

3

Circles

TOPIC COVERED

1. Tangent to a Circle and Number of Tangents from a Point on a Circle

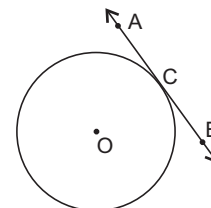
1. TANGENT TO A CIRCLE AND NUMBER OF TANGENTS FROM A POINT ON A CIRCLE

Understanding Tangent to a Circle

A tangent to a circle is a line that intersects or touches the circle at only one point. The point of intersection is called the **point of contact** or the **point of tangency**.

In the given figure, line AB is a tangent and point C is the point of contact.

Theorem 1. *The tangent at any point of a circle is perpendicular to the radius through the point of contact.*



Given: A circle (O, r) and a tangent AB to the circle at the point P.

To prove: $OP \perp AB$.

Proof: Take a point 'Q' (say) other than P on AB. Join OQ. Suppose OQ intersects the circle at R.

$$OP = OR \quad \dots(i) \text{ (Radii of the same circle)}$$

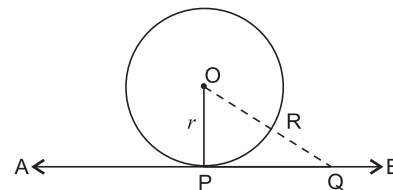
$$OQ = OR + RQ$$

$$\Rightarrow OQ > OR$$

$$\Rightarrow OQ > OP$$

$$\Rightarrow OP < OQ$$

[Using (i)]



Thus, OP is shorter than any other line segments joining O to any point of AB, other than P.

Therefore, OP is the shortest distance between the point O and the line AB.

We know that the shortest distance between a point and line is the perpendicular distance.

$\therefore OP \perp AB$

- Note:** 1. Converse of this theorem is also true, i.e., a line drawn through the end point of a radius and perpendicular to it, is a tangent to the circle.
2. The line containing the radius through the point of contact is also sometimes called the 'normal' to the circle at the point.

Number of Tangents from a Point on a Circle

The number of tangents that can be drawn to a circle from a point depends upon the position of the point with respect to the circle.

There are three cases possible.

Case 1: There is no tangent to a circle passing through a point lying inside the circle (see Fig. 1).

Case 2: There is one and only one tangent to a circle passing through a point lying on the circle (see Fig. 2).

Case 3: There are exactly two tangents to a circle passing through a point lying outside the circle (see Fig. 3).

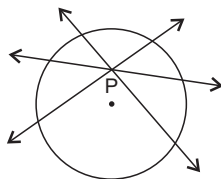


Fig. 1

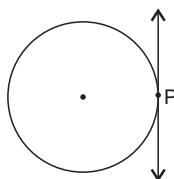


Fig. 2

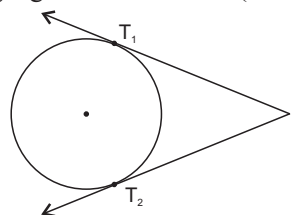


Fig. 3

Length of the Tangent

The length of the segment of the tangent from an external point (P in Fig. 3 above) and the point of contact with the circle (T_1 or T_2 in Fig. 3 above) is called the **length of the tangent** from the point P to the circle.

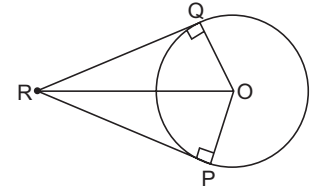
Theorem 2. *The lengths of tangents drawn from an external point to a circle are equal.* [CBSE 2018, AI 2017]

Given: A circle with centre O. R is an external point. Two tangents RP and RQ are drawn from point R to the circle.

To prove: $RQ = RP$

Proof: Consider ΔRQO and ΔRPO .

Here, $OQ = OP$ (Radii of a circle)
 $OR = OR$ (Common)
 $\angle OQR = \angle OPR = 90^\circ$
 (Tangents are perpendiculars to the radius of a circle at the point of contact.)
 $\therefore \Delta RQO \cong \Delta RPO$ (RHS congruency)
 $\Rightarrow RQ = RP$ (CPCT)



Note: $\angle ORQ = \angle ORP$. Therefore, OR bisects $\angle QRP$. Thus, the centre lies on the bisector of the angle between the two tangents.

Theorem 3. *If two tangents are drawn to a circle from an external point, then*

- they subtend equal angles at the centre
- they are equally inclined to the line segment joining the centre to that point.

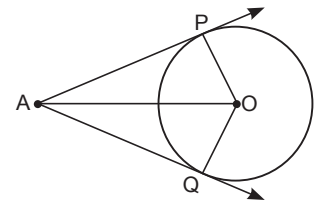
Given: A circle with centre O and a point A outside it.

AP and AQ are tangents drawn to the circle from point A.

To prove: $\angle AOP = \angle AOQ$ and $\angle OAP = \angle OAQ$

Proof: In ΔOAP and ΔOAQ , we have

$OP = OQ$ (Radii of the same circle)
 $AP = AQ$ (Tangents from an external point are equal)
 $OA = OA$ (Common)
 $\therefore \Delta OAP \cong \Delta OAQ$ (By SSS congruency)
 $\Rightarrow \angle AOP = \angle AOQ$ (CPCT)
 and $\angle OAP = \angle OAQ$ (CPCT)



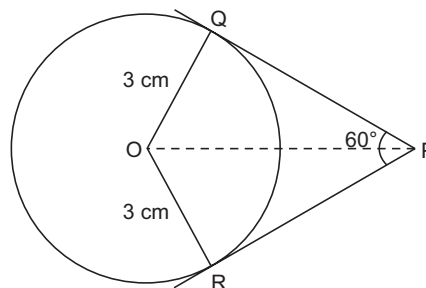
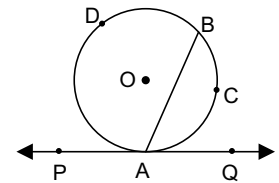
Angles of Alternate Segment of Circle

Look at the figure drawn along side.

Here, chord AB of any circle, is drawn from point of contact A of the tangent PQ which makes $\angle BAP$ and $\angle BAQ$ with PQ. The chord AB divides the circle into two segments ADB and ACB. The segments ADB and ACB of the circle are alternate segments of $\angle BAQ$ and $\angle BAP$.

Example 1. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

[CBSE Standard SP 2020-21] [Imp.]



Solution. In ΔQPO and ΔRPO ,

$OQ = OR$ (Radii of same circle)
 $OP = OP$ (Common)
 $QP = RP$ (Tangents)
 $\therefore \Delta QPO \cong \Delta RPO$ (By SSS congruency)

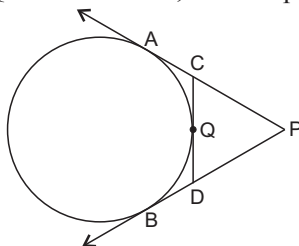
$\Rightarrow \angle QPO = \angle RPO = \frac{60^\circ}{2} = 30^\circ$

In ΔQPO , $\angle OQP = 90^\circ$ (Tangent is perpendicular at the point of contact)

$\therefore \tan 30^\circ = \frac{OQ}{QP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{QP} \Rightarrow QP = 3\sqrt{3} \text{ cm}$

Thus, $QP = RP = 3\sqrt{3} \text{ cm}$

Example 2. In the given figure, PA and PB are tangents to the circle drawn from an external point P. CD is the third tangent touching the circle at Q. If PA = 15 cm, find the perimeter of $\triangle PCD$.



Solution. Since PA and PB are tangents from same external point P.

$$\therefore PA = PB = 15 \text{ cm}$$

$$\begin{aligned} \text{Now, Perimeter of } \triangle PCD &= PC + CD + DP = PC + CQ + QD + DP = PC + CA + DB + DP \\ &= PA + PB = 15 \text{ cm} + 15 \text{ cm} = 30 \text{ cm} \end{aligned}$$

Example 3. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. [NCERT] [Imp.]

Solution. Let O be the common centre of two concentric circles and let AB be a chord of larger circle touching the smaller circle at P. Join OP.

Since OP is the radius of the smaller circle and AB is a tangent to this circle at P.

$$\therefore OP \perp AB$$

Since perpendicular drawn from the centre of a circle to any chord of the circle bisects the chord.

$$\Rightarrow AP = PB$$

Now, in $\triangle AOP$, right-angled at P,

$$OA^2 = AP^2 + OP^2 \Rightarrow 5^2 = AP^2 + 3^2 \Rightarrow 25 - 9 = AP^2$$

$$\Rightarrow AP^2 = 16 \Rightarrow AP = 4$$

$$\text{Now } AB = 2 \times AP = 2 \times 4 = 8$$

Hence, the length of the chord of the larger circle which touches the smaller circle is 8 cm.

