RECTILINEAR MOTION

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RECTILINEAR MOTION

MECHANICS

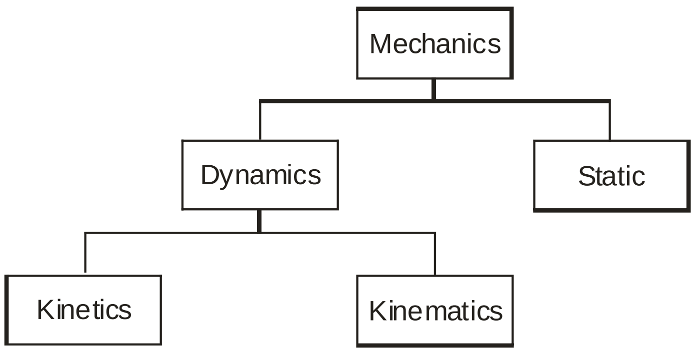
Mechanics is the branch of physics in which we deals with the cause and effects of motion of a particle, rigid objects and deformable bodies etc.

Mechanics may be divided into three branches:

1. Statics: which deals with forces acting on and in a body at rest

2. Kinematics: which describes the possible motions of a body or system of bodies

3. Kinetics: which attempts to explain or predict the motion that will occur in a given situation



KINEMATICS

In kinematics we study how a body moves without knowing why it moves. All particles of a rigid body in translation motion move in identical fashion hence any of the particles of a rigid body in translation motion can be used to represent translation motion of the body. This is why, while analyzing its translation motion, a rigid body is considered a particle and kinematics of translation motion as particle kinematics.

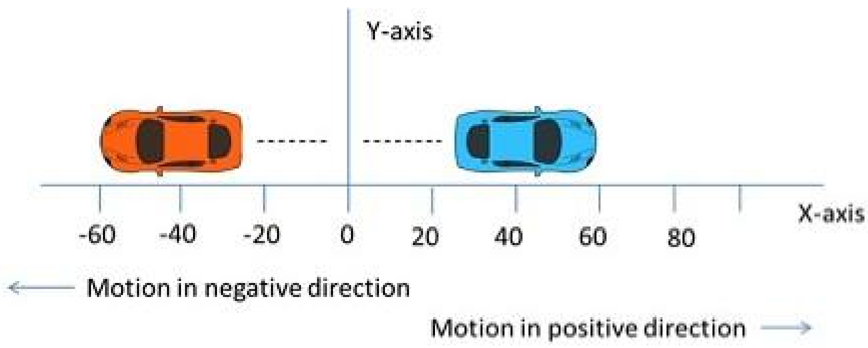
Types of Translation Motion A body in translation motion can move on either a straight line path or curvilinear path.

MOTION AND REST

An object is said to be in motion with respect to an observer (frame of refrence) if its position changes with respect to that observer.

Motion is a combined property of the object and the observer There is no meaning of rest or motion without the observer. Nothing is in absolute rest or in absolute motion.

It may happen by both ways either observer moves or object moves.



RECTILINEAR MOTION (ONE DIMENSIONAL MOTION)

Rectilinear motion is another name for straight-line motion.
It deals with the kinematics of a particle in one dimension.

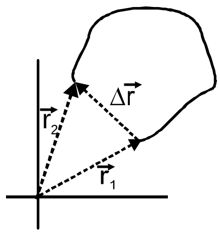
Position of a particle in space

The position of a particle refers to its location in the space at a certain moment of time. It is concerned with the question – "where is the particle at a particular moment of time?"

Displacement

The change in the position of a moving object is known as displacement.

It is the vector joining the initial position ($\vec{r}_1$) of the particle to its final position ($\vec{r}_2$) during an interval of time.



$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

Displacement can be negative positive or zero.

(i) Displacement is a vector quantity
(ii) Dimension : [$M^0 L^1 T^0$]
(iii) Unit : metre (S.I.)

Distance Travelled

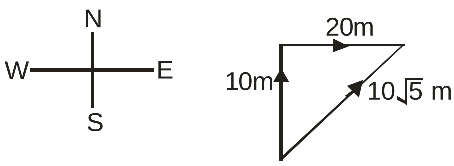
The length of the actual path travelled by a particle during a given time interval is called as distance travelled. The distance travelled is a scalar quantity which is quite different from displacement. In general, the distance travelled between two points may not be equal to the magnitude of the displacement between the same points.

Distance travelled is a scalar quantity.

SOLVED EXAMPLE

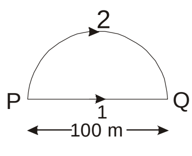
Example 1. A man goes 10m towards North, then 20m towards east then find the magnitude of displacement ?

Solution:



Ans. $10\sqrt{5}$ **m**

Example 2. Satish takes path 1 (straight line) to go from P to Q and manoj takes path 2(semicircle).



(a) Find the distance travelled by satish and manoj ?
(b) Find the displacement of satish and manoj ?

Solution : (a) Distance travelled by satish = 100 m
Distance travelled by manoj = π(50 m) = 50π m
(b) Displacement of satish = 100 m
Displacement of manoj = 100 m

Comparison between distance travelled and displacement :

(i) For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means body is moving towards the initial position.

(ii) For a moving particle distance can never be negative or zero while displacement can be. (zero displacement means that body after motion has came back to initial position)

Average Velocity (in an interval) :

The average velocity of a moving particle over a certain time interval is defined as the displacement divided by the total time taken in the process .

Average Velocity = $\dfrac{\text{displacement}}{\text{time interval}}$

for straight line motion, along x–axis, we have

$$v_{av} = \bar{v} = \langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The dimension of average velocity is $[LT^{-1}]$ and its SI unit is m/s.

The average velocity is a vector in the direction of displacement.

For motion in a straight line, directional aspect of a vector can be taken care of by +ve and -ve sign of the quantity.

Instantaneous Velocity (at an instant) :

The velocity at a particular instant of time is known as instantaneous velocity.

OR

Instantaneous velocity is defined as rate of change of position vector of particles with time at a certain instant of time.

The term "**velocity**" usually means instantaneous velocity.

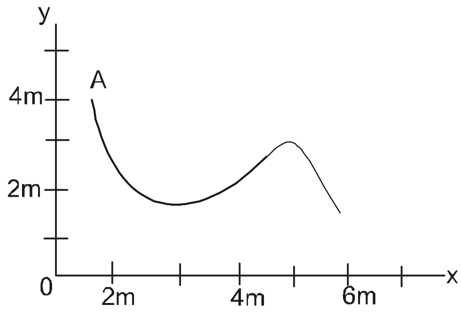
Instantaneous velocity $\vec{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta \vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt}$

Note :

- The magnitude of instantaneous velocity and instantaneous speed are equal.
- The determination of instantaneous velocity by using the definition usually involves calculation of derivative. We can find $v = \dfrac{dx}{dt}$ by using the standard results from differential calculus.
- Direction of Instantaneous velocity is always tangential to the path.

SOLVED EXAMPLE

Example 3. A particle starts from a point A and travels along the solid curve shown in figure. Find approximately the position B of the particle such that the average velocity between the positions A and B has the same direction as the instantaneous velocity at B.

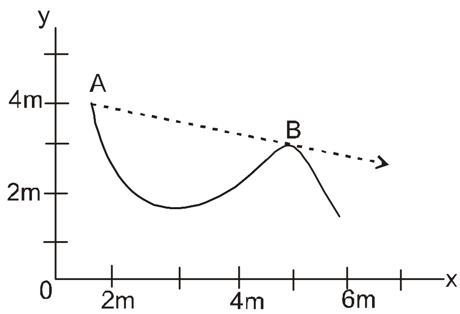


Answer : x = 5m, y = 3m

Solution : The given curve shows the path of the particle starting at y = 4 m.

Average velocity = $\frac{\text{displacement}}{\text{time taken}}$ **;** where displacement is straight line distance between points

Instantaneous velocity at any point is the tangent drawn to the curve at that point.



Now, as shown in the graph, line AB shows displacement as well as the tangent to the given curve. Hence, point B is the point at which direction of AB shows average as well as instantaneous velocity.

Example 4. The motion of a particle is described by the equation $x = a + bt^2$ where a = 15cm and b = 3cm. Its instantaneous velocity at time 3 sec will be

Answer : 18 cm/sec

Solution : $x = a + bt^2$ $\therefore v = \frac{dx}{dt} = 0 + 2bt$

At t = 3sec, v = 2 × 3 × 3= 18 cm/sec

Average Speed (in an interval)

Average speed is defined as the total path length travelled divided by the total time interval during which the motion has taken place. It helps in describing the motion along the actual path.

Average Speed = $\frac{\text{distance travelled}}{\text{time interval}}$

The dimension of velocity is $[LT^{-1}]$ and its SI unit is m/s.

Instantaneous speed :

It is the speed of a particle at particular instant. When we say "speed", it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (i.e., $\Delta t \to 0$).

Thus Instantaneous speed $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$

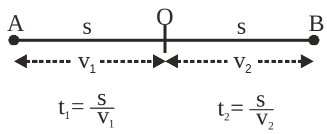
Note :

- Average speed is always positive in contrast to average velocity which being a vector, can be positive or negative.
- If the motion of a particle is along a straight line and in same direction then,average velocity = average speed.
- Average speed is, in general, greater than the magnitude of average velocity.

SOLVED EXAMPLE

Example 5. If a particle travels the first half distance with speed v_1 and second half distance with speed v_2.Find its average speed during journey.

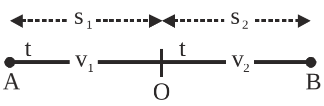
Solution: $v_{avg.} = \dfrac{s+s}{t_1+t_2} = \dfrac{2s}{\dfrac{s}{v_1}+\dfrac{s}{v_2}} = \dfrac{2v_1v_2}{v_1+v_2}$



Example 6. If a particle travels with speed v_1 during first half time interval and with v_2 speed during second half time interval. Find its average speed during its journey.

Solution: Total distance = $s_1 + s_2 = v_1t+v_2t = (v_1 + v_2)t$

Total time = t + t = 2t



$v_{avg.} = \dfrac{s_1+s_2}{t+t} = \dfrac{(v_1+v_2)t}{2t} = \dfrac{v_1+v_2}{2}$

Example 7. A car travels a distance A to B at a speed of 40 km/h and returns to A at a speed of 30 km/h.

(i) What is the average speed for the whole journey

(ii) What is the average velocity?

Solution: (i) Let AB = s, time taken to go from A to B, $t_1 = \dfrac{s}{40}$ h

and time taken to go from B to A, $t_2 = \dfrac{s}{30}$ h

$\therefore$ total time taken = $t_1 + t_2 = \dfrac{s}{40} + \dfrac{s}{30} = \dfrac{(3+4)\ s}{120} = \dfrac{7s}{120}$ h

Total distance travelled = s + s = 2

$\therefore$ Average speed $= \dfrac{\text{total distance travelled}}{\text{total time taken}} = \dfrac{2s}{\frac{7s}{120}} = \dfrac{120 \times 2}{7}$ = 34.3 km/h.

(ii) Total displacement = zero, since the car returns to the original position.

Therefore, average velocity = $\dfrac{\text{total displacement}}{\text{time taken}} = \dfrac{0}{2t} = 0$

Example 8. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. On reaching the market he instantly turns and walks back with a speed of 7.5 km/h. What is the (a) magnitude of average velocity and (b) average speed of the man, over the interval of time (i) 0 to 30 min. (ii) 0 to 50 min (iii) 0 to 40 min.

Solution : Time taken by man to go from his home to market, $t_1 = \dfrac{\text{distance}}{\text{speed}} = \dfrac{2.5}{5} = \dfrac{1}{2}$ h

Time taken by man to go from market to his home, $t_2 = \dfrac{2.5}{7.5} = \dfrac{1}{3}$h

$\therefore$ Total time taken = $t_1 + t_2 = \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{5}{6}$h = 50 min.

(i) 0 to 30 min

Average speed = $\dfrac{\text{distance}}{\text{time interval}} = \dfrac{2.5}{\frac{30}{60}}$ = 5 km/h

(ii) 0 to 50 min

Total displacement = zero so average velocity = 0

So, average speed = $\dfrac{5}{50/60}$ = 6 km/h

Total distance travelled = 2.5 + 2.5 = 5 km.

(iii) 0 to 40 min

Distance moved in 30 min (from home to market) = 2.5 km.

Distance moved in 10 min (from market to home) with speed 7.5 km/h = 7.5 × $\dfrac{10}{60}$ = 1.25 km

So, displacement = 2.5 – 1.25 = 1.25 km (towards market)

Distance travelled = 2.5 + 1.25 = 3.75 km

Average velocity $= \dfrac{1.25}{\frac{40}{60}}$ = 1.875 km/h. (towards market)

Average speed $= \dfrac{3.75}{\frac{40}{60}}$ = 5.625 km/h.

Average Acceleration :

When an object is moving with a variable acceleration, then the average acceleration of the object for the given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken

$$\text{Average Acceleration} = \frac{\text{Total change in velocity}}{\text{total time taken}}$$

Suppose the velocity of a particle is $\vec{v}_1$ at time t_1 and $\vec{v}_2$ at time t_2. Then $\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$

Instantaneous Acceleration :

The acceleration of the object at a given instant of time or at a given point of motion, is called its instantaneous acceleration. Suppose the velocity of a particle at time $t_1 = t$ is $\vec{v}_1 = \vec{v}$ and becomes $\vec{v}_2 = \vec{v} + \Delta\vec{v}$ at time $t_2 = t + \Delta t$, Then, $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$

If Δt approaches to zero then the rate of change of velocity will be instantaneous acceleration.

$$\text{Instantaneous acceleration } \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

The dimension of acceleration is $[LT^{-2}]$ and its SI unit is m/s^2.

SOLVED EXAMPLE

Example 9. Position of a particle as a function of time is given as $x = 8t^2 + 7t + 3$. Find the velocity and acceleration of the particle at t = 2second?

Solution : Velocity; $v = \frac{dx}{dt} = 16t + 7$

At $t = 2$ second

$v = 16(2) + 7$

$v = 39$ m/s

Acceleration; $a = \frac{d^2x}{dt^2} = 16 \text{ m/s}^2$

Acceleration is constant, so at t = 2 s $a = 16 \text{ m/s}^2$

MOTION WITH UNIFORM VELOCITY (UNIFORM MOTION)

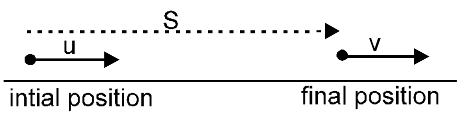
if a particle is moving with constant velocity v, then motion is called uniform motion. distance travelled by particle in t seconds is given by

Distance travelled = constant speed × time

UNIFORMLY ACCELERATED MOTION :

If a particle is accelerated with constant acceleration in an interval of time, then the motion is termed as uniformly accelerated motion in that interval of time.

For uniformly accelerated motion along a straight line (x–axis) during a time interval of t seconds, the following important results can be used.



Equations of motion (motion with constant acceleration)

If a particle moves with acceleration $\vec{a}$, then by definition $\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow d\vec{v} = \vec{a}dt$. Let at starting (t = 0) initial velocity of the particle $\vec{u}$ and at time t its final velocity = $\vec{v}$ then $\int_{\vec{u}}^{\vec{v}} d\vec{v} = \int_0^t \vec{a}dt$

If acceleration is constant

$$\int_{\vec{u}}^{\vec{v}} d\vec{v} = \vec{a}\int_0^t dt \Rightarrow [\vec{v}]_{\vec{u}}^{\vec{v}} = \vec{a}[t]_0^t \Rightarrow \vec{v} - \vec{u} = \vec{a}\,t \Rightarrow \vec{v} = \vec{u} + \vec{a}t \qquad ....(1)$$

Now by definition of velocity, equation (1) reduces to

$$\vec{v} = \frac{d\vec{s}}{dt} = \vec{u} + \vec{a}t \Rightarrow \int_0^{\vec{s}} d\vec{s} = \int_0^t (\vec{u} + \vec{a}t)dt \Rightarrow \vec{s} = \left[\vec{u}t + \frac{1}{2}\vec{a}t^2\right]_0^t \Rightarrow \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \qquad ....(2)$$

Now substituting the value of t from equation (1) to equation (2)

$$s = u\frac{(v-u)}{a} + \frac{1}{2}a\left[\frac{v-u}{a}\right]^2 \Rightarrow 2as = 2uv - 2u^2 + v^2 + u^2 - 2uv \Rightarrow v^2 = u^2 + 2as \qquad ....(iii)$$

vector form of equation (iii) $\boxed{v^2 = u^2 + 2\vec{a}.\vec{s}}$ ....(3)

These three equation are called equations of motion and are applicable only and only when acceleration is constant.

