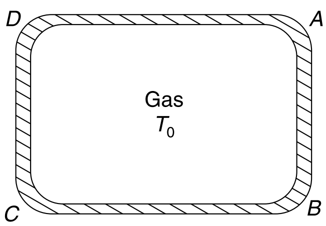
# Chapter 3

# Kinetic Theory of Gases and Gas Laws

## LEVEL 1

**Q. 1:** An ideal gas at temperature $T_0$ is contained in a container. By some mechanism, the temperature of the wall $AB$ is suddenly increased to $T(> T_0)$. Will the pressure exerted by the gas on wall $AB$ change suddenly? Will it be higher or lower than pressure on wall $CD$?




**Q. 2:** Find the number of atoms in molecule of a gas for which the ratio of specific heats at constant pressure and constant volume ($\gamma$) becomes $\frac{25}{21}$ times if the rotational degree of freedom of its molecules are frozen. Assume that the gas molecules originally had translational and rotational degree of freedom.

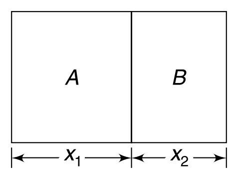
**Q. 3:** An ideal gas expands following a relation $\frac{P^2}{\rho}$ = constant, where $P$ = pressure and $\rho$ = density of the gas. The gas is initially at temperature $T$ and density $\rho$ and finally its density becomes $\frac{\rho}{3}$.

(a) Find the final temperature of the gas.

(b) Draw the $P - T$ graph for the process.

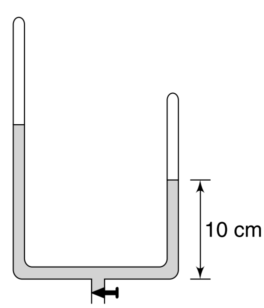
**Q. 4:** A container having volume $V$ contains $N$ atoms of a gas $X$. $2N$ atoms of another gas $Y$ is injected into the container and temperature is raised to $T$ and maintained constant. At this temperature the atoms of $X$ and $Y$ combine to form molecule $X_2Y$. Find the pressure inside the container after the reaction is completed.

**Q. 5:** A conducting piston separates a cylindrical tube into two compartments $A$ and $B$. The two compartments contain equal mass of two different gases. The molar mass of two gases are $M_A$ = 32 g and $M_B$ = 28 g. Find the ratio of lengths $(X_1 : X_2)$ of the two compartments in equilibrium.



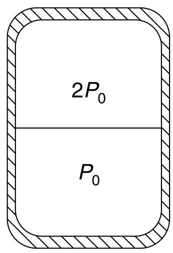

**Q. 6:** A container of volume $V_1$ has an ideal gas of molar mass $M_1$ at pressure $P_1$ and temperature $T_1$. Another container of volume $V_2$ has another ideal gas of molar mass $M_2$ at pressure $P_2$ and temperature $T_2$. Two gases are mixed in a vessel and acquire an equilibrium temperature and pressure of $T_0$ and $P_0$ respectively. Find the density of the mixture.

**Q. 7:** A $U$ shaped tube has two arms of equal cross section and lengths $\ell_1$ = 80 cm and $\ell_2$ = 40 cm. The open ends are sealed with air in the tube at a pressure of 80 cm of mercury. Some mercury is now introduced in the tube through a stopcock connected at the bottom (the air is not allowed to leak out). In steady condition the length of mercury column in the shorter arm was found to be 10 cm. Find the length of the mercury column in the longer arm. Neglect the volume of the part of the tube connecting two arms and assume that the temperature is constant.

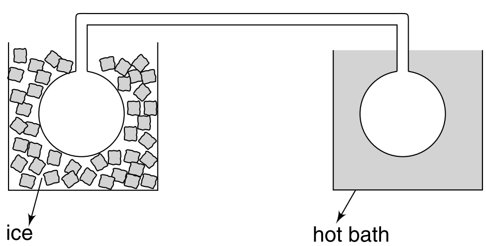



**Q. 8:** Tidal volume is that volume of air which is inhaled (and exhaled) in one breath by a human during quiet breathing. For a normal man this volume is close to 0.6 litre. 35% of oxygen (in terms of molecular count) gets converted into carbon – di – oxide in the exhaled air. Nearly 21% of the air that we breath in is oxygen. Assume that a man is breathing when the air is at STP and the moisture content of inhaled and exhaled air is same. Estimate the difference in mass of an inhaled breath and exhaled breath.

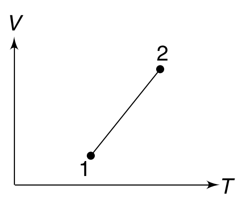
**Q. 9:** A cylindrical container of volume $V_0$ is divided into two parts by a thin conducting separator of negligible mass. The walls of the container are adiabatic. Ideal gases are filled in the two parts such that the pressures are $P_0$ and $2P_0$ when the separator is held in the middle of the container (see figure). Now the separator is slowly slid and released in a position where it stays in equilibrium. Find the volume of the two parts.




**Q. 10:** Two identical glass bulbs are interconnected by a thin tube of negligible volume. An ideal gas is filled in the bulbs at STP. One bulb is placed in a tub of melting ice and the other bulb is placed in a hot bath. The gas pressure in the bulbs becomes 1.5 times. Find the tem perature of the hot bath. Which bulb has more gas?




**Q. 11:** Two states –1 and 2 – of an ideal gas has been shown in *VT* graph. In which state is the pressure of the gas higher?




**Q. 12:** A cylindrical tube of cross sectional area $A$ and length $L$ is filled with an ideal gas. Temperature of the gas varies linearly from $T_0$ at one end to $2T_0$ at the other end. Calculate the number of mole of the gas in the tube assuming pressure to be uniform throughout equal to $P$.

**Q. 13:** Calculate mean free path length ($\lambda$) for molecules of an ideal gas at STP. It is known that molecular diameter is $2 \times 10^{-10}$ m.

**Q. 14:** A container contains a gas and a few drops of water at $T$K. The pressure in the container is 830 mm of Hg. The temperature of the container is reduced by 1%. Calculate the new pressure in the container. It is known that the gas is saturated with water vapour and the saturated vapour pressure at two temperatures are 30 mm and 25 mm of Hg.

**Q. 15:** A close container of volume 0.02 $m^3$ contains a mixture of neon and another gas $A$ of unknown molecular mass. The container contains 4 g of Neon and 24 g of gas $A$. At a temperature of 27°C the pressure of the mixture is $10^5$ N/$m^2$. Is there any possibility of finding gas $A$ in the atmosphere of a planet of radius 600 km and mean density $\rho = 5 \times 10^3$ kg/$m^3$?

Temperature of the planet is 2200°C.

Molar molecular mass of neon = 20 g;

Gas constant $R = 8.314$ J $mol^{-1}$ $K^{-1}$

Gravitational constant $G = 6.67 \times 10^{-11}$ $N \cdot m^2$ $kg^{-2}$

**Q. 16:** We know that volume ($V$) occupied by a substance scales as $x^3$, where $x$ is the average distance between its molecules. Assume that water vapour at 100°C and atmospheric pressure has average intermolecular separation equal to $x_v$ and for liquid water at 100°C the intermolecular separation is $x_w$. Find the ratio $\frac{x_v}{x_w}$ considering density of water at 100°C to be $1.0 \times 10^3$ kg/$m^3$ and taking water vapour as an ideal gas.

Atmospheric pressure $P_0 = 1.0 \times 10^5$ N/$m^2$; Gas constant $R = 8.3$ J $mol^{-1}$ $K^{-1}$.

**Q. 17:** The diameter of molecules of a gas, considered as sphere, is about $3 \times 10^{-10}$ m. Assume the entire volume occupied by the gas to be divided into cubic cells with one molecule per cell. Estimate the distance between molecules in terms of molecular diameter under standard conditions.

**Q. 18:** The core of the sun is a plasma. It is at a temperature of the order of $10^7$ K and contains equal number of protons and electrons. The density of the core of the Sun is $10^5$ kg $m^{-3}$. Assume that molar mass of proton is 1 g $mol^{-1}$ and the mass of an electron to be negligible compared to the mass of a proton. Estimate the pressure at the core of the sun. Assume that plasma behaves like an ideal gas.

**Q. 19:** Calculate the *rms* speed of He and $N_2$ molecules in the atmosphere at 300 K. Explain why our atmosphere has only a small amount of helium though it has large nitrogen content. The sun has temperature close to 6000 K. How can you explain the existence of He in the Sun.

**Q. 20:** A cylinder of oxygen, used as medical aid, has a volume of 16 L. It is filled to a pressure of $1.37 \times 10^7$ $Nm^{-2}$ above the atmospheric pressure at room temperature of 300 K. When in use, the flow rate at atmospheric pressure is 2.4 L/min. How long will the cylinder last? Atmospheric pressure is $10^5$ $Nm^{-2}$.

**Q. 21.** It is known that pressure of a gas increases if–

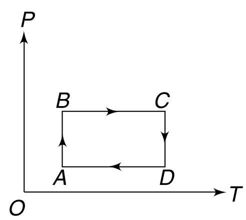
(A) You increase the temperature of the gas while holding its volume constant.

