27. Hyperbola

Exercise 27.1

1. Question

The equation of the directrix of a hyperbola is x – y + 3 = 0. Its focus is (-1, 1) and eccentricity 3. Find the equation of the hyperbola.

Answer

Given: Equation of directrix of a hyperbola is x – y + 3 = 0. Focus of hyperbola is (-1, 1) and eccentricity (e) = 3

To find: equation of the hyperbola

Let M be the point on directrix and P(x, y) be any point of the hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is an eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x+1)^2 + (y-1)^2} = 3\left|\frac{(x-y+3)}{\sqrt{1^2 + (-1)^2}}\right|$$

$$\Rightarrow \sqrt{(x+1)^2 + (y-1)^2} = 3\left|\frac{(x-y+3)}{\sqrt{1+1}}\right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x+1)^2 + (y-1)^2}\right)^2 = \left(3\left|\frac{(x-y+3)}{\sqrt{1+1}}\right|\right)^2$$

$$\Rightarrow (x+1)^2 + (y-1)^2 = \frac{3^2(x-y+3)^2}{2}$$

{∵ $(a - b)^2 = a^2 + b^2 + 2ab$ &

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$}

$\Rightarrow 2\{x^2 + 1 + 2x + y^2 + 1 - 2y\} = 9\{x^2 + y^2 + 9 + 6x - 6y - 2xy\}$

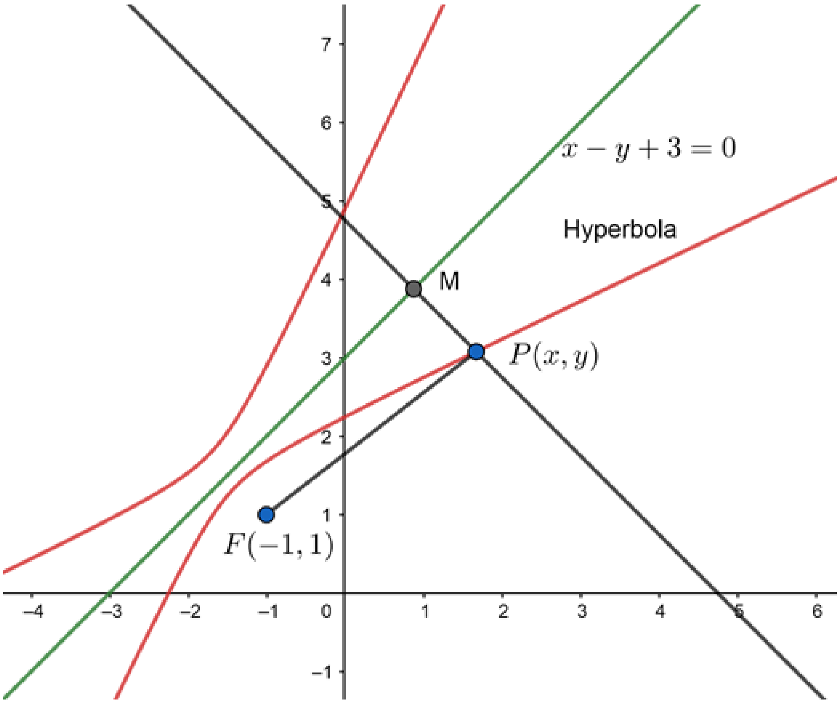
$\Rightarrow 2x^2 + 2 + 4x + 2y^2 + 2 - 4y = 9x^2 + 9y^2 + 81 + 54x - 54y - 18xy$

$\Rightarrow 2x^2 + 4 + 4x + 2y^2 - 4y - 9x^2 - 9y^2 - 81 - 54x + 54y + 18xy = 0$

$\Rightarrow -7x^2 - 7y^2 - 50x + 50y + 18xy - 77 = 0$

$\Rightarrow 7x^2 + 7y^2 + 50x - 50y - 18xy + 77 = 0$

This is the required equation of hyperbola



2 A. Question

Find the equation of the hyperbola whose

focus is (0, 3), directrix is x + y – 1 = 0 and eccentricity = 2

Answer

Given: Equation of directrix of a hyperbola is x + y – 1 = 0. Focus of hyperbola is (0, 3) and eccentricity (e) = 2

To find: equation of the hyperbola

Let M be the point on directrix and P(x, y) be any point of the hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is an eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-0)^2 + (y-3)^2} = 2\left|\frac{(x+y-1)}{\sqrt{1^2+1^2}}\right|$$

$$\Rightarrow \sqrt{(x-0)^2 + (y-3)^2} = 2\left|\frac{(x+y-1)}{\sqrt{1+1}}\right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x-0)^2 + (y-3)^2}\right)^2 = \left(2\left|\frac{(x+y-1)}{\sqrt{1+1}}\right|\right)^2$$

$$\Rightarrow (x-0)^2 + (y-3)^2 = \frac{2^2(x+y-1)^2}{2}$$

{∵ $(a - b)^2 = a^2 + b^2 + 2ab$ &

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$}

$\Rightarrow 2\{x^2 + y^2 + 9 - 6y\} = 4\{x^2 + y^2 + 1 - 2x - 2y + 2xy\}$

$\Rightarrow 2x^2 + 2y^2 + 18 - 12y = 4x^2 + 4y^2 + 4 - 8x - 8y + 8xy$

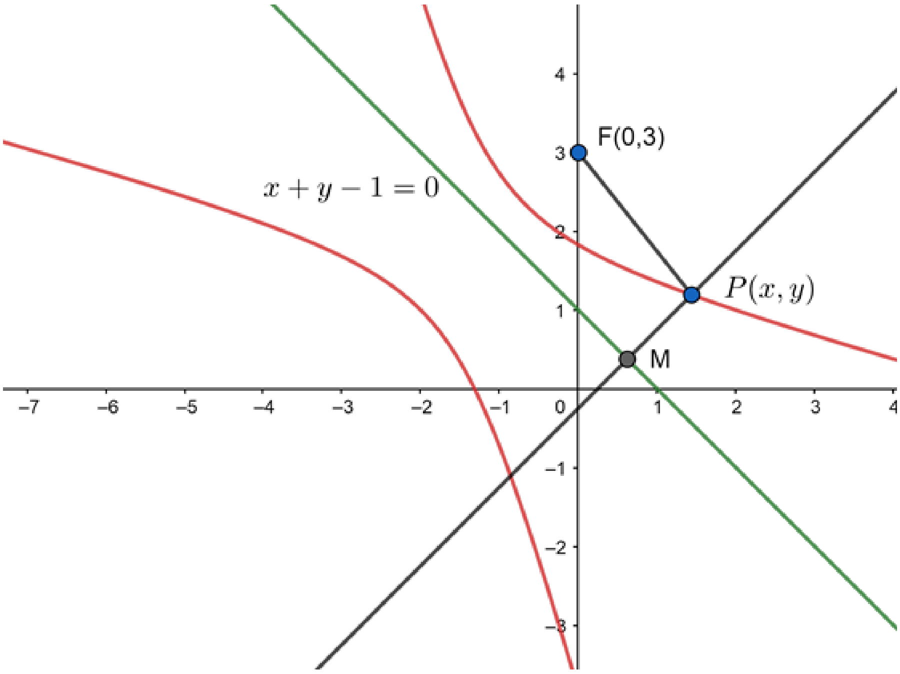
$\Rightarrow 2x^2 + 2y^2 + 18 - 12y - 4x^2 - 4y^2 - 4 - 8x + 8y - 8xy = 0$

$\Rightarrow - 2x^2 - 2y^2 - 8x - 4y - 8xy + 14 = 0$

$\Rightarrow -2(x^2 + y^2 - 4x + 2y + 4xy - 7) = 0$

$\Rightarrow \mathbf{x^2 + y^2 - 4x + 2y + 4xy - 7 = 0}$

This is the required equation of hyperbola



2 B. Question

Find the equation of the hyperbola whose

focus is (1, 1), directrix is 3x + 4y + 8 = 0 and eccentricity = 2

Answer

Given: Equation of directrix of a hyperbola is 3x + 4y + 8 = 0. Focus of hyperbola is (1, 1) and eccentricity (e) = 2

To find: equation of hyperbola

Let M be the point on directrix and P(x, y) be any point of hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-1)^2 + (y-1)^2} = 2\left|\frac{(3x + 4y + 8)}{\sqrt{3^2 + 4^2}}\right|$$

$$\Rightarrow \sqrt{(x-1)^2 + (y-1)^2} = 2\left|\frac{(3x + 4y + 8)}{\sqrt{9 + 16}}\right|$$

Squaring both sides:

$\Rightarrow \left(\sqrt{(x-1)^2+(y-1)^2}\right)^2 = \left(2\left|\frac{(3x+4y+8)}{\sqrt{25}}\right|\right)^2$

$\Rightarrow (x-1)^2+(y-1)^2 = \frac{2^2(3x+4y+8)^2}{25}$

{∵ $(a - b)^2 = a^2 + b^2 + 2ab$ &

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$}

$\Rightarrow 25\{x^2 + 1 - 2x + y^2 + 1 - 2y\} = 4\{9x^2 + 16y^2 + 64 + 24xy + 64y + 48x\}$

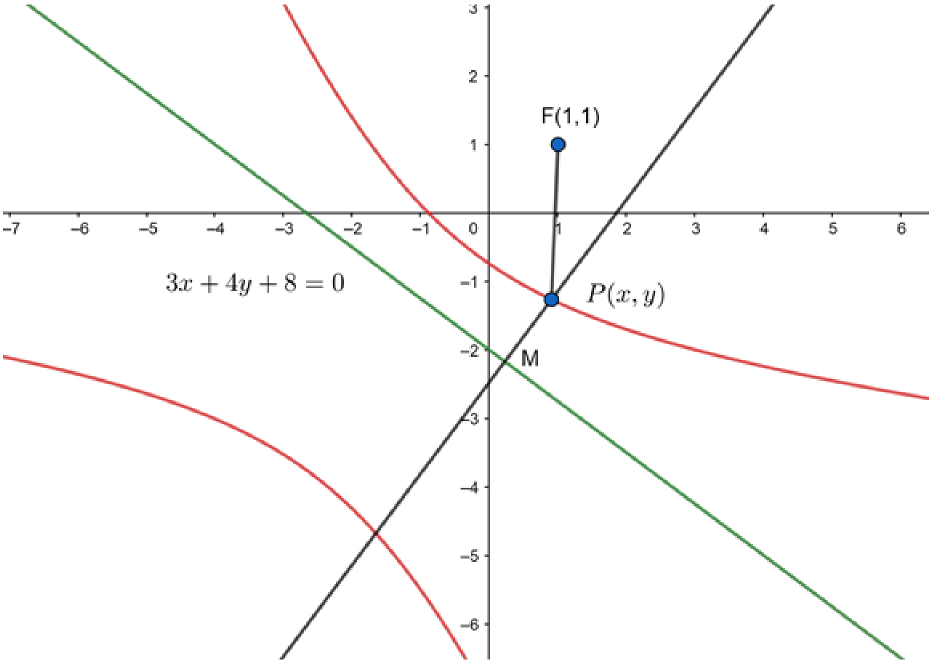
$\Rightarrow 25x^2 + 25 - 50x + 25y^2 + 25 - 50y = 36x^2 + 64y^2 + 256 + 96xy + 256y + 192x$

$\Rightarrow 25x^2 + 25 - 50x + 25y^2 + 25 - 50y - 36x^2 - 64y^2 - 256 - 96xy - 256y - 192x = 0$

$\Rightarrow - 11x^2 - 39y^2 - 242x - 306y - 96xy - 206 = 0$

$\Rightarrow \mathbf{11x^2 + 39y^2 + 242x + 306y + 96xy + 206 = 0}$

This is the required equation of hyperbola



2 C. Question

Find the equation of the hyperbola whose

focus is (1, 1) directrix is 2x + y = 1 and eccentricity = $\sqrt{3}$

Answer

Given: Equation of directrix of a hyperbola is 2x + y – 1 = 0. Focus of hyperbola is (1, 1) and eccentricity (e) = $\sqrt{3}$

To find: equation of hyperbola

Let M be the point on directrix and P(x, y) be any point of hyperbola

Formula used:

$e = \frac{PF}{PM} \Rightarrow PF = ePM$

where e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-1)^2+(y-1)^2} = \sqrt{3}\left|\frac{(2x+y-1)}{\sqrt{2^2+1^2}}\right|$$

$$\Rightarrow \sqrt{(x-1)^2+(y-1)^2} = \sqrt{3}\left|\frac{(2x+y-1)}{\sqrt{4+1}}\right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x-1)^2+(y-1)^2}\right)^2 = \left(\sqrt{3}\left|\frac{(2x+y-1)}{\sqrt{5}}\right|\right)^2$$

$$\Rightarrow (x-1)^2+(y-1)^2 = \frac{3(2x+y-1)^2}{5}$$

{∵ $(a - b)^2 = a^2 + b^2 + 2ab$ &

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$}

$\Rightarrow 5\{x^2 + 1 - 2x + y^2 + 1 - 2y\} = 3\{4x^2 + y^2 + 1 + 4xy - 2y - 4x\}$

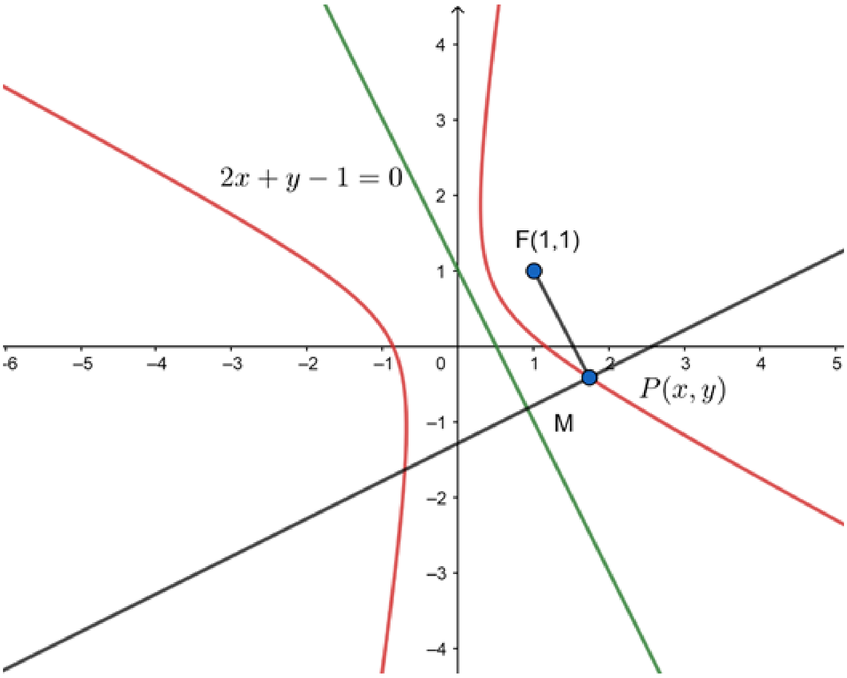
$\Rightarrow 5x^2 + 5 - 10x + 5y^2 + 5 - 10y = 12x^2 + 3y^2 + 3 + 12xy - 6y - 12x$

$\Rightarrow 5x^2 + 5 - 10x + 5y^2 + 5 - 10y - 12x^2 - 3y^2 - 3 - 12xy + 6y + 12x = 0$

$\Rightarrow -7x^2 + 2y^2 + 2x - 4y - 12xy + 7 = 0$

$\mathbf{\Rightarrow 7x^2 - 2y^2 - 2x + 4y + 12xy - 7 = 0}$

This is the required equation of hyperbola.



