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Sets, Relations & Functions

SETS, RELATIONS & FUNCTIONS

SETS

1. SET

A set is a collection of well-defined and well distinguished objects of our perception or thought.

1.1 Notations

The sets are usually denoted by capital letters A, B, C, etc. and the members or elements of the set are denoted by lower-case letters a, b, c, etc. If x is a member of the set A, we write $x \in A$ (read as 'x belongs to A') and if x is not a member of the set A, we write $x \notin A$ (read as 'x does not belong to A,). If x and y both belong to A, we write $x, y \in A$.

2. REPRESENTATION OF A SET

Usually, sets are represented in the following two ways :

(i) Roster form or Tabular form

(ii) Set Builder form or Rule Method

2.1 Roster Form

In this form, we list all the member of the set within braces (curly brackets) and separate these by commas. For example, the set A of all odd natural numbers less that 10 in the Roster form is written as :

$A = \{1, 3, 5, 7, 9\}$

Note...

(i) In roster form, every element of the set is listed only once.

(ii) The order in which the elements are listed is immaterial.

For example, each of the following sets denotes the same set $\{1, 2, 3\}$, $\{3, 2, 1\}$, $\{1, 3, 2\}$

2.2 Set-Builder Form

In this form, we write a variable (say x) representing any member of the set followed by a property satisfied by each member of the set.

For example, the set A of all prime numbers less than 10 in the set-builder form is written as

$A = \{x \mid x \text{ is a prime number less that } 10\}$

The symbol '|' stands for the words 'such that'. Sometimes, we use the symbol ':' in place of the symbol '|'.

3. TYPES OF SETS

3.1 Empty Set or Null Set

A set which has no element is called the null set or empty set. It is denoted by the symbol ϕ .

For example, each of the following is a null set :

(a) The set of all real numbers whose square is –1.

(b) The set of all rational numbers whose square is 2.

(c) The set of all those integers that are both even and odd.

A set consisting of atleast one element is called a non-empty set.

3.2 Singleton Set

A set having only one element is called singleton set.

For example, $\{0\}$ is a singleton set, whose only member is 0.

3.3 Finite and Infinite Set

A set which has finite number of elements is called a finite set. Otherwise, it is called an infinite set.

For example, the set of all days in a week is a finite set whereas the set of all integers, denoted by $\{\ldots\ldots\ldots\ldots -2, -1, 0, 1, 2, \ldots\}$ or $\{x \mid x \text{ is an integer}\}$, is an infinite set.

An empty set ϕ which has no element in a finite set A is called empty of void or null set.

3.4 Cardinal Number

The number of elements in finite set is represented by n(A), known as Cardinal number.

3.5 Equal Sets

Two sets A and B are said to be equals, written as A = B, if every element of A is in B and every element of B is in A.

3.6 Equivalent Sets

Two finite sets A and B are said to be equivalent, if n (A) = n (B). Clearly, equal sets are equivalent but equivalent sets need not be equal.

For example, the sets A = { 4, 5, 3, 2} and B = {1, 6, 8, 9} are equivalent but are not equal.

3.7 Subset

Let A and B be two sets. If every elements of A is an element of B, then A is called a subset of B and we write $A \subset B$ or $B \supset A$ (read as 'A is contained in B' or B contains A'). B is called superset of A.

Note...

(i) Every set is a subset and a superset itself.

(ii) If A is not a subset of B, we write $A \not\subset B$.

(iii) The empty set is the subset of every set.

(iv) If A is a set with $n(A) = m$, then the number of subsets of A are 2^m and the number of proper subsets of A are 2^m -1.

For example, let A = {3, 4}, then the subsets of A are ϕ, {3}, {4}. {3, 4}. Here, $n(A) = 2$ and number of subsets of A = $2^2 = 4$. Also, $\{3\} \subset \{3,4\}$ and $\{2,3\} \not\subset \{3,4\}$

3.8 Power Set

The set of all subsets of a given set A is called the power set of A and is denoted by P(A).

For example, if A = {1, 2, 3}, then

P(A) = { ϕ, {1}, {2}, {3}, {1,2} {1, 3}, {2, 3}, {1, 2, 3}}

Clearly, if A has n elements, then its power set P (A) contains exactly 2^n elements.

4. OPERATIONS ON SETS

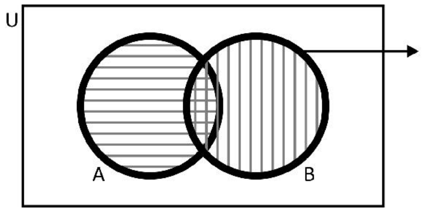
4.1 Union of Two Sets

The union of two sets A and B, written as $A \cup B$ (read as 'A union B'), is the set consisting of all the elements which are either in A or in B or in both Thus,

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Clearly, $x \in A \cup B \Rightarrow x \in A$ **or** $x \in B$, and

$x \notin A \cup B \Rightarrow x \notin A$ **and** $x \notin B$.



Combination of the shaded regions together represent the union of sets.

For example, if A = {a, b, c, d} and B = {c, d, e, f}, then $A \cup B$ = {a, b, c, d, e, f}

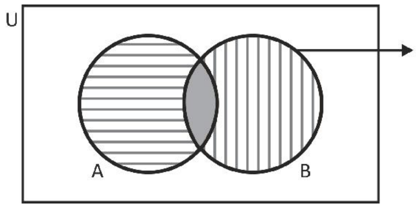
4.2 Intersection of Two sets

The intersection of two sets A and B, written as $A \cap B$ (read as 'A' intersection 'B') is the set consisting of all the common elements of A and B. Thus,

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Clearly, $x \in A \cap B \Rightarrow x \in A$ **and** $x \in B$, and

$x \notin A \cap B \Rightarrow x \notin A$ **or** $x \notin B$.

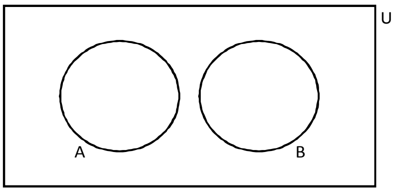


The shaded region which is common to both the shaded regions represents intersection of sets

For example, if A = {a, b, c, d) and B = {c, d, e, f}, then $A \cap B$ = {c, d}.

4.3 Disjoint Sets

Two sets A and B are said to be disjoint, if $A \cap B = \phi$, i.e. A and B have no element in common.



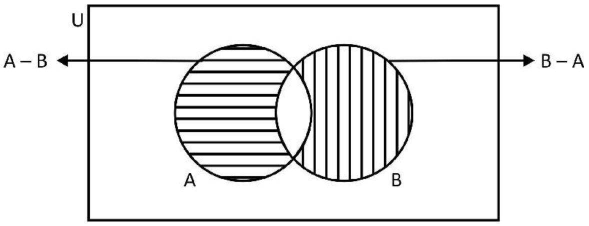
For example, if A = {1, 3, 5} and B = {2, 4, 6}, then $A \cap B = \phi$, so A and B are disjoint sets.

4.4 Difference of Two Sets

If A and B are two sets, then their difference A - B is defined as :

$A - B = \{x : x \in A \text{ and } x \notin B\}$.

Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$.



For example, if A = {1, 2, 3, 4, 5} and B = {1, 3, 5, 7, 9} then A - B = {2, 4} and B - A = {7, 9}.

Important Results

(a) $A - B \neq B - A$

(b) The sets A - B , B - A and $A \cap B$ are disjoint sets

(c) $A - B \subseteq A$ and $B - A \subseteq B$

(d) $A - \phi = A$ and $A - A = \phi$

4.5 Symmetric Difference of Two Sets

The symmetric difference of two sets A and B , denoted by $A \,\Delta\, B$, is defined as

$A \,\Delta\, B = (A - B) \cup (B - A)$.

For example, if A = {1,2,3,4,5} and B = {1, 3,5,7,9} then

$A \,\Delta\, B = (A - B) \cup (B - A) = \{2,4\} \cup \{7,9\} = \{2,4,7,9\}$.

4.6 Complement of a Set

If U is a universal set and A is a subset of U, then the complement of A is the set which contains those elements of U, which are not contained in A and is denoted by A'or A^c. Thus,

$A^c = \{x : x \in U \text{ and } x \notin A\}$

For example, if U = {1,2,3,4 ...} and A {2,4,6,8,...}, then, A^c = {1,3,5,7, ...}

Important Results

a) $U^c = \phi$ b) $\phi^c = U$ c) $A \cup A^c = U$

d) $A \cap A^c = \phi$

5. ALGEBRA OF SETS

1. For any set A , we have

   a) $A \cup A = A$ b) $A \cap A = A$

2. For any set A, we have

   c) $A \cup \phi = A$ d) $A \cap \phi = \phi$

   e) $A \cup U = U$ f) $A \cap U = A$

3. For any two sets A and B, we have

   g) $A \cup B = B \cup A$ h) $A \cap B = B \cap A$

4. For any three sets A, B and C, we have

   i) $A \cup (B \cup C) = (A \cup B) \cup C$

   j) $A \cap (B \cap C) = (A \cap B) \cap C$

5. For any three sets A, B and C, we have

   k) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   l) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6. If A is any set, we have $(A^c)^c = A$.

7. Demorgan's Laws For any three sets A, B and C, we have

   m) $(A \cup B)^c = A^c \cap B^c$

   n) $(A \cap B)^c = A^c \cup B^c$

   o) $A - (B \cup C) = (A - B) \cap (A - C)$

   p) $A - (B \cap C) = (A - B) \cup (A - C)$

Important Results on Operations on Sets

(i) $A \subseteq A \cup B,\ B \subseteq A \cup B,\ A \cap B \subseteq A,\ A \cap B \subseteq B$

(ii) $A - B = A \cap B^c$ (iii) $(A - B) \cup B = A \cup B$

(iv) $(A - B) \cap B = \phi$ (v) $A \subseteq B \Leftrightarrow B^c \subseteq A^c$

(vi) $A - B = B^c - A^c$ (vii) $(A \cup B) \cap (A \cup B^c) = A$

(viii) $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

(ix) $A - (A - B) = A \cap B$

(x) $A - B = B - A \Leftrightarrow A = B$ (xi) $A \cup B = A \cap B \Leftrightarrow A = B$

(xii) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Example – 1

Write the set of all positive integers whose cube is odd.

Sol. The elements of the required set are not even.

[$\because$ Cube of an even integer is also an even integer]

Moreover, the cube of a positive odd integer is a positive odd integer.

$\Rightarrow$ The elements of the required set are all positive odd integers.

Hence, the required set, in the set builder form, is :

$$\{2k + 1 \ : \ k \geq 0,\ k \in Z\}.$$

Example – 2

Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}\right\}$ in the set builder form.

Sol. In each element of the given set the denominator is one more than the numerator.

Also the numerators are from 1 to 7.

Hence the set builder form of the given set is :

$$\{x : x = n/n+1,\ n \in N \text{ and } 1 \leq n \leq 7\}.$$

Example – 3

Write the set $\{x : x$ is a positive integer and $x^2 < 30\}$ in the roster form.

Sol. The squares of positive integers whose squares are less than 30 are : 1, 2, 3, 4, 5.

Hence the given set, in roster form, is $\{1, 2, 3, 4, 5\}$.

Example – 4

Write the set $\{0, 1, 4, 9, 16, ...\}$ in set builder form.

Sol. The elements of the given set are squares of integers :

$0,\ \pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ ....$

Hence the given set, in set builder form, is $\{x^2 : x \in Z\}$.

Example – 5

State which of the following sets are finite and which are infinite

(i) $A = \{x : x \in N$ and $x^2 - 3x + 2 = 0\}$

(ii) $B = \{x : x \in N$ and $x^2 = 9\}$

(iii) $C = \{x : x \in N$ and x is even$\}$

(iv) $D = \{x : x \in N$ and $2x - 3 = 0\}$.

Sol. (i) $A = \{1, 2\}$.

[$\because x^2 - 3x + 2 = 0 \Rightarrow (x - 1)(x - 2) = 0 \Rightarrow x = 1, 2$]

Hence A is finite.

(ii) $B = \{3\}$.

[$\because x^2 = 9 \Rightarrow x = \pm 3$. But $3 \in N$]

Hence B is finite.

(iii) $C = \{2, 4, 6, ...\}$

Hence C is infinite.

(iv) $D = \phi$. $\left[\because\ 2x - 3 = 0 \Rightarrow x = \frac{3}{2} \notin N\right]$

Hence D is finite.

Example – 6

Which of the following are empty (null) sets ?

(i) Set of odd natural numbers divisible by 2

(ii) $\{x : 3 < x < 4, x \in N\}$

(iii) $\{x : x^2 = 25$ and x is an odd integer$\}$

(iv) $[x : x^2 - 2 = 0$ and x is rational$]$

(v) $\{x :$ x is common point of any two parallel lines$\}$.

Sol. (i) Since there is no odd natural number, which is divisible by 2.

∴ it is an empty set.

(ii) Since there is no natural number between 3 and 4.

∴ it is an empty set.

(iii) Now $x^2 = 25 \Rightarrow x = \pm 5$, both are odd.

∴ The set $\{-5, 5\}$ is non-emptry.

(iv) Since there is no rational number whose square is 2,

∴ the given set is an empty set.

(v) Since any two parallel lines have no common point,

∴ the given set is an empty set.

Example – 7

Find the pairs of equal sets from the following sets, if any, giving reasons :

$A = \{0\}$, $B = \{x : x > 15$ and $x < 5\}$,

$C = \{x : x - 5 = 0\}$, $D = \{x : x^2 = 25\}$,

$E = \{x :$ x is a positive integral root of the equation $x^2 - 2x - 15 = 0\}$.

Sol. Here we have,

$A = \{0\}$

$B = \phi$

[∵ There is no number, which is greater than 15 and less than 5]

$C = \{5\}$ $[\because x - 5 = 0 \Rightarrow x = 5]$

$D = \{-5, 5\}$ $[\because x^2 = 25 \Rightarrow x = +5]$

and $E = \{5\}$.

$[\because x^2 - 2x - 15 = 0 \Rightarrow (x-5)(x+3) = 0 \Rightarrow x = 5, -3$. Out of these two,

5 is positive integral]

Clearly $C = E$.

Example – 8

Are the following pairs of sets equal ? Give reasons.

(i) $A = \{1, 2\}$, $B = \{x :$ x is a solution of $x^2 + 3x + 2 = 0\}$

(ii) $A = \{x :$ x is a letter in the word FOLLOW$\}$,

$B = \{y :$ y is a letter in the word WOLF$\}$.

Sol. (i) $A = \{1, 2\}$, $B = \{-2, -1\}$

$[\because x^2 + 3x + 2 = 0 \Rightarrow (x+2)(x+1) = 0 \Rightarrow x = -2, —1]$

Clearly $A \neq B$.

(ii) $A = \{F, O, L, L, O, W\} = \{F, O, L, W\}$

$B = \{W, O, L, F\} = \{F, O, L, W\}$.

Clearly $A = B$.

Example – 9

Let $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5, 6, 7\}$, $C = \{6, 7, 8, 9\}$ and $D = \{7, 8, 9, 10\}$. Find :

(a) **(i)** $A \cup B$ **(ii)** $B \cup D$

(iii) $A \cup B \cup C$ **(iv)** $B \cup C \cup D$

(b) **(i)** $A \cap B$ **(ii)** $B \cap D$ **(iii)** $A \cap B \cap C$.

Sol. (a) (i) $A \cup B = \{1, 2, 3, 4, 5\} \cup \{3, 4, 5, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$.

