Chapter – 5
Arithmetic Progressions

Exercise 5.4

Q. 1 Which term of the AP: 121, 117, 113, . . ., isits first negative term? [Hint: Find *n* for $a_n < 0$]

Answer: Here, first term, a = 121common difference, $d = a_2 - a_1 = 117 - 121 = -4$

Let the first negative term be 'a_n'.

Also, we know that, nth term of an AP is given by $a_n = a + (n - 1)d$

We have to find least value of n, such that $a_n < 0$

$\Rightarrow a + (n - 1)d < 0$

$\Rightarrow 121 + (n - 1)(-4) < 0$

$\Rightarrow 121 - 4(n - 1) < 0$

$\Rightarrow 4(n - 1) > 121$

$\Rightarrow 4n - 4 > 121$

$\Rightarrow 4n > 125$

$\Rightarrow n > 31.25$

Therefore, n is 32 [least positive integer greater than 31.25 is 32]

Hence, the 32nd term of AP is first negative term. Also, $a_{32} = a + 31d$

$= 121 + 31(-4)$ $= 121 - 124 = -3$

Q. 2 The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Answer: To find: S_{16}

Given $a_3 + a_7 = 6$

$a_3 \times a_7 = 8$

nth term of an AP is given by the formula $a_n = a + (n - 1)\,d$ where, $a_n =$ nth term

n = number of term d = common difference. So now its given that sum of third and seventh term is 6, thus we need to find 3rd and 7th term first,$a_3 = a + 2d$

$a_7 = a + 6d$

As per question;

$a_3 + a_7 = 6$

So now,

a + 2d + a + 6d = 6
2a + 8d = 6
a + 4d = 3
a = 3 – 4d ....eq (i)
Similarly,
Product of third and seventh term is given as 8. So,

(a + 2d)(a + 6d) = 8
$a^2 + 6ad + 2ad + 12d^2 = 8$ ....eq(ii)
Substituting the value of a in equation (ii), we get;

$(3 - 4d)^2 + 8(3 - 4d)d + 12d^2 = 8$
$9 - 24d + 16d^2 + 24d - 32d^2 + 12d^2 = 8$
$9 - 4d^2 = 8$
2d = 1
d = ± 1/2
Using the value of d in equation (1), we get;
a = 3 – 4d

$$a = 3 - 4 \times \frac{1}{2}$$

or, a = 3 – 2 =1

Sum of first 16 terms is calculated as follows:

$$S = \frac{n}{2}[2a + (n-1)d]$$

$$S_{16} = \frac{16}{2}[2 \times 1 + (16 - 1) \times \frac{1}{2}]$$

S_{16} = 8[2 + (15/2)]

= 4 × 19
S_{16} = 76
Thus, sum of first 16 terms of this AP is 76.

Now by taking d = -1/2, we get, a = 3 - 4 (-1/2)a = 3 + 2 = 5

$$S = \frac{16}{2}[2 \times 5 + (16 - 1)\frac{-1}{2}]$$

S = 8[10 - 15/2]S = 4[20 - 15]S = 4[5] = 20So, another possible value of sum is 20.

Q. 3 A ladder has rungs 25 cm apart.(see Fig. 5.7). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$m apart, what is the length of the wood required for the rungs?

[Hint: Number of rungs = $\frac{250}{25}$]

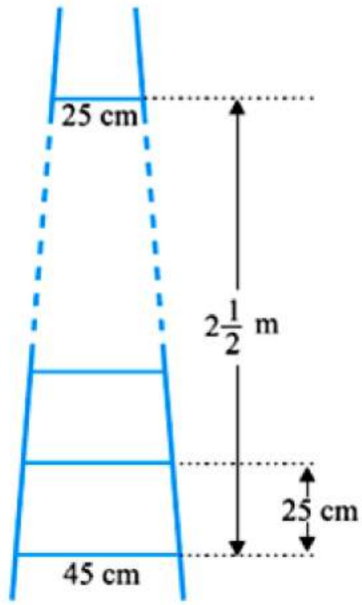

Fig. 5.7

Answer: Total distance between top and bottom rung = 2m 50 cm

$$\text{Number of rungs} = \frac{250}{25} + 1$$

Distance between any two rungs = 25

Number of rungs = 11

And it is also given that bottom most rungs is of 45 cm length and top most is of 25 cm length. As it is given that the length of rungs decrease uniformly, it will for an AP with a = 25, a_{11} = 45 and n = 11