2 D. Question

Find the equation of the hyperbola whose

focus is (2, -1), directrix is 2x + 3y = 1 and eccentricity = 2

Answer

Given: Equation of directrix of a hyperbola is 2x + 3y – 1 = 0. Focus of hyperbola is (2, -1) and eccentricity (e) = 2

To find: equation of hyperbola

Let M be the point on directrix and P(x, y) be any point of hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-2)^2 + (y+1)^2} = 2\left|\frac{(2x+3y-1)}{\sqrt{2^2+3^2}}\right|$$

$$\Rightarrow \sqrt{(x-2)^2 + (y+1)^2} = 2\left|\frac{(2x+3y-1)}{\sqrt{4+9}}\right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x-2)^2 + (y+1)^2}\right)^2 = \left(2\left|\frac{(2x+3y-1)}{\sqrt{13}}\right|\right)^2$$

$$\Rightarrow (x-2)^2 + (y+1)^2 = \frac{4(2x+3y-1)^2}{13}$$

{∵ $(a - b)^2 = a^2 + b^2 + 2ab$ &

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$}

$\Rightarrow 13\{x^2 + 4 - 4x + y^2 + 1 + 2y\} = 4\{4x^2 + 9y^2 + 1 + 12xy - 6y - 4x\}$

$\Rightarrow 13x^2 + 52 - 52x + 13y^2 + 13 + 26y = 16x^2 + 36y^2 + 4 + 48xy - 24y - 16x$

$\Rightarrow 13x^2 + 52 - 52x + 13y^2 + 13 + 26y - 16x^2 - 36y^2 - 4 - 48xy + 24y + 16x = 0$

$\Rightarrow - 3x^2 - 23y^2 - 36x + 50y - 48xy + 61 = 0$

$\Rightarrow$ **$3x^2 + 23y^2 + 36x - 50y + 48xy - 61 = 0$**

This is the required equation of hyperbola.

2 E. Question

Find the equation of the hyperbola whose

focus is (a, 0), directrix is 2x + 3y = 1 and eccentricity = 2

Answer

Given: Equation of directrix of a hyperbola is 2x – y + a = 0. Focus of hyperbola is (a, 0) and eccentricity (e) $= \frac{4}{3}$

To find: equation of hyperbola

Let M be the point on directrix and P(x, y) be any point of hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-a)^2 + (y-0)^2} = \frac{4}{3}\left|\frac{(2x-y+a)}{\sqrt{2^2+(-1)^2}}\right|$$

$$\Rightarrow \sqrt{(x-a)^2+(y)^2} = \frac{4}{3}\left|\frac{(2x-y+a)}{\sqrt{4+1}}\right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x-a)^2+(y)^2}\right)^2 = \left(\frac{4}{3}\left|\frac{(2x-y+a)}{\sqrt{5}}\right|\right)^2$$

$$\Rightarrow (x-a)^2+(y)^2 = \frac{16(2x-y+a)^2}{9\times 13}$$

{∵ $(a - b)^2 = a^2 + b^2 + 2ab$ &

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$}

$\Rightarrow 117\{x^2 + a^2 - 2ax + y^2\} = 16\{4x^2 + y^2 + a^2 - 4xy - 2ay + 4ax\}$

$\Rightarrow 117x^2 + 117a^2 - 234ax + 117y^2 = 64x^2 + 16y^2 + 16a^2 - 64xy - 32ay + 64ax$

$\Rightarrow 117x^2 + 117a^2 - 234ax + 117y^2 - 64x^2 - 16y^2 - 16a^2 + 64xy + 32ay - 64ax = 0$

$\Rightarrow$ **$53x^2 + 101y^2 - 298ax + 32ay + 64xy + 111a^2 = 0$**

This is the required equation of hyperbola.

2 F. Question

Find the equation of the hyperbola whose

focus is (2, 2), directrix is x + y = 9 and eccentricity = 2

Answer

Given: Equation of directrix of a hyperbola is x + y – 9 = 0. Focus of hyperbola is (2, 2) and eccentricity (e) = 2

To find: equation of hyperbola

Let M be the point on directrix and P(x, y) be any point of hyperbola

Formula used:

$$e = \frac{PF}{PM} \Rightarrow PF = ePM$$

where e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$$\sqrt{(x-2)^2+(y-2)^2} = 2\left|\frac{(x+y-9)}{\sqrt{1^2+1^2}}\right|$$

$$\Rightarrow \sqrt{(x-2)^2+(y-2)^2} = 2\left|\frac{(x+y-9)}{\sqrt{1+1}}\right|$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x-2)^2+(y-2)^2}\right)^2 = \left(2\left|\frac{(x+y-9)}{\sqrt{2}}\right|\right)^2$$

$$\Rightarrow (x-2)^2+(y-2)^2 = \frac{4(x+y-9)^2}{2}$$

{∵ $(a - b)^2 = a^2 + b^2 + 2ab$ &

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$}

$\Rightarrow x^2 + 4 - 4x + y^2 + 4 - 4y = 2\{x^2 + y^2 + 81 + 2xy - 18y - 18x\}$

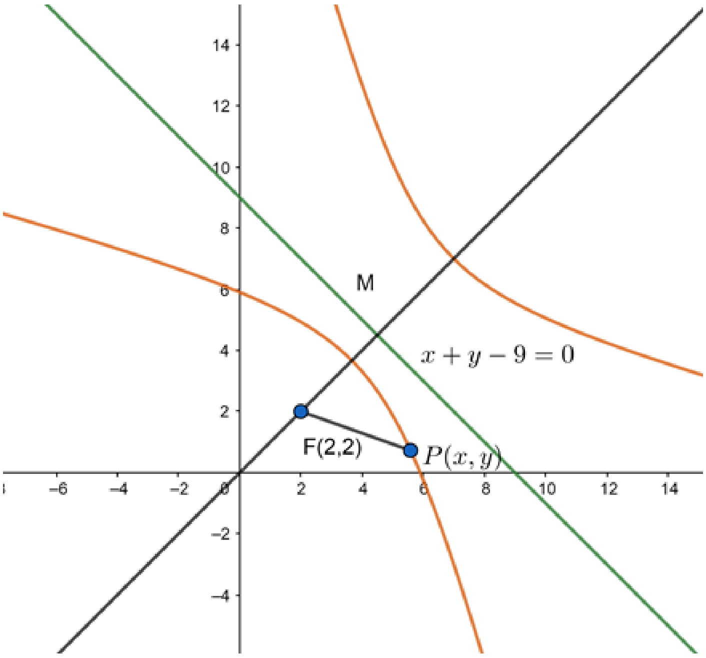
$\Rightarrow x^2 - 4x + y^2 + 8 - 4y = 2x^2 + 2y^2 + 162 + 4xy - 36y - 36x$

$\Rightarrow x^2 - 4x + y^2 + 8 - 4y - 2x^2 - 2y^2 - 162 - 4xy + 36y + 36x = 0$

$\Rightarrow -x^2 - y^2 + 32x + 32y + 4xy - 154 = 0$

$\Rightarrow \mathbf{x^2 + y^2 - 32x - 32y + 4xy + 154 = 0}$

This is the required equation of hyperbola.



3 A. Question

Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola.

$9x^2 - 16y^2 = 144$

Answer

Given: $9x^2 - 16y^2 = 144$

To find: eccentricity(e), coordinates of the foci f(m,n), equation of directrix, length of latus-rectum of hyperbola.

$9x^2 - 16y^2 = 144$

$$\Rightarrow \frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{where } c = \sqrt{a^2 + b^2}$$

Foci is given by (±ae, 0)

Equation of directrix are: $x = \pm\frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, a = 4 and b = 3

$c = \sqrt{4^2 + 3^2}$

$\Rightarrow c = \sqrt{16 + 9}$

$\Rightarrow c = \sqrt{25}$

⇒ c = 5

Therefore,

$\mathbf{e = \frac{5}{4}}$

$\Rightarrow ae = 4 \times \frac{5}{4} = 5$

Foci: (±5, 0)

Equation of directrix are:

$x = \pm\frac{a}{e}$

$\Rightarrow x = \pm\frac{4}{\frac{5}{4}}$

$\Rightarrow x = \pm\frac{16}{5}$

$\Rightarrow 5x = \pm 16$

$\Rightarrow \mathbf{5x \mp 16 = 0}$

Length of latus rectum,

$= \frac{2b^2}{a}$

$= \frac{2 \times (3)^2}{4}$

$= \frac{9}{2}$

3 B. Question

Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola.

$16x^2 – 9y^2 = -144$

Answer

Given: $16x^2 – 9y^2 = -144$

To find: eccentricity(e), coordinates of the foci f(m,n), equation of directrix, length of latus-rectum of hyperbola.

$$\frac{9y^2}{144} - \frac{16x^2}{144} = 1$$

$$\Rightarrow \frac{y^2}{16} - \frac{x^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{4^2} = -1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$:

Eccentricity(e) is given by,

$$e = \frac{c}{b}, \text{where } c = \sqrt{a^2 + b^2}$$

Foci is given by (0, ±be)

The equation of directrix are: $y = \pm\frac{b}{e}$

Length of latus rectum is $\frac{2a^2}{b}$

Here, a = 3 and b = 4

$$c = \sqrt{4^2 + 3^2}$$

$$\Rightarrow c = \sqrt{16 + 9}$$

$$\Rightarrow c = \sqrt{25}$$

⇒ c = 5

Therefore,

$$\mathbf{e = \frac{5}{4}}$$

$$\Rightarrow be = 4 \times \frac{5}{4} = 5$$

Foci: (0, ±5)

The equation of directrix are:

$$y = \pm\frac{b}{e}$$

$$\Rightarrow y = \pm\frac{4}{\frac{5}{4}}$$

$$\Rightarrow y = \pm\frac{16}{5}$$

$$\Rightarrow 5y = \pm 16$$

$$\Rightarrow \mathbf{5y \mp 16 = 0}$$

Length of latus rectum,

$$= \frac{2a^2}{b}$$

$$= \frac{2 \times (3)^2}{4}$$

$$= \frac{9}{2}$$

3 C. Question

Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola.

$4x^2 - 3y^2 = 36$

Answer

Given: $4x^2 - 3y^2 = 36$

To find: eccentricity(e), coordinates of the foci f(m,n), equation of directrix, length of latus-rectum of hyperbola.

$$\frac{4x^2}{36} - \frac{3y^2}{36} = 1$$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{12} = 1$$

$$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{(\sqrt{12})^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{where } c = \sqrt{a^2 + b^2}$$

Foci are given by (±ae, 0)

The equation of directrix are $x = \pm\frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, a = 3 and b = $\sqrt{12}$

$$c = \sqrt{3^2 + \left(\sqrt{12}\right)^2}$$

$$\Rightarrow c = \sqrt{9 + 12}$$

$$\Rightarrow c = \sqrt{21}$$

Therefore,

$$\mathbf{e = \frac{\sqrt{21}}{3}}$$

$$\Rightarrow ae = 3 \times \frac{\sqrt{21}}{3} = \sqrt{21}$$

Foci: $(\pm\sqrt{21}, 0)$

The equation of directrix are:

$x = \pm\frac{a}{e}$

$\Rightarrow x = \pm\frac{3}{\frac{\sqrt{21}}{3}}$

$\Rightarrow x = \pm\frac{9}{\sqrt{21}}$

$\Rightarrow \sqrt{21}x = \pm 9$

$\Rightarrow \boldsymbol{\sqrt{21}x \mp 9 = 0}$

Length of latus rectum,

$= \frac{2b^2}{a}$

$= \frac{2 \times (\sqrt{12})^2}{3}$

$= \frac{2 \times 12}{3}$

= 8

3 D. Question

Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola.

$3x^2 - y^2 = 4$

Answer

Given: $3x^2 - y^2 = 4$

To find: eccentricity(e), coordinates of the foci f(m,n), equation of directrix, length of latus-rectum of hyperbola.

$\frac{3x^2}{4} - \frac{y^2}{4} = 1$

$\Rightarrow \frac{x^2}{\frac{4}{3}} - \frac{y^2}{4} = 1$

$\Rightarrow \frac{x^2}{\left(\frac{2}{\sqrt{3}}\right)^2} - \frac{y^2}{(2)^2} = 1$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$e = \frac{c}{a}$, where $c = \sqrt{a^2 + b^2}$

Foci are given by (±ae, 0)

The equation of directrix are $x = \pm\frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, a = $\frac{2}{\sqrt{3}}$ and b = 2

$$c = \sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 + (2)^2}$$

$$\Rightarrow c = \sqrt{\frac{4}{3} + 4}$$

$$\Rightarrow c = \sqrt{\frac{4 + 12}{3}}$$

$$\Rightarrow c = \sqrt{\frac{16}{3}}$$

$$\Rightarrow c = \frac{4}{\sqrt{3}}$$

Therefore,

$$\mathbf{e} = \frac{\frac{4}{\sqrt{3}}}{\frac{2}{\sqrt{3}}}$$

$$\Rightarrow \mathbf{e = 2}$$

$$\Rightarrow ae = \frac{2}{\sqrt{3}} \times 2 = \frac{4}{\sqrt{3}}$$

$$\text{Foci: } \left(\pm\frac{4}{\sqrt{3}}, 0\right)$$

The equation of directrix are:

$$x = \pm\frac{a}{e}$$

$$\Rightarrow x = \pm\frac{\frac{2}{\sqrt{3}}}{2}$$

$$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}x = \pm 1$$

$$\Rightarrow \mathbf{\sqrt{3}x \mp 1 = 0}$$

Length of latus rectum,

$$= \frac{2(2)^2}{\frac{2}{\sqrt{3}}}$$

$$= 4\sqrt{3}$$

3 E. Question

Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola.