(ii) $B \cup D = \{3, 4, 5, 6, 7\} \cup \{7, 8, 9, 10\}$

$= \{3, 4, 5, 6, 7, 8, 9, 10\}$.

(iii) $A \cup B \cup C = \{1, 2, 3, 4, 5\} \cup \{3, 4, 5, 6, 7\} \cup \{6, 7, 8, 9\}$.

$= \{1, 2, 3, 4, 5, 6, 7\} \cup \{6, 7, 8, 9\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

(iv) $B \cup C \cup D = \{3, 4, 5, 6, 7\} \cup \{6, 7, 8, 9\} \cup \{7, 8, 9, 10\}$.

$= \{3, 4, 5, 6, 7, 8, 9\} \cup \{7, 8, 9, 10\} = \{3, 4, 5, 6, 7, 8, 9, 10\}$.

(b) (i) $A \cap B = \{1, 2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7\} = \{3, 4, 5\}$.

(ii) $B \cap D = \{3, 4, 5, 6, 7\} \cap \{7, 8, 9, 10\} = \{7\}$.

(iii) $A \cap B \cap C = \{1, 2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7\} \cap \{6, 7, 8, 9\} = \{3, 4, 5\} \cap \{6, 7, 8, 9\} = \phi$.

Example – 10

If $A_1 = \{2, 3, 4, 5\}$, $A_2 = \{3, 4, 5, 6\}$, $A_3 = \{4, 5, 6, 7\}$, find $\cup A_i$ and $\cap A_i$, where $i = \{1, 2, 3\}$.

Sol. (i) $\cup A_i = A_1 \cup A_2 \cup A_3 = \{2,3,4,5\} \cup \{3,4,5,6\} \cup \{4,5,6,7\}$

$= \{2,3,4,5\} \cup \{3,4,5,6,7\} = \{2,3,4,5,6,7\}$.

(ii) $\cap A_i = A_1 \cap A_2 \cap A_3 = \{2,3,4,5\} \cap \{3,4,5,6\} \cap \{4,5,6,7\}$

$= \{2,3,4,5\} \cap \{4,5,6\} = \{4,5\}$.

Example – 11

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8}. Find :

(i) A^C (ii) B^C (iii) $(A^C)^C$ (iv) $(A \cup B)^C$

Sol. (i) A^C = Set of those elements of U, which are not in A = {5, 6, 7, 8, 9}.

(ii) B^C = Set of those elements of U, which are not in B = {1, 3, 5, 7, 9}.

(iii) $(A^C)^C$ = Set of those elements of U, which are not in A' = {1, 2, 3, 4} = A.

(iv) $A \cup B = \{1,2,3,4\} \cup \{2,4,6,8\} = \{1,2,3,4,6,8\}$.

∴ $(A \cup B)^C$ = Set of those elements of U, which are not in $(A \cup B) = \{5,7,9\}$.

Example – 12

If U = {x : x is a letter in English alphabet}, A = {x : x is a vowel in English alphabet}.

Find A^C and $(A^C)^C$.

Sol. (i) Since A = {x : x is a letter in English alphabet},

∴ A^C is the set of those elements of U, which are not vowels = {x : x is a consonant in English alphabet}.

(ii) $(A^C)^C$ is the set of those elements of U, which are not consonants = {x : x is a vowel in English alphabet} = A.

Hence $(A^C)^C = A$.

Example – 13

Let A = {1, 2, 3, 4, 5, 6}, B = {3, 4, 5, 6, 7, 8}. Find (A – B) ∪ (B – A).

Sol. We have, A = {1, 2, 3, 4, 5, 6} and B = {3, 4, 5, 6, 7, 8}.

∴ A – B = {1, 2} and B – A = {7, 8}

∴ (A – B) ∪ (B – A) = {1, 2} ∪ {7, 8} = {1, 2, 7, 8}.

Some Basis Results about Cardinal Number

If A, B and C are finite sets and U be the finite universal set, then

(i) $n(A^c) = n(U) - n(A)$

(ii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(iii) $n(A \cup B) = n(A) + n(B)$, where A and B are disjoint non - empty sets.

(iv) $n(A \cap B^c) = n(A) - n(A \cap B)$

(v) $n(A^c \cap B^c) = n(A \cup B)^c = n(U) - n(A \cup B)$

(vi) $n(A^c \cup B^c) = n(A \cap B)^c = n(U) - n(A \cap B)$

(vii) $n(A - B) = n(A) - n(A \cap B)$

(viii) $n(A \cap B) = n(A \cup B) - n(A \cap B^c) - n(A^c \cap B)$

(ix) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

(x) If $A_1, A_2, A_3, \ldots A_n$ are disjoint sets , then

$n(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n) = n(A_1) + n(A_2) + n(A_3) + \ldots + n(A_n)$

(xi) $n(A \Delta B)$ = number of elements which belong to exactly one of A or B.

Example – 14

If A = {1, 2, 3}, B = {4, 5, 6} and C = {7, 8, 9}, verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Sol. We have, A = {1, 2, 3}, B = {4, 5, 6} and C = {7, 8, 9}.

∴ $A \cup B = \{1,2,3\} \cup \{4,5,6\} = \{1,2,3,4,5,6\}$ ...(1)

$A \cup C = \{1,2,3\} \cup \{7,8,9\}$

$= \{1,2,3,7,8,9\}$ ...(2)

and $B \cap C = \{4,5,6\} \cap \{7,8,9\} = \phi$ ...(3)

Now $A \cup (B \cap C) = \{1, 2, 3\} \cup \phi = \{1,2,3\}$ ...(4)

and $(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6\} \cap \{1,2,3,7,8,9\}$

$= \{1,2,3\}$ ...(5)

From (4) and (5), $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, which verifies the result.

Example – 15

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}. Verify that

(i) $(A \cup B)^C = A^C \cap B^C$ **(ii)** $(A \cap B)^C = A^C \cup B^C$.

Sol. We have, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}.

(i) $A \cup B = \{2, 4, 6, 8\} \cup \{2, 3, 5, 7\}$

$\therefore$ $= \{2, 3, 4, 5, 6, 7, 8\}$

$\therefore$ $(A \cup B)^C = \{1, 9\}$ ...(1)

Also $A^C = \{1, 3, 5, 7, 9\}$

and $B^C = \{1, 4, 6, 8, 9\}$

$\therefore$ $A^C \cap B^C = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\}$

$= \{1, 9\}$ ...(2)

From (1) and (2), $(A \cup B)^C = A^C \cap B^C$, which verifies the result.

(ii) $A \cap B = \{2, 4, 6, 8\} \cap \{2, 3, 5, 7\} = \{2\}$

$\therefore$ $(A \cap B)^C = \{1, 3, 4, 5, 6, 7, 8, 9\}$ ...(3)

and $A^C \cup B^C = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9\}$ ...(4)

From (3) and (4), $(A \cap B)^C = A^C \cup B^C$, which verifies the result.

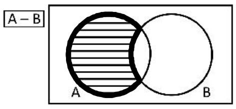
Example – 16

If A and B are any two sets, prove using Venn Diagrams

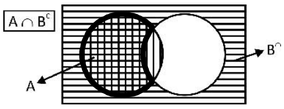
(i) A – B = $A \cap B^C$ **(ii) (A – B)** $\cup B = A \cup B$.

Sol.

(i)

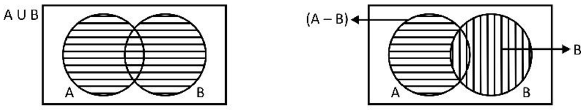


Shaded region in both are the same.



The common shaded area represents the intersection.

(ii)



Combination of all the shaded regions represents the union.

Example – 17

Prove that :

$A \cap (B - C) = (A \cap B) - (A \cap C)$

Sol. Let x be an arbitrary element of $A \cap (B - C)$.

Then $x \in A \cap (B - C)$

$\Rightarrow$ $x \in A$ and $x \in (B - C)$

$\Rightarrow$ $x \in A$ and ($x \in B$ and $x \notin C$)

$\Rightarrow$ ($x \in A$ and $x \in B$) and ($x \in A$ and $x \notin C$)

$\Rightarrow$ $x \in (A \cap B)$ and $x \notin (A \cap C)$

$\Rightarrow$ $x \in \{(A \cap B) - (A \cap C)\}$

$\therefore$ $A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$ ... (1)

Let y be an arbitrary element of $(A \cap B) - (A \cap C)$.

Then $y \in (A \cap B) - (A \cap C)$

$\Rightarrow$ $y \in (A \cap B)$ and $y \notin (A \cap C)$

$\Rightarrow$ ($y \in A$ and $y \in B$) and ($y \in A$ and $y \notin C$)

$\Rightarrow$ $y \in A$ and ($y \in B$ and $y \notin C$)

$\Rightarrow$ $y \in A$ and $y \in (B - C)$

$\Rightarrow$ $y \in A \cap (B - C)$

$\therefore$ $(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$ ... (2)

Combining (1) and (2).

$A \cap (B - C) = (A \cap B) - (A \cap C)$.

Example – 18

Prove the following :

$A \subset B \Leftrightarrow B^c \subset A^c$

Sol. Let $x \in B^c$, where x is arbitrary.

Now $x \in B^c$

$\Rightarrow$ $x \notin B$

$\Rightarrow$ $x \notin A [\because A \subset B]$

$\Rightarrow$ $x \in A^c$

$\therefore$ $B^c \subset A^c$ ... (1)

Conversely : Let $x \in A$, where x is arbitrary.

Now $x \in A$

$\Rightarrow$ $x \notin A^c$

$\Rightarrow$ $x \notin B_c$ $[\because B^c \subset A^c]$

$\Rightarrow$ $x \in B$

$\therefore$ $A \subset B$

Combining (1) and (2), $A \subset B \Leftrightarrow B^c \subset A^c$.

Example – 19

Prove the following :

$A - B = A - (A \cap B)$

where U is the universal set.

Sol. Let $x \in (A - B)$, where x is arbitrary.

Now $x \in (A - B)$

$\Leftrightarrow$ $x \in A$ and $x \notin B$

$\Leftrightarrow$ $(x \in A$ and $x \in A)$ and $x \notin B$

[Note this step]

$\Leftrightarrow$ $x \in A$ and $(x \in A$ and $x \notin B)$

[Associative Law]

$\Leftrightarrow$ $x \in A$ and $x \notin (A \cap B)$

$\Leftrightarrow$ $x \in A - (A \cap B)$

Hence $A - B = A - (A \cap B)$.

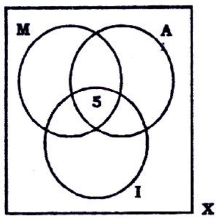
Example – 20

In a class of 200 students who appeared in a certain examination. 35 students failed in MHTCET, 40 in AIEEE, 40 in IIT, 20 failed in MHTCET and AIEEE, 17 in AIEEE and IIT, 15 in MHTCET and IIT and 5 failed in all three examinations. Find how many students

(i) Did not fail in any examination.

(ii) Failed in AIEEE or IIT.

Sol.



$n(M) = 35, n(A) = 40, n(I) = 40$

$n(M \cap A) = 20, n(A \cap I) = 17,$

$n(I \cap M) = 15, n(M \cap A \cap I) = 5$

$n(X) = 200$

$n(M \cup A \cup I) = n(M) + n(A) + n(I) -$

$n(M \cap A) - n(A \cap I) - n(M \cap I) + n(M \cap A \cap I)$

$= 35 + 40 + 40 - 20 - 17 - 15 + 5 = 68$

(i) Number of students passed in all three examination

$= 200 - 68 = 132$

(ii) Number of students failed in IIT or AIEEE

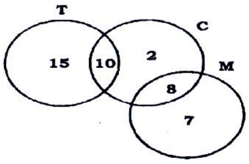
$= n(I \cup A) = n(I) + n(A) - n(I \cap A)$

$= 40 + 40 - 17 = 63$

Example – 21

In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 students take both tea and coffee, 8 students take both milk and coffee. None of the them take tea and milk both and everyone takes atleast one beverage, find the number of students in the hostel.

Sol.



Let the sets, T and C and set M are the students who drink tea, coffee and milk respectively. This problem can be solved by Venn diagram.

$n(T) = 25; n(C) = 20; n(M) = 15$

$n(T \cap C) = 10; n(M \cap C) = 8$

Number of students in hostel

$= n(T \cup C \cup M)$

$\therefore n(T \cup C \cup M) = 15 + 10 + 2 + 8 + 7 = 42$

RELATION & FUNCTION-I

1. INTRODUCTION

In this chapter, we will learn how to create a relation between two sets by linking pairs of objects from two sets. We will learn how a relation qualifies for being a function. Finally, we will see kinds of function, some standard functions etc.

2. RELATIONS

2.1 Cartesian product of sets

Definition : Given two non-empty sets P & Q. The cartesian product P × Q is the set of all ordered pairs of elements from P & Q i.e.

$$P \times Q = \{(p, q); p \in P; q \in Q\}$$

2.2 Relations

2.2.1 Definition : Let A & B be two non-empty sets. Then any subset 'R' of A × B is a relation from A to B.

If $(a, b) \in R$, then we write it as a R b which is read as a is related to b' by the relation R', 'b' is also called image of 'a' under R.

2.2.2 Domain and range of a relation : If R is a relation from A to B, then the set of first elements in R is called domain & the set of second elements in R is called range of R. symbolically.

$$\text{Domain of } R = \{x : (x, y) \in R\}$$

$$\text{Range of } R = \{y : (x, y) \in R\}$$

The set B is called co-domain of relation R.

Note that range $\subset$ co-domain.

Note...

Total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B. If n(A) = p and n(B) = q, then n(A × B) = pq and total number of relations is 2^{pq}.

2.2.3 Inverse of a relation : Let A, B be two sets and let R be a relation from a set A to set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by

$$R^{-1} = \{(b, a) : (a, b) \in R\}$$

Clearly, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$

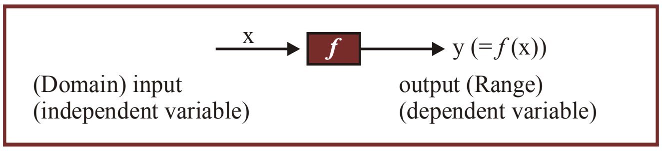
Also, Dom (R) = Range (R^{-1}) and Range (R) = Dom (R^{-1}).

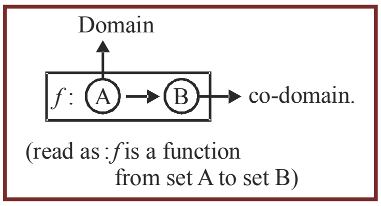
3. FUNCTIONS

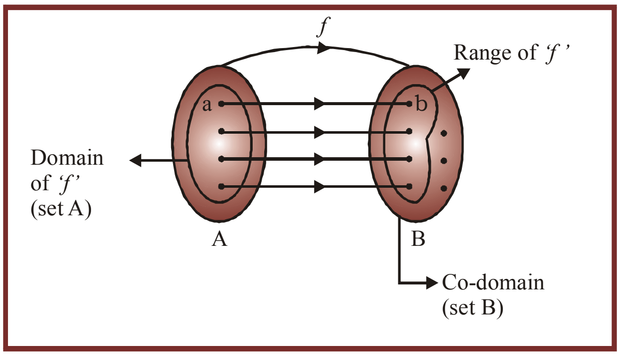
3.1 Definition

A relation 'f' from a set A to set B is said to be a function if every element of set A has one and only one image in set B.