Displacement of body in n^{th} second

$$s_{n^{th}} = s_n - s_{n-1} = un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n-1)^2 = un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + an - \frac{a}{2}$$

$$\boxed{s_{n^{th}} = u + \frac{a}{2}(2n-1)} \qquad ....(4)$$

Point to rememeber (constant acceleration)

(a) $a = \frac{v-u}{t}$

(b) $V_{av} = \frac{v+u}{2}$

(c) $S = (v_{av})t$

(d) $S = \left(\frac{v+u}{2}\right)t$

(e) $v = u + at$

(f) $s = ut + 1/2\ at^2$

$s = vt - 1/2\ at^2$

$x_f = x_i + ut + 1/2\ at^2$

(g) $v^2 = u^2 + 2as$

(h) $s_n = u + a/2\ (2n - 1)$

u = initial velocity (at the beginning of interval)

a = acceleration

v = final velocity (at the end of interval)

s = displacement $(x_f - x_i)$

x_f = final coordinate (position)

x_i = initial coordinate (position)

s_n = displacement during the n^{th} sec

DIRECTIONS OF VECTORS IN STRAIGHT LINE MOTION

In straight line motion, all the vectors (position, displacement, velocity & acceleration) will have only one component (along the line of motion) and there will be only two possible directions for each vector.

• For example, if a particle is moving in a horizontal line (x–axis), the two directions are right and left. Any vector directed towards right can be represented by a positive number and towards left can be represented by a negative number.

Note :

• If acceleration is in same direction as velocity, then speed of the particle increases and motion is called accelerated motion.

• If acceleration is in opposite direction to the velocity then speed decreases i.e. the particle slows down. This situation is known as retarded motion

SOLVED EXAMPLE

Example 10. A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0m/s^2, find the distance travelled by the car after he sees the need to put the brakes on

Solution: Distance covered by the car during the application of brakes by driver –

$$s_1 = ut = \left(54 \times \frac{5}{18}\right)(0.2) = 15 \times 0.2 = 3.0 \text{ meter}$$

After applying the brakes; $v = 0$ $u = 15$ m/s, $a = 6$ m/s^2 $s_2 = ?$

Using $v^2 = u^2 - 2as \Rightarrow 0 = (15)^2 - 2 \times 6 \times s_2 \Rightarrow 12\ s_2 = 225 \Rightarrow s_2 = \frac{225}{12} = 18.75 \text{ metre}$

Distance travelled by the car after driver sees the need for it $s = s_1 + s_2 = 3 + 18.75 = 21.75$ metre.

Example 11. A particle moving rectilinearly with constant acceleration is having initial velocity of 20 m/s. After some time, its velocity becomes 40 m/s. Find out velocity of the particle at the mid point of its path?

Solution : Let the total distance be 2x.

$\therefore$ distance upto midpoint = x

Let the velocity at the mid point be v

and acceleration be a.

From equations of motion

$v^2 = 20^2 + 2ax$ ____ (1)

$40^2 = v^2 + 2ax$ ____ (2)

(2) – (1) gives

$v^2 - 40^2 = 20^2 - v^2$

$\Rightarrow v^2 = 1000 \quad \Rightarrow \quad v = 10\sqrt{10}$ m/s

Example 12. A body A moves with a uniform acceleration and zero initial velocity. Another body B, starts from the same point at the same instant moves in the same direction with a constant velocity v. The two bodies meet after a time t. Find the value of t?

Solution: Let the meet after time 't'. Distance covered by body A is $\frac{1}{2}at^2$ and Distance covered by body B is vt

$\frac{1}{2}at^2 = vt \qquad \therefore\ t = \frac{2v}{a}$

Example 13. A police inspector in a jeep is chasing a pickpocket an a straight road. The jeep is going at its maximum speed v (assumed uniform). The pickpocket rides on the motorcycle of a waiting friend when the jeep is at a distance d away, and the motorcycle starts with a constant acceleration a. Show that the pick pocket will be caught if $v \geq \sqrt{2ad}$.

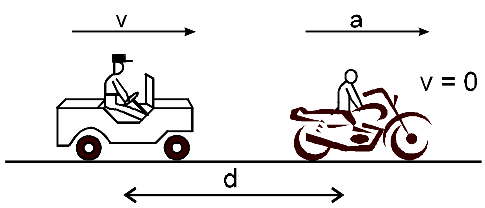
Solution : Suppose the pickpocket is caught at a time t after motorcycle starts. The distance travelled by the motorcycle during this interval is

$s = \frac{1}{2}at^2$ ____ (1)

During this interval the jeep travels a distance

$s + d = vt$ ____ (2)

By (1) and (2),



$\frac{1}{2}at^2 + d = vt$

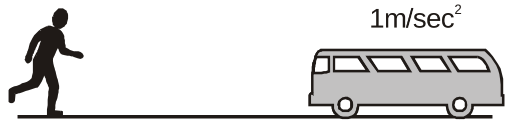
or, $t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}$

The pickpocket will be caught if t is real and positive.

This will be possible if

$v^2 \geq 2ad$ or, $v \geq \sqrt{2ad}$

Example 14. Harish is standing 40 m behind the bus. Bus starts with 1m/sec^2 constant acceleration and also at the same instant the man starts moving with constant speed 9 m/s. Find the time taken by Harish to catch the bus.



Solution : Let after time 't' Harish will catch the bus

For bus

$x = x_0 + ut + \frac{1}{2}at^2$, $x = 40 + 0(t) + \frac{1}{2}(1)t^2$

$x = 40 + \frac{t^2}{2}$ .... (i)

For Harish, $x = 9t$ .... (ii)

From (i) & (ii)

$40 + \frac{t^2}{2} = 9t$ or $t = 8$ s or $t = 10$s.

MOTION UNDER GRAVITY

The force of attraction of earth on bodies, is called force of gravity. Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

In the absence of air resistance, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude (h << R) is called free fall.

An ideal one-dimensional motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

SOLVED EXAMPLE

Example 15. A body is freely dropped from a height h above the ground. Find the ratio of distances fallen in first one second, first two seconds, first three seconds, also find the ratio of distances fallen in 1st second, in 2nd second, in 3rd second etc.

Solution: From second equation of motion, i.e. $h = \frac{1}{2}gt^2$ ($h = ut + \frac{1}{2}gt^2$ and $u = 0$)

$h_1 : h_2 : h_3 ...... = \frac{1}{2}g(1)^2 : \frac{1}{2}g(2)^2 : \frac{1}{2}g(3)^2 = 1^2 : 2^2 : 3^2 .... = 1 : 4 : 9 : ......$.

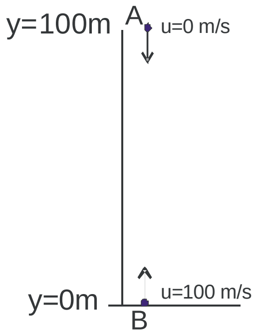
Now from the of distance travelled in nth second

$s_n = u + \frac{1}{2}a(2n - 1)$ here $u = 0$, $a = g \Rightarrow s_n = \frac{1}{2}g(2n - 1)$

$\Rightarrow s_1 : s_2 : s_3 .... = \frac{1}{2}g(2 \times 1 - 1) : \frac{1}{2}g(2 \times 2 - 1) : \frac{1}{2}g(2 \times 3 - 1) = 1 : 3 : 5 ....$

Example 16. A particle is dropped from height 100 m and another particle is projected vertically up with velocity 100 m/s from the ground along the same line. Find out the position where two particle will meet? (take g = 10 m/s^2)

Solution : Let the upward direction is positive.



Let the particles meet at a distance y from the ground.

For particle A,

$y_0 = +100$ m

$u = 0$ m/s

$a = -10$ m/s^2

$y = 100 + 0(t) - \frac{1}{2} \times 10 \times t^2$ $\quad [y = y_0 + ut + \frac{1}{2}at^2]$

$= 100 - 5t^2$ ---- (1)

For particle B,

$y_0 = 0$ m

$u = +100$ m/s

$a = -10$ m/s^2

$y = 100(t) - \frac{1}{2} \times 10 \times t^2$

$= 100t - 5t^2$ ---- (2)

According to the problem;

$100t - 5t^2 = 100 - 5t^2$

$t = 1$ sec

Putting t = 1 sec in equation. (1),

$y = 100 - 5 = 95$ m

Hence, the particles will meet at a height 95 m above the ground.

Example 17. A particle is dropped from a tower. It is found that it travels 30 m in the last second of its journey. Find out the height of the tower? (take g = 10 m/s^2)

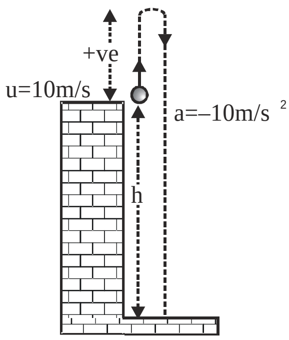
Solution : Let the total time of journey be n seconds.

Using; $s_n = u + \frac{a}{2}(2n-1) \Rightarrow 30 = 0 + \frac{10}{2}(2n-1)$

$n = 3.5$ sec

Height of tower; $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 3.5^2 = 61.25$ m

Example 18. A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s, find the time when it strikes the ground (g = 10 m/s^2)



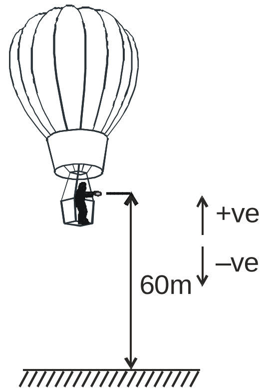
Solution: In the problem u = + 10 m/s, a = – 10 m/s^2 and s = –40m (at the point where ball strikes the ground)

Substituting in $s = ut + \frac{1}{2}at^2$

$$-40 = 10t - 5t^2 \Rightarrow 5t^2 - 10t - 40 = 0 \Rightarrow t^2 - 2t - 8 = 0$$

Solving this we have t = 4 s and – 2s. Taking the positive value t = 4s.

Example 19. A stone is dropped from a balloon going up with a uniform velocity of 5 m/s. If the balloon was 60 m high when the stone was dropped, find its height when the stone hits the ground. Take g = 10 m/s^2.



Solution :

$$S = ut + \frac{1}{2}at^2$$

$$-60 = 5(t) + \frac{1}{2}(-10)t^2$$

$$-60 = 5t - 5t^2$$

$$5t^2 - 5t - 60 = 0$$

$$t^2 - t - 12 = 0$$

$$t^2 - 4t + 3t - 12 = 0$$

$$(t - 4)(t + 3) = 0$$

$$\therefore \quad t = 4$$

Height of balloon from ground at this instant

$$= 60 + 4 \times 5 = 80 \text{ m}$$

Note :

- As the particle is detached from the balloon it is having the same velocity as that of balloon, but its acceleration is only due to gravity and is equal to g.

REACTION TIME

When a situation demands our immediate action. It takes some time before we really respond. Reaction time is the time an observer takes to observe, think and act.

STRAIGHT LINE-EQUATION, GRAPH

Slope of a line

It is the tan of angle made by a line with the positive direction of x-axis, measured in anticlockwise direction.

Slope = tan θ (In 1st quadrant tan θ is +ve & 2nd quadrant tan θ is –ve)

In Figure - 1 slope is positive
$\theta < 90°$ (1st quadrant)

In Figure - 2 slope is negative
$\theta > 90°$ (2nd quadrant)

Equation of straight line is $y = mx + c$

where m(slope of line) = $\tan\theta$ and c is y intercept.

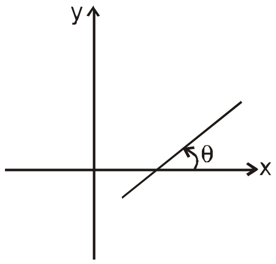


Figure - 1

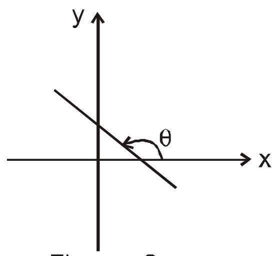
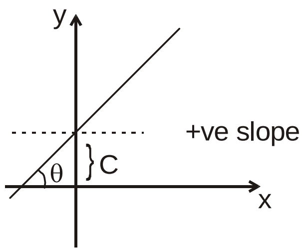
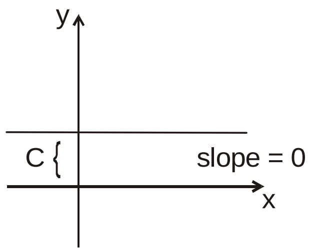
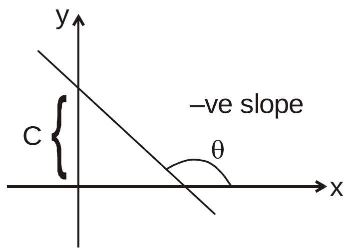


Figure - 2

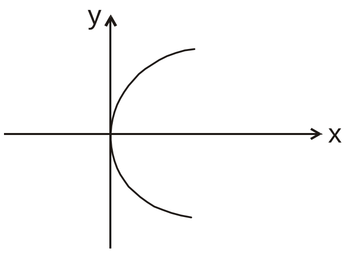




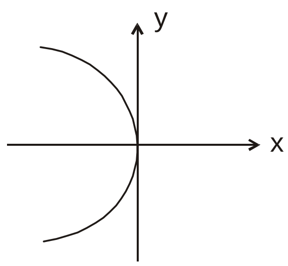


PARABOLIC CURVE-EQUATION, GRAPH

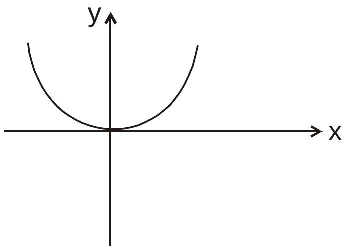
$y^2 = kx$



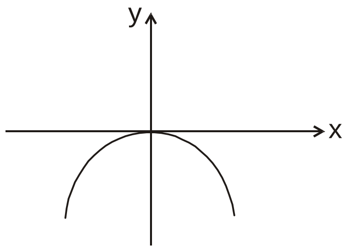
$y^2 = -kx$



$x^2 = ky$

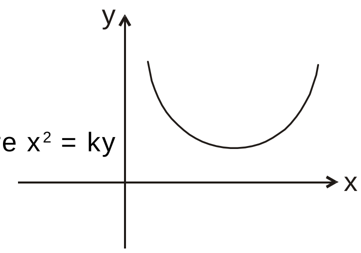


$x^2 = -ky$



Where k is a positive constant.

Equation of parabola :



Case (i) : $y = ax^2 + bx + c$

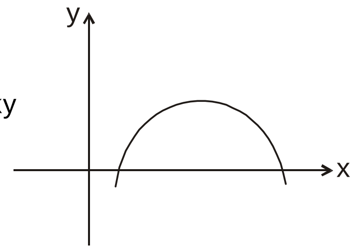
For $a > 0$

The nature of the parabola will be like that of the of nature $x^2 = ky$

Minimum value of y exists at the vertex of the parabola.

$$y_{min} = \frac{-D}{4a} \text{ where } D = b^2 - 4ac$$

Coordinates of vertex = $\left(\frac{-b}{2a}, \frac{D}{4a}\right)$



Case (ii) : $a < 0$

The nature of the parabola will be like that of the nature of $x^2 = -ky$

Maximum value of y exists at the vertex of parabola.

$$y_{max} = \frac{D}{4a} \text{ where } D = b^2 - 4ac$$

GRAPHS IN UNIFORMLY ACCELERATED MOTION ($a \neq 0$)

• x is a quadratic polynomial in terms of t. Hence x – t graph is a parabola.

Position-time graph

• v is a linear polynomial in terms of t. Hence v–t graph is a straight line of slope a.

Velocity-time graph

• a–t graph is a horizontal line because a is constant.

Acceleration-time graph

INTERPRETATION OF SOME MORE GRAPHS

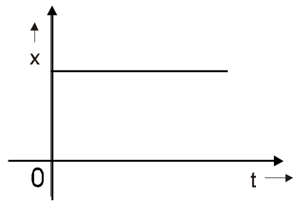
Position vs Time graph

(i) Zero Velocity

As position of particle is fixed at all the time, so the body is at rest.

Slope; $\frac{dx}{dt}$ = tanθ = tan 0º = 0

Velocity of particle is zero



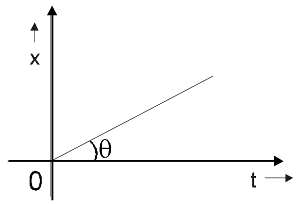
(ii) Uniform Velocity

Here tanθ is constant

$$\tan\theta = \frac{dx}{dt}$$

$\therefore$ $\frac{dx}{dt}$ is constant.

$\therefore$ velocity of particle is constant.



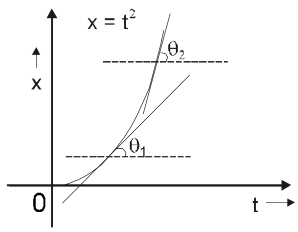
(iii) Non uniform velocity (increasing with time)

In this case;

As time is increasing, θ is also increasing.

$\therefore$ $\frac{dx}{dt}$ = tanθ is also increasing

Hence, velocity of particle is increasing.



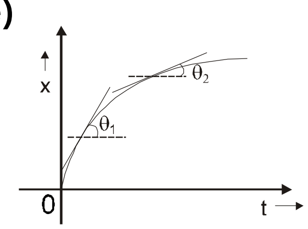
(iv) Non uniform velocity (decreasing with time)

In this case;

As time increases, θ decreases.