$2x^2 - 3y^2 = 5$

Answer

Given: $2x^2 - 3y^2 = 5$

To find: eccentricity(e), coordinates of the foci f(m,n), equation of directrix, length of latus-rectum of hyperbola.

$$\frac{2x^2}{5} - \frac{3y^2}{5} = 1$$

$$\Rightarrow \frac{x^2}{\frac{5}{2}} - \frac{y^2}{\frac{5}{3}} = 1$$

$$\Rightarrow \frac{x^2}{\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^2} - \frac{y^2}{\left(\frac{\sqrt{5}}{\sqrt{3}}\right)^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{where } c = \sqrt{a^2 + b^2}$$

Foci are given by (±ae, 0)

The equation of directrix are $x = \pm\frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, a = $\frac{\sqrt{5}}{\sqrt{2}}$ and b = $\frac{\sqrt{5}}{\sqrt{3}}$

$$c = \sqrt{\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{3}}\right)^2}$$

$$\Rightarrow c = \sqrt{\frac{5}{2} + \frac{5}{3}}$$

$$\Rightarrow c = \sqrt{\frac{15 + 10}{6}}$$

$$\Rightarrow c = \sqrt{\frac{25}{6}}$$

$$\Rightarrow c = \frac{5}{\sqrt{6}}$$

Therefore,

$$\mathbf{e} = \frac{\frac{5}{\sqrt{6}}}{\frac{\sqrt{5}}{\sqrt{2}}}$$

$$\Rightarrow \mathbf{e} = \frac{\sqrt{\mathbf{5}}}{\sqrt{\mathbf{3}}}$$

$$\Rightarrow \mathrm{ae} = \frac{\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{5}}{\sqrt{3}} = \frac{5}{\sqrt{6}}$$

$$\textbf{Foci}: \left(\pm\frac{\mathbf{5}}{\sqrt{\mathbf{6}}}, \mathbf{0}\right)$$

The equation of directrix are:

$$x = \pm\frac{a}{e}$$

$$\Rightarrow x = \pm\frac{\frac{\sqrt{5}}{\sqrt{2}}}{\frac{\sqrt{5}}{\sqrt{3}}}$$

$$\Rightarrow x = \pm\frac{1}{\sqrt{6}}$$

$$\Rightarrow \sqrt{6}x = \pm 1$$

$$\Rightarrow \sqrt{\mathbf{6}}\mathbf{x} \mp \mathbf{1} = \mathbf{0}$$

Length of latus rectum,

$$= \frac{2b^2}{a}$$

$$= \frac{2\left(\frac{\sqrt{5}}{\sqrt{3}}\right)^2}{\frac{\sqrt{5}}{\sqrt{2}}}$$

$$= \frac{2 \times \frac{5}{3}}{\frac{\sqrt{5}}{\sqrt{2}}}$$

$$= \frac{2\sqrt{10}}{3}$$

4. Question

Find the axes, eccentricity, latus-rectum and the coordinates of the foci of the hyperbola $25x^2 - 36y^2 = 225$.

Answer

Given: $2x^2 - 3y^2 = 5$

To find: eccentricity(e), coordinates of the foci f(m,n), equation of directrix, length of latus-rectum of hyperbola.

$$\frac{25x^2}{225} - \frac{36y^2}{225} = 1$$

$$\Rightarrow \frac{x^2}{\left(\frac{15}{5}\right)^2} - \frac{y^2}{\left(\frac{15}{6}\right)^2} = 1$$

$$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{where } c = \sqrt{a^2 + b^2}$$

Foci are given by (±ae, 0)

The equation of directrix are $x = \pm\frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, a = 3 and b = $\frac{5}{2}$

$$c = \sqrt{(3)^2 + \left(\frac{5}{2}\right)^2}$$

$$\Rightarrow c = \sqrt{9 + \frac{25}{4}}$$

$$\Rightarrow c = \sqrt{\frac{36 + 25}{4}}$$

$$\Rightarrow c = \sqrt{\frac{61}{2}}$$

$$\Rightarrow c = \frac{\sqrt{61}}{2}$$

Therefore,

$$\mathbf{e} = \frac{\frac{\sqrt{61}}{2}}{3}$$

$$\Rightarrow \mathbf{e} = \frac{\sqrt{61}}{6}$$

$$\Rightarrow ae = 3 \times \frac{\sqrt{61}}{6} = \frac{\sqrt{61}}{2}$$

$$\textbf{Foci}: \left(\pm\frac{\sqrt{61}}{2}, \mathbf{0}\right)$$

The equation of directrix are:

$x = \pm\frac{a}{e}$

$\Rightarrow x = \pm\frac{3}{\frac{\sqrt{61}}{6}}$

$\Rightarrow x = \pm\frac{18}{\sqrt{61}}$

$\Rightarrow \sqrt{61}x = \pm 18$

$\Rightarrow \mathbf{\sqrt{61}x \mp 18 = 0}$

Length of latus rectum,

$= \frac{2b^2}{a}$

$= \frac{2\left(\frac{5}{2}\right)^2}{3}$

$= \frac{2 \times \frac{25}{4}}{3}$

$= \frac{25}{6}$

5 A. Question

Find the centre, eccentricity, foci and directions of the hyperbola

$16x^2 - 9y^2 + 32x + 36y - 164 = 0$

Answer

Given: $16x^2 - 9y^2 + 32x + 36y - 164 = 0$

To find: center, eccentricity(e), coordinates of the foci f(m,n), equation of directrix.

$16x^2 - 9y^2 + 32x + 36y - 164 = 0$

$\Rightarrow 16x^2 + 32x + 16 - 9y^2 + 36y - 36 - 16 + 36 - 164 = 0$

$\Rightarrow 16(x^2 + 2x + 1) - 9(y^2 - 4y + 4) - 16 + 36 - 164 = 0$

$\Rightarrow 16(x^2 + 2x + 1) - 9(y^2 - 4y + 4) - 144 = 0$

$\Rightarrow 16(x + 1)^2 - 9(y - 2)^2 = 144$

$\Rightarrow \frac{16(x+1)^2}{144} - \frac{9(y-2)^2}{144} = 1$

$\Rightarrow \frac{(x+1)^2}{9} - \frac{(y-2)^2}{16} = 1$

$\Rightarrow \frac{(x+1)^2}{3^2} - \frac{(y-2)^2}{4^2} = 1$

Here, **center of the hyperbola is (-1, 2)**

Let x + 1 = X and y – 2 = Y

$\Rightarrow \frac{X^2}{3^2} - \frac{Y^2}{4^2} = 1$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$e = \frac{c}{a}$, where $c = \sqrt{a^2 + b^2}$

Foci are given by (±ae, 0)

The equation of directrix are $x = \pm\frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, a = 3 and b = 4

$c = \sqrt{(3)^2 + (4)^2}$

$\Rightarrow c = \sqrt{9 + 16}$

$\Rightarrow c = \sqrt{25}$

$\Rightarrow c = 5$

Therefore,

$\mathbf{e = \frac{5}{3}}$

$\Rightarrow ae = 3 \times \frac{5}{3} = 5$

$\mathbf{Foci: (\pm ae, 0)}$

⇒ X = ±5 and Y = 0

⇒ x + 1 = ±5 and y – 2 = 0

⇒ x = ±5 – 1 and y = 2

So, Foci: (±5 – 1, 2)

Equation of directrix are:

$X = \pm\frac{a}{e}$

$\Rightarrow X = \pm\frac{3}{\frac{5}{3}}$

$\Rightarrow X = \pm\frac{9}{5}$

$\Rightarrow 5X = \pm 9$

$\Rightarrow 5X \mp 9 = 0$

$\Rightarrow 5(x + 1) \mp 9 = 0$

$\Rightarrow 5x + 5 \mp 9 = 0$

$\Rightarrow 5x + 5 - 9 = 0$ and $5x + 5 + 9 = 0$

$\Rightarrow \mathbf{5x - 4 = 0}$ and $\mathbf{5x + 14 = 0}$

5 B. Question

Find the centre, eccentricity, foci and directions of the hyperbola

$x^2 - y^2 + 4x = 0$

Answer

Given: $x^2 - y^2 + 4x = 0$

To find: center, eccentricity(e), coordinates of the foci f(m,n), equation of directrix.

$x^2 - y^2 + 4x = 0$

$\Rightarrow x^2 + 4x + 4 - y^2 - 4 = 0$

$\Rightarrow (x + 2)^2 - y^2 = 4$

$$\Rightarrow \frac{(x+2)^2}{4} - \frac{y^2}{4} = 1$$

$$\Rightarrow \frac{(x+2)^2}{2^2} - \frac{y^2}{2^2} = 1$$

Here, **center of the hyperbola is (2, 0)**

Let x – 2 = X

$$\Rightarrow \frac{X^2}{2^2} - \frac{y^2}{2^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{where } c = \sqrt{a^2 + b^2}$$

Foci are given by (±ae, 0)

The equation of directrix are $x = \pm\frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, a = 2 and b = 2

$$c = \sqrt{(2)^2 + (2)^2}$$

$$\Rightarrow c = \sqrt{4 + 4}$$

$$\Rightarrow c = \sqrt{8}$$

$$\Rightarrow c = 2\sqrt{2}$$

Therefore,

$$e = \frac{2\sqrt{2}}{2}$$

$$\Rightarrow e = \sqrt{2}$$

$\Rightarrow ae = 2 \times \sqrt{2} = 2\sqrt{2}$

Foci: $(\pm \mathbf{ae}, \mathbf{0})$

$\Rightarrow X = \pm 2\sqrt{2}$ and y = 0

$\Rightarrow x + 2 = \pm 2\sqrt{2}$ and y = 0

$\Rightarrow x = \pm 2\sqrt{2} - 2$ and y = 0

So, Foci: $(\pm 2\sqrt{2} - 2, 0)$

Equation of directrix are:

$$X = \pm \frac{a}{e}$$

$$\Rightarrow X = \pm \frac{2}{\sqrt{2}}$$

$$\Rightarrow X = \pm \frac{2}{\sqrt{2}}$$

$$\Rightarrow X = \pm \sqrt{2}$$

$$\Rightarrow X \mp \sqrt{2} = 0$$

$$\Rightarrow x + 2 \mp \sqrt{2} = 0$$

$\Rightarrow \mathbf{x + 2 - \sqrt{2} = 0}$ and $\mathbf{x + 2 + \sqrt{2} = 0}$

5 C. Question

Find the centre, eccentricity, foci and directions of the hyperbola

$x^2 - 3y^2 - 2x = 8$

Answer

Given: $x^2 - 3y^2 - 2x = 8$

To find: center, eccentricity(e), coordinates of the foci f(m,n), equation of directrix.

$x^2 - 3y^2 - 2x = 8$

$\Rightarrow x^2 - 2x + 1 - 3y^2 - 1 = 8$

$\Rightarrow (x - 1)^2 - 3y^2 = 9$

$$\Rightarrow \frac{(x-1)^2}{9} - \frac{3y^2}{9} = 1$$

$$\Rightarrow \frac{(x-1)^2}{3^2} - \frac{y^2}{(\sqrt{3})^2} = 1$$

Here, **center of the hyperbola is (1, 0)**

Let x – 1 = X

$$\Rightarrow \frac{X^2}{3^2} - \frac{y^2}{(\sqrt{3})^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$:

Eccentricity(e) is given by,

$e=\frac{c}{a}$, where $c=\sqrt{a^2+b^2}$

Foci is given by (±ae, 0)

Equation of directrix are: $x=\pm\frac{a}{e}$

Length of latus rectum is $\frac{2b^2}{a}$

Here, a = 3 and b = $\sqrt{3}$

$c=\sqrt{(3)^2+\left(\sqrt{3}\right)^2}$

$\Rightarrow c=\sqrt{9+3}$

$\Rightarrow c=\sqrt{12}=2\sqrt{3}$

Therefore,

$e=\frac{2\sqrt{3}}{3}$

$\Rightarrow ae=3\times\frac{2\sqrt{3}}{3}=2\sqrt{3}$

$\mathbf{Foci}:(\pm\mathbf{ae},\mathbf{0})$

$\Rightarrow X=\pm2\sqrt{3}$ and y = 0

$\Rightarrow x-1=\pm2\sqrt{3}$ and y = 0

$\Rightarrow x=\pm2\sqrt{3}+1$ and y = 0

So, Foci: $(\pm2\sqrt{3}+1, 0)$

Equation of directrix are:

$X=\pm\frac{a}{e}$

$\Rightarrow X=\pm\frac{3}{\frac{2\sqrt{3}}{3}}$

$\Rightarrow X=\pm\frac{9}{2\sqrt{3}}$

$\Rightarrow 2\sqrt{3}X\mp9=0$

$\Rightarrow 2\sqrt{3}(x-1)\mp9=0$

$\Rightarrow 2\sqrt{3}x-2\sqrt{3}\mp9=0$

$\Rightarrow \mathbf{2\sqrt{3}x-2\sqrt{3}-9=0}$ and $\mathbf{2\sqrt{3}x-2\sqrt{3}+9=0}$

6 A. Question

Find the equation of the hyperbola, referred to its principal axes as axes of coordinates, in the following cases:

the distance between the foci = 16 and eccentricity = $\sqrt{2}$

Answer

Given: the distance between the foci = 16 and eccentricity $= \sqrt{2}$

To find: the equation of the hyperbola

Formula used:

For hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between the foci is 2ae and $b^2 = a^2(e^2 - 1)$

Therefore

2ae = 16

$$\Rightarrow ae = \frac{16}{2}$$

$$\Rightarrow a \times \sqrt{2} = 8$$

$$\{\because e = \sqrt{2}\}$$

$$\Rightarrow a = \frac{8}{\sqrt{2}}$$

$$\Rightarrow a^2 = \frac{64}{2} = 32$$

$b^2 = a^2(e^2 - 1)$

$$\Rightarrow b^2 = 32\left\{\left(\sqrt{2}\right)^2 - 1\right\}$$

$$\Rightarrow b^2 = 32(2 - 1)$$

$$\Rightarrow b^2 = 32$$

Equation of hyperbola is:

$$\frac{x^2}{32} - \frac{y^2}{32} = 1$$

$$\Rightarrow \mathbf{x^2 - y^2 = 32}$$

Hence, required equation of hyperbola is $\mathbf{x^2 - y^2 = 32}$

6 B. Question

Find the equation of the hyperbola, referred to its principal axes as axes of coordinates, in the following cases:

conjugate axis is 5 and the distance between foci = 13

Answer

Given: the distance between the foci = 13 and conjugate axis is 5

To find: the equation of the hyperbola

Formula used:

For hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between the foci is 2ae and $b^2 = a^2(e^2 - 1)$

Length of conjugate axis is 2b

Therefore

$$2b = 5 \Rightarrow b = \frac{5}{2}$$

$$\Rightarrow b^2 = \frac{25}{4}$$

2ae = 13

$$\Rightarrow ae = \frac{13}{2}$$

$$\Rightarrow a^2e^2 = \frac{169}{4}$$

$b^2 = a^2(e^2 - 1)$

$\Rightarrow b^2 = a^2e^2 - a^2$

$$\Rightarrow \frac{25}{4} = \frac{169}{4} - a^2$$

$$\Rightarrow a^2 = \frac{169}{4} - \frac{25}{4}$$

$$\Rightarrow a^2 = \frac{144}{4} = 36$$

Equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{36} - \frac{y^2}{\frac{25}{4}} = 1$$

$$\Rightarrow \frac{x^2}{36} - \frac{4y^2}{25} = 1$$

$$\Rightarrow \frac{25x^2 - 144y^2}{900} = 1$$

$$\Rightarrow 25x^2 - 144y^2 = 900$$

Hence, required equation of hyperbola is **$25x^2 - 144y^2 = 900$**

6 C. Question

Find the equation of the hyperbola, referred to its principal axes as axes of coordinates, in the following cases:

conjugate axis is 7 and passes through the point (3, -2).

