Linear Inequalities

Introduction

In Class10 we have studied about Linear Equations in one or two variables and also how to solve such equations. Example: $2x + 6 = 0$, $4x - 8 = 0$, etc...

Here we will get certain situations involving a sign '<' (less than), '>' (greater than), '≤'(less than or equal) and ≥ (greater than or equal) which are known as "inequalities".

Inequalities

"Two real numbers or two algebraic expressions related by the symbol '<' , '>' , '≤' or '≥' form an inequality".

Examples: $2x - 5 < 13$,

$2x + 3 \geq 15$,

$4x + 2y < 12$,

$5x - 3 > 17$

Types of Inequalities

Strict Inequality

(has only one condition)

Example: (i) $2x + 4 > 12$

(ii) $3x - 7 < 11$

Slack Inequality

(has two conditions)

Example: (i) $5x + 2 \leq 10$

(ii) $4x - 3 \geq 13$

*matlab strict me ya toh less than hoga ya greater than but slack inequality me less than equal to hoga ya phir greater than equal to OK...

Algebraic Solutions of Linear Inequalities

Any solution of an inequality in one variable is a value of the variable which makes it a true statement.
(*matlab simple x ki value find karni hai bas)

SOME IMPORTANT RULES

Rule1: Equal numbers can be added or subtracted from both sides of an inequality without affecting the sign of inequality.

Example: Suppose we have an inequality $6 > 2$.

Add : $6 + 2 > 2 + 2$	Subtract : $6 - 2 > 2 - 2$
$8 > 4$	$4 > 0$

here note that sign of the inequality remains same i.e >.

Rule2: Both sides of an inequality can be multiplied or divided by the same positive number, without affecting sign of the inequality.

Example: Suppose we have an inequality $8 > 6$.

Multiply : $8 \times 2 > 6 \times 2$	Divide : $8 \div 2 > 6 \div 2$
$16 > 12$	$4 > 3$

here note that sign of the inequality remains same i.e >.

Rule3: Both sides of an inequality can be multiplied or divided by a negative number, but the sign of the inequality will get reversed.

Example: Suppose we have an inequality $8 > 6$.

Multiply : $8 \times -2 < 6 \times -2$ | Divide : $8 \div -2 > 6 \div -2$

$-16 < -12$ | $-4 < -3$

here note that sign of the inequality has changed i.e. <

Rule4: If we remove negative sign from both sides of an Inequality, then the sign of inequality will change.

Example: Suppose we have $-20 > -24$

removing negative sign from both sides we get $20 < 24$.

Rule5 : If we do reciprocal of an Inequality then the sign of inequality changes

Example: Suppose we have an Inequality $10 > 6$

if we do reciprocal we have $\frac{1}{10} < \frac{1}{6}$

note here the sign of the Inequality has changed i.e. <

(*kuch questions karte hain abb...)

TYPE 1 Questions :

Example: Solve the following inequalities for real x :

(i) $4x + 3 < 5x + 7$

$4x - 5x < 7 - 3$

$-X < 4$

multiply b/s by -1, we get

$\Rightarrow \quad -(-x) > -4$

$\Rightarrow \quad x > -4$

$X \quad (-4, \quad)$

(ii) $3(2 - x) \geq 2(1 - x)$

$6 - 3x > 2 - 2x$

$6 - 2 > -2x + 3x$

$4 > x$

or $x < 4$

$X \quad (- \quad, 4]$

(iii) $x > x + 1$

(iv) $x < (5x - 2) - (7x - 3)$

SOLUTION: (iii) Taking LCM on RHS

$$\frac{x}{3} > \frac{x+2}{2}$$

Cross multiplying

$$2 \times X > 3(x+2)$$

$$2x > 3x + 6$$

$$-6 > 3x - 2x$$

$$-6 > x$$

or $x < -6$

$X \in (-\infty, -6)$

SOLUTION: (iv) Taking LCM on RHS

$$\frac{4}{x} < \frac{3}{5(5x-2) - 3(7x-3)}$$

$$\frac{4}{x} < \frac{5}{15}$$

$$\frac{4}{x} < \frac{25x - 10 - 21x + 9}{15}$$

$$\frac{4}{x} < \frac{4x - 1}{15}$$

Cross multiplying

$$15 \times X < 4(4x - 1)$$

$$15x < 16x - 4$$

$$4 < 16x - 15x$$

$$4 < x$$

or $x > 4$

$X \in (4, \infty)$

TYPE 2 Questions :