$\therefore$ $\frac{dx}{dt}$ = tanθ also decreases.

Hence, velocity of particle is decreasing.



Velocity vs time graph

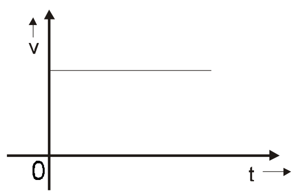
(i) Zero acceleration

Velocity is constant.

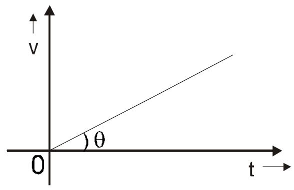
tanθ = 0

$\therefore$ $\frac{dv}{dt} = 0$

Hence, acceleration is zero.



(ii) Uniform acceleration

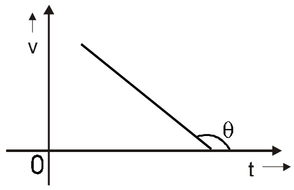


$\tan\theta$ is constant.

$\therefore \dfrac{dv}{dt}$ = constant

Hence, it shows constant acceleration.

(iii) Uniform retardation



Since $\theta > 90^\circ$

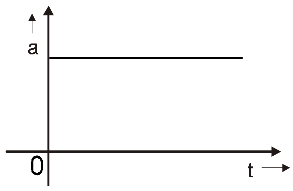
$\therefore$ $\tan\theta$ is constant and negative.

$\therefore \dfrac{dv}{dt}$ = negative constant

Hence, it shows constant retardation.

Acceleration vs time graph

(i) Constant acceleration

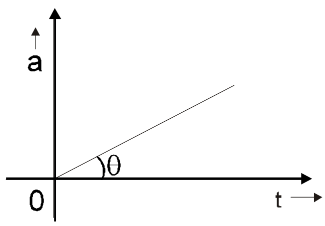


$\tan\theta = 0$

$\therefore \dfrac{da}{dt} = 0$

Hence, acceleration is constant.

(ii) Uniformly increasing acceleration



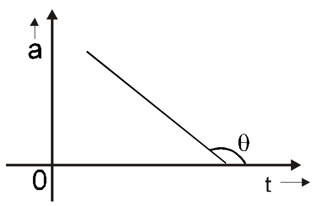
θ is constant.

$0^\circ < \theta < 90^\circ \Rightarrow \tan\theta > 0$

$\therefore \dfrac{da}{dt} = \tan\theta$ = constant > 0

Hence, acceleration is uniformly increasing with time.

(iii) Uniformly decreasing acceleration



Since $\theta > 90^\circ$

$\therefore$ $\tan\theta$ is constant and negative.

$\therefore \dfrac{da}{dt}$ = negative constant

Hence, acceleration is uniformly decreasing with time

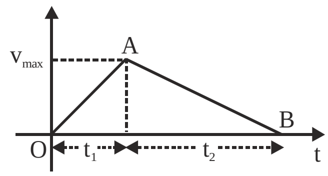
SOLVED EXAMPLE

Example 20. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β, to come to rest. If the total time elapsed is t evaluate (a) the maximum velocity attained and (b) the total distance travelled.

Solution: (a) Let the car accelerates for time t_1 and decelerates for time t_2 then

$t = t_1 + t_2$ ......(i)

and corresponding velocity–time graph will be as shown in. fig.



From the graph α = slope of line AB = $\dfrac{v_{max}}{t_1} \Rightarrow t_1 = \dfrac{v_{max}}{\alpha}$

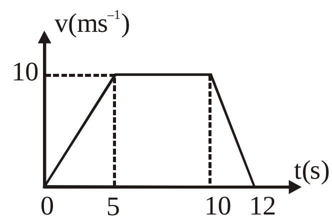
and β = – slope of line OB = $\dfrac{v_{max}}{t_2} \Rightarrow t_2 = \dfrac{v_{max}}{\beta}$

$$\Rightarrow \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} = t \Rightarrow v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t \Rightarrow v_{max} = \frac{\alpha\beta t}{\alpha+\beta}$$

(b) Total distance = area under v–t graph = $\dfrac{1}{2} \times t \times v_{max} = \dfrac{1}{2} \times t \times \dfrac{\alpha\beta t}{\alpha+\beta} = \dfrac{1}{2}\left(\dfrac{\alpha\beta t^2}{\alpha+\beta}\right)$

Note: This problem can also be solved by using equations of motion (v = u + at, etc.).

Example 21. Draw displacement-time and acceleration – time graph for the given velocity–time graph



Solution: For $0 \le t \le 5$ $v \propto t \Rightarrow s \propto t^2$ and a_1 = constant $\dfrac{10}{5}$ = 2 ms^{-2}

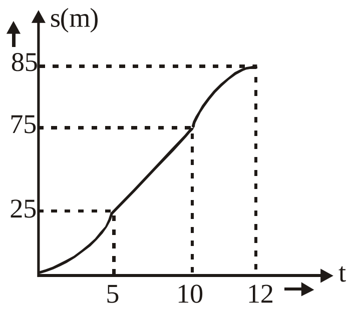
for whole interval s_1 = Area under the curve = $\dfrac{1}{2} \times 5 \times 10 = 25$ m

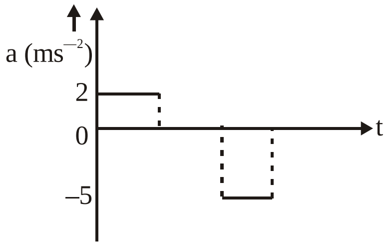
For $5 \le t \le 10$ $v = 10ms^{-1}$ $\Rightarrow a = 0$

for whole interval s_2 = Area under the curve = $\dfrac{1}{2} \times 5 \times 10 = 50$ m

For $10 \le t \le 12$ v linearly decreases with time $\Rightarrow a_3 = -\dfrac{10}{2} = -5\ ms^{-1}$

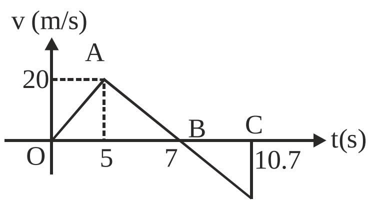
for whole interval s_3 = Area under the curve = $\dfrac{1}{2} \times 2 \times 10 = 10$ m

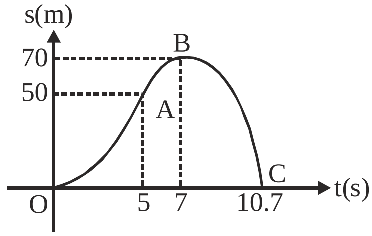




Example 22. A rocket is fired upwards vertically with a net acceleration of 4 m/s^2 and initial velocity zero. After 5 seconds its fuel is finished and it decelerates with g. At the highest point its velocity becomes zero. Then it accelerates downwards with acceleration g and return back to ground. Plot velocity–time and displacement–time graphs for the complete journey. Take g = 10 m/s^2.

Solution:





In the graphs, $v_A = at_{OA}$ = (4) (5) = 20 m/s $v_B = 0 = v_A - gt_{AB}$

$\therefore\ t_{AB} = \frac{v_A}{g} = \frac{20}{10} = 2s$ $\qquad \therefore\ t_{OAB}$ = (5 + 2)s = 7s

Now, s_{OAB} = area under v–t graph between 0 to 7 s = $\frac{1}{2}$ (7) (20) = 70 m

Now, $s_{OAB} = s_{BC} = \frac{1}{2} gt^2_{BC}$ $\qquad \therefore\ 70 = \frac{1}{2}$ (10) t^2_{BC}

$\therefore\ t_{BC} = \sqrt{14} = 3.7s$ $\qquad \therefore\ t_{OAB}$ = 7 + 3.7 = 10.7s

Also s_{OA} = area under v–t graph between OA = $\frac{1}{2}$ (5) (20) = 50 m

Example 23. At the height of 500m, a particle A is thrown up with v = 75 ms^{-1} and particle B is released from rest. Draw, accelearation –time, velocity–time, speed–time and displacement–time graph of each particle.

Solution: **For particle A** :

Time of flight

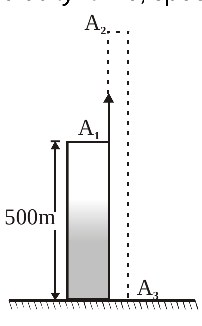
$-500 = +75\ t - \frac{1}{2} \times 10t^2$

$\Rightarrow t^2 - 15t - 100 = 0$

$\Rightarrow$ t = 20 s

Time taken for A_1A_2

= 75 – 10t $\Rightarrow$ t = 7.5 s



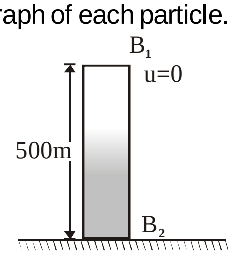
For Particle B

Time of flight

$500 = \frac{1}{2}(10)t^2 \Rightarrow t = 10$ s

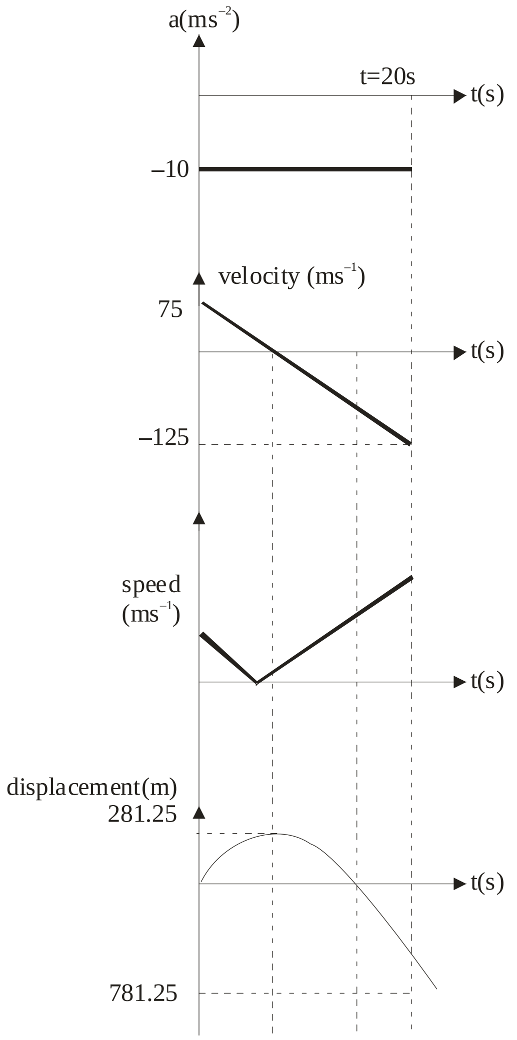
Velocity at B_2

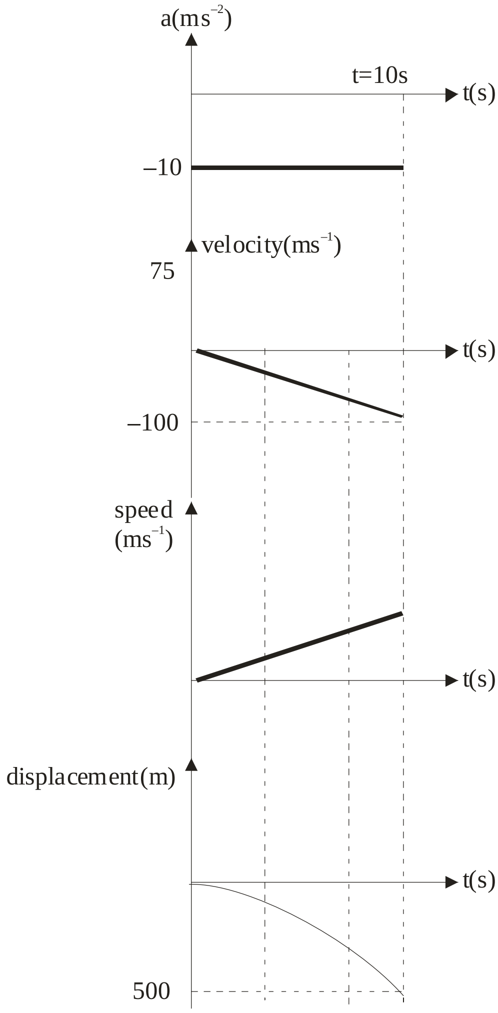
v = 0 – (10) (10) = – 100 ms^{-1}



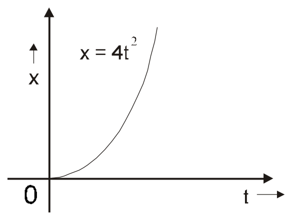
Velocity at A_3, $v = 75 - 10 \times 20 = -125\ ms^{-1}$

Height $A_2A_1 = \frac{1}{2}(10)(7.5)^2 = 281.25$ m





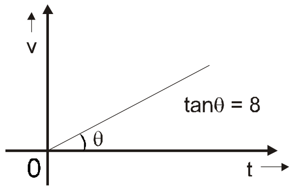
Example 24. The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



Solution : $x = 4t^2$

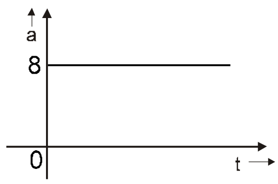
$$v = \frac{dx}{dt} = 8t$$

Hence, velocity-time graph is a straight line having slope i.e. $\tan\theta = 8$.



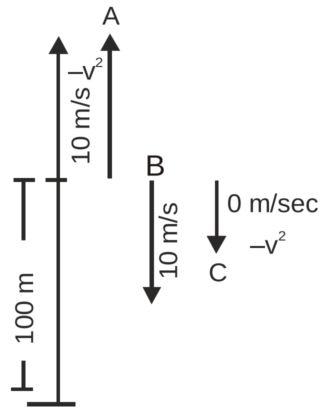
$$a = \frac{dv}{dt} = 8$$

Hence, acceleration is constant throughout and is equal to 8.



Example 25. At the height of 100 m, a particle A is thrown up with V = 10 m/s, B particle is thrown down with V =10m/s and C particle released with V = 0 m/s. Draw graphs of each particle.

(i) Displacement–time
(ii) Speed–time
(iii) Velocity–time
(iv) Acceleration–time



Solution : **For particle A :**

(i) Displacement vs time graph is

$$y = ut + \frac{1}{2}at^2$$

$u = + 10$ m/sec^2

$$y = 10t - \frac{1}{2}\times 10t^2$$

$= 10\ t - 5t^2$

$$v = \frac{dy}{dt} = 10 - 10\ t = 0$$

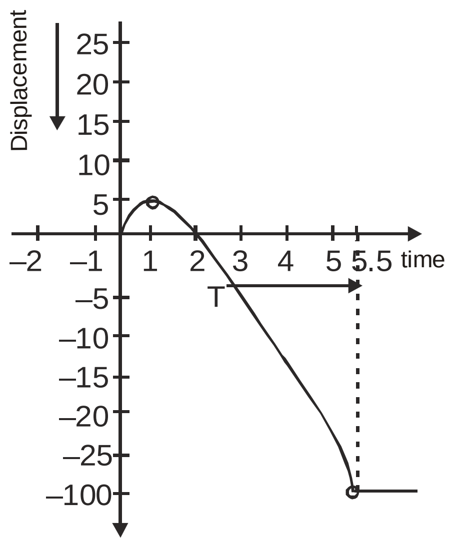
t = 1 ; hence, velocity is zero at t = 1

$10\ t - 5\ t^2 = -100$

$t^2 - 2t - 20 = 0$

t = 5.5 sec.

i.e. particle travels up till 5.5 seconds.



(ii) Speed vs time graph :

Particle has constant acceleration = g ↓ throughout the motion, so v-t curve will be straight line.

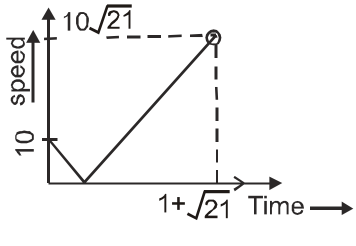
when moving up, v = u + at

0 = 10 – 10 t or t = 1 is the time at which speed is zero.

there after speed increases at constant rate of 10 m/s^2 .

Resulting Graph is : (speed is always positive).

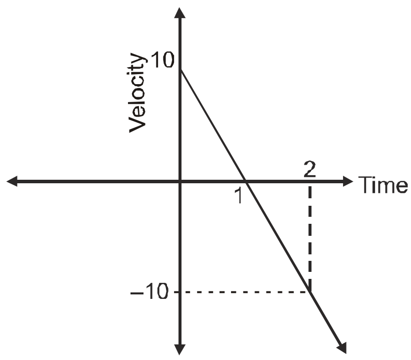
This shows that particle travels till a time of $1 + \sqrt{21}$ seconds



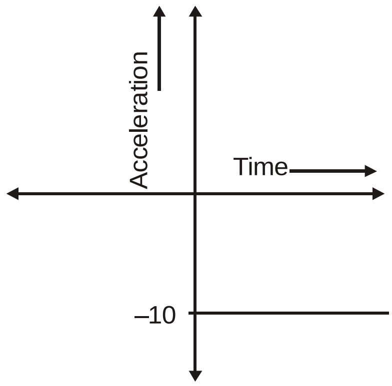
(iii) Velocity vs time graph :

V = u + at

V = 10 – 10t ; this shows that velocity becomes zero at t = 1 sec and thereafter the velocity is negative with slope g.