Notations







3.2 Domain, Co-domain and Range of a function

Domain : When we define $y = f(x)$ with a formula and the domain is not stated explicitly, the domain is assumed to be the largest set of x–values for which the formula gives real y–values.

The domain of $y = f(x)$ is the set of all real x for which $f(x)$ is defined (real).

Algo Check : Rules for finding Domain :

(i) Expression under even root (i.e. square root, fourth root etc.) should be non–negative.

(ii) Denominator $\neq 0$.

(iii) $\log_a x$ is defined when $x > 0$, $a > 0$ and $a \neq 1$.

(iv) If domain of $y = f(x)$ and $y = g(x)$ are D_1 and D_2 respectively, then the domain of $f(x) \pm g(x)$ or $f(x) \cdot g(x)$ is $D_1 \cap D_2$. While domain of $\dfrac{f(x)}{g(x)}$ is $D_1 \cap D_2 - \{x : g(x) = 0\}$.

Range : The set of all f-images of elements of A is known as the range of f & denoted by $f(A)$.

Range $= f(A) = \{f(x) : x \in A\}$;

$f(A) \subseteq B$ {Range $\subseteq$ Co-domain}.

Algo Check : Rule for finding range :

First of all find the domain of $y = f(x)$

(i) If domain $\in$ finite number of points

$\Rightarrow$ range $\in$ set of corresponding $f(x)$ values.

(ii) If domain $\in$ R or R – {some finite points}

Put $y = f(x)$

Then express x in terms of y. From this find y for x to be defined. (i.e., find the values of y for which x exists).

(iii) If domain $\in$ a finite interval, find the least and greater value for range using monotonocity.

Note...

1. Question of format :

$$\left(y = \frac{Q}{Q};\ y = \frac{L}{Q};\ y = \frac{Q}{L}\right) \quad \begin{matrix} Q \rightarrow \text{quadratic} \\ L \rightarrow \text{Linear} \end{matrix}$$

Range is found out by cross-multiplying & creating a quadratic in 'x' & making $D \geq 0$ (as $x \in R$)

2. Questions to find range in which-the given expression $y = f(x)$ can be converted into x (or some function of x) = expression in 'y'.

Do this & apply method (ii).

Note...

Two functions f & g are said to be equal iff

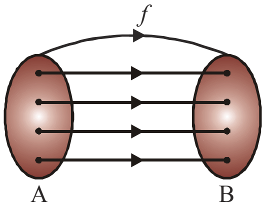
1. Domain of f = Domain of g

2. Co-domain of f = Co-domain of g

3. $f(x) = g(x)$ $\forall$ $x \in$ Domain.

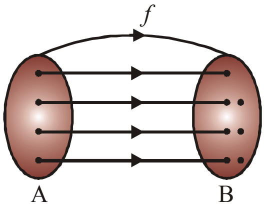
3.3 Kinds of Functions

Range = Co-domain



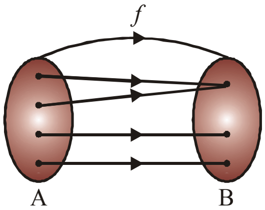
(one-to-one) & (onto)

Range $\subset$ Co-domain



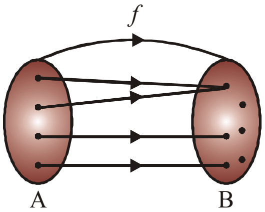
(one-to-one) & (into)

Range = Co-domain



(many-to-one) & (onto)

Range $\subset$ Co-domain



(many-to-one) & (into)

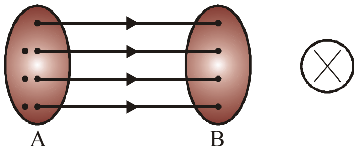
Note...

(a) One-to-One functions are also called Injective functions.

(b) Onto functions are also called Surjective

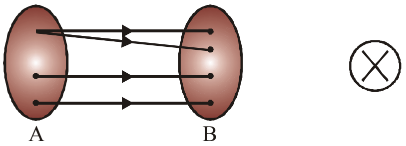
(c) (one-to-one) & (onto) functions are also called Bijective Functions.

Relations which can not be catagorized as a function



Not a function

As not all elements of set A are associated with some elements of set B. (violation of– point (i)– definition 2.1)



Not a function ← one-to-many

An element of set A is not associated with a unique element of set B, (violation of point (ii) definition 2.1)

Methods to check one-one mapping

1. **Theoretically :** If $f(x_1) = f(x_2)$

   $\Rightarrow \quad x_1 = x_2$, then $f(x)$ is one-one.

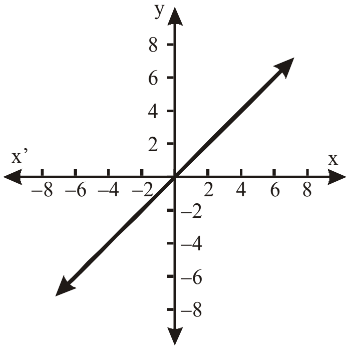
2. **Graphically :** A function is one-one, iff no line parallel to x-axis meets the graph of function at more than one point.

3. **By Calculus :** For checking whether $f(x)$ is One-One, find whether function is only increasing or only decreasing in their domain. If yes, then function is one-one, i.e. if $f'(x) \geq 0, \ \forall \ x \in$ domain or i.e., if $f'(x) \leq 0, \ \forall \ x \in$ domain, then function is one-one.

3.4 Some standard real functions & their graphs

3.4.1 Identity Function :

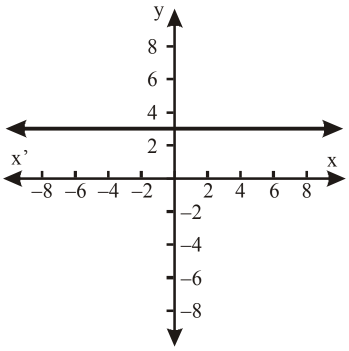
The function $f : R \to R$ defined by $y = f(x) = x \ \forall \ x \in R$ is called identity function.



$f(x) = x$

3.4.2 Constant Function :

The function $f : R \to R$ defined by $y = f(x) = c, \ \forall \ x \in R$ where c is a constant is called constant function



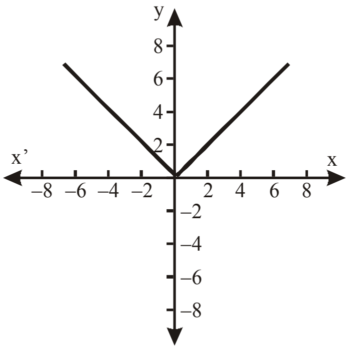
$f(x) = 3$

3.4.3 Modulus Function :

The function $f : R \to R$ defined by

$$f(x) = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$$

is called modulus function. It is denoted by $y = f(x) = |x|$.



$f(x) = |x|$

Its also known as "Absolute value function'.

Properties of Modulus Function :

The modulus function has the following properties :

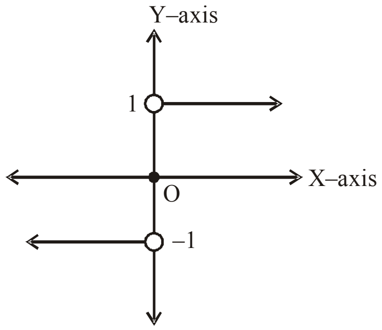
1. For any real number x, we have $\sqrt{x^2} = |x|$
2. $|x\ y| = |x|\,|y|$
3. $|x + y| \leq |x| + |y|$
4. $|x - y| \geq \big||x| - |y|\big|$

$\left.\begin{matrix} 3. \\ 4. \end{matrix}\right\}$ triangle inequality

3.4.4 Signum Function : The function $f: R \rightarrow R$ defined by

$$f(x) = \begin{cases} 1; & x > 0 \\ 0; & x = 0 \\ -1; & x < 0 \end{cases}$$

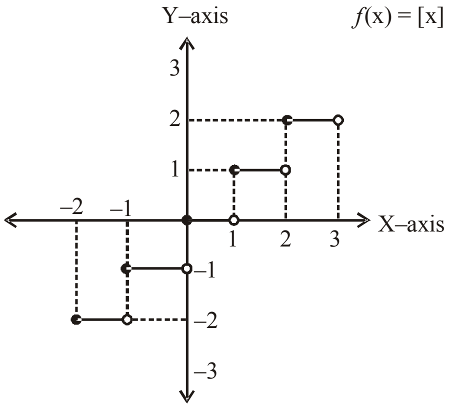
is called signum function. It is usually denoted by $y = f(x) = \text{sgn}(x)$.



Note...

$$\text{Sgn}(x) = \begin{cases} \frac{|x|}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$$

3.4.5 Greatest Integer Function : The function $f: R \rightarrow R$ defined as the greatest integer less than or equal to x. It is usually denoted as $y = f(x) = [x]$



Properties of Greatest Integer Function :

If n is an integer and x is any real number between n and n + 1, then the greatest integer function has the following properties :

(1) $[-n] = -[n]$

(2) $[x+n] = [x] + n$

(3) $[-x] = -[x] - 1$

(4) $[x] + [-x] = \begin{cases} -1, & \text{if } x \notin I \\ 0, & \text{if } x \in I \end{cases}$

Note...

Fractional part of x, denoted by $\{x\}$ is given by $x - [x]$. So,

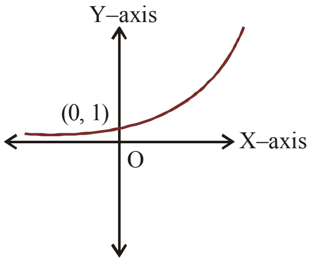
$$\{x\} = x - [x] = \begin{cases} x-1; & 1 \leq x < 2 \\ x\ ; & 0 \leq x < 1 \\ x+1; & -1 \leq x < 0 \end{cases}$$

3.4.6 Exponential Function :

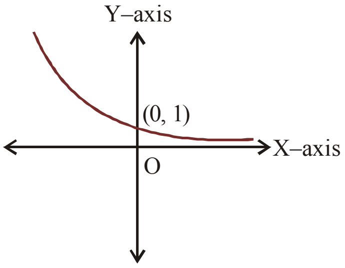
$f(x) = a^x, \quad a > 0, \quad a \neq 1$

Domain : $x \in R$

Range : $f(x) \in (0, \infty)$



$f(x) = a^x$, when $a > 1$



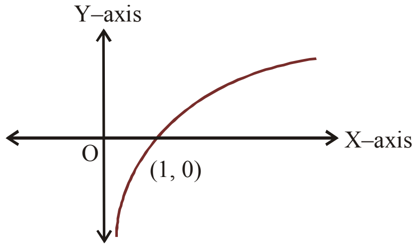
$f(x) = a^x$, when $0 < a < 1$

3.4.7 Logarithm Function :

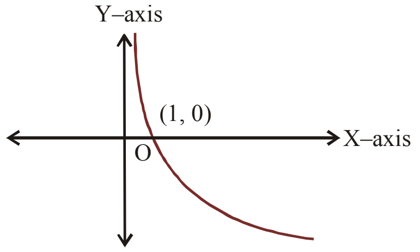
$f(x) = \log_a x, \qquad a > 0, \ a \neq 1$

Domain : $x \in (0, \infty)$

Range : $y \in R$



$f(x) = \log_a x$, when $a > 1$



$f(x) = \log_a x$, when $0 < a < 1$

(a) The Principal Properties of Logarithms

Let M & N are arbitrary positive numbers, $a > 0, \ a \neq 1, \ b > 0, b \neq 1$.

(i) $\log_b a = c \quad \Rightarrow \quad a = b^c$

(ii) $\log_a (M \,.\, N) = \log_a M + \log_a N$

(iii) $\log_a (M/N) = \log_a M - \log_a N$

(iv) $\log_a M^N = N \ \log_a M$

(v) $\log_b a = \dfrac{\log_c a}{\log_c b}, \ c > 0, \ c \neq 1.$

(vi) $a^{\log_c b} = b^{\log_c a}, \ a, b, c > 0, \ c \neq 1.$

Note. . .

(a) $\log_a a = 1$

(b) $\log_b a \,.\, \log_c b \,.\, \log_a c = 1$

(c) $\log_a 1 = 0$

(d) $e^{x \ln a} = e^{\ln a^x} = a^x$

(b) Properties of Monotonocity of Logarithm

(i) If $a > 1, \ \log_a x < \log_a y \quad \Rightarrow \quad 0 < x < y$

(ii) If $0 < a < 1, \ \log_a x < \log_a y \quad \Rightarrow \quad x > y > 0$

(iii) If $a > 1$ then $\log_a x < p \quad \Rightarrow \quad 0 < x < a^p$

(iv) If $a > 1$ then $\log_a x > p \quad \Rightarrow \quad x > a^p$

(v) If $0 < a < 1$ then $\log_a x < p \quad \Rightarrow \quad x > a^p$

(vi) If $0 < a < 1$ then $\log_a x > p \quad \Rightarrow \quad 0 < x < a^p$

Note. . .

If the exponent and the base are on same side of the unity, then the logarithm is positive.

If the exponent and the base are on different sides of unity, then the logarithm is negative.

4. ALGEBRA OF REAL FUNCTION

In this section, we shall learn how to add two real functins, subtract a real function from another, multiply a real function by a scalar (here by a scalar we mean a real number), multiply two real functions and divide one real function by another.

4.1 Addition of two real functions

Let $f : X \to R$ and $g : X \to R$ by any two real functions, whre $X \subset R$. Then, we define $(f + g) : X \to R$ by

$(f + g)(x) = f(x) + g(x)$, for all $x \in X$.

4.2 Subtraction of a real function from another

Let $f : X \to R$ be any two any two real functions, whre $X \subset R$. Then, we define $(f - g) : X \to R$ by

$(f - g)(x) = f(x) - g(x)$, for all $x \in X$.

4.3 Multiplication by a scalar

Let $f : X \to R$ be a real valued function and α be a scalar. Here by scalar, we mean a real number. Then the product αf is a function from X to R defined by $(\alpha f)(x) = \alpha f(x), x \in X$.

4.4 Multiplication of two real functions

The product (or multiplication) of two real functions $f : X \to R$ and $g : X \to R$ is a function $fg : X \to R$ defined by $(fg)(x) = f(x)\, g(x)$, for all $x \in X$.

This is also called *pointwise multiplication*.

4.5 Quotient of two real functions

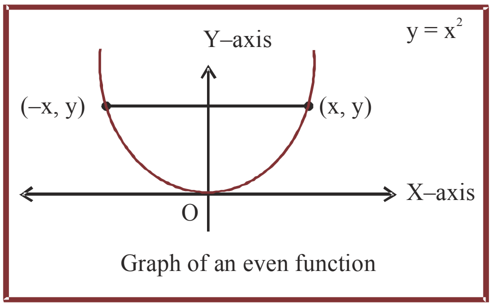
Let f and g be two real functions defined from $X \to R$ where $X \subset R$. The quotient of f by g denoted by f/g is a function defined by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{provided } g(x) \neq 0, x \in X.$$

4.6 Even and Odd Functions

Even Function : $f(-x) = f(x), \ \forall \ x \in$ Domain

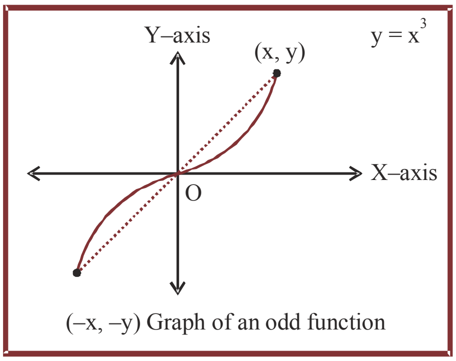
The graph of an even function $y = f(x)$ is symmetric about the y–axis. i.e., (x, y) lies on the graph ⇔ (–x, y) lies on the graph.