Answer

Given: conjugate axis is 5 and passes through the point (3, -2)

To find: the equation of the hyperbola

Formula used:

For hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Conjugate axis is 2b

Therefore

$$2b = 7 \Rightarrow b = \frac{7}{2}$$

$$\Rightarrow b^2 = \frac{49}{4}$$

The equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since it passes through (3, -2)

$$\Rightarrow \frac{(3)^2}{a^2} - \frac{(-2)^2}{\frac{49}{4}} = 1$$

$$\Rightarrow \frac{9}{a^2} - \frac{4(4)}{49} = 1$$

$$\Rightarrow \frac{9}{a^2} - \frac{16}{49} = 1$$

$$\Rightarrow \frac{9}{a^2} = 1 + \frac{16}{49}$$

$$\Rightarrow \frac{9}{a^2} = \frac{49 + 16}{49}$$

$$\Rightarrow \frac{9}{a^2} = \frac{65}{49}$$

$$\Rightarrow a^2 = \frac{49}{65} \times 9$$

$$\Rightarrow a^2 = \frac{441}{65}$$

The equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\text{Since, } a^2 = \frac{441}{65} \text{ and } b^2 = \frac{49}{4}$$

$$\Rightarrow \frac{x^2}{\frac{441}{65}} - \frac{y^2}{\frac{49}{4}} = 1$$

$$\Rightarrow \frac{65x^2}{441} - \frac{4y^2}{49} = 1$$

$\Rightarrow \frac{65x^2 - 36y^2}{441} = 1$

$\Rightarrow 65x^2 - 36y^2 = 441$

Hence, required equation of hyperbola is **$65x^2 - 36y^2 = 441$**

7 A. Question

Find the equation of the hyperbola whose

foci are (6, 4) and (-4, 4) and eccentricity is 2.

Answer

Given: Foci are (6, 4) and (-4, 4) and eccentricity is 2

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1$ where center is (x_1, y_1)

Center is the mid-point of two foci.

Distance between the foci is 2ae and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m-a)^2 + (n-b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$\left(\frac{m+a}{2}, \frac{n+b}{2}\right)$

Center of hyperbola having foci (6, 4) and (-4, 4) is given by

$= \left(\frac{6-4}{2}, \frac{4+4}{2}\right)$

$= \left(\frac{2}{2}, \frac{8}{2}\right)$

= (1, 4)

The distance between the foci is 2ae, and Foci are (6, 4) and (-4, 4)

$\Rightarrow \sqrt{(6+4)^2 + (4-4)^2} = 2ae$

$\Rightarrow \sqrt{(10)^2 + (0)^2} = 2ae$

$\Rightarrow \sqrt{100} = 2ae$

$\Rightarrow 10 = 2ae$

$\Rightarrow \frac{10}{2} = ae$

$\Rightarrow ae = 5$

$\{\because e = 2\}$

$\Rightarrow a \times 2 = 5$

$\Rightarrow a = \frac{5}{2}$

$\Rightarrow a^2 = \frac{25}{4}$

$b^2 = a^2(e^2 - 1)$

$\Rightarrow b^2 = \frac{25}{4}\{(2)^2 - 1\}$

$\Rightarrow b^2 = \frac{25}{4}(4 - 1)$

$\Rightarrow b^2 = \frac{25}{4}(3)$

$\Rightarrow b^2 = \frac{75}{4}$

The equation of hyperbola:

$$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1$$

$$\Rightarrow \frac{(x - 1)^2}{\frac{25}{4}} - \frac{(y - 4)^2}{\frac{75}{4}} = 1$$

$$\Rightarrow \frac{4(x - 1)^2}{25} - \frac{4(y - 4)^2}{75} = 1$$

$$\Rightarrow \frac{12(x - 1)^2 - 4(y - 4)^2}{75} = 1$$

$\Rightarrow 12(x - 1)^2 - 4(y - 4)^2 = 75$

$\Rightarrow 12(x^2 + 1 - 2x) - 4(y^2 + 16 - 8y) = 75$

$\Rightarrow 12x^2 + 12 - 24x - 4y^2 - 64 + 32y - 75 = 0$

$\Rightarrow 12x^2 - 4y^2 - 24x + 32y - 127 = 0$

Hence, required equation of hyperbola is **$12x^2 - 4y^2 - 24x + 32y - 127 = 0$**

7 B. Question

Find the equation of the hyperbola whose

vertices are (-8, -1) and (16, -1) and focus is (17, -1)

Answer

Given: Vertices are (-8, -1) and (16, -1) and focus is (17, -1)

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two vertices

The distance between two vertices is 2a

The distance between the foci and vertex is ae – a and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m-a)^2 + (n-b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m+a}{2}, \frac{n+b}{2}\right)$$

Center of hyperbola having vertices (-8, -1) and (16, -1) is given by

$$= \left(\frac{-8+16}{2}, \frac{-1-1}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{-2}{2}\right)$$

= (4, -1)

The distance between two vertices is 2a and vertices are (-8, -1) and (16, -1)

$$\Rightarrow \sqrt{(16+8)^2 + (-1+1)^2} = 2a$$

$$\Rightarrow \sqrt{(24)^2 + (0)^2} = 2a$$

$$\Rightarrow \sqrt{576} = 2a$$

$$\Rightarrow 24 = 2a$$

$$\Rightarrow a = 12$$

The distance between the foci and vertex is ae – a, Foci is (17, -1) and the vertex is (16, -1)

$$\Rightarrow \sqrt{(17-16)^2 + (-1+1)^2} = ae - a$$

$$\Rightarrow \sqrt{(1)^2 + (0)^2} = a(e-1)$$

$$\Rightarrow \sqrt{1} = 12(e-1)$$

$$\Rightarrow \frac{1}{12} = e - 1$$

$$\Rightarrow e = 1 + \frac{1}{12}$$

$$\Rightarrow e = \frac{13}{12}$$

$b^2 = a^2(e^2 - 1)$

$$\left\{\because\ a\ =\ 12 \text{ and } e\ = \frac{13}{12}\right\}$$

$$\Rightarrow b^2 = (12)^2\left\{\left(\frac{13}{12}\right)^2 - 1\right\}$$

$$\Rightarrow b^2 = 144\left(\frac{169}{144} - 1\right)$$

$$\Rightarrow b^2 = 144\left(\frac{169-144}{144}\right)$$

$\Rightarrow b^2 = 25$

The equation of hyperbola:

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1$$

$$\Rightarrow \frac{(x-4)^2}{144} - \frac{(y+1)^2}{25} = 1$$

$$\Rightarrow \frac{25(x-4)^2 - 144(y+1)^2}{3600} = 1$$

$$\Rightarrow 25(x-4)^2 - 144(y+1)^2 = 3600$$

$\Rightarrow 25(x^2 + 16 – 8x) – 144(y^2 + 1 + 2y) = 3600$

$\Rightarrow 25x^2 + 400 – 200x – 144y^2 – 144 – 288y – 3600 = 0$

$\Rightarrow 25x^2 – 144y^2 – 200x – 288y – 3344 = 0$

Hence, required equation of hyperbola is **$25x^2 – 144y^2 – 200x – 288y – 3344 = 0$**

7 C. Question

Find the equation of the hyperbola whose

foci are (4, 2) and (8, 2) and eccentricity is 2.

Answer

Given: Foci are (4, 2) and (8, 2) and eccentricity is 2

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two foci.

Distance between the foci is 2ae and $b^2 = a^2(e^2 – 1)$

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m-a)^2 + (n-b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m+a}{2}, \frac{n+b}{2}\right)$$

Center of hyperbola having foci (4, 2) and (8, 2) is given by

$$= \left(\frac{4+8}{2}, \frac{2+2}{2}\right)$$

$$= \left(\frac{12}{2}, \frac{4}{2}\right)$$

= (6, 2)

The distance between the foci is 2ae and Foci are (4, 2) and (8, 2)

$$\Rightarrow \sqrt{(4-8)^2 + (2-2)^2} = 2ae$$

$\Rightarrow \sqrt{(-4)^2 + (0)^2} = 2ae$

$\Rightarrow \sqrt{16} = 2ae$

$\Rightarrow 4 = 2ae$

$\Rightarrow \frac{4}{2} = ae$

$\Rightarrow ae = 2$

$\{\because e = 2\}$

$\Rightarrow a \times 2 = 2$

$\Rightarrow a = 1$

$\Rightarrow a^2 = 1$

$b^2 = a^2(e^2 - 1)$

$\Rightarrow b^2 = 1\{(2)^2 - 1\}$

$\Rightarrow b^2 = 1(4 - 1)$

$\Rightarrow b^2 = 3$

The equation of hyperbola:

$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1$

$\Rightarrow \frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$

$\Rightarrow \frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$

$\Rightarrow \frac{3(x-6)^2 - (y-2)^2}{3} = 1$

$\Rightarrow 3(x-6)^2 - (y-2)^2 = 3$

$\Rightarrow 3(x^2 + 36 - 12x) - (y^2 + 4 - 4y) = 3$

$\Rightarrow 3x^2 + 108 - 36x - y^2 - 4 + 4y - 3 = 0$

$\Rightarrow 3x^2 - y^2 - 36x + 4y + 101 = 0$

Hence, required equation of hyperbola is **$3x^2 - y^2 - 36x + 4y + 101 = 0$**

7 D. Question

Find the equation of the hyperbola whose

vertices are at (0. ± 7) and foci at $\left(0, \pm\frac{28}{3}\right)$

Answer

Given: Vertices are (0, ± 7) and foci are $\left(0, \pm\frac{28}{3}\right)$

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$$

Vertices of the hyperbola are given by (0, ±b)

Foci of the hyperbola are given by (0, ±be)

Vertices are (0, ±7) and foci are $(0, \pm\frac{28}{3})$

Therefore,

$$b = 7 \text{ and } be = \frac{28}{3}$$

$$\Rightarrow 7 \times e = \frac{28}{3}$$

$$\Rightarrow e = \frac{4}{3}$$

$a^2 = b^2(e^2 - 1)$

$$\left\{\because\ b = 7 \text{ and } e\ = \frac{4}{3}\right\}$$

$$\Rightarrow a^2 = 7^2\left\{\left(\frac{4}{3}\right)^2 - 1\right\}$$

$$\Rightarrow a^2 = 49\left(\frac{16}{9} - 1\right)$$

$$\Rightarrow a^2 = 49\left(\frac{16-9}{9}\right)$$

$$\Rightarrow a^2 = 49 \times \frac{7}{9}$$

$$\Rightarrow\ a^2\ = \frac{343}{9}$$

The equation of hyperbola:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$$

$$\Rightarrow \frac{x^2}{\frac{343}{9}}-\frac{y^2}{49}=-1$$

$$\Rightarrow \frac{9x^2}{343}-\frac{y^2}{49}=-1$$

$$\Rightarrow \frac{9x^2-7y^2}{343}=-1$$

$\Rightarrow 9x^2 - 7y^2 = -343$

$\Rightarrow 9x^2 - 7y^2 + 343 = 0$

Hence, required equation of hyperbola is $\mathbf{9x^2 - 7y^2 + 343 = 0}$

7 E. Question

Find the equation of the hyperbola whose vertices are at (±6, 0) and one of the directrices is x = 4.

Answer

Given: Vertices are (± 6, 0) and one of the directrices is x = 4

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices of the hyperbola are given by (±a, 0)

The equation of the directrices: $x = \pm\frac{a}{e}$

Vertices are (± 6, 0) and one of the directrices is x = 4

Therefore,

$$a = 6 \text{ and } \frac{a}{e} = 4$$

$$\Rightarrow \frac{6}{e} = 4$$

$$\Rightarrow e = \frac{6}{4}$$

$$\Rightarrow e = \frac{3}{2}$$

$b^2 = a^2(e^2 - 1)$

$$\left\{\because a = 6 \text{ and } e = \frac{3}{2}\right\}$$

$$\Rightarrow b^2 = (6)^2\left\{\left(\frac{3}{2}\right)^2 - 1\right\}$$

$$\Rightarrow b^2 = 36\left(\frac{9}{4} - 1\right)$$

$$\Rightarrow b^2 = 36\left(\frac{9-4}{4}\right)$$

$$\Rightarrow b^2 = 36 \times \frac{5}{4}$$

$\Rightarrow b^2 = 45$

The equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{36} - \frac{y^2}{45} = 1$$

$$\Rightarrow \frac{5x^2 - 4y^2}{180} = 1$$

$\Rightarrow 5x^2 - 4y^2 = 180$

$\Rightarrow 5x^2 - 4y^2 - 180 = 0$

Hence, required equation of hyperbola is $\mathbf{5x^2 - 4y^2 - 180 = 0}$

7 F. Question

Find the equation of the hyperbola whose

foci at (± 2, 0) and eccentricity is 3/2.

Answer

Given: Foci are (2, 0) and (-2, 0) and eccentricity is $\frac{3}{2}$

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two foci.