(B) You compress the gas holding its temperature constant.

(i) In which of the two cases [$A$ or $B$] the average impulse imparted by a gas molecule to the container wall during a collision increases?

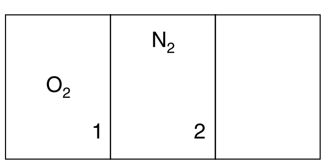
(ii) In which of the two cases the frequency of collisions of gas molecules with the container wall increases.

**Q. 22:** An ideal gas undergoes a cyclic process $A \to B \to C \to D$ for which $P - T$ graph is as shown. Draw $P - V$ and $V - T$ of graph to represent the same process.

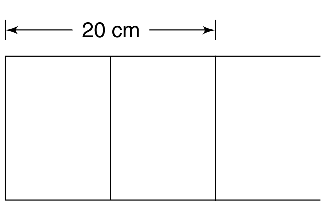



**Q. 23:** An insulated cylindrical vessel is divided into three identical parts by two partitions 1 and 2. The left part contains $O_2$ gas, the middle part has $N_2$ and the third chamber has vacuum. The average molecular speed in oxygen chamber is $V_0$ and that in nitrogen chamber is $\sqrt{\frac{8}{7}}V_0$. Pressure of the gases in two chambers is same. Partition 1 is removed and the gases are allowed to mix. Now the stopper holding the partition 2 is removed and it slides to the right wall of the container, so that the mixture of gases occupy the entire volume of the container.

Find the average speed of $O_2$ molecules now.




**Q. 24:** In a cylindrical container two pistons enclose gas in two compartments as shown in the figure. The pistons have negligible thickness and can move without friction. The system is originally in equilibrium. The outer piston is slowly moved out by 10 cm and the inner piston is found to move by 4 cm. Find the distance of the inner piston in equilibrium from the closed end of the cylinder if the outer piston is slowly moved out of the cylinder. Assume temperature of the gas to remain constant.

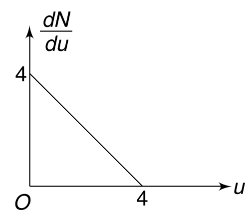



## LEVEL 2

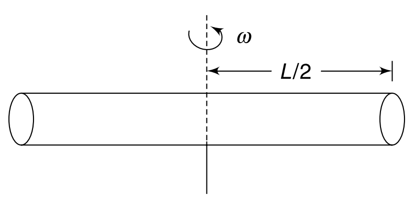
**Q. 25:** The *rms* speed of molecules of an ideal gas is $v_{rms}$. Obtain the expression for *rms* value of relative speed for all pair of molecules in a gas sample.

**Q. 26:** On a cold winter night you switch on a room heater to keep your room warm. Does it mean that the total energy of the air inside the room increases after you switch on the heater?

**Q. 27:** A hypothetical gas sample has its molecular speed distribution graph as shown in the figure. The speed ($u$) and $\frac{dN}{du}$ have appropriate units. Find the root mean square speed of the molecules. Do not worry about units.




**Q. 28:** A cylindrical container of length 2L is rotating with an angular speed $\omega$ about an axis passing through its centre and perpendicular to its length. Its contains an ideal gas of molar mass $M$. Calculate the ratio of gas pressure at the end of the container to the pressure at its centre. Neglect gravity and assume that temperature of the gas throughout the container is $T$.

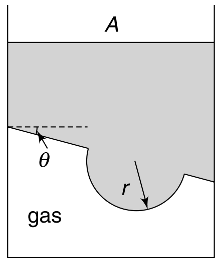



**Q. 29:** A container of volume $V$ = 16.6 litre contains a mixture of hydrogen and helium at a temperature of 300 K. The pressure in the container is $6 \times 10^5$ Pa and mass of the mixture is 10 g. Let $f_1$ and $f_2$ be the number of collisions made by the hydrogen and helium molecules with unit area of the container wall in unit time. Calculate the ratio $\frac{f_1}{f_2}$. [$R = 8.3$ J mol$^{-1}$ K$^{-1}$].

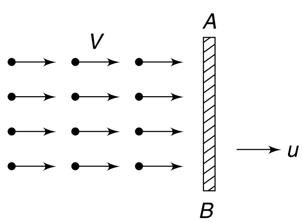
**Q. 30:** An ideal gas is enclosed in a cylinder having cross sectional area $A$. The piston of mass $M$ has its lower face inclined at $\theta$ to the horizontal. The lower face of the piston also has a hemispherical bulge of radius $r$. The atmospheric pressure is $P_0$.

(a) Find pressure of the gas.

(b) The piston is slowly pulled up by a distance $x$. During the process the piston is always maintained in equilibrium by adding heat to the gas. [It means if the piston is left at any stage it will stay there in equilibrium]. Find change in temperature of the gas. Number of mole of the gas in the cylinder is one.




**Q. 31:** A beam of gas molecules is incident normally on a plate $AB$. Each gas molecule has mass $m$ and velocity $V$. The incident beam falls on an area $A$ on the plate and all the molecule strike the plate elastically. Number of molecules in unit volume of the beam is $n$. When the plate is moved to right (see fig.) a force $F_1$ is needed to keep it moving with constant velocity $u(< V)$. When the plate is moved to left with constant velocity $u$, an external force $F_2$ is needed. Find $F_2 - F_1$.




**Q. 32:** Atmosphere of a planet contains only an ideal gas of molar mass $M_0$. The temperature of the atmosphere varies with height such that the density of atmosphere remains same throughout. The planet is a uniform sphere of mass $M$ and radius '$a$'. Thickness of atmosphere is small compared to '$a$' so that acceleration due to gravity can be assumed to be uniform throughout the atmosphere.

(a) Find the temperature difference between the surface of the planet and a point at height $H$ in its atmosphere

(b) If the *rms* speed of gas molecules near the surface of the planet is half the escape speed, calculate the temperature of the atmosphere at a height $H$ above the surface. Assume $\frac{C_P}{C_V} = \gamma$ for the atmospheric gas.

**Q. 33:** The Maxwell-Boltzmann distribution of molecular speeds in a sample of an ideal gas can be expressed as

$$f = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} \cdot dv$$

Where $f$ represent the fraction of total molecules that have speeds between $v$ and $v + dv \cdot m$, $k$ and $T$ are mass of each molecule, Boltzmann constant and temperature of the gas.

(a) What will be value of $\int_{v=0}^{v=\infty} f dv$?

(b) It is given that $\int_0^{\infty} v^3 e^{-av^2} dv = \frac{1}{2a^2}$

Find the average speed of gas molecules at temperature $T$.

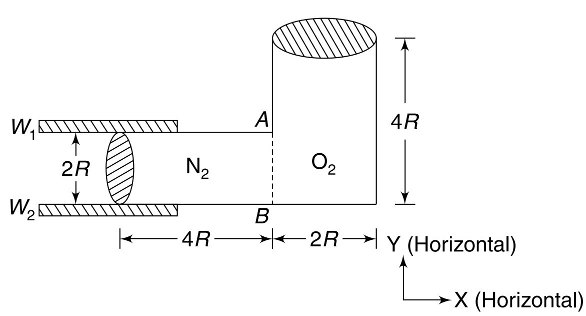
**Q. 34.** A large cylindrical tower is kept vertical with its ends closed. An ideal gas having molar mass $M$ fills the tower. Assume that temperature is constant throughout, acceleration due to gravity ($g$) is constant throughout and the pressure at the top of the tower is zero.

(a) Calculate the fraction of total weight of the gas inside the tower that lies above certain height $h$.

(b) At what height $h_0$ the quantity of gas above and below is same. How does the value of $h_0$ change with temperature of the gas in the tower?

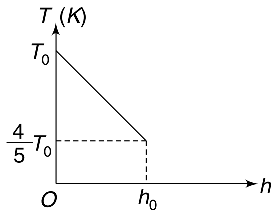
**Q. 35:** A physics book was found on a newly discovered island in 18th century. A problem in the book was as follows. "1 *pinch* of an ideal gas is kept in a container of volume 1.5 *volka*. When the temperature is 40 *tapu*, the gas pressure is 25 *phatka*. When the temperatue is reduced to –20 *tapu* the gas pressure becomes 10 *phatka*. Find the temperature of absolute zero in *tapu*.

**Q. 36:** A $L$ shaped container has dimensions shown in the fig. It has circular cross section of radius $R$. It is placed on a smooth horizontal table. The container is divided into two equal sections by a membrane $AB$. One section contains nitrogen and the other one contains oxygen. Temperature of both sides is same but pressure in the compartment having $N_2$ is 4 times that in the other compartment. Due to some reason the membrane gets punctured. Find the distance moved by the cylinder if the cylinder is constrained to move in $x$ direction only [with the help of guiding walls $W_1$ and $W_2$]. There is no friction anywhere and mass of the cylinder and membrane is negligible.



**Q. 37:** (a) Assume the atmosphere as an ideal gas in static equilibrium at constant temperature $T_0$. The pressure on the ground surface is $P_0$. The molar mass of the atmosphere is $M$. Calculate the atmospheric pressure at height $h$ above the ground.