Graph of an even function

Odd Function : $f(-x) = -f(x), \ \forall \ x \in$ Domain

The graph of an odd function $y = f(x)$ is symmetric about origin i.e. if point (x, y) is on the graph of an odd function, then (–x, –y) will also lie on the graph.



Graph of an odd function

5. PERIODIC FUNCTION

Definition : A function $f(x)$ is said to be periodic function, if there exists a positive real number T, such that

$$f(x+T) = f(x), \ \forall x \in R.$$

Then, $f(x)$ is a periodic function where least positive value of T is called fundamental period.

Graphically : If the graph repeats at fixed interval, then function is said to be periodic and its period is the width of that interval.

Some standard results on periodic functions :

	Functions	Periods
(i)	$\sin^n x, \cos^n x, \sec^n x, \operatorname{cosec}^n x$	π ; if n is even. 2π ; (if n is odd or fraction)
(ii)	$\tan^n x, \cot^n x$	π ; n is even or odd.
(iii)	$\|\sin x\|, \|\cos x\|, \|\tan x\|$ $\|\cot x\|, \|\sec x\|, \|\operatorname{cosec} x\|$	π
(iv)	$x - [x]$, [.] represents greatest integer function	1
(v)	Algebraic functions e.g., $\sqrt{x}$, $x^2, x^3 + 5$, ...etc.	period does not exist

Properties of Periodic Function

(i) If $f(x)$ is periodic with period T, then

(a) $c \cdot f(x)$ is periodic with period T.

(b) $f(x \pm c)$ is periodic with period T.

(c) $f(x) \pm c$ is periodic with period T.

where c is any constant.

(ii) If $f(x)$ is periodic with period T, then

$k f(cx + d)$ has period $T/|c|$,

i.e. Period is only affected by coefficient of x where k, c, d ∈ constant.

(iii) If $f_1(x), f_2(x)$ are periodic functions with periods T_1, T_2 respectively, then we have, $h(x) = a f_1(x) \pm b f_2(x)$ has period as, LCM of $\{T_1, T_2\}$

Note..

(a) $$\text{LCM of} \left(\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right) = \frac{\text{LCM of } (a, c, e)}{\text{HCF of } (b, d, f)}$$

(b) LCM of rational and rational always exists.

LCM of irrational and irrational sometime exists.

But LCM of rational and irrational never exists.

e.g., LCM of (2π, 1, 6π) is not possible as

$2\pi, 6\pi \in$ irrational and $1 \in$ rational.

SOLVED EXAMPLES

RELATION & FUNCTION-I

Example – 1

If $\left(\frac{x}{3}+1,\ y-1\right)=(2,\ 1)$ find values of x and y.

Sol. $\frac{x}{3}+1=2\ \ \&\ \ y-1=1$

x = 3 and y = 2.

Example – 2

If A = {1, 2}, find A × A × A

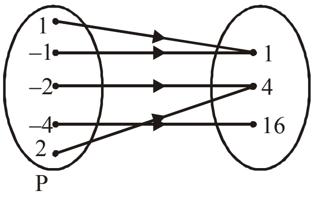
Sol. A × A × A = {(x, y, z), x ∈ A, y ∈ A, z ∈ A}

so, A × A × A = {(1, 1, 1), (1, 1,), (1, 2, 1), (2, 1, 1),

(2, 2, 2), (2, 2, 1), (2, 1, 2), (1, 2, 2)}

Example – 3

Following figure shows a relation between sets P and Q. Write this relation in (i) set builder form, (ii) roster form



Sol. It is clear, that relation R is "y is the square of x".

(i) In set builder form, R = {(x, y) : $y=x^2$, x ∈ P, y ∈ Q}

(ii) In roster form,

R = {(1, 1), (–1, 1), (2, 4), (–2, 4) (4, 16)}

Example – 4

Let R be the relation on Z defined by R = {(a, b); a, b, ∈ Z, a – b is an integer}. Find domain and range of R.

Sol. As for any two integers a & b, a – b an integer hence domain and range is all integers.

Example – 5

Determine domain and range of :-

$$R=\left\{\left(x-4,\ \frac{2+x}{2-x}\right):4\le x\le 6,\ x\in N\right\}$$

Sol. $R=\left\{(8,-3),\left(9,\frac{7}{3}\right),(10,-2)\right\}$

so, domain = {8, 9, 10}

range = $\left\{-3,\ -\frac{7}{3},\ -2\right\}$

Example – 6

Let A = {1, 2}. List all the relation on A.

Sol. Given A = {1, 2}

A × A = {(1, 1), (1, 2), (2, 1), (2, 2)}

Since relation R from set A to set A is a subset of A × A

∴ All the relations on A are :

ϕ, {(1, 1)}, {(1, 2)}, {(2, 1)}, {(2, 2)}, {(1, 1), (1, 2)}, {(1, 1), (2, 1)}, {(1, 1), (2, 2)}, {(1, 2), (2, 1)}, {(1, 2), (2, 2)}, {(2, 1), (2, 2)}, {(1, 1), (1, 2), (2, 1)}, {(1, 1), (1, 2), (2, 2)}, {(1, 1), (2, 1), (2, 2)}, {(1, 2), (2, 1), (2, 2)}, {(1, 1), (1, 2), (2, 1), (2, 2)}.

Since n(A × A) = 4, the number of all relations in the set A = 2^4 i.e., 16.

Example – 7

Find the domain and range of the following functions

(i) $\left\{\left(x,\ \frac{x^2-1}{x-1}\right):x\in R,x\ne 1\right\}$

(ii) $\left\{\left(x,\ \frac{1}{1-x^2}\right):x\in R,x\ne \pm 1\right\}$

Sol. (i) Let $f(x)=\left\{\left(x,\ \frac{x^2-1}{x-1}\right):x\in R,x\ne 1\right\}$

Clearly, f is not defined when $x = 1$

∴ f is defined for all real values of x except $x = 1$

∴ Domain $= R - \{1\}$

Let $y = \dfrac{x^2 - 1}{x - 1} = x + 1 (\text{as } x \neq 1)$

∴ $x = y - 1$

Clearly, x is not defined when $y = 2$ as $x \neq 1$

∴ Range $= R - \{2\}$.

(ii) Let $f(x) = \left\{\left(x, \dfrac{1}{1 - x^2}\right) : x \in R, x = \pm 1\right\}$

Clearly, $f(x) = \dfrac{1}{1 - x^2}$ is not defined when $1 - x^2 = 0$

i.e., when $x = \pm 1$

∴ Domain $= R - \{1, -1\}$

Further, $y = \dfrac{1}{1 - x^2}$

$\Rightarrow \left(1 - x^2\right) = \dfrac{1}{y} \qquad \Rightarrow x = \sqrt{\left(1 - \dfrac{1}{y}\right)} = \sqrt{\dfrac{y - 1}{y}}$

∴ x is defined when $y \in (-\infty, 0) \cup [1, \infty)$.

Range $= (-\infty, 0) \cup [1, \infty)$.

Example – 8

Let $f = \left\{\left(x, \dfrac{x^2}{1 + x^2}\right) : x \in R\right\}$ be a function from R to R.

Determine the range of f.

Sol. Clearly $f : R \to R$ is a function such that

$$f(x) = \frac{x^2}{1 + x^2}$$

Let $y = \dfrac{x^2}{1 + x^2}$. So $x^2 = y(1 + x^2)$. Therefore, $x^2(1 - y) = y$ which

implies $x = \pm\sqrt{\dfrac{y}{1 - y}}$. Since $x \in R$, $\dfrac{y}{1 - y} \geq 0$.

i.e. $y \in [0, 1)$. Thus range is $[0, 1)$

Example – 9

Let $f, g : R \to R$ be defined respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and f/g.

Sol. Let $f(x) = x + 1, g(x) = 2x - 3$

∴ $f + g = f(x) + g(x) = (x + 1) + (2x - 3)$

$= 3x - 2$

$f - g = f(x) - g(x) = (x + 1) - (2x - 3)$

$= x + 1 - 2x + 3 = -x + 4$

$\dfrac{f}{g} = \dfrac{f(x)}{g(x)} = \dfrac{x + 1}{2x - 3}; \; x \neq \dfrac{3}{2}$

Example – 10

A function f is defined on the set $\{1, 2, 3, 4, 5\}$ as follows :

$$f(x) = \begin{cases} 1 + x & \text{if } 1 \leq x < 2 \\ 2x - 1 & \text{if } 2 \leq x < 4 \\ 3x - 10 & \text{if } 4 \leq x < 6 \end{cases}$$

(i) Find the domain of the function.

(ii) Find the range of the function.

(iii) Find the values of $f(2), f(3), f(4), f(6)$.

Sol. (i) Domain : $\{1, 2, 3, 4, 5\}$

(ii) Range :

$f(1) = 1 + 1 = 2 \qquad f(4) = 3(4) - 10 = 2$

$f(2) = 2(2) - 1 = 3 \qquad f(5) = 3(5) - 10 = 5$

$f(3) = 2(3) - 1 = 5$

So, range is $\{2, 3, 5\}$

Example – 11

Find the domain of definition of the following

function : $f(x) = \sqrt{\dfrac{(x - 1)(x + 2)}{(x - 3)(x - 4)}}$

Sol. For $f(x)$ to be defined $\dfrac{(x - 1)(x + 2)}{(x - 3)(x - 4)} \geq 0$ and $x \neq 3, 4$

By wavy – curve method the domain of definition of $f(x)$ is the set

$x \in (-\infty, -2] \cup [1, 3) \cup (4, \infty)$.

Example – 12

Find the domain of definition of the following function : $f(x) = \sqrt{\log_{\frac{1}{2}}(2x-3)}$

Sol. For f(x) to be defined $\log_{1/2}(2x-3) \geq 0$

$\Rightarrow$ $2x - 3 \leq 1$

$\Rightarrow$ $x \leq 2$ ....(1)

Also $2x - 3 > 0$

$\Rightarrow$ $x > \frac{3}{2}.$ ....(2)

Combining (1) and (2) we get the required values of x.

Hence the domain of definition of f(x) is the set $\left(\frac{3}{2}, 2\right]$

Example – 13

If y =3[x] +1 =2[x – 3] +5, then find the value of [x + y], where [.] represents greatest integer function.

Sol. We are given that 3[x] + 1 = 2([x] – 3) + 5

$\Rightarrow$ [x] = –2

$\Rightarrow$ y = 3(–2) + 1 = –5

Hence [x + y] = [x] + y = – 2 – 5 = – 7

Example –14

Solve $\frac{|x+3| + x}{x+2} > 1$

Sol. $\frac{|x+3| + x}{x+2} - 1 > 0$

$\Rightarrow$ $\frac{|x+3| + x - x - 2}{x+2} > 0$

$\Rightarrow$ $\frac{|x+3| - 2}{x+2} > 0$ ...(i)

Now two cases arises :

Case I : When $x + 3 \geq 0$ ...(ii)

$\Rightarrow$ $\frac{x + 3 - 2}{x + 2} > 0$

$\Rightarrow$ $\frac{x + 1}{x + 2} > 0$

$\Rightarrow$ $x \in (-\infty, -2) \cup (-1, \infty)$ using number line rule as shown in figure.

But $x \geq -3$ {from (ii)}

$\Rightarrow$ $x \in [-3, -2) \cup (-1, \infty)$ ...(a)

Case II : When $x + 3 < 0$ ...(iii)

$\Rightarrow$ $\frac{-(x + 3) - 2}{x + 2} > 0$

$\Rightarrow$ $\frac{-(x + 5)}{(x + 2)} > 0$

$\Rightarrow$ $\frac{(x + 5)}{(x + 2)} < 0$

$\Rightarrow$ $x \in (-5, -2)$ using number line rule as shown in figure.

But $x < -3$ {from (iii)}

$\therefore$ $x \in (-5, -3)$ ...(b)

Thus from (a) and (b), we have;

$x \in [-3, -2) \cup (-1, \infty) \cup (-5, -3)$

$\Rightarrow$ $x \in (-5, -2) \cup (-1, \infty)$

Example – 15

The value of x if | x + 3 | > | 2 x – 1 | is

(a) $\left(-\frac{2}{3}, 4\right)$ (b) $\left(-\frac{2}{3}, \infty\right)$

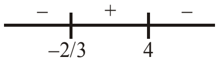
(c) (0, 1) (d) None of these

Sol. Squaring both sides, we get

$|x+3|^2 > |2x-1|^2$

or $\{(x+3) - (2x-1)\}\ \{(x+3) + (2x-1)\} > 0$

$\Rightarrow$ $\{(-x+4)(3x+2)\} > 0$



$\Rightarrow$ $x \in \left(-\frac{2}{3}, 4\right)$

Hence, (a) is the correct answer.

Example – 16

The value of x, $\log_e (x-3) < 1$ is

(a) (0, 3) (b) (0, e)

(c) (0, e + 3) (d) (3, 3 + e)

Sol. From definition of logarithms $x - 3 > 0$

or $x > 3$ ...(i)

Also, e > 1, given inequality may written as

$\because$ $x - 3 < (e)^1$

or $x < 3 + e$ ...(ii)

Using (i) & (ii)

$\Rightarrow$ $x \in (3, 3 + e)$

Hence, (d) is the correct answer.

Example – 17

The value of x if $\log_{1/2} x \geq \log_{1/3} x$ is

(a) (0, 1] (b) (0, 1)

(c) [0, 1) (d) None of these

Sol. **Case I.** When $x \neq 1$ and $x > 0$.

$\log_{1/2} x \geq \log_{1/3} x$

$$\Rightarrow \frac{\log_x x}{\log_x\left(\frac{1}{2}\right)} \geq \frac{\log_x x}{\log_x\left(\frac{1}{3}\right)}$$

$$\Rightarrow \frac{1}{-\log_x 2} \geq + \frac{1}{-\log_x 3}$$

$$\Rightarrow \frac{1}{\log_x 2} \leq \frac{1}{\log_x 3}$$

$\Rightarrow$ $\log_x 2 \geq \log_x 3$ where $x \neq 1$

which is only possible, if $0 < x < 1$

Case II. When x = 1.

$\Rightarrow$ $\log_{1/2} x = \log_{1/3} x$, equality sign holds true.

Combining the above cases,

$0 < x \leq 1$ or $x \in (0, 1]$.

Hence, (a) is correct answer.

Example – 18

Solve $(x + 1)^2 + (x^2 + 3x + 2)^2 = 0$

Sol. Here, $(x + 1)^2 + (x^2 + 3x + 2)^2 = 0$ if and only if each term is zero simultaneously,

$(x + 1) = 0$ and $(x^2 + 3x + 2) = 0$

i.e., $x = -1$ and $x = -1, -2$

The common solution is $x = -1$

Hence, solution of above equation is $x = -1$

Example – 19

Find the domain of the function;

$$f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{(x+2)}$$

Sol. $f(x) = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

{as we know; $\log_a x$ is defined when x, a > 0 and a ≠ 1 also $\log_a 1 = 0$}

Thus, $\log_{10}(1 - x)$ exists when, $1 - x > 0$ ...(i)

also $\dfrac{1}{\log_{10}(1-x)}$ exists when, $1 - x > 0$

and $1 - x \neq 1$ ...(ii)

$\Rightarrow$ $x < 1$ and $x \neq 0$ ...(iii)

also we have $\sqrt{x+2}$ exists when $x + 2 \geq 0$

or $x \geq -2$ ....(iv)

Thus, $f(x) = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$ exists when (iii) and (iv) both holds true.