Example 4. Prove that the parallelogram circumscribing a circle is a rhombus. [NCERT] [CBSE 2014] [Imp.]

Solution. Given: ABCD is a parallelogram.

The sides AB, BC, CD and DA touch a given circle at the point P, Q, R and S respectively.

Proof: Since lengths of the two tangents from an external point to a circle are equal.

$$\therefore AP = AS \quad \dots(i) \text{ [Tangents from external point A]}$$

$$BP = BQ \quad \dots(ii) \text{ [Tangents from external point B]}$$

$$CR = CQ \quad \dots(iii) \text{ [Tangents from external point C]}$$

$$\text{and } DR = DS \quad \dots(iv) \text{ [Tangents from external point D]}$$

$$\begin{aligned} \text{Now, } AB + CD &= AP + BP + CR + DR = AS + BQ + CQ + DS \\ &= (AS + DS) + (BQ + CQ) \Rightarrow AB + CD = AD + BC \end{aligned}$$

$$\text{But } AB = CD \text{ and } AD = BC \quad [\because \text{Opposite sides of a parallelogram are equal}]$$

$$\therefore AB + AB = AD + AD \Rightarrow 2AB = 2AD \Rightarrow AB = AD.$$

$$\therefore AB = BC = CD = AD$$

Since all the four sides of a parallelogram ABCD are equal.

$\therefore$ ABCD is a rhombus.

Example 5. In the figure given, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.

[NCERT] [AI 2017] [CBSE SP 2018] [Imp.]

Solution. Given: A circle C(O, r) in which XY and X'Y' are parallel tangents. AB is another tangent intersecting XY and X'Y' at A and B respectively.

To prove: $\angle AOB = 90^\circ$.

Prove: Join O and C.

$$\begin{aligned} \text{In } \triangle AOP \text{ and } \triangle AOC, \quad OP &= OC && \text{(Radii of a circle)} \\ OA &= OA && \text{(Common)} \\ \angle OPA &= \angle OCA && \text{(Each } 90^\circ) \end{aligned}$$

$$\therefore \triangle AOP \cong \triangle AOC$$

$$\Rightarrow \angle 1 = \angle 2$$

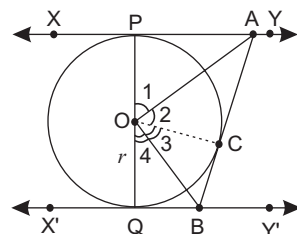
$$\text{Similarly, } \triangle BOQ \cong \triangle BOC$$

$$\Rightarrow \angle 3 = \angle 4$$

(By RHS Congruency)
... (i) (CPCT)

(By RHS Congruency)

... (ii) (CPCT)



Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$

$\Rightarrow 2\angle 2 + 2\angle 3 = 180^\circ$

$\Rightarrow 2(\angle 2 + \angle 3) = 180^\circ \Rightarrow \angle 2 + \angle 3 = 90^\circ \Rightarrow \angle AOB = 90^\circ$

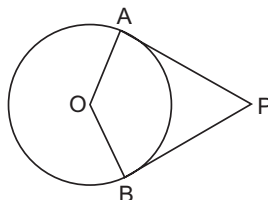
Hence Proved.

(Angles in a straight line)

[From (i) and (ii)]

Example 6. In the given figure, OP is equal to diameter of the circle. Prove that ABP is an equilateral triangle.

[CBSE 2009]



Solution. Join OP and let it meets the circle at point Q.

Since $OP = 2r$

(Diameter of the circle)

$\Rightarrow OQ = QP = r$

Consider $\triangle AOP$ in which $OA \perp AP$ and OP is the hypotenuse.

$\therefore OQ = AQ = OA$

(Mid-point of the hypotenuse is equidistant from the vertices)

$\Rightarrow \triangle OAQ$ is an equilateral triangle.

$\Rightarrow \angle AOQ = 60^\circ$

(Each angle of an equilateral triangle is 60°)

Consider right-angled triangle OAP .

$\angle AOQ = 60^\circ$

(Proved above)

$\angle OAP = 90^\circ \Rightarrow \angle APO = 30^\circ$

$\angle APB = 2\angle APO = 2 \times 30^\circ = 60^\circ$

Also $PA = PB$

(Tangents to a circle from an external point are equal.)

$\Rightarrow \angle PAB = \angle PBA$

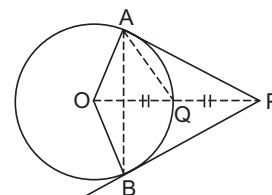
(Angles opposite to equal sides in $\triangle PAB$)

In $\triangle ABP$, $\angle APB = 60^\circ$

$\Rightarrow \angle PAB = \angle PBA = \frac{180^\circ - 60^\circ}{2} = 60^\circ$

$\Rightarrow$ Each angle of $\triangle PAB$ is 60°

$\Rightarrow \triangle PAB$ is an equilateral triangle. Hence Proved.



Example 7. In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC. [AI 2014]

Solution. Let circle touches AC at point E and AB at point F. Join OA, OB, OC, OE and OF.

Since, the tangents to a circle from an exterior point are equal, therefore

$BD = BF = 8 \text{ cm}, CD = CE = 6 \text{ cm}$

and $AE = AF = x \text{ cm}$ (let)

$\Rightarrow AC = (x + 6) \text{ cm}, AB = (x + 8) \text{ cm}$

and $BC = (6 + 8) \text{ cm} = 14 \text{ cm}$

Now, $s = \frac{AB + BC + AC}{2} = \frac{x + 8 + 14 + x + 6}{2}$

$= (x + 14) \text{ cm}$

Also $a = BC = 14 \text{ cm},$

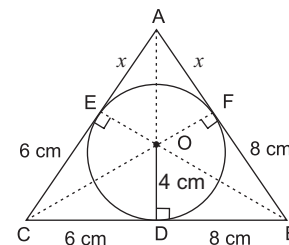
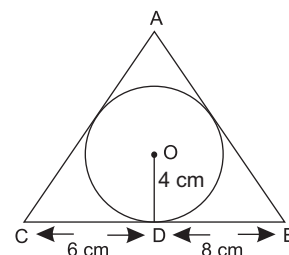
$b = AC = (x + 6) \text{ cm}$

and $c = AB = (x + 8) \text{ cm}$

Using Heron's formula, we get

Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(x+14)(x+14-14)(x+14-x-6)(x+14-x-8)}$



$$= \sqrt{(x+14) \cdot x \cdot 8 \cdot 6} = \sqrt{48 \cdot x(x+14)} \quad \dots(i)$$

Also, Area of ABC = Area of $\triangle OBC$ + Area of $\triangle OCA$ + Area of $\triangle OAB$

$$\begin{aligned} &= \frac{1}{2} BC \times OD + \frac{1}{2} AC \times OE + \frac{1}{2} AB \times OF \\ &= \frac{1}{2} \times 14 \times 4 + \frac{1}{2} \times (x+6) \times 4 + \frac{1}{2} (x+8) \times 4 \\ &= (28 + 2x + 12 + 2x + 16) \text{ cm}^2 = (4x + 56) \text{ cm}^2 \end{aligned} \quad \dots(ii)$$

From (i) and (ii), we get

$$\begin{aligned} \sqrt{48x(x+14)} &= 4x + 56 = 4(x+14) \\ \Rightarrow 48x(x+14) &= 16(x+14)^2 && \text{(Squaring both sides)} \\ \Rightarrow 3x(x+14) &= (x+14)^2 \\ \Rightarrow 3x(x+14) - (x+14)^2 &= 0 \\ \Rightarrow (x+14)(3x-x-14) &= 0 \\ \Rightarrow (x+14)(2x-14) &= 0 \\ \Rightarrow x+14=0 \quad \text{or} \quad 2x-14=0 \\ \Rightarrow x=-14 \quad \text{or} \quad x=7. \end{aligned}$$

As the length of any side of triangle can never be negative, $x \neq -14$.

$$\therefore x=7 \Rightarrow AB=(x+8)=15 \text{ cm, and } AC=x+6=13 \text{ cm}$$

Exercise 3.1

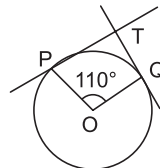
I. Very Short Answer Type Questions

[1 Mark]

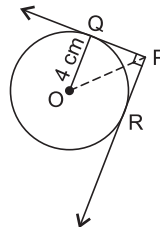
1. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

- (1) In the given figure, if TP and TQ are tangents to a circle with centre O, so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is



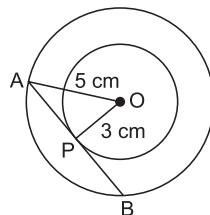
- (a) 110° (b) 90° (c) 80° (d) 70°
- (2) In the given figure, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR = 90^\circ$, then length of PQ is [CBSE Standard 2020]



- (a) 3 cm (b) 4 cm (c) 2 cm (d) $2\sqrt{2}$ cm
- (3) From an external point A, two tangents AB and AC are drawn to the circle with centre O. Then OA is the perpendicular bisector of
- (a) BC (b) AB (c) AC (d) none of these
- (4) If the radii of two concentric circles are 6 cm and 10 cm, the length of chord of the larger circle which is tangent to other is
- (a) 14 cm (b) 16 cm (c) 18 cm (d) 12 cm

(5) In the given figure, the length PB is equal to

- (a) 2 cm
- (b) 7 cm
- (c) 6 cm
- (d) 4 cm



2. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- (c) Assertion (A) is true but reason (R) is false.
- (d) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** When two tangents are drawn to a circle from an external point, they subtend equal angles at the centre.

