Level-I

Chapter 11

Straight Lines

Solutions (Set-1)

Very Short Answer Type Questions :

1. Find the inclination of the line $x - y + 3 = 0$ with the positive direction of x-axis.

Sol. $x - y + 3 = 0$

$\Rightarrow \quad y = x + 3$

Comparing with $y = mx + c$

$\therefore \quad \tan\theta = m = 1$

$\Rightarrow \quad \theta = 45°$

2. Find the angle between the lines joining the points (3, –1) & (2, 3) & the points (5, 2) and (9, 3).

Sol. Let the slope of the line through (3, –1) & (2, 3) is m_1.

$$m_1 = \frac{3-(-1)}{2-3} = -4$$

Let the slope of the line through (5, 2) and (9, 3) is m_2.

$$m_2 = \frac{3-2}{9-5} = \frac{1}{4}$$

Now, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left|\frac{-4-\frac{1}{4}}{1+(-4)\left(\frac{1}{4}\right)}\right|$$

$$= \left|\frac{\frac{-17}{4}}{0}\right|$$

$\Rightarrow \quad \theta = 90°$

3. Show that the line $5x - 2y - 1 = 0$ is the mid-parallel to the line $5x - 2y - 9 = 0$ and $5x - 2y + 7 = 0$.

Sol. Slope of all three lines is $\frac{5}{2}$.

$\therefore$ All lines are parallel

$\Rightarrow$ $5x - 2y - 1 = 0$

If $x = 1,\ -y = 2$

Now, distance of (1, 2) from $5x - 2y - 9 = 0$

$$= \left|\frac{5(1)-2(2)-9}{\sqrt{5^2+2^2}}\right| = \frac{8}{\sqrt{29}}$$

Also, distance of (1, 2) from $5x - 2y + 7 = 0$

$$= \left|\frac{5(1)-2(2)+7}{\sqrt{5^2+2^2}}\right| = \frac{8}{\sqrt{29}}$$

Since the perpendicular distance of point (1, 2) from both the lines is equal, thus it is mid parallel between the two lines.

4. If the lines $2x + y - 3 = 0$, $5x + ky - 3 = 0$ and $3x - y - 2 = 0$ are concurrent, find the value of k.

Sol. Here given lines are

$2x + y - 3 = 0$ ...(i)

$5x + ky - 3 = 0$ ...(ii)

$3x - y - 2 = 0$ ...(iii)

Solving (i) and (iii), by cross multiplication method, we get

$$\frac{x}{-2-3} = \frac{y}{-9+4} = \frac{1}{-2-3} \text{ or, } x = 1,\ y = 1$$

Therefore, the point of intersection of two lines is (1, 1), Since above three lines are concurrent, the point (1, 1) will satisfy equation (i)

i.e., $5 \cdot 1 + k \cdot 1 - 3 = 0$ or $k = -2$.

5. Name the figure formed by the lines $ax \pm by \pm c = 0$.

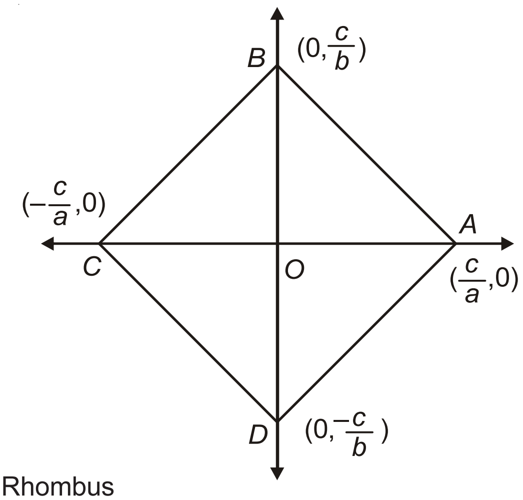
Sol. Equations are

$$ax + by + c = 0 \Rightarrow \left(\frac{-c}{a}, 0\right) \& \left(0, \frac{-c}{b}\right)$$

$$ax - by + c = 0 \Rightarrow \left(\frac{-c}{a}, 0\right) \& \left(0, \frac{c}{b}\right)$$

$$ax + by - c = 0 \Rightarrow \left(\frac{c}{a}, 0\right) \& \left(0, \frac{c}{b}\right)$$

$$ax - by - c = 0 \Rightarrow \left(\frac{c}{a}, 0\right) \& \left(0, \frac{-c}{b}\right)$$



Since $AB = BC = CD = AD$ and $AC \perp BD$, it is a Rhombus

6. Find the equation of line passing through origin and equally inclined to positive y-axis and negative x-axis.

Sol. $\theta = 90° + 45° \Rightarrow \tan\theta = -\cot 45° = -1$

$\therefore$ Equation $\Rightarrow$ $y - 0 = -1\,(x - 0)$

$x + y = 0$

7. Find equation of line which is equidistant and parallel to both $x = 2$ and $x = -4$.

Sol. $x = -1$

8. Find the angle between the following lines.

$(a^2 - ab)\, y = (ab + b^2)\, x + b^3$ and $(ab + b^2)\, y = (ab - b^2)\, x + a^3$.

Sol. Ans. 45° [**Hint**- use: $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$]

9. Find the angle between the lines $x = a$ and $by + c = 0$.

Sol. $x = a$ is a line parallel to y-axis and $y = \dfrac{-c}{b}$ is a line parallel to x-axis. Thus, angle is 90°

Short Answer Type Questions :

10. Show that the points $(a, 0)$, $(0, b)$ and $(3a, -2b)$ are collinear. Also, find the equation of the line containing them.

Sol. Equation of line through $(a, 0)$ & $(0, b)$ is

$$y - 0 = \frac{b - 0}{0 - a}(x - a)$$

$\Rightarrow$ $-ay = bx - ab$.

$\Rightarrow$ $bx + ay - ab = 0$

Clearly $(3a, -2b)$ satisfies $b(3a) + a(-2b) - ab = 0$

Thus, all three points are collinear.

And the equation containing them is $bx + ay - ab = 0$

11. Through the point $P(3, -5)$, a line is drawn inclined at 45° with the positive direction of x-axis. It meets the line $x + y - 6 = 0$ at the point Q. Find the length of PQ.

Sol. Equation of line through $P(3, -5)$ and slope = $\tan\theta = \tan 45° = 1$ is

$y - (-5) = 1(x - 3)$

$x - y - 8 = 0$

Now, $x - y - 8 = 0$ and $x + y - 6 = 0$ intersect at $(7, -4)$

Now, distance between $P(3,-5)$ and $Q(7,-1)$ is

$$\sqrt{(7-3)^2 + (-1+5)^2}$$

$$= \sqrt{4^2 + 4^2}$$

$$= 4\sqrt{2}$$

12. If the points (a, 0), (0, b) and (3, 4) are collinear, then show that $\frac{3}{a}+\frac{4}{b}=1$.

Sol. $A(a, 0)$, $B(0, b)$, $C(3, 4)$

Slope of $AB = \frac{b-0}{0-a}=\frac{-b}{a}$

Slope of $BC = \frac{4-b}{3-0}=\frac{4-b}{3}$

Since A,B and C are collinear,

$\therefore$ Slope of AB = Slope of BC

$\frac{-b}{a} = \frac{4-b}{3}$

$\Rightarrow -3b = 4a - ab$

$\Rightarrow 4a + 3b = ab$

$\Rightarrow \frac{4a}{ab}+\frac{3b}{ab} = 1$

$\Rightarrow \frac{4}{b}+\frac{3}{a} = 1$ Hence proved.

13. A line cuts off intercepts of – 3 and 4 on x and y-axis respectively. Find the slope of the line.

Sol. $\frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow \frac{x}{-3}+\frac{y}{4}=1$

$\Rightarrow -4x + 3y = 12 \Rightarrow 3y = 4x + 12$

$\Rightarrow y=\frac{4}{3}x+12$

$\therefore$ Slope $= \frac{4}{3}$

14. A line passes through point (1, 5) and cuts off intercept 7 units on x-axis. Find the slope of the line.

Sol. Line cuts off intercept of 7 units

$\Rightarrow$ Point (7, 0) lies on the line

$\therefore$ Slope $= \frac{0-5}{7-1}=\frac{-5}{6}$

15. Find the equation of the line which passes through the point (– 4, 3) and the portion of the line intercepted between the axes is divided internally in the ratio 5 : 3 by this point.

Sol. (– 4,3) divides the line segment AB in the ratio 5:3.