Distance between the foci is 2ae and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m-a)^2 + (n-b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m+a}{2}, \frac{n+b}{2}\right)$$

Center of hyperbola having Foci (2, 0) and (-2, 0) is given by

$$= \left(\frac{2-2}{2}, \frac{0-0}{2}\right)$$

$$= \left(\frac{0}{2}, \frac{0}{2}\right)$$

= (0, 0)

The distance between the foci is 2ae, and Foci are (2, 0) and (-2, 0)

$$\Rightarrow \sqrt{(2+2)^2 + (0-0)^2} = 2ae$$

$$\Rightarrow \sqrt{(4)^2 + (0)^2} = 2ae$$

$$\Rightarrow \sqrt{16} = 2ae$$

$$\Rightarrow 4 = 2ae$$

$$\Rightarrow \frac{4}{2} = ae$$

$$\Rightarrow ae = 2$$

$$\left\{\because e = \frac{3}{2}\right\}$$

$$\Rightarrow a \times \frac{3}{2} = 2$$

$\Rightarrow a = \frac{4}{3}$

$\Rightarrow a^2 = \frac{16}{9}$

$b^2 = a^2(e^2 - 1)$

$\Rightarrow b^2 = \frac{16}{9}\left\{\left(\frac{3}{2}\right)^2 - 1\right\}$

$\Rightarrow b^2 = \frac{16}{9}\left(\frac{9}{4} - 1\right)$

$\Rightarrow b^2 = \frac{16}{9}\left(\frac{9-4}{4}\right)$

$\Rightarrow b^2 = \frac{16}{9}\left(\frac{5}{4}\right)$

$\Rightarrow b^2 = \frac{20}{9}$

The equation of hyperbola:

$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1$

$\Rightarrow \frac{(x-0)^2}{\frac{16}{9}} - \frac{(y-0)^2}{\frac{20}{9}} = 1$

$\Rightarrow \frac{9x^2}{16} - \frac{9y^2}{20} = 1$

$\Rightarrow \frac{45x^2 - 36y^2}{80} = 1$

$\Rightarrow 45x^2 - 36y^2 = 80$

$\Rightarrow 45x^2 - 36y^2 - 80 = 0$

Hence, required equation of hyperbola is **$45x^2 - 36y^2 - 80 = 0$**

8. Question

Find the eccentricity of the hyperbola, the length of whose conjugate axis is $\frac{3}{4}$ of the length of the transverse axis.

Answer

Given: the length of whose conjugate axis is $\frac{3}{4}$ of the length of the transverse axis

To find: eccentricity of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Length of the conjugate axis is 2b and length of transverse axis is 2a

According to question:

$$2b = \frac{3}{4} \times 2a$$

$$\Rightarrow \frac{b}{a} = \frac{3}{4}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{9}{16}$$

We know,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{9}{16}}$$

$$\Rightarrow e = \sqrt{\frac{16+9}{16}}$$

$$\Rightarrow e = \sqrt{\frac{25}{16}}$$

$$\Rightarrow e = \frac{5}{4}$$

Hence, the eccentricity of the hyperbola is $\frac{5}{4}$

9 A. Question

Find the equation of the hyperbola whose

the focus is at (5, 2), vertices at (4, 2) and (2, 2) and centre at (3, 2)

Answer

Given: Vertices are (4, 2) and (2, 2), the focus is (5, 2) and centre (3, 2)

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two vertices

The distance between two vertices is 2a

The distance between the foci and vertex is ae – a and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m-a)^2 + (n-b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m+a}{2}, \frac{n+b}{2}\right)$$

The distance between two vertices is 2a and vertices are (4, 2) and (2, 2)

$\Rightarrow \sqrt{(4-2)^2+(2-2)^2}=2a$

$\Rightarrow \sqrt{(2)^2+(0)^2}=2a$

$\Rightarrow \sqrt{4}=2a$

$\Rightarrow 2=2a$

$\Rightarrow a=1$

The distance between the foci and vertex is ae – a, Foci is (5, 2) and the vertex is (4, 2)

$\Rightarrow \sqrt{(5-4)^2+(2-2)^2}=ae-a$

$\Rightarrow \sqrt{(1)^2+(0)^2}=a(e-1)$

$\Rightarrow \sqrt{1}=1(e-1)$

$\Rightarrow 1 = e - 1$

$\Rightarrow e = 1 + 1$

$\Rightarrow e = 2$

$b^2 = a^2(e^2 - 1)$

$\left\{\because\ a\ =\ 12 \text{ and } e\ =\frac{13}{12}\right\}$

$\Rightarrow b^2=(1)^2\{(2)^2-1\}$

$\Rightarrow b^2=1(4-1)$

$\Rightarrow b^2=1(3)$

$\Rightarrow b^2=3$

The equation of hyperbola:

$\frac{(x-x_1)^2}{a^2}-\frac{(y-y_1)^2}{b^2}=1$

{∵ Centre (3, 2)}

$\Rightarrow \frac{(x-3)^2}{1}-\frac{(y-2)^2}{3}=1$

$\Rightarrow \frac{3(x-3)^2-(y-2)^2}{3}=1$

$\Rightarrow 3(x-3)^2-(y-2)^2=3$

$\Rightarrow 3(x^2 + 9 - 6x) - (y^2 + 4 - 4y) = 3$

$\Rightarrow 3x^2 + 27 - 18x - y^2 - 4 + 4y - 3 = 0$

$\Rightarrow 3x^2 - y^2 - 18x + 4y + 20 = 0$

Hence, required equation of hyperbola is $\mathbf{3x^2 - y^2 - 18x + 4y + 20 = 0}$

9 B. Question

Find the equation of the hyperbola whose focus is at (4, 2), centre at (6, 2) and e = 2.

Answer

Given: Foci is (4, 2), e = 2 and center at (6, 2)

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$\frac{(x-x_1)^2}{a^2}-\frac{(y-y_1)^2}{b^2}=1$ where center is (x_1, y_1)

Center is the mid-point of two vertices

The distance between two vertices is 2a

The distance between the foci and vertex is ae – a and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m-a)^2+(n-b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$\left(\frac{m+a}{2}, \frac{n+b}{2}\right)$

Therefore

Let one of the two foci is (m, n) and the other one is (4, 2)

Since, Centre(6, 2)

$\left(\frac{m+4}{2}=6, \frac{n+2}{2}=2\right)$

$\Rightarrow (m+4=12, n+2=4)$

$\Rightarrow (m=8, n=2)$

Foci are (4, 2) and (8, 2)

The distance between the foci is 2ae and Foci are (4, 2) and (8, 2)

$\Rightarrow \sqrt{(4-8)^2+(2-2)^2}=2ae$

$\Rightarrow \sqrt{(-4)^2+(0)^2}=2ae$

$\Rightarrow \sqrt{16}=2ae$

$\Rightarrow 4=2ae$

$\Rightarrow \frac{4}{2}=ae$

$\Rightarrow ae=2$

$\{\because\ e=2\}$

$\Rightarrow a\times 2=2$

$\Rightarrow a=\frac{2}{2}=1$

$\Rightarrow a^2 = 1$

$b^2 = a^2(e^2 - 1)$

$\Rightarrow b^2 = 1\{(2)^2 - 1\}$

$\Rightarrow b^2 = 4 - 1$

$\Rightarrow b^2 = 3$

The equation of hyperbola:

$$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1$$

$$\Rightarrow \frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$$

$$\Rightarrow \frac{3(x-6)^2 - (y-2)^2}{3} = 1$$

$\Rightarrow 3(x^2 + 36 - 12x) - (y^2 + 4 - 4y) = 3$

$\Rightarrow 3x^2 + 108 - 36x - y^2 - 4 + 4y - 3 = 0$

$\Rightarrow 3x^2 - y^2 - 36x + 4y + 101 = 0$

Hence, required equation of hyperbola is $\mathbf{3x^2 - y^2 - 36x + 4y + 101 = 0}$

10. Question

If P is any point on the hyperbola whose axis are equal, prove that SP.S'P = CP^2

Answer

Given: Axis of the hyperbola are equal, i.e. a = b

To prove: SP.S'P = CP^2

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{a^2}{a^2}}$$

$\Rightarrow e = \sqrt{1+1}$

$\Rightarrow e = \sqrt{2}$

Foci of the hyperbola are given by (±ae, 0)

$\Rightarrow$ Foci of hyperbola are given by $(\pm\sqrt{2}a, 0)$

So, $S(\sqrt{2}a, 0)$ and $S'(-\sqrt{2}a, 0)$

Let P (m, n) be any point on the hyperbola

The distance between two points (m, n) and (a, b) is given by $\sqrt{(m-a)^2 + (n-b)^2}$

$SP = \sqrt{(m - \sqrt{2}a)^2 + (n - 0)^2}$

$\Rightarrow SP^2 = m^2 + 2a^2 - 2\sqrt{2}am + n^2$

$S'P = \sqrt{(m + \sqrt{2}a)^2 + (n - 0)^2}$

$\Rightarrow S'P^2 = m^2 + 2a^2 + 2\sqrt{2}am + n^2$

C is Centre with coordinates (0, 0)

$CP = \sqrt{(m - 0)^2 + (n - 0)^2}$

$\Rightarrow CP^4 = (m^2 + n^2)^2$

$\Rightarrow CP^4 = m^4 + n^4 + 2m^2n^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

Now,

$SP^2.S'P^2 = (m^2 + 2a^2 + n^2 - 2\sqrt{2}am)(m^2 + 2a^2 + n^2 + 2\sqrt{2}am)$

$\Rightarrow SP^2.S'P^2 = (m^2 + 2a^2 + n^2)^2 - (2\sqrt{2}am)^2$

$\Rightarrow SP^2.S'P^2 = m^4 + 4a^4 + n^4 + 4a^2m^2 + 4a^2n^2 + 2m^2n^2 - 8a^2m^2$

$\Rightarrow SP^2.S'P^2 = m^4 + 4a^4 + n^4 + 4a^2n^2 + 2m^2n^2 - 4a^2m^2$

$\Rightarrow SP^2.S'P^2 = m^4 + n^4 + 2m^2n^2 + 4a^2(a^2 + n^2 - m^2)$

{$\because a^2 = m^2 - n^2$}

$\Rightarrow SP^2.S'P^2 = m^4 + n^4 + 2m^2n^2 + 4a^2(m^2 - n^2 + n^2 - m^2)$

$\Rightarrow SP^2.S'P^2 = m^4 + n^4 + 2m^2n^2 + 4a^2(0)$

$\Rightarrow SP^2.S'P^2 = m^4 + n^4 + 2m^2n^2$

From (i):

$\Rightarrow SP^2.S'P^2 = CP^4$

Taking square root both sides:

$\Rightarrow \sqrt{SP^2.S'P^2} = \sqrt{CP^4}$

$\Rightarrow SP.S'P = CP^2$

Hence Proved

11 A. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

vertices (± 2, 0), foci (± 3, 0)

Answer

Given: Vertices are (±2, 0) and foci are (±3, 0)

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices of the hyperbola are given by (±a, 0)

Foci of the hyperbola are given by (±ae, 0)

Vertices are (±2, 0) and foci are (±3, 0)

Therefore,

a = 2 and ae = 3

⇒ 2 × e = 3

$$\Rightarrow e = \frac{3}{2}$$

$b^2 = a^2(e^2 - 1)$

$$\left\{\because\ a = 2 \text{ and } e\ = \frac{3}{2}\right\}$$

$$\Rightarrow b^2 = 2^2\left\{\left(\frac{3}{2}\right)^2 - 1\right\}$$

$$\Rightarrow b^2 = 4\left(\frac{9}{4} - 1\right)$$

$$\Rightarrow b^2 = 4\left(\frac{9-4}{4}\right)$$

$$\Rightarrow b^2 = 4 \times \frac{5}{4}$$

$\Rightarrow b^2 = 5$

The equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{4} - \frac{y^2}{5} = 1$$

$$\Rightarrow \frac{5x^2 - 4y^2}{20} = 1$$

$\Rightarrow 5x^2 - 4y^2 = 20$

$\Rightarrow 5x^2 - 4y^2 - 20 = 0$

Hence, required equation of hyperbola is $\mathbf{5x^2 - 4y^2 - 20 = 0}$

11 B. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

vertices (0, ± 5), foci (0, ± 8)

Answer

Given: Vertices are (0, ±5) and foci are (0, ±8)

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Vertices of the hyperbola are given by (0, ±b)

Foci of the hyperbola are given by (0, ±be)

Vertices are (0, ±5) and foci are (0, ±8)

Therefore,

$$b = 5 \text{ and } be = 8$$

$$\Rightarrow 5 \times e = 8$$

$$\Rightarrow e = \frac{8}{5}$$

$a^2 = b^2(e^2 - 1)$

$$\left\{\because\ b = 5 \text{ and } e\ = \frac{8}{5}\right\}$$

$$\Rightarrow a^2 = 5^2\left\{\left(\frac{8}{5}\right)^2 - 1\right\}$$

$$\Rightarrow a^2 = 25\left(\frac{64}{25} - 1\right)$$

$$\Rightarrow a^2 = 25\left(\frac{64 - 25}{25}\right)$$

$$\Rightarrow a^2 = 25 \times \frac{39}{25}$$

$\Rightarrow a^2 = 39$

The equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \frac{x^2}{39} - \frac{y^2}{25} = -1$$

Hence, the required equation of the hyperbola is $\mathbf{\frac{x^2}{39} - \frac{y^2}{25} = -1}$

11 C. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

vertices (0, ± 3), foci (0, ± 5)

Answer

Given: Vertices are (0, ±3) and foci are (0, ±5)

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Vertices of the hyperbola are given by (0, ±b)

Foci of the hyperbola are given by (0, ±be)

Vertices are (0, ±3) and foci are (0, ±5)

Therefore,

$b = 3 \text{ and } be = 5$

$\Rightarrow 3 \times e = 5$

$\Rightarrow e = \frac{5}{3}$

$a^2 = b^2(e^2 - 1)$

$\left\{\because\ b = 3 \text{ and } e = \frac{5}{3}\right\}$

$\Rightarrow a^2 = 3^2\left\{\left(\frac{5}{3}\right)^2 - 1\right\}$

$\Rightarrow a^2 = 9\left(\frac{25}{9} - 1\right)$

$\Rightarrow a^2 = 9\left(\frac{25-9}{9}\right)$

$\Rightarrow a^2 = 9 \times \frac{16}{9}$

$\Rightarrow a^2 = 16$

The equation of hyperbola:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = -1$

Hence, the required equation of the hyperbola is $\mathbf{\frac{x^2}{16} - \frac{y^2}{9} = -1}$

11 D. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

foci (±5, 0), transverse axis = 8

Answer

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Length of transverse axis is 2a

Coordinates of the foci for a standard hyperbola is given by (±ae, 0)

According to question:

2a = 8 and ae = 5

2a = 8

$\Rightarrow a = \frac{8}{2}$

⇒ a = 4

⇒ $a^2 = 16$

ae = 5

$\Rightarrow 4 \times e = 5$

$\Rightarrow e = \frac{5}{4}$

We know,

$b^2 = a^2(e^2 - 1)$

$\Rightarrow b^2 = 16\left\{\left(\frac{5}{4}\right)^2 - 1\right\}$

$\Rightarrow b^2 = 16\left(\frac{25}{4} - 1\right)$

$\Rightarrow b^2 = 16\left(\frac{25-4}{4}\right)$

$\Rightarrow b^2 = 16\left(\frac{21}{4}\right)$

⇒ $b^2 = 4(21)$

⇒ $b^2 = 84$

Hence, the equation of the hyperbola is:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \mathbf{\frac{x^2}{16} - \frac{y^2}{84} = 1}$

11 E. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

foci (0, ±13), conjugate axis = 24

Answer

Given: foci (0, ±13) and the conjugate axis is 24

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Length of the conjugate axis is 2b

Coordinates of the foci for a standard hyperbola is given by (0, ±be)

According to question:

2b = 24 and be = 13

2b = 24

$\Rightarrow b = \frac{24}{2}$

⇒ b = 12

⇒ b^2 = 144

be = 12

$\Rightarrow 12 \times e = 13$

$\Rightarrow e = \frac{13}{12}$

We know,

$a^2 = b^2(e^2 - 1)$

$$\Rightarrow a^2 = 144\left\{\left(\frac{13}{12}\right)^2 - 1\right\}$$

$$\Rightarrow b^2 = 144\left(\frac{169}{144} - 1\right)$$

$$\Rightarrow b^2 = 144\left(\frac{169 - 144}{144}\right)$$

$$\Rightarrow b^2 = 144\left(\frac{25}{144}\right)$$

⇒ b^2 = 25

Hence, the equation of the hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \mathbf{\frac{x^2}{144} - \frac{y^2}{25} = -1}$$

11 F. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

foci $(\pm 3\sqrt{5}, 0)$, the latus-rectum = 8

Answer

Given: Foci $(\pm 3\sqrt{5}.0)$ and the latus-rectum = 8

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Coordinates of the foci for a standard hyperbola is given by (±ae, 0)

Length of latus rectum is $\frac{2b^2}{a}$

According to the question:

$$ae = 3\sqrt{5} \text{ and } \frac{2b^2}{a} = 8$$

$$ae = 3\sqrt{5}$$

$$\Rightarrow e = \frac{3\sqrt{5}}{a}$$

$$\Rightarrow e^2 = \left(\frac{3\sqrt{5}}{a}\right)^2$$

$$\Rightarrow e^2 = \frac{45}{a^2}$$

$$\frac{2b^2}{a} = 8$$

$$b^2 = \frac{8a}{2}$$

$$b^2 = 4a$$

We know,

$b^2 = a^2(e^2 - 1)$

$$\Rightarrow 4a = a^2\left\{\frac{45}{a^2} - 1\right\}$$

$$\Rightarrow 4a = a^2\left(\frac{45 - a^2}{a^2}\right)$$

$\Rightarrow 4a = 45 - a^2$

$\Rightarrow a^2 + 4a - 45 = 0$

$\Rightarrow a^2 + 9a - 5a - 45 = 0$

$\Rightarrow a(a + 9) - 5(a + 9) = 0$

$\Rightarrow (a + 9)(a - 5) = 0$

$\Rightarrow a = -9$ or $a = 5$

Since a is a distance, and it can’t be negative

$\Rightarrow a = 5$

$\Rightarrow a^2 = 25$

$b^2 = 4a$

$\Rightarrow b^2 = 4(5)$

$\Rightarrow b^2 = 20$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{25} - \frac{y^2}{20} = 1$$

11 G. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

foci (±4, 0), the latus-rectum = 12

Answer

Given: Foci (±4, 0), the latus-rectum = 12

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Coordinates of the foci for a standard hyperbola is given by (±ae, 0)

Length of latus rectum is $\frac{2b^2}{a}$

According to the question:

$$ae = 4 \text{ and } \frac{2b^2}{a} = 12$$

ae = 4

$$\Rightarrow e = \frac{4}{a}$$

$$\Rightarrow e^2 = \left(\frac{4}{a}\right)^2$$

$$\Rightarrow e^2 = \frac{16}{a^2}$$

$$\frac{2b^2}{a} = 12$$

$$b^2 = \frac{12a}{2}$$

$$b^2 = 6a$$

We know,

$b^2 = a^2(e^2 – 1)$

$$\Rightarrow 6a = a^2\left\{\frac{16}{a^2} - 1\right\}$$

$$\Rightarrow 6a = a^2\left(\frac{16 - a^2}{a^2}\right)$$

$\Rightarrow 6a = 16 – a^2$

$\Rightarrow a^2 + 6a – 16 = 0$

$\Rightarrow a^2 + 8a – 2a – 16 = 0$

$\Rightarrow a(a + 8) – 2(a + 8) = 0$

⇒ (a + 8)(a – 2) = 0

⇒ a = -8 or a = 2

Since a is a distance, and it can't be negative,

⇒ a = 2

⇒ $a^2 = 4$

$b^2 = 6a$

⇒ $b^2 = 6(2)$

⇒ $b^2 = 12$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \mathbf{\frac{x^2}{4} - \frac{y^2}{12} = 1}$$

1 H. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

vertices (0, ± 6), $e = \frac{5}{3}$

Answer

Given: Vertices (0, ±6), $e = \frac{5}{3}$

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Coordinates of the vertices for a standard hyperbola is given by (0, ±b)

According to question:

b = 6 ⇒ $b^2 = 36$

We know,

$a^2 = b^2(e^2 - 1)$

$$\Rightarrow a^2 = 6^2\left\{\left(\frac{5}{3}\right)^2 - 1\right\}$$

$$\left\{\because e = \frac{5}{3}\right\}$$

$$\Rightarrow a^2 = 36\left(\frac{25}{9} - 1\right)$$

$$\Rightarrow a^2 = 36\left(\frac{25 - 9}{9}\right)$$

$\Rightarrow a^2 = 36\left(\frac{16}{9}\right)$

$\Rightarrow a^2 = 64$

Hence, equation of hyperbola is:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

$\Rightarrow \mathbf{\frac{x^2}{64} - \frac{y^2}{36} = -1}$

11 I. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

foci $(0, \pm\sqrt{10})$, passing through (2, 3)

Answer

Given: Foci $(0, \pm\sqrt{10})$, passing through (2, 3)

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Coordinates of the foci for a standard hyperbola is given by (0, ±be)

According to the question:

$be = \sqrt{10}$

$\Rightarrow b^2e^2 = 10$

Since (2, 3) passing through hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Therefore,

$\frac{2^2}{a^2} - \frac{3^2}{b^2} = -1$

$\Rightarrow \frac{4}{a^2} - \frac{9}{b^2} = -1$

$\Rightarrow \frac{4}{a^2} - \frac{9}{b^2} = -1$

{∵ $a^2 = b^2(e^2 - 1)$}

$\Rightarrow \frac{4}{b^2(e^2 - 1)} - \frac{9}{b^2} = -1$

$\Rightarrow \frac{4}{b^2e^2 - b^2} - \frac{9}{b^2} = -1$

$\Rightarrow \frac{4}{10 - b^2} - \frac{9}{b^2} = -1$

$$\Rightarrow \frac{4b^2 - 9(10 - b^2)}{(10 - b^2)b^2} = -1$$

$$\Rightarrow \frac{4b^2 - 90 + 9b^2}{(10 - b^2)b^2} = -1$$

$$\Rightarrow \frac{-(90 - 13b^2)}{(10 - b^2)b^2} = -1$$

$\Rightarrow 90 - 13b^2 = (10 - b^2)b^2$

$\Rightarrow 90 - 13b^2 = 10b^2 - b^4$

$\Rightarrow 90 - 13b^2 - 10b^2 + b^4 = 0$

$\Rightarrow b^4 - 23b^2 + 90 = 0$

$\Rightarrow b^4 - 18b^2 - 5b^2 + 90 = 0$

$\Rightarrow b^2(b^2 - 18) - 5(b^2 - 18) = 0$

$\Rightarrow (b^2 - 18)(b^2 - 5) = 0$

$\Rightarrow b^2 = 18$ or 5

Case 1: $b^2 = 18$ and $b^2e^2 = 10$

$a^2 = b^2(e^2 - 1)$

$\Rightarrow a^2 = b^2e^2 - b^2$

$\Rightarrow a^2 = 10 - 18$

$\Rightarrow a^2 = -8$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \frac{x^2}{-8} - \frac{y^2}{10} = -1$$

$$\Rightarrow -\left(\frac{x^2}{8} + \frac{y^2}{10}\right) = -1$$

$$\Rightarrow \frac{x^2}{8} + \frac{y^2}{10} = 1$$

Case 2: $b^2 = 5$ and $b^2e^2 = 10$

$a^2 = b^2(e^2 - 1)$

$\Rightarrow a^2 = b^2e^2 - b^2$

$\Rightarrow a^2 = 10 - 5$

$\Rightarrow a^2 = 5$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \frac{x^2}{5} - \frac{y^2}{10} = -1$$

11 J. Question

In each of the following find the equations of the hyperbola satisfying the given conditions

foci (0, ± 12), latus-rectum = 36.

Answer

Given: Foci (0, ±12), the latus-rectum = 36

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Coordinates of the foci for a standard hyperbola is given by (0, ±be)

Length of latus rectum is $\frac{2a^2}{b}$

According to the question:

$$be = 12 \text{ and } \frac{2a^2}{b} = 36$$

be = 12

$$\Rightarrow e = \frac{12}{b}$$

$$\Rightarrow e^2 = \left(\frac{12}{b}\right)^2$$

$$\Rightarrow e^2 = \frac{144}{b^2}$$

$$\frac{2a^2}{b} = 36$$

$$\Rightarrow a^2 = \frac{36b}{2}$$

$$\Rightarrow a^2 = 18b$$

We know,

$a^2 = b^2(e^2 - 1)$

$$\Rightarrow 18b = b^2\left\{\frac{144}{b^2} - 1\right\}$$

$$\Rightarrow 18b = b^2\left(\frac{144 - b^2}{b^2}\right)$$

$\Rightarrow 18b = 144 - b^2$

$\Rightarrow b^2 + 18b - 144 = 0$

$\Rightarrow b^2 + 24b - 6b - 144 = 0$

$\Rightarrow b(b + 24) - 6(b + 24) = 0$

⇒ (b + 24)(b – 6) = 0

⇒ b = -24 or b = 6

Since b is a distance, and it can't be negative

⇒ b = 6

⇒ b^2 = 36

a^2 = 18b

⇒ a^2 = 18(6)

⇒ b^2 = 108

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \mathbf{\frac{x^2}{36} - \frac{y^2}{108} = -1}$$

12. Question

If the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then obtain its equation.

Answer

Given: Distance between foci is 16 and eccentricity is $\sqrt{2}$

To find: equation of the hyperbola

Formula used:

The standard form of the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The distance between foci is given by 2ae

According to question:

2ae = 16

$$\Rightarrow a = \frac{16}{2e}$$

$$\{\because e = \sqrt{2}\}$$

$$\Rightarrow a = \frac{8}{\sqrt{2}}$$

$$\Rightarrow a = 4\sqrt{2}$$

⇒ a^2 = 32

We know,

$b^2 = a^2(e^2 - 1)$

$$\Rightarrow b^2 = 32\left\{\left(\sqrt{2}\right)^2 - 1\right\}$$

$$\Rightarrow b^2 = 32(2 - 1)$$

$\Rightarrow b^2 = 32$

Hence, the equation of the hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{32} - \frac{y^2}{32} = 1$$

$\Rightarrow x^2 - y^2 = 32$

13. Question

Show that the set of all points such that the difference of their distances from (4, 0) and (-4, 0) is always equal to 2 represents a hyperbola.

Answer

To prove: the set of all points under given conditions represents a hyperbola

Let a point P be (x, y) such that the difference of their distances from (4, 0) and (-4, 0) is always equal to 2.

Formula used:

The distance between two points (m, n) and (a, b) is given by

$$\sqrt{(m-a)^2 + (n-b)^2}$$

The distance of P(x, y) from (4, 0) is $\sqrt{(x-4)^2 + (y-0)^2}$

The distance of P(x, y) from (-4, 0) is $\sqrt{(x+4)^2 + (y-0)^2}$

Since, the difference of their distances from (4, 0) and (-4, 0) is always equal to 2

Therefore,

$$\sqrt{(x+4)^2 + (y-0)^2} - \sqrt{(x-4)^2 + (y-0)^2} = 2$$

$$\Rightarrow \sqrt{(x+4)^2 + (y-0)^2} = 2 + \sqrt{(x-4)^2 + (y-0)^2}$$

Squaring both sides:

$$\Rightarrow \left(\sqrt{(x+4)^2 + (y-0)^2}\right)^2 = \left(2 + \sqrt{(x-4)^2 + (y-0)^2}\right)^2$$

$$\Rightarrow \left(\sqrt{(x+4)^2 + (y-0)^2}\right)^2 = 2^2 + \left(\sqrt{(x-4)^2 + (y-0)^2}\right)^2 + 4\sqrt{(x-4)^2 + (y-0)^2}$$

$$\Rightarrow (x+4)^2 + (y)^2 = 4 + (x-4)^2 + (y)^2 + 4\sqrt{(x-4)^2 + (y)^2}$$

$$\Rightarrow x^2 + 16 + 8x + y^2 = 4 + x^2 + 16 - 8x + y^2 + 4\sqrt{(x-4)^2 + (y)^2}$$

$$\Rightarrow x^2 + 16 + 8x + y^2 - 4 - x^2 - 16 + 8x - y^2 = 4\sqrt{(x-4)^2 + (y)^2}$$

$$\Rightarrow 16x - 4 = 4\sqrt{(x-4)^2 + (y)^2}$$

$$\Rightarrow 4(4x-1) = 4\sqrt{(x-4)^2 + (y)^2}$$

$$\Rightarrow 4x - 1 = \sqrt{(x-4)^2 + (y)^2}$$

Squaring both sides:

$\Rightarrow (4x - 1)^2 = \left(\sqrt{(x-4)^2 + (y)^2}\right)^2$

$\Rightarrow 16x^2 + 1 - 8x = (x - 4)^2 + y^2$

$\Rightarrow 16x^2 + 1 - 8x = x^2 + 16 - 8x + y^2$

$\Rightarrow 16x^2 + 1 - 8x - x^2 - 16 + 8x - y^2 = 0$

$\Rightarrow 15x^2 - y^2 - 15 = 0$

Hence, required equation of hyperbola is $\mathbf{15x^2 - y^2 - 15 = 0}$

Very Short Answer

1. Question

Write the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.