Reason (R): A parallelogram circumscribing a circle is a rhombus.

(2) **Assertion (A):** If in a cyclic quadrilateral, one angle is 40° , then the opposite angle is 140° .

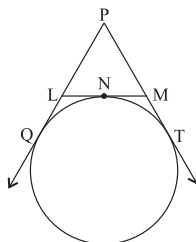
Reason (R): Sum of opposite angles in a cyclic quadrilateral is equal to 360° .

(3) **Assertion (A):** PA and PB are two tangents to a circle with centre O such that $\angle AOB = 110^\circ$, then $\angle APB = 90^\circ$.

Reason (R): The length of two tangents drawn from an external point are equal.

3. Answer the following:

- (1) Two concentric circles of radii a and b ($a > b$) are given. Find the length of the chord of the larger circle which touches the smaller circle. [AI 2019, Foreign 2015]
- (2) From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50^\circ$, then find $\angle AOB$. [Delhi 2016]
- (3) If the angle between two tangents drawn from an external point 'P' to a circle of radius ' r ' and centre O is 60° , then find the length of OP. [CBSE Standard SP 2019-20, AI 2017]
- (4) If the radii of two concentric circles are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle. [CBSE Standard SP 2019-20]
- (5) If $PQ = 28$ cm, then find the perimeter of $\triangle PLM$. [CBSE Standard SP 2020-21]



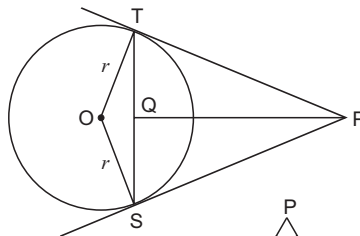
(6) PQ is a tangent to a circle with centre O at point P. If $\triangle OPQ$ is an isosceles triangle, then find $\angle OQP$.

[CBSE Standard SP 2020-21]

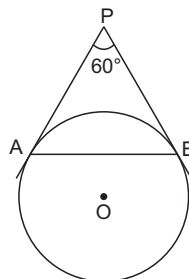
II. Short Answer Type Questions -I

[2 Marks]

- 4. Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre. [CBSE 2014]
- 5. In the given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r . If $OP = 2r$, show that $\angle OTS = \angle OST = 30^\circ$. [CBSE 2016]



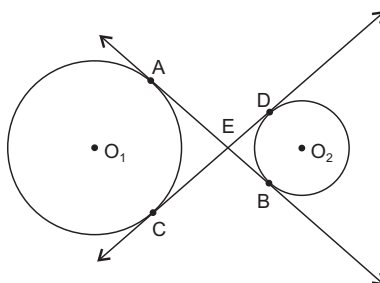
- 6. In the given figure, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60^\circ$. Find the length of chord AB.



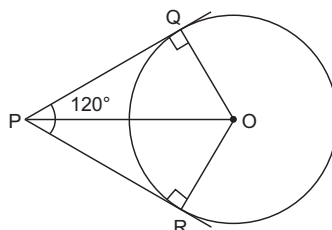
[CBSE 2016]

7. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q, so that $OQ = 12$ cm. Find the length of PQ. [NCERT]
8. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle. [NCERT] [Imp.]
9. Prove that the tangents at the extremities of a chord of a circle make equal angles with the chord. [NCERT Exemplar][AI 2017]

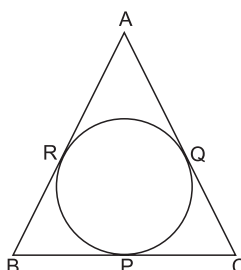
10. In the given figure, common tangents AB and CD to two circles with centres O_1 and O_2 intersect at E. Prove that $AB = CD$. [AI 2015]



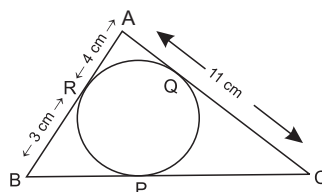
11. If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that $\angle QPR = 120^\circ$, prove that $2PQ = PO$. [Delhi 2014, Foreign 2016]



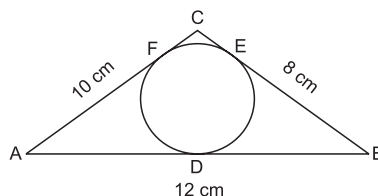
12. In the given figure, a circle inscribed in $\triangle ABC$, touches its sides BC, CA and AB at the points P, Q and R respectively. If $AB = AC$, then prove that $BP = CP$. [Foreign 2013]



13. In the given figure, $\triangle ABC$ is circumscribing a circle. Find the length of BC. [CBSE 2009]

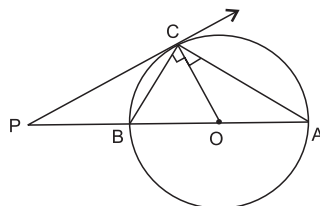


14. A circle is inscribed in a $\triangle ABC$ having sides 8 cm, 10 cm and 12 cm as shown in the following figure. Find AD, BE and CF. [CBSE 2009, 2012, 2013] [Imp.]

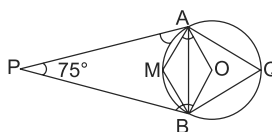


15. PA and PB are tangents from P to the circle with centre O. At point M, a tangent is drawn cutting PA at K and PB at N. Prove that $KN = AK + BN$.

16. PC is a tangent to the circle at C. AOB is the diameter which when extended meets the tangent at P. Find $\angle CBA$, $\angle AOC$ and $\angle BCO$, if $\angle PCA = 110^\circ$. [CBSE 2013]

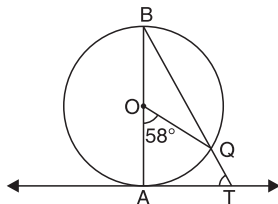


17. In the given figure, O is the centre of the circle. Determine $\angle AQB$ and $\angle AMB$, if PA and PB are tangents and $\angle APB = 75^\circ$. [CBSE 2012]



18. In the given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^\circ$, find $\angle ATQ$.

[Delhi 2016]

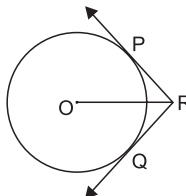


19. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of the line segment PQ.

[Delhi 2016]

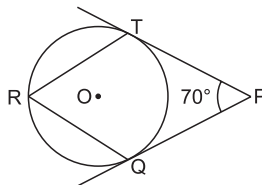
20. In the given figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120^\circ$, then prove that $OR = PR + RQ$.

[AI 2016]



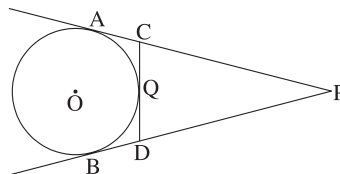
21. In the given figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ = 70^\circ$, find $\angle TRQ$.

[Foreign 2016]



22. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If $PA = 12$ cm, $QC = QD = 3$ cm, then find $PC + PD$.

[Delhi 2017]



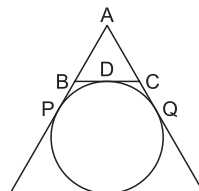
23. A circle touches all the four sides of a quadrilateral ABCD. Prove that

$$AB + CD = BC + DA$$

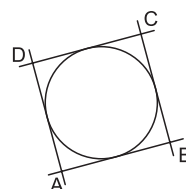
[CBSE Standard 2020, AI 2017, CBSE 2016, NCERT]

24. In the given figure, find the perimeter of $\triangle ABC$, if $AP = 12$ cm.

[CBSE Standard 2020]

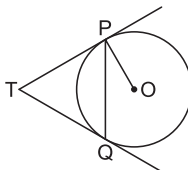


25. In the given figure, a circle touches all the four sides of a quadrilateral ABCD in which $AB = 6$ cm, $BC = 7$ cm and $CD = 4$ cm. Find AD.