$\therefore -4 = \frac{5\times 0+3(-a)}{5+3}$

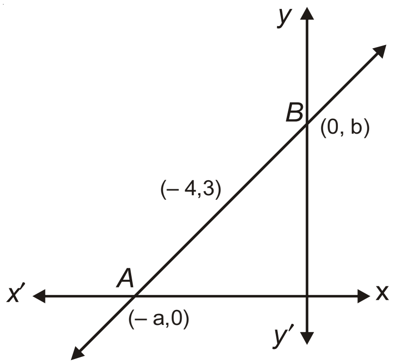
$\Rightarrow \quad -4 \times 8 = -3a$

$\Rightarrow \quad a = \dfrac{32}{3}$

$3 = \dfrac{5(b) + 3(0)}{5+3}$

$\Rightarrow \quad 5b = 8 \times 3$

$\Rightarrow \quad b = \dfrac{24}{5}$



Now, the points are $\left(\dfrac{-32}{3}, 0\right)$ and $\left(0, \dfrac{24}{5}\right)$

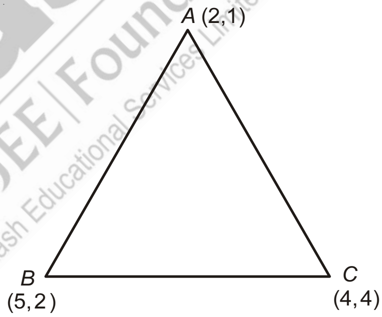
$\therefore$ Equation of the line is

$$\left(y - \frac{24}{5}\right) = \left(\frac{0 - \frac{24}{5}}{\frac{-32}{3} - 0}\right)(x - 0)$$

$\Rightarrow \quad 18x - 40y + 192 = 0$

16. Find the lengths of the perpendicular segments drawn from the vertex of the triangle with vertices $A(2,1)$, $B(5, 2)$ and $C(4, 4)$.

Sol. Equation of AC



$$y - 4 = \left(\frac{1-4}{2-4}\right)(x - 4)$$

$\Rightarrow \quad 3x - 2y - 4 = 0$

Now, perpendicular distance of B from AC is

$$\left|\frac{3(5) - 2(2) - 4}{\sqrt{3^2 + 2^2}}\right| = \left|\frac{15 - 4 - 4}{\sqrt{13}}\right| = \frac{7}{\sqrt{13}}$$

17. Find the ratio in which the line $3x + 4y + 2 = 0$ divides the distance between the lines $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$.

Sol. Distance between $3x + 4y + 2 = 0$ and $3x + 4y + 5 = 0$ is

$$\left|\frac{5-2}{\sqrt{3^2 + 4^2}}\right| = \frac{3}{5}$$

Distance between $3x + 4y + 2 = 0$ and $3x + 4y - 5 = 0$ is

$$\left|\frac{2-(-5)}{\sqrt{3^2 + 4^2}}\right| = \frac{7}{5}$$

$\therefore$ Required ratio $= \dfrac{3}{5} : \dfrac{7}{5}$

$= 3 : 7$

18. Find the coordinates of the points which are at a distance of $4\sqrt{2}$ from the point (3, 4) and which lie on the line passing through the point (3, 4) and inclined at an angle of 45° with positive x-axis.

Sol. Let the point be (a, b)

$\therefore \quad \sqrt{(a-3)^2+(b-4)^2} = 4\sqrt{2}$

$\Rightarrow \quad (a-3)^2 + (b-4)^2 = 32$ ...(i)

Now, equation of line with slope tan 45° = 1 & passing through (3, 4) is

$y - 4 = 1(x - 3)$

$x - y + 1 = 0$ ...(ii)

since (a, b) lies on (ii)

$\therefore \quad a - b + 1 = 0$

$b = a + 1$

Putting value of b in (i)

$(a-3)^2 + (a + 1 - 4)^2 = 32$

$\Rightarrow \quad 2(a-3)^2 = 32$

$\Rightarrow \quad (a-3)^2 = 16$

$\Rightarrow \quad a - 3 = \pm 4$

$\Rightarrow \quad a - 3 = 4$	$\Rightarrow \quad a - 3 = -4$
$\Rightarrow \quad a = 7$	$\Rightarrow \quad a = -1$
$\Rightarrow \quad b = 8$	$\Rightarrow \quad b = 0$

$\therefore$ The required points are (7, 8) or (– 1, 0)

19. If the slope of a line passing through point $P(3, 2)$ is $\frac{3}{4}$, then find the points on the line which are 5 units away from point P.

Sol. Equation of line passing through (3, 2) having slope $\frac{3}{4}$ is

$$y - 2 = \frac{3}{4}(x - 3)$$

$\Rightarrow \quad 4y - 3x + 1 = 0$

Let (a, b) be the points such that $(a, -3)^2 + (b-2)^2 = 25$ ...(i)

Since (a, b) lies on $4y - 3x + 1 = 0$, then $4b - 3a + 1 = 0$

$\Rightarrow \quad b = \dfrac{3a-1}{4}$ ...(ii)

Solving (i) & (ii), we get

$b = -1, 7$

$\therefore \quad a = -1, 5$

$\therefore$ Points are (–1, –1) and (7, 5)

20. Two lines cut x-axis at distance of 4 units and -4 units and y-axis at distance of 2 units and 6 units respectively. Find the coordinates of their point of intersection.

Sol. $a_1 = 4, \quad b_1 = 2$

$\therefore$ Equation is

$$\frac{x}{4}+\frac{y}{2}=1$$

$x + 2y = 4$ ...(i)

$a_2 = -4, \quad b_2 = 6$

Equation is

$$\frac{x}{-4}+\frac{y}{6}=1$$

$-3x + 2y = 12$ ...(ii)

Multiply (i) by 3

$$3x + 6y = 12$$

$$\underline{-3x+2y=12}$$

$$8y = 24$$

$$y = 3$$

$$x = 4 - 2 \times 3$$

$$= -2$$

$\therefore$ Point of intersection is $(-2, 3)$.

21. Find the equation of a straight line on which length of perpendicular from the origin is 4 units and the line makes an angle of 120° with positive x-axis.

Sol. $\sin 120° = \sin (90 + 30°)$

$$= \cos 30° = \frac{\sqrt{3}}{2}$$

$\cos 120° = \cos(90°+30°)$

$$= -\sin 30°$$

$$= -\frac{1}{2}$$

$\therefore$ The equation is $\cos 120°\, x + \sin 120°\, y = 4$

$$-\frac{x}{2}+\frac{\sqrt{3}}{2}y=4$$

$$\Rightarrow \quad x-\sqrt{3}y+4=0$$

22. Find the points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$.

Sol. Let the point be (a, b)

$\therefore \quad a + b = 4 \Rightarrow b = 4 - a$

$$\Rightarrow \left|\frac{4a+3b-10}{\sqrt{4^2+3^2}}\right| = 1$$

$\Rightarrow |4a + 3(4 - a) - 10| = 5$

$\Rightarrow |4a+12-3a-10| = 5$

$\Rightarrow$ $|a + 2| = 5$

$\Rightarrow$ $a + 2 = \pm 5$

$a + 2 = 5$	$a + 2 = -5$
$a = 3$	$a = -7$
$b = 4 - 3 = 1$	$b = 4 - (-7) = 11$

$\therefore$ The points are (3, 1) and (– 7, 11)

23. Find the coordinates of the foot of perpendicular from the point (2, 3) on the line $y = 3x + 4$.

Sol. Let the point be (a, b)

Since (a, b) lies on $y = 3x + 4$

$\therefore$ $3a - b + 4 = 0$ ...(i)

Slope of the line through (2, 3) and (a, b)

$$= m_1 = \frac{3-b}{2-a}$$

Slope of $y = 3x + 4 = m_2 = 3$

Now, $m_1 m_2 = -1$

$$\left(\frac{3-b}{2-a}\right)3 = -1$$

$\Rightarrow$ $a - 3b = a - 2$

$\Rightarrow$ $a + 3b = 11$ ...(ii)

Solving (i) & (ii), we get

$$a = \frac{-1}{10} \text{ and } b = \frac{37}{10}$$

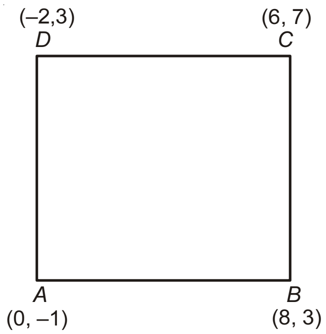
$\therefore$ The coordinates of the foot are $\left(\frac{-1}{10}, \frac{37}{10}\right)$

24. Show that 4 points (0, –1), (8, 3), (6, 7) and (–2, 3) are the vertices of a rectangle.

Sol. Slope of $AB = m_1$

$$= \frac{3-(-1)}{8-0}$$

$$= \frac{4}{8} = \frac{1}{2}$$



Slope of $BC = m_2 = \dfrac{7-3}{6-8} = \dfrac{4}{-2} = -2$

Slope of $CD = m_3 = \dfrac{7-3}{6-(-2)} = \dfrac{1}{2}$

Slope of $AD = m_4 = \dfrac{3-(-1)}{-2-0} = -2$

$m_1 = m_3$ and $m_2 = m_4$

$\Rightarrow$ $AB || CD$ and $BC || DA$

$\therefore$ $ABCD$ is a parallelogram

Also $m_1 m_2 = -1$

$\therefore$ $DA \perp AB$

$\therefore$ $ABCD$ is a rectangle.