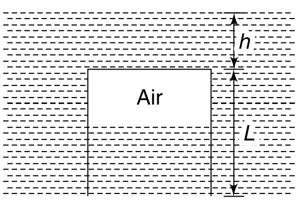
(b) To be more realistic, let as assume that the temperature in the troposphere (lower part of the atmosphere) decreases with height as shown in the figure. Now calculate the atmospheric pressure at a height $h_0$ above the ground.




**Q. 38:** A cylindrical container has cross sectional area of $A = 0.05\ m^2$ and length $L = 0.775$ m. Thickness of the wall of the container as well as mass of the container is negligible. The container is pushed into a water tank with its open end down. It is held in a position where its closed end is $h$ = 5.0 m below the water surface. What force is required to hold the container in this position? Assume temperature of air to remain constant.

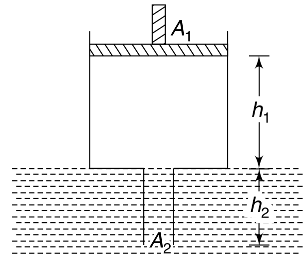
Atmospheric pressure $P_0 = 1 \times 10^5$ Pa; Acceleration due to gravity $g = 10\ m/s^2$

Density of water = $10^3\ kg/m^3$




**Q. 39:** A syringe has two cylindrical parts of cross sectional area $A_1 = 4\ cm^2$ and $A_2 = 1\ cm^2$. A mass less piston can slide on the inner wall of the part having cross section $A_1$. The end of the part having cross section $A_2$ is open. Syringe is dipped in a large water tank such that the narrower part remains completely submerged and the wider part is filled with air. The length shown in figure are $h_1$ = 55 cm and $h_2$ = 100 cm. The piston is pushed down by a distance $x$ such that 25% of the air inside the syringe is expelled out through the open end of the narrower part. Assume that the temperature of air remains constant and calculate $x$.

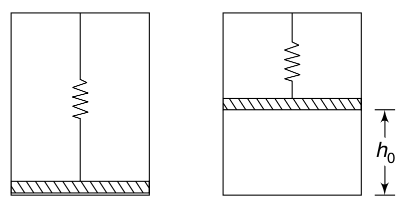
Density of water = $10^3$ k $gm^{-3}$; g = 10 $ms^{-2}$; Atmospheric pressure = $10^5\ Nm^{-2}$.




**Q. 40:** If an ideal gas is allowed to undergo free expansion it does not cool. But a real gas cools during free expansion. Why?

**Q. 41:** A container has a gas that consists of positively charged ions. The gas undergoes a free expansion in which there is no heat exchange with surrounding. Does the temperature of the gas increases or decreases? Why?

**Q. 42:** A freely sliding massive piston is supported by a spring inside a vertical cylinder as shown. When all air is pumped out of the container, the piston remains in equilibrium with only a tiny gap between the piston and the bottom surface of the cylinder. An ideal gas at temperature $T_0$ is slowly injected under the piston so that it rises to height $h_0$ (see figure). Calculate the height of the piston from the bottom of the container if the temperature of the gas is slowly raised to $4T_0$.



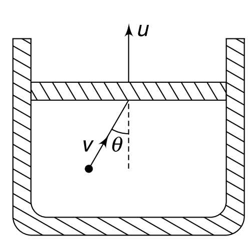

## LEVEL 3

**Q. 43:** An ideal gas is inside a cylinder with a piston that can move freely. The walls of the cylinder and piston are non- conducting. The piston is being moved out of the cylinder at a constant speed $u$.

(a) Consider a gas molecule of mass $m$ moving with speed $v (>> u)$. It hits the piston elastically at an angle of incidence $\theta$. Calculate the loss in kinetic energy of the molecule.

(b) If the area of piston is A and pressure of the gas is $P$, calculate the rate of decreases of molecular kinetic energy of the gas sample.

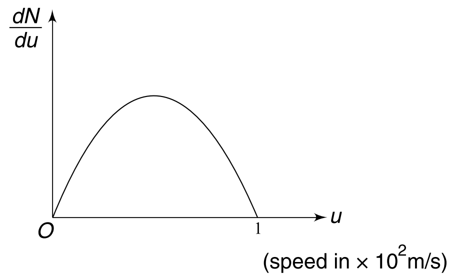
(c) If $u$ >> molecular velocities, at what rate will the gas lose its molecular kinetic energy?



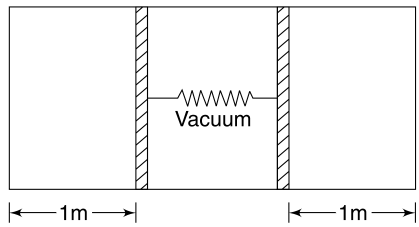
**Q. 44:** The speed distribution of molecules in a sample of a gas is shown in the figure. The graph between $\frac{dN}{du}$ and $u$ is a parabola and total number of molecules in the sample is $N_0$.

(a) Calculate the *rms* speed of the molecules.

(b) Calculate the total translational kinetic energy of molecules if mass of the sample is 10 g.




**Q. 45:** A cylindrical container is divided into three parts by two tight fitting pistons. The pistons are connected by a spring. The region between the pistons is vacuum and the other two parts have same number of moles of an ideal gas. Initially, both the gas chambers are at temperature $T_0$ and the spring is compressed by 1 m. Length of both gas chambers is 1 m in this position. Now the temperature of the left and right chambers are raised to $\frac{4T_0}{3}$ and $\frac{5T_0}{3}$ respectively. Find the final compression in the spring in equilibrium. Assume that the pistons slide without friction.




**Q. 46:** A helium balloon has its rubber envelope of weight $w_{\text{Rubber}}$ and its is used to lift a weight of $w_{\text{Load}}$. The volume of fully inflated balloon is $V_0$. The atmospheric temperature is $T_0$ throughout and the atmospheric pressure is known to change with height $y$ according to equation $P = P_0 e^{-ky}$ where $k$ is a positive constant and $P_0$ is atmospheric pressure at ground level. The balloon is inflated with sufficient amount of helium so that the net upward force on is $F_0$ at the ground level. Assume that pressure inside the balloon is always equal to the outside atmospheric pressure. Density of air and helium at ground level is $\rho_a$ and $\rho_{\text{He}}$ respectively.

(a) Find the number of moles ($n$) of the helium in the balloon.

For following two questions assume $n$ to be a known quantity.

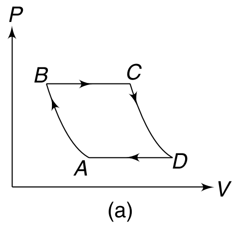
(b) Find the height ($y_0$) at which the balloon is fully inflated.

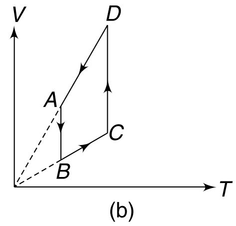
(c) Prove that the balloon will be able to rise to height $y_0$ if

$$\frac{nRT_0 g}{P_0}(\rho_a - \rho_{\text{Hg}}) > w_{\text{Rubber}} + w_{\text{Load}}.$$

## ANSWERS

1. Pressure on $AB$ will be higher than pressure on $CD$.
2. Two
3. (a) $\sqrt{3}\,T$
4. $P = \left(\frac{2N}{N_A}\right)\frac{RT}{V}$
5. $\frac{7}{8}$
6. $\frac{(P_1V_1M_1T_2 + P_2V_2M_2T_1)P_0}{(P_1V_1T_2 + P_2V_2T_1)RT_0}$
7. 16.27 cm
8. 0.24 mg
9. $\frac{2V_0}{3}$ and $\frac{V_0}{3}$
10. 819 K
11. State 1
12. $\frac{PAL}{RT_0}\ln 2$
13. $2.0 \times 10^{-7}$ m
14. 817 mm of Hg
15. No
16. 12
17. Ten times the molecular diameter
18. $2 \times 10^{16}\ P_a$
20. 920 min
21. (a) $A$ (ii) $A$ and $B$

**22.** 


**23.** $V_0$

**24.** 8 cm.

**25.** $\sqrt{2}\,v_{rms}$

**26.** No

**27.** $u_{rms} = \sqrt{\frac{8}{3}}$

**28.** $e^{\frac{M\omega^2L^2}{2RT}}$

**29.** $3\sqrt{2}$

**30.** (a) $P_0 + \frac{\text{Mg}}{A}$ (b) $\frac{(P_0A + \text{Mg})x}{R}$

**31.** 8 m *An Vu*

**32.** (a) $\Delta T = \frac{GMM_0H}{Ra^2}$ (b) $T_H = \frac{GMM_0}{R_a}\left[\frac{1}{2\gamma} - \frac{H}{a}\right]$

**33.** (a) 1 (b) $\sqrt{\frac{8\text{kT}}{\pi m}}$

**34.** (a) $e^{\frac{-Mgh}{RT}}$ (b) $\left(\frac{RT}{\text{Mg}}\right)\ln 2$

(c) $h_0$ increase with $T$.

