,\frac{h}{3}\right)$

(c) $\left(0,0,\frac{h}{2}\right)$ (d) $\left(0,0,\frac{3h}{8}\right)$

Angular Velocity and its Relationship with Linear Velocity

3. The linear velocity of a rotating body is given by $\vec{v}=\vec{\omega}\times\vec{r}$, where $\vec{\omega}$ is the angular velocity and $\vec{r}$ is the radius vector. The angular velocity of a body is $\vec{\omega}=\hat{i}-2\hat{j}+2\hat{k}$ and the radius vector $\vec{r}=4\hat{j}-3\hat{k}$, then $|\vec{v}|$ is:
(a) $\sqrt{29}$ units (b) $\sqrt{31}$ units
(c) $\sqrt{37}$ units (d) $\sqrt{41}$ units

4. Initial angular velocity of a circular disc of mass M is ω_1. The two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$

(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$

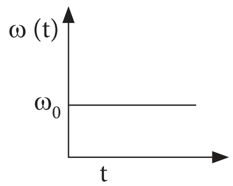
Torque and Angular Momentum

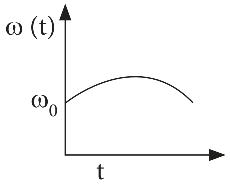
5. A rod of weight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is:

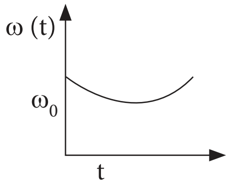
(a) $\frac{Wx}{d}$ (b) $\frac{Wd}{x}$

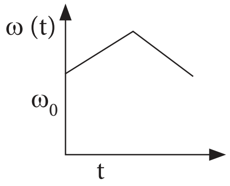
(c) $\frac{W(d-x)}{x}$ (d) $\frac{W(d-x)}{d}$

6. A circular platform is free to rotate in horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform ω_t will vary with time t as:

(a)


(b) 

(c) 

(d) 

Equilibrium of a Rigid body

7. Choose the correct option:

	Column I		Column II
A	The equilibrium of a body is said to stable if	(i)	on being slightly disturbed, it shows no tendency to come back to its original position and moves away from it
B	The body is said to be in unstable equilibrium if	(ii)	on being slightly displaced, it remains in the new position
C	A body is said to be in neutral equilibrium if	(iii)	on being slightly disturbed, it tends to come back to its original position

(a) A-(ii), B-(iii), C-(i) (b) A-(iii), B-(i), C-(ii)
(c) A-(i), B-(ii), C-(iii) (d) None of these

Moment of Inertia

8. Let I_1 and I_2 be the moments of inertia of two bodies of identical geometrical shape. If the first body is made of aluminium and the second of iron, then:

(a) $I_1 < I_2$ (b) $I_1 = I_2$

(c) $I_1 > I_2$ (d) $I_1 = \frac{I_2}{2}$

9. From a solid sphere of mass M and radius R, a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:

(a) $\frac{MR^2}{32\sqrt{2}\pi}$ (b) $\frac{MR^2}{16\sqrt{2}\pi}$

(c) $\frac{4MR^2}{9\sqrt{3}\pi}$ (d) $\frac{4MR^2}{3\sqrt{3}\pi}$

10. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_x and I_y is:

(a) $I_Y = 32\, I_X$ (b) $I_Y = 16\, I_X$

(c) $I_Y = I_X$ (d) $I_Y = 64\, I_X$

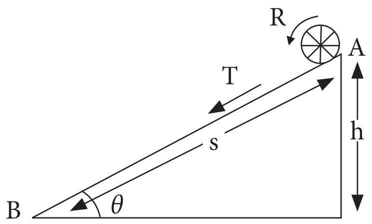
Kinetic Energy, Work and Power of Rotational Motion

11. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy is halved, then the new angular momentum is:
(a) $L/4$ (b) $2L$
(c) $4L$ (d) $L/2$

Rolling Motion

12. Suppose a body of mass M and radius R is allowed to roll on an inclined plane without slipping from its top most point A. The velocity acquired by the body, as it reaches the bottom of the inclined plane, is given by:

$$\left(\text{Given:} \quad 1+\frac{1}{MR^2}=\beta\right)$$



(a) $\sqrt{2gh}$ (b) $\sqrt{\beta \times 2gh}$

(c) $\sqrt{\frac{2gh}{\beta}}$ (d) $\frac{2gh}{\beta}$

NCERT EXEMPLAR PROBLEMS

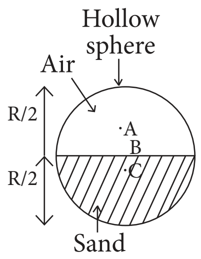
Centre of Mass and its Motion

1. For which of the following does the centre of mass lie outside the body?

(a) A pencil (b) A shotput
(c) A dice (d) A bangle

[***Hint:*** *Centre of mass of a system (body) is a point that moves as though all the mass were concentrated there and all external forces were applied there.*]

2. Which of the following points is the likely position of the centre of mass of the system shown in figure?



(a) A (b) B
(c) C (d) D

Angular Velocity and its Relationship with Linear Velocity

3. When a disc rotates with uniform angular velocity, which of the following is not true?

(a) The sense of rotation remains same.
(b) The orientation of the axis of rotation remains same.
(c) The speed of rotation is non-zero and remains same.
(d) The angular acceleration is non-zero and remains same.

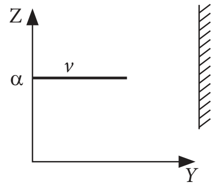
[***Hint:*** *The rate of change of angular velocity is defined as angular acceleration. If particle has angular velocity ω_1 at time t_1, and angular velocity ω_2 at time t_2, then angular acceleration $\alpha = \omega_2 - \omega_1/t_2 - t_1$.*]

4. A Merry-go-round, made of a ring-like platform of radius R and mass M, is revolving with angular speed ω. A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round). The speed of the round afterwards is:

(a) 2ω (b) ω (c) $\omega/2$ (d) 0

Torque and Angular Momentum

5. A particle of mass m is moving in yz-plane with a uniform velocity v with its trajectory running parallel to positive y-axis and intersecting z-axis at $z = a$. The change in its angular momentum about the origin as it bounces elastically from a wall at:

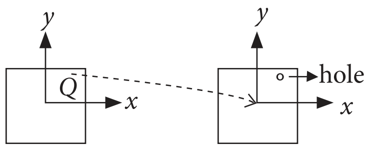


(a) $mva\ \hat{e}_x$ (b) $2mva\ \hat{e}_x$
(c) $ymv\ \hat{e}_x$ (d) $ymv\ \hat{e}_x$

[***Hint:*** *Angular momentum is an axial vector, i.e., always directed perpendicular to the plane of rotation and along the axis of rotation.*]

Moment of Inertia

6. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind. The moment of inertia about the z-axis is then



(a) increased.
(b) decreased.
(c) the same.
(d) changed in unpredicted manner.

ASSERTION AND REASONS

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as:
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

Centre of Mass and its Motion

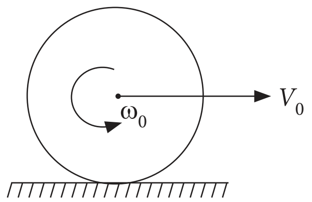
1. **Assertion:** Toppling is the application of the centre of mass.
Reason: Toppling stability determines the maximum angle that an object can be tilted before it will topple over.
2. **Assertion:** The reference frame does not decide the position of centre of mass.
Reason: Centre of mass depends only upon the rest mass of the body.
3. **Assertion:** The centre of mass of a body may lie where there is no mass.
Reason: The centre of mass does not depend upon the mass of the body.

Angular Velocity and its Relationship with Linear Velocity

4. **Assertion:** If the head of a right-handed screw rotates with the body, the screw advances in the direction of the angular velocity.
Reason: For rotation about a fixed axis, the angular velocity vector lies along the axis of rotation.

Torque and Angular Momentum

5. **Assertion:** A man sitting on a rolling table, when he stretches his arms horizontally, his speed is reduced.
Reason: Principle of conservation of angular momentum is applicable in this situation.
6. **Assertion:** A ladder is more apt to slip, when you are high up on it than when you just begin to climb.
Reason: At the high up on a ladder, the torque is large and on climbing up the torque is small.
7. **Assertion:** If a particle moves with a constant velocity, then angular momentum of this particle about any point remains constant.
Reason: Angular momentum has the units of Planck's constant.
8. **Assertion:** A uniform disc of radius R is performing impure rolling motion on a rough horizontal plane as shown in figure. After some time, the disc comes to rest. It is possible only when $v_0 = \frac{\omega_0 R}{2}$.



Reason: For a body performing pure rolling motion, the angular momentum is conserved about any point in space.
9. **Assertion:** For system of particles under central force field, the total angular momentum is conserved.
Reason: The torque acting on such a system is zero.
10. **Assertion:** A person standing on a rotating platform suddenly stretched his arms, the platform slows down.
Reason: A person by stretching his arms increases the moment of inertia and decreases angular velocity.