25. Find the coordinates of the points on the line $x + 5y = 13$ which are at a distance of 2 units from the line $12x - 5y + 26 = 0$.

Sol. Let the point be (a, b)

Since (a, b) lies on $x + 5y = 13$

$\therefore$ $a + 5b = 13$

$\Rightarrow$ $5b = 13 - a$

Now, distance of (a, b) from $12x - 5y + 26 = 0$ is 2 units

$\therefore$ $\left|\dfrac{12a - 5b + 26}{\sqrt{12^2 + 5^2}}\right| = 2$

$\Rightarrow$ $|12a - 5b + 26| = 13 \times 2$

$\Rightarrow$ $|12a - 13 + a + 26| = 26$

$\Rightarrow$ $|13a + 13| = 26$

$\Rightarrow$ $|a + 1| = 2$

$\Rightarrow$ $a + 1 = \pm 2$

$\Rightarrow$ $a = 1, -3$

If $a = 1$, then $b = \dfrac{12}{5}$ and if $a = -3$, then $b = \dfrac{16}{5}$

$\therefore$ Points are $\left(1, \dfrac{12}{5}\right)$ & $\left(-3, \dfrac{16}{5}\right)$

26. Find the x and y-intercept of the line that passes through (2, 2) and is perpendicular to the line $3x + y = 3$.

Sol. Slope of $3x + y = 3 = m_1 = -3$

$\therefore$ Slope of required line $= m_2 = \dfrac{1}{3}$

$\therefore$ Equation of the required line is

$(y - 2) = \dfrac{1}{3}(x - 2)$

$\Rightarrow$ $3y - 6 = x - 2$

$\Rightarrow$ $x - 3y + 4 = 0$

$\Rightarrow$ $\dfrac{-x}{4} + \dfrac{3y}{4} = 1$

$\Rightarrow$ $\dfrac{x}{-4} + \dfrac{y}{\frac{4}{3}} = 1$

$\therefore$ x intercept = – 4

y intercept = $\dfrac{4}{3}$

27. Find the value of k if the straight line $2x + 3y + 4 + k(6x - y + 12) = 0$ is perpendicular to the line $7x + 5y = 4$.

Sol. $2x + 3y + 4 + k(6x - y + 12) = 0$

$(2 + 6k)x + (3 - k)y + 4 + 2k = 0$

Let slope be m_1 $\quad\therefore m_1 = -\left(\dfrac{2+6k}{3-k}\right)$

Let slope of $7x + 5y = 4$ is m_2

$m_2 = \dfrac{-7}{5}$

Now, $m_1 \times m_2 = -1$

$\Rightarrow$ $-\left(\dfrac{2+6k}{3-k}\right)\left(\dfrac{-7}{5}\right) = -1$

$\Rightarrow$ $7(2 + 6k) = -5(3 - k)$

$\Rightarrow$ $37k = -29$

$\Rightarrow$ $k = \dfrac{-29}{37}$

28. Find the equations of the straight lines passing through the point of intersection of $x + 3y + 4 = 0$ and $3x + y + 4 = 0$ and equally inclined to the axes.

Sol. Point of intersection of $x + 3y + 4 = 0$ and $3x + y + 4 = 0$ is $x = -1$, $y = -1$

The required lines pass through (–1, –1) and are equally inclined with the axes *i.e.* slopes are ± 1

Equation of requaired lines are

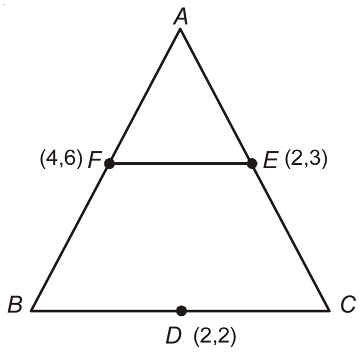
$y + 1 = \pm 1(x + 1)$

$y + 1 = x + 1$ and $y + 1 = -(x + 1)$

$\Rightarrow$ $x - y = 0$ $\Rightarrow$ $x + y + 2 = 0$

Long Answer Type Questions :

29. Mid-points of the sides of a triangle are (2, 2), (2, 3) and (4, 6). Find the equations of the sides of a triangle.



Sol. Slope of $EF = \frac{6-3}{4-2} = \frac{3}{2}$

$BC \parallel EF \Rightarrow$ Slope of $BC = \frac{3}{2}$, Point (2, 2)

Equation of BC is $y - 2 = \frac{3}{2}(x-2) \Rightarrow 3x - 2y - 2 = 0$

Similarly, find for AB, AC

Equations are $3x - 2y - 2 = 0$, $2x - y - 1 = 0$, $x = 4$

30. The sides AB, BC, CD, DA of a quadrilateral have the equations $x + 2y = 3$, $x = 1$, $x - 3y = 4$, $5x + y = -12$ respectively. Find the angle between the diagonals AC and BD.

Sol. Solving the equations in pairs we get the co-ordinates of the vertices as $A(-3, 3)$, $B(1,1)$, $C(1, -1)$ and $D(-2, -2)$.

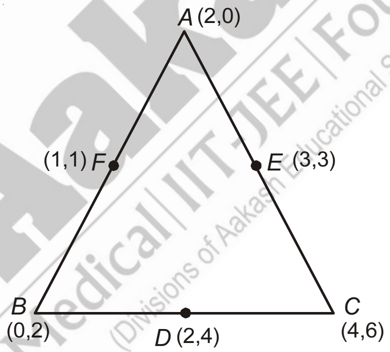
m_1 = slope of $AC = -1$, m_2 = slope of $BD = 1$

$\therefore \quad m_1m_2 = -1$

Hence the diagonals AC and BD are perpendicular.

31. Find the equations of the medians of the triangle whose vertices are (2, 0), (0, 2) and (4, 6).

Sol. Using mid-point formula, we will find D, E and F.

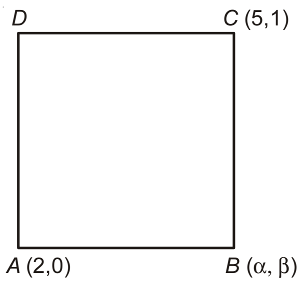


Then, using two point form we find the equations

$x = 2$, $5x - 3y = 2$, $x - 3y + b = 0$

32. The opposite angular points of a square are (2, 0) and (5, 1). Find the remaining points.

Sol. Let the unknown vertex be $B(\alpha, \beta)$



We know that,

$AB^2 = BC^2 \Rightarrow (\alpha - 2)^2 + (\beta - 0)^2 = (\alpha - 5)^2 + (\beta - 1)^2$

$\Rightarrow\ 6\alpha + 2\beta = 22$

$\Rightarrow\ \beta = 11 - 3\alpha$ ...(i)

As, $B = 90°$, we get

$AB^2 + BC^2 = AC^2$

$\Rightarrow\ (\alpha - 2)^2 + (\beta - 0)^2 + (\alpha - 5)^2 + (\beta - 1)^2 = (5 - 2)^2 + (1 - 0)^2$

$\Rightarrow\ \alpha^2 + \beta^2 - 7\alpha - \beta + 10 = 0$

$\Rightarrow\ \alpha^2 + (11 - 3\alpha)^2 - 7x - (11 - 3\alpha) + 10 = 0$

$\Rightarrow\ \alpha^2 - 7\alpha + 12 = 0$

$\Rightarrow\ \alpha = 4, 3$

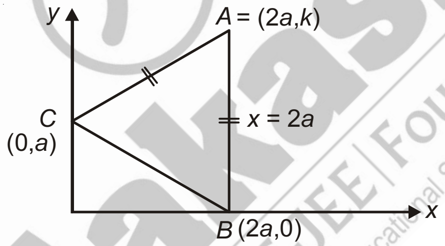
From (i),

When $\alpha = 4$, $\beta = -1$ and when $\alpha = 3$, $\beta = 2$

Therefore, the required vertices are (4, –1) and (3, 2).