(iv) Acceleration vs time graph : throughout the motion, particle has constant acceleration = –10 m/s^2.



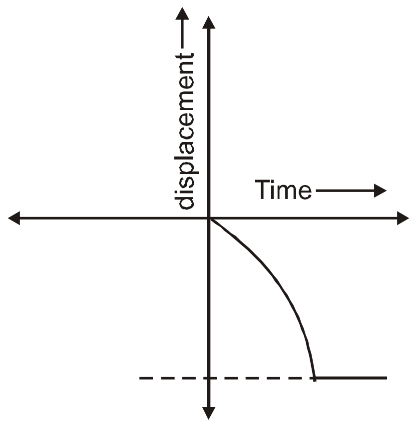
For particle B :

$$u = -10 \text{ m/s. } y = -10t - \frac{1}{2}(10)\,t^2 = -10t - 5t^2$$

(i) Displacement time graph :

$y = 10t - 5t^2$

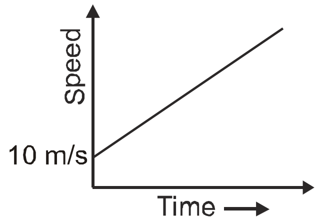
$\frac{dy}{dt} = -10t - 5t^2 = -10 - 10t$ this shows that slope becomes more negative with time.



(ii) Speed time graph :

$$\frac{dy}{dt} = -10t - 5t^2 = -10 - 10t$$

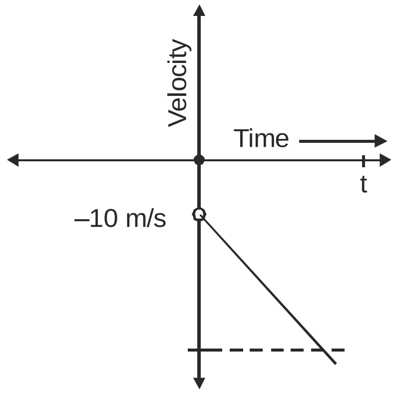
hence, speed is directly proportional to time with slope of 10. initial speed = 10 m/s



(iii) Velocity time graph :

$$\frac{dy}{dt} = -10t - 5t^2 = -10 - 10t$$

hence, velocity is directly proportional to time with slope of –10. Initial velocity = –10 m/s

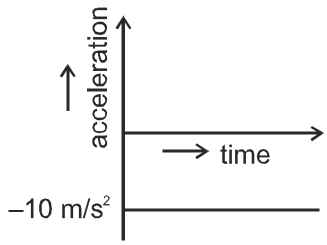


(iv) Acceleration vs time graph :

throughout the motion,

particle has constant acceleration = –10 m/s^2.

$$a = \frac{dv}{dt} = -10$$

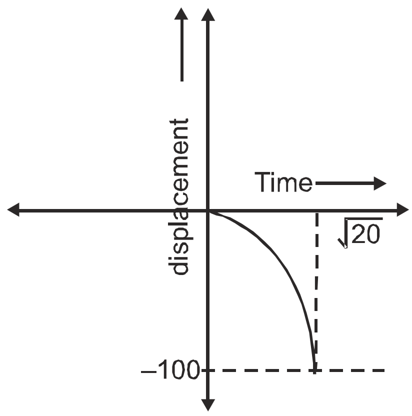


For Particle C :

(i) Displacement time graph :

$$u = 0 \quad , \; y = -\frac{1}{2} \times 10t^2 = -5t^2$$

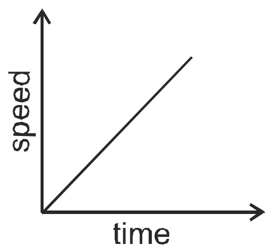
this shows that slope becomes more negative with time.



(ii) Speed vs time graph :

$$v = \frac{dy}{dt} = -10\,t$$

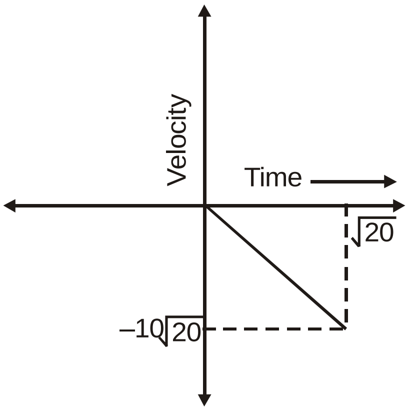
hence, speed is directly proportional to time with slope of 10.



(iii) Velocity time graph :

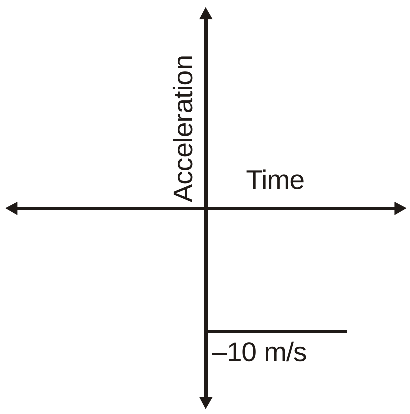
$V = u + at$

$V = -10t$; hence, velocity is directly proportional to time with slope of –10.



(iv) Acceleration vs time graph :

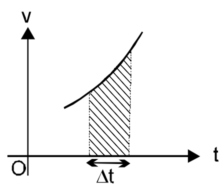
throughout the motion, particle has constant acceleration = $-10\ m/s^2$.



DISPLACEMENT FROM v – t GRAPH & CHANGE IN VELOCITY FROM a -t GRAPH

Displacement = Δx = area under v–t graph.

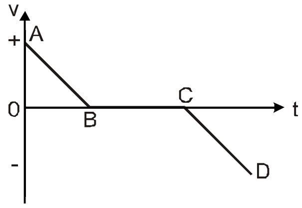
Since a negative velocity causes a negative displacement, areas below the time axis are taken negative. In similar way, can see that $\Delta v = a \Delta t$ leads to the conclusion that **area under a – t graph gives the change in velocity Δv during that interval.**



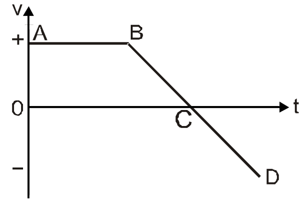
SOLVED EXAMPLE

Example 26. Describe the motion shown by the following velocity-time graphs.

(a)



(b)



Solution : (a) **During interval AB:** velocity is +ve so the particle is moving in +ve direction, but it is slowing down as acceleration (slope of v-t curve) is negative. **During interval BC:** particle remains at rest as velocity is zero. Acceleration is also zero. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

(b) **During interval AB:** particle is moving in +ve direction with constant velocity and acceleration is zero. **During interval BC:** particle is moving in +ve direction as velocity is +ve, but it slows down until it comes to rest as acceleration is negative. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

Important Points to Remember

- For uniformly accelerated motion ($a \neq 0$), x–t graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.
- For uniformly accelerated motion ($a \neq 0$), v–t graph is a straight line whose slope gives the acceleration of the particle.
- In general, the slope of tangent in x–t graph is velocity and the slope of tangent in v–t graph is the acceleration.
- The area under a–t graph gives the change in velocity.
- The area between the v–t graph gives the distance travelled by the particle, if we take all areas as positive.
- Area under v–t graph gives displacement, if areas below the t–axis are taken negative.

MOTION WITH NON-UNIFORM ACCELERATION

(USE OF DEFINITE INTEGRALS)

$$\Delta x = \int_{t_i}^{t_f} v(t)\,dt$$ (displacement in time interval $t = t_i$ to t_f)

The expression on the right hand side is called the definite integral of v(t) between $t = t_i$ and $t = t_f$. . Similarly change in velocity

$$\Delta v = v_f - v_i = \int_{t_i}^{t_f} a(t)\,dt$$

Solving Problems which Involves Non uniform Acceleration:

(i) Acceleration depending on velocity v or time t

By definition of acceleration, we have $a = \frac{dv}{dt}$. If a is in terms of t,

$\int_{v_0}^{v} dv = \int_{0}^{t} a(t)\,dt$. If a is in terms of v, $\int_{v_0}^{v} \frac{dv}{a(v)} = \int_{0}^{t} dt$.

On integrating, we get a relation between v and t, and then using $\int_{x_0}^{x} dx = \int_{0}^{t} v(t)\,dt$, x and t can also be related.

(ii) Acceleration depending on velocity v or position x

$$a = \frac{dv}{dt} \Rightarrow a = \frac{dv}{dx}\frac{dx}{dt} \Rightarrow a = \frac{dx}{dt}\frac{dv}{dx}$$

$$\Rightarrow a = v\frac{dv}{dx}$$

This is another important expression for acceleration.

If a is in terms of x,

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx.$$

If a is in terms of v, $\int_{v_0}^{v} \frac{v\,dv}{a(v)} = \int_{x_0}^{x} dx$

On integrating, we get a relation between x and v.

Using $\int_{x_0}^{x} \frac{dx}{v(x)} = \int_{0}^{t} dt$, we can relate x and t.

SOLVED EXAMPLE

Example 27. The velocity of any particle is related with its displacement as; $x = \sqrt{v+1}$, Calculate acceleration at x = 5 m.

Solution: $\therefore x = \sqrt{v+1}$ $\therefore x^2 = v + 1 \Rightarrow v = (x^2 - 1)$

Therefore $a = \frac{dv}{dt} = \frac{d}{dt}\left(x^2 - 1\right) = 2x\frac{dx}{dt} = 2x\ v = 2x\left(x^2 - 1\right)$

At x = 5 m, a = 2 × 5 (25 – 1) = 240 m/s^2

Example 28. The velocity of a particle moving in the positive direction of x–axis varies as v = $\alpha\sqrt{x}$ where α is positive constant. Assuming that at the moment t = 0, the particle was located at x = 0 find, (i) the time dependance of the velocity and the acceleration of the particle and (ii) the average velocity of the particle averaged over the time that the particle takes to cover first s metres of the path.

Solution: (i) Given that $v = \alpha\sqrt{x}$

$$\Rightarrow \frac{dx}{dt} = \alpha\sqrt{x} \quad \therefore \frac{dx}{\sqrt{x}} = \alpha\, dt \Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt \quad 2\sqrt{x} = \alpha t \Rightarrow x = \left(\alpha^2 t^2/4\right)$$

Velocity $\frac{dx}{dt} = \frac{1}{2}\alpha^2 t$ and Acceleration $\frac{d^2x}{dt^2} = \frac{1}{2}\alpha^2$

(ii) Time taken to cover first s metres s = $\frac{\alpha^2 t^2}{4} \Rightarrow t^2 = \frac{4s}{\alpha^2} \Rightarrow t = \frac{2\sqrt{s}}{\alpha}$;

average velocity = $\frac{\text{total displacement}}{\text{total time}} = \frac{s\alpha}{2\sqrt{s}} = \frac{1}{2}\sqrt{s}\alpha$

Example 29. A particle moves in the plane xy with constant acceleration a directed along the negative y–axis. The equation of motion of the particle has the form y = px – qx² where p and q are positive constants. Find the velocity of the partcle at the origin of coordinates.

Solution: Given that $y = px - qx^2$

$$\therefore \frac{dy}{dt} = p\frac{dx}{dt} - q.2x\frac{dx}{dt} \text{ and } \frac{d^2y}{dt^2} = p\frac{d^2x}{dt^2} - 2qx\frac{d^2x}{dt^2} - 2q\left(\frac{dx}{dt}\right)^2 = (p-2qx)\frac{d^2x}{dt^2} - 2q\left(\frac{dx}{dt}\right)^2$$

$\because \frac{d^2x}{dt^2} = 0$ (no acceleration along x–axis) and $\frac{d^2y}{dt^2} = -a$

$$\therefore v_x^2 = \frac{a}{2q} \Rightarrow v_x = \sqrt{\frac{a}{2q}} \text{ Further, } \left(\frac{dy}{dt}\right)_{x=0} = p\frac{dx}{dt} \Rightarrow v_y = p\sqrt{\left(\frac{a}{2q}\right)}$$

$$\text{Now } v = \sqrt{\left(v_x^2 + v_y^2\right)} = \sqrt{\left(\frac{a}{2q} + \frac{ap^2}{2q}\right)} \Rightarrow v = \sqrt{\left[\frac{a\left(p^2+1\right)}{2q}\right]}$$

Example 30. For a particle moving along x-axis, acceleration is given as a = v. Find the position as a function of time? Given that at t = 0 , x = 0 v = 1.

Solution : $a = v \Rightarrow \frac{dv}{dt} = v \Rightarrow \int \frac{dv}{v} = \int dt$

$\ell nv = t + c \Rightarrow 0 = 0 + c$

$v = e^t \Rightarrow \frac{dx}{dt} = e^t \Rightarrow \int dx = \int e^t dt$

$\Rightarrow x = e^t + c \Rightarrow 0 = 1 + c$

$x = e^t - 1$

MISCELLANEOUS SOLVED PROBEME

Problem 1. A particle covers $\frac{3}{4}$ of total distance with speed v_1 and next $\frac{1}{4}$ with v_2. Find the average speed of the particle?

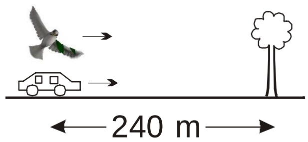
Answer : $\frac{4v_1v_2}{v_1+3v_2}$

Solution : Let the total distance be s

average speed (< v >) = $\frac{\text{Total distance}}{\text{Total time taken}}$

$$< v > = \frac{s}{\frac{3s}{4v_1}+\frac{s}{4v_2}} = \frac{1}{\frac{3}{4v_1}+\frac{1}{4v_2}} = \frac{4v_1v_2}{v_1+3v_2}$$

Problem 2. A car is moving with speed 60 Km/h and a bird is moving with speed 90 km/h along the same direction as shown in figure. Find the distance travelled by the bird till the time car reaches the. tree?



Answer : 360 m

Solution : Time taken by a car to reaches the tree (t) = $\frac{240\,m}{60\,km/hr} = \frac{0.24}{60}$ hr

Now, the distance travelled by the bird during this time interval (s)

$$= 90\times\frac{0.24}{60} = 0.12 \times 3 \text{ km} = 360 \text{ m}.$$

Problem 3. The position of a particle moving on X-axis is given by

$$x = At^3 + Bt^2 + Ct + D.$$

The numerical values of A, B, C, D are 1, 4, -2 and 5 respectively and SI units are used. Find (a) the dimensions of A, B, C and D, (b) the velocity of the particle at t = 4 s, (c) the acceleration of the particle at t = 4s, (d) the average velocity during the interval t =0 to t = 4s, (e) the average acceleration during the interval t = 0 to t = 4 s.

Answer : **(a)** [A] = $[LT^{-3}]$, [B] = $[LT^{-2}]$, [C] = $[LT^{-1}]$ and [D] = [L] ;
(b) 78 m/s ; **(c)** 32 m/s^2 ; **(d)** 30 m/s ; **(e)** 20 m/s^2

Solution : As $x = At^3 + Dt^2 + Ct + D$

(a) Dimensions of A, B, C and D ,

$[At^3] = [x]$ (by principle of homogeneity)

$[A] = [LT^{-3}]$

similarly, $[B] = [LT^{-2}]$, $[C] = [LT^{-1}]$ and $[D] = [L]$;

(b) As $v = \frac{dx}{dt} = 3At^2 + 2Bt + C$

velocity at t = 4 sec.

$v = 3(1)(4)^2 + 2(4)(4) - 2 = 78$ m/s.

(c) Acceleration $(a) = \frac{dv}{dt} = 6At + 2B$; $a = 32$ m/s^2

(d) average velocity as $x = At^3 + Bt^2 + Ct = D$

position at t = 0, is x = D = 5m.

Position at t = 4 sec is $(1)(64) + (4)(16) - (2)(4) + 5 = 125$ m

Thus the displacement during 0 to 4 sec. is $125 - 5 = 120$ m

$\therefore \langle v \rangle = 120 / 4 = 30$ m/s

(e) $v = 3At^2 + 20t + C$, velocity at t = 0 is $c = -2$ m/s

velocity at t = 4 sec is 78 m/s $\therefore \langle a \rangle = \frac{v_2 - v_1}{t_2 - t_1} = 20$ m/s^2

Problem 4. For a particle moving along x-axis, velocity is given as a function of time as $v = 2t^2 + \sin t$. At t = 0, particle is at origin. Find the position as a function of time?

Solution : $v = 2t^2 + \sin t \Rightarrow \frac{dx}{dt} = 2t^2 + \sin t$

$$\int_0^x dx = \int_0^t (2t^2 + \sin t)\,dt = x = \frac{2}{3}t^3 - \cos t + 1$$ **Ans.**

Problem 5. A car decelerates from a speed of 20 m/s to rest in a distance of 100 m. What was its acceleration, assumed constant?