**35.** $-60$ *tapu*

**36.** 0.83 $R$ towards left

**37.** (a) $P = P_0 e^{\frac{-Mgh}{RT}}$ (b) $P = P_0\left(\frac{5}{4}\right)^{-\frac{5Mgh_0}{RT_0}}$

**38.** 250 N

**39.** 42.5 cm

**41.** Temperature increases

**42.** $2h_0$

**43.** (a) $2mvu\cos\theta$ (b) $PAu$ (c) zero

**44.** (a) $v_{rms} = \sqrt{\frac{3}{10}} \times 100$ m/s

(b) $K_T = 15$ J

**45.** $\frac{\sqrt{13} - 1}{2}$ m

**46.** (a) $n = \frac{P_0(F_0 + w_{\text{Rubber}} + w_{\text{Load}})}{RT_0(\rho_a - \rho_{\text{He}})g}$

(b) $y_0 = \frac{1}{k}\ln\left(\frac{P_0V_0}{nRT_0}\right)$

## SOLUTIONS

**1.** **Hint:** When a molecule strikes $AB$, it will gain energy & rebound with higher speed. Change in momentum will be higher.

**2.** Let degree of freedom for gas molecules be $f$

$$\gamma = \frac{f+2}{f}$$

After freezing of rotational degree of freedoms the molecules can have translational kinetic energy only and degree of freedom will become = 3

$$\gamma' = \frac{3+2}{3} = \frac{5}{3}$$

Given $$\frac{25}{21}\frac{f+2}{f} = \frac{5}{3} \Rightarrow f = 5$$

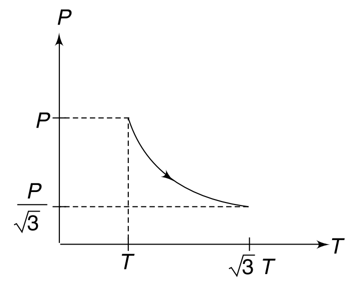
In means the gas is diatomic

**3.** (a) $$\frac{P^2}{\rho} = \text{a const} \qquad \text{...(i)}$$

From $$PV = nRT$$

$$\frac{P}{\rho} = \frac{RT}{M} \qquad \text{...(ii)}$$

Using (i) and (ii) $PT$ = a constant ...(iii)




Using (i) we can say that when density changes from $\rho$ to $\frac{\rho}{3}$, the pressure will change from $P$ to $\frac{P}{\sqrt{3}}$ and using (iii) it can be concluded that temperature will change from $T$ be $\sqrt{3}\,T$

(b) $PT$ = constant

**4.** 2 atoms of $X$ combine with 1 atoms of $Y$

$\therefore$ $\frac{N}{2}$ molecule of $X_2Y$ is formed and $\frac{3N}{2}$ atoms of $Y$ is left as it is.

$\therefore$ Total number of particles $= \frac{N}{2} + \frac{3N}{2} = 2N$

Pressure is given by $PV = nRT$

$$\Rightarrow \quad P = \left(\frac{2N}{N_A}\right)\frac{RT}{V}$$

**5.** For an ideal gas $PV = nRT$

$$\Rightarrow \quad PV = \frac{m}{M}RT \quad \therefore \quad MV = \frac{mRT}{P} = \text{a constant}$$

This is because the two gases have equal mass, same pressure and temperature in equilibrium.

$$\therefore \quad M_A V_A = M_B V_B$$

$$32\,(X_1 A) = 28\,(X_2 A) \quad \therefore \quad \frac{X_1}{X_2} = \frac{7}{8}$$

**6.** $$n_1 = \frac{P_1 V_1}{RT_1} \text{ and } n_2 = \frac{P_2 V_2}{RT_2}$$

For the mixture $$P_0 V_0 = (n_1 + n_2)RT_0$$

$$V_0 = (n_1 + n_2)\frac{RT_0}{P_0}$$

$\therefore$ Density of mixture $$\rho = \frac{n_1 M_1 + n_2 M_2}{V_0} = \frac{(n_1 M_1 + n_2 M_2)P_0}{(n_1 + n_2)RT_0}$$

$$= \frac{\left(\frac{P_1 V_1 \mathrm{M}_1}{RT_1} + \frac{P_2 V_2 \mathrm{M}_2}{RT_2}\right)P_0}{\left(\frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2}\right)RT_0} = \frac{(P_1 V_1 T_2 M_1 + P_2 V_2 T_1 M_2)P_0}{(P_1 V_1 T_2 + P_2 V_2 T_1)RT_0}$$

**7.** Let $P_1$ and $P_2$ be pressure (in cm of Hg) of the air trapped in the shorter and the longer arms respectively. Let the required height of Hg column in the longer tube be $h$.

For air in shorter arm ($PV$ = const)

$$(80\text{ cm})(40\text{ cm. }A) = P_1(40 - 10)A \quad \Rightarrow \quad P_1 = \frac{320}{3}\text{ cm}$$

Similarly, for the other arm

$$(80\text{ cm})(80\text{ cm. A}) = P_2(80 - h)A$$

$$\therefore \quad P_2 = \frac{80 \times 80}{80 - h}$$

Equating the pressure in Hg just below the surface in shorter arm to the pressure at same horizontal level in the other arm–

$$P_2 + (h - 10) = P_1$$

$$\frac{80 \times 80}{80 - h} + h - 10 = \frac{320}{3}$$

$$3[6400 + 90h - 800 - h^2] = 25600 - 320h$$

$$3h^2 - 590h + 8800 = 0$$

$$h = \frac{590 \pm \sqrt{348100 - 105600}}{6}$$

$$= \frac{590 \pm 492.4}{6} = \frac{97.6}{6} = 16.27 \text{ cm}$$

[+ve sign is unacceptable]

**8.** Number of moles of air in one breath $= \dfrac{0.6 \text{ litre}}{22.4 \text{ litre}} = 0.027$ mole

Number of moles of $O_2$ in inhaled air $= 0.027 \times 0.21$

No. of moles of $O_2$ converted into $CO_2$ is

$$= 0.027 \times 0.21 \times 0.35 = 1.98 \times 10^{-3}$$

$\therefore$ Difference in mass of air

$$= 1.98 \times 10^{-3}\left[M_{CO_2} - M_{O_2}\right]$$

$$= 1.98 \times 10^{-3}\left[44\frac{\text{g}}{\text{mol}} - 32\frac{\text{g}}{\text{mol}}\right]$$

$$= 2.4 \times 10^{-2} \text{ g} = 0.24 \text{ mg}$$

**9.** Since the separator is conducting, the temperature in both parts is same (say, $T_0$). Number of moles in two chambers are

$$n_1 = \frac{2P_0V_0}{2RT} = \frac{P_0V_0}{RT} \text{ and } n_2 = \frac{P_0V_0}{2RT}$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{2}{1}$$

When the separator is in equilibrium, the pressure on two sides is same.

Let volume of upper and lower parts be $V_1$ and $V_2$ respectively and the common pressure and temperature be $P$ and $T$

$$PV_1 = n_1RT \quad \text{and} \quad PV_2 = n_2RT$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{2}{1}$$

Also $$V_1 + V_2 = V_0$$

$$\therefore \quad V_1 = \frac{2V_0}{3} \quad \text{and} \quad V_2 = \frac{V_0}{3}$$

**10.** Let volume of each bulb be $V_0$

Number of moles in each bulb $n_0 = \dfrac{P_0V_0}{RT_0}$

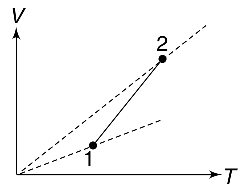
Number of moles in the bulb kept in ice $= \dfrac{1.5P_0V_0}{RT_0}$

Number of moles in the other bulb = $\dfrac{1.5 P_0 V_0}{RT}$

$$\therefore \quad \frac{1.5 P_0 V_0}{RT} + \frac{1.5 P_0 V_0}{RT_0} = \frac{2 P_0 V_0}{RT_0}$$

$$\frac{1.5}{T} + \frac{1.5}{T_0} = \frac{2}{T_0}$$

$$\Rightarrow \quad \frac{1.5}{T} = \frac{0.5}{T_0} \quad \Rightarrow \quad T = 3T_0 = 819 \text{ K}$$

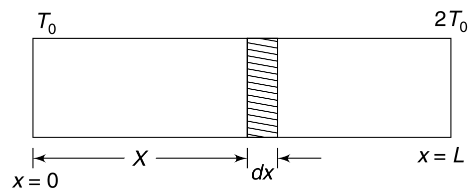



**11.** $$PV = nRT$$

$$\Rightarrow \quad V = \frac{nR}{P} T$$

If we draw constant pressure line on $VT$ graph, the slope will be higher for lower pressure. Hence $P_2 < P_1$.

**12.** Temperature at distance $x$ from the colder end is–




$$T = T_0 + \frac{T_0}{L}x$$

Consider a volume $A\,dx$ as shown in the figure.