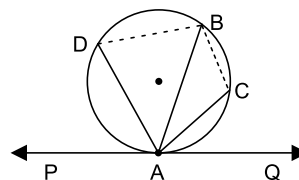


26. In the given figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2 \angle OPQ$.

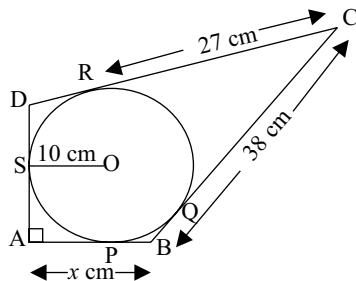
[Standard 2020]



27. Using the given figure, answer the following questions:
 (i) Name the alternate segment of circle of $\angle BAQ$
 (ii) Name the alternate segment of circle of $\angle DAP$.
 (iii) If B is joined with C then $\angle ACB$ is equal to which angle?
 (iv) $\angle ABD$ and $\angle ADB$ is equal to which angles?

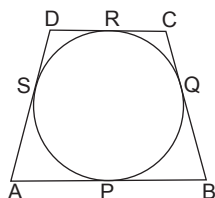


28. In the figure, quadrilateral ABCD is circumscribing a circle with centre O and $AD \perp AB$. If radius of incircle is 10 cm, then find the value of x .



[CBSE Standard SP 2020-21]

29. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$.



III. Short Answer Type Questions -II

[3 Marks]

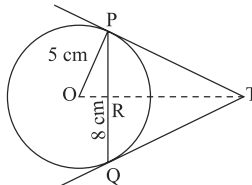
30. The incircle of a $\triangle ABC$ touches the sides AB, BC and CA at P, Q, R respectively. Show that

$$AP + BQ + CR = PB + QC + RA = \frac{1}{2} (\text{Perimeter of } \triangle ABC)$$

[Imp.]

31. If a chord AB of the larger of the two concentric circles is a tangent to the smaller circle at P, prove that $PA = PB$.

32. In the given figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of TP.



[AI 2019]

33. The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle. BD is a tangent to the smaller circle, touching it at D and intersecting the larger circle at P on producing. Find the length of AP.

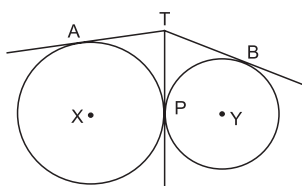
[CBSE SP 2018-19]

34. A circle is touching the side BC of $\triangle ABC$ at X and touching AB and AC produced at P and Q respectively. Prove that

$$AP = AQ = \frac{1}{2} (\text{Perimeter of } \triangle ABC). \text{ Given } AP = 10 \text{ cm, find the perimeter of } \triangle ABC.$$

[CBSE 2001, 2002]

35. In the given figure, two circles with centres X and Y touch externally at P. If tangents AT and BT meet the common tangent at T, then prove that $AT = BT$.



[CBSE 2013] [Imp.]

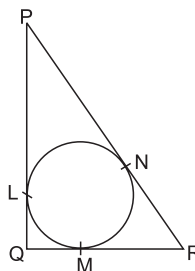
36. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre. [NCERT]

37. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. [NCERT, CBSE 2014] [Imp.]

38. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact. [Foreign 2012]

39. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. [AI 2019, Delhi 2013]

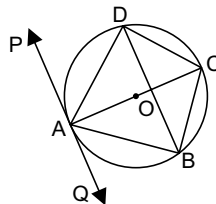
40. In the given figure, a circle is inscribed in a triangle PQR with $PQ = 10$ cm, $QR = 8$ cm and $PR = 12$ cm. Find the lengths of QM, RN and PL.



[AI 2013]

41. Two concentric circles are of radii 7 cm and r cm respectively, where $r > 7$ cm. A chord of the larger circle, of length 48 cm, touches the smaller circle. Find the value of r . [Delhi 2009]

42. If $d_1, d_2, (d_2 > d_1)$ be the diameters of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, prove that $d_2^2 = c^2 + d_1^2$.
[Foreign 2009] [NCERT Exemplar]
43. In a cyclic quadrilateral ABCD, diagonal AC bisects the angle C (see the given figure). Then prove that diagonal BD is parallel to the tangent PQ of a circle which passes through the points A, B, C and D.
[Delhi 2020]



IV. Long Answer Type Questions

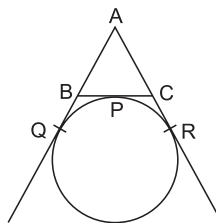
[5 Marks]

44. Two tangents PA and PB are drawn to a circle with centre O, such that $\angle APB = 120^\circ$. Prove that $OP = 2AP$.
[CBSE 2016]
45. Tangents AP and AQ are drawn to circle with centre O from an external point A. Prove that $\angle PAQ = 2\angle OPQ$.
[CBSE 2017, 2013, 2012, 2011, 2009] [Imp.]

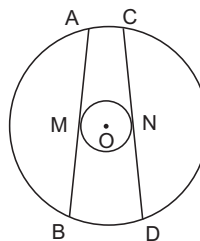
46. A circle touches the side BC of a $\triangle ABC$ at P and AB and AC when produced at Q and R respectively as shown in the figure.

Show that $AQ = \frac{1}{2}$ (Perimeter of $\triangle ABC$)

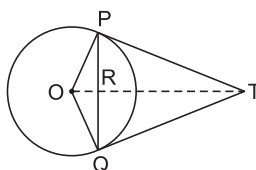
or show that $AQ = \frac{1}{2} (BC + CA + AB)$.



47. In two concentric circles, prove that all chords of the outer circle which touch the inner circle are of equal length (see figure).



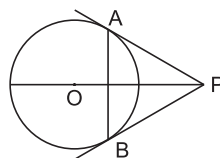
48. In the given figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the lengths of TP and TQ.



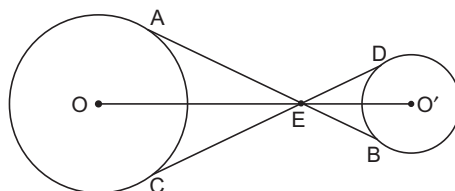
49. If a, b, c , are the sides of a right-angled triangle, where c is the hypotenuse, then prove that the radius r of the circle which touches the sides of the triangle is given by $r = \frac{a + b - c}{2}$.
[NCERT Exemplar; CBSE 2012]

50. Two concentric circles of radii 13 cm and 12 cm, are given. Find the length of chord of the larger circle which touches smaller circle.
[CBSE 2012, 2011]

51. In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre O. Prove that OP is the right bisector of line segment AB.
[CBSE 2011]



52. In the given figure, common tangents AB and CD of two circles with centres O and O' intersect at E. Prove that the points O, E and O' are collinear.



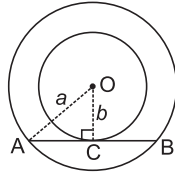
53. If a number of circles pass through the end points P and Q of a line segment PQ, then show that their centres lie on the perpendicular bisector of PQ.
[NCERT Exemplar]

Answers and Hints

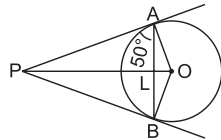
1. (1) (d) 70° (1) (2) (b) 4 cm (1)
 (3) (a) BC (1) (4) (b) 16 cm (1)
 (5) (d) 4 cm (1)

2. (1) (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A). (1)
 (2) (c) Assertion (A) is true but reason (R) is false. (1)
 (3) (d) Assertion (A) is false but reason (R) is true. (1)

3. (1) $AC = BC$
 $\Rightarrow AB = 2AC$
 Now in $\triangle OCA$,
 $AO^2 = OC^2 + AC^2$
 $\Rightarrow a^2 = b^2 + AC^2$
 $\Rightarrow AC = \sqrt{a^2 - b^2}$
 $\therefore AB = 2\sqrt{a^2 - b^2} = 2AC$
 Thus, length of chord $= 2\sqrt{a^2 - b^2}$ (1)



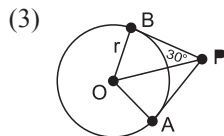
- (2) Given, $\angle PAB = 50^\circ$
 $\angle PAB + \angle OAB = 90^\circ$
 $[\because \text{an angle between radius OA and tangent PA is } 90^\circ]$