Solution : v = 0 u = 20 m/s s = 100 m $\Rightarrow$ as $v^2 = u^2 + 2as$

$0 = 400 + 2a \times 100 \Rightarrow a = -2$ m/s

$\therefore$ acceleration = **2 m/s²** **Ans.**

Problem 6. A 150 m long train accelerates uniformly from rest. If the front of the train passes a railway worker 50 m away from the station at a speed of 25 m/s, what will be the speed of the back part of the train as it passes the worker?

Solution : $v^2 = u^2 + 2as$

$25 \times 25 = 0 + 100a$

$a = \frac{25}{4}$ m/s^2

Now, for time taken by the back end of the train to pass the worker

we have $v'^2 = v^2 + 2al = (25)^2 + 2 \times \frac{25}{48} \times 150$

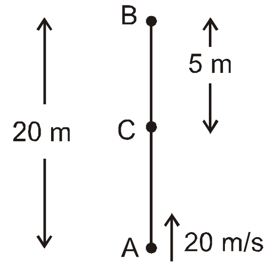
$v'^2 = 25 \times 25 \times 4$

v' = **50 m/s.** **Ans.**

Problem 7. A particle is thrown vertically with velocity 20 m/s . Find (a) the distance travelled by the particle in first 3 seconds, (b) displacement of the particle in 3 seconds.

Answer : 25m, 15m

Solution : Highest point say B



$V_B = 0$

$v = u + gt$

$0 = 20 - 10\ t$

$t = 2$ sec.

$\therefore$ distance travel in first 2 seconds.

s = s(t =0 to 2sec) + s (2sec. to 3sec.)

$s = [ut + 1/2\ at^2]_{t=0 \text{ to } t=2s} + [ut + 1/2 at^2]_{t=2 \text{ to } t=3s}$

$s = 20 \times 2 - 1/2 \times 10 \times 4 + 1/2 \times 10 \times 1^2$

$= (40 - 20) + 5 =$ **25 m.**

and displacement = 20 – 5 = **15 m.**

Problem 8. The acceleration of a particle moving in a straight line varies with its displacement as, a = 2s velocity of the particle is zero at zero displacement. Find the corresponding velocity displacement equation.

Solution: $a = 2s \Rightarrow \frac{dv}{dt} = 2s \quad \Rightarrow \quad \frac{dv}{ds}.\frac{ds}{dt} = 2s \Rightarrow \frac{dv}{ds}.v = 2s$

$\Rightarrow \int vdv = 2\int sds \quad \Rightarrow \quad \left(\frac{v^2}{2}\right)_0^v = 2\left(\frac{s^2}{2}\right)_0^s$

$\Rightarrow \frac{v^2}{2} = s^2 \Rightarrow v = s\sqrt{2}$

Problem 9. If a body travels half its total path in the last second of its fall from rest, find :

(a) The time and

(b) height of its fall. Explain the physically unacceptable solution of the quadratic time equation. ($g = 9.8\ m/s^2$)

Solution: If the body falls a height h in time t, then

$h = (1/2)gt^2$ [u = 0 as the body starts from rest] ... (1)

Now, as the distance covered in (t – 1) second is $h' = \frac{1}{2}g(t-1)^2$ ... (2)

So from Equations (1) and (2) distance travelled in the last second.

$h - h' = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2$ i.e., $h - h' = \frac{1}{2}g(2t-1)$

But according to given problem as $(h - h') = \frac{h}{2}$

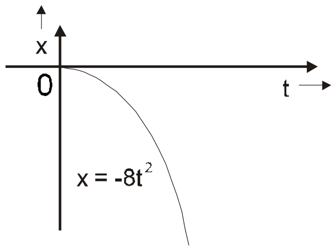
i.e., $\left(\frac{1}{2}\right)h = \left(\frac{1}{2}\right)g(2t-1)$ or $\left(\frac{1}{2}\right)gt^2 = g(2t-1)$ [as from equation (1) $h = \left(\frac{1}{2}\right)gt^2$]

or $t^2 - 4t + 2 = 0$ or $t = [4 \pm \sqrt{(4^2 - 4\times 2)}]/2$ or $t = 2 \pm \sqrt{2} \Rightarrow t = 0.59$ s or 3.41 s

0.59 s is physically unacceptable as it gives the total time t taken by the body to reach ground lesser than one sec while according to the given problem time of motion must be greater than 1s.

so t = 3.41s and $h = \frac{1}{2} \times (9.8) \times (3.41)^2 = 57$ m

Problem 10. The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



Upwards direction is taken as positive, downwards direction is taken as negative.

Solution : (a) The equation of motion is : $x = -8t^2$

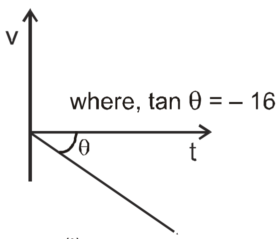
$\therefore\ v = \frac{dx}{dt} = -16\ t$; this shows that velocity is directly proportional to time and slope of velocity-time curve is negative i.e. – 16.

Hence, resulting graph is (i)

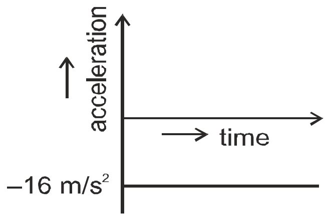
(b) Acceleration of particle is : $a = \frac{dv}{dt} = -16.$

This shows that acceleration is constant but negative.

Resulting graph is (ii)

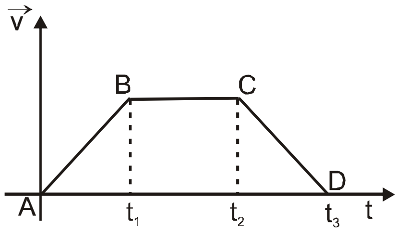


(i)



(ii)

Problem 11. Draw displacement–time and acceleration–time graph for the given velocity–time graph.

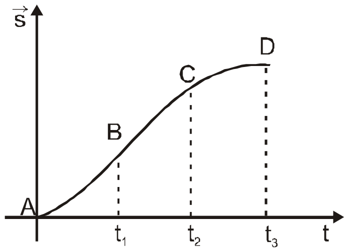


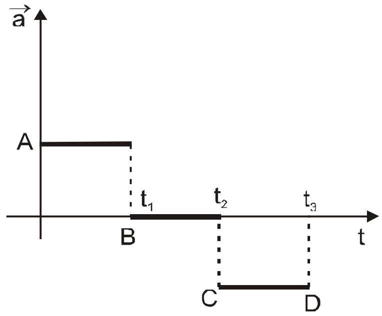
Solution : **Part AB :** v-t curve shows constant slope
i.e. constant acceleration or Velocity increases at constant rate with time.
Hence, s-t curve will show constant increase in slope
and a-t curve will be a straight line.

Part BC : v-t curve shows zero slope i.e. constant velocity. So, s-t curve will show constant slope and acceleration will be zero.

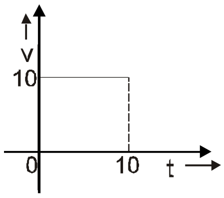
Part CD : v-t curve shows negative slope i.e. velocity is decreasing with time or acceleration is negative.
Hence , s-t curve will show decrease in slope becoming zero in the end.
and a-t curve will be a straight line with negative intercept on y-axis.

RESULTING GRAPHS ARE:

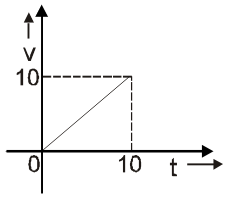




Problem 12. For a particle moving along x-axis, following graphs are given. Find the distance travelled by the particle in 10 s in each case?



(a)



(b)

Solution : **(a)** Distance area under the v - t curve

$\therefore$ distance = $10 \times 10 = 100$ m **Ans.**

(b) Area under v – t curve

$\therefore$ distance = $\frac{1}{2} \times 1 \times 10 = 50$ m **Ans.**

Problem 13. For a particle moving along x-axis, acceleration is given as $a = 2v^2$. If the speed of the particle is v_0 at x = 0, find speed as a function of x.

Solution : $a = 2v^2 \Rightarrow$ or $\frac{dv}{dt} = 2v^2$ or $\frac{dv}{dx} \times \frac{dx}{dt} = 2v^2$

$v\frac{dv}{dx} = 2v^2 \qquad \Rightarrow \qquad \frac{dv}{dx} = 2v$

$\int_{v_0}^{v} \frac{dv}{v} = \int_0^x 2\,dx \qquad \Rightarrow \qquad [\ell n v]_{v_0}^{v} = [2x]_0^x$

$\ell n \frac{v}{v_0} = 2x \qquad \Rightarrow \qquad v = v_0 e^{2x}$ **Ans.**

Problem 14. A balloon is rising with constant acceleration 2 m/sec^2. Two stones are released from the balloon at the interval of 2 sec. Find out the distance between the two stones 1 sec. after the release of second stone.

Solution : Acceleration of balloon = 2 m/sec^2

Let at t = 0, y = 0 when the first stone is released.

By the question, $y_1 = 0\, t_1 + \frac{1}{2} g t_1^2$

(taking vertical upward as – ve and downward as + ve)

$\therefore$ Position of Ist stone = $\frac{9}{2}g$

(1 second after release of second stone will be the 3rd second for the 1st stone)

For second stone $y_2 = ut_2 + \frac{1}{2}gt_2^2$

$u = 0 + at = -2 \times 2 = -4$m/s (taking vertical upward as – ve and downward as + ve)

$\therefore \quad y_2 = -4 \times 1 + \frac{1}{2} g \times (1)^2 \quad (t_2 = 1 \text{ second})$

The 2nd stone is released after 2 second

$\therefore \quad y = -\frac{1}{2}at^2 = -\frac{1}{2} \times 2 \times 4 = -4$

So, Position of second stone from the origin $= -4 + \frac{1}{2}g - 4$

Distance between two stones $= \frac{1}{2} g \times 9 - \frac{1}{2} g \times 1 + 8 = 48$ m.

Problem 15. A rocket is fired vertically up from the ground with a resultant vertical acceleration of 10m/s^2. The fuel is finished in 1 minute and it continues to move up.

(a) What is the maximum height reached?

(b) After finishing fuel, calculate the time for which it continues its upwards motion. (Take g = 10 m/s^2)

Solution: (a) The distance travelled by the rocket during burning interval (1minute= 60s) in which resultant acceleration 10 m/s^2 is vertically upwards will be $h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000$ m = 18 km and velocity acquired by it will be $v = 0 + 10 \times 60 = 600$ m/s

Now after 1 minute the rocket moves vertically up with initial velocity of 600 m/s and acceleration due to gravity opposes its motion. So, it will go to a height h_2 from this point, till its velocity becomes zero such that

$0 = (600)^2 - 2gh_2 \quad \Rightarrow \quad h_2 = 18000$ m = 18 km [$g = 10$ ms^{-2}]

So the maximum height reached by the rocket from the ground, $H = h_1 + h_2 = 18 + 18 = 36$ km

(b) As after burning of fuel the initial velocity 600m/s and gravity opposes the motion of rocket, so from 1st equation of motion time taken by it till it velocity v =0

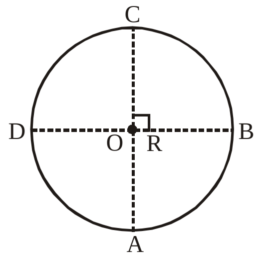
$0 = 600 - gt \quad \Rightarrow \quad t = 60$ s

Exercise # 1

PART-I : SUBJECTIVE QUESTIONS

SECTION (A) : DISTANCE AND DISPLACEMENT

A-1. A particles starts from point A with constant speed v on a circle of radius R. Find magnitude of displacement during its journey from :-

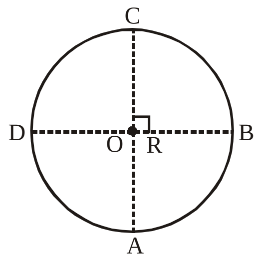


(a) A to B (b) A to C (c) A to D

A-2. A man moves to go 50 m due south, 40 m due west and 20 m due north to reach a field.
(a) What distance does he have to walk to reach the field ?
(b) What is his displacement from his house to the field?

SECTION (B) : AVERAGE SPEED AND AVERAGE VELOCITY

B-1. A particle starts from point A with constant speed v on a circle of radius R. Find magnitude of average velocity during its journey from :-



(a) A to B (b) A to C (c) A to D

B-2. A particle covers each $\frac{1}{3}$ of the total distance with speed v_1, v_2 and v_3 respectively. Find the average speed of the particle ?

SECTION (C) : VELOCITY, ACCELERATION, AVERAGE ACCELERATION

C-1. A particle moving with initial velocity of 5 m/s in one-dimension with constant acceleration of 10 m/s^2 is observed to cover a distance of 100 m during a 4s interval. The direction of velocity and acceleration are same. How far will the particle move in the next 4s?

C-2. The position of a body is given by $x = At + 4Bt^3$, where A and B are constants, x is position and t is time. Find (a) acceleration as a function of time, (b) velocity and acceleration at t = 5 s.

C-3. A boy start towards east with uniform speed 5m/s. After t = 2 second he turns right and travels 40 m with same speed. Again he turns right and travels for 8 second with same speed. Find out the displacement; average speed, average velocity and total distance travelled.

SECTION (D) : EQUATIONS OF MOTION AND MOTION UNDER GRAVITY

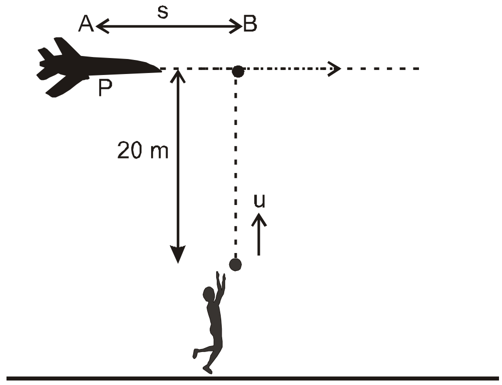
D-1. A car accelerates from 36 km/h to 90 km/h in 5 s on a straight rod. What was its acceleration in m/s^2 and how far did it travel in this time? Assume constant acceleration and direction of motion remains constant.

D-2. A particle moving along a straight line with constant acceleration is having initial and final velocity as 5 m/s and 15 m/s respectively in a time interval of 5 s. Find the distance travelled by the particle and the acceleration of the particle. If the particle continues with same acceleration, find the distance covered by the particle in the 8th second of its motion. (direction of motion remains same)

D-3. A ball is dropped from a tower. In the last second of its motion it travels a distance of 15 m. Find the height of the tower. [take g = $10 m/sec^2$]

D-4. A train starts from rest and moves with a constant acceleration of 2.0 m/s^2 for half a minute. The brakes are then applied and the train comes to rest in one minute after applying brakes. Find (a) the total distance moved by the train, (b) the maximum speed attained by the train and (c) the position(s) of the train at half the maximum speed. (Assume retardation to be constant)

D-5. A toy plane P starts flying from point A along a straight horizontal line 20 m above ground level starting with zero initial velocity and acceleration 2 m/s^2 as shown. At the same instant, a man P throws a ball vertically upwards with initial velocity 'u'. Ball touches (coming to rest) the base of the plane at point B of plane's journey when it is vertically above the man. 's' is the distance of point B from point A. Just after the contact of ball with the plane, acceleration of plane increases to 4 m/s^2. Find:

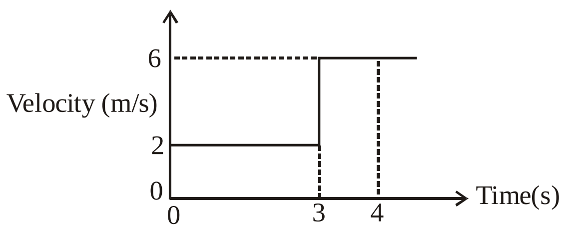


(i) Initial velocity 'u' of ball.
(ii) Distance 's'.
(iii) Distance between man and plane when the man catches the ball back. (g = 10 m/s^2)

D-6. A car travelling at 72 km/h deaccelerates uniformly at 2 m/s^2. Calculate (a) the distance it goes before it stops, (b) the time it takes to stop, and (c) the distance it travels during the first and third seconds.

SECTION (E) : GRAPH RELATED QUESTIONS

E-1. In the following graph variation with time (t), in velocity (v) of a particle moving rectilinearly is shown. What is average velocity in m/s of the particle in time interval from 0 s to 4 s?

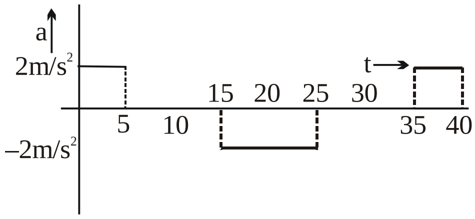


E-2 A tiger running 100 m race, accelerates for one third time of the total time and then moves with uniform speed. Then find the total time taken by the tiger to run 100 m if the acceleration of the tiger is $8 m/s^2$.