$\Rightarrow$ $-2 \leq x < 1$ and $x \neq 0$

$\Rightarrow$ $x \in [-2, 0) \cup (0, 1)$

Example – 20

Find value of 'x' so that $\log_{|x|} |x - 1| \geq 0$.

Sol. It is clear that $|x| > 0$ and $|x| \neq 1$

$\Rightarrow$ $x \neq 0, -1, 1$

Also, $|x - 1| > 0 \Rightarrow x \neq 1$,

Case I : For $0 < |x| < 1$

$\Rightarrow$ $x \in (-1, 0) \cup (0, 1)$ ...(1)

and $\log_{|x|}|x-1| \ge 0$

⇒ $\log_{|x|}|x-1| \ge \log_{|x|} 1$ so that

$0 < |x-1| \le 1$

⇒ $-1 \le x-1 \le 1$ and $x \ne 1$

⇒ $0 \le x \le 2$ and $x \ne 1$

⇒ $x \in [0, 1) \cup (1, 2]$. ...(2)

From (1) and (2) we have $x \in (0, 1)$

Case II : For $|x| > 1$

⇒ $x < -1$ or $x > 1$

i.e. $x \in (-\infty, -1) \cup (1, \infty)$ ...(3)

and $\log_{|x|}|x-1| \ge 0$

⇒ $|x-1| \ge 1$

⇒ $x-1 \ge 1$ or $x-1 \le -1$

⇒ $x \ge 2$ or $x \le 0$

i.e. $x \in (-\infty, 0] \cup [2, \infty)$ ...(4)

From (3) and (4), we find that $x \in (-\infty, -1) \cup [2, \infty)$

Hence $x \in (0, 1) \cup (-\infty, -1) \cup [2, \infty)$

Example – 21

If $y = 3[x] + 1 = 2[x-3] + 5$, then find the value of $[x + y]$, where [.] represents greatest integer function.

Sol. We are given that $3[x] + 1 = 2([x] - 3) + 5$

⇒ $[x] = -2$

⇒ $y = 3(-2) + 1 = -5$

Hence $[x + y] = [x] + y = -2 - 5 = -7$

Example – 22

Solve the equation $|2x-1| = 3[x] + 2\{x\}$ for x.
where [.] represents greatest integer function and {} represents fraction part function.

Sol. **Case I :** For $x < \frac{1}{2}$, $|2x-1| = 1-2x$

⇒ $1-2x = 3[x] + 2\{x\}$.

⇒ $1-2x = 3(x - \{x\}) + 2\{x\}$.

⇒ $\{x\} = 5x-1$.

Now $0 \le \{x\} < 1$

⇒ $0 \le 5x-1 < 1$.

⇒ $\frac{1}{5} \le x < \frac{2}{5}$ ⇒ $[x] = 0$

⇒ $x = \{x\}$ ⇒ $x = 5x-1$

⇒ $x = \frac{1}{4}$, which is a solution.

Case II : For $x \ge \frac{1}{2}$, $|2x-1| = 2x-1$

⇒ $2x-1 = 3[x] + 2\{x\}$.

⇒ $2x-1 = 3(x - \{x\}) + 2\{x\}$.

$\{x\} = x+1$

Now $0 \le \{x\} < 1$

⇒ $0 \le x+1 < 1.$ ⇒ $-1 \le x < 0.$

which is not possible since $x \ge \frac{1}{2}$.

Hence $x = \frac{1}{4}$ is the only solution.

Example – 23

Find the domain of definition of the following

function : $f(x) = \sqrt{\log_{\frac{1}{2}}(2x-3)}$

Sol. For f(x) to be defined $\log_{1/2}(2x-3) \ge 0$

⇒ $2x-3 \le 1$

⇒ $x \le 2$ ....(1)

Also $2x-3 > 0$

⇒ $x > \frac{3}{2}$. ....(2)

Combining (1) and (2) we get the required values of x.

Hence the domain of definition of f(x) is the set $\left(\frac{3}{2}, 2\right]$

Example – 24

Find the range of the following function :

$f(x) = \ln\sqrt{x^2 + 4x + 5}$

Sol. Here $f(x) = \ln\sqrt{x^2 + 4x + 5} = \ln\sqrt{(x+2)^2 + 1}$

i.e. $x^2 + 4x + 5$ takes all values in $[1, \infty)$

⇒ $f(x)$ will take all values in $[0, \infty)$.

Hence range of $f(x)$ is $[0, \infty)$.

Example – 25

Let $A = \{x : -1 \le x \le 1\} = B$ for a mapping $f : A \to B$. For each of the following functions from A to B, find whether it is surjective or bijective.

(a) $f(x) = |x|$ (b) $f(x) = x\,|x|$

(c) $f(x) = x^3$ (d) $f(x) = [x]$

(e) $f(x) = \sin\dfrac{\pi x}{2}$

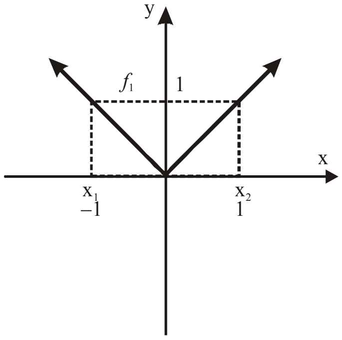
Sol. **(a)** $f(x) = |x|$

Graphically we can see that for $x \in [-1, 1]$

$y = |x| \in [0, 1]$

Since, Range $([0, 1]) \subset$ co-domain $(B = [-1, 1])$

many-to-one

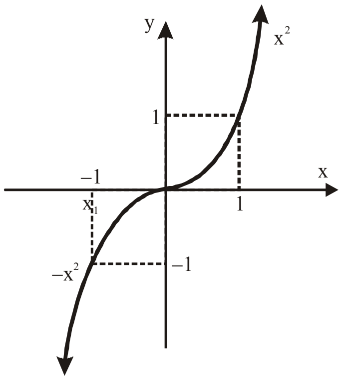


$\Rightarrow$ into function

$\Rightarrow$ $f : [-1, 1] \to [-1, 1],\ f(x) = |x|$

is many-to-one & into

(b) $f(x) = x\,|x| = \begin{cases} x^2, & x \ge 0 \\ -x^2, & x < 0 \end{cases}$



By calculus

$$f'(x) = \begin{cases} 2x, & x > 0 \\ -2x, & x < 0 \end{cases}$$

Since, $f'(x) \ge 0$ always over the entire domain.

$\Rightarrow$ one-to-one / injective function.

Also Range for domain $x \in [-1, 1] = [-1, 1] =$ co-domain

$\Rightarrow$ Surjective Function

Bijective Function

$\Rightarrow$ Similar approach for other questions.

Example – 26

Check whether the function :

$f(x) = 2x^3 + 3x^2 + 6x + 5$ is

one-to-one or many-to-one

Sol. $f(x) = 2x^3 + 3x^2 + 6x + 5$

$f'(x) = 6(x^2 + x + 1) > 0\ \forall\ x \in R$

as $(a > 0\ \&\ D < 0)$ for $x^2 + x + 1$

$\Rightarrow$ $f(x)$ is increasing function on its entire domain

$\Rightarrow$ one-to-one function.

Example – 27

Find the range of the function $y = \dfrac{1}{2 - \sin 3x}$

Sol. Clearly, as Denominator $(2 - \sin 3x) \ne 0$

$(\sin 3x \ne 2)$

$\Rightarrow$ Domain : $x \in R$

We have, $y = \dfrac{1}{2 - \sin 3x}$

Note : (sin 3x) can be seperated & written as a function of y

$\Rightarrow$ $2 - \sin 3x = \dfrac{1}{y}$

$\Rightarrow$ $\sin 3x = \dfrac{2y - 1}{y}$

for x to be real

$\Rightarrow$ $-1 \le \dfrac{2y-1}{y} \le 1$ (since, $-1 \le \sin 3x \le 1$)

Ask : Can we cross multiply ‘y’ across inequality?

Yes!! as $y > 0$ (why??) $\rightarrow \because\ y = \dfrac{1}{2 - \sin 3x}$

$$-1 \le \frac{2y-1}{y} \le 1 \quad \xrightarrow{\text{Alternate}} \quad -1 \le \frac{2y-1}{y} \le 1$$

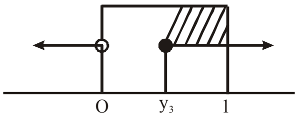
$-y \le 2y - 1 \le y$

$\dfrac{2y-1}{y} + 1 \ge 0 \ \cap\ \dfrac{2y-1}{y} - 1 \le 0$

$2y - 1 \ge -y$ & $2y - 1 \le y$ $\qquad \dfrac{3y-1}{y} \ge 0 \cap \dfrac{y-1}{y} \le 0$

$\Rightarrow \quad y \ge \dfrac{1}{3} \cap y \le 1$

$\Rightarrow$ Range : $y \in \left[\dfrac{1}{3}, 1\right] \leftarrow$



Alternate Method :

$$y = \frac{1}{2 - \sin 3x}$$

we know, $\quad -1 \le \sin 3x \le 1$

$\Rightarrow \quad 1 \ge -\sin 3x \ge 1$

$\Rightarrow \quad 1 \le 2 - \sin 3x \le 1$

$\Rightarrow \quad \dfrac{1}{1}\left(\dfrac{1}{2 - \sin 3x}\right) \ge \dfrac{1}{3}$

$\Rightarrow$ Range $y \in \left[\dfrac{1}{3}, 1\right]$

Inequality changes upon reciprocating as all expressions across inequality are (positive).

Example – 28

Find domain for $f(x) = \sqrt{\cos(\sin x)}$.

Sol. $f(x) = \sqrt{\cos(\sin x)}$ is defined, if

$\cos(\sin x) \ge 0$

As, we know

$-1 \le \sin x \le 1 \qquad$ for all x

$\cos \theta \ge 0$

(Here, $\theta = \sin x$ lies in the 1st and 4th quadrants)

i.e. $\quad \cos(\sin x) \ge 0$, for all x

i.e. $\quad x \in R$.

Thus, domain $f(x) \in R$

Example – 29

Solve for x

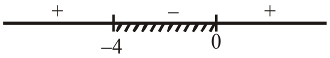
$$|x| + |x+4| = 4$$

Sol. $|x| + |x+4| = 4$

As we know, $\qquad |x| + |y| = |x - y|$, iff $xy \le 0$

$x(x+y) \le 0$

Using number line rule,



$\Rightarrow \quad x \in [-4, 0]$

Example – 30

Solve $\left|\dfrac{x}{x-1}\right| + |x| = \dfrac{x^2}{|x-1|}$

Sol. Let $f(x) = \dfrac{x}{x-1}$ and $g(x) = x$

$\therefore \quad f(x) + g(x) = \dfrac{x}{x-1} + x = \dfrac{x^2}{x-1}$

Using, $|f(x)| + |g(x)| = |f(x) + g(x)|$

i.e. $\qquad f(x) \cdot g(x) \ge 0$

$\Rightarrow \quad \dfrac{x}{x-1} . x \ge 0 \ \Rightarrow \dfrac{x^2}{x-1} \ge 0$

$\Rightarrow \quad x \in \{0\} \cup (1, \infty)$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

SETS

Definition and Types of Sets

1. Which of the following is the empty set?

(a) $\{x : x$ is a real number and $x^2 - 1 = 0\}$

b) $\{x : x$ is a real number and $x^2 + 1 = 0\}$

(c) $\{x : x$ is a real number and $x^2 - 9 = 0\}$

(d) $\{x : x$ is a real number and $x^2 = x + 2\}$

2. Let $A = \{2, 3, 4\}$ and $X = \{0, 1, 2, 3, 4\}$, then which of the following statements is correct

(a) $\{0\} \in A^c$ w.r.t. X
(b) $\phi \in A^c$ w.r.t. X
(c) $\{0\} \subset A^c$ w.r.t.X
(d) $0 \subset A^c$ w.r.t. X.

3. In a city 20% of the population travels by car, 50% travels by bus and 10% travels by both car and bus. Then, persons travelling by car or bus is

(a) 80 %
(b) 40%
(c) 60%
(d) 70%

4. If $A = \{1, 2, 3\}$, $B = \{a, b\}$, then $A \times B$ is given by

(a) $\{(1, a), (2, b), (3, b)\}$

(b) $\{(1, b), (2, a)\}$

(c) $\{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

(d) $\{(1, a), (2, a), (2, b), (3, b)\}$

5. The set of intelligent students in a class is

(a) a null set

(b) a singleton set

(c) a finite set

(d) not a well defined collection

Operations on sets (Union and Intersection)

6. If $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, $B = \{2, 4, ...,18\}$and N is the universal set, then $A^c \cup ((A \cup B) \cap B^c)$ is

(a) A
(b) N
(c) B
(d) None of these

7. Let $A = \{x : x$ is a multiple of $3\}$ and $B = \{x : x$ is a multiple of $5\}$. Then $A \cap B$ is given by

(a) $\{3, 6, 9...\}$
(b) $\{5, 10, 15, 20, ...\}$
(c) $\{15, 30, 45, ...\}$
(d) None of these

8. If $Y \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$, then

(a) The smallest set of Y is $\{3, 5, 9\}$

(b) The smallest set of Y is $\{2, 3, 5, 9\}$

(c) The largest set of Y is $\{1, 2, 3, 5, 9\}$

(d) The largest set of Y is $\{2, 3, 5, 9\}$

9. Given the sets $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$, then $[A \cup (B \cap C)]$ is

(a) $\{1, 2, 3, 4, 5, 6\}$
(b) $\{1, 2, 4, 5\}$
(c) $\{1, 2, 3, 4\}$
(d) $\{3\}$

10. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to

(a) B^c
(b) A^c
(c) B
(d) A

11. If $A = \{x : x = 4n + 1, 2 \le n \le 5\}$, then number of subsets of A is

(a) 16
(b) 15
(c) 4
(d) None of these

12. If $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 6\}$ and $C = \{3, 4, 6, 7\}$, then

(a) $A - (B \cap C) = \{1, 3, 4\}$
(b) $A - (B \cap C) = \{1, 2, 4\}$
(c) $A - (B \cup C) = \{2, 3\}$
(d) $A - (B \cup C) = \{1\}$

Properties of cardinality and its practical applications

13. X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$, $n(X \cup Y) = 38$ then $n(X \cap Y)$ is

(a) 4
(b) 2
(c) 6
(d) None of these

14. If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements; then number of elements in $X \cap Y$ are

(a) 5
(b) 8
(c) 6
(d) None of these

15. If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, then number of elements $S \cup T$ has

(a) 42
(b) 50
(c) 48
(d) None of these

16. In a committee 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. The number of persons speaking at least one of these two languages is

(a) 60 (b) 40
(c) 38 (d) None of these

17. In a group of 1000 people, there are 750 people who can speak Hindi and 400 who can speak English .Then number of persons who can speak Hindi only is

(a) 300 (b) 400
(c) 600 (d) None of these

18. In a statistical investigation of 1,003 families of Calcutta, it was found that 63families had neither a radio nor a T.V, 794 families had a radio and 187 had a T.V. The number of families in that group having both a radio and a T.V is