- $\Rightarrow 50^\circ + \angle OAB = 90^\circ$
 $\Rightarrow \angle OAB = 90^\circ - 50^\circ = 40^\circ$
 $\Rightarrow \angle OBA = 40^\circ$ $[\because OA = OB]$

Now in $\triangle AOB$, we have
 $\angle AOB + \angle ABO + \angle BAO = 180^\circ$
 $[\because \text{sum of angles in a triangle is } 180^\circ]$

- $\Rightarrow \angle AOB + 40^\circ + 40^\circ = 180^\circ$
 $\Rightarrow \angle AOB = 180^\circ - 80^\circ = 100^\circ$ (1)



- In $\triangle OBP$, $\frac{OB}{OP} = \sin 30^\circ$
 $\Rightarrow \frac{r}{OP} = \frac{1}{2}$
 $\therefore OP = 2r$ (1)

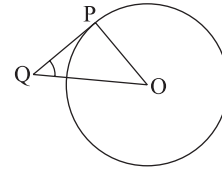
- (4) Length of chord
 $= 2 \times \sqrt{5^2 - 4^2}$
 $= 2 \times 3 \text{ cm} = 6 \text{ cm}$ (1)

- (5) $PQ = PT$
 $PL + LQ = PM + MT$
 $PL + LN = PM + MN$
 Perimeter ($\triangle PLM$) $= PL + LM + PM$ (1/2)
 $= PL + LN + MN + PM$
 $= 2(PL + LN)$

$$= 2(PL + LQ)$$

$$= 2 \times 28 = 56 \text{ cm} \quad (1/2)$$

- (6) In $\triangle OPQ$



$$\angle P + \angle Q + \angle O = 180^\circ$$

$$2\angle Q + \angle P = 180^\circ \quad (1/2)$$

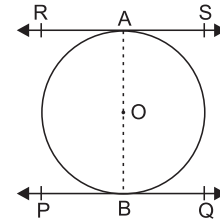
$$2\angle Q + 90^\circ = 180^\circ$$

$$2\angle Q = 90^\circ$$

$$\angle Q = 45^\circ \quad (1/2)$$

4. **Construction:** Join OA and OB.

Proof: As we know, OB is perpendicular to PQ.
 [Tangent is perpendicular to radius at the point of contact.]



Now, given, $PQ \parallel RS$

$\Rightarrow$ BO (produced to RS) is perpendicular to RS. ... (i)
 [A line perpendicular to one of the two parallel lines is perpendicular to other line also]

Also, OA is perpendicular to RS.

$[\because \text{Tangent perpendicular to radius}] \dots (ii)$ (1)

From (i) and (ii), OA and OB must coincide as only one line can be drawn perpendicular to the line from a point outside the line.

$\therefore$ AOB is straight line.

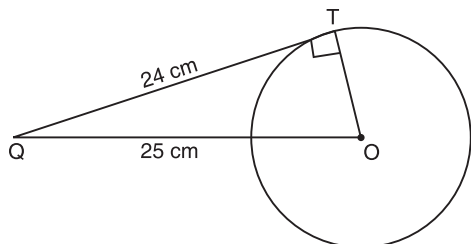
$\therefore$ A, O, B are collinear.

$\Rightarrow$ AB passes through O, the centre of the circle. (1)

6. 5 cm (2)

7. Since $PQ = \sqrt{OQ^2 - OP^2} = \sqrt{12^2 - 5^2}$ (1)
 $= \sqrt{144 - 25} = \sqrt{119} \text{ cm}$ (1)

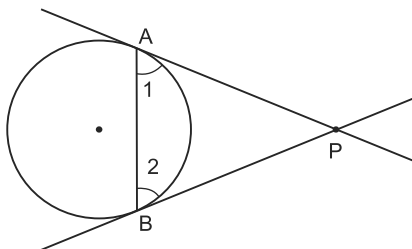
8. $\therefore$ Using Pythagoras theorem, we get
 $OQ^2 = QT^2 + OT^2$
 $\Rightarrow OT^2 = OQ^2 - QT^2$
 $= 25^2 - 24^2$
 $= (25 - 24)(25 + 24)$
 $= 1 \times 49 = 49$ (1)



$$\Rightarrow OT = 7 \text{ cm}$$

Thus, the required radius is 7 cm. (1)

9. We know that tangents drawn from an external point P to a circle are equal so $PA = PB$. (1)



$$\Rightarrow \angle 2 = \angle 1$$

[Angles opposite to equal sides of a triangle are equal]
Hence, tangents PA and PB make equal angles with chord AB.

Hence proved. (1)

10. AB and CD are common tangents to the two given circles with centres O_1 and O_2 respectively (refer to question for figure).

We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal in length.

$$\therefore AE = EC \text{ and } EB = ED \quad (1)$$

$$\Rightarrow AE + EB = EC + ED \Rightarrow AB = CD \quad (1)$$

11. **Proof:** In triangle OQP and ORP,

$$OQ = OR = r \text{ (say)} \quad [\text{Radii}]$$

$$OP = OP \quad (\text{Common})$$

$$PQ = PR$$

[The lengths of the tangents drawn from an external point to a circle are equal]

$$\therefore \Delta OQP \cong \Delta ORP \quad (\text{by SSS})$$

$$\therefore \angle OPQ = \angle OPR \quad (\text{CPCT}) \quad (1)$$

$$\text{Now, given } \angle QPR = 120^\circ$$

$$\Rightarrow \angle OPQ + \angle OPR = 120^\circ$$

$$\Rightarrow 2\angle OPQ = 120^\circ$$

$$\Rightarrow \angle OPQ = 60^\circ = \angle OPR$$

$$\text{Now, in } \Delta OQP, \angle Q = 90^\circ$$

[$\because$ Tangent is perpendicular to radius]

$$\text{Then, } \frac{PQ}{OP} = \cos 60^\circ = \frac{1}{2}$$

$$\Rightarrow OP = 2PQ \quad (1)$$

12. $AB = AC$

$$\Rightarrow AR + BR = AQ + CQ$$

$$\Rightarrow AR + BR = AR + CQ \quad [AQ = AR \text{ equal tangents}]$$

$$\Rightarrow BR = CQ \quad (1)$$

$$\text{Now, } BR = BP \quad [\text{Length of equal tangents}]$$

$$\text{and } CQ = CP$$

$$\Rightarrow BP = CP \quad (1)$$

13. $BP = BR = 3 \text{ cm}$

$$AQ = AR = 4 \text{ cm} \quad (1)$$

$$\Rightarrow CQ = 11 \text{ cm} - 4 \text{ cm} = 7 \text{ cm}$$

$$\Rightarrow CP = CQ = 7 \text{ cm}$$

$$\text{Hence, } BC = BP + CP = (3 + 7) = 10 \text{ cm} \quad (1)$$

14. Let $AD = x_1$, $BE = x_2$ and $CF = x_3$;

$$\text{then } AF = AD = x_1, BD = BE = x_2$$

$$\text{and } CE = CF = x_3.$$

$$\therefore x_1 + x_2 = 12; x_2 + x_3 = 8; x_1 + x_3 = 10 \quad (1)$$

Adding,

$$2(x_1 + x_2 + x_3) = 30$$

$$\Rightarrow x_1 + x_2 + x_3 = 15$$

Solve for x_1 , x_2 and x_3 to get

$$AD = 7 \text{ cm}, BE = 5 \text{ cm},$$

$$CF = 3 \text{ cm} \quad (1)$$

16. $\angle AOC = 140^\circ$, $\angle CBA = 70^\circ = \angle BCO$ (2)

17. $\angle AQB = 52.5^\circ$, $\angle AMB = 127.5^\circ$ (2)

18. $\because$ AT is a tangent and BA is a diameter.

$$\text{So } OA \perp AT$$

$$\Rightarrow \angle OAT = 90^\circ \text{ or } \angle BAT = 90^\circ$$

Arc AQ subtends an angle of 58° at the circle.