E-3. Two particles A and B start from rest and move for equal time on a straight line. The particle A has an acceleration 'a' for the first half of the total time and '2a' for the second half. The particle B has an acceleration '2a' for the first half and 'a' for the second half. Which particle has covered larger distance?

E-4. Figure shows a graph of acceleration of a particle moving on the x-axis. Plot the following graphs if the particle is at origin and at rest at t = 0.

(i) velocity-time graph (ii) displacement-time graph (iii) distance-time graph.



PART-II : OBJECTIVE QUESTIONS

SECTION (A) : DISTANCE AND DISPLACEMENT

A-1. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1m long and requires 1s. Determine how long the drunkard takes to fall in a pit 9m away from the start.

(A) 15 sec. (B) 21 sec. (C) 9 sec. (D) 16 sec

A-2. A hall has the dimensions 10 m × 10 m × 10 m. A fly starting at one corner ends up at a farthest corner. The magnitude of its displacement is:

(A) $5\sqrt{3}$ m (B) $10\sqrt{3}$ m (C) $20\sqrt{3}$ m (D) $30\sqrt{3}$ m

SECTION (B) : AVERAGE SPEED AND AVERAGE VELOCITY

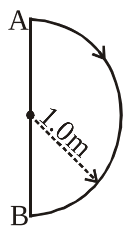
B-1. A body covers first $\frac{1}{3}$ part of its journey with a velocity of 2 m/s, next $\frac{1}{3}$ part with a velocity of 3 m/s and rest of the journey with a velocity 6m/s. The average velocity of the body will be

(A) 3 m/s (B) $\frac{11}{3}$ m/s (C) $\frac{8}{3}$ m/s (D) $\frac{4}{3}$ m/s

B-2. A person travelling on a straight line without changing direction moves with a uniform speed v_1 for half distance and next half distance he covers with uniform speed v_2 . The average speed v is given by

(A) $v = \frac{2v_1v_2}{v_1 + v_2}$ (B) $v = \sqrt{v_1 v_2}$ (C) $\frac{2}{v} = \frac{1}{v_1} - \frac{1}{v_2}$ (D) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$

B-3. In 1.0 sec. a particle goes from point A to point B moving in a semicircle of radius 1.0 m. The magnitude of average velocity is :



(A) 3.14 m/sec
(B) 2.0 m/sec
(C) 1.0 m/sec
(D) zero

B-4. An object is tossed vertically into the air with an initial velocity of 8 m/s. Using the sign convention upwards as positive, how does the vertical component of the acceleration a_y of the object (after leaving the hand) vary during the flight of the object? [assume air exerts a resistive force in the opposite direction of its motion]

(A) On the way up $a_y > 0$, on the way down $a_y > 0$

(B) On the way up $a_y < 0$, on the way down $a_y > 0$

(C) On the way up $a_y > 0$, on the way down $a_y < 0$

(D) On the way up $a_y < 0$, on the way down $a_y < 0$

SECTION (C) : VELOCITY, ACCELERATION AND AVERAGE ACCELERATION

C-1. If a body starts from rest and travels 120 cm in the 6th second, with constant acceleration then what is the acceleration :

(A) 0.20 m/s^2 (B) 0.027 m/s^2 (C) 0.218 m/s^2 (D) 0.03 m/s^2

C-2. A stone is thrown vertically upward with an initial speed u from the top of a tower, reaches the ground with a speed 3u. The height of the tower is:

(A) $\frac{3u^2}{g}$ (B) $\frac{4u^2}{g}$ (C) $\frac{6u^2}{g}$ (D) $\frac{9u^2}{g}$

C-3. The displacement of a body is given by $2s = gt^2$ where g is a constant. The velocity of the body at any time t is:

(A) gt (B) gt/2 (C) $gt^2/2$ (D) $gt^3/6$

C-4✍. A particle starts from rest with uniform acceleration a. Its velocity after n seconds is v. The displacement of the particle in the last two seconds is :

(A) $\frac{2v(n-1)}{n}$ (B) $\frac{v(n-1)}{n}$ (C) $\frac{v(n+1)}{n}$ (D) $\frac{2v(2n+1)}{n}$

SECTION (D) : EQUATIONS OF MOTION AND MOTION UNDER GRAVITY

D-1.✍ A particle performs rectilinear motion in such a way that its initial velocity has opposite direction with its uniform acceleration. Let x_A and x_B be the magnitude of displacements in the first 10 seconds and the next 10 seconds, then:

(A) $x_A < x_B$

(B) $x_A = x_B$

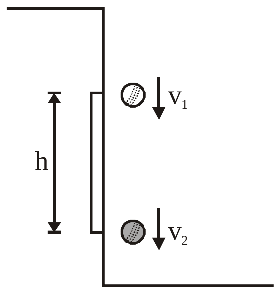
(C) $x_A > x_B$

(D) the information is insufficient to decide the relation of x_A with x_B.

D-2. A stone is released from an elevator going up with an acceleration a and speed u. The acceleration and speed of the stone just after the release is

(A) a upward, zero (B) (g-a) upward, u (C) (g-a) downward, zero (D) g downward, u

D-3. A ball dropped from the top of a building passes past a window of height h in time t. If its speeds at the top and the bottom edges of the window are denoted by v_1 and v_2 respectively, which of the following set of equations are correct?



(A) $v_2 - v_1 = gt$ and $(v_2 - v_1)t = h$
(B) $v_2 - v_1 = gt$ and $(v_2 + v_1)t = 2h$
(C) $v_2 + v_1 = gt$ and $(v_2 - v_1)t = h$
(D) None of the above.

D-4. A stone is dropped into a well in which the level of water is h below the top of the well. If v is velocity of sound, the time T after dropping the stone at which the splash is heard is given by

(A) T = 2h/v
(B) $T = \sqrt{\frac{2h}{g}} + \frac{h}{v}$
(C) $T = \sqrt{\frac{2h}{g}} + \frac{h}{2v}$
(D) $T = \sqrt{\frac{h}{2g}} + \frac{2h}{v}$

D-5. A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of :
(A) 3 s (B) 5 s (C) 7 s (D) 9 s

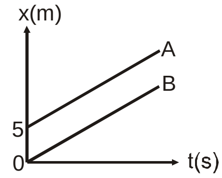
D-6. A student determined to test the law of gravity for himself walks off a sky scraper 320 m high with a stopwatch in hand and starts his free fall (zero initial velocity). 5 second later, superman arrives at the scene and dives off the roof to save the student. What must be superman's initial velocity in order that he catches the student just before reaching the ground ?
[Assume that the superman's acceleration is that of any freely falling body.] ($g = 10\ m/s^2$)

(A) 98 m/s (B) $\frac{275}{3}$ m/s (C) $\frac{187}{2}$ m/s (D) It is not possible

D-7. In the above question, what must be the maximum height of the skyscraper so that even superman cannot save him.
(A) 65 m (B) 85 m (C) 125 m (D) 145 m

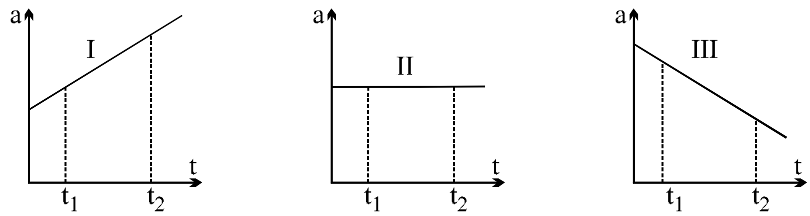
SECTION (E) : GRAPH RELATED QUESTIONS

E-1. Figure shows position-time graph of two cars A and B. Lines are parallel.



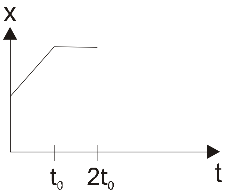
(A) Car A is faster than car B.
(B) Car A always leads Car B.
(C) Both cars are moving with same velocity.
(D) Both cars have positive acceleration.

E-2. Each of the three graphs represents acceleration versus time for an object that already has a positive velocity at time t_1. Which graphs show an object whose speed is increasing for the entire time interval between t_1 and t_2?



(A) graph I, only
(B) graphs I and II, only
(C) graphs I and III, only
(D) graphs I, II, and III

E-3. Figure shows the position time graph of a particle moving on the X-axis.

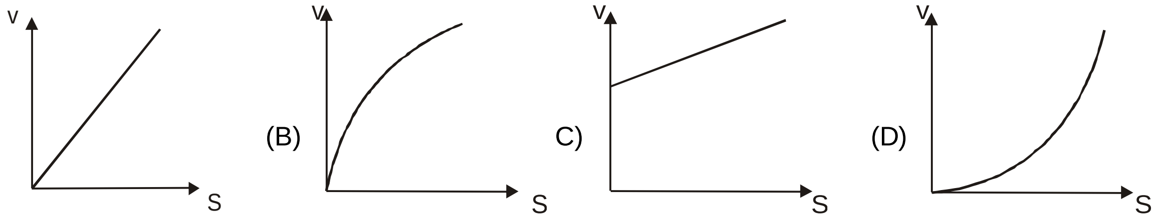


(A) the particle is continuously going in positive x direction
(B) area under x–t curve shows the displacement of particle
(C) the velocity increases up to a time t_o, and then becomes constant.
(D) the particle moves at a constant velocity up to a time t_o, and then stops.

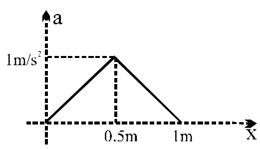
E-4. The displacement time graphs of two particles A and B are straight lines making angles of respectively 30^0 and 60^0 with the time axis. If the velocity of A is v_A and that of B is v_B, then the value of $\frac{v_A}{v_B}$ is

(A) $\frac{1}{2}$
(B) $\frac{1}{\sqrt{3}}$
(C) $\sqrt{3}$
(D) $\frac{1}{3}$

E-5. A particle starts from rest and moves along a straight line with constant acceleration. The variation of velocity v with displacement S is :



E-6 A body initially at rest, starts moving along x-axis in such a way so that its acceleration vs displacement plot is as shown in figure. The maximum velocity of particle is :-

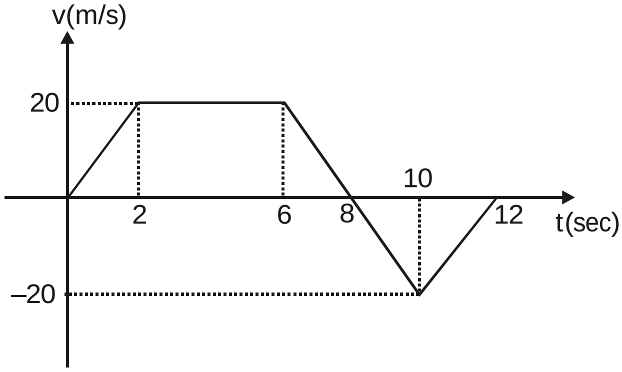


(A) 1 m/s
(B) 6 m/s
(C) 2 m/s
(D) none

PART - III : MATCH THE COLUMN

MM-1.

The velocity-time graph of an object moving along a straight line is given below.



	Column–I	**Column-II**		**Column-III**	
	Time interval	**Average velocity**		**Average acceleration**	
(I)	0 to 2 sec	(i)	10 m/s	(P)	zero
(II)	2 to 6 sec	(ii)	$-\frac{10}{3}$ m/s	(Q)	–2 m/s^2
(III)	0 to 10 sec	(iii)	15 m/s	(R)	$-\frac{10}{3}$ m/s^2
(IV)	6 to 12 sec	(iv)	20 m/s	(S)	–4 m/s^2

1. Which of the following combination is correctly matched :-

(A) (III) (iv) (S) (B) (III) (i) (R) (C) (III) (i) (S) (D) (III) (i) (Q)

2. Which of the following combination is correctly matched :-

(A) (II) (iv) (P) (B) (I) (i) (Q) (C) (II) (iv) (Q) (D) (I) (i) (S)

3. Which of the following combination is correctly matched :-

(A) (IV) (ii) (S) (B) (IV) (iii) (R) (C) (IV) (i) (S) (D) (IV) (ii) (R)

MM-2.

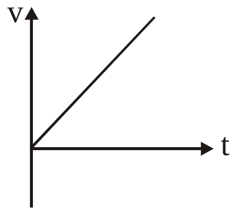
A balloon rises up with constant net acceleration of 10m/s^2. After 2 s a particle drops from the balloon. After further 2 s match the following : (g = 10 m/s^2)

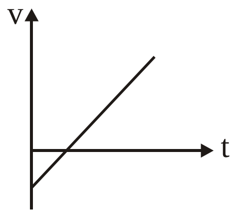
	Column-I		**Column-II**
(A)	Height of particle from ground	(P)	Zero
(B)	Speed of particle	(Q)	10 SI units
(C)	Displacement of Particle	(R)	40 SI units
(D)	Acceleration of particle	(S)	20 SI units

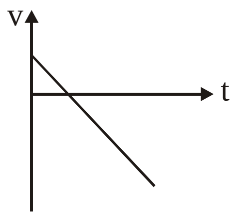
MM-3.

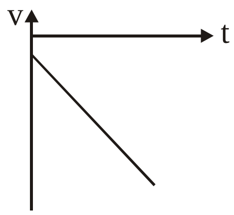
In the first column of the given table, some velocity-time (v-t) graphs and in the second column some position-time (x-t) graphs are shown. Suggest suitable match or matches.

Column - I

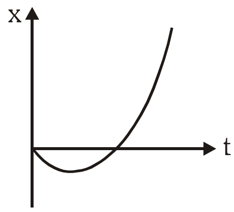
(A) 

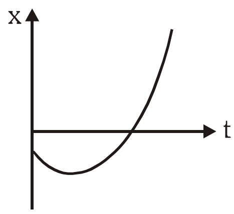
(B) 

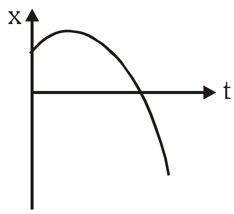
(C) 

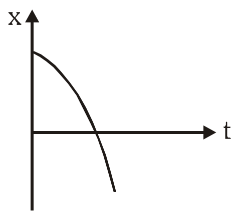
(D) 

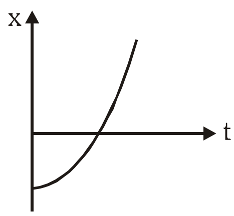
Column - II

(P) 

(Q) 

(R) 

(S) 

(T) 

Exercise # 2

PART - I : OBJECTIVE QUESTIONS

1. A particle goes from A to B with a speed of 40km/h and B to C with a speed of 60km/h. If AB = 6BC, the average speed in km/h between A and C is.

(A) 42 km/hr (B) 50 km/hr (C) 48 km/hr (D) 52 km/hr

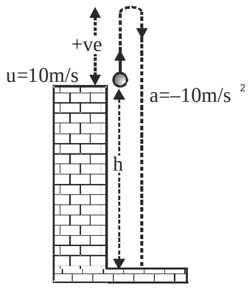
2. A point moves in a straight line so that its displacement is x m at time t sec, given by $x^2 = t^2 + 1$. Its acceleration in m/s^2 at time t sec is :

(A) $\frac{1}{x}$ (B) $\frac{1}{x} - \frac{1}{x^2}$ (C) $-\frac{t}{x^2}$ (D) $\frac{1}{x^3}$

3. Two balls of equal masses are thrown upward, along the same vertical line at an interval of 2 seconds, with the same initial velocity of 40 m/s. Then these collide at a height of (Take g = 10 m/s^2)

(A) 120 m (B) 75 m (C) 200 m (D) 45 m

4. A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s, find the time when it strikes the ground (g = 10 m/s^2)



(A) 6 (B) 2 (C) 4 (D) 8

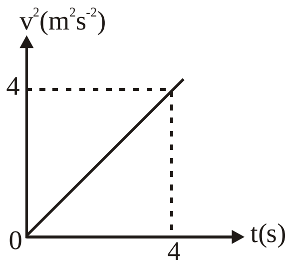
5. Water drops fall at regular intervals from a tap which is 5m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant ? (Take g = 10 ms^{-2})

(A) $\frac{5}{4}$ m (B) 4 m (C) $\frac{5}{2}$ m (D) $\frac{15}{4}$ m

6. A balloon is moving upwards with velocity 10 ms^{-1}. It releases a stone which comes down to the ground in 11 s. The height of the balloon from the ground at the moment when the stone was dropped is :

(A) 495 m (B) 592 m (C) 460 m (D) 500 m

7. A particle is moving along a straight line such that square of its velocity varies with time as shown in the figure. What is the acceleration of the particle at t = 4 s?



(A) 4 m/s^2 (B) 1/4 m/s^2 (C) 1/2 m/s^2 (D) 0

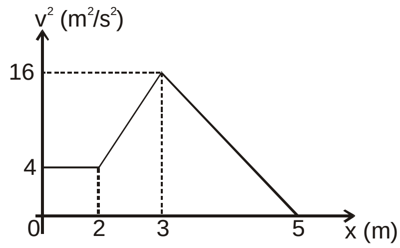
PART - II : SUBJECTIVE QUESTIONS

1. The velocity of a particle moving along the x-axis is given by equation $x = \sqrt{v} + 3$, where x is the position in meter and v is velocity (in m/s). The acceleration of particle at x = 4m is p m/s^2. Fill the value of 3p.