Equilibrium of a Rigid Body

11. **Assertion:** For a body to be in equilibrium it must satisfy both the conditions stated simultaneously.
Reason: (a) the vector sum of all the external forces acting on the body should vanish.
(b) the vector sum of all the external torque acting on the body should vanish.

Moment of Inertia.

12. **Assertion:** There are very small sporadic changes in the speed of rotation of the Earth.
Reason: Shifting of large air masses in the Earth's atmosphere produce a change in the moment of inertia of the Earth causing its speed of rotation to change.
13. **Assertion:** Value of radius of gyration of a uniform rigid body depends on axis of rotation.
Reason: Radius of gyration is root mean square distance of particles of the body from the axis of rotation.
14. **Assertion:** The speed of whirlwind in a tornado is alarmingly high.
Reason: If no external torque acts on a body, its angular velocity remains conserved.

Kinetic Energy, Work and Power of Rotational Motion

15. **Assertion:** When a body rolls down an inclined plane it has both kinetic and potential energy.
Reason: The rolling body follow the conservation law of mechanical energy.
16. **Assertion:** Power associated with torque is product of torque and angular speed of the body about the axis of rotation.
Reason: Torque in rotational motion is analogue to force in translatory motion.

Rolling Motion

17. **Assertion:** A sphere cannot roll on a smooth inclined surface.
Reason: The motion of a rigid body which is pivoted or fixed in some way is rotation.
18. **Assertion:** A motion that is a combination of rotational and translational motion is called rolling motion.
Reason: Example of rolling motion is a wheel rolling down the road.
19. **Assertion:** The velocity of a body at the bottom of an inclined plane of given height is more when it slides down the plane compared to when it rolling down the same plane.
Reason: In rolling down a body acquires both kinetic energy of translation and rotation.
20. **Assertion:** When an object experiences pure translational motion, all of its points move with the same velocity as the centre of mass; that is in the same direction and with the same speed $v(r) = v_{\text{centre of mass}}$.
Reason: When an object experiences pure rotational motion about its centre of mass, all of its points move at right angles to the radius in a plane perpendicular to the axis of rotation with a speed proportional to the distance from the axis of rotation, $v(r) = r\omega$.

ANSWER KEYS

Practice Time

1	(b)	2	(b)	3	(b)	4	(d)	5	(c)	6	(b)	7	(c)	8	(c)	9	(a)	10	(a)
11	(a)	12	(b)	13	(d)	14	(d)	15	(a)	16	(a)	17	(c)	18	(b)	19	(c)	20	(a)
21	(d)	22	(b)	23	(b)	24	(c)	25	(a)	26	(b)	27	(c)	28	(b)	29	(b)	30	(a)
31	(a)	32	(b)	33	(c)	34	(c)	35	(d)	36	(d)	37	(b)	38	(a)	39	(c)	40	(c)
41	(a)	42	(c)	43	(b)	44	(a)	45	(a)	46	(d)	47	(c)	48	(b)	49	(d)	50	(b)
51	(c)	52	(c)	53	(b)	54	(b)	55	(a)	56	(d)	57	(b)	58	(c)	59	(d)	60	(b)
61	(a)	62	(b)	63	(a)	64	(b)	65	(a)	66	(c)	67	(d)	68	(c)	69	(c)	70	(d)
71	(a)	72	(a)	73	(d)	74	(d)	75	(a)	76	(d)	77	(d)	78	(b)	79	(d)	80	(b)
81	(b)	82	(b)	83	(b)	84	(a)	85	(d)	86	(c)	87	(d)	88	(b)	89	(a)	90	(b)
91	(a)	92	(c)	93	(a)	94	(b)	95	(c)	96	(a)	97	(d)	98	(a)	99	(d)		

High-Order Thinking Skill

1	(c)	2	(a)	3	(a)	4	(c)	5	(d)	6	(b)	7	(b)	8	(a)	9	(c)	10	(d)
11	(a)	12	(c)																

NCERT Exemplar Problems

1	(d)	2	(c)	3	(d)	4	(b)	5	(b)	6	(b)

Assertion and Reasons

1	(a)	2	(c)	3	(c)	4	(a)	5	(a)	6	(a)	7	(b)	8	(c)	9	(a)	10	(a)
11	(a)	12	(a)	13	(a)	14	(c)	15	(a)	16	(b)	17	(b)	18	(a)	19	(a)	20	(b)

HINTS AND EXPLANATIONS

Practice Time

1 (b) Centre of mass depends on the distribution of mass in the body.

2 (b) Insert the given values,

$$x_{CM} = \frac{1\times 0 + 1\times PQ + 1\times PR}{1+1+1} = \frac{PQ+PR}{3}$$

and, $y_{CM} = 0$

3 (b) As we know that,

$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} = \frac{m(\vec{x}+\vec{y})}{2m} = \frac{\vec{x}+\vec{y}}{2}$$

4 (d) As we know that,

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3} = \frac{1\times 0 + 1\times 3 + 1\times 0}{1+1+1} = 1$$

$$Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1+m_2+m_3} = \frac{1\times 0 + 1\times 0 + 1\times 4}{1+1+1} = \frac{4}{3}$$

Therefore, the coordinates of centre of mass is $\left(1, \frac{4}{3}\right)$.

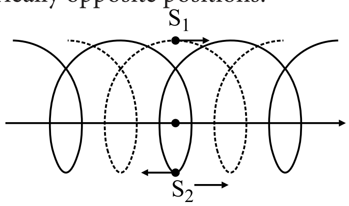
5 (c) The CM of the system lies at the point of contact.

6 (b) The motion of a rigid body which is not pivoted or fixed in some way is either a pure translation or a combination of translation and rotation. The motion of a rigid body which is pivoted or fixed in some way is rotation.

7 (c) In rotation of a rigid body about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis.

8 (c) Centre of mass does not necessarily lie only where there is mass. It can lie outside the body as well. For example, centre of mass of circular ring lies in the centre of the ring where there is no mass.

9 (a) When no external force acts on the binary star, its CM will move like a free particle [see figure below]. From the CM frame, the two stars will seem to move in a circle about the CM with diametrically opposite positions.



(i) Trajectories of two stars S_1 (dotted line) and S_2 (solid line) forming a binary system with their centre of mass C in uniform motion.

(ii) The same binary system, with the centre of mass, C at rest. So, to understand the motion of a complicated system, we can separate the motion of the system into two parts. So, the combination of the motion of the CM and motion about the CM could be described the motion of the system.

10 (a) As we know that,

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3}$$

$$X_{CM} = \frac{300\times(0) + 500(40) + 400\times 70}{300+500+400}$$

$$X_{CM} = \frac{500\times 40 + 400\times 70}{1200}$$

$$= 40 \text{ cm}$$

11 (a) Does not shift as no external force acts. The centre of mass of the system continues its original path. It is only the internal forces which come into play while breaking.

12 (b) The coordinate of CM of three particles are,

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3}$$

and,

$$y = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1+m_2+m_3}$$

If, $m_1 = m_2 = m_3 = m$

Then,

$$x = \frac{(x_1+x_2+x_3)m}{m+m+m} = 2$$

and

$$y=\frac{(y_1+y_2+y_3)m}{m+m+m}$$
$$=2$$

So, coordinate of C of three particles are (2,2).

13 (d) By the definition, position vector of centre of mass of two particle system is such that the product of total mass of the system and position vector of centre of mass is equal to the sum of products of masses of two particles and their respective position vectors, i.e.,

$$(m_1+m_2)\vec{R}=m_1\vec{r}_1+m_2\vec{r}_2, \rho\cdot\vec{R}=\frac{N_i\vec{r}+m_2\vec{r}_2}{m_1+m_2}$$

14 (d) The position of centre of mass of a system depends upon mass, position and symmetry of the body.

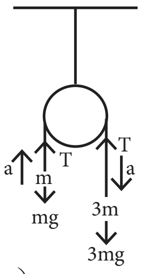
$$R_{CM}=\frac{\sum m_ir_i}{\sum m_i}$$

15 (a) As we know that,

$$X_{CM}=\frac{\int_0^L(dm)x}{\int_0^L(dm)}$$
$$=\frac{\int_0^2(\lambda_xd_x)x}{\int_0^L(\lambda_xd_x)}$$
$$=\frac{2L}{3}$$

16 (a) The motion of the centre of mass depends on total external forces exerted on the body.

17 (c) When the system is released, the heavier mass moves downwards and the lighter one upwards. Thus, centre of mass will move towards the heavier mass with acceleration.



$$a=\left(\frac{3m-m}{3m+m}\right)g$$
$$=\frac{g}{2}$$

18 (b) In rotatory motion, linear velocities of all the particles of the body are different.