d can be calculated as follows:

nth term of an AP is given by

$a_n = a + (n - 1)d$

$a_{11} = a + 10d$

$45 = 25 + 10d$

$10d = 45 – 25 = 20$

$d = 2$

Total length of wood will be equal to the sum of 11 terms:

$S = \frac{N}{2}[2 + (n - 1)d$

$= \frac{11}{2}[2 * 25 + 10 * 2]$

$= 11[\ 25 + (10)]$

$= 11 \times 35$

= 385 cm

Therefore, total wood required for rungs is equal to 385 m

Q. 4 The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x. [Hint: $S_{x-1} = S_{49} - S_x$]

Answer: The AP in the above problem is

1, 2, 3, - - - , 49

With first term, a = 1

Common difference, d = 1

nth term of AP = a + (n - 1)d

$a_n = 1 + (n - 1)1$

$a_n = n$ [1]

Suppose there exist a mth term such that, (m<49)

Sum of first m - 1 terms of AP = Sum of terms following the mth term

Sum of first m - 1 terms of AP = Sum of whole AP - Sum of first m terms of AP

As we know sum of first n terms of an AP is,

$S_n = \frac{n}{2}[a + a_n]$ if last term a_n is given

$$\frac{m-1}{2}(a + a_{m-1}) = \frac{49}{2}(a + a_{49}) - \frac{m}{2}(a + a_m)$$

$(m - 1)(1 + m - 1) = 49(1 + 49) - m(1+m)$ [using 1]

$(m - 1)m = 2450 - m(1 + m)$

$m^2 - m = 2450 - m + m^2$

$2m^2 = 2450$

$m^2 = 1225$
$m = 35$ or $m = -35$ [not possible as no of terms can't be negative.]
and $a_m = m = 35$ [using 1]
So, sum of no of houses preceding the house no 35 is equal to the sum of no of houses following the house no 35.

Q. 5 A small terrace at a football ground comprises of 15 steps each of which is 50 m long andbuilt of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. (see Fig. 5.8). Calculate the total volume of concrete required to build the terrace. [Hint: Volume of concrete required to build the first step $= \frac{1}{4} \times \frac{1}{2} \times 50\ m^3$]

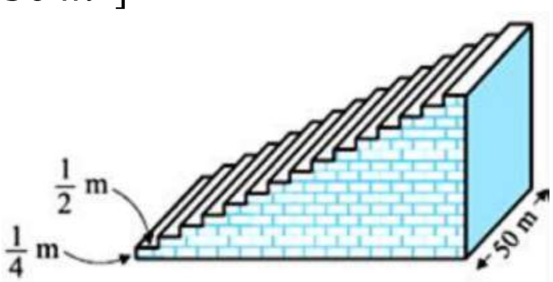

Fig. 5.8

Dimensions of 1st step = 50 m × 0.25 m × 0.5 m

Volume of 1^{st} step = 6.25 m^3

Dimensions of 2^{nd} step = 50 m × 0.5 m × 0.5 m

Volume of 2^{nd}step = 12.5 m^3

Dimensions of 3^{rd}step = 50 m × 0.75 m × 0.5 m

Volume of 3^{rd}step = 18.75 m^3

Clearly, volumes of respective steps are in AP
Now, we have a = 6.25, d = 6.25 and n = 15

Sum of 15 terms can be calculated as follows:

$$S_{15} = \frac{15}{2}[2 * 6.25 + (14)6.25]$$

$$= \frac{15}{2}(100)$$

= 750

Hence, the volume of concrete will be 750m^3.