2. A particle moving in straight line, traversed half the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time and with velocity v_2 for the other half of the time. The mean velocity of the particle averaged over the whole time of motion comes out to be $av_0\left(\frac{v_1 + v_2}{b\,v_0 + v_1 + v_2}\right)$, where a and b are positive integers. Find a + b.

3. From the top of a tower, a ball is thrown vertically upwards. When the ball reaches h below the tower, its speed is double of what it was at height h above the tower. The greatest height attained by the ball from the tower is $x\frac{h}{3}$. Find x.

4. The graph shows the variation of v^2 v/s x. The magnitude of maximum acceleration is α m/s^2. Value of α is.



5. A lift is descending with uniform acceleration. To measure the acceleration , a person in the lift drops a coin at the moment when lift was descending with speed 6 ft/s. The coin is 5 ft above the floor of the lift at time it is dropped. The person observes that the coin strikes the floor in 1 second. Calculate from these data, the acceleration of the lift in ft/s^2. [Take g = 32 ft/s^2]

6. Two particle A and B are moving in same direction on same straight line. A is ahead of B by 20m. A has constant speed 5 m/sec and B has initial speed 30 m/sec and retardation of 10 m/sec^2. Then if x (in m) is total distance travelled by B as it meets A for second time. Then value of x will be.

7. A rocket is fired vertically upwards with initial velocity 40 m/s at the ground level. Due to fuel propulsion it is accelerated by 2 m/s^2 until it reaches an altitude of 1000 m. At that point the engines shut off and the rocket goes into free-fall. If the velocity (in m/s) just before it collides with the ground is 40α. Then fill the value of α. Neglect air resistance (g = 10m/s^2).

8. A particle is thrown upwards from ground. It experiences a constant air resistance which can produce a retardation of 2 m/s^2 opposite to the direction of velocity of particle. The ratio of time of ascent to the time of descent is $\sqrt{\frac{\alpha}{\beta}}$. Where α and β are integers. Find minimum value of $\alpha + \beta$. [g = 10 m/s^2]

9. A lift starts from the top of a mine shaft and descends with a constant speed of 10 m/s. 4 s later a boy throws a stone vertically upwards from the top of the shaft with a speed of 30 m/s. If stone hits the lift at a distance x below the shaft write the value of $\frac{x}{3}$ (in m) [Take : g = 10 m/s^1]

[Given value of $20\sqrt{6} = 49$]

10. The maximum possible acceleration of a train starting from the rest and moving on straight track is 10 m/s^2 and maximum possible retardation is 5 m/s^2. Find the minimum time in which the train can complete a journey of 1000m ending at rest, is n$\sqrt{\frac{2}{3}}$ sec. Where n is integer. Find n.

PART - III : ONE OR MORE THAN ONE CORRECT QUESTION

1. Mark the correct statements for a particle going on a straight line (x–position coordinate, v–velocity, a–acceleration):

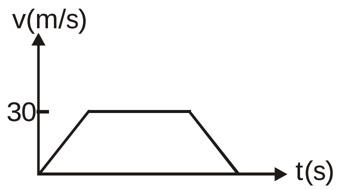
(A) If the v and a have opposite sign, the object is slowing down.

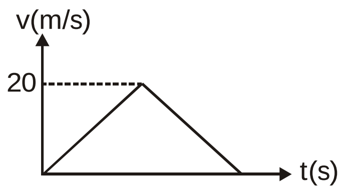
(B) If the x and v have opposite sign, the particle is moving towards the origin.

(C) If the v is zero at an instant, then a should also be zero at that instant.

(D) If the v is zero for a time interval, then a is zero at every instant within the time interval.

2. A car has to cover a distance of 400 m along a straight line starting from rest and stopping at the end. The maximum acceleration and deceleration the car can attain both have magnitude equal to 1 m/s^2, and the car can attain a maximum speed of 30 m/s. Car has to complete its journey in shortest time, then :-

(A) Velocity–time graph of car is 

(B) Velocity–time graph of car is 

(C) The time of journey of car is 20 s.

(D) The time of journey of car is 40 s.

3. A particle moving along a straight line with uniform acceleration has velocities 7m/s at A and 17 m/s at C. B is the mid point of AC. Then

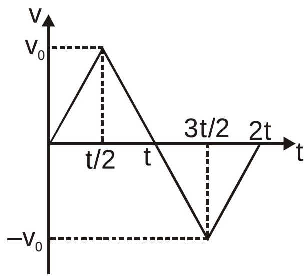
(A) The velocity at B is 12 m/s.

(B) The average velocity between A and B is 10 m/s.

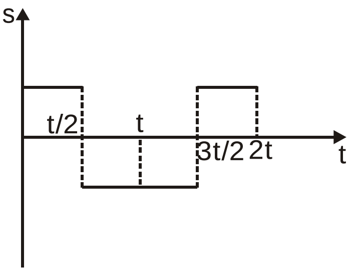
(C) The ratio of the time to go from A to B to that from B to C is 3 : 2.

(D) The average velocity between B and C is 15 m/s.

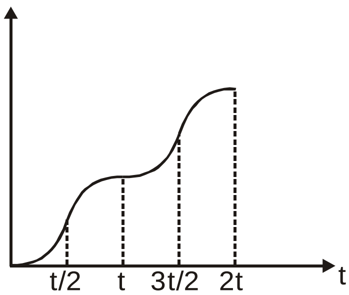
4. Velocity–time graph of a particle moving along x axis is given as shown. Which of the following will not be displacement–time graph for given particle :-



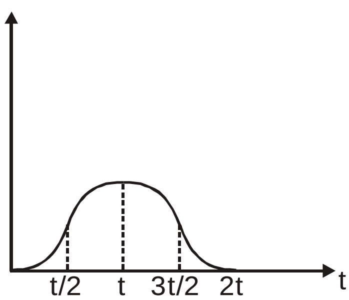
(A)



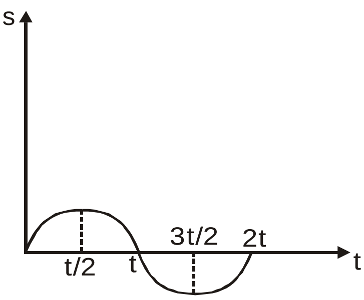
(B)



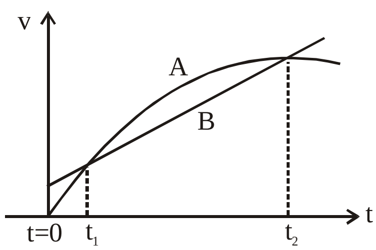
(C)



(D)



5. The graph given shows the velocity of two cars, A & B as a function of time. The cars move along the x-axis on parallel but separate tracks, so that they can pass each other's position without colliding. Initially x-coordination of cars are same. Mark the correct statement(s) :



(A) At time t_1 car A is overtaking car B

(B) Between $t = 0$ & $t = t_1$ distance travelled by car B is greater than distance travelled by car A.

(C) Between $t = t_1$ & $t = t_2$ distance travelled by car A is greater than distance travelled by car B.

(D) Car B is moving with constant velocity.

PART - IV : COMPREHENSION

Comprehension # 1

The position of a particle is given by

$x = 2 (t - t^2)$

where t is expressed in seconds and x is in meter. Positive direction is towards right.

1. The acceleration of the particle is

(A) 0 (B) 4 m/s^2 (C) -4 m/s^2 (D) None of these.

2. The maximum value of position co-ordinate of particle on positive x-axis is

(A) 1 m (B) 2 m (C) $\frac{1}{2}$ m (D) 4 m

3. The particle

(A) never goes to negative x-axis

(B) never goes to positive x-axis

(C) starts from the origin then goes upto x = 1/2 in the positive x-axis then goes to left

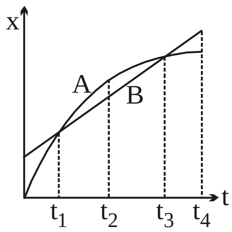
(D) final velocity of the particle is zero

4. The total distance travelled by the particle between t = 0 to t = 1 s is :

(A) 0 m (B) 1 m (C) 2 m (D) $\frac{1}{2}$ m

Comprehension # 2

The graph given shows the **POSITION** of two cars, A and B, as a function of time. The cars move along the x-axis on parallel but separate tracks, so that they can pass each other's position without colliding.



5. At which instant in time is car-A overtaking the car-B ?

(A) t_1 (B) t_2 (C) t_3 (D) t_4

6. At time t_3, which car is moving faster?

(A) car A (B) car B (C) same speed (D) None of these

7. At which instant do the two cars have the same velocity ?

(A) t_1 (B) t_2 (C) t_3 (D) t_4

8. Which one of the following best describes the motion of car A as shown on the graphs?

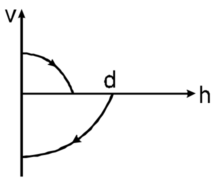
(A) speeding up (B) constant velocity

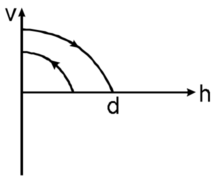
(C) slowing down (D) first speeding up, then slowing down

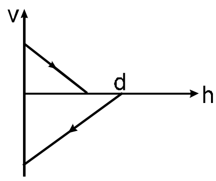
Exercise # 3

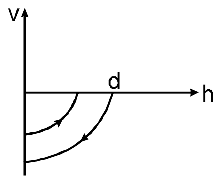
PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as **[JEE '2000, 1/35]**

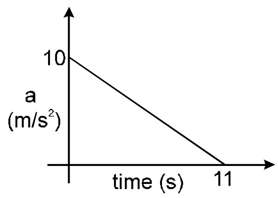
(A) 

(B) 

(C) 

(D) 

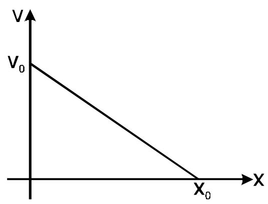
2. A particle is initially at rest, It is subjected to a linear acceleration a , as shown in the figure. The maximum speed attained by the particle is



[JEE (Scr.) 2004; 3 Marks]

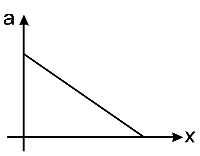
(A) 605 m/s (B) 110 m/s (C) 55 m/s (D) 550 m/s

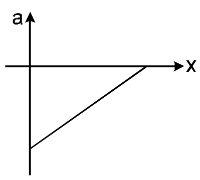
3. The velocity displacement graph of a particle moving along a straight line is shown.

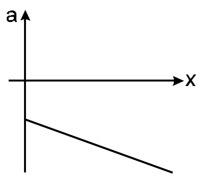


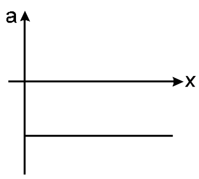
[JEE (Scr.) 2005; 3 Marks]

The most suitable acceleration-displacement graph will be

(A) 

(B) 

(C) 

(D) 

PART - II : JEE(MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. An object moving with a speed of 6.25 m/s, is decelerated at a rate given by : **[AIEEE - 2011; 4/120, –1]**

$$\frac{d\upsilon}{dt} = -2.5\sqrt{\upsilon}$$

where υ is the instantaneous speed. The time taken by the object, to come to rest, would be :

(1) 1 s (2) 2 s (3) 4 s (4) 8 s

2. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is a n times that taken by it to reach the highest point of its path.

The relation between H, u and n is : **[JEE (Main) 2014; 4/120, –1]**

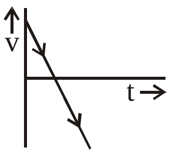
(1) $2gH = n^2u^2$

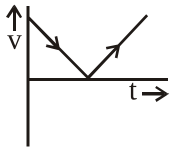
(2) $gH = (n-2)^2u^2$

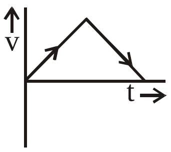
(3) $2gH = nu^2(n-2)$

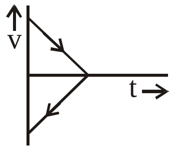
(4) $gH = (n-2)u^2$

3. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time? **[JEE (Main) 2017; 4/120, –1]**

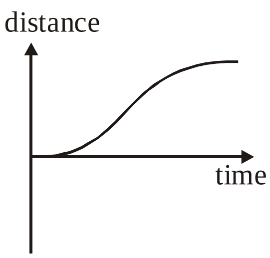
(1) 

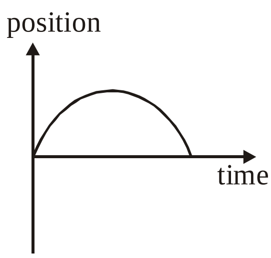
(2) 

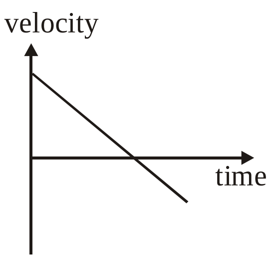
(3) 

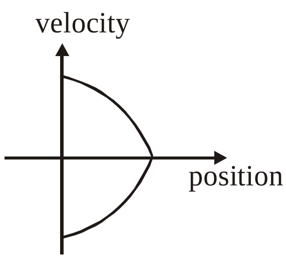
(4) 

4. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **[JEE (Main) 2018; 4/120, –1]**

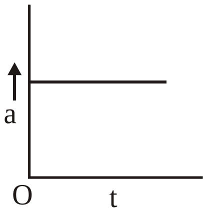
(1) 

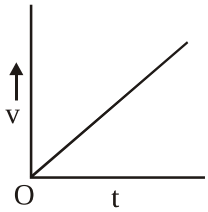
(2) 

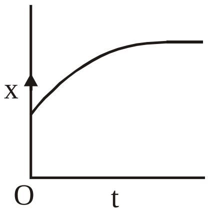
(3) 

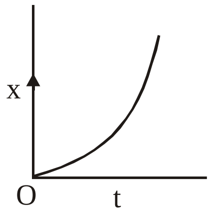
(4) 

5. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time) **[JEE (Main) 2019, January; 4/120, –1]**

(A)


(B)


(C)


(D)


(1) (A), (B), (C) (2) (A),(B) (3) (A), (B), (D) (4) (B), (C)

6. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time t = τ(assume that the particle is at origin at t = 0). **[JEE (Main) 2019, January; 4/120, –1]**

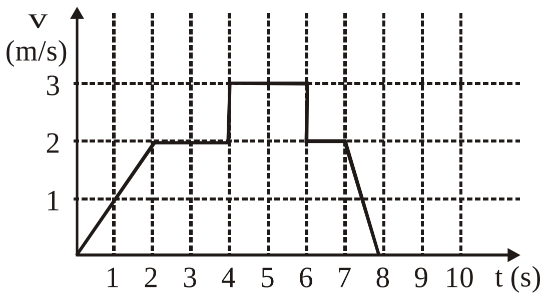
(1) $\frac{b^2\tau}{4}$ (2) $\frac{b^2\tau}{2}$ (3) $b^2 \tau$ (4) $\frac{b^2\tau}{\sqrt{2}}$

7. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to : **[JEE (Main) 2020, January; 4/100, –1]**

(1) $\frac{a_1 + a_2}{2}t$ (2) $\sqrt{2a_1a_2}\,t$ (3) $\frac{2a_1a_2}{a_1 + a_2}t$ (4) $\sqrt{a_1a_2}\,t$

8. A particle starts from the origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s ?

[JEE (Main) 2020, January; 4/100, –1]



(1) 6 m (2) 9 m (3) 3 m (4) 10 m

Answers

Exercise - 1

PART - I

SECTION (A) :

A-1. $(a)\sqrt{2}R,\ (b)2R,\ (c)\sqrt{2}R$

A-2. (a) 110 m (b) 50 m, $\tan^{-1}\frac{4}{3}$ west of south;

SECTION (B) :

B-1. $(a)2\sqrt{2}\frac{v}{\pi},(b)\frac{2v}{\pi},(c)\frac{2\sqrt{2}v}{3\pi}$

B-2. $\frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_1v_3}$

SECTION (C) :

C-1. 260 m

C-2. (a) 24 Bt ; (b) A + 300 B, 120 B

C-3. 50m at 53° S of W, 5m/s, 25/9 m/s at 53° S of W, 90 m]

SECTION (D) :

D-1. a = 3 m/s^2 ; $\frac{175}{2}$ = 87.5 m

D-2. 50m ; 2m/s^2 ; 20 m

D-3. 20m

D-4. (a) 2700 m = 2.7 km , (b) 60 m/s, (c) 225 m and 2.25 km

D-5. (i) 20 m/s (ii) 4 m (iii) $\sqrt{656}$ m.