19 (c) Given that,

$$I=2.5 \text{ kg m}^2$$
$$\omega_0=40 \text{ rad s}^{-1}$$
$$\tau=10 \text{ Nm}$$
$$\omega=60 \text{ rad s}^{-1}$$

As $\tau=I\alpha$

or
$$\alpha=\frac{\tau}{I}$$
$$=\frac{10}{2.5}$$
$$=4 \text{ rad s}^{-2}$$

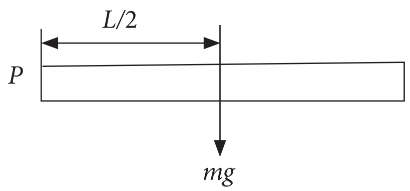
Using, $\omega=\omega_0+\alpha t$

$$t=\frac{\omega-\omega_0}{\alpha}$$

Substituting the values, we get

$$t=\frac{60-40}{4}$$
$$=5 \text{ s}$$

20 (a) From the figure,



Torque on the rod = Moment of weight of the rod about P

$$\tau=Mg\frac{L}{2} \quad \ldots\text{(i)}$$

$\therefore$ Moment of inertia of rod about

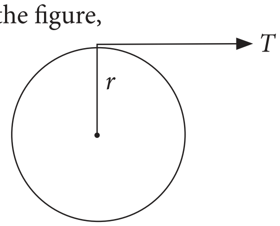
$$P=\frac{ML^2}{3} \quad \ldots\text{(ii)}$$

As, $\tau=l.\alpha$

From equations (i) and (ii), we get

$$\text{Mg}\frac{L}{2}=\frac{ML^2}{3}\alpha$$
$$\alpha=\frac{3g}{2L}$$

21 (d) From the figure,



$$Tr=l\alpha$$
$$T=\frac{l\alpha}{r}$$
$$=\frac{mr^2}{2}\times\frac{\alpha}{r}$$
$$=\frac{mr\alpha}{2}$$
$$=\frac{50\times0.5\times2\times2\pi}{2} \text{ N}$$
$$=157 \text{ N}$$

22 (b) As we know that,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$= 0 + \frac{1}{2} \times 2 \times 10^2$$
$$= 100 \text{ radian}$$

No. of revolutions $= \dfrac{\theta}{2\pi}$

$$= \frac{100}{2\pi}$$
$$\simeq 16$$

23 (b) Given that,

$$\omega = a - bt$$

At time,

$$t = 0, \quad \omega = \omega_0 = a$$
$$\alpha = \frac{d\omega}{dt}$$
$$= \frac{d}{dt}(a - bt)$$
$$= -b$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

If ω is zero,

$$\therefore \quad 0 = \omega_0^2 + 2\alpha\theta$$
$$\text{or} \quad \theta = -\frac{\omega_0^2}{2\alpha}$$
$$= \frac{a^2}{2b}$$

24 (c) The angular momentum of a disc of moment of inertia I_1 and rotating about its axis with angular velocity ω is $L_1 = I_1\omega$ when a round disc of moment of inertia I_2 is placed on first disc, then angular momentum of the combination is,

$$L_2 = \left(I_1 + I_2\right)\omega'$$

In the absence of any external torque, angular momentum remains conserved, i.e., $L_1 = L_2$

$$I_1\omega = \left(I_1 + I_2\right)\omega'$$
$$\Rightarrow \quad \omega' = \frac{I_1\omega}{I_1 + I_2}$$

25 (a) As we know that,

$$\omega = \omega_0 + \alpha t$$

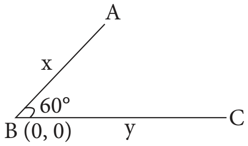
or

$$\omega = 0 + \alpha t$$
$$\text{or} \quad \alpha = \frac{\omega}{t}$$
$$= \frac{15}{0.270} \text{ rad } s^{-2}$$
$$\therefore \quad a = r\alpha$$
$$= 0.81 \times \frac{15}{0.270}$$
$$= 45 \text{ ms}^{-2}$$

26 (b) As we know that,

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$
$$= \frac{2 \times 2 + 4 \times 10}{2 + 4}$$
$$= 7.3 \text{ m/s}$$

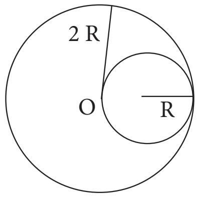
27 (c) From the figure,



Centre of mass, $x_{CM} = \dfrac{x}{2}\dfrac{(\rho x)\left(\dfrac{x}{2}\right)\dfrac{1}{2} + \rho y\left(\dfrac{y}{2}\right)}{\rho(x + y)}$

$$\Rightarrow \quad \frac{1}{2} + \frac{y}{x} = \frac{y^2}{x^2}$$
$$\therefore \quad \frac{BC}{AB} = \frac{y}{x}$$
$$= \frac{1 + \sqrt{3}}{2}$$
$$= 1.37$$

28 (b) Let the mass per unit area be α. Then the mass of the complete disc,



$$= \sigma\left[\pi(2R)^2\right]$$
$$= 4\pi\sigma R^2$$

The mass of the removed disc $= \sigma\left(\pi R^2\right)$

$$= \pi\sigma R^2$$

Let us consider the above situation to be a complete disc of radius $2R$ on which a disc of radius R of negative mass is superimposed. Let O be the origin. Then the above figure can be redrawn keeping in mind the concept of centre of mass as

$4\pi\sigma R^2 \xleftrightarrow{R}$

O $\quad \pi\sigma R^2$

$$X_{CM} = \frac{\left(4\pi\sigma R^2\right) \times 0 + \left(-\pi\sigma R^2\right)R}{4\pi\sigma R^2 - \pi\sigma R^2}$$
$$\therefore \quad X_{CM} = \frac{-\pi\sigma R^2 \times R}{3\pi\sigma R^2}$$
$$\therefore \quad X_{CM} = -\frac{R}{3}$$
$$\Rightarrow \quad \alpha = \frac{1}{3}$$

29 (b) The incorrect statement is $\vec{a} = \vec{v} \times \vec{r}$.

30 (a) As we know that,

For just complete rotation $v = \sqrt{Rg}$ at top point

The rotational speed of the drum,

$$\omega = \frac{v}{R} = \sqrt{\frac{g}{R}} = \sqrt{\frac{10}{1.25}}$$

The maximum rotational speed of the drum in revolutions per minute,

$$\omega\ (\text{rpm}) = \frac{60}{2\pi}\sqrt{\frac{10}{1.25}} = 27$$

31 (a) Angular velocity of each particle of a rotating rigid body about axis of rotation is same.

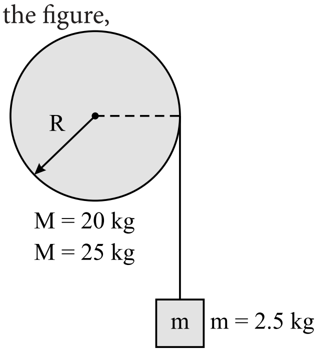
32 (b) As we know that,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 4 \times 10 \times 10$$

$$\Rightarrow \quad \theta = 200 \text{ rad}$$

$$\therefore \text{ Number of revolutions } = \frac{200}{2\pi} = 31 \text{ (approx.)}$$

33 (c) From the figure,



Moment of inertia of flywheel about its axis,

$$I = \frac{MR^2}{2} = \frac{(20\text{ kg})(25 \times 10^{-2}\text{ m})^2}{2} = 25 \times 25 \times 10^{-3}\text{ kgm}^2$$

Torque acting on the flywheel,

$$\tau = FR = mgR$$

$$= (2.5\text{ kg})(10\text{ m/s}^2)(25 \times 10^{-2}\text{ m}) = 25 \times 25 \times 10^{-2}\text{ Nm}$$

Angular acceleration of the flywheel, $\alpha = \frac{\tau}{I}$

$$= \frac{25 \times 25 \times 10^{-2}}{25 \times 25 \times 10^{-3}} = 10 \text{ rad s}^2$$

34 (c) Conservation of angular momentum gives,

$$\frac{1}{2}MR^2\omega_1 = \left(\frac{1}{2}MR^2 + 2mR^2\right)\omega_2$$

$$\frac{1}{2}MR^2\omega_1 = \frac{1}{2}R^2(M + 4m)\omega_2$$

$$\omega_2 = \left(\frac{M}{M + 4m}\right)\omega_1$$

35 (d) Given that,

$$M = 3 \text{ kg},$$

$$R = 40 \text{ cm} = 0.4 \text{ m}$$

Moment of inertia of the hollow cylinder about its axis is,

$$I = MR^2 = 3\text{ kg}\,(0.4\text{ m})^2 = 0.48\text{ kgm}^2$$

Force applied, $F = 30N$

$\therefore$ Torque, $\tau = F \times R = 30\text{ N} \times 0.4\text{ m} = 12\text{ Nm}$

Also, $\tau = I\alpha$

$$\alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25 \text{ rad } s^{-2}$$

36 (d) Since, the disc rotates with uniform angular velocity. So the angular acceleration $\propto \frac{dw}{dt}$ must be zero. So the option (d) is the only correct option.