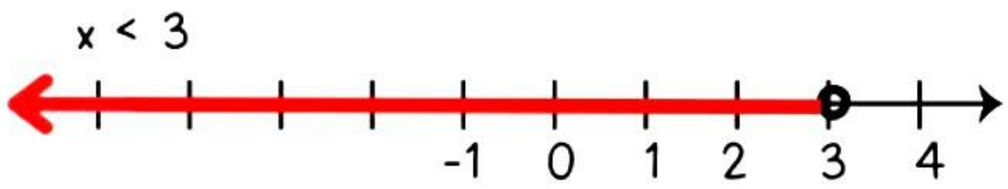
Example: Solve $3x - 2 < 2x + 1$ for real x and show the graph of solutions on number line.

SOLUTION: We have $3x - 2 < 2x + 1$

$$3x - 2x < 1 + 2$$

$x < 3 \quad \Rightarrow X \in (-\infty, 3)$

NUMBER LINE :



TYPE 3 Questions :

Example:

Ravi obtained 70 and 75 marks in first two unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

Let the marks obtained by Ravi in third test be x.

It is given that 70 and 75 are the marks of first two unit tests.

A.T.Q. Average of marks is to be atleast 60

so, $70 + 75 + x \geq 60$

Cross multiplying, we get

$$70 + 75 + x \geq 60 \times 3$$

$$145 + x \geq 180$$

$$x \geq 180 - 145$$

$$x \geq 35$$

Hence, the minimum marks Ravi should get is 35.

Example:

SOLUTION:

Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

(*consecutive odd positive integers jaise 1, 3, 5, 7, 9, etc. matlab Jo ek ke baad ek aate hain)

Let the first odd positive integer be x,

then the other consecutive odd positive integer will be $x + 2$.

We need to find out the pairs of such numbers i.e. $(x, x + 2)$

A.T.Q. both integers are smaller than 10 i.e.

$$x < 10$$

$$x + 2 < 10 \Rightarrow x < 10 - 2 \Rightarrow x < 8$$

Also, the sum of these integers is more than 11

i.e. $x + (x + 2) > 11$

$$2x + 2 > 11$$

$$2x > 9$$

$$x > 9/2 \text{ or } x > 4.5$$

Now odd integers between 4.5 and 8 are 5 and 7 so, $x = 5$ or $x = 7$

Hrnce, required pairs of numbers are (5,7) and (7,9).

Question:

Solve $5x - 3 < 3x + 1$ when

(i) x is an integer, (ii) x is a real number.

Solution

We have, $5x - 3 < 3x + 1$

or $5x - 3 + 3 < 3x + 1 + 3$

or $5x < 3x + 4$

or $5x - 3x < 3x + 4 - 3x$

or $2x < 4$

or $x < 2$

(i) When x is an integer,

the solutions of the given inequality are ..., - 4, - 3, - 2, - 1, 0, 1

(ii) When x is a real number, the solutions of the inequality are given by

$x < 2$, i.e., all real numbers x which are less than 2.

Therefore, the solution set of the inequality is x (- , 2).

Question:

Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.

Solution

$x > 10$ and $x + (x + 2) < 40$

Solving (2),

we get $2x + 2 < 40$

i.e., $x < 19$

From (1) and (3), we get

$10 < x < 19$

Since x is an odd number, x can take the values 11, 13, 15, and 17.

So, the required possible pairs will be (11, 13), (13, 15), (15, 17), (17, 19)

Question:

Solve $-8 \quad 5x - 3 < 7$.

Solution

In this case, we have two inequalities, $-8 \quad 5x - 3$ and $5x - 3 < 7$, which we will solve simultaneously.

We have $-8 \quad 5x - 3 < 7$

or $-5 \quad 5x < 10$

or $-1 \quad x < 2$

Question:

A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

Solution

Let x litres of 30% acid solution is required to be added.

Then Total mixture = (x + 600) litres

Therefore $30\%\ x + 12\%$ of $600 > 15\%$ of $(x + 600)$

and $30\%\ x + 12\%$ of $600 < 18\%$ of $(x + 600)$

or $30x + 7200 > 15x + 9000$

and $30x + 7200 < 18x + 10800$

or $15x > 1800$ and $12x < 3600$

or $x > 120$ and $x < 300$,

i.e. $120 < x < 300$

Thus, the number of litres of the 30% solution of acid will have to be more than 120 litres but less than 300 litres.