Answer

Given: $9x^2 - 16y^2 = 144$

To find: eccentricity(e)

$9x^2 - 16y^2 = 144$

$\Rightarrow \frac{9x^2}{144} - \frac{16y^2}{144} = 1$

$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$

$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$e = \frac{c}{a}, \text{where } c = \sqrt{a^2 + b^2}$

Here, a = 4 and b = 3

$c = \sqrt{4^2 + 3^2}$

$\Rightarrow c = \sqrt{16 + 9}$

$\Rightarrow c = \sqrt{25}$

$\Rightarrow c = 5$

Therefore,

$\mathbf{e = \frac{5}{4}}$

Hence, eccentricity is $\frac{5}{4}$

2. Question

Write the eccentricity of the hyperbola whose latus-rectum is half of its transverse axis.

Answer

Given: Latus-rectum is half of its transverse axis

To find: eccentricity of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Length of transverse axis is 2a

Latus-rectum of the hyperbola is $\frac{2b^2}{a}$

According to question:

Latus-rectum is half of its transverse axis

$$\Rightarrow \frac{2b^2}{a} = \frac{1}{2} \times 2a$$

$$\Rightarrow 2b^2 = a^2$$

We know,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{b^2}{2b^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{1}{2}}$$

$$\Rightarrow e = \sqrt{\frac{2+1}{2}}$$

$$\Rightarrow e = \sqrt{\frac{3}{2}}$$

Hence, eccentricity is $\sqrt{\frac{3}{2}}$

3. Question

Write the coordinates of the foci of the hyperbola $9x^2 - 16y^2 = 144$.

Answer

Given: $9x^2 - 16y^2 = 144$

To find: coordinates of the foci f(m,n)

$9x^2 - 16y^2 = 144$

$$\Rightarrow \frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$e = \frac{c}{a}, \text{where } c = \sqrt{a^2 + b^2}$

Foci is given by (±ae, 0)

Here, a = 4 and b = 3

$c = \sqrt{4^2 + 3^2}$

$\Rightarrow c = \sqrt{16 + 9}$

$\Rightarrow c = \sqrt{25}$

⇒ c = 5

Therefore,

$\mathbf{e = \frac{5}{4}}$

$\Rightarrow ae = 4 \times \frac{5}{4} = 5$

Foci: (±5, 0)

4. Question

Write the equation of the hyperbola of eccentricity $\sqrt{2}$, if it is known that the distance between its foci is 16.

Answer

Given: Distance between foci is 16 and eccentricity is $\sqrt{2}$

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Distance between foci is given by 2ae

According to question:

2ae = 16

$\Rightarrow a = \frac{16}{2e}$

$\{\because e = \sqrt{2}\}$

$\Rightarrow a = \frac{8}{\sqrt{2}}$

$\Rightarrow a = 4\sqrt{2}$

⇒ $a^2 = 32$

We know,

$b^2 = a^2(e^2 - 1)$

$$\Rightarrow b^2 = 32\left\{\left(\sqrt{2}\right)^2 - 1\right\}$$

$$\Rightarrow b^2 = 32(2 - 1)$$

⇒ $b^2 = 32$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{32} - \frac{y^2}{32} = 1$$

⇒ $x^2 - y^2 = 32$

5. Question

If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, write the value of b^2

Answer

To find: value of b^2

Given: foci of given ellipse and hyperbola coincide

$$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{4^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{25x^2}{441} - \frac{25y^2}{81} = 1$$

$$\Rightarrow \frac{x^2}{\frac{441}{25}} - \frac{y^2}{\frac{81}{25}} = 1$$

$$\Rightarrow \frac{x^2}{\left(\frac{12}{5}\right)^2} - \frac{y^2}{\left(\frac{9}{5}\right)^2} = 1$$

Formula used:

Coordinates of the foci for standard ellipse is given by $(\pm c_1, 0)$ where $c_1^2 = a_1^2 - b_1^2$

Coordinates of the foci for standard hyperbola is given by $(\pm c_2, 0)$ where $c_2^2 = a_2^2 + b_2^2$

Since, their foci coincide

$$\Rightarrow c_1^2 = c_2^2$$

$$\Rightarrow a_1^2 - b_1^2 = a_2^2 + b_2^2$$

Here $a_1 = 4$, $b_1 = b$, $a_2 = \frac{12}{5}$ and $b_2 = \frac{9}{5}$

$\Rightarrow 4^2 - b^2 = \frac{12^2}{5^2} + \frac{9^2}{5^2}$

$\Rightarrow 16 - b^2 = \frac{144}{25} + \frac{81}{25}$

$\Rightarrow 16 - b^2 = \frac{225}{25}$

$\Rightarrow 16 - b^2 = 9$

$\Rightarrow b^2 = 16 - 9$

$\Rightarrow$ **$b^2 = 7$**

6. Question

Write the length of the latus-rectum of the hyperbola $16x^2 - 9y^2 = 144$.

Answer

Given: $16x^2 - 9y^2 = 144$

To find: length of latus-rectum of hyperbola.

$16x^2 - 9y^2 = 144$

$\Rightarrow \frac{16x^2}{144} - \frac{9y^2}{144} = 1$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$

$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{4^2} = 1$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Length of latus rectum is $\frac{2b^2}{a}$

Here, a = 3 and b = 4

Length of latus rectum,

$= \frac{2b^2}{a}$

$= \frac{2 \times (4)^2}{3}$

$= \frac{32}{3}$

7. Question

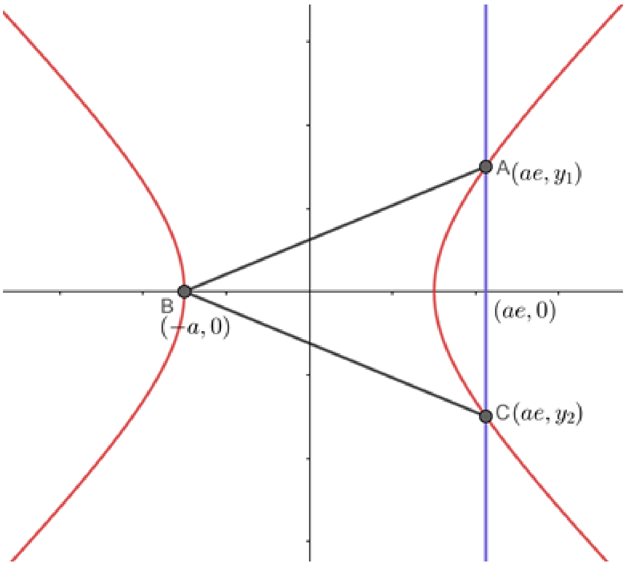
If the latus-rectum through one focus of a hyperbola subtends a right angle at the farther vertex, then write the eccentricity of the hyperbola.

Answer

Given: latus-rectum through one focus of a hyperbola subtends a right angle at the farther vertex

To find: eccentricity of hyperbola

Let B is vertex of hyperbola and A and C are point of intersection of latus-rectum and hyperbola



Standard equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since, A and C lie on hyperbola

Therefore

$$\frac{(ae)^2}{a^2} - \frac{y_1^2}{b^2} = 1$$

$$\Rightarrow \frac{a^2e^2}{a^2} - \frac{y_1^2}{b^2} = 1$$

$$\Rightarrow e^2 - \frac{y_1^2}{b^2} = 1$$

$$\Rightarrow \frac{y_1^2}{b^2} = e^2 - 1$$

$$\Rightarrow y_1^2 = b^2(e^2 - 1) \ldots\ldots\ldots\ldots\ldots..(i)$$

As angle between AB and AC is 90°

$\Rightarrow \text{Slope}_{AB} \times \text{Slope}_{BC} = -1$

$$\Rightarrow \frac{y_1}{ae + a} \times -\frac{y_1}{ae + a} = -1$$

$$\Rightarrow \left(\frac{y_1}{ae + a}\right)^2 = 1$$

$$\Rightarrow \frac{y_1^2}{\{a(e + 1)\}^2} = 1$$

From (i):

$$\Rightarrow \frac{b^2(e^2 - 1)}{a^2(e + 1)^2} = 1$$

$$\Rightarrow \frac{a^2(e^2 - 1)(e^2 - 1)}{a^2(e + 1)^2} = 1$$

$\{\because b^2 = a^2(e^2 - 1)\}$

$\Rightarrow \frac{(e^2-1)^2}{(e+1)^2} = 1$

$\Rightarrow (e^2 - 1)^2 = (e + 1)^2$

$\Rightarrow e^4 + 1 - 2e^2 = e^2 + 1 + 2e$

$\Rightarrow e^4 + 1 - 2e^2 - e^2 - 1 - 2e = 0$

$\Rightarrow e^4 - 3e^2 - 2e = 0$

$\Rightarrow e(e^3 - 3e - 2) = 0$

$\Rightarrow e(e- 2)(e + 1)^2 = 0$

$\Rightarrow$ e = 0 or 2 or -1

But e should be greater than or equal to 1 for hyperbola

$\Rightarrow$ e = 2

Hence, eccentricity of hyperbola is 2

8. Question

Write the distance between the directrices of the hyperbola

x = 8 sec θ, y = 8 tan θ.

Answer

Given: Hyperbola is x = 8 sec θ and y = 8 tan θ

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between directrix is given by $\frac{2a}{e}$

x = 8 sec θ and y = 8 tan θ

$\Rightarrow \sec\theta = \frac{x}{8}$ and $\tan\theta = \frac{y}{8}$

We know,

$\sec^2 \theta - \tan^2 \theta = 1$

$\Rightarrow \left(\frac{x}{8}\right)^2 - \left(\frac{y}{8}\right)^2 = 1$

$\Rightarrow \frac{x^2}{8^2} - \frac{y^2}{8^2} = 1$

Here a = 8 and b = 8

Now,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$\Rightarrow e = \sqrt{1 + \frac{8^2}{8^2}}$

$\Rightarrow e = \sqrt{1 + 1}$

$\Rightarrow e = \sqrt{2}$

Hence, distance between directrix,

$= \frac{2a}{e}$

$= \frac{2(8)}{\sqrt{2}}$

$= \mathbf{8\sqrt{2}}$

9. Question

Write the equation of the hyperbola whose vertices are (±3, 0) and foci at (±5, 0).

Answer

Given: Vertices are (±3, 0) and foci are (±5, 0)

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Vertices of hyperbola are given by (±a, 0)

Foci of hyperbola are given by (±ae, 0)

Vertices are (±3, 0) and foci are (±5, 0)

Therefore,

a = 3 and ae = 5

⇒ 3 × e = 5

$\Rightarrow e = \frac{5}{3}$

$b^2 = a^2(e^2 - 1)$

$\left\{\because\ a = 3 \text{ and } e = \frac{5}{3}\right\}$

$\Rightarrow b^2 = 3^2\left\{\left(\frac{5}{3}\right)^2 - 1\right\}$

$\Rightarrow b^2 = 9\left(\frac{25}{9} - 1\right)$

$\Rightarrow b^2 = 9\left(\frac{25 - 9}{9}\right)$

$\Rightarrow b^2 = 9 \times \frac{16}{9}$

$\Rightarrow b^2 = 16$

Equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \mathbf{\frac{x^2}{9} - \frac{y^2}{16} = 1}$$

Hence, required equation of hyperbola is $\mathbf{\frac{x^2}{9} - \frac{y^2}{16} = 1}$

10. Question

If e_1 and e_2 are respectively the eccentricities of the ellipse $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, then write the value of $2e_1^2 + e_2^2$.

Answer

Given: e_1 and e_2 are respectively the eccentricities of the ellipse $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$

To find: value of $2e_1^2 + e_2^2$

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

$$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{2^2} = 1$$

Eccentricity(e) of hyperbola is given by,

$$e_2 = \frac{c}{a} \text{where } c = \sqrt{a^2 + b^2}$$

Here a = 3 and b = 2

$$c = \sqrt{3^2 + 2^2}$$

$$\Rightarrow c = \sqrt{9 + 4}$$

$$\Rightarrow c = \sqrt{13}$$

Therefore,

$$e_2 = \frac{\sqrt{13}}{3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . . (1)$$

For ellipse:

$$\frac{x^2}{18} + \frac{y^2}{4} = 1$$

$$\frac{x^2}{\left(\sqrt{18}\right)^2} + \frac{y^2}{2^2} = 1$$

Eccentricity(e) of ellipse is given by,

$$e_1 = \frac{c}{b} \text{where } c = \sqrt{b^2 - a^2}$$

Here $a = \sqrt{18}$ and $b = 2$

$c = \sqrt{\left(\sqrt{18}\right)^2 - 2^2}$

$\Rightarrow c = \sqrt{18 - 4}$

$\Rightarrow c = \sqrt{14}$

Therefore,

$e_1 = \frac{\sqrt{14}}{\sqrt{18}}$

$\Rightarrow e_1 = \sqrt{\frac{14}{18}}$

$\Rightarrow e_1 = \sqrt{\frac{7}{9}} \dots \dots \dots \dots \dots \dots . (2)$

Substituting values from (1) and (2) in $2e_1^2 + e_2^2$

$2e_1^2 + e_2^2$

$= 2\left(\sqrt{\frac{7}{9}}\right)^2 + \left(\frac{\sqrt{13}}{3}\right)^2$

$= 2\left(\frac{7}{9}\right) + \frac{13}{9}$

$= \frac{14}{9} + \frac{13}{9}$

$= \frac{14 + 13}{9}$

$= \frac{27}{9}$

= 3

Hence, value of $2e_1^2 + e_2^2$ is **3**

MCQ

1. Question

Equation of the hyperbola whose vertices are (± 3, 0) and foci at (± 5, 0), is

A. $16x^2 - 9y^2 = 144$

B. $9x^2 - 16y^2 = 144$

C. $25x^2 - 9y^2 = 225$

D. $9x^2 - 25y^2 = 81$

Answer

Given: Vertices are (±3, 0) and foci are (±5, 0)

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

Vertices of hyperbola are given by (±a, 0)

Foci of hyperbola are given by (±ae, 0)

Vertices are (±3, 0) and foci are (±5, 0)

Therefore,

a = 3 and ae = 5

⇒ 3 × e = 5

$$\Rightarrow e=\frac{5}{3}$$

$b^2 = a^2(e^2 - 1)$

$$\left\{\because\ a=3 \text{ and } e\ =\frac{5}{3}\right\}$$

$$\Rightarrow b^2=3^2\left\{\left(\frac{5}{3}\right)^2-1\right\}$$

$$\Rightarrow b^2=9\left(\frac{25}{9}-1\right)$$

$$\Rightarrow b^2=9\left(\frac{25-9}{9}\right)$$

$$\Rightarrow b^2=9\times\frac{16}{9}$$

$\Rightarrow b^2 = 16$

Equation of hyperbola:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\Rightarrow \frac{x^2}{9}-\frac{y^2}{16}=1$$

$$\Rightarrow \frac{16x^2-9y^2}{144}=1$$

$\Rightarrow 16x^2 - 9y^2 = 144$

Hence, required equation of hyperbola is **$16x^2 - 9y^2 = 144$**

2. Question

If e_1 and e_2 are respectively the eccentricities of the ellipse $\frac{x^2}{18}+\frac{y^2}{4}=1$ and the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, then the relation between e_1 and e_2 is