37 (b) Given that,

Moment of inertia, $I = 3 \times 10^2\text{ kgm}^2$

Torque, $\tau = 6.9 \times 10^2\text{ Nm}$

Initial angular speed, $\omega_0 = 4.6 \text{ rad } \mathbf{s}^{-1}$

Final angular speed, $\omega = 0 \text{ rad } s^{-1}$

As $\omega = \omega_0 + \alpha t$

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$\therefore \quad = \frac{0-4.6}{t}$$

$$= -\frac{4.6}{t} \text{ rad } s^{-2}$$

Negative sign is used for retardation,

Torque, $\tau = I\alpha$

$$6.9\times10^2 = 3\times10^2 \times \frac{4.6}{t}$$

$$t = \frac{3\times10^2\times4.6}{6.9\times10^2}$$

$$= 2 \text{ s}$$

38 (a) Initial angular momentum of ring, $L = I\omega = Mr^2\omega$

Final angular momentum of ring and four particles system, $L = \left(Mr^2 + 4mr^2\right)\omega'$

As there is no torque on the system, therefore angular momentum remains constant,

$$Mr^2\omega = \left(Mr^2 + 4mr^2\right)\omega'$$

$$\Rightarrow \quad \omega' = \frac{M\omega}{M+4m}$$

39 (c) Area swept out per second = Area swept in one rotation × Number of rotations per unit time

$$= \pi r^2 v$$

40 (c) According to conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

$$\frac{1}{2}MR^2\omega = \left(\frac{1}{2}MR^2 + \frac{1}{2}\left(\frac{M}{4}\right)R^2\right)\omega_1$$

So, $\quad \omega_1 = \frac{4}{5}\omega$

41 (a) The moment of inertia of the uniform rod about an axis through one end and perpendicular to length is $\frac{ml^2}{3}$. Where m is mass of rod and l is its length.

Torque, $\tau = I\alpha$ acting on the centre of gravity of rod is given by

Put the values,

$$\tau = mg\frac{l}{2}$$

$$I\alpha = mg\frac{l}{2}$$

$$\frac{ml^2}{3}\alpha = mg\frac{l}{2}$$

$$\alpha = \frac{3g}{2l}$$

42 (c) Given that,

Angle turned in 3s, $\theta_{3s} = 2\pi\times10$

$$= 20\pi \text{ rad}$$

As we know that,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$20\pi = 0 + \frac{1}{2}\alpha\times(3)^2$$

$$\alpha = \frac{40\pi}{9} \text{ rad/s}^2$$

Now angle turned in 6s from the starting,

$$\theta_{6s} = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 0 + \frac{1}{2}\times\left(\frac{40\pi}{9}\right)\times(6)^2$$

$$= 80\pi \text{ rad}$$

$\therefore$ Angle turned between $t = 3s$ to $t = 6s$,

$$\theta_{\text{last } 3s} = \theta_{6s} - \theta_{3s}$$

$$= 80\pi - 20\pi$$

$$= 60\pi$$

Number of revolution $= \frac{60\pi}{2\pi}$

$= 30$ revolutions

43 (b) Given that, $F = (3i + 2j + 6k)$

$$r = 2i$$

As we know that, for a uniform rod

$$I = \frac{ML^2}{12}$$

$$= \frac{6\times4\times4}{12}$$

$$= 8 \text{ kgm}^{-2}$$

And, we also know that,

$$\tau = r\times F$$

$$= [2i\times(3i+2j+6k)]$$

$$= 4k - 12j$$

As the rod can rotate along Y-axis, so

$$\tau_y = I\alpha$$

$$-12j = 8\alpha$$

$$\alpha = -\frac{3}{2}j \text{ rad/s}^{-2}$$

44 (a) The centre of mass is at $L/2$ distance from the axis,

Hence centripetal force, $F_C = M\left(\frac{L}{2}\right)\omega^2$.

45 (a) Mass (m) = 2 kg

Initial radius of the path (r_1) = 0.8 m

Initial angular velocity (ω_1) = 44 rad/s

Final radius of the path $(r_2) = 1$ m

Initial moment of inertia, $I_1 = mr_1^2$

$$= 2 \times (0.8)^2$$
$$= 1.28 \ \text{kgm}^2$$

and final moment of inertia, $I_2 = mr_2^2$

$$= 2 \times (1)^2$$
$$= 2 \ \text{kgm}^2$$

From the law of conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

$$\Rightarrow \qquad \omega_2 = \frac{I_1\omega_1}{I_2}$$
$$= \frac{1.28 \times 44}{2}$$
$$= 28.16 \text{ rad/sec}$$

46 (d) As we know that,

Angular momentum $= I\omega$

where, $I =$ moment of inertia

$\omega =$ angular velocity

47 (c) Given that,

$$\omega = 3600 \text{ rpm}$$
$$\omega = 3600 \text{ rpm}$$
$$= \frac{3600}{60} \times 2\pi \ \text{rads}^{-1}$$
$$= 120\pi \ \text{rad s}^{-1}$$
$$P = 80 \text{ HP}$$
$$= 80 \times 746 \text{ W}$$
$$= 59680 \text{ Nms}^{-1}$$

As we know that,

$$P = \tau\omega$$

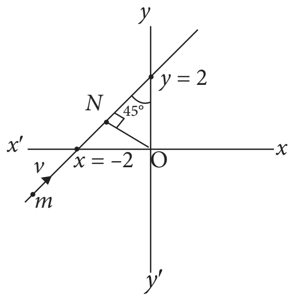
So, $\qquad \tau = \dfrac{P}{\omega}$

Put the value, we have

$$\tau = \frac{59680}{120\pi}$$

So, $\qquad \tau = 158.30611$ Nm

48 (b) The given equation of the line is $y = x + 2$.
At $x = -2$, $y = 0$ and at $x = 0$, $y = 2$
The line is drawn as shown in the figure.



Length of perpendicular drawn from origin to this line is,

$$ON = r_\perp$$
$$= \frac{0 + 0 + 2}{\sqrt{1^2 + 1^2}}$$
$$= \sqrt{2}$$

Angular momentum of particle about O is,

$$L = mvr_\perp$$
$$= (1)(2)\sqrt{2}$$
$$= 2\sqrt{2} \ \text{kgm}^2\text{s}^{-1}$$

49 (d) If p be the linear momentum of a particle and r is its position vector from the point of rotation, then angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$
$$= rp\sin\theta\,\hat{n}$$

Where $\hat{n}$ is a unit vector in the direction of rotation. Hence, angular momentum vector is perpendicular to orbital plane.

50 (b) According to the conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

$$\therefore \left[\frac{ML^2}{12} + 2mr^2\right]\omega_1 = \left[\frac{ML^2}{12} + 2m\left(\frac{L}{2}\right)^2\right]\omega_2$$

or $\quad \left[\dfrac{0.75 \times (0.4)^2}{12} + 2 \times 1 \times (0.1)^2\right] \times 30$

$$= \left[\frac{0.75 \times (0.4)^2}{12} + 2 \times 1 \times (0.2)^2\right] \times \omega_2$$

or $\quad \omega_2 = \dfrac{(0.01 + 0.02)30}{(0.01 + 0.08)}$

$$= \frac{0.03 \times 30}{0.09}$$
$$= 10 \text{ rad/sec}$$

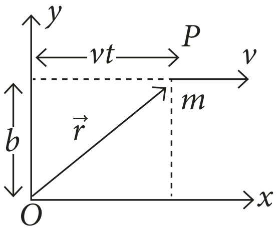
51 (c) Torque acting on disc is,

$$\tau = F_1 r + F_2 r$$
$$= 5 \times \frac{1}{2} + 5 \times \frac{1}{2}$$
$$= 5 \text{ Nm}$$

From angular impulse momentum theorem,

$$\int_{t_i}^{t_f} \tau dt = L_t - L_i$$
$$5 \times 5 = L_f - 0$$
$$L_f = 25 \text{ Nms}$$

52 (c) Suppose the particle of mass m is moving with speed v parallel to x-axis as shown in figure, then at any time t coordinates of P will be



$$x = vt,\ y = b \text{ and } z = 0$$

While components of velocity will be

$$v_x = v,\ v_y = 0 \text{ and } v_z = 0$$

(As it is moving parallel to x-axis)

So, $\vec{L} = \vec{r} \times \vec{p}$

$$= m\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ vt & b & 0 \\ v & 0 & 0 \end{vmatrix}$$

$$= \hat{k}m[vt \times 0 - vb]$$

$$= -mvb\hat{k}$$

53 (b) Taking the same mass of sphere, if radius is increased, then moment of inertia, rotational kinetic energy and angular velocity will change but according to law of conservation of momentum, angular momentum will not change.

54 (b) Given that,

radius vector, $\vec{r} = 2i + \hat{j} + \hat{k}$

Linear momentum, $\vec{p} = 2\hat{i} - 3\hat{j} + \hat{k}$

As angular momentum, $\vec{L} = \vec{r} \times \vec{p}$

$$\therefore \quad \vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= \hat{i}(1-(-3)) - \hat{j}(2-2) + \hat{k}(-6-2)$$

$$= 4\hat{i} - 8\hat{k}$$

55 (a) As we know that,

$L = mr^2\omega.$

For given m and ω,

$L \propto r^2,$

If r is halved the angular momentum

L becomes one-fourth.