Number of moles in this volume is

$$dn = \frac{P(A\,dx)}{RT} = \frac{PA}{RT_0} \frac{dx}{\left(1 + \frac{x}{L}\right)}$$

$$\therefore \quad n = \frac{PA}{RT_0} \int_0^L \frac{dx}{1 + \frac{x}{L}} = \frac{PAL}{RT_0} \left[\ln\left(1 + \frac{x}{L}\right)\right]_0^L$$

$$= \frac{PAL}{RT_0} \ln 2$$

**13.** 22.4 litre volume of the gas has $6.02 \times 10^{23}$ molecules.

Number of molecule per unit volume is $n = \dfrac{6.02 \times 10^{23}}{22.4 \times 10^{-3}} = 2.7 \times 10^{25}$ molecules m$^{-3}$

Mean free path length is given by $\lambda = \dfrac{1}{\sqrt{2}\,\pi n d^2}$

$$= \frac{1}{1.414 \times 3.14 \times 2.7 \times 10^{25} (2 \times 10^{-10})^2} = 2.0 \times 10^{-7} \text{ m}$$

**14.** By Dalton's law of partial pressure, the partial pressure of gas at $TK$ is = (830 – 30) mm of Hg

Let the pressure in container be $P$ at new temperature of $\left(T - \dfrac{T}{100}\right)K$

The partial pressure of gas is ($P$ – 25) mm of Hg

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\therefore \qquad \frac{800}{T} = \frac{(P-25)}{T\cdot\frac{99}{100}}$$

$$\Rightarrow \qquad P = 25 + \frac{800\times 99}{100} = 817\,\text{mm of Hg}$$

**15.** Number of moles of gas mixture in the container

$$n = \frac{PV}{RT} = \frac{10^5 \times 0.02}{8.314\times 300}$$

$$n_{\text{Ne}} + n_A = 0.8$$

$$\frac{4}{20} + n_A = 0.8 \quad \Rightarrow \quad n_A = 0.6$$

$\therefore$ Molar mass of gas $A$ is $\quad m = \frac{24}{0.6} = 40$ g

Let's find the *rms* speed of gas molecules of $A$ at temperature of 2200°C = 2473 $K$

$$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\times 8.314\times 2473}{40\times 10^{-3}}} = 1242 \text{ m/s}$$

The escape speed on the surface of a planet is

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\frac{4}{3}\pi R^3\cdot\rho}{R}} = \sqrt{\frac{8\pi}{3}G\rho}\,.R$$

$$= \sqrt{\frac{8\times 3.14\times 6.67\times 10^{-11}\times 5\times 10^3}{3}}\times 600\times 10^3$$

$$= 16.7\times 10^{-4}\times 6\times 10^5 \simeq 1000\,\text{m/s}$$

Because $V_{rms} > V_e$, it is unlikely that gas $A$ will be found in atmosphere of the planet.

**16.** Consider $m$ mass of water as well as vapour. Volumes are

$$V_w = \frac{m}{\rho_w} \text{ and}$$

$$V_v = \frac{nRT}{P_0} \qquad [\because \; PV = nRT]$$

$$= \frac{mRT}{MP_0} \quad [M = \text{molar mass of water} = 1.8\times 10^{-2}\text{ kg mol}^{-1}]$$

$$\therefore \qquad \frac{V_v}{V_w} = \frac{RT\rho_w}{MP_0}$$

$$\therefore \qquad \left(\frac{x_v}{x_w}\right)^3 = \frac{RT\rho_w}{MP_0} = \frac{8.3\times 373\times 1.0\times 10^3}{1.8\times 10^{-2}\times 1.0\times 10^5} = 1720$$

$$\therefore \qquad \frac{x_v}{x_w} \simeq 12$$

**17.** Volume of 1 mole at STP is 22.4 litre $= 22.4 \times 10^{-3}\,\text{m}^3$

Number of molecules in one mole $= 6.02 \times 10^{23}$

$\therefore$ Volume of each cubic cell $= \dfrac{22.4 \times 10^{-3}}{6.02 \times 10^{23}} = 3.7 \times 10^{-26}\,\text{m}^3$

Side length of each cell $= (3.7 \times 10^{-26})^{1/3} = 3.3 \times 10^{-9}\,\text{m}$

Molecular diameter $= 3 \times 10^{-10}\,\text{m}$

$\therefore$ Distance between molecules is about 10 times the molecular diameter.

**18.** $1\,\text{m}^3$ volume of matter has a mass of $10^5$ kg and is mostly due to protons.

$n_p$ = number of moles of proton in $1\,\text{m}^3$ volume.

$$= \frac{\text{mass of protons}}{\text{Molar mass of proton}} = \frac{10^5}{10^{-3}} = 10^8 \text{ mol.}$$

The matter has equal number of electrons. Hence total count of particles is

$$2n_p = 2 \times 10^8 \text{ mol.}$$

$$\therefore \quad P = \frac{nRT}{V} = \frac{2 \times 10^8 \times 8.314 \times 10^7}{1} = 2 \times 10^{16} P_a$$

**19.**

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{\text{He}} = \sqrt{\frac{3 \times 8.214 \times 300}{4 \times 10^{-3}}} = 1.37 \times 10^3 \text{ ms}^{-1} = 1.37 \text{ kms}^{-1}$$

$$V_{N_2} = \sqrt{\frac{3 \times 8.314 \times 300}{28 \times 10^{-3}}} = 0.5 \times 10^3 \text{ ms}^{-1} = 0.5 \text{ kms}^{-1}$$

The escape speed from the surface of the earth is 11.2 $\text{kms}^{-1}$. A significant fraction of He molecules will have speed in excess of escape speed. Thus, they leak away into space. Over time the He content has decreased to almost nothing.

Escape speed on the sun is too high due to its large mass.

**20.** Let the volume of entire gas at atmospheric pressure be $V$.

$$P_1 V_1 = P_2 V_2 \qquad [P_2 = 1.37 \times 10^7 + 10^5 \simeq 1.38 \times 10^7]$$

$$10^5 V = 1.38 \times 10^7 \times 16$$

$$\therefore \quad V = 1.38 \times 16 \times 10^2 \text{ litre}$$

$\therefore$ Time for which the cylinder can last is $t = \dfrac{1.38 \times 16 \times 10^2}{2.4} = 920$ min.

**21.** (i) In case (A) the speed of molecules increase. Therefore, they impart more impulse during each collision. In case (B) the speed of molecules do not change since temperature is held constant.

(ii) Frequency of collisions will increase in both cases. In case (A) it happens because of more speed and in case (B) it happens because of lesser distance to be travelled by molecules between two collisions.

**23.** $$V_0 \propto \sqrt{\frac{T_0}{M_0}} \quad \text{and} \quad V_N \propto \sqrt{\frac{T_N}{M_N}}$$

$$\therefore \quad \frac{V_N}{V_0} = \sqrt{\frac{M_0 T_N}{M_N T_0}}$$

$$\frac{8}{7} = \frac{32\,T_N}{28\,T_0} \quad \therefore \quad T_N = T_0$$

Since pressure and volume of two chambers are also same we conclude that number of moles of both gases must be same.

$\therefore$ After mixing the temperature remains unchanged. In pushing partition 2 the gas does no work since there is vacuum to the right of the partition. It means internal energy of gas mixture does not change. Thus average speed of molecules do not change.

**24.** Initial pressure in both chambers = atmospheric pressure ($P_0$)

Let initial volume of the left and right chambers be $V_1$ and $V_2$ respectively, and the area of cross section of the cylinder be $A$.

For left chamber using $PV$ = constant we get–

$$P(V_1 + 4A) = P_0 V_1 \qquad \text{...(i)}$$

$4A$ is change in volume of the compartment.

Similarly for right chamber $P(V_2 + 6A) = P_0 V_2$ ...(ii)

Where $P$ = final common pressure of the two chambers after the outer piston has been moved by 10 cm

(i) ÷ (ii) gives $$\frac{V_1 + 4A}{V_2 + 6A} = \frac{V_1}{V_2}$$

$$\Rightarrow \quad 4V_2 = 6V_1 = \frac{V_1}{V_2} = \frac{2}{3}$$

$\therefore$ If original distance of inner piston from the closed end is $x_1$ then $x_1 = \frac{20}{5} \times 2 = 8$ cm

After the outer piston is removed, the inner piston will come to equilibrium at its original location only; because pressure will become equal to atmospheric pressure.

Hence answer is 8 cm.