33. The extremities of the base of an isosceles triangle are the points $(2a, 0)$ and $(0, a)$ and the equation of one of the equal sides is $x = 2a$. Find the equation of the other equal side.

Sol. In the given figure $x = 2a$ is the line BA passing through $B(2a, 0)$ and let the point A on it as$(2a, k)$.



Now, $AC = AB \Rightarrow 4a^2 + (k - a)^2 = k^2$

$\therefore\ 5a^2 - 2ak = 0$

$\therefore\ k = \dfrac{5a}{2}$ as $a \neq 0$

$\therefore$ Point A is $\left(2a, \dfrac{5a}{2}\right)$

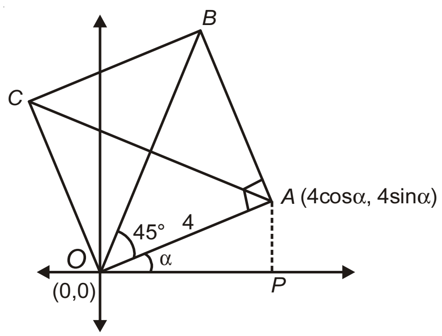
Hence by two point formula, the equation of side AC is $3x - 4y + 4a = 0$.

34. One side of a square is inclined to the x-axis at an angle α and one of its extremities is at origin. If the side of a square is 4, find the equations of the diagonals of the square.

Sol. Slope of $OB = \tan(45° + \alpha)$

$$= \frac{1 + \tan\alpha}{1 - \tan\alpha}$$

$$= \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha}$$



$\therefore$ Equation of OB is

$$y - 0 = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha}(x - 0)$$

$\Rightarrow$ $x(\sin\alpha + \cos\alpha) - y(\cos\alpha - \sin\alpha) = 0$

slope of $AC = -\dfrac{1}{\dfrac{\sin\alpha+\cos\alpha}{\cos\alpha-\sin\alpha}} = \dfrac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}$

$\therefore$ Equation of AC is

$$y - 4\sin\alpha = \frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}(x - 4\cos\alpha)$$

$\Rightarrow$ $x(\cos\alpha - \sin\alpha) + y(\sin\alpha + \cos\alpha) = 4$

35. Find the equations of the lines passing through the point (1, 0) and at a distance of $\dfrac{\sqrt{3}}{2}$ from the origin.

Sol. Let the equation of the line be

$$\frac{x}{a}+\frac{y}{b}=1$$

Point (1, 0) lies on the line

$\therefore$ $\dfrac{1}{a}+\dfrac{0}{b}=1$

$\Rightarrow$ $a = 1$

Now, distance of line from origin is $\dfrac{\sqrt{3}}{2}$

$\therefore$ $\left|\dfrac{\dfrac{0}{a}+\dfrac{0}{b}-1}{\sqrt{\dfrac{1}{a^2}+\dfrac{1}{b^2}}}\right| = \dfrac{\sqrt{3}}{2}$

$\Rightarrow$ $\dfrac{-1}{\sqrt{1+\dfrac{1}{b^2}}} = \pm\dfrac{\sqrt{3}}{2}$

Squaring both sides

$\Rightarrow$ $\dfrac{1}{1+\dfrac{1}{b^2}} = \dfrac{3}{4}$

$\Rightarrow$ $1+\dfrac{1}{b^2} = \dfrac{4}{3}$

$\Rightarrow$ $\dfrac{1}{b^2} = \dfrac{1}{3}$

$\Rightarrow$ $b^2 = 3$ $\Rightarrow$ $b = \pm\sqrt{3}$

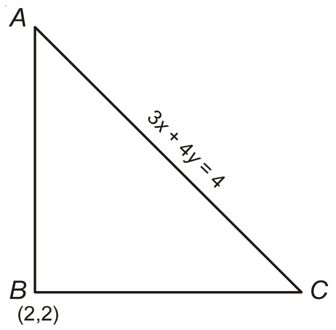
$\therefore$ Equations are $\dfrac{x}{1}+\dfrac{y}{\pm\sqrt{3}} = 1$

$x+\dfrac{y}{\sqrt{3}} = 1$ $\qquad$ $x-\dfrac{y}{\sqrt{3}} = 1$

$\sqrt{3}x + y - \sqrt{3} = 0$ $\qquad$ $\sqrt{3}x - y - 1 = 0$

36. Find the equation of one of the sides of an isosceles right angled triangle whose hypotenuse is given by $3x + 4y = 4$ and the opposite vertex of the hypotenuse is (2, 2).

Sol. $\angle A = \angle C = 45°$



$3x + 4y = 4$

slope $= m_1 = \dfrac{-3}{4}$

Let slope of AB be m_2.

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$\tan 45° = \left|\frac{m_2 + \frac{3}{4}}{1 - \frac{3}{4}m_2}\right|$$

$$\frac{4m_2 + 3}{4 - 3m_2} = \pm 1$$

$4m^2 + 3 = 4 - 3m_2$	$4m_2 + 3 = -4 + 3m_2$
$\Rightarrow\ 7m_2 = 1$	$\Rightarrow\ m_2 = -7$
$\Rightarrow\ m_2 = \dfrac{1}{7}$	

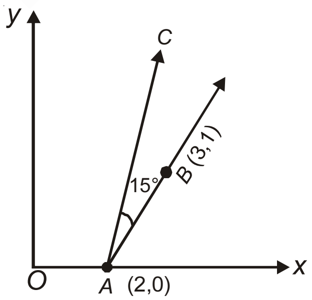
$\therefore$ Equation are $y - 2 = \dfrac{1}{7}(x - 2)$ and $y - 2 = -7(x - 2)$

$\Rightarrow\ 7y - 14 = x - 2$	$\Rightarrow\ y - 2 = -7x + 14$
$\Rightarrow\ x - 7y + 12 = 0$	$\Rightarrow\ 7x + y - 16 = 0$

37. If the line joining two points $A(2, 0)$ and $B(3, 1)$ is rotated about A in anticlockwise direction through an angle of 15°, then find the equation of the line in new position.

Sol. The slope of the line AB is $\dfrac{1-0}{3-2} = 1$ or $\tan 45°$.

After rotation of the line through 15°, the slope of the line AC in new position is $\tan 60° = \sqrt{3}$



Therefore, the equation of the new line AC is

$$y - 0 = \sqrt{3}(x - 2)$$

or $\quad y - \sqrt{3}x + 2\sqrt{3} = 0$

38. Find the reflection of the point (4, –13) about the line $5x + y + 6 = 0$.

Sol. Let (h,k) be the point of reflection of the given point (4, –13) about the line $5x + y + 6 = 0$. The mid-point of the line segment joining points (h, k) and (4, –13) is given by

$$\left(\frac{h+4}{2}, \frac{k-13}{2}\right)$$

This point lies on the given line, so we have

$$5\left(\frac{h+4}{2}\right) + \frac{k-13}{2} + 6 = 0$$

or $\quad 5h + k + 19 = 0 \qquad$ ...(i)

Again the slope of the line joining points (h, k) and (4, –13) is given by $\frac{k+13}{h-4}$.

This line is perpendicular to the given line and hence $(-5)\left(\frac{k+3}{h-4}\right) = -1$

This gives $5k + 65 = h - 4$

or $\quad h - 5k - 69 = 0 \qquad$ ...(ii)

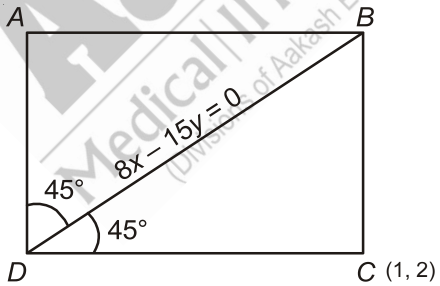
On solving (i) and (ii), we get

$h = -1$ and $k = -14$.

Thus the point (–1, –14) is the reflection of the given point.

39. If one diagonal of a square is along the line $8x - 15y = 0$ and one of its vertex is at (1, 2), then find the equation of sides of the square passing through this vertex.

Sol. Let $ABCD$ be the given square and the coordinates of the vertex D be (1, 2).



$\therefore \quad BD$ is along the line $8x - 15y = 0$

Its slope is $\frac{8}{15}$. The angles made by BD with sides AD and DC is 45°. Let the slope of DC be m.

$$\therefore \quad \tan 45° = \frac{m - \frac{8}{15}}{1 + \frac{8m}{15}}$$

or $\quad 15 + 8m = 15m - 8$

or $\quad 7m = 23$, which gives $m = \frac{23}{7}$

Therefore, the equation of the side DC is given by

$$y - 2 = \frac{23}{7}(x - 1) \quad \text{or} \quad 23x - 7y - 9 = 0$$

Similarly, the equation of another side AD is given by

$$y - 2 = \frac{-7}{23}(x - 1) \quad \text{or} \quad 7x + 23y - 53 = 0$$

40. For what values of a and b the intercepts cut off on the coordinate axes by the line $ax + by + 8 = 0$ are equal in length but opposite in signs to those cut off by the line $2x - 3y + 6 = 0$ on the axes.