(a) 36 (b) 41
(c) 32 (d) None of these

19. If A has 3 elements and B has 6 elements, then the minimum number of elements in the set $A \cup B$ is

(a) 6 (b) 3
(c) ϕ (d) None of these

20. The set $A = \{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ equals

(a) ϕ (b) $\{14, 3, 4\}$
(c) $\{3\}$ (d) $\{4\}$

INEQUALITIES

21. The set of values of x satisfying the inequalities $(x-1)(x-2) < 0$ and $(3x-7)(2x-3) > 0$ is

(a) (1, 2) (b) (2, 7/3)
(c) (1, 7/3) (d) (1, 3/2)

22. If $x^2 + 6x - 27 > 0$ and $x^2 - 3x - 4 < 0$, then

(a) $x > 3$ (b) $x < 3$
(c) $3 < x < 4$ (d) $x = 7/2$

23. Find the set of values of 'x' for which the given condition is true $(x-1)(x-3)(x+5) > 0$

(a) $(-5, 1) \cup (3, \infty)$ (b) $(-1, 5)$
(c) $[-5, 1] \cup [3, \infty)$ (d) none of these

24. The value for which $\frac{(x-1)}{x} \ge 2$

(a) (0, 1) (b) $(-\infty, -1)$
(c) $(-\infty, 0)$ (d) $[-1, 0)$

25. The value of x for which $12x - 6 < 0$ and $12 - 3x < 0$

(a) ϕ (b) R
(c) $R - \{0\}$ (d) none of these

26. The value of x for which $\frac{x-3}{4} - x < \frac{x-1}{2} - \frac{x-2}{3}$ and $2 - x > 2x - 8$

(a) [–1, 10/3] (b) (–1, 10/3)
(c) R (d) none of these

27. Solve for x :

(i) $\frac{(2x-1)(x-1)^4(x-2)^4}{(x-2)(x-4)^4} \le 0$

(a) [½, 2) (b) R
(c) ϕ (d) (½, 2)

(ii) $(x-2)^4(x-3)^3(x-4)^2(1-x) \le 0$

(a) (1, 3) (b) $(-\infty, 1) \cup (3, \infty)$
(c) $(-\infty, 1] \cup [3, \infty)$ (d) none

(iii) $x + \frac{1}{x} \ge 2$

(a) $(0, \infty)$ (b) R
(c) ϕ (d) $[0, \infty)$

(iv) $\frac{x^2}{x-1} \ge 0$

(a) $(1, \infty)$ (b) $[1, \infty)$
(c) $\{0\} \cup (1, \infty)$ (d) none

28. If $c < d$, $x^2 + (c + d)x + cd < 0$, then x belongs to.

(a) (–d, –c] (b) (–d, –c)
(c) R (d) ϕ

29. Solution of $\frac{x-7}{x+3} > 2$ is

(a) $(-3, \infty)$ (b) $(-\infty, -13)$
(c) (–13, –3) (d) none of these

30. The set of values of x which satisfy the inequations $5x + 2 < 3x + 8$ and $\frac{x+2}{x-1} < 4$ is

(a) $(-\infty, 1)$ (b) (2, 3)
(c) $(-\infty, 3)$ (d) $(-\infty, 1) \cup (2, 3)$

31. If $x^2 - 1 \le 0$ and $x^2 - x - 2 \ge 0$, then x lies in the interval set

(a) $(1, -1)$ (b) $(-1, 1)$

(c) $(1, 2)$ (d) $\{-1\}$

32. The solution set of $\frac{x^2 - 3x + 4}{x + 1} > 1,\ x \in R$ is

(a) $(3, \infty)$ (b) $(-1, 1) \cup (3, \infty)$

(c) $[-1, 1] \cup [3, \infty)$ (d) none

33. If $\frac{1}{a} < \frac{1}{b}$, then :

(a) $|a| > |b|$ (b) $a < b$

(c) $a > b$ (d) none of these

34. If $-2 < x < 3$, then :

(a) $4 < x^2 < 9$ (b) $0 \le |x| < 5$

(c) $0 \le x^2 < 9$ (d) None of these

35. $x > \sqrt{2 - x^2}$

(a) $x \in (1, \infty)$ (b) $x \in (-\infty, -1) \cup (1, \infty)$

(c) $x \in \left(1, \sqrt{2}\right]$ (d) $x \in \left[\sqrt{2}, \infty\right)$

Functions : Definition, Domain, Range and Types of Functions

36. Let $A = [-1, 1]$ and $f : A \to A$ be defined as $f(x) = x|x|$ for all $x \in A$, then $f(x)$ is

(a) many-one into function

(b) one-one into function

(c) many-one onto function

(d) one-one onto function

37. The function $f : R \to R$ defined by

$f(x) = (x - 1)(x - 2)(x - 3)$ is

(a) one-one but not onto

(b) onto but not one-one

(c) both one-one and onto

(d) neither one-one nor onto

38. Find the domain of $f(x) = \frac{1}{\sqrt{x - 5}} + x^2 + \frac{1}{\sqrt{x + 7}}$

(a) $x \in [-7, 5]$ (b) $x \in (5, \infty)$

(c) $x \in (-\infty, 7)$ (d) none of these

39. Find the domain $y = \sqrt{1 - x} + \sqrt{x - 5}$

(a) $x \in \phi$ (b) $y \in (-\infty, 1]$

(c) $x \in (-\infty, 1] \cup [5, \infty)$ (d) none of these

40. The domain of the function

$f(x) = \sqrt{x - 3 - 2\sqrt{x - 4}} - \sqrt{x - 3 + 2\sqrt{x - 4}}$ is

(a) $[4, \infty)$ (b) $(-\infty, 4]$

(c) $(4, \infty)$ (d) $(-\infty, 4)$

41. The domain of the function $f(x) = \sqrt{\log_{10} \frac{3 - x}{x}}$

(a) $\left(0, \frac{3}{2}\right)$ (b) $(0, 3)$

(c) $\left(-\infty, \frac{3}{2}\right)$ (d)) $\left(0, \frac{3}{2}\right]$

42. The domain of the function

$f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}$

where [ ] denotes greatest integer function

(a) $R - [-2, 4)$ (b) $R - \{-3, 2\}$

(c) R (d) $R - \{2, 3\}$

43. The domain of the function $f(x) = \log_e(x - [x])$, where [.] denotes the greatest integer function, is

(a) R (b) R – Ƶ

(c) $(0, +\infty)$ (d) None of these

44. Find the Range $y = \frac{2x + 1}{x - 5}$

(a) $y \ne 2$ (b) $x \ne 5$

(c) $y \ne 5$ (d) none of these

45. Range of the function $f(x) = \frac{x}{1 + x^2}$ is

(a) $(-\infty, \infty)$ (b) $[-1, 1]$

(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $\left[-\sqrt{2}, \sqrt{2}\right]$

46. The range of the function $f(x)=x^2+\frac{1}{x^2+1}$

(a) $[1,\infty)$
(b) $[2,\infty)$
(c) $\left[\frac{3}{2},\infty\right)$
(d) None of these

Standard Functions Modulus, Greatest Integer & Logrithm Function

47. $|3x+7| < 5$, then x belongs to

(a) $(-4,-3)$
(b) $(-4,-2/3)$
(c) $(-5,5)$
(d) $(-5/3,5/3)$

48. The least integer satisfying,

$$49.4-\frac{(27-x)}{10}<47.4-\left(\frac{27-9x}{10}\right)$$ is

(a) 2
(b) 3
(c) 4
(d) none of these

49. Solution of $|3x-2|\ge 1$ is

(a) $[1/3,1]$
(b) $(1/3,1)$
(c) $\{1/3,1\}$
(d) $\left(-\infty,\frac{1}{3}\right]\cup[1,\infty)$

50. If $3<|x|<6$, then x belongs to :

(a) $(-6,-3)\cup(3,6)$
(b) $(-6,6)$
(c) $(-3,+3)\cup(3,6)$
(d) None of these

51. If $-5<x<4$, then :

(a) $0\le|x|<4$
(b) $4<|x|<5$
(c) $0\le|x|<5$
(d) none of these

52. If $|x|<x$, then :

(a) x is a positive real number
(b) x is a non-negative real number
(c) there is no x satisfying this inequality
(d) x is a negative real number

53. $|2x-3|<|x+5|$, then x belongs to

(a) $(-3,5)$
(b) $(5,9)$
(c) $(-2/3,8)$
(d) $(-8,2/3)$

54. Solve for x :

(i) $|x+4|>5$

(a) $(-\infty,1)$
(b) $(-\infty,9)$
(c) $R-[-9,1]$
(d) none

(ii) $|x+2|<4$

(a) $(-6,2)$
(b) $(-6,0)$
(c) $(-6,2]$
(d) $(0,2)$

(iii) $\left|\frac{x^2+6}{5x}\right|\ge 1$

(a) $(-\infty,-3)$
(b) $(-\infty,-3)\cup(3,\infty)$
(c) R
(d) $(-\infty,-3]\cup[-2,0)\cup(0,2]\cup[3,\infty)$

55. Solution of $\left|x+\frac{1}{x}\right|<4$ is

(a) $\left(2-\sqrt{3},2+\sqrt{3}\right)\cup\left(-2-\sqrt{3},-2+\sqrt{3}\right)$
(b) $R-\left(2-\sqrt{3},2+\sqrt{3}\right)$
(c) $R-\left(-2-\sqrt{3},-2+\sqrt{3}\right)$
(d) none of these

56. If $f(x)=\frac{1}{\sqrt{|x|-x}}$, then domain of f(x) is

(a) $(-\infty,0)$
(b) $(-\infty,2)$
(c) $(-\infty,\infty)$
(d) None of the above

57. If $|x|>5$, then

(a) $0<x<5$
(b) $x<-5$ or $x>5$
(c) $-5<x<5$
(d) $x>5$

58. If $[x]^2=[x+2]$, where $[x]$ = the greatest integer less than or equal to x, then x must be such that

(a) $x=2,-1$
(b) $x\in[2,3)$
(c) $x\in[-1,0)$
(d) none of these

59. The value of x, $\log_e(x-3)<1$ is

(a) $(0,3)$
(b) $(0,e)$
(c) $(0,e+3)$
(d) $(3,3+e)$

60. Solve for x :

(i) $log_{0.2}(x+5)>0$

(a) $(-5,-4)$ (b) $(-5,4)$
(c) $(0,4)$ (d) $(0,5)$

(ii) $log_x 0.5>2$

(a) $\left(\sqrt{½}\,,1\right)$ (b) $\left(-\infty,1\right)$
(c) $(-1,0)$ (d) $(-1,1)$

(iii) $log_x(x+7)<0$

(a) $(0,1)$ (b) $\left(-\infty,1\right)$
(c) $(-1,0)$ (d) $(-1,1)$

61. Let $f(x)=log_{x^2}25$ and $g(x) = log_x 5$ then $f(x) = g(x)$ holds for x belonging to

(a) R (b) $(0,1)\cup(1,+\infty)$
(c) ϕ (d) None of these

62. Solve for x : $3^{\left(x^2-2\right)}<\left(\frac{1}{3}\right)^{\left(1-\frac{3}{2}|x|\right)}$

(a) $\left(-\sqrt{2},-1\right)$ **(b)** $\left(-\sqrt{2},2\right)$
(c) $\left(-2,-\sqrt{2}\right)$ **(d)** None of these

63. The domain of the function $f(x)=\log_2(\log_3(\log_4 x))$ is

(a) $(-\infty,4)$ (b) $(4,\infty)$
(c) $(0,4)$ (d) $(1,\infty)$

64. The value of x, $\log_{½} x\geq\log_{1/3} x$ is

(a) $(0,1]$ (b) $(0,1)$
(c) $[0,1)$ (d) none

Miscellanious

65. The number of real solutions of

$\sqrt{x^2-4x+3}=\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$ is

(a) one (b) two
(c) three (d) none of these

66. The largest interval among the following for which $x^{12}-x^9+x^4-x+1>0$ is

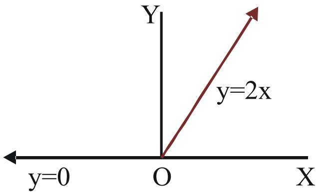
(a) $-4<x\leq0$ (b) $0<x<1$
(c) $-100<x<100$ (d) $-\infty<x<\infty$

67. If $f(x)=x^2-3x+1$ and $f(2\alpha)=2f(\alpha)$, then α is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$ (d) none of these

68. The graph of a real-valued function $f(x)$ is the following. The function is



(a) $f(x)=x-|x|$ (b) $f(x)=x+|x|$
(c) $f(x)=2x$ (d) None of these

69. The number of real solutions of the equation $e^x = x$ is

(a) 1 (b) 2
(c) 0 (d) none of these

70. The number of real solutions of the equation $\log_{0.5} x=|x|$ is

(a) 1 (b) 2
(c) 0 (d) none of these

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. A function f from the set of natural numbers to integers defined by

$$f(n)=\begin{cases}\frac{n-1}{2}, & \text{when n is odd}\\ -\frac{n}{2}, & \text{when n is even}\end{cases} \quad \text{is}$$ **(2003)**

(a) onto but no one-one

(b) one-one and onto both

(c) neither one-one nor onto

(d) one-one but not onto

2. Domain of definition of the function is

$$f(x)=\frac{3}{4-x^2}+\log_{10}\left(x^3-x\right),$$ **(2003)**

(a) $(-1,0)\cup(1,2)$

(b) $(1,2)\cup(2,\infty)$

(c) $(-1,0)\cup(1,2)\cup(2,\infty)$

(d) $(1,2)$

3. If $f: R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all x, $y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is **(2003)**

(a) $\frac{7(n+1)}{2}$ (b) $7n(n+1)$

(c) $\frac{7n(n+1)}{2}$ (d) $\frac{7n}{2}$

4. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then **(2004)**

(a) $f(x)=f(-x)$ (b) $f(2+x) = f(2-x)$

(c) $f(x+2)=f(x-2)$ (d) $f(x) = -f(-x)$

5. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **(2009)**

(a) $A = C$ (b) $B = C$

(c) $A \cap B = \phi$ (d) $A = B$

6. For real x, let $f(x) = x^3 + 5x + 1$, then **(2009)**

(a) f is one-one but not onto R

(b) f is onto R but not one-one

(c) f is one-one and onto R

(d) f is neither one-one nor onto R

7. The domain of the function $f(x)=\frac{1}{\sqrt{|x|-x}}$ is **(2011)**

(a) $(0, \infty)$ (b) $(-\infty, 0)$

(c) $(-\infty, \infty) - \{0\}$ (d) $(-\infty, \infty)$

8. Let for $a \neq a_1 \neq 0$, $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(2)$ is **(2011)**

(a) 18 (b) 3

(c) 9 (d) 6

9. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to : **(2014)**

(a) Y (b) N

(c) $Y - X$ (d) X

10. Let P be the relation defined on the set of all real numbers such that

$P = \{(a, b): \sec^2 a - \tan^2 b = 1\}$. Then P is: **(2014/Online Set–1)**

(a) reflexive and symmetric but not transitive.

(b) reflexive and transitive but not symmetric.

(c) symmetric and transtive but not reflexive.

(d) an equivalence relation.