**25.** Consider two molecules having velocities $\vec{v}_1$ and $\vec{v}_2$. Relative velocity of these two molecules

$$\overrightarrow{v_{12}} = \vec{v}_1 - \vec{v}_2$$

$$\therefore \quad \overrightarrow{v_{12}} \cdot \overrightarrow{v_{12}} = (\vec{v}_1 - \vec{v}_2) \cdot (\vec{v}_1 - \vec{v}_2)$$

$$v_{12}^2 = v_1^2 + v_2^2 - 2\vec{v}_1 \cdot \vec{v}_2$$

Taking average for all pair of molecules

$$\langle v_{12}^2 \rangle = \langle v_1^2 \rangle + \langle v_2^2 \rangle - 2\langle \vec{v}_1 \cdot \vec{v}_2 \rangle$$

But $$\langle \vec{v}_1 \cdot \vec{v}_2 \rangle = \langle v_1 \cdot v_2 \cos\theta \rangle = 0$$

This is because $\cos\theta$ will be negative for obtuse angle between $\vec{v}_1$ and $\vec{v}_2$ and positive for acute angles. Number of molecules is large.

$$\therefore \quad \langle v_{12}^2 \rangle = \langle v_1^2 \rangle + \langle v_2^2 \rangle$$

$$\langle v_1^2 \rangle = \langle v_2^2 \rangle = \langle v^2 \rangle \text{ [say]} = \langle v_{rms}^2 \rangle$$

$$\therefore \qquad \langle v_{12}^2 \rangle = v_{rms}^2$$

$$\sqrt{\langle v_{12}^2 \rangle} = \sqrt{2}\, v_{rms}$$

**26.** The pressure inside the room = outside pressure ($P_0$)
Volume of the room $V_0$ is a constant.

$$\therefore \qquad P_0 V_0 = nRT$$

$$\Rightarrow \qquad nT = \frac{P_0 V_0}{R} = \text{a constant}$$

$$\therefore \qquad U = nC_v T = \text{a constant}$$

Total energy of air in the room does not change (!)
However, with rise in temperature the average energy of molecules does increase. Actually, some molecules are expelled out of the room and total number of molecules in the room decreases.

**27.** The equation of the straight line is

$$\frac{dN}{du} = -u + 4$$

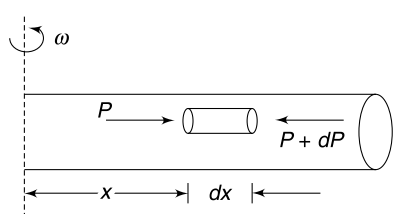
$$u_{rms}^2 = \frac{\int u^2 dN}{\int dN} = \frac{\int_0^4 u^2 \frac{dN}{du} du}{\int_0^4 \frac{dN}{du} du} = \frac{\int_0^4 u^2(-u+4)\, du}{\text{area under the graph}}$$

$$= \frac{\int_0^4 (-u^3)\, du + 4\int_0^4 u^2 du}{\frac{1}{2} \times 4 \times 4} = \frac{\left[-\frac{u^4}{4} + 4 \cdot \frac{u^3}{3}\right]_0^4}{8}$$

$$u_{rms}^2 = \frac{8}{3} \quad \Rightarrow u_{rms} = \sqrt{\frac{8}{3}}$$

**28.** For an ideal gas $\qquad \dfrac{P}{\rho} = \dfrac{RT}{M}$

$$\therefore \qquad \rho = \frac{PM}{RT} \qquad \text{...(i)}$$




Consider a cylindrical element of the gas at a distance $x$ from the axis of rotation.
Mass of element = $\rho\, A\, dx$
[$A$ = cross section of the element]
Net force on the element towards centre is = $A\, dP$

This must be equal to the centripetal force

$$\therefore \quad AdP = (\rho A\,dx)\,\omega^2 x$$

$$dP = \rho\omega^2 x dx$$

$$dP = \frac{PM}{RT}\omega^2 x\,dx \quad \text{[using (i)]}$$

$$\therefore \quad \int_{P_0}^{P} \frac{dP}{P} = \frac{M\omega^2}{RT}\int_0^L x\,dx$$

[$P_0$ = pressure at centre, $P$ = pressure at the end]

$$\therefore \quad \ln\frac{P}{P_0} = \frac{M\omega^2 L^2}{2RT}$$

$$\therefore \quad \frac{P}{P_0} = e^{\frac{M\omega^2 L^2}{2RT}}$$

**29.** For the gas mixture $nRT = pV$

$$\Rightarrow \quad \left(\frac{m_H}{m_H} + \frac{m_{He}}{M_{He}}\right)RT = PV$$

$$\Rightarrow \quad \left(\frac{m_H}{2} + \frac{m_{He}}{4}\right) \times 8.3 \times 300 = 6 \times 10^5 \times 16.6 \times 10^{-3}$$

$$\Rightarrow \quad \frac{m_H}{2} + \frac{m_{He}}{4} = \frac{6 \times 16.6 \times 10^2}{8.3 \times 300} = 4 \text{ gram}$$

$$\Rightarrow \quad 2m_H + m_{He} = 16 \qquad \text{...(i)}$$

Given $\quad m_H + m_{He} = 10$

Solving $\quad m_H = 6\text{g};\ m_{He} = 4\text{g}$

$$n_H = 3\,\text{mol.};\ n_{He} = 1 \text{ mol.}$$

Partial pressure of the two gases

$$\frac{P_H}{P_{He}} = \frac{n_H}{n_{He}} = \frac{3}{1}$$

Now pressure = (change in momentum of a molecule during one collision) × (frequency of collisions per unit area)

$$\therefore \quad P = 2mV_{rms}\cdot f$$

$$\therefore \quad \frac{P_H}{P_{He}} = \frac{(2mV_{rms}f)_H}{(2mV_{rms}f)_{He}}$$

$$\Rightarrow \quad \frac{3}{1} = \frac{M_H}{M_{He}}\sqrt{\frac{M_{He}}{M_H}}\cdot\frac{f_1}{f_{He}}$$

$$\Rightarrow \quad \frac{f_1}{f_2} = \frac{3}{1}\sqrt{\frac{M_{He}}{M_H}} = 3\sqrt{2}$$

**30.** (a) The gas applies force on the piston, normal to its surface. When we take projection of this force in vertical direction it will be simply $PA$.

$$\therefore \quad PA = P_0A + Mg$$

$$\therefore \quad P = P_0 + \frac{Mg}{A} \quad \text{...(i)}$$

(b) Pressure of the gas is always $P$ as given by (i)

Change in volume $\Delta V = Ax$

$$PV = nRT$$

$$\therefore \quad P\Delta V = nR\Delta T \quad [\because P = \text{a constant}]$$

$$\therefore \quad \Delta T = \frac{P\Delta V}{nR} = \frac{\left(P_0 + \frac{Mg}{A}\right)A \cdot x}{nR}$$

$$= \frac{(P_0A + Mg)x}{nR} = \frac{(P_0A + Mg)x}{R}$$

**31. When plate is moving to right:**

Molecules bounce back with speed $V - 2u$

$\therefore$ Change in momentum of one molecule

$$\Delta P_1 = m(V - 2u) - (-mV) = 2mV - 2mu$$

Number of molecule hitting per unit time $n_1 = A(V - u)n$

$$\therefore \quad F_1 = n_1\Delta P_1 = An(V - u)2m(V - u) = 2mnA(V - u)^2$$

**When plate is moving to left:**

Molecule bounce back with speed $V + 2u$

$\therefore$ Change in momentum of one molecule

$$\Delta P_2 = m(V + 2u) - (-mV) = 2mV + 2mu$$

And number of molecule hitting per unit time

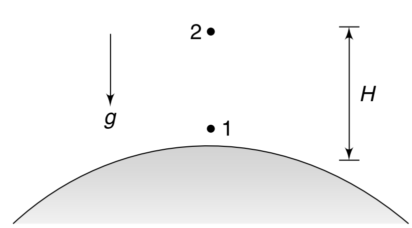
$$n_2 = A(V + u) \cdot n$$

$$\therefore \quad F_2 = n_2\Delta P_2 = An(V + u)2m(V + u) = 2mAn(V + u)^2$$

$$\therefore \quad F_2 - F_1 = 2mnA[(V + u)^2 - (V - u)^2]$$

$$= 8mnAV \cdot u$$

**32.** (a) For an ideal gas $PV = nRT$




$$PV = \frac{m}{M_0}RT$$

$$\frac{P}{\rho} = \frac{RT}{M_0} \quad \left[\rho = \frac{m}{V} = \text{density}\right]$$

Pressure difference between a point on surface and a point at a height $H$ is given by

$$P_1 - P_2 = \rho g H \quad \Rightarrow \quad \frac{P_1}{\rho} - \frac{P_2}{\rho} = gH$$

$$\frac{RT_1}{M_0} - \frac{RT_2}{M_0} = gH \quad \therefore \quad T_1 - T_2 = gH\,\frac{M_0}{R}$$

But $$g = \frac{GM}{a^2}$$

$\therefore$ $$T_1 - T_2 = \frac{GMM_0H}{Ra^2} \qquad \text{...(i)}$$

(b) Escape speed $$V_e = \sqrt{\frac{2GM}{a}}$$

$\therefore$ $$\sqrt{\frac{\gamma RT_1}{M_0}} = \sqrt{\frac{GM}{2a}}$$

$\therefore$ $$T_1 = \frac{GMM_0}{2\gamma Ra}$$

From (i) $$T_2 = \frac{GMM_0}{Ra}\left[\frac{1}{2\gamma} - \frac{H}{a}\right]$$

**33.** (a) The given integral must be equal to 1. $f$ is fraction of molecules having speed between $v$ and $v + dv$. If all such fractions are summed up it must be 1.