Sol. $ax + by + 8 = 0$

Let x & y intercepts be x_1 and y_1 respectively

$$\therefore \quad x_1 = \frac{-8}{a} \quad \text{and} \quad y_1 = \frac{-8}{b}$$

$2x - 3y + 6 = 0$

Let x & y intercepts be x_2 and y_2 respectively

$$x_2 = \frac{-6}{2} = -3 \qquad y_2 = \frac{-6}{-3} = 2$$

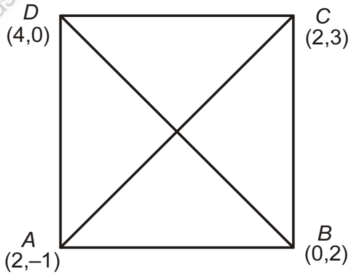
According to the question,

$x_1 = -x_2$	$y_1 = -y_2$
$\Rightarrow -\frac{8}{a} = 3$	$\Rightarrow \frac{-8}{b} = 2$
$\Rightarrow a = \frac{-8}{3}$	$\Rightarrow b = -4$

41. Show that $A(2, -1)$, $B(0, 2)$, $C(2, 3)$ & $D(4,0)$ are coordinates of the angular points of a parallelogram and find the angle between the diagonals.

Sol. Slope of line through $AB = m_1$

$$= \frac{2-(-1)}{0-2} = \frac{-3}{2}$$



Slope of line through $BC = m_2 = \frac{3-2}{2-0} = \frac{1}{2}$

Slope of line through $CD = m_3 = \frac{3-0}{2-4} = \frac{-3}{2}$

Slope of line through $AD = m_4 = \frac{0-(-1)}{4-2} = \frac{1}{2}$

Since $m_1 = m_3$ and $m_2 = m_4$

$\therefore \quad AB \parallel CD,\ BC \parallel AD$

$\therefore \quad ABCD$ is a parallelogram

Slope of $AC = m_5 = \frac{3-(-1)}{2-2} = \frac{1}{0}$

$$\frac{1}{m_5} = 0$$

Slope of $BD = m_6 = \dfrac{0-2}{4-0} = \dfrac{-1}{2}$

Now, If angle between AC & BC is θ

$$\therefore \quad \tan\theta = \left|\frac{m_5 - m_6}{1 + m_5 m_6}\right| = \left|\frac{1 - \frac{m_6}{m_5}}{\frac{1}{m_5} + m_6}\right| = \left|\frac{1 - \left(\frac{-1}{2}\right)0}{0 + \left(\frac{-1}{2}\right)}\right|$$

$$= \left|\frac{1}{\frac{-1}{2}}\right|$$

$$= 2$$

$$\therefore \quad \theta = \tan^{-1}(2)$$

❑ ❑ ❑

Chapter 11

Straight Lines

Solutions (Set-2)

[Distance Formula and Section Formula]

1. If the area of the triangle formed by the points (0, 0), (h, 0) and (0, 4) is 2, then the sum of values of h is

(1) 1 (2) 2 (3) 0 (4) 4

Sol. Answer (3)

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ h & 0 & 1 \\ 0 & 4 & 1 \end{vmatrix} = \pm 2$$

$\Rightarrow$ $4h = \pm 4$

$\Rightarrow$ $h = \pm 1$

Hence sum = 1 – 1 = 0

2. If the points (a, b), (1, 1) and (2, 2) are collinear then

(1) $a = 2b$ (2) $a = 3b$ (3) $2a = b$ (4) $a = b$

Sol. Answer (4)

$$\begin{vmatrix} a & b & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $a(1-2) - b(1-2) + 1(2-2) = 0$

$\Rightarrow$ $-a + b = 0 \Rightarrow a = b$

3. If the sides of triangle ABC are such that a = 4, b = 5, c = 6, then the ratio in which incentre divide the angle bisector of B is

(1) 2 : 3 (2) 2 : 1 (3) 5 : 2 (4) 1 : 1

Sol. Answer (2)

Ratio = $\dfrac{c+a}{b} = \dfrac{4+6}{5} = 2:1$

4. Which of the following is not always inside a triangle ?

(1) Incentre (2) Centroid

(3) Intersection of altitudes (4) Intersection of medians

Sol. Answer (3)

Orthocentre is not always inside the triangle.

5. If the coordinates of vertices of a triangle is always rational then the triangle cannot be

(1) Scalene (2) Isosceles (3) Rightangle (4) Equilateral

Sol. Answer (4)

Let $A = (x_1, y_1), B = (x_2, y_2), C = (x_3, y_3)$

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \text{Rational number}$$

But the area of equilateral triangle is also calculated by

$$A = \frac{\sqrt{3}}{4} \times \text{(side)}^2 = \text{Irrational}$$

Hence triangle cannot be equilateral.

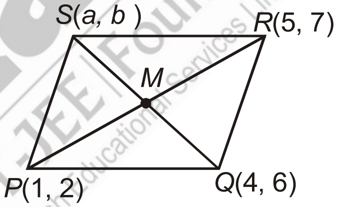
6. If $P(1, 2)$, $Q(4, 6)$, $R(5, 7)$ and $S(a, b)$ are the vertices of a parallelogram $PQRS$ then

(1) $a = 2, b = 4$ (2) $a = 3, b = 4$ (3) $a = 2, b = 3$ (4) $a = 3, b = 3$

Sol. Answer (3)

In a parallelogram, mid-points coincide

Mid-point of PR = Mid-point of SQ



$$\left(\frac{5+1}{2}, \frac{2+7}{2}\right) = \left(\frac{a+4}{2}, \frac{b+6}{2}\right)$$

$\Rightarrow$ $a = 2, b = 3$

[Slope, Angle between Lines and Different form of Straight Line]

7. If the angle between two lines is 45° and the slope of one line is 2, then the product of possible slopes of other line is

(1) –1 (2) 1 (3) 2 (4) 6

Sol. Answer (1)

$$\tan 45° = \pm\frac{m-2}{1+2m}$$

$$\Rightarrow m = 3, -\frac{1}{3}$$

Product = –1

8. If m is the slope of a line and $m + 2 = m + 3$, then the possible angle between line and x-axis is

(1) 0° (2) 90° (3) 60° (4) 45°

Sol. Answer (2)

In this case the line will be parallel to y-axis. Hence the angle = 90°.

9. The intercept on x-axis of the line $y = mx + c$ is

(1) c (2) $-c$ (3) $-\frac{c}{m}$ (4) $-\frac{m}{c}$

Sol. Answer (3)

$0 = mx + c$

$\Rightarrow \quad x = -\frac{c}{m}$

10. If $x = x_1 \pm r\cos\theta,\ y = y_1 \pm r\sin\theta$ be the equation of straight line then, the parameter in this equation is

(1) θ (2) x_1 (3) y_1 (4) r

Sol. Answer (4)

11. The digonals of parallelogram $PQRS$ are along the lines $x + 3y = 4$, $6x - 2y = 7$, then $PQRS$ must be a

(1) Rectangle (2) Square (3) Rhombus (4) Trapezium

Sol. Answer (3)

Slopes of diagonals are $-\frac{1}{3}$ and 3.

$\therefore \quad m_1 m_2 = \left(-\frac{1}{3}\right)(3) = -1$

$\therefore$ Diagonals are perpendicular.

$\therefore$ Parallelogram is a Rhombus.

12. What is/are the point(s) on the line $x + y = 4$ that lie(s) at unit distance from the line $4x + 3y = 10$?

(1) (11, –7) (2) (3, 1) (3) (–7, 11) & (3, 1) (4) (7, 11) & (1, –3)

Sol. Answer (3)

Let point on line $x + y = 4$ be $(x, 4 - x)$.

$\therefore \quad 1 = \left|\frac{4x + 3(4 - x) - 10}{\sqrt{16 + 9}}\right|$

$|x + 2| = 5 \Rightarrow x = 3$ & -7

$\therefore$ Points are (3, 1) & (–7, 11).