56 (d) As we know that,

Rotational kinetic energy, $K_R = \frac{1}{2}I\omega^2$

As angular momentum, $L = I\omega$

Therefore $K_R = \frac{1}{2}\left(\frac{L}{\omega}\right)\omega^2$

$$= \frac{L\omega}{2}$$

or $L = \frac{2K_R}{\omega}$

$$= \frac{2K_R}{2\pi v}$$

$$= \frac{K_R}{\pi v}$$

When v is doubled and K_R is halved, angular momentum becomes

$$L' = \frac{\frac{1}{2}K_R}{\pi(2v)}$$

$$= \frac{1}{4}\frac{K_R}{\pi v}$$

$$= \frac{1}{4}L$$

57 (d) As we know that,

$$\vec{L} = \vec{r} \times \vec{p}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix}$$

$$= \hat{i}(-4+4) - \hat{j}(-2+3) + \hat{k}(4-6)$$

$$= 0\hat{i} - 1\hat{j} - 2\hat{k}$$

58 (c) From Kepler's law of planetary motion,

$$L = mvr$$

$$= m\sqrt{\frac{GM}{r}}r$$

$$L \propto \sqrt{r}$$

59 (d) As we know that,

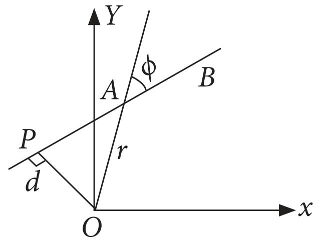
As $E = \frac{1}{2}I\omega^2$

$$= \frac{1}{2}\frac{I^2\omega^2}{I}$$

$$= \frac{L^2}{2I}$$

$$L = \sqrt{2EI}$$

60 (b) From the definition of angular momentum,



$$L = r \times p = mv\sin\phi(-k)$$

Therefore, the magnitude of L is,

$$L = mvr\sin\phi$$
$$= mvd$$

where, $d = r\sin\phi$ is the distance of closest approach of the particle to the origin. As d is same for both the particles.

Hence, $L_A = L_B$

61 (a) Torque and rate of change of angular momentum,

$$\text{or } \tau = \frac{d}{dt}(\text{constant } L)$$

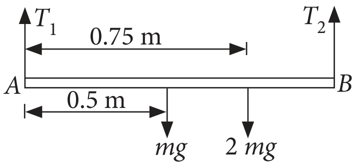
$$\text{or } \tau = 0$$

62 (b) Since, upward forces on the rod are equal to downward forces hence, the resultant force on the rod is zero and there is no linear motion. Moment of forces about one end of rod,

$$= -F \times 20 + F \times 40 - F \times 60 + F \times 80$$
$$= F \times 40$$

As it is not zero, hence the torque acts on the rod.

63 (a) From the figure,



$$T_1 + T_2 = 3\,mg$$

Taking torques about A, we get

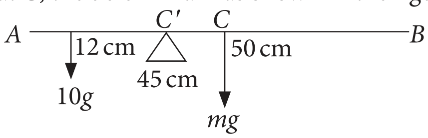
$$0.5\,mg + 0.75 \times 2\,mg = 1 \times T_2$$

Similarly, taking torque about B,

we get $\quad T_1 = mg$

$$\therefore \quad \frac{T_1}{T_2} = \frac{1}{2}$$

64 (b) Let m be the mass of the metre stick concentrated at C, the 50 cm mark as shown in the figure.



In equilibrium, taking moments of forces about C, we get

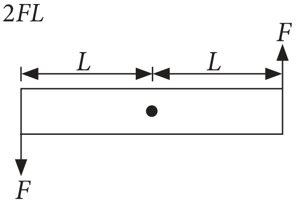
$$10\,g(45 - 12) = mg(50 - 45)$$
$$10\,g \times 33 = mg \times 5$$

$$m = \frac{10 \times 33}{5}$$
$$= 66\text{ g}$$

65 (a) By sign convention anticlockwise moment (or torque) is taken as positive while clockwise moment (or torque) is taken as negative.

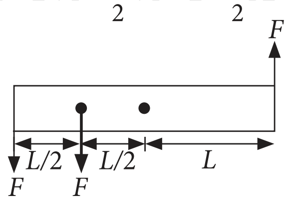
In case I, net torque about its centre is

$$\tau = F \times L + F \times L$$
$$= 2FL$$



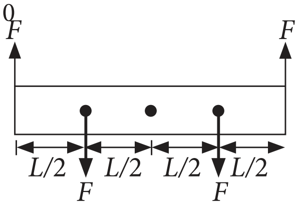
In case II, net torque about its centre is

$$\tau = F \times L + F \times \frac{L}{2} + F \times L = \frac{5}{2}FL$$



In case III, net torque about its centre is

$$\tau = -F \times L + F \times \frac{L}{2} - F \times \frac{L}{2} + F \times L$$
$$= 0$$

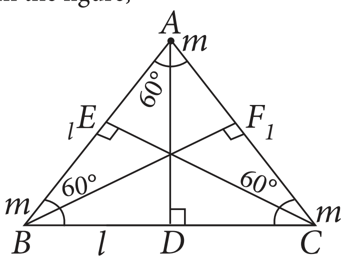


66 (c) Planar, perpendicular and perpendicular. The moment of inertia of a planar body about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

67 (d) Hence, net moment of inertia through its middle point O is.

$$I = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 + \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2$$
$$= \frac{1}{3}\left[\frac{ML^2}{8} + \frac{ML^2}{8}\right]$$
$$= \frac{ML^2}{12}$$

68 (c) From the figure,



Moment of inertia about BC is,

$$I_{BC} = m(0)^2 + m(0)^2 + m(AD)^2$$

$$AD = \sqrt{AB^2 - \left(\frac{BC}{2}\right)^2}$$

$$= \sqrt{l^2 - \frac{l^2}{4}}$$

$$= \frac{\sqrt{3}}{2}l$$

$$\therefore \quad I_{BC} = m\left(\frac{\sqrt{3}}{2}l\right)^2$$

$$= \frac{3}{4}ml^2$$

Similarly, we find the moment of inertia about AB is

$$I_{AB} = m(0)^2 + m(0)^2 + m(CE)^2$$

In right-angled ΔBEC,

$$\sin 60^\circ = \frac{CE}{BC}$$

$$= \frac{CE}{l}$$

or $\quad CE = l \sin 60^\circ$

or $\quad CE = \frac{\sqrt{3}}{2}l$

$$\therefore \quad I_{AB} = m\left(\frac{\sqrt{3}}{2}l\right)^2$$

$$= \frac{3}{4}ml^2$$

Moment of inertia along AC is

$$I_{AC} = m(0)^2 + m(0)^2 + m(BF)^2$$

In right angle ΔBFC

$$\sin 60^\circ = \frac{BF}{BC}$$

$$= \frac{BF}{l}$$

or $\quad BF = l \sin 60^\circ$

or $\quad BF = l\frac{\sqrt{3}}{2}$

$$\therefore \quad I_{AC} = m\left(\frac{\sqrt{3}}{2}l\right)^2$$

$$= \frac{3}{4}ml^2$$

69 (c) MI of uniform circular disc of radius R and mass M about an axis passing through CM and normal to the disc is

$$I_{CM} = \frac{1}{2}MR^2$$

From parallel axis theorem,

$$I_T = I_{COM} + MR^2$$

$$= \frac{1}{2}MR^2 + MR^2$$

$$= \frac{3}{2}MR^2$$

70 (d) Moment of inertia of a hollow cylinder of mass M and radius r about its own axis is MR^2.

71 (a) Given that,

$$\tau = 3.14 \text{ Nm}$$

$$\alpha = 4\pi \text{ rad/s}^2$$

As $\quad \tau = I\alpha$

$$\therefore \quad I = \frac{\tau}{\alpha}$$

$$= \frac{3.14}{4\pi}$$

$$= \frac{3.14}{4 \times 3.14}$$

$$= 0.25 \text{ kgm}^2$$

72 (a) As we know that,

Moment of inertia of a ring about its axis and perpendicular to $= MR^2$

Moment of inertia of disc about its axis and perpendicular to its plane $= \frac{1}{2}MR^2$

$= 0.5\ MR^2$

Moment of inertia of a solid sphere about one of its diameter $= \frac{2}{5}MR^2$

$= 0.4\ MR^2$

Moment of inertia of a spherical shell about one of its diameter $= \frac{2}{3}MR^2$

$= 0.66\ MR^2$

73 (d) The moment of inertia of a thin uniform rod of mass A and length l about an axis perpendicular to the rod through its centre is,

$$I = \frac{Ml^2}{12} \quad \text{...(i)}$$

The moment of inertia of the rod through its end point is

$$I' = \frac{Ml^2}{3} \qquad \text{[From (i)]}$$

$$= 4\left(\frac{Ml^2}{12}\right)$$

$$= 4I$$

74 (d) The correct relation between moment of inertia I, radius of gyration K and mass M of the body is

$$K = \sqrt{\frac{I}{M}}$$

75 (a) As we know that,

$$I = MK^2$$

$$= \frac{ML^2}{12} \qquad \left[\therefore K = \frac{L}{\sqrt{12}}\right]$$

76 (d) As we know that,

$$\pi r = l$$

$$\therefore \qquad r = \frac{l}{\pi}$$

Moment of inertia of a ring about its

$$\text{diameter } = \frac{1}{2} mR^2$$

$\therefore$ Moment of inertia of semicircle

$$= \frac{1}{2}\left[m\left(\frac{l}{\pi}\right)^2\right]$$

$$= \frac{ml^2}{2\pi^2}$$

77 (d) According to the condition,

$$\frac{M_A R^2}{M_B (2R)^2} = \frac{I_A}{I_B}$$

$$\frac{I_A}{I_B} = \frac{1}{4} \qquad [\because M_A = M_B]$$

$$\text{or,} \qquad I_A = \frac{I_B}{4}$$

78 (b) Because the entire mass of a ring is at its periphery at maximum distance from the centre and $I = MR^2$.