(b) $$v_{av} = \int_0^\infty v f\,dv$$

$$= \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}\int_0^\infty v^3\, e^{-\frac{mv^2}{2kT}}\,dv$$

Let $$\frac{m}{2kT} = a$$

$$v_{av} = \frac{4}{\sqrt{\pi}}\,a^{3/2}\int_0^\infty v^3 e^{-av^2}\,dv$$

$$= \frac{4}{\sqrt{\pi}}\,a^{3/2}\,\frac{a^{-2}}{2} = \frac{2}{\sqrt{\pi a}}$$

$$v_{av} = \sqrt{\frac{8kT}{\pi m}}$$

**34.** $P$ = pressure at height $y$

$P + dP$ = pressure at height $y + dy$

$$dP = -\rho g\,dy \qquad \text{...(i)} \quad [\rho = \text{density of gas}]$$

From ideal gas equation $$PV = nRT$$

$$\Rightarrow \qquad \frac{P}{\rho} = \frac{RT}{M}$$

$$\therefore \qquad \text{from (i) } dP = -\frac{Mg}{RT} P\,dy$$

$$\int_{P_0}^{P} \frac{dP}{P} = -\frac{Mg}{RT}\int_0^h dy \quad \Rightarrow \quad \ln\frac{P}{P_0} = -\frac{Mgh}{RT}$$

$$\therefore \qquad P = P_0 e^{-\frac{Mgh}{RT}} \qquad \text{...(ii)}$$

(a) Pressure at height $h = \dfrac{\text{wt. of gas above}}{\text{area}}$

Pressure at bottom $= \dfrac{\text{wt. of complete gas}}{\text{area}}$

$$\therefore \qquad \frac{P}{P_0} = \frac{\text{wt. of gas above '}h\text{'}}{\text{Total weight of gas}}$$

$$\therefore \qquad \text{Required fraction} = \frac{P}{P_0} = e^{-\frac{Mgh}{RT}}$$

(b)

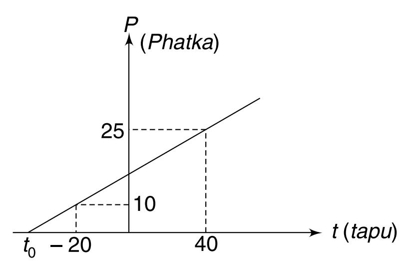
$$\frac{P}{P_0} = \frac{1}{2}$$

$$\Rightarrow \qquad e^{-\frac{Mgh_0}{RT}} = \frac{1}{2}$$

$$\frac{Mgh_0}{RT} = \ln 2 \quad \therefore \quad h_0 = \frac{RT}{Mg}\ln 2$$

**35.** *Pinch*, *volka*, *tapu* and *phatka* are unit of quantity of gas, volume, temperature and pressure.

Volume is constant. Hence $P$ change linearly with temperature '$t$'. Graph of $P$ (in *phatka*) vs temperature $t$ (in *tapu*) is as shown.




The equation of the line is

$$P = \frac{1}{4}t + 15$$

At absolute zero temperature, $P = 0$

$$\therefore \qquad t_0 = -60 \textit{ tapu}$$

**36.** Pressure in $N_2$ chamber = 4 times the pressure in $O_2$ chamber

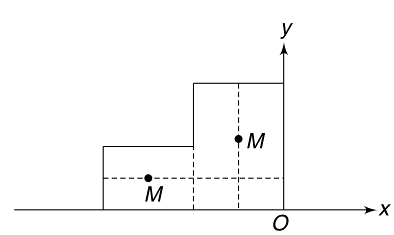
$\therefore$ Number of moles in $N_2$ chamber = 4 times the number of moles in $O_2$ chamber

$$n_{N_2} = 4n_{O_2} = 4n\,(\text{say})$$

Mass of gas in $N_2$ chamber

$$M_{N_2} = 28 \times 4n = 112\,n$$

Mass of gas in $O_2$ chamber, $M_{O_2} = 32\,n$

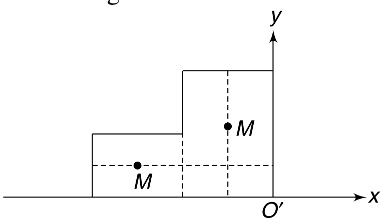


Let's find the $x$ co-ordinate of the COM of the gaseous system with edge $O$ as origin

$$x_{cm} = \frac{M_{N_2}(-4R) + M_{O_2}(-R)}{M_{N_2} + M_{O_2}} = -\frac{112n \times 4R + 32n \times R}{112n + 32n}$$

$$= -\frac{480R}{144} = -3.33R$$

After mixing of gases, the $x$ co-ordinate of COM with respect to new position $O'$ of the edge will be–




$$x'_{cm} = \frac{M(-4R) + M(-R)}{2M} = -2.5\ R$$

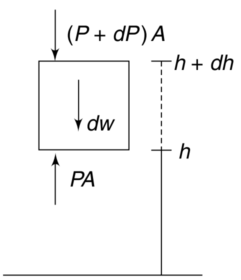
But $x$ co-ordinate of COM will not get displaced in absence of any external force in $x$ direction.

For this, distance travelled by edge $O$ must be

$$(3.33 - 2.5)R = 0.83\ R \text{ towards left.}$$

**37.** (a) For an ideal gas density $\rho = \frac{MP}{RT}$

Consider a cylindrical element of atmospheric air at height $h$. For its equilibrium




$$PA - (P + dP)A = dw$$

$$-dP \cdot A = \rho \cdot A\,dhg$$

$$-dP = -\rho g dh$$

$$\frac{dP}{P} = -\frac{Mg}{RT}\,dh \quad \text{...(A)}$$

$$\int_{P_0}^{P} \frac{dP}{P} = -\frac{Mg}{RT}\int_0^h dh$$

$$\Rightarrow \quad \ln\frac{P}{P_0} = -\frac{Mgh}{RT} \Rightarrow P = P_0 e^{-\frac{Mgh}{RT}}$$

(b) The temperature changes with height as

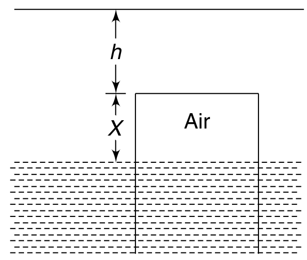
$$T = -\left(\frac{T_0}{5h_0}\right)h + T_0$$

$$\therefore \text{ from (A)} \quad \frac{dP}{P} = -\frac{Mg}{R}\frac{dh}{T_0 - \left(\frac{T_0}{5h_0}\right)h}$$

$$\int_{P_0}^{P}\frac{dP}{P} = -\frac{Mg}{RT_0}\int_0^{h_0}\frac{dh}{\left(1 - \frac{h}{5h_0}\right)}$$

$$\ell n\frac{P}{P_0} = \frac{-\frac{Mg}{RT_0}}{-\frac{1}{5h_0}}\left[\ell n\left[1-\frac{h}{5h_0}\right]\right]_0^{h_0}$$

$$\ln\frac{P}{P_0} = -\frac{5Mgh_0}{RT_0}\ln\frac{5}{4} \Rightarrow P = P_0\left(\frac{5}{4}\right)^{-\frac{5Mgh_0}{RT_0}}$$




**38.** Volume of air in the container before it is placed inside water $V_1 = A\cdot L$

When inside water, let the length of air column be $x$.

Volume of air $V_2 = A\cdot x$

Pressure of air $= P_2 = P_0 + \rho g(x+h)$

For isothermal condition $P_2V_2 = P_1V_1$

$$P_2\cdot A\cdot x = P_0AL$$

$$[P_0 + \rho g(x+h)]x = P_0L \Rightarrow [10^5 + 10^4(x+5)]x = 10^5\times 0.775$$

$$10x + x^2 + 5x = 7.75 \Rightarrow x^2 + 15x - 7.75 = 0$$

$\Rightarrow$ $$x = 0.5 \text{ m}$$

Force required = Buoyancy on 0.5 m length of air column $= Ax\rho\cdot g$

$$= 0.05\times 0.5\times 10^3\times 10 = 250\ N$$

**39.** Initial volume of air $= A_1h_1 = 4\times 55$ cm$^3$

Final volume of air $= V_2$

Initial air pressure $P_1 = P_0 = 10^5$ Nm$^{-2}$

Initial number of moles of air $= n$

Final no. of moles $= 0.75\ n$

Final air pressure

$$P_2 = 10^5 + \rho gh_2 = 10^5 + 10^3\times 10\times 1 = 1.1\times 10^5 \text{ Nm}^{-2}$$

Now $$\frac{P_2V_2}{n_2} = \frac{P_1V_1}{n_1}$$

$$\frac{1.1\times 10^5\times V_2}{0.75n} = \frac{10^5\times 4\times 55}{n}$$

$\therefore$ $$V_2 = 150\,\text{cm}^3$$

Volume of air in narrow part = 100 cm$^3$,

$\therefore$ air in wider part = 50 cm$^3$

$\therefore$ length of air column in wider part $= \frac{50 \text{ cm}^3}{4 \text{ cm}^2} = 12.5$ cm

$\therefore$ $$x = 55 - 12.5 = 42.5 \text{ cm}$$

**40.** In case of an ideal gas we assume no inter-molecular force. When gas undergoes free expansion, the kinetic energy of molecules do not change, and hence the temperature does not change.