13. The line $5x + 4y = 0$ passes through the point of intersection of straight lines

(1) $x + 2y - 10 = 0,\ 2x + y = -5$ (2) $x + 2y + 10 = 0,\ 2x - y + 5 = 0$

(3) $x - 2y - 10 = 0,\ 2x + y - 5 = 0$ (4) $x = y,\ 2x = y + 1$

Sol. Answer (1)

Equation of a lines which passes through the intersection of two lines

$L_1 + \lambda L_2 = 0$

Let $L_1 \equiv x + 2y - 10 = 0$

$L_2 \equiv 2x + y + 5 = 0$

$L_1 + 2L_2 = 0$

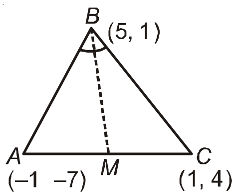
$(x + 2y - 10) + 2(2x + y + 5) = 0$

$5x + 4y = 0$

14. The vertices of a triangle are $A(-1, -7)$, $B(5, 1)$ and $C(1, 4)$. The equation of angle bisector of $\angle ABC$ is

(1) $x + 7y + 2 = 0$ (2) $x - 7y + 2 = 0$ (3) $x - 7y - 2 = 0$ (4) $x + 7y - 2 = 0$

Sol. Answer (2)



$$\frac{AM}{CM} = \frac{AB}{BC} = \frac{10}{5} = \frac{2}{1}$$

$\therefore$ By section formula the coordinates of m

$$\left(\frac{2-1}{2+1}, \frac{8-7}{2+1}\right) = \left(\frac{1}{3}, \frac{1}{3}\right)$$

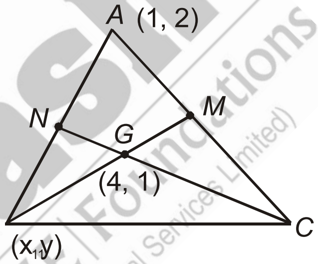
$\therefore$ Equation of BM = $\dfrac{x-5}{\frac{1}{3}-5} = \dfrac{y-1}{\frac{1}{3}-1}$

$x - 7y + 2 = 0$

15. In ΔABC, if $A \equiv (1, 2)$ and equations of the medians through B and C are $x + y = 5$ and $x = 4$, then point B must be

(1) $(1, 4)$ (2) $(7, -2)$ (3) $(4, 1)$ (4) $(-2, 7)$

Sol. Answer (2)



Equation of CN be $x = 4$

$\therefore$ Let coordinate of N be $(4, b)$.

N is mid point of AB.

$\therefore$ Coordinate of B $(7, 2b - 2)$.

B lie on the line $x + y = 5$

$\therefore$ $7 + 2b - 2 = 5$

$b = 0$

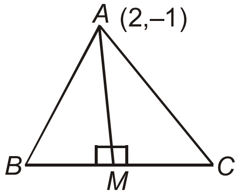
$\therefore$ B $(7, -2)$

[Length of Perpendicular, Angle Bisectors, Reflection and Analysis of Three Lines]

16. The equation of base of an equilateral triangle is $x + y = 2$ and vertex is $(2, -1)$, then the length of the side of the triangle is equal to

(1) $\sqrt{\frac{2}{3}}$ (2) $\sqrt{\frac{1}{3}}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\sqrt{3}$

Sol. Answer (1)



Equation of BC

$x + y - 2 = 0$

$$AM = \left|\frac{2-1-2}{\sqrt{1+1}}\right| = \frac{1}{\sqrt{2}}$$ [Altitude of equatorial $\Delta = \frac{\sqrt{3}}{2}$ side]

$$\frac{\sqrt{3}}{2}(\text{side}) = \frac{1}{\sqrt{2}}$$

Side = $\sqrt{\frac{2}{3}}$

17. The equation of a straight line equally inclined to the axes and equidistant from the points (1, –2) and (3, 4) is

(1) $x + y + 1 = 0$ (2) $x - y + 1 = 0$ (3) $x - y - 1 = 0$ (4) $x + y - 1 = 0$

Sol. Answer (3)

Slope of line = ± 1

Let equation of straight line be $y = x + C$

$\therefore\ \ x - y + C = 0$ ...(1)

Line (1) is equidistance from points (1, –2) & (3, 4)

$$\therefore\ \ \left|\frac{1+2+C}{\sqrt{2}}\right| = \left|\frac{3-4+C}{\sqrt{2}}\right|$$

$| C + 3 | = | C - 1|$

$C + 3 = -C + 1 \Rightarrow C = -1$

From equation (1) line is $x - y - 1 = 0$

18. A line passing through (0, 0) and perpendicular to $2x + y + 6 = 0$, $4x + 2y - 9 = 0$ then the origin divides the line in the ratio of

(1) 1 : 2 (2) 2 : 1 (3) 4 : 3 (4) 3 : 4

Sol. Answer (3)

Perpendicular distances of the lines from origin are

$$OM = \frac{6}{\sqrt{5}} \text{ and } ON = \left|\frac{-9}{\sqrt{20}}\right| = \frac{9}{2\sqrt{5}}$$

O divides MN in the ratio = $\dfrac{6}{\sqrt{5}} : \dfrac{9}{2\sqrt{5}} = 2 : \dfrac{3}{2}$ = 4 : 3

19. A ray of light is sent along the line $x - 2y - 3 = 0$ on reaching the line $3x - 2y - 5 = 0$ the ray is reflected from it, then the equation of the line containing the reflected ray is

(1) $2x - 29y - 30 = 0$ (2) $29x - 2y - 31 = 0$ (3) $3x - 31y + 37 = 0$ (4) $31x - 3y + 37 = 0$

Sol. Answer (2)

Let A be the point of incidence.

$\therefore\ \ A$ is intersection of

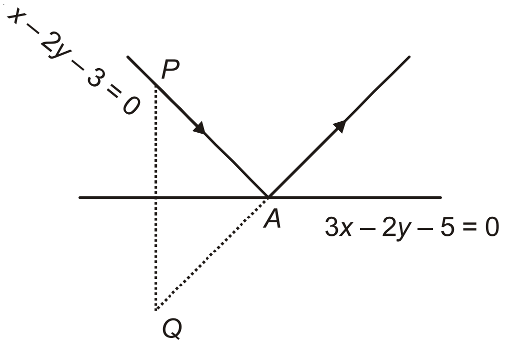
$x - 2y - 3 = 0$ ...(i)

and $3x - 2y - 5 = 0$ ...(ii)

$\therefore\ \ A = (1, -1)$

Let P be any point on the line of incidence $x - 2y - 3 = 0$. So we take $P = (3, 0)$

Let $Q(\alpha, \beta)$ be angle of P in the line $3x - 2y - 5 = 0$



$\therefore\ \ PQ \perp$ the line $3x - 2y - 5$

$$\therefore\ \ \frac{\beta}{\alpha - 3} \times \frac{B}{2} = -1 \quad \text{...(iii)}$$

$$\text{And } 3\left(\frac{\alpha+3}{2}\right) - 2\left(\frac{\beta}{2}\right) - 5 = 0 \quad \text{...(iv)}$$

Equation (iii) $\Rightarrow 3\beta + 2\alpha = 6$

Equation (iv) $\Rightarrow 3\alpha - 2\beta - 1 = 0$

Solving these, we get

$$\alpha = \frac{15}{13}, \beta = \frac{16}{13}$$

$\therefore$ $Q = \left(\frac{15}{13}, \frac{16}{13}\right)$

$\therefore$ Line containing the reflected ray is the line joining the points $A(1, -1)$ and $Q\left(\frac{15}{13}, \frac{16}{13}\right)$

$\therefore$ Required equation is $y+1 = \dfrac{\frac{16}{13}+1}{\frac{16}{13}-1}(x-1)$

$\Rightarrow$ $29x - 2y - 31 = 0$

20. The equation of the bisector of the acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$ is

(1) $11x + 3y - 9 = 0$ (2) $3x - 11y + 9 = 0$ (3) $11x - 3y - 9 = 0$ (4) $11x - 3y + 9 = 0$

Sol. Answer (4)

Lines are $3x - 4y + 7 = 0$

$-12x - 5y + 2 = 0$

$a_1 a_2 + b_1 b_2 = -36 + 20 < 0$

Positive sign gives acute angle bisector, $\dfrac{3x-4y+7}{\sqrt{9+16}} = +\dfrac{-12x-5y+2}{\sqrt{144+25}}$

$11x - 3y + 9 = 0$

21. If a, b and c are in A.P. then $ax + by + c = 0$ will always pass through a fixed point whose co-ordinates are

(1) $(1, -2)$ (2) $(1, 2)$ (3) $(0, -2)$ (4) $(1, -1)$

Sol. Answer (1)

$$b = \frac{a+c}{2}$$

$\therefore$ $ax + \left(\frac{a+c}{2}\right) y + c = 0$

$$a\left(x + \frac{1}{2}y\right) + c\left(\frac{1}{2}y + 1\right) = 0$$

$$\left(x + \frac{1}{2}y\right) + \frac{c}{a}\left(\frac{1}{2}y + 1\right) = 0$$

This line will always pass through the intersection point of two lines

$x + \frac{1}{2}y = 0$ and $\frac{1}{2}y + 1 = 0$

Solve these equations $y = -2$, $x = 1$

Fixed point $(1, -2)$.