79 (d) As we know that,

Kinetic energy of

$$\text{rotation } = \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} \times \left(\frac{1}{2} MR^2\right)\omega^2$$

$$= \frac{1}{4} \times 20 \times (0.25)^2 \times 100 \times 100$$

$$= 3125 \text{ J}$$

80 (b) According to the principle of conservation of angular momentum, $I\omega =$ Constant

As I is doubled, ω becomes half.

Now, KE of rotation, $K = \frac{1}{2} I\omega^2$

Since I is doubled and ω is halved, KE will become half K/2.

81 (b) Rotational kinetic energy,

$$K_R = \frac{1}{2} I\omega^2$$

$$K_R = \frac{1}{2} \times \frac{MR^2}{2} \times \omega^2$$

$$= \frac{1}{4} mv^2 \left[\because \omega = \frac{v}{R}\right]$$

Translational kinetic energy, $K_T = \frac{1}{2} mv^2$

Total kinetic energy, $= K_T + K_R$

$$= \frac{1}{2} mv^2 + \frac{1}{4} mv^2$$

$$= \frac{3}{4} mv^2$$

$$\frac{\text{Rotational kinetic energy}}{\text{Total kinetic energy}} = \frac{\frac{1}{4} mv^2}{\frac{3}{4} mv^2}$$

$$= \frac{1}{3}$$

82 (b) Angular momentum remains constant because of the torque of tension is zero.

$$L_i = L_f$$

$$mv_0 R = mv\frac{R}{2}$$

$$v = 2 v_0$$

$$KE_F = \frac{1}{2} m(2v_0)^2$$

$$= 2mv_0^2$$

83 (b) Let f be the force of friction between the shell and horizontal surface.

For translational motion,

$$F + f = ma \qquad \ldots\text{(i)}$$

For rotational motion,

$$FR - fR = I\alpha$$

$$= I\frac{a}{R} \; [a = R\alpha \text{ for pure rolling}]$$

$$F - f = I\frac{a}{R^2} \qquad \ldots\text{(ii)}$$

On adding equations (i) and (ii), we get

$$2F = \left(m + \frac{I}{R^2}\right)a$$

$$= \left(m + \frac{2}{3}m\right)a \; \left[\therefore I_{\text{shell}} = \frac{2}{3}mR^2\right]$$

$$= \frac{5}{3} ma$$

$$F = \frac{5}{6} ma$$

$$a = \frac{6F}{5m}$$

84 (a) Because its MI (or value of K^2/R^2) is minimum for sphere.

85 (d) As we know that,

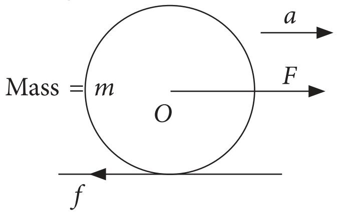
$$a = \frac{g \sin A}{1 + \frac{K^2}{R^2}}$$

$$= \frac{g\sin\theta}{1+\frac{2}{5}} \qquad \left[\text{As, } \theta = 30^\circ \text{ and } \frac{K^2}{R^2} = \frac{2}{5}\right]$$

$$= \frac{g/2}{7/5}$$

$$= \frac{5g}{14}$$

86 (c) From the figure,



$$ma = F - f \qquad \text{...(i)}$$

And, torque $\tau = I\alpha$

$$\Rightarrow \qquad \frac{mR^2}{2}\alpha = fR$$

$$\frac{mR^2}{2}\frac{a}{R} = fR \qquad \left[\because \alpha = \frac{a}{R}\right]$$

$$\Rightarrow \qquad \frac{ma}{2} = f \qquad \text{...(ii)}$$

Put this value in equation (i)

$$ma = F - \frac{ma}{2}$$

$$\text{or,} \qquad F = \frac{3ma}{2}$$

87 (d) As we know that,

$$\frac{KE_{\text{rot}}}{KE_{\text{trans}}} = \frac{\frac{1}{2}mv^2\left(\frac{K^2}{R^2}\right)}{\frac{1}{2}mv^2}$$

$$0.4 = \frac{K^2}{R^2}$$

$$\frac{K^2}{R^2} = \frac{2}{5}$$

$$\frac{K^2}{R^2} = \frac{2}{5}$$

$$K^2 = \frac{2}{5}R^2$$

Therefore, it is a solid sphere.

88 (b) As we know that,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v}{R}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$= \frac{3}{4}mv^2$$

$$\text{Rotational K.E.} = \frac{1}{4}mv^2$$

$$= \frac{1}{4}\times\frac{4}{3}mgh$$

$$= \frac{mgh}{3}$$

$$= 2\times 9.8\times(3/3)$$

$$= 19.6\text{ J}$$

89 (a) For disc rolling without slipping on inclined plane, acceleration

$$a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$$

$$= \frac{g\sin\theta}{1+\frac{(R/\sqrt{2})^2}{R^2}}$$

$$= \frac{2}{3}g\sin\theta$$

90 (b) As we know that,

$$\frac{\text{Rotational K.E.}}{\text{Total K.E.}} = \frac{\frac{1}{2}mv^2\left(\frac{K^2}{R^2}\right)}{\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)}$$

$$= \frac{K^2}{K^2+R^2}$$

91 (a) As we know that,

$$a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$$

$$= \frac{g\sin\theta}{1+\frac{2}{5}}$$

$$= \frac{5}{7}g\sin\theta$$

92 (c) After collision velocity of CM of A becomes zero and that of B becomes equal to initial velocity of CM of A. But angular velocity of A remains unchanged as the two spheres are smooth.

93 (a) As we know that,

$$\text{For solid sphere, } \frac{K^2}{R^2} = \frac{2}{5}$$

For disc and solid cylinder, $\dfrac{K^2}{R^2}=\dfrac{1}{2}$

Since acceleration of a body, which is rolling on an inclined plane at angle θ with horizontal is

$$a=\frac{g\sin\theta}{1+K^2/R^2} \qquad \text{...(i)}$$

It is clear from equation (i) that a solid sphere

$a_{\text{solid sphere}} > a_{\text{disc}} = a_{\text{solid cylinder}}$

Hence solid sphere takes least time in reaching the bottom of the inclined plane.

94 (b) By energy conservation,

$$(K.E.)_i+(P.E.)_i=(K.E.)_f+(P.E.)_f$$

$$(K.E.)_i=0,$$

$$(P.E.)_i=mgh,$$

$$(P.E.)_f=0$$

$$(K.E.)_f=1/2I\omega^2+1/2mv_{COM}^2$$

Where I (moment of inertia) = $1/2mR^2$ (for solid cylinder)

So, $$mgh=1/2\left(1/2mR^2\right)\left(\frac{v_{COM}^2}{R^2}\right)+1/2mv_{COM}^2$$

$$\Rightarrow \quad v_{COM}=\sqrt{4gh/3}$$

95 (c) Option c is correct-A-(i), (ii), (iv), B-(i), (ii), C-(i), (ii), (iii), D-(i), (ii)

96 (a) As we know that,

$$a_{\text{slipping}}=g\sin\theta$$

$$a_{\text{rolling}}=\frac{g\sin\theta}{1+\dfrac{K^2}{R^2}}$$

$$=\frac{5}{7}g\sin\theta$$

$$\frac{a_{\text{rolling}}}{a_{\text{slipping}}}=\frac{5}{7}$$

97 (d) The object is disc.

98 (a) Just before collision between two balls,

Potential energy lost by A = kinetic energy gained by ball A

$$mg\frac{h}{2}=\frac{1}{2}I_{CM}\omega^2+\frac{1}{2}mv_{CM}^2$$

$$=\frac{1}{2}\times\frac{2}{5}mR^2\times\left(\frac{v_{CM}}{R}\right)^2+\frac{1}{2}mv_{CM}^2+$$

$$\frac{1}{2}mv_{CM}^2$$

$$\Rightarrow \frac{5}{7}mgh=mv_{CM}^2$$

$$\Rightarrow \frac{mgh}{7}=\frac{1}{5}mv_{CM}^2$$

After collision, only translational kinetic energy is transferred to ball B.