In real gases molecules do exert force on each other. These forces increases the potential energy during expansion [in fact, two attracting particles will have more potential energy the farther apart they are]. Increase in potential energy results in decrease in kinetic energy. This results in fall in temperature.

**41.** Repelling particles will have lesser potential energy the farther they are. A decrease in potential energy means an increase in kinetic energy. This results in rise in temperature.

**42.** Let initial extension in the spring be $x_0$

$$kx_0 = mg \qquad ...(i)$$

Let pressure of the gas at temperature $T_0$ be $P_0$

For equilibrium of the piston $P_0A + k(x_0 - h_0) = mg$

Using (i) we get
$$P_0A = kh_0 \qquad ...(ii)$$

After the temperature is raised to $4T_0$ let the pressure be $P$ and height be $h$.

A similar working, as above, can show that

$$PA = kh \qquad ...(iii)$$

(iii) ÷ (ii)
$$\frac{P}{P_0} = \frac{h}{h_0} \qquad ...(iv)$$

But
$$\frac{PV}{T} = \text{constant}$$

$$\therefore \quad \frac{P(Ah_0)}{T_0} = \frac{PAh}{4T_0} \Rightarrow \frac{P}{P_0} = \frac{4h_0}{h} \qquad ...(iv)$$

From (iv) and (v)
$$\frac{h}{h_0} = \frac{4h_0}{h}$$

$$\Rightarrow \quad h = 2h_0$$

**43.** (a) The velocity component of the molecule parallel to the wall do not change. Velocity component perpendicular to the wall is $v\cos\theta$ and after the collision it becomes $v\cos\theta - 2u$.

[since, speed of approach = speed of separation in an elastic collision]

Loss in kinetic energy of the molecule is

$$\frac{1}{2}m(v\cos\theta)^2 - \frac{1}{2}m(v\cos\theta - 2u)^2$$

$$= \frac{1}{2}m(v\cos\theta)^2 - \frac{1}{2}m(v\cos\theta)^2 - \frac{1}{2}m(2u)^2 + 2mvu\cos\theta \simeq 2mvu\cos\theta.$$

(b) The product $Fu = PAu$ is the rate at which work is done on the piston or the power developed by the expanding gas. This is the rate at which the molecules will lose kinetic energy as they are not receiving energy from any other source.

(c) Molecules will not collide with the piston in this case. There is no loss in kinetic energy and no change in temperature.

**44.** (a) Let the equation of parabola be

$$\frac{dN}{du} = a(u - u^2)$$

$$\therefore \qquad \int_0^1 \left(\frac{dN}{du}\right) du = N_0 \quad \Rightarrow \quad a\left[\frac{u^2}{2} - \frac{u^3}{3}\right]_0^1 = N_0$$

$$\Rightarrow \qquad \frac{a}{6} = N_0 \quad \Rightarrow \quad a = 6N_0$$

$$\frac{dN}{du} = 6N_0(u - u^2)$$

$$v_{rms} = \sqrt{\frac{\int u^2\, dN}{N_0}}$$

But
$$\int u^2\, dN = \int u^2 \left(\frac{dN}{du}\right) du = a\int_0^1 (u^3 - u^4)\, du$$

$$= a\left[\frac{1}{4} - \frac{1}{5}\right] = \frac{a}{20} = \frac{6N_0}{20} = \frac{3N_0}{10}$$

$$\therefore \qquad v_{rms} = \sqrt{\frac{3}{10}} \text{ unit}$$

Since the speed axis shows speed in $10^2$ m/s

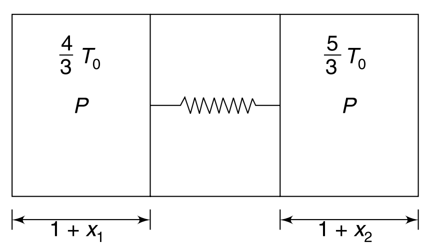
$$\therefore \qquad v_{rms} = \sqrt{\frac{3}{10}} \times 100 \text{ m/s}$$

(b)
$$K_T = \frac{1}{2} mv_{rms}^2 = \frac{1}{2} \times (10 \times 10^{-3}) \times 3000 = 15 \text{ J}$$

**45.** Let the left piston move by $x_1(\rightarrow)$ ad the right piston move by $x_2(\leftarrow)$

Further compression in the spring $x = x_1 + x_2$

Let initial and final pressure of both gases be $P_0$ and $P$ respectively.




$$\frac{P_0 A(1)}{T_0} = \frac{PA(1 + x_1)}{\frac{4}{3}T_0} = \frac{PA(1 + x_2)}{\frac{5}{3}T_0}$$

$$\Rightarrow \qquad \frac{P_0}{P} = \frac{3}{4}(1 + x_1) \qquad \text{...(i)}$$

And
$$5(1 + x_1) = 4(1 + x_2) \quad \Rightarrow \quad 5x_1 = 4x_2 - 1 \qquad \text{...(ii)}$$

But
$$x_2 = x - x_1$$

$$5x_1 = 4(x - x_1) - 1 \quad \Rightarrow \quad x_1 = \frac{4x - 1}{9} \qquad \text{...(iii)}$$

Put this in (i) to get
$$\frac{P_0}{P} = \frac{2 + x}{3} \qquad \text{...(iv)}$$

For initial and final equilibrium

$$P_0 A = kx_0 \quad \Rightarrow \quad P_0 = \frac{k(1)}{A} = \frac{k}{A} \text{ and } PA = k(1 + x) \quad \Rightarrow \quad P = \frac{k(1 + x)}{A}$$

$$\therefore \qquad \frac{P_0}{P} = \frac{1}{1 + x} \qquad \text{...(v)}$$

From (iv) and (v) $\frac{2+x}{3} = \frac{1}{1+x}$

$\Rightarrow$ $x^2 + 3x - 1 = 0$

$\Rightarrow$ $x = \frac{\sqrt{13} - 3}{2}$

$\therefore$ Final compression $= x + 1 = \frac{\sqrt{13} - 1}{2}$ m

**46.** (a) Net upward force on the balloon is

$$F_0 = F_B - w_{\text{Rubber}} - w_{\text{Load}} - w_{\text{He}}$$

$$F_0 = V\rho_a g - w_{\text{Rubber}} - w_{\text{Load}} - V\rho_{\text{He}} g$$

$$V = \frac{F_0 + w_{\text{Rubber}} + w_{\text{Load}}}{(\rho_a - \rho_{\text{He}})g} \quad \text{...(i)}$$

$$n = \frac{PV}{RT} = \frac{P_0(F_0 + w_{\text{Rubber}} + w_{\text{Load}})}{RT_0(\rho_a - \rho_{\text{He}})g}$$

(b) Atmospheric pressure at height $y$

$$P = P_0 e^{-ky}$$

$\Rightarrow$ $$\ln\left(\frac{P}{P_0}\right) = -ky$$

$\Rightarrow$ $$y = \frac{1}{k}\ln\left(\frac{P_0}{P}\right) \quad \text{...(ii)}$$

But $$P = \frac{nRT}{V}$$

For fully inflated balloon $$P = \frac{nRT_0}{V_0}$$

$\therefore$ $$y_0 = \frac{1}{k}\ln\left(\frac{P_0 V_0}{nRT_0}\right) \quad \text{...(iii)}$$

(c) Density of air decreases with height. If temperature remains constant then

$$\frac{P}{\rho} = \text{a constant}$$

$\Rightarrow$ $$\frac{P}{P_0} = \frac{\rho_y}{\rho_a} \Rightarrow \rho_y = \frac{P}{P_0}\rho_a \quad \text{...(iv)}$$

$\therefore$ Buoyancy at height $y$ is

$$F_{By} = \frac{P}{P_0}\rho_a V g = \rho_a V g e^{-ky}$$

Net upward force on the balloon at height $y$ is

$$F_{\text{net}} = F_{\text{By}} - w_{\text{Rubber}} - w_{\text{Load}} - w_{\text{He}}$$

$$= \rho_a V g e^{-ky} - w_{\text{Rubber}} - w_{\text{Load}} - \rho_{\text{He}} V g e^{-ky}$$

[since density of He will also change in a way similar to the air]

$\therefore$ $$F_{\text{net}} = V g e^{-ky}(\rho_a - \rho_{\text{He}}) - w_{\text{Rubber}} - w_{\text{Load}}$$

Balloon will rise to height $y = y_0 = \dfrac{1}{k}\ln\left(\dfrac{P_0 V_0}{nRT_0}\right)$

If $\qquad F_{\text{net}} > 0$

$$\Rightarrow \qquad V_0 g\, e^{-\ln\left(\frac{P_0 V_0}{nRT_0}\right)}(\rho_a - \rho_{\text{He}}) > w_{\text{Rubber}} + w_{\text{Load}}$$

$$\Rightarrow \qquad V_0 \mathrm{g} \frac{nRT_0}{P_0 V_0}(\rho_a - \rho_{\text{He}}) > w_{\text{Rubber}} + w_{\text{Load}}$$

$$\Rightarrow \qquad \frac{nRT_0 g}{P_0}(\rho_a - \rho_{\text{Hg}}) > w_{\text{Rubber}} + w_{\text{Load}}$$