22. Sum of the possible values of λ for which the following three lines $x + y = 1$, $\lambda x + 2y = 3$, $\lambda^2 x + 4y + 9 = 0$ are concurrent is

(1) – 13 (2) 14 (3) 16 (4) –14

Sol. Answer (1)

The given lines are concurrent if

$$\begin{vmatrix} 1 & 1 & -1 \\ \lambda & 2 & -3 \\ \lambda^2 & 4 & 9 \end{vmatrix} = 0$$

Solving, we get

$\lambda^2 + 13\lambda - 30 = 0$

which gives two values of λ whose sum is –13.

[Pair of Straight Lines]

23. The equation $2x^2 + 3y^2 - 8x - 18y + 35 = K$ represents

(1) No locus if $K > 0$ (2) An ellipse if $K < 0$ (3) A point if $K = 0$ (4) A hyperbola if $K > 0$

Sol. Answer (3)

By complete squaring method $2(x-2)^2 + 3(y-3)^2 = k$

If $k = 0$

$2(x-2)^2 + 3(y-3)^2 = 0$

Then necessarily $(x-2)^2 = 0$ & $(y-3)^2 = 0$ $\therefore$ $x = 2$ & $y = 3$

$\therefore$ Equation represents a point if $k = 0$

24. The straight line $ax + by = 1$ makes with the curve $px^2 + 2axy + qy^2 = r$, a chord which subtends a right angle at the origin. Then

(1) $r(b^2 + q^2) = p + a$ (2) $r(b^2 + p^2) = p + q$ (3) $r(a^2 + b^2) = p + q$ (4) $(a^2 + p^2)r = q + b$

Sol. Answer (3)

$px^2 + 2axy + qy^2 = r(1)^2$

$px^2 + 2axy + qy^2 = r\,[ax + by]^2$

$(p - ra^2)\,x^2 + (q - rb^2)y^2 + (a - rab)\,2xy = 0$

These lines are perpendicular

$\therefore$ $p - ra^2 + q - rb^2 = 0$

$p + q = r(a^2 + b^2)$

25. The straight lines joining the origin to the point of intersection of two curves

$ax^2 + 2hxy + by^2 + 2gx = 0$

and $a'x^2 + 2h'xy + b'y^2 + 2g'x = 0$ will be at right angles to one another if

(1) $g'(a+b) = g(a'+b')$ (2) $g'(a'+b) = g(a+b')$ (3) $g'(a-b) = g(a'-b')$ (4) $g'(a-b') = g(a'-b)$

Sol. Answer (1)

Any curve passing through the intersection of the given curves is

$ax^2 + 2hxy + by^2 + 2gx + \lambda(a'x^2 + 2h'xy + b'y^2 + 2g'x) = 0$ …(i)

This will be pair of straight lines passing through origin if it is II[nd] degree homogeneous in x and y. For this the condition on (i) is

Coefficient of $x = 2g + 2\lambda g' = 0 \Rightarrow \lambda = \dfrac{-g}{g'}$

Also, the lines are perpendicular

i.e., coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow \quad a + \lambda a' + b + \lambda b' = 0$

$\Rightarrow \quad a + b + \lambda(a' + b') = 0$

$\Rightarrow \quad a + b = -\lambda(a' + b')$

$\Rightarrow \quad g'(a + b) = g(a' + b')$

26. The middle point of the line segment joining (3, –1) and (1, 1) is shifted by two units (in the sense increasing y) perpendicular to the line segment. Then the coordinates of the point in the new position is

(1) $(2-\sqrt{2}, \sqrt{2})$ (2) $(\sqrt{2}, 2+\sqrt{2})$ (3) $(2+\sqrt{2}, \sqrt{2})$ (4) $(\sqrt{2}, 2-\sqrt{2})$

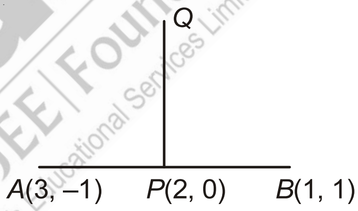
Sol. Answer (3)

Let P be the middle point of the line segment joining $A(3, -1)$ and $B(1, 1)$.

Then $P = (2, 0)$

Let P be shifted to Q where $PQ = 2$ and y–coordinate of Q is greater than that of P (from graph)

Now, Slope of $AB = \dfrac{1+1}{1-3} = -1$



$\therefore \quad$ Slope of $PQ = 1$

$\therefore \quad$ Coordinates in Q by distance formula

$= (2 \pm 2\cos\theta, 0 \pm 2\sin\theta)$, where $\tan\theta = 1$

$= (2 \pm \sqrt{2}, \pm\sqrt{2})$

As y-coordinate of Q is greater than that of P

$\therefore \quad Q = (2+\sqrt{2}, \sqrt{2})$, which is the required point.

27. Distance between the pair of straight lines

$x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ is given by

(1) $\dfrac{3}{\sqrt{10}}$ (2) $\dfrac{5}{\sqrt{10}}$ (3) $\dfrac{6}{\sqrt{10}}$ (4) $\dfrac{7}{\sqrt{10}}$

Sol. Answer (3)

The given equation is

$x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0 \quad \ldots$(i)

Here $abc + 2gfh - af^2 - bg^2 - ch^2 = 0$

And $h^2 - ab = 0$

$\therefore \quad$ Equation (i) Represents the parallel straight lines.

From (i), we know

$$9y^2 + 6(x+2)y + (x^2+4x-5) = 0$$

$$y = \frac{-6(x+2) \pm \sqrt{86(x+2)^2 - 36(x^2+4x-5)}}{2\times 9}$$

Or, $3y + x = 1$ and $3y + x + 5 = 0$

There are two parallel lines and distance between these two lines is $\frac{5-(-1)}{\sqrt{3^2+1^2}} = \frac{6}{\sqrt{10}}$.

[Miscellaneous]

28. The limiting position of the point of intersection of the straight line $3x + 5y = 1$ and $(2 + c)x + 5c^2y = 1$ as c tends to one is

(1) $\left(\frac{1}{2}, -\frac{1}{10}\right)$ (2) $\left(\frac{3}{8}, \frac{1}{-40}\right)$ (3) $\left(\frac{2}{5}, \frac{1}{25}\right)$ (4) $\left(\frac{2}{5}, \frac{1}{-25}\right)$

Sol. Answer (4)

Solve these equations $x = \frac{(C^2-1)}{(3C+2)(C-1)}$ and $y = \frac{(C-1)}{-5(3C+2)(C-1)}$

Here $C \neq 1$ at $C = 1$ lines are coincident.

$$\therefore \quad x = \lim_{C\to 1} \frac{(C^2-1)}{(3C+2)(C-1)} = \frac{C+1}{3C+2} \Rightarrow x = \frac{2}{5}$$

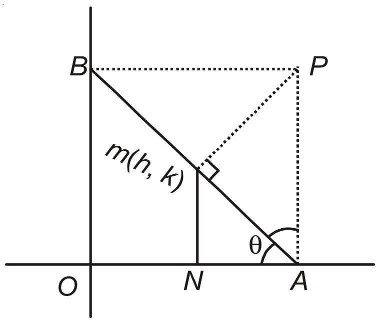
$$\therefore \quad y = \lim_{C\to 1} \frac{(C-1)}{-5(3C+2)(C-1)} = \lim_{C\to 1} \frac{1}{-5(3C+2)} = \frac{-1}{25}$$

$\therefore$ Point of intersection is $\left(\frac{2}{5}, \frac{-1}{25}\right)$.

29. The ends A, B of a straight line segment of constant length c slides on the fixed rectangular axes OX, OY respectively. If the rectangle $OAPB$ be completed. Then the locus of the foot of the perpendicular drawn from P upon AB is

(1) $x^{2/3} - y^{2/3} = c^{2/3}$ (2) $x^{1/3} + y^{1/3} = c^{1/3}$ (3) $x^{2/3} + y^{2/3} = c^{2/3}$ (4) $x^{1/3} - y^{2/3} = c^{1/3}$

Sol. Answer (3)



Let $\angle BAO = \theta$, then

$OA = c \cos\theta$

$OB = c \sin\theta$

Let m (h, k) be foot of the perpendicular from P on AB.