Therefore, P is transitive

11. Let f be a function defined on the set of real number such taht for $x \geq 0$, $f(x) = 3 \sin x + 4 \cos x$. Then (x) at $x = -\frac{11\pi}{6}$ is equal to : **(2014/Online Set–2)**

(a) $\frac{3}{2}+2\sqrt{3}$ (b) $-\frac{3}{2}+2\sqrt{3}$

(c) $\frac{3}{2}-2\sqrt{3}$ (d) $-\frac{3}{2}-2\sqrt{3}$

12. A relation on the set $A = \{x : |x| < 3, \in xZ\}$, where Z is the set of integers is defined by $R = \{(x, y) : y = |x|, x \neq -1\}$. Then the number of elements in the power se of R is: **(2014/Online Set–3)**

(a) 32 (b) 16

(c) 8 (d) 64

13. Let $f : R \rightarrow R$ be defined by $f(x) = \frac{|x|-1}{|x|+1}$ then f is: **(2014/Online Set–4)**

(a) both one - one and onto

(b) one - one but not onto

(c) onto but not one - one

(d) neither one - one nor onot.

14. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is: **(2015)**

(a) 275 (b) 510

(c) 219 (d) 256

15. In a certain town, 25% of the families own a phone and 15% own a car; 65% families own neither a phone nor a car and 2,000 families own both a car and a phone. Consider the following three statements :

(a) 5% families own both a car and a phone

(b) 35% families own either a car or a phone

(c) 40,000 families live in the town

Then, **(2015/Online Set–1)**

(a) Only (b) and (c) are correct

(b) Only (a) and (c) are correct

(c) All (a), (b) and (c) are correct

(d) Only (a) and (b) are correct

16. If $f : R \rightarrow S$, defined by

$f(x) = \sin x - \sqrt{3} \cos x + 1$, is onto, then the interval of S is **(2015/Online Set–2)**

(a) [0, 1] (b) [-1, 1]

(c) [0, 3] (d) [-1, 3]

17. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then: **(2016/Online Set–2)**

(a) $P \subset Q$ and $Q - P \neq \phi$

(b) $Q \not\subset P$

(c) $P \not\subset Q$

(d) $P = Q$

18. Let a, b, c ∈ R. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy$, $\forall$ x, y ∈ R, then $\sum_{n=1}^{10} f(n)$ is equal to: **(2017)**

(a) 330 (b) 165

(c) 190 (d) 255

19. Let $S = \{x \in R : x \geq 0 \text{ and } 2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0\}$ Then S : **(2018)**

(a) Contain exactly four element

(b) is an empty set.

(c) contain exactly one element

(d) contains exactly two elements.

20. Two sets A and B are as under :

$$A = \left\{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\right\}$$

$$B = \left\{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\right\}.$$

Then : **(2018)**

(a) neither $A \subset B$ nor $B \subset A$

(b) $B \subset A$

(c) $A \subset B$

(d) $A \cap B = \phi$ (an empty set)

21. Consider the following two binary relations on the set $A = \{a, b, c\}$:

$R_1 = \{(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)\}$ and

$R_2 = \{(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c)\}$. Then :

(2018/Online Set–1)

(a) both R_1 and R_2 are not symmetric.

(b) R_1 is not symmetric but it is transitive.

(c) R_2 is symmetric but it is not transitive.

(d) both R_1 and R_2 are transitive.

22. Let N denote the set of all natural numbers. Define two binary relations on N as

$R_1 = \{(x, y) \in N \times N : 2x + y = 10\}$ and

$R_2 = \{(x, y)\ N\ N : x + 2y = 10\}$. Then :

(2018/Online Set–3)

(a) Range of R_1 is $\{2, 4, 8\}$.

(b) Range of R_2 is $\{1, 2, 3, 4\}$.

(c) Both R_1 and R_2 are symmetric relations.

(d) Both R_1 and R_2 are transitive relations.

23. Let A, B and C be three events, which are pair-wise independent and $\overline{E}$ denotes the complement of an event E. If $P(A \cap B \cap C) = 0$ and $P(C) > 0$, then $P\left[\left(\overline{A} \cap \overline{B}\right) | C\right]$ is equal to : **(2018/Online Set–3)**

(a) $P(\overline{A}) - P(B)$

(b) $P(A) + P(\overline{B})$

(c) $P(\overline{A}) - P(\overline{B})$

(d) $P(\overline{A}) + P(\overline{B})$

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. Find the set of values of 'x' for which the given conditions are true :

(a) $-(x-1)(x-3)(x+5)<0$

(b) $\dfrac{(x-1)(x-2)}{(x-3)} \le 0$

2. If $|x-1|+|x|+|x+1| \ge 6$; then x lies in

(a) $(-\infty, 2]$ (b) $(-\infty,-2]\cup[2,\infty)$

(c) R (d) ϕ

3. Solution set of, $\log_3(x^2-2) < \log_3\left(\frac{3}{2}|x|-1\right)$ is

(a) $\left(-\sqrt{2},-1\right)$ (b) $\left(-2,-\sqrt{2}\right)$

(c) $\left(-\sqrt{2},2\right)$ (d) none of these

4. Solution set of the inequality : $\dfrac{1}{2^x-1} > \dfrac{1}{1-2^{(x-1)}}$ is

(a) $(1,\infty)$ (b) $\left(0,\log_2\left(\frac{4}{3}\right)\right)$

(c) $(-1,\infty)$ (d) $\left(0,\log_2\left(\frac{4}{3}\right)\right)\cup(1,\infty)$

5. Solution of the inequality $x > \sqrt{(1-x)}$ is given by

(a) $\left(-\infty,\left(-1-\sqrt{5}\right)/2\right)$

(b) $\left(\left(\sqrt{5}-1\right)/2,\infty\right)$

(c) $\left(-\infty,\left(-1-\sqrt{5}\right)/2\right)\cup\left(\left(\sqrt{5}-1\right)/2,\infty\right)$

(d) $\left(\left(\sqrt{5}-1\right)/2,1\right]$

6. If $x^2-1\le 0$ and $x^2-x-2\ge 0$, then x lies in the interval set

(a) $(1,-1)$ (b) $(-1,1)$

(c) $(1,2)$ (d) $\{-1\}$

7. The least integer satisfying,

$$49.4-\frac{(27-x)}{10} < 47.4-\left(\frac{27-9x}{10}\right) \text{ is}$$

(a) 2 (b) 3

(c) 4 (d) none of these

8. Solution of $|3x-2|\ge 1$ is

(a) [1/3, 1] (b) (1/3, 1)

(c) {1/3, 1} (d) $\left(-\infty,\frac{1}{3}\right]\cup[1,\infty)$

9. Solution of $|x-1|\ge|x-3|$ is

(a) $x\le 2$ (b) $x\ge 2$

(c) [1, 3] (d) none of these

10. Solution of $|1/x-2|<4$ is

(a) $\left(-\infty,-\frac{1}{2}\right)$ (b) $\left(\frac{1}{6},\infty\right)$

(c) $\left(-\frac{1}{2},\frac{1}{6}\right)$ (d) $\left(-\infty,-\frac{1}{2}\right)\cup\left(\frac{1}{6},\infty\right)$

11. Solution of $\left|x+\frac{1}{x}\right|<4$ is

(a) $\left(2-\sqrt{3},2+\sqrt{3}\right)\cup\left(-2-\sqrt{3},-2+\sqrt{3}\right)$

(b) $R-\left(2-\sqrt{3},2+\sqrt{3}\right)$

(c) $R-\left(-2-\sqrt{3},-2+\sqrt{3}\right)$

(d) none of these

12. The solution set of $x^2+2\le 3x\le 2x^2-5$ is

(a) ϕ (b) [1, 2]

(c) $(-\infty,-1)\cup[5/2,\infty)$ (d) none

13. The solution set of $\dfrac{x^2-3x+4}{x+1}>1,\ x\in R$ is

(a) $(3,\infty)$ (b) $(-1,1)\cup(3,\infty)$

(c) $[-1,1]\cup[3,\infty)$ (d) none

14. The number of integral solutions of $\frac{x+2}{x^2+1} > \frac{1}{2}$ is

(a) 4 (b) 5

(c) 3 (d) none of these

15. If for $x \in R$, $\frac{1}{3} \le \frac{x^2-2x+4}{x^2+2x+4} \le 3$, then $\frac{9.3^{2x}+6.3^{x}+4}{9.3^{2x}-6.3^{x}+4}$ lies b/w

(a) 1 and 2 (b) 1/3 and 3

(c) 0 and 4 (d) none of these

16. The solution set of $log_2|4-5x| > 2$ is

(a) $\left(8/5, \infty\right)$ (b) $\left(4/5, 8/5\right)$

(c) $(-\infty, 0) \cup \left(8/5, \infty\right)$ (d) none

17. Solution of $2^x + 2^{|x|} \geq 2\sqrt{2}$ is

(a) $\left(-\infty, \log_2\left(\sqrt{2}+1\right)\right)$

(b) $(0, \infty)$

(c) $\left(\frac{1}{2}, \log_2\left(\sqrt{2}-1\right)\right)$

(d) $\left(-\infty, \log_2\left(\sqrt{2}-1\right)\right] \cup [½, \infty)$

18. The largest interval among the following for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is

(a) $-4 < x \leq 0$ (b) $0 < x < 1$

(c) $-100 < x < 100$ (d) $-\infty < x < \infty$

19. Solution of $|x-1| + |x-2| + |x-3| \geq 6$ is

(a) $[0, 4]$ (b) $(-\infty, -2) \cup [4, \infty)$

(c) $(-\infty, 0] \cup [4, \infty)$ (d) none

20. If $f(x) = \cos[\pi]x + \cos[\pi x]$, where $[y]$ is the greatest integer function of y then $f(\pi/2)$ is equal to

(a) cos 3 (b) 0

(c) cos 4 (d) none of these

21. Let $f(x) = \begin{cases} 1+|x| & x < -1 \\ [x] & x \geq -1 \end{cases}$

where [.] denotes the greatest integer function. Then $f(f(-2.3))$ is equal to

(a) 4 (b) 2

(c) –3 (d) 3

22. The domain of the function $y = log_{10}\, log_{10}\, log_{10} \ldots log_{10} x$ is

⋮……. n times …….⋮

(a) $[10^n, +\infty)$ (b) $(10^{n-1}, +\infty)$

(c) $[10^{n-2}, +\infty)$ (d) None of these

23. The largest set of real values of x for which

$f(x) = \sqrt{(x+2)(5-x)} - \frac{1}{\sqrt{x^2-4}}$ is a real function is

(a) $[1, 2) \cup (2, 5]$ (b) $(2, 5]$

(c) $[3, 4]$ (d) None of these

24. The domain of the function $f(x) = \sqrt{x - \sqrt{1-x^2}}$ is

(a) $\left[-1, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, 1\right]$ (b) $[-1, 1]$

(c) $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right)$

(d) $\left[\frac{1}{\sqrt{2}}, 1\right]$

25. The domain of the function $f(x) = log_{10}\, log_{10}(1 + x^3)$ is

(a) $(-1, \infty)$ (b) $(0, \infty)$

(c) $[0, \infty)$ (d) $(-1, 0)$

26. The domain of the function $f(x) = \sqrt{x^2 - [x]^2}$, where $[x]$ = the greatest integer less than or equal to x is

(a) R (b) $[0, +\infty)$

(c) $(-\infty, 0]$ (d) None of these

27. The domain of $f(x) = \sqrt{log_{x^2-1}(x)}$ is

(a) $(\sqrt{2}, +\infty)$ (b) $(0, \infty)$

(c) $(1, +\infty)$ (d) None of these

28. The domain of the real-valued function $f(x) = log_e|log_e x|$ is

(a) $(1, +\infty)$ (b) $(0, +\infty)$

(c) $(e, +\infty)$ (d) None of these

29. If [.] denotes the greatest integer function then the domain of the real-valued function $\log_{[x+1/2]}|x^2 - x - 2|$ is

(a) $\left[\frac{3}{2}, +\infty\right)$ (b) $\left[\frac{3}{2}, 2\right) \cup (2, +\infty)$

(c) $\left(\frac{1}{2}, 2\right) \cup (2, +\infty)$ (d) None of these

30. Let $f(x) = log_{x^2} 25$ and $g(x) = log_x 5$ then $f(x) = g(x)$ holds for x belonging to

(a) R (b) $(0, 1) \cup (1, +\infty)$

(c) ϕ (d) None of these

31. Let $f: \{x, y, z\} \to \{a, b, c\}$ be a one-one function and only one of the conditions (i) $f(x) \neq b$, (ii) $f(y), = b$ (iii) $f(z) \neq a$ is true then the function f is given by the set

(a) $\{(x, a), (y, b), (z, c)\}$ (b) $\{(x, a), (y, c), (z, b)\}$

(c) $\{(x, b), (y, a), (z, c)\}$ (d) $\{(x, c), (y, b), (z, a)\}$

32. The function $f: R \to R$ defined by $f(x) = 6^x + 6^{|x|}$ is

(a) one-one and onto (b) many-one and onto

(c) one-one and into (d) many-one and into

33. A function whose graph is symmetrical about the y-axis is given by

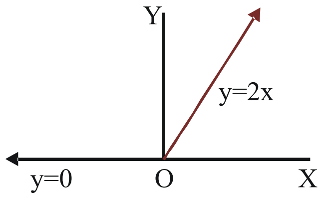
(a) $f(x) = log_e\left(x + \sqrt{x^2+1}\right)$

(b) $f(x+y) = f(x) + f(y)$ for all $x, y \in R$

(c) $f(x) = \cos x + \sin x$

(d) None of these

34. The graph of a real-valued function $f(x)$ is the following. The function is



(a) $f(x) = x - |x|$ (b) $f(x) = x + |x|$

(c) $f(x) = 2x$ (d) None of these

35. Let $f(x) = \begin{cases} x^2, & 0 < x < 2 \\ 2x - 3, & 2 \leq x < 3 \\ x + 2, & x \geq 3 \end{cases}$. Then

(a) $f\left(f\left(f\left(\frac{3}{2}\right)\right)\right) = f\left(\frac{3}{2}\right)$

(b) $1 + f\left(f\left(f\left(\frac{5}{2}\right)\right)\right) = f\left(\frac{5}{2}\right)$

(c) $f(f(1)) = f(1) = 1$

(d) None of these

36. If $a^2 + b^2 + c^2 = 1$, then ab + bc + ca lies in the interval

(a) $\left[-\frac{1}{2}, 1\right]$ (b) $\left[0, \frac{1}{2}\right]$

(c) [0, 1] (d) [1, 2]

37. $log_2(x^2 - 3x + 18) < 4$, then x belongs to

(a) (1, 2) (b) (2, 16)

(c) (1, 16) (d) none of these

38. If $x = log_a(bc)$, $y = log_b(ca)$ and $z = log_c(ab)$ then which of the following is equal to 1?

(a) $x + y + z$ (b) $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1}$

(c) xyz (d) none of these

39. If $0 < x < 1000$ and $\left[\frac{x}{2}\right] + \left[\frac{x}{3}\right] + \left[\frac{x}{5}\right] = \frac{31}{30}x$, where [x] is the greatest integer less than or equal to x, the number of possible values of x is

(a) 34 (b) 32

(c) 33 (d) none of these

40. For a real number x, [x] denotes the integral part of x. The value of

$$\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{100}\right] + \left[\frac{1}{2} + \frac{2}{100}\right] + ... + \left[\frac{1}{2} + \frac{99}{100}\right] \text{is}$$