So just after collision, rotational kinetic energy of ball,

$$A=\frac{1}{5}mv_{CM}^2$$

$$=\frac{mgh}{7}$$

100 (d) As we know that,

$$a=\frac{mg\sin\theta}{m+\dfrac{I}{r^2}}$$

For cylinder, $$a_c=\frac{M_cg\sin\theta_c}{M_c+\dfrac{1}{2}\dfrac{M_cR^2}{R^2}}$$

$$=\frac{M_cg\sin\theta_c}{M_C+\dfrac{M_cR^2}{2R^2}}$$

or, $$a_c=\frac{2}{3}g\sin\theta_c$$

For sphere,

$$a_s=\frac{M_sg\sin\theta_s}{M_s+\dfrac{I_s}{r^2}}$$

$$=\frac{M_sg\sin\theta_s}{M_s+\dfrac{2}{5}\dfrac{MsR^2}{R^2}}$$

or, $$a_s=\frac{5}{7}g\sin\theta_s \quad [\text{Given } a_c=a_s]$$

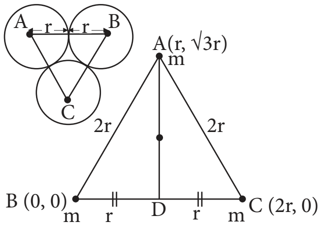
i.e., $$\frac{2}{3}g\sin\theta_c=\frac{5}{7}g\sin\theta_s$$

$$\frac{\sin\theta_c}{\sin\theta_s}=\frac{\frac{5}{7}g}{\frac{2}{3}g}$$

$$=\frac{15}{14}$$

High-Order Thinking Skill

1 (c) From the figure,



In ΔABD,

$$AD = \frac{\sqrt{3}}{2} \times AB$$

$$= \frac{\sqrt{3}}{2} \times 2r$$

$$\therefore \quad AD = \sqrt{3}r$$

$\therefore$ Position of centre of mass,

$$x = \frac{m\times 0 + m\times 2r + m\times r}{3m}$$

$$= \frac{3mr}{3m}$$

$$= r$$

A

and,

$$y = \frac{m\times 0 + m\times 0 + m\times \sqrt{3}r}{3m}$$

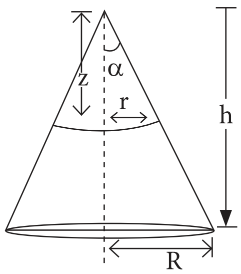
$$= \frac{\sqrt{3}mr}{3m}$$

$$= \frac{1}{\sqrt{3}}r$$

$$\approx 0.6r$$

Co-ordinates of centre of mass = $(r, 0.6r)$ These are also co-ordinates of point of interaction of medians.

2 (a) As we know that,



$$dm = \rho\pi r^2 dz$$

from figure,

$$\tan\alpha = \frac{r}{z} = \frac{R}{h}$$

$$\therefore \quad r = \frac{R}{h}z$$

Now, $z_{CM} = \dfrac{\int z\,dm}{\int dm} = \dfrac{\int_0^h \rho\pi r^2 z\,dz}{\frac{1}{3}\pi R^2 h\rho}$

$$= \frac{3}{R^2 h}\int_0^h \left(\frac{R}{h}z\right)^2 z\,dz$$

$$= \frac{3}{hR^2}\left(\frac{R^2}{h^2}\right)\int_0^h z^3\,dz$$

$$= \frac{3}{h^3}\left[\frac{z^4}{4}\right]_0^h = \frac{3h}{4}$$

So that, the distance of centre of mass from base is,

$$h - \frac{3h}{4} = \frac{h}{4}$$

Centre of mass has co-ordinates $\left(0, 0, \frac{h}{4}\right)$

3 (a) As we know that,

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix}$$

$$= \hat{i}(6-8) - \hat{j}(-3) + 4\hat{k}$$

$$= -2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\therefore |\vec{v}| = \sqrt{(-2)^2 + (3)^2 + 4^2}$$

$$= \sqrt{29} \text{ units}$$

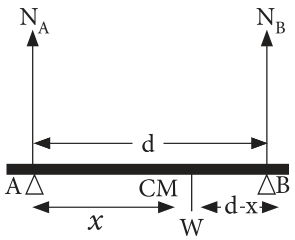
4 (c) Angular momentum of the system is conserved,

$$\therefore \quad \frac{1}{2}MR^2\omega_1 = 2mR^2\omega + \frac{1}{2}MR^2\omega$$

or $\quad M\omega_1 = (4m + M)\omega$

or, $\quad \omega = \dfrac{M\omega_1}{M + 4m}$

5 (d) From the figure,



$$N_A + N_B = W$$

Torque balances at CM of rod

$$N_A\,x = N_B(d - x)$$

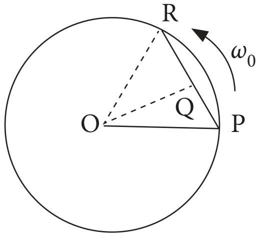
$$\therefore \quad N_A x = (W - N_A)(d - x)$$

$$\therefore \quad N_A x = Wd - Wx - N_A d + N_A x$$

$$\therefore \quad N_A d = W(d - x)$$

$$\therefore \quad N_A = \frac{W(d - x)}{d}$$

6 (b) Let the tortoise be initially sitting at the point P. Its distance from the centre = OP. Let x be the distance at any instant t, when the tortoise is moving along the chord PR.



By conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

$$\therefore\ m(\text{OP})^2\omega_0 = mx^2\omega$$

$$\therefore\quad \omega = \frac{(\text{OP})^2\omega_0}{x^2}$$

$$\therefore\quad \omega \propto \frac{1}{x^2}$$

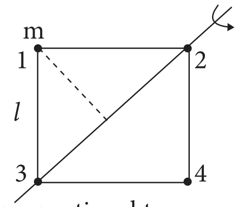
Variation of ω with x is non-linear. Now, x decreases, takes a minimum value and again increases to maximum value. Thus, ω first increases take a maximum value at Q and again falls to original value at R option (b) is correct.

7 (b) **Stable equilibrium:** The equilibrium of a body is said to stable if, on being slightly disturbed, it tends to come back to its original position.

Unstable equilibrium: The body is said to be in unstable equilibrium if on being slightly disturbed, it shows no tendency to come back to its original position and moves away from it.

Neutral equilibrium: A body is said to be in neutral equilibrium if on being slightly displaced, it remains in the new position.

8 (a) From the figure,



I is always proportional to mass and radius of the body. Here, two bodies having same shape and same radius are given. I is directly proportional to mass of the body.

$$I \propto M$$

and, $\quad I \propto \rho V$

But $\quad V =$ constant

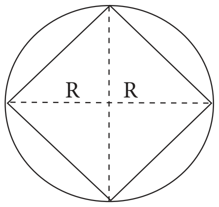
$$\therefore\quad I \propto \rho$$

$$\therefore\quad \frac{I_1}{I_2} = \frac{\rho_1}{\rho_2} = \frac{\rho_{AL}}{\rho_{\text{iron}}}$$

But $\quad \rho_{\text{Al}} < \rho_{\text{iron}}$

$$\therefore\quad I_1 < I_2$$

9 (c) From the figure,



$$R = \frac{\sqrt{3}}{2}a$$

$$\Rightarrow\quad a = \frac{2}{\sqrt{3}}R$$

Let M and M' be mass of sphere and cube respectively

$$\therefore\quad \frac{M}{M'} = \frac{\left(\frac{4}{3}\pi R^3\right)\rho}{\left(\frac{2}{\sqrt{3}}R\right)^3\rho} = \frac{\sqrt{3}}{2}\pi$$

$$\Rightarrow\quad M' = \frac{2M}{\sqrt{3}\pi}$$

$$I = \frac{M'a^2}{6}$$

$$= \frac{2\text{M}}{\sqrt{3}\pi}\times\frac{4}{3}\text{R}^2\times\frac{1}{6}$$

$$I = \frac{4\text{MR}^2}{9\sqrt{3}\pi}$$

10 (d) As we know that,

Mass of disc $X = (\pi R^2 t)\sigma$, where $\sigma =$ density

$$\therefore\quad I_X = \frac{MR^2}{2} = \frac{(\pi R^2 t\sigma)R^2}{2} = \frac{\pi R^4\sigma t}{2}$$