Let $MN \perp OX$.

$ON = h = OA - NA$

$= c \cos\theta - MA.\cos\theta$

$= c \cos\theta - PA.\cos\left(\frac{\pi}{2} - \theta\right).\cos\theta$

$= c \cos\theta - c \sin\theta \cdot \sin\theta \cdot \cos\theta$

$= c \cos\theta\ (1 - \sin^2\theta)$

$\Rightarrow$ $h = c\cos^3\theta$ …(i)

$k = MN = MA\sin\theta$

$\Rightarrow$ $k = c.\sin^3\theta$ …(ii)

$\Rightarrow$ $h^{2/3} + k^{2/3} = c^{2/3}(\sin^2\theta + \cos^2\theta) = c^{2/3}$

Replacing (h, k) by (x, y), we get

$x^{2/3} + y^{2/3} = c^{2/3}$

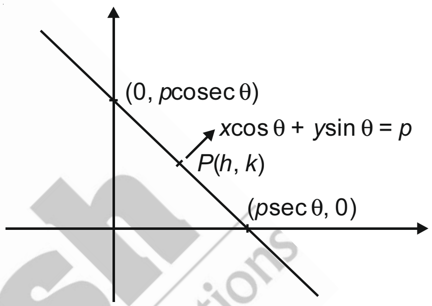
30. Locus of the middle point of the portion of the line $x\cos\theta + y\sin\theta = p$ which is intercepted between the axes, given that p is constant, is

(1) $x^2 + y^2 = p^2$ (2) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{p^2}$ (3) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$ (4) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{2}{p^2}$

Sol. Answer (3)

$p\sec\theta = 2h$

and $p\operatorname{cosec}\theta = 2k$



By $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \frac{p^2}{4h^2}+\frac{p^2}{4k^2} = 1$$

$$\Rightarrow \frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$$

31. A variable straight line, drawn through the point of intersection of the straight line $\frac{x}{4}+\frac{y}{2}=1$ and $\frac{x}{2}+\frac{y}{4}=1$ meets the co-ordinate axes in A and B. The locus of midpoint of AB is

(1) $x + y = xy$ (2) $2(x + y) = 3xy$ (3) $2(x + y) = 2xy$ (4) $(x - y)^2 = xy$

Sol. Answer (2)

Intersection of $x + 2y = 4$ and $2x + y = 4$ is $\left(\frac{4}{3},\frac{4}{3}\right)$

$\therefore$ Variable straight line is

$$y-\frac{4}{32}=m\left(x-\frac{4}{3}\right)$$

x-intercept $=\left(\frac{4}{3}\frac{-4}{3m},0\right)$

and y-intercept $=\left(0,\frac{4}{3}\frac{-4m}{3}\right)$

Locus of mid point

$\frac{2}{3}-\frac{2}{3m}=h$ ...(1)

and $\frac{2}{3}-\frac{2m}{3}=k$ ...(2)

$\Rightarrow 2m-2=3mh$

$\Rightarrow m=\dfrac{2}{2-3h}$

Put m in equation (2), we get

$$\frac{2}{3}-k=\frac{2\cdot 2}{3(2-3h)}$$

$$\Rightarrow (2-3k)(2-3h)=4$$

$$\Rightarrow 4-6k-6h+9hk=4$$

$$\Rightarrow 2(x+y)=3xy$$

32. A straight line passes through a fixed point (2, 3). Locus of the foot of the perpendicular on it drawn from the origin is

(1) $x^2 + y^2 + 2x - 3y = 0$

(2) $x^2 + y^2 - 2x - 3y = 0$

(3) $x^2 + y^2 - 2x + 3y = 0$

(4) $x^2 + y^2 + 2x + 3y = 0$

Sol. Answer (2)

Let foot of perpendicular be (h, k)

and line be $y - 3 = m(x - 2)$

$\Rightarrow nx-y+3-2m=0$

$\Rightarrow \dfrac{h-0}{m}=\dfrac{k-0}{-1}=\dfrac{-(3-2m)}{1+m^2}$

$\therefore$ $x^2 + y^2 - 2x - 3y = 0$

33. $y = mx$ be a variable line (m is variable) intersect the lines $2x + y - 2 = 0$ and $x - 2y + 2 = 0$ in P and Q. The locus of midpoint of segment of PQ is

(1) $2x^2 + 3xy - 2y^2 + x + 3y = 0$

(2) $2x^2 - 3xy - 2y^2 + x + 3y = 0$

(3) $2x^2 + 3xy + 2y^2 + x + 3y = 0$

(4) $2x^2 + 3xy - 2y^2 - x - 3y = 0$

Sol. Answer (2)

$P=\left(\dfrac{2}{2+m},\dfrac{2m}{2+m}\right)$ and $Q\left(\dfrac{2}{2m-1},\dfrac{2m}{2m-1}\right)$

Let mid point be (h, k)

$\Rightarrow \quad \dfrac{2}{m+2}+\dfrac{2}{2m-1}=2h$ ...(1)

and $\dfrac{2m}{m+2}+\dfrac{2m}{2m-1}=2k$ ...(2)

Elimunating m from (1) and (2), we get locus as

$2x^2 - 3xy - 2y^2 + x + 3y = 0$

34. A variable straight line passes through the points of intersection of the lines $x + 2y = 1$ and $2x - y = 1$ and meets the co-ordinate axes in A and B. The locus of middle point of AB is

(1) $3xy = (x - y)$ (2) $3xy = x + y$ (3) $10xy = x + 3y$ (4) $xy = x + 3y$

Sol. Answer (3)

Intersection point of $x + 2y = 1$ and $2x - y = 1$ is $\left(\frac{3}{5}, \frac{1}{5}\right)$

Let variable line be $y\frac{-1}{5} = 3\left(x - \frac{3}{5}\right)$

y-intercept $= \frac{1}{5}\frac{-3m}{5}$

and x-intercept $= \frac{3}{5}\frac{-1}{5m}$

Applying condition of locus (assuming (h, k) as required locus)

$\frac{3m-1}{5m} = 2h$ and $\frac{1-3m}{5} = 2k$

$\therefore$ $10xy = x + 3y$

35. If $A(\cos\alpha, \sin\alpha)$, $B(\sin\alpha, -\cos\alpha)$, $C(2, 1)$ are vertices of a triangle ABC. The locus of its centroid if α varies is

(1) $9x^2 + 9y^2$ (2) $9x^2 + 9y^2 - 12x + 6y - 3 = 0$

(3) $9x^2 + 9y^2 - 12x - 6y + 3 = 0$ (4) $9x^2 + 9y^2 + 12x + 6y - 3 = 0$

Sol. Answer (3)

Let centroid be (h, k)

$\Rightarrow$ $2 + \cos\alpha + \sin\alpha = 3h$ and $\sin\alpha - \cos\alpha + 1 = 3k$

Elimunating α, we get locus as

$9x^2 + 9y^2 + 12x + 6y - 3 = 0$

36. Two fixed points A and B have co-ordinates (x_1, y_1) and (x_2, y_2). A point P moves such that AP is perpendicular to BP, then locus of P is

(1) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ (2) $(x + x_1)(x + x_2) + (y + y_1)(y + y_2) = 0$

(3) $(x - x_1)(y - y_1) + (x - x_2)(y - y_2) = 0$ (4) $(x + x_1)(y + y_1) + (x + x_2)(y + y_2) = 0$

Sol. Answer (1)

Let $P(h, k)$

Hence $\frac{K - y_1}{h - x_1} \cdot \frac{K - y_2}{h - x_2} = -1$

$\Rightarrow (x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

37. Locus of the middle point of the intercept on the line $y = x + c$ made by the lines $2x + 3y = 5$ and $2x + 3y = 8$, c being a parameter, is

(1) $2x + 3y + 13 = 0$ (2) $4x + 6y + 13 = 0$ (3) $4x + 6y - 13 = 0$ (4) $2x + 3y - 13 = 0$

Sol. Answer (3)

$$\therefore \quad \frac{8-3c}{5}+\frac{5-3c}{5}=h \text{ and } \frac{5+2c}{5}+\frac{8+2c}{5}=k$$

$\Rightarrow$ $4x + 6y - 13$ (on elimunating c)

38. If the pair of straight lines $x^2 - y^2 + 2x + 1 = 0$ meet the y-axis in A and B, $|AB|$ equals to

(1) 1 (2) 0 (3) 2 (4) 4

Sol. Answer (3)

$x = 0 \Rightarrow -y^2 = 1 \Rightarrow y = \pm 1$

Let $A(0, -1)$ and $B(0, 1) \Rightarrow |AB| = 2$

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