$$\angle AOQ = 2\angle ABQ$$

$$\text{So, } \angle ABQ = 29^\circ \quad (1)$$

In ΔABT ,

$$\angle A + \angle ABT + \angle ATB = 180^\circ$$

$$\Rightarrow 90^\circ + 29^\circ + \angle ATB = 180^\circ \Rightarrow \angle ATB = 61^\circ$$

$$\Rightarrow \angle ATQ = 61^\circ$$

$$\text{Hence, } \angle ATQ = 61^\circ. \quad (1)$$

19. TP and TQ are tangents at P and Q respectively.

$$\text{So, } OP \perp PT \text{ and } OQ \perp QT$$

$$\therefore \angle OPT = \angle OQT = 90^\circ$$

In ΔOPT and ΔOQT

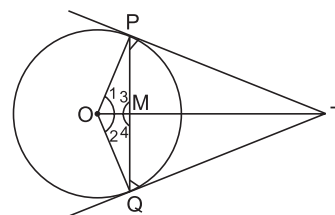
$$OP = OQ \quad (\text{Radius})$$

$$\angle P = \angle Q \quad (\text{Each } 90^\circ)$$

$$OT = OT \quad (\text{Common})$$

$$\text{So, } \Delta OPT \cong \Delta OQT \quad (\text{RHS})$$

$$\Rightarrow \angle 1 = \angle 2 \quad (\text{CPCT}) \quad (1)$$



Now, in ΔOMP and ΔOMQ ,

$$OP = OQ \quad (\text{Radius})$$

$$\angle 1 = \angle 2 \quad (\text{Proved above})$$

$$OM = OM \quad (\text{Common})$$

$$\text{So, } \Delta OMP \cong \Delta OMQ \quad (\text{SAS})$$

$$\Rightarrow PM = MQ \text{ and } \angle 3 = \angle 4 \quad (\text{By CPCT})$$

$$\text{Now } \angle 3 + \angle 4 = 180^\circ \quad (\because \text{Linear Pair Axiom})$$

$$\Rightarrow 2\angle 3 = 180^\circ \Rightarrow \angle 3 = 90^\circ$$

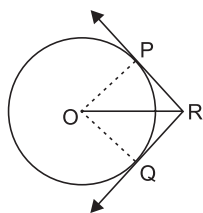
$$\Rightarrow \angle OMP = 90^\circ$$

Hence, OT is the right bisector of the line segment PQ. (I)

20. OR bisects $\angle PRQ$

$$\therefore \angle PRO = \angle QRO = 60^\circ$$

$$[\because \angle PRQ = \angle ORP + \angle ORQ = 120^\circ]$$



In right $\triangle OPR$,
 $OP \perp PR$

$[\because \text{radius is perpendicular to the tangent at point of contact}]$

$$\therefore \cos \angle ORP = \frac{PR}{OR} = \cos 60^\circ$$

$$\Rightarrow OR = 2PR \quad \dots(i) \quad (1)$$

Similarly, in right $\triangle OQR$,

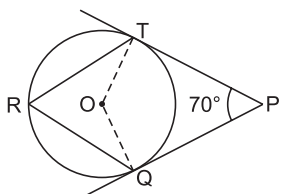
$$\frac{QR}{OR} = \cos 60^\circ \Rightarrow OR = 2QR \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2OR = 2PR + 2QR$$

$$\Rightarrow OR = PR + RQ \quad (1)$$

21. We know that tangent is perpendicular to radius. Hence,
 $\angle OTP = \angle OQP = 90^\circ$



In quadrilateral PQOT,

$$\angle QOT + \angle OTP + \angle TPQ + \angle OQP = 360^\circ$$

$$\Rightarrow \angle TOQ + \angle TPQ = 180^\circ \Rightarrow \angle TOQ = 110^\circ$$

$$\text{Also } \angle TOQ = 2\angle TRQ$$

$$\Rightarrow 110^\circ = 2\angle TRQ$$

$$\Rightarrow \angle TRQ = 55^\circ \quad (1)$$

22. $PA = PC + CA = PC + CQ$

$$\Rightarrow 12 = PC + 3$$

$$\Rightarrow PC = 9 \text{ cm} \quad (1)$$

$$\text{Similarly, } PD = 9 \text{ cm}$$

$$\therefore PC + PD = 18 \text{ cm} \quad (1)$$

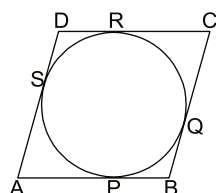
23. Here

$$AP = AS$$

$$BP = BQ$$

$$CR = CQ$$

$$DR = DS \quad (1)$$

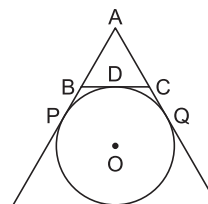


$$\text{Adding } (AP + PB) + (CR + RD)$$

$$= (AS + SD) + (BQ + QC)$$

$$\Rightarrow AB + CD = AD + BC \quad (1)$$

24. Since AP and AQ are two tangents drawn from common external point A.



$$\therefore AP = AQ = 12 \text{ cm} \quad (\text{Given})$$

$$\text{Similarly, } BP = BD \quad \dots(i)$$

$$\text{and } CQ = CD \quad \dots(ii) \quad (1)$$

Now perimeter of

$$\triangle ABC = AB + BC + AC$$

$$= AB + BD + CD + AC$$

$$= AB + BP + CQ + AC$$

$$= AP + AQ = 12 + 12 \quad [\text{From (i) and (ii)}]$$

$$= 24 \text{ cm} \quad (1)$$

25. 3 cm

27. (i) $\angle ADB$ (ii) $\angle ABD$

(iii) $\angle BAP$ (iv) $\angle DAP$ and $\angle BAQ$ (2)

28. $\angle A = \angle OPA = \angle OSA = 90^\circ$ ($\frac{1}{2}$)

$$\text{Hence, } \angle SOP = 90^\circ$$

$$\text{Also, } AP = AS$$

Hence, OSAP is a square

$$AP = AS = 10 \text{ cm} \quad (\frac{1}{2})$$

$$CR = CQ = 27 \text{ cm}$$

$$BQ = BC - CQ$$

$$= 38 - 27 = 11 \text{ cm} \quad (\frac{1}{2})$$

$$BP = BQ = 11 \text{ cm}$$

$$x = AB = AP + BP$$

$$= 10 + 11 = 21 \text{ cm} \quad (\frac{1}{2})$$

29. To prove: $AB + CD = AD + BC$

Proof: $AS = AP$ (Length of tangents from an external point to a circle are equal)

$$BQ = BP$$

$$CQ = CR$$

$$DS = DR \quad (1)$$

$$AS + BQ + CQ + DS = AP + BP + CR + DR$$

$$(AS + DS) + (BQ + CQ)$$

$$= (AP + BP) + (CR + DR)$$

$$AD + BC = AB + CD \quad (1)$$

32. Given: $PQ = 8 \text{ cm}$ and $PO = 5 \text{ cm}$

Now, $PR = RQ$ (Perpendicular from the centre bisects the chord)

$$= \frac{8}{2} = 4 \text{ cm} \quad (1)$$

$$\text{In } \triangle OPR, OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2}$$

$$= \sqrt{25 - 16} = \sqrt{9} = 3 \text{ cm}$$

Let $\angle POR$ be θ .

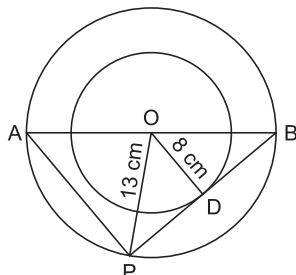
$$\text{In } \triangle POR, \tan \theta = \frac{PR}{RO} = \frac{4}{3} \quad (1)$$

We know, $OP \perp TP$ as point of contact of a tangent is perpendicular to the line from the centre.

$$\text{In } \triangle OTP, \tan \theta = \frac{OP}{TP} \Rightarrow \frac{4}{3} = \frac{5}{TP} \Rightarrow TP = \frac{15}{4}$$

$$\Rightarrow TP = 3.75 \text{ cm} \quad (1)$$

33. $\angle APB = 90^\circ$ (Angle in semi-circle) $(\frac{1}{2})$
 $\angle ODB = 90^\circ$ (Radius is perpendicular to tangent)



(1)

$$\triangle ABP \sim \triangle OBD$$

$$\Rightarrow \frac{AB}{OB} = \frac{AP}{OD} \Rightarrow \frac{26}{13} = \frac{AP}{8}$$

$$\Rightarrow AP = 16 \text{ cm} \quad (1\frac{1}{2})$$

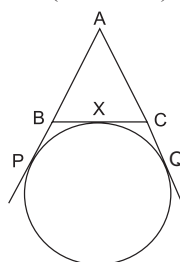
34. In the given figure,

$$AP = AQ, BP = BX \text{ and } CX = CQ$$

$$\Rightarrow AB + BX = AC + CX \quad \dots(i) \quad (1)$$

$\therefore$ Perimeter of

$$\begin{aligned} \triangle ABC &= AB + BC + CA \\ &= AB + (BX + XC) + CA \end{aligned}$$



$$\begin{aligned} &= (AB + BX) + (XC + CA) \\ &= 2(AB + BX), \text{ using (i).} \end{aligned} \quad (1)$$

$$\text{Hence, } AP = \frac{1}{2} \times (\text{perimeter of } \triangle ABC) = AQ.$$

$$\text{If } AP = 10 \text{ cm, then perimeter of } \triangle ABC = 20 \text{ cm.} \quad (1)$$