D-6. (a) 100 m ; (b) 10 s ; (c) 19 m, 15 m

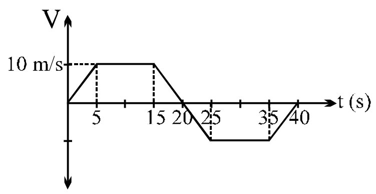
SECTION (E) :

E-1. 3 m/s

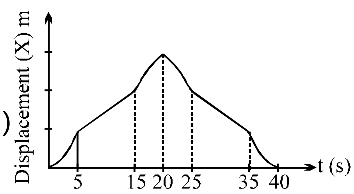
E-2. T = $3\sqrt{5}$ sec

E-3. Particle B

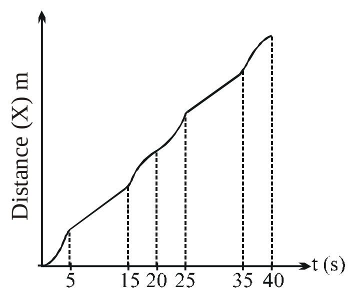
E-4. (i)



(ii)



(iii)



PART - II

SECTION (A) :

A-1. (B) **A-2.** (B)

SECTION (B) :

B-1. (A) **B-2. (A)** **B-3.** (B) **B-4.** (D)

SECTION (C) :

C-1. (C) **C-2.** (B) **C-3.** (A) **C-4.** (A)

SECTION (D) :

D-1. (D) **D-2.** (D) **D-3.** (B) **D-4.** (B)
D-5. (B) **D-6.** (B) **D-7.** (C)

SECTION (E) :

E-1. (B,C) **E-2.** (D) **E-3.** (D) **E-4.** (D)
E-5. (B) **E-6.** (A)

PART - III

1. (D) **2.** (A) **3.** (D)
4. (A) - (R); (B) - (P) ; (C) - (S) ; (D) - (Q) **5.** (A) $\rightarrow$ (T); (B) $\rightarrow$ (P,Q); (C) $\rightarrow$ (R); (D) $\rightarrow$ (S)

Exercise - 2

PART - I

1. (A) **2.** (D) **3.** (B) **4.** (C)
5. (D) **6.** (A) **7.** (B)

PART - II

1. 6 **2.** 4.00 **3.** (5) **4.** 6
5. 22 ft/s^2 **6.** 50 **7.** 4 **8.** 30
9. 43 **10.** 30

PART - III

1. (A,B,D) **2.** (B,D) **3.** (B,C,D) **4.** (A,B,D)
5. (B,C)

PART - IV

1. (C) **2.** (C) **3.** (C) **4.** (B)
5. (A) **6.** (B) **7.** (B) **8.** (C)

Exercise - 3

PART - I

1. (A) **2.** (C) **3.** (B)

PART - II

1. (2) **2.** (3) **3.** (1) **4.** (1)
5. (3) **6.** (2) **7.** (4) **8.** (2)

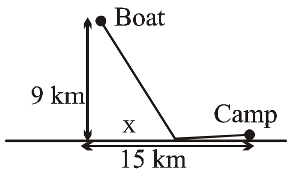
Ranker Problems

SUBJECTIVE QUESTIONS

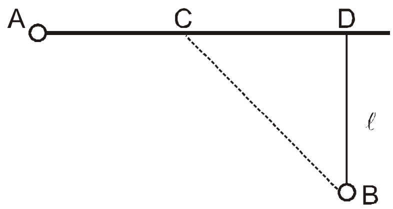
1. A fishing boat is anchored 9 km away from the nearest point on shore. A messenger must be sent from the fishing boat to a camp, 15 km from the point on shore closest to the boat. If the messenger can walk at a speed of 5km per hour and can row at 4 km per hour.

(i) Form an expression relating time taken to reach the camp t with distance x on shore where he lands.

(ii) At what point on shore must he land in order to reach the camp in the shortest possible time?



2. From point A located on a highway as shwon in figure, one has to get by car as soon as possible to point B located in the field at a distance ℓ from the highway. It is known that the car moves in the field η times slower than on the highway. At what distance from point D one must turn off the highway?



3. On a 2–lane road, car A is travelling with a speed of 36 km/h. Two cars B and C approach car A in opposite directions with a speed of 54 km/h each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?

4. A motorboat going downstream overcame a raft (A wooden block) at a point A; τ = 60 min later it turned back and after some time passed the raft at a distance ℓ = 6.0 km from the point A. Find the flow velocity assuming the duty of the engine to be constant.

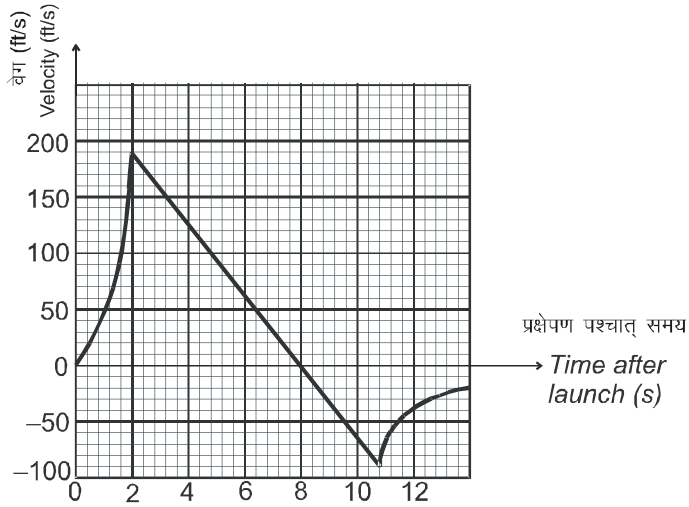
5. A police jeep is chasing a culprit going on a moter bike. The motor bike crosses a turn at a speed of 72 km/h. The jeep follows it at a speed of 108 km/h, crossing the turn 10 seconds later than bike (keeping constant speed). After crossing the turn, jeep acclerates with constant accleration 2 m/s^2. Assuming bike travels at constant speed, how far from the turn will the jeep catch the bike?

6. The velocity of a particle moving in the positive direction of the x axis varies as $v = \alpha\sqrt{x}$ where α is a positive constant. Assuming that at the moment t = 0 the particle was located at the point x = 0, find:

(a) the time dependence of the velocity and the acceleration of the particle;

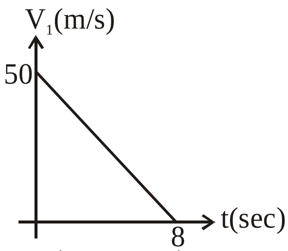
(b) the mean velocity of the particle averaged over the time that the particle takes to cover the first s metres of the path.

7. When a model rocket is launched, the propellant burns for a few seconds, accelerating the rocket upward. After burnout, the rocket moves upward for a while and then begins to fall. A parachute opens shortly after the rocket starts down. The parachute slows the rocket to keep it from breaking when it lands. The figure here shows velocity data from the flight of the model rocket. Use the data to answer the following.

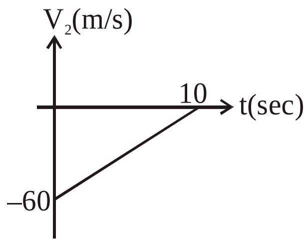


(a) How fast was the rocket climbing when the engine stopped?
(b) For how many seconds did the engine burn?
(c) When did the rocket reach its highest point? What was its velocity then?
(d) When did the parachute open up ? How fast was the rocket falling then?
(e) How long did the rocket fall before the parachute opened?
(f) When was the rocket's acceleration greatest?
(g) When was the acceleration constant? What was its value then (to the nearest integer)?

8. Two trains are moving in opposite direction on same track. When their separation was 600 m their drivers notice the mistake and start slowing down to avoid collision. Graphs of their velocities as function of time is as shown. If separation between the drivers when first train stops is 'x' m then find the value of $\frac{x}{16}$.



(First train)



(Second train)

9. A helicopter takes off along the vertical with an acceleration of 3 m/sec^2 & zero initial velocity. In a certain time, the pilot switches off the engine. At the point of takeoff, the sound dies away in 30 sec. Determine the velocity of the helicopter at the moment when its engine is switched off, assuming the velocity of sound is 320 m/sec.

10. A particle of mass 10^{-2} kg is moving along the positive x–axis under the influence of a force

$F(x) = -\frac{K}{2x^2}$ where $K = 10^{-2}$ N m^2. At time t = 0 it is at x = 1.0 m and its velocity is v = 0. Find

(i) its velocity when it reaches x = 0.50 m
(ii) the time at which it reaches x = 0.25 m.

Answers

Ranker Problems

1. (i) $t = \frac{\sqrt{x^2 + (9)^2}}{4} + \frac{15 - x}{5}$ (ii) 3 km from the camp.

2. $CD = \frac{\ell}{\sqrt{\eta^2 - 1}}$

3. 1 m/sec^2

4. $v = l/2\tau$ = 3.0 km per hour

5. 400 m

6. (a) $v = \alpha^2 t / 2$, $a = \alpha^2 / 2$; (b) $\langle v \rangle = \frac{\alpha\sqrt{s}}{2}$

7. (a) 190 ft/s (b) 2 s

(c) 8 s, 0 ft/s (d) $\frac{54}{5}$ = 10.8 s, 90 ft/s

(e) $\frac{14}{5}$ = 2.8 s (f) greatest acceleration happens 2 s after launch

(g) constant acceleration between 2 and $\frac{54}{5}$ = 10.8 s, – 32 ft/s^2.

8. 7

9. 80 m/sec

10. (i) $\vec{V} = -1\,\hat{i}$ m/s (ii) $t = \frac{\pi}{3} + \frac{\sqrt{3}}{4}$ s.

Self Assessment Test

JEE (ADVANCED) PAPER - 1

SECTION-1 : ONE OPTION CORRECT (Maximum Marks - 12)

1. Two objects moving along the same straight line are leaving point A with an acceleration a, 2a & velocity 2u, u respectively at time t = 0. The distance moved by the object with respect to point A when one object overtakes the other is :

(A) $\frac{6u^2}{a}$ (B) $\frac{2u^2}{a}$ (C) $\frac{4u^2}{a}$ (D) none of these

2. A particle is moving along x axis with constant acceleration. At t = 0 , the particle is at x = 3 m and $\frac{dx}{dt} = +4$ m/s. The maximum value of x co-ordinate of the particle is observed 2 seconds later. Starting from t = 0 sec after what time particle reaches its initial position again?

(A) 4 sec. (B) 6 sec. (C) 8 sec. (D) 12 sec.

3. A particle is projected from ground in vertical direction at t = 0. At t = 0.8 sec, it reaches h = 14m. It will again come to same height at t = [g = 10 m/s²]

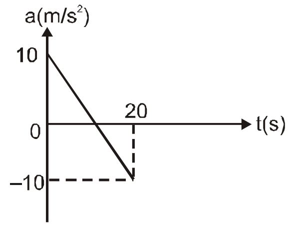
(A) 2 sec. (B) $\frac{14}{5}$ sec. (C) 3 sec. (D) $\frac{7}{2}$ sec.

4. A particle is projected with velocity v_0 along x–axis. The deceleration on the particle is proportional to the square of the distance from the origin i.e., $a = -\alpha x^2$. The distance at which the particle stops is:–

(A) $\sqrt{\frac{3v_0}{2\alpha}}$ (B) $\left(\frac{3v_0}{2\alpha}\right)^{\frac{1}{3}}$ (C) $\sqrt{\frac{3v_0^2}{2\alpha}}$ (D) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$

SECTION-2 : ONE OR MORE THAN ONE CORRECT (Maximum Marks - 32)

5. The acceleration time plot for a particle (starting from rest) moving on a straight line is shown in figure. For given time interval,



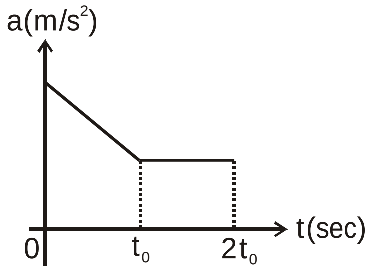
(A) The particle has zero average acceleration

(B) The particle has never turned around.

(C) The particle has zero displacement

(D) The average speed in the interval 0 to 10s is the same as the average speed in the interval 10s to 20s.

6. The acceleration-time graph of a particle moving along the x-axis is given below. Initial velocity of particle is along the positive x-axis. Mark the **CORRECT** statement(s) :



(A) Velocity of particle is decreasing in time interval t = 0 to t = t_0.

(B) Velocity of particle is increasing in time interval t = 0 to t = t_0.

(C) Velocity of particle is constant in time interval t = t_0 to t = $2t_0$.

(D) Velocity of particle is increasing in time interval t = t_0 to t = $2t_0$.

7. A particle moves along the X–axis as $x = u(t-2) + a(t-2)^2$ (t is in sec.)

(A) The initial velocity of the particle is u

(B) The acceleration of the particle is a

(C) The acceleration of the particle is 2a

(D) At t = 2s particle is at the origin.

8. A body travelling along a straight line with a uniform acceleration has velocities 5 m/s at a point A and 15 m/s at a point B respectively. If M is the mid point of AB, then :

(A) The ratio of times taken by the body to cover distance MB and AM is $\left[\frac{\sqrt{5}-1}{2}\right]$

(B) The velocity at M is $5\sqrt{5}$ m/s

(C) Average velocity over AM is $\frac{5\left(\sqrt{5}+1\right)}{2}$ m/s

(D) The product of the acceleration and the distance AB is 100 m^2/s^2.

9. Which of the following statement(s) is/are **CORRECT** :

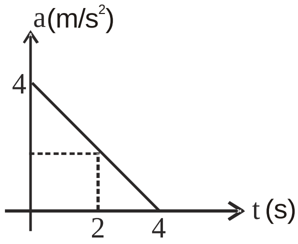
(A) The velocity-time graph does not give any information about initial position.

(B) The acceleration-time graph never gives information about the change in velocity.

(C) From the velocity-time graph of a moving particle we can get the average velocity and average acceleration in a time interval.

(D) The area under the curve of the acceleration (a) versus position (x) graph with position (x) axis gives $\frac{v_2^2 - v_1^2}{2}$. Where v_1 and v_2 are the velocities of the particle at initial and final positions respectively.

10. Acceleration time graph of particle is as shown in figure. Initial velocity of particle is 8m/s.



(A) maximum velocity of particle is 16 m/s

(B) displacement of particle from t = 2 sec to t = 4 sec is 92/3 m

(C) particle changes its direction of motion at t = 4 sec

(D) velocity time graph is a parabola.

11. A particle is moving along X-axis whose position is given by $x = 2 - 4t + \frac{t^3}{3}$. Mark the correct statement(s) in relation to its motion :

(A) Direction of motion is not changing at any of the instants

(B) Direction of motion is changing at t = 2s

(C) For 0 < t < 2s, the particle is slowing down

(D) For 0 < t < 2s, the particle is speeding up

12. The displacement of a moving particle is proportional to the square of the time. for this particle :

(A) the velocity is constant

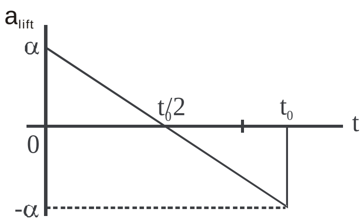
(B) the velocity is variable

(C) the acceleration is constant

(D) the acceleration is variable

SECTION-3 : NUMERICAL VALUE TYPE (Maximum Marks - 18)

13. A person stands in an elevator. Starting at rest at t = 0 the elevator moves upward, coming to rest again at time $t = t_0$. The acceleration of the elevator during this period is shown graphically below. The total distance traveled by the elevator is s_0. The value of $\frac{\alpha t_0^2}{s_0}$ is.



14. A body starts from the origin and moves along the X-axis such that the velocity at any instant is given by $(4t^3 - 2t)$, where t is in second and velocity is in m/s. What is acceleration of the particle, when it is at distance 2m from the origin.

15. A healthy youngman standing at a distance of 6 m from a 11.5 m high building sees a kid slipping from the top floor. With what uniform acceleration (starting from rest) should he run to catch the kid at the arms height (1.5 m)? Take g = 10 m/s^2.

16. A body is thrown vertically up from the top of a tower. When the body descends through a distance 3m below the top of the tower, its velocity is double what it was at the same height above the top of the tower. The maximum height (above the top) reached by the body is (in m)

17. A body starts from rest and moves for 'n' seconds with uniform acceleration 'a', its velocity after n seconds is 8 m/s. The displacement of the body in last 3 seconds is 15 m. Find a (in m/s^2).

18. Between two stations a train accelerates uniformally at first, then moves with a constant speed and finally retards uniformlly. If the ratio of the time taken are 1 : 8 : 1 and the greatest speed attained by the train is 60 kmh^{-1}, if the average speed, in ms^{-1}, over the whole journey is 5α then find the value of α.

Answers

SAP (SELF ASSESSMENT PAPER)

1.	(A)	**2.**	(A)	**3.**	(D)	**4.**	(D)	**5.**	(A,B,D)
6.	(B,D)	**7.**	(C, D)	**8.**	(A, B,C,D)	**9.**	(A,C,D)	**10.**	(A,B,D)
11.	(B, C)	**12.**	(B,C)	**13.**	06.00	**14.**	22.00	**15.**	06.00
16.	05.00	**17.**	02.00	**18.**	03.00				