Similarly, $I_Y = \dfrac{(\text{Mass})(4R)^2}{2} = \dfrac{\pi(4R)^2}{2}\dfrac{t}{4}\sigma\times 16R^2$

or, $\quad I_Y = 32\pi R^4 t\sigma$

$$\frac{I_X}{I_Y} = \frac{\pi R^4\sigma t}{2}\times\frac{1}{32\pi R^4\sigma t} = \frac{1}{64}$$

$$\therefore\quad I_Y = 64 I_X$$

11 (a) Angular momentum,

$$L = I\omega$$

Rotational kinetic energy, $KE = \dfrac{1}{2}I\omega^2$

$$\therefore\quad \frac{L}{KE} = \frac{I\omega\times 2}{I\omega^2}$$

$$= \frac{2}{\omega}$$

$$L = \frac{2KE}{\omega}$$

or $\quad \dfrac{L_1}{L_2} = \dfrac{KE_1}{KE_2}\times\dfrac{\omega_2}{\omega_1}$

$$\Rightarrow\quad 2\times 2 = 4$$

$$\therefore \quad L_2 = \frac{L_1}{4} = \frac{L}{4}$$

12 (c) As the body rolls down the inclined plane, it loses potential energy. However, in rolling, it acquires both linear and angular speeds and hence, gain in kinetic energy of translation and that of rotation. So, by conservation of mechanical energy,

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

But as in rolling, $v = R\omega$

$$\therefore \quad Mgh = \frac{1}{2}Mv^2\left[1 + \frac{I}{MR^2}\right]$$

$$\because \quad 1 + \frac{I}{MR^2} = \beta$$

$$\therefore \quad Mgh = \frac{1}{2}\beta Mv^2$$

$$\therefore \quad v = \sqrt{\frac{2gh}{\beta}}$$

NCERT Exemplar Problems

1 (d) Out of the four given bodies, the centre of mass of a bangle lies outside it, other three bodies it lies within the body. A bangle is in the form of a ring. Position of centre of mass depends upon the shape of the body and distribution of mass. So, out of four given bodies, the centre of mass lies at the centre, which is outside the body (boundary) whereas in all other three bodies it lies within the body because they are completely solid.

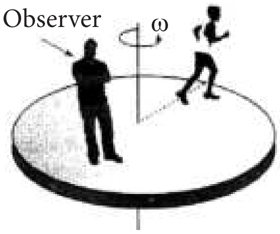
2 (c) The position of centre of mass of the system in this problem is closer to heavier mass or masses or it depends upon distribution of mass. So it is likely to be at *C*. In the above diagram, lower part of the sphere containing sand is heavier than upper part containing air. Hence CM of the system lies below the horizontal diameter.

3 (d) When the disc is rotated with constant angular velocity, angular acceleration of the disc is zero. Because we know that angular acceleration

$\alpha = \Delta\omega/\Delta t$

Here ω is constant, so $\Delta\omega = 0$.

4 (b) As no torque is exerted by the person jumping, radially away from the centre of the round (as seen from the round), let the total moment of inertia of the system is $2I$ (round + Person (because the total mass is $2M$) and the round is revolving with angular speed ω. Since the angular momentum of the person when it jumps off the round is $I\omega$ the actual momentum of round seen from ground is $2\,I\omega - I\omega = I\omega$. So, we conclude that the angular speed remains same, i.e. ω.



5 (b) As we know that,

The initial velocity is, $\vec{v}_i = v\hat{e}_y$

After reflection from the wall, the final velocity is,

$$\vec{v}_f = -v\hat{e}_y$$

The trajectory is given as $\vec{r} = y\hat{e}_y + a\hat{e}_z$

Hence, the change in angular momentum is,

$$\Delta\vec{L} = \vec{r} \times m\left(\vec{v}_f - \vec{v}_i\right)$$

$$= \left(y\hat{e}_y + a\hat{e}_z\right) \times \left(-mv\hat{e}_y - mv\hat{e}_y\right)$$

$$= \left(y\hat{e}_y + a\hat{e}_z\right) \times \left(-2mv\hat{e}_y\right)$$

$$= 2mva\hat{e}_x \quad [\hat{\beta}_y \times \hat{e}_y = 0 \text{ and } \hat{e}_z \times \hat{e}_y = -\hat{e}_x]$$

6 (b) According to the theorem of perpendicular axes,

$$I_z = I_x + I_y$$

With the hole, I_x and I_y both decreases. Gluing the removed piece at the centre of square plate does not affect I. So that, I_z decreases overall.

Assertion and Reasons

1 (a) One useful application of the centre of mass is determining the maximum angle that an object can be tilted before it will topple over.

2 (c) As we know that the centre of mass of the object depends upon the consistent masses and its relative distribution. So that the reference frame will give the same position for the centre of mass.

3 (c) Centre of mass depends upon the distribution of mass of the body.

4 (a) If the direction of rotation of the body changes (clockwise or anticlockwise), so the direction of angular velocity also changes.

5 (a) There is no external torque about the axis of rotation of the chair, if friction in the rotational mechanism is neglected, and as we know that $I\omega$ is constant, where I is moment of inertia and ω is angular velocity.

As he stretch, his arms the moment of inertia I increases about the rotation, which results in decreasing the angular speed to conserve the angular momentum.

6 (a) A ladder is more apt to slip, as a body high up on it than when body just begin to climb and at the high up on a ladder, the torque is large and on climbing up the torque is small.

7 (b) As we know that,

$L = mvr\sin\theta$ or mvd,

where d is the perpendicular distance.

In case of constant velocity m, v, and d all are constants. From Bohr's theory

$L = \frac{nh}{2\pi}$, therefore L and h have same units.

8 (c) Angular momentum about bottommost point,

$$L = mv_0R - \frac{1}{2}mR^2\omega_0$$

If $L = 0$

or $v_0 = \frac{\omega_0 R}{2}$

Then disc will come to rest after sometime.

9 (a) When a body rotates about an axis under the action of an external torque τ, then the rate of change of angular momentum of the body is equal to the torque, that is $\frac{dL}{dt} = \tau$.

If external torque is zero $(\tau = 0)$, then

$$\frac{dL}{dt} = 0$$

$$dL = 0$$

$$L = \text{constant}$$

The central force field, torque

$$\tau = F \times r = 0$$

So, L remains conserved in this case.

10 (a) When a person stretches his arms, his moment of inertia $(= mR^2)$ increases.

Since, no external torque acts on the system,

$$\tau = 0$$

$\therefore$ $I\omega =$ constant

Therefore, as I increases, ω will decrease and the platform slows down.

11 (a) For the body to be in translatory equilibrium vector sum of all external forces acting on body is zero. For body to be in rotatory equilibrium, vector sum of all external torques acting on body is zero.

12 (a) Moon effects tides, hence sporadic changes of speed of rotation of Earth happens. From law of conservation of angular momentum, when no external torque is acting upon body rotating about axis, then angular momentum remains constant.

$$L = I\omega = \text{constant}$$

When large air passes in the Earth's atmosphere they cause a change in moment of inertia, and since angular momentum is to be maintained constant, the angular velocity or speed of rotation changes.

13 (a) Radius of gyration of a body about a given axis is

$$K = \frac{\sqrt{r_1^2 + r_2^2 + \ldots\ldots r_n^2}}{n}$$

It thus depends upon shape and size of body, position and configuration of axis of rotation and distribution of mass of body wrt axis of rotation.

14 (c) In whirlwind in tornado, nearby air gets concentrated in small space decreasing its moment of inertia. Since, $I\omega = \text{constant}$, so due to decrease in moment of inertia of air, its angular speed increases to high value.

If no external torque acts, then $\tau = 0$

$$\frac{dL}{dt} = 0$$

$$L = \text{constant}$$

$$I\omega = \text{constant}$$

As in the rotational motion, the moment of inertia of the body can change due to the change in position of the axis of rotation, the angular speed may not remain conserved.

15 (a) It has both till it reaches the lowest level, where it has zero potential energy. The total mechanical energy remains constant during the motion downwards.

16 (b) In rotatory motion, torque is analogous to force in translatory motion.

In rotatory motion,

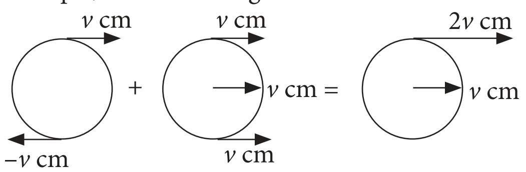
$$P = \frac{\text{Work done}}{\text{Time}}$$

$$= \frac{\tau\theta}{t} = \tau \times \omega$$

In translatory motion, $P = \frac{W}{t} = F \times v$

17 (b) A sphere cannot roll on a smooth inclined plane, because force of friction is zero for smooth surface and in case of sphere friction provides the torque for rolling.

18 (a) A motion that is a combination of rotational and translational motion is called rolling motion. For example, a wheel rolling downs the road.



19 (a) The velocity of a body at the bottom of an inclined plane of given height is more when it slides down the plane compared to when it rolling down the same plane and in rolling down a body acquires both kinetic energy of translation and rotation.

20 (b) When an object experiences pure translational motion, all points move with same velocity as the centre of mass; in the same direction and with the same speed $v(r) = v_{\text{centre of mass}}$ and object will also move in straight line in absence of net external force.

When an object experiences pure rotational motion about its centre of mass, all points move at right angle to the radius in a plane which is perpendicular to the axis of rotation with speed proportional to the distance from the axis of rotation.