36. In right $\triangle OAP$ and right $\triangle OBP$, we have

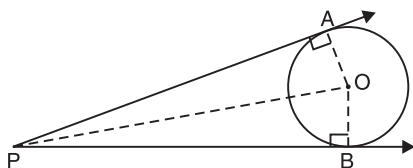
$$PA = PB \quad [\text{Tangents to circle from an external point P}]$$

$$OA = OB \quad [\text{Radii of the same circle}]$$

$$OP = OP \quad [\text{Common}]$$

$\therefore$ By SSS congruency,

$$\triangle OAP \cong \triangle OBP \quad (1)$$



$\therefore$ Their corresponding parts are equal.

$$\therefore \angle OPA = \angle OPB$$

$$\text{and } \angle AOP = \angle BOP$$

$$\Rightarrow \angle APB = 2 \angle OPA$$

$$\text{and } \angle AOB = 2 \angle AOP \quad (1)$$

$$\text{But } \angle AOP = 90^\circ - \angle OPA$$

$$\Rightarrow 2 \angle AOP = 180^\circ - 2 \angle OPA$$

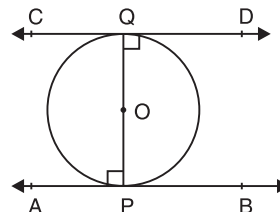
$$\Rightarrow \angle AOB = 180^\circ - \angle APB$$

$$\Rightarrow \angle AOB + \angle APB = 180^\circ. \quad (1)$$

37. In the figure, we have:

PQ is diameter of the given circle and O is its centre.

Let tangents AB and CD be drawn at the end points of the diameter PQ.



Since the tangent at a point to a circle is perpendicular to the radius through the point. (1)

$$\therefore PQ \perp AB \Rightarrow \angle APQ = 90^\circ$$

$$\text{and } PQ \perp CD \Rightarrow \angle PQD = 90^\circ \quad (1)$$

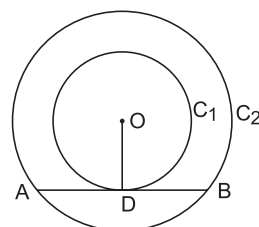
$$\Rightarrow \angle APQ = \angle PQD$$

But they form a pair of alternate angles.

$$\therefore AB \parallel CD. \quad (1)$$

38. **Given:** Let O be the centre of two concentric circles C_1 and C_2 .

Let AB be the chord of larger circle C_2 which is a tangent to the smaller circle C_1 at D.



To prove: Now we have to prove that the chord AB is bisected at D that is $AD = BD$. (1)

Construction: Join OD.

Proof: Now since OD is the radius of the circle C_1 and AB is the tangent to the circle C_1 at D.

$$\text{So, } OD \perp AB$$

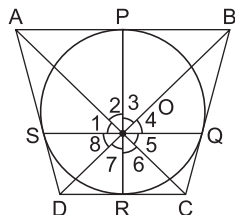
[Radius of the circle is perpendicular to tangent at any point of contact] (1)

Since AB is the chord of the circle C_2 and $OD \perp AB$.

$$\therefore AD = DB$$

[Perpendicular drawn from the centre to the chord always bisects the chord] (1)

39. **Given:** ABCD is a quadrilateral circumscribing the circle with centre O touching it at P, Q, R, S.



To prove:

$$\angle AOB + \angle DOC = 180^\circ$$

$$\angle AOD + \angle BOC = 180^\circ$$

Construction: Join AO, PO, BO, QO, CO, RO, DO, SO. (1)

Proof: In $\triangle AOS$ and $\triangle AOP$,

$$AO = AO \quad (\text{Common})$$

$$AS = AP$$

(Tangents from external point)

$$OS = OP \quad (\text{Radii of same circle})$$

By SSS congruence rule,

$$\triangle AOS \cong \triangle AOP$$

$$\angle 1 = \angle 2 \quad \dots(i) \text{ (by CPCT) (1)}$$

$$\text{Similarly, } \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8 \quad \dots(ii)$$

Now,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

[$\because$ ASP of quadrilateral]

$$\Rightarrow \angle 2 + \angle 2 + \angle 3 + \angle 3 + \angle 6 + \angle 6 + \angle 7 + \angle 7 = 360^\circ$$

[By (i) and (ii)]

$$\Rightarrow 2[\angle 2 + \angle 3 + \angle 6 + \angle 7] = 360^\circ$$

$$\angle AOB + \angle COD = 180^\circ$$

$$\text{Similarly, } \angle AOD + \angle BOC = 180^\circ \quad (1)$$

40. Let $QM = x = QL$

$$MR = y = RN$$

$$\text{and } PL = z = PN$$

$$\text{Now } PQ = 10 \text{ cm,}$$

$$QR = 8 \text{ cm,}$$

$$PR = 12 \text{ cm}$$

$$\Rightarrow x + y = 8, y + z = 12, z + x = 10 \quad (1)$$

$$\Rightarrow 2x + 2y + 2z = 8 + 12 + 10 = 30$$

$$\Rightarrow x + y + z = 15 \Rightarrow 8 + z = 15$$

$$\Rightarrow z = 7 \quad (1)$$

$$\Rightarrow x + 12 = 15 \Rightarrow x = 3$$

$$\Rightarrow y + 10 = 15 \Rightarrow y = 5$$

$$\text{Hence, } QM = 3 \text{ cm, } RN = 5 \text{ cm and } PL = 7 \text{ cm. (1)}$$

41. **Given:** $OP = 7 \text{ cm, } OA = r \text{ cm}$

$$AB = 48 \text{ cm}$$

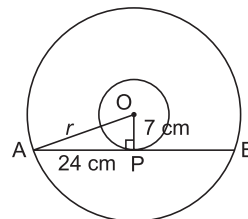
$$\text{Now } OP \perp AB$$

(as radius makes an angle of 90° with the tangent at point of contact)

$$\text{Also, } AP = PB$$

(Perpendicular drawn from centre to the chord bisects the chord)

$$\text{So, } AP = 24 \text{ cm} \quad (1)$$



$$\text{In } \triangle OPA, \quad \angle P = 90^\circ \quad (1)$$

By Pythagoras theorem in $\triangle OPA$,

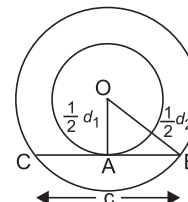
$$OA^2 = AP^2 + OP^2$$

$$r^2 = 24^2 + 7^2$$

$$= 576 + 49 = 625$$

$$\Rightarrow r = 25 \text{ cm} \quad (1)$$

42. $\therefore$ Diameter of bigger circle = d_2



$$\text{So, radius of bigger circle} = \frac{1}{2}d_2 = OB$$

$$\text{Diameter of smaller circle} = d_1 \quad (1)$$

$$\text{So, radius of smaller circle} = \frac{1}{2}d_1 = OA$$

$$AB = \frac{c}{2}$$

$$\text{In right } \triangle OAB, \quad \angle A = 90^\circ \quad [\because \text{Radius is perpendicular to the tangent at point of contact}] \quad (1)$$

By Pythagoras theorem

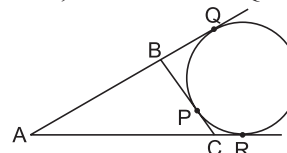
$$OB^2 = AB^2 + OA^2$$

$$\Rightarrow \left(\frac{1}{2}d_2\right)^2 = \left(\frac{1}{2}c\right)^2 + \left(\frac{1}{2}d_1\right)^2$$

$$\Rightarrow \frac{1}{4}d_2^2 = \frac{1}{4}c^2 + \frac{1}{4}d_1^2$$

$$\Rightarrow d_2^2 = c^2 + d_1^2 \quad (1)$$

46. We have $BQ = BP$, $CP = CR$ and $AQ = AR$ (1)



$$\text{Now, } 2AQ = AQ + AR$$

$$= (AB + BQ) + (AC + CR) \quad (1)$$

$$= AB + BP + AC + CP$$

$$= (BP + CP) + AC + AB \quad (1)$$

$$= BC + CA + AB \quad (1)$$

$$\text{i.e. } AQ = \frac{1}{2}(BC + CA + AB) \quad (1)$$

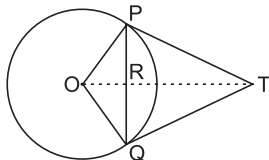
48. Join OT intersecting PQ at R.