(a) 49 (b) 50

(c) 48 (d) 51

41. If $f(n+1) = \frac{2f(n)+1}{2}$, n = 1, 2, ... and $f(1) = 2$, then $f(101)$ equals

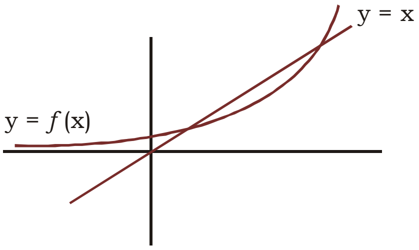
(a) 52 (b) 49

(c) 48 (d) 51

42. The domain of function $f(x) = \dfrac{1}{\sqrt{x^2 - \{x\}^2}}$, where $\{x\}$ denotes fraction part of x.

(a) $R - [0, 1)$
(b) $R - \left[\dfrac{1}{2}, 1\right]$
(c) $(-\infty, \dfrac{1}{2}] \cup (1, \infty)$
(d) none of these

43. If graph of $y = f(x)$ is



Then $f(x)$ can be

(a) $y = 2\,e^x$
(b) $y = 4\,e^x$
(c) $y = e^{x+\frac{1}{2}}$
(d) $y = \dfrac{1}{4}e^x$

44. The equation $|\,|x - 1| + a\,| = 4$ can have real solutions for x if **'a'** belongs to the interval

(a) $(-\infty, 4]$
(b) $(-\infty, -4]$
(c) $(4, +\infty)$
(d) $[-4, 4]$

45. If $\log_k x \cdot \log_5 k = \log_x 5$, $k \neq 1$, $k > 0$, then x is equal to

(a) k
(b) $\dfrac{1}{5}$
(c) 5
(d) None of these

46. If $x^4 f(x) - \sqrt{1 - \sin 2\pi x} = |f(x)| - 2f(x)$, then f(-2) equals:

(a) $\dfrac{1}{17}$
(b) $\dfrac{1}{11}$
(c) $\dfrac{1}{19}$
(d) 0

47. If $\dfrac{1}{2} \leq \log_{0.1} x \leq 2$ then

(a) the maximum value of x is $\dfrac{1}{\sqrt{10}}$

(b) x lies between $\dfrac{1}{100}$ and $\dfrac{1}{\sqrt{10}}$

(c) x does not lie between $\dfrac{1}{100}$ and $\dfrac{1}{\sqrt{10}}$

(d) the minimum value of x is $\dfrac{1}{100}$

48. If f is an even function defined on the interval (–5, 5) then a value of x satisfying the equation $f(x) = f\left(\dfrac{x+1}{x+2}\right)$ is

(a) $\dfrac{-1+\sqrt{5}}{2}$
(b) $\dfrac{-3+\sqrt{5}}{2}$
(c) $\dfrac{-1-\sqrt{5}}{2}$
(d) $\dfrac{-3-\sqrt{5}}{2}$

49. Let $f(x) = [x]$ = the greatest integer less than or equal to x and $g(x) = x - [x]$. Then for any two real numbers x and y.

(a) $f(x + y) = f(x) + f(y)$
(b) $g(x + y) = g(x) + g(y)$
(c) $f(x + y) = f(x) + f(y + g(x))$
(d) none of these

50. Let $x \in N$ and let x be a perfect square. Let $f(x)$ = the quotient when x is divided by 5 and $g(x)$ = the remainder when x is divided by 5. Then $\sqrt{x} = f(x) + g(x)$ holds for x equal to

(a) 0
(b) 16
(c) 25
(d) None of these

51. Let $f(x) = [x]^2 + [x + 1] - 3$, where [x] = the greatest integer $\leq$ x. Then

(a) $f(x)$ is a many- one and into function
(b) $f(x) = 0$ for inifinite number of values of x
(c) $f(x) = 0$ for only two real values
(d) none of these

52. Which of the following functions is not injective ?

(a) $f(x) = |x + 1|$, $x \in [-1, 0]$
(b) $f(x) = x + 1/x$, $x \in (0, \infty)$
(c) $f(x) = x^2 + 4x - 5$
(d) $f(x) = e^{-x}$, $x \in [0, \infty)$

53. Let $f : R \to R$ be defined by $f(x) = [x]$ and $g(x) = \frac{3-2x}{4}$, [.] represents greatest integer function, then

(a) f is neither one-one nor onto

(b) g is one-one but f is not one-one

(c) f is one-one and g is onto

(d) neither f nor g is onto

Assertion Reason

(A) If both ASSERTION and REASON are true and reason is the correct explanation of the assertion.

(B) If both ASSERTION and REASON are true but reason is not the correct explanation of the assertion.

(C) If ASSERTION is true but REASON is false.

(D) If both ASSERTION and REASON are false.

(E) If ASSERTION is false but REASON is true.

54. **Assertion** : The function $\frac{ax+b}{cx+d}$, $(ad - bc \neq 0)$ cannot attain the value $\frac{a}{c}$.

Reason : The domain of the function $g(y) = \frac{b-dy}{cy-a}$ is all the reals except a/c.

(a) A (b) B

(c) C (d) D

(e) E

55. **Assertion** : The domain of the function $\sin^{-1} x + \cos^{-1} x + \tan^{-1} x$ is $[-1, 1]$

Reason : $\sin^{-1} x$, $\cos^{-1} x$ are defined for $|x| \leq 1$ and $\tan^{-1} x$ is defined for all x.

(a) A (b) B

(c) C (d) D

(e) E

56. **Assertion** : The range of the function $f(x) = \sin^2 x + p \sin x + q$, where $|p| > 2$, will be real numbers between $q - \frac{p^2}{4}$ and $q + p + 1$.

Reason : The function $g(t) = t^2 + pt + 1$, where $t \in [-1, 1]$ and $|p| > 2$, will attain minimum and maximum values at -1 and 1.

(a) A (b) B

(c) C (d) D

(e) E

57. **Assertion** : Let A and B be two sets each with a finite number of elements. Assume that there is an injective mapping from A to B and that there is an injective mapping from B to A. Then there is a bijective mapping from A to B.

Reason : An onto function is not necessarily one-one.

(a) A (b) B

(c) C (d) D

(e) E

58. **Assertion** : The domain of a function $y = f(x)$ will be all reals if for every real x there exist y.

Reason : The range of a function $y = f(x)$ will be all reals if for every real y there exists a real x such that $f(x) = y$.

(a) A (b) B

(c) C (d) D

(e) E

Using the following passage, solve Q.59 to Q.61

Passage –1

A rational function is defined as quotient of two polynomials, p(x) and q(x). The domain of the rational function must be all reals except the roots of the equation q(x) = 0. The range of rational function can be found by finding minimum and maximum values of the function. In case p(x) and q(x) have a common factor x – β. Then after cancelling the common factor, the rational function must assume a value at x = β which should be deleted from the found range since β is not there in the domain of the rational function.

59. The range of the rational function $f(x)=\dfrac{3x+1}{2x+1}$ must be

(a) $R-\left\{-\frac{1}{2}\right\}$ (b) $R-\left\{-\frac{1}{3}\right\}$

(c) $R-\left\{\frac{3}{2}\right\}$ (d) R

60. The range of the rational function $f(x)=\dfrac{(2x+1)}{2x^2+5x+2}$ must be

(a) R – {0} (b) R – {–2}

(c) R – {0, –2} (d) $R-\left\{0,\frac{2}{3}\right\}$

61. The range of the rational function $f(x)=\dfrac{2x^2+5x+2}{2x+1}$ must be

(a) R – {0} (b) R – {–2}

(c) $R-\left\{0,-2,\frac{2}{3}\right\}$ (d) $R-\left\{\frac{3}{2}\right\}$

Using the following passage, solve Q.62 to Q.64

Passage –2

Let $f(x)$ be a function defined by

f(x) = 1! + 2! + 3! + 4! .... + x!, where x is a positive integer. Answer the following questions :

62. The last digit of $f(2007)$ will be

(a) 3 (b) 7

(c) 5 (d) 1

63. The number of solutions of equation $f(x)=y^2$ where y is a positive integer and x < 5 is

(a) 5 (b) 2

(c) 3 (d) None of these

64. The number of solutions of $f(x)=y^2$ where x > 5 must be

(a) 1 (b) 2

(c) 3 (d) None of these

Match the column

65.

Column–I	**Column–II**
(A) $f(x+y)=f(x)+f(y)$	(P) $log_3 x$
(B) $f(xy)=f(x)+f(y)$	(Q) $\tan^{-1}x$
(C) $f(x+y)=f(x).f(y)$	(R) 3x
(D) $f(x)+f(y)=f\left(\dfrac{x+y}{1-xy}\right)$	(S) 3^x

66.

Column–I	**Column–II**
(A) odd function	(P) $x-[x]$
(B) even function	(Q) $log\left(x+\sqrt{1+x^2}\right)$
(C) neither even nor odd	(R) $x\,log\dfrac{1+x}{1-x}$
	(S) $\dfrac{2^{x/2}}{1+2^{x/2}}$

Subjective

67. When $0 \le x < 2\pi$ and [x] denotes greatest integer $\le$ x, then [sin x] + [cos x] + [sin x + cos x] takes exactly k integer values. Then k must be

68. The range of the function $\sqrt{x-6}+\sqrt{12-x}$ is an interval of length $2\sqrt{3}-\sqrt{k}$ then *k* must be

69. If $f\left(x+\dfrac{1}{x}\right)=x^3+x^{-3}$ then f (5) must be equal to

70. The least period of the function

$\cos(\cos x)+\sin(\cos x)+\sin 4x$ is $k\dfrac{\pi}{2}$

then value of k must be

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Single Answer Type Questions

1. Indicate the correct alternative : The number $log_2 7$ is **(1990)**

(a) an integer (b) a rational number

(c) an irrational number (d) a prime number

2. The domain of definition of the function y(x) is given by the equation $2^x + 2^y = 2$ is **(2000)**

(a) $0 < x \le 1$ (b) $0 \le x \le 1$

(c) $-\infty < x \le 0$ (d) $-\infty < x < 1$

3. Number of solutions of $log_4 (x - 1) = log_2 (x - 3)$ is **(2001)**

(a) 3 (b) 1

(c) 2 (d) 0

4. Let $f(x) = \frac{\alpha x}{x+1}$, $x \ne -1$. Then, for what value of α is $f(f(x)) = x$? **(2002)**

(a) $\sqrt{2}$ (b) $-\sqrt{2}$

(c) 1 (d) –1

5. If $f : [0, \infty) \to [0, \infty)$ and $f(x) = \frac{x}{1+x}$, then f is **(2003)**

(a) one-one and onto

(b) one-one but not onto

(c) onto but not one-one

(d) neither one-one nor onto

6. Range of the function $f(x) = \frac{x^2+x+2}{x^2+x+1}; x \in R$ is **(2003)**

(a) $(1, \infty)$ (b) (1, 11/7)

(c) (1, 7/3] (d) (1, 7/5)

7. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is **(2012)**

(a) one-one and onto

(b) onto but not one-one

(c) one-one but not onto

(d) neither one-one nor onto

Multiple Type Question

8. If $f(x) = \cos [\pi^2] x + \cos [-\pi^2] x$, where [x] stands for the greatest integer function, then **(1990)**

(a) $f\left(\frac{\pi}{2}\right) = -1$ (b) $f(\pi) = 1$

(c) $f(-\pi) = 0$ (d) $f\left(\frac{\pi}{4}\right) = 1$

Match the Columns

Match the conditions/expressions in Column I with statement in Column II.

9. Let $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$

	Column I	Column II
(A)	If $-1 < x < 1$, then f(x) satisfies	(p) $0 < f(x) < 1$
(B)	If $1 < x < 2$, then f(x) satisfies	(q) $f(x) < 0$
(C)	If $3 < x < 5$, then f(x) satisfies	(r) $f(x) > 0$
(D)	If $x > 5$, then f(x) satisfies	(s) $f(x) < 1$

(2007)

Integer Answer Type Questions

10. Find the values of x satisfying the equation **(1990)**
$|x - 1|^A = (x - 1)^7$ where $A = log_3 x^2 - 2\ log_x 9$.

11. Find all real numbers x which satisfy the equation, **(1999)**

$$2\ log_2\ log_2 x + log_{1/2}\ log_2 \left(2\sqrt{2}\ x\right) = 1.$$

12. $log_{3/4}\ log_8 (x^2 + 7) + log_{1/2}\ log_{1/4} (x^2 + 7)^{-1} = -2.$ **(2000)**

ANSWER KEY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (b) 2. (c) 3. (c) 4. (c) 5. (d) 6. (b) 7. (c) 8. (a,c) 9. (c) 10. (d)

11. (a) 12. (b,d) 13. (b) 14. (a) 15. (a) 16. (a) 17. (c) 18. (b) 19. (a) 20. (a)

21. (d) 22. (c) 23. (a) 24. (d) 25. (a) 26. (b) 27. (i) (a); (ii) (d); (iii) (a); (iv) (c) 28. (b)

29. (c) 30. (d) 31. (d) 32. (b) 33. (d) 34. (c) 35. (c) 36. (d) 37. (b) 38. (b)

39. (a) 40. (a) 41. (d) 42. (a) 43. (b) 44. (a) 45. (c) 46. (a) 47. (b) 48. (b)

49. (d) 50. (a) 51. (c) 52. (c) 53. (c) 54. (i) (c) (ii) (a) (iii) (d) 55. (a) 56. (a)

57. (b) 58. (d) 59. (d) 60. (i) (a) (ii) (a) (iii) (a) 61. (b) 62. (d) 63. (b) 64. (a)

65. (a) 66. (d) 67. (c) 68. (b) 69. (c) 70. (a)

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. (b) 2. (c) 3. (c) 4. (b) 5. (b) 6. (c) 7. (b) 8. (a) 9. (a) 10. (d)

11. (c) 12. (b) 13. (d) 14. (c) 15. (c) 16. (d) 17. (d) 18. (a) 19. (d) 20. (c)

21. (c) 22. (b) 23. (a)

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (a) $(-5, 1) \cup (3, \infty)$; (b) $(-\infty, 1] \cup [2, 3)$ 2. (b) 3. (d) 4. (d) 5. (d) 6. (d) 7. (b)

8. (d) 9. (b) 10. (d) 11. (a) 12. (a) 13. (b) 14. (c) 15. (b) 16. (c) 17. (d)

18. (d) 19. (c) 20. (c) 21. (d) 22. (d) 23. (b) 24. (d) 25. (b) 26. (d) 27. (a)

28. (d) 29. (b) 30. (b) 31. (c) 32. (d) 33. (d) 34. (b) 35. (a,b,c) 36. (a) 37. (a)

38. (b) 39. (c) 40. (b) 41. (a) 42. (d) 43. (d) 44. (a) 45. (b,c) 46. (a) 47. (a,b,d)

48. (a,b,c,d) 49. (c) 50. (b,c) 51. (a,b) 52. (b,c) 53. (a,b) 54. (a) 55. (a) 56. (d) 57. (b)

58. (b) 59. (c) 60. (d) 61. (d) 62. (a) 63. (b) 64. (d)

65. (A) → (R), (B) → (P), (C) → (S), (D) → (Q) 66. (A) → (Q), (B) → (R, S), (C) → (P)

67. (0005) 68. (0006) 69. (0110) 70. (0004)

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (c) 2. (d) 3. (b) 4. (d) 5. (b) 6. (c) 7. (b) 8. (a,c)

9. (A–p; B–q; C–q; D–p) 10. $x = 2$ or 81 11. $x = 8$ 12. $x = 3$ or -3

Dream on !!