OT bisects $\angle PTQ$

$$\therefore \angle PTO = \angle QTO$$

$$\therefore \angle PTR = \angle QTR \quad \dots(i) \quad (1)$$

In $\triangle PTR$ and $\triangle QTR$,
 $PT = QT$ [Length of tangents drawn from common external point are equal]
 $RT = RT$ [common] (1)
 $\angle PTR = \angle QTR$ [$\therefore$ from (i)]
 $\therefore \triangle PTR \cong \triangle QTR$ [By SAS]
 $\Rightarrow PR = RQ$ [$\therefore$ By CPCT]



$\Rightarrow R$ is mid-point of PQ
 $OR \perp PQ$

In right triangle ORP ,
 $OP^2 = PR^2 + OR^2$ [$\therefore$ Given $OP = 5$ cm,
 $PQ = 8$ cm, $PR = QR = 4$ cm]

$\Rightarrow 25 = 16 + OR^2$
 $OR = 3$ cm (1)

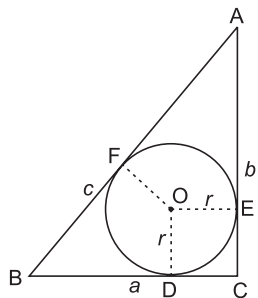
In $\triangle ORQ$ and $\triangle OQT$
 $\angle ORQ = \angle OQT$ (Each 90°)
 $\angle ROQ = \angle TOQ$ (Common)
 $\triangle ORQ \sim \triangle OQT$ (By AA criterion)

$\Rightarrow \frac{OR}{OQ} = \frac{RQ}{QT}$ (By C.P. of similar triangles)

$\Rightarrow \frac{3}{5} = \frac{4}{QT} \Rightarrow QT = \frac{20}{3}$ cm

Also $PT = QT \Rightarrow PT = \frac{20}{3}$ cm (1)

49. Let the circle touches the sides BC , CA , AB of the right triangle ABC at D , E and F respectively, where $BC = a$, $CA = b$ and $AB = c$ (see figure). Then $AE = AF$ and $BD = BF$. Also $CE = CD = r$. (2)



i.e. $b - r = AF$, $a - r = BF$

or $AB = c = AF + BF = b - r + a - r$ (1)

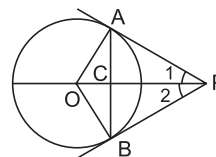
This gives $r = \frac{a+b-c}{2}$. (1)

50. 10 cm

51. Join OA and OB .

In $\triangle PAO$ and $\triangle PBO$,
 $OA = OB$ [Radii]
 $OP = OP$ [Common]

and $AP = BP$ [Tangents from P]
 $\therefore \triangle PAO \cong \triangle PBO$ (SSS congruence rule) (2)
 $\Rightarrow \angle 1 = \angle 2$ [CPCT]



In $\triangle APC$ and $\triangle BPC$,
 $\angle 1 = \angle 2$ [Proved]

$AP = BP$ and $PC = PC$ (1)

$\therefore \triangle APC \cong \triangle BPC$ (SSS congruence rule)

$\Rightarrow AC = BC$ [CPCT]

and $\angle ACP = \angle BCP$ [CPCT] (1)

Also, $\angle ACP + \angle BCP = 180^\circ$

$\Rightarrow \angle ACP = \angle BCP = 90^\circ$

$\therefore OP$ is the right bisector of AB . (1)

52. In $\triangle AEO$ and $\triangle CEO$,
 $OE = OE$ [Common]

$OA = OC$ [Radii of same circle]

$EA = EC$ [Tangents from an external point to a circle are equal in length]

$\therefore \triangle OEA \cong \triangle OEC$
[By SSS criterion of congruence] (1)

$\Rightarrow \angle OEA = \angle OEC$ [CPCT]

$\therefore \angle 1 = \angle 2$ [CPCT] (1)

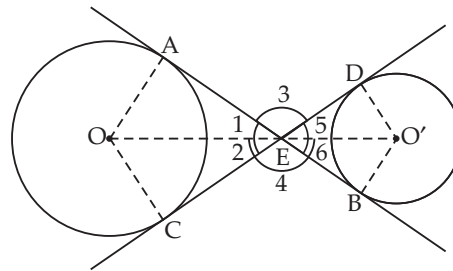
Similarly, $\angle 5 = \angle 6$

and $\angle 3 = \angle 4$ [Vertically opposite angles] (1)

Since sum of angles at a point = 360°

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360^\circ$

$\Rightarrow 2(\angle 1 + \angle 3 + \angle 5) = 360^\circ$



(1)

$\Rightarrow \angle 1 + \angle 3 + \angle 5 = 180^\circ$

$\Rightarrow \angle OEO' = 180^\circ$

$\therefore OEO'$ is a straight line.

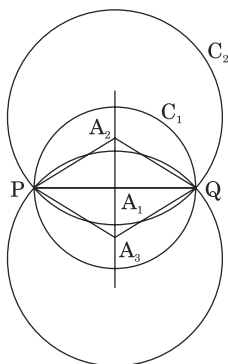
Hence, O , E and O' are collinear. (1)

53. Centre of any circle passing through the end points P and Q of a line segment are equidistant from P and Q .

$\therefore A_1P = A_1Q$

$A_2P = A_2Q$

$A_3P = A_3Q$ (2)



(1) As we know that any point on perpendicular bisector of a segment is equidistant from the end points of the segment. Hence, A_1, A_2, A_3 points are the centres of circles passing through the end points P and Q of a segment PQ or the centres of circles lie on the perpendicular bisector of PQ.

(2)

54. (a) $PA \cdot PB = (PN - AN)(PN + BN)$
 $= (PN - AN)(PN + AN)$ (As $AN = BN$)
 $\Rightarrow PA \cdot PB = PN^2 - AN^2$... (i) (1)
- (b) $PN^2 - AN^2 = (OP^2 - ON^2) - AN^2$ (As $ON \perp PN$)
 $= OP^2 - (ON^2 + AN^2)$ (1)
 $= OP^2 - OA^2$ (As $ON \perp AN$)
 $\Rightarrow PN^2 - AN^2 = OP^2 - OT^2$ (As $OA = OT$) ... (ii) (1)
- (c) From (i) and (ii)
 $PA \cdot PB = OP^2 - OT^2 = PT^2$ (As $\angle OTP = 90^\circ$) (1)

Case Study Based Questions

- I. 1. (c) 150° 2. (a) 75°
 3. (b) 75° 4. (a) 90°
 5. (c) $5\sqrt{3}$ m
- II. 1. (a) 7 cm 2. (b) 5 cm
 3. (d) 3 cm 4. (c) 24 sq cm
 5. (b) 60 sq cm

EXPERTS' OPINION

Questions based on following types are very important for Exams. So, students are advised to revise them thoroughly.

- To prove and use the theorem "The tangent at any point of a circle is perpendicular to the radius through the point of contact."
- To prove and use the theorem "The lengths of tangents drawn from an external point to a circle are equal."

QUICK REVISION NOTES

- A tangent to a circle is a line that intersects the circle at only one point.
- There is only one tangent at a point of the circle.
- The tangent to a circle is a special case of the secant.
- There is no tangent to a circle passing through a point lying inside the circle.
- There is one and only one tangent to a circle passing through a point lying on the circle.
- There are exactly two tangents to a circle through a point lying outside the circle.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- The lengths of tangents drawn from an external point to a circle are equal.

COMMON ERRORS

Errors	Corrections
(i) Interpreting incorrectly that all secants are tangents.	(i) Remember that the tangent to a circle is a special case of secant, when the two end points of its corresponding chord coincide.
(ii) Interpreting incorrectly that many tangents can be drawn to a circle passing through a point lying on the circle.	(ii) There is one and only one tangent to a circle passing through a point lying on the circle.
(iii) Distinguishing incorrectly the terms 'inscribed' and 'circumscribed'.	(iii) Understand the difference between terms 'inscribed' and 'circumscribed' clearly.   Inscribed Circumscribed
(iv) Not recognising corresponding parts of congruent triangles to find the values of angles.	(iv) Write corresponding parts of congruent triangles correctly which are equal.
(v) Do not using angle sum property of triangle and quadrilateral to find the unknown angles.	(v) Must use angle sum property of triangle and quadrilateral to find the unknown angles when sum are given.