A. $3e_1^2 + e_2^2 = 2$

B. $e_1^2 + 2e_2^2 = 3$

C. $2e_1^2 + e_2^2 = 3$

D. $e_1^2 + 3e_2^2 = 2$

Answer

Given: e_1 and e_2 are respectively the eccentricities of the ellipse $\frac{x^2}{18}+\frac{y^2}{4}=1$ and the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$

To find: value of $2e_1^2 + e_2^2$

$$\frac{x^2}{9}-\frac{y^2}{4}=1$$

$$\Rightarrow \frac{x^2}{3^2}-\frac{y^2}{2^2}=1$$

Eccentricity(e) of hyperbola is given by,

$$e_2=\frac{c}{a}\text{where } c=\sqrt{a^2+b^2}$$

Here a = 3 and b = 2

$$c=\sqrt{3^2+2^2}$$

$$\Rightarrow c=\sqrt{9+4}$$

$$\Rightarrow c=\sqrt{13}$$

Therefore,

$$e_2=\frac{\sqrt{13}}{3}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

For ellipse:

$$\frac{x^2}{18}+\frac{y^2}{4}=1$$

$$\frac{x^2}{\left(\sqrt{18}\right)^2}+\frac{y^2}{2^2}=1$$

Eccentricity(e) of ellipse is given by,

$$e_1=\frac{c}{a}\text{where } c=\sqrt{a^2-b^2}$$

Here $a=\sqrt{18}$ and $b=2$

$$c=\sqrt{\left(\sqrt{18}\right)^2-2^2}$$

$$\Rightarrow c=\sqrt{18-4}$$

$$\Rightarrow c=\sqrt{14}$$

Therefore,

$$e_1=\frac{\sqrt{14}}{\sqrt{18}}$$

$$\Rightarrow e_1=\sqrt{\frac{14}{18}}$$

$\Rightarrow e_1 = \sqrt{\frac{7}{9}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots . (2)$

Substituting values from (1) and (2) in $2e_1^2 + e_2^2$

$2e_1^2 + e_2^2$

$= 2\left(\sqrt{\frac{7}{9}}\right)^2 + \left(\frac{\sqrt{13}}{3}\right)^2$

$= 2\left(\frac{7}{9}\right) + \frac{13}{9}$

$= \frac{14}{9} + \frac{13}{9}$

$= \frac{14 + 13}{9}$

$= \frac{27}{9}$

= 3

Hence, value of $2e_1^2 + e_2^2$ is **3**

3. Question

The distance between the directrices of the hyperbola x = 8 sec θ, y = 8 tan θ, is

A. $8\sqrt{2}$

B. $16\sqrt{2}$

C. $4\sqrt{2}$

D. $6\sqrt{2}$

Answer

Given: Hyperbola is x = 8 sec θ and y = 8 tan θ

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Distance between directrix is given by $\frac{2a}{e}$

x = 8 sec θ and y = 8 tan θ

$\Rightarrow \sec\theta = \frac{x}{8} \text{ and } \tan\theta = \frac{y}{8}$

We know,

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \left(\frac{x}{8}\right)^2 - \left(\frac{y}{8}\right)^2 = 1$

$\Rightarrow \frac{x^2}{8^2} - \frac{y^2}{8^2} = 1$

Here a = 8 and b = 8

Now,

$e = \sqrt{1 + \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 + \frac{8^2}{8^2}}$

$\Rightarrow e = \sqrt{1+1}$

$\Rightarrow e = \sqrt{2}$

Hence, distance between directrix,

$= \frac{2a}{e}$

$= \frac{2(8)}{\sqrt{2}}$

$= \mathbf{8\sqrt{2}}$

4. Question

The equation of the conic with focus at (1, -1) directrix along x – y + 1= 0 and eccentricity $\sqrt{2}$ is

A. xy = 1

B. 2xy + 4x – 4y – 1 = 0

C. $x^2 - y^2 = 1$

D. 2xy – 4x + 4y + 1 = 0

Answer

Given: Equation of directrix of a hyperbola is x – y + 1= 0. Focus of hyperbola is (1, -1) and eccentricity (e) is $\sqrt{2}$

To find: equation of conic

Let M be the point on directrix and P(x, y) be any point of hyperbola

Formula used:

$e = \frac{PF}{PM} \Rightarrow PF = ePM$

where e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix

Therefore,

$\sqrt{(x-1)^2 + (y+1)^2} = \sqrt{2}\left|\frac{(x-y+1)}{\sqrt{1^2 + (-1)^2}}\right|$

$\Rightarrow \sqrt{(x-1)^2 + (y+1)^2} = \sqrt{2}\left|\frac{(x-y+1)}{\sqrt{1+1}}\right|$

Squaring both sides:

$\Rightarrow \left(\sqrt{(x-1)^2 + (y+1)^2}\right)^2 = \left(\sqrt{2}\left|\frac{(x-y+1)}{\sqrt{1+1}}\right|\right)^2$

$\Rightarrow (x-1)^2 + (y+1)^2 = \frac{(\sqrt{2})^2 (x-y+1)^2}{2}$

$\Rightarrow (x-1)^2 + (y+1)^2 = \frac{2(x-y+1)^2}{2}$

{∵ $(a - b)^2 = a^2 + b^2 + 2ab$}

$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + y^2 + 1 - 2xy + 2x - 2y$

$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y - x^2 - y^2 + 2xy - 1 - 2x + 2y = 0$

⇒ $2xy - 4x + 4y + 1 = 0$

This is the required equation of hyperbola

5. Question

The eccentricity of the conic $9x^2 - 16y^2 = 144$ is

A. $\frac{5}{4}$

B. $\frac{4}{3}$

C. $\frac{4}{5}$

D. $\sqrt{7}$

Answer

Given: $9x^2 - 16y^2 = 144$

To find: eccentricity(e)

$9x^2 - 16y^2 = 144$

$\Rightarrow \frac{9x^2}{144} - \frac{16y^2}{144} = 1$

$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$

$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$e = \frac{c}{a}$, where $c = \sqrt{a^2 + b^2}$

Here, a = 4 and b = 3

$c = \sqrt{4^2 + 3^2}$

$\Rightarrow c = \sqrt{16 + 9}$

$\Rightarrow c = \sqrt{25}$

⇒ c = 5

Therefore,

$\mathbf{e = \frac{5}{4}}$

Hence, eccentricity is $\frac{5}{4}$

6. Question

A point moves in a plane so that its distances PA and PB from two fixed points A and B in the plane satisfy the relation PA – PB = k(k ≠ 0), then the locus of P is

A. a hyperbola

B. a branch of the hyperbola

C. a parabola

D. an ellipse

Answer

We know it by the fact that when difference in distances is constant, it forms as hyperbola

7. Question

The eccentricity of the hyperbola whose latus-rectum is half of its transverse axis, is

A. $\frac{1}{\sqrt{2}}$

B. $\sqrt{\frac{2}{3}}$

C. $\sqrt{\frac{3}{2}}$

D. none of these

Answer

Given: Latus-rectum is half of its transverse axis

To find: eccentricity of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Length of transverse axis is 2a

Latus-rectum of the hyperbola is $\frac{2b^2}{a}$

According to question:

Latus-rectum is half of its transverse axis

$$\Rightarrow \frac{2b^2}{a} = \frac{1}{2} \times 2a$$

$$\Rightarrow 2b^2 = a^2$$

We know,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{b^2}{2b^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{1}{2}}$$

$$\Rightarrow e = \sqrt{\frac{2+1}{2}}$$

$$\Rightarrow e = \sqrt{\frac{3}{2}}$$

Hence, eccentricity is $\sqrt{\frac{3}{2}}$

8. Question

The eccentricity of the hyperbola $x^2 - 4y^2 = 1$ is

A. $\frac{\sqrt{3}}{2}$

B. $\frac{\sqrt{5}}{2}$

C. $\frac{2}{\sqrt{3}}$

D. $\frac{2}{\sqrt{5}}$

Answer

Given: $x^2 - 4y^2 = 1$

To find: eccentricity(e)

$x^2 - 4y^2 = 1$

$$\Rightarrow \frac{x^2}{1} - \frac{y^2}{\frac{1}{4}} = 1$$

$$\Rightarrow \frac{x^2}{1} - \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{where } c = \sqrt{a^2 + b^2}$$

Here, a = 1 and $b = \frac{1}{2}$

$$c = \sqrt{1^2 + \left(\frac{1}{2}\right)^2}$$

$$\Rightarrow c = \sqrt{1 + \frac{1}{4}}$$

$$\Rightarrow c = \sqrt{\frac{5}{4}}$$

$$\Rightarrow c = \frac{\sqrt{5}}{2}$$

Therefore,

$$e = \frac{\frac{\sqrt{5}}{2}}{1}$$

$$\Rightarrow \mathbf{e} = \frac{\sqrt{\mathbf{5}}}{\mathbf{2}}$$

Hence, eccentricity is $\frac{\sqrt{5}}{2}$

9. Question

The difference of the focal distances of any point on the hyperbola is equal to

A. length of the conjugate axis

B. eccentricity

C. length of the transverse axis

D. Latus-rectum

Answer

This is definition of eccentricity.

Eccentricity is difference of the focal distances of any point on the hyperbola

10. Question

the foci of the hyperbola $9x^2 - 16y^2 = 144$ are

A. (± 4, 0)

B. (0, ± 4)

C. (± 5, 0)

D. (0, ± 5)

Answer

Given: $9x^2 - 16y^2 = 144$

To find: coordinates of the foci f(m,n)

$9x^2 - 16y^2 = 144$

$\Rightarrow \frac{9x^2}{144} - \frac{16y^2}{144} = 1$

$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$

$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$e = \frac{c}{a}$, where $c = \sqrt{a^2 + b^2}$

Foci is given by (±ae, 0)

Here, a = 4 and b = 3

$c = \sqrt{4^2 + 3^2}$

$\Rightarrow c = \sqrt{16 + 9}$

$\Rightarrow c = \sqrt{25}$

⇒ c = 5

Therefore,

$\mathbf{e = \frac{5}{4}}$

$\Rightarrow ae = 4 \times \frac{5}{4} = 5$

Foci: (±5, 0)

11. Question

The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then equation of the hyperbola is

A. $x^2 + y^2 = 32$

B. $x^2 - y^2 = 16$

C. $x^2 + y^2 = 16$

D. $x^2 - y^2 = 32$

Answer

Given: Distance between foci is 16 and eccentricity is $\sqrt{2}$

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between foci is given by 2ae

According to question:

2ae = 16

$$\Rightarrow a = \frac{16}{2e}$$

$$\{\because e = \sqrt{2}\}$$

$$\Rightarrow a = \frac{8}{\sqrt{2}}$$

$$\Rightarrow a = 4\sqrt{2}$$

$\Rightarrow a^2 = 32$

We know,

$b^2 = a^2(e^2 - 1)$

$$\Rightarrow b^2 = 32\left\{\left(\sqrt{2}\right)^2 - 1\right\}$$

$$\Rightarrow b^2 = 32(2 - 1)$$

$\Rightarrow b^2 = 32$

Hence, equation of hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{32} - \frac{y^2}{32} = 1$$

$\Rightarrow x^2 - y^2 = 32$

12. Question

If e_1 is the eccentricity of the conic $9x^2 + 4y^2 = 36$ and e_2 is the eccentricity of the conic $9x^2 - 4y^2 = 36$, then

A. $e_1^2 - e_2^2 = 2$

B. $2 < e_2^2 - e_1^2 < 3$

C. $e_2^2 - e_1^2 = 2$

D. $e_2^2 - e_1^2 > 3$

Answer

Given: e_1 and e_2 are respectively the eccentricities of $9x^2 + 4y^2 = 36$ and $9x^2 - 4y^2 = 36$ respectively

To find: $e_1^2 - e_2^2$

$9x^2 - 4y^2 = 36$

$$\Rightarrow \frac{9x^2}{36} - \frac{4y^2}{36} = 1$$

$$\Rightarrow \frac{x^2}{4} - \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{2^2} - \frac{y^2}{3^2} = 1$$

Eccentricity(e) of hyperbola is given by,

$$e_2 = \frac{c}{a} \text{where } c = \sqrt{a^2 + b^2}$$

Here a = 2 and b = 3

$$c = \sqrt{2^2 + 3^2}$$

$$\Rightarrow c = \sqrt{4 + 9}$$

$$\Rightarrow c = \sqrt{13}$$

Therefore,

$$e_2 = \frac{\sqrt{13}}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

For ellipse:

$9x^2 + 4y^2 = 36$

$$\Rightarrow \frac{9x^2}{36} + \frac{4y^2}{36} = 1$$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{2^2} + \frac{y^2}{3^2} = 1$$

Eccentricity(e) of ellipse is given by,

$$e_1 = \frac{c}{b} \text{where } c = \sqrt{b^2 - a^2}$$

Here a = 2 and b = 3

$$c = \sqrt{3^2 - 2^2}$$

$$\Rightarrow c = \sqrt{9 - 4}$$

$$\Rightarrow c = \sqrt{5}$$

Therefore,

$$e_1 = \frac{\sqrt{5}}{3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

Substituting values from (1) and (2) in $2e_1^2 + e_2^2$

$e_1^2 - e_2^2$

$$= \left(\frac{\sqrt{5}}{3}\right)^2 - \left(\frac{\sqrt{13}}{2}\right)^2$$

$$= \frac{5}{9} - \frac{13}{4}$$

$$= \frac{20 - 117}{36}$$

$$= \frac{-97}{36}$$

$\Rightarrow e_2^2 - e_2^2$

$$= \frac{97}{36}$$

Hence, value of $2 < e_2^2 - e_1^2 < 3$

13. Question

If the eccentricity of the hyperbola $x^2 - y^2 \sec^2\propto = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2 \sec^2\propto + y^2 = 25$, then $\propto =$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer

Given: e_1 and e_2 are respectively the eccentricities of $x^2 - y^2 \sec^2 \alpha = 5$ and $x^2 \sec^2 \alpha + y^2 = 25$ respectively

To find: value of α

$x^2 - y^2 \sec^2 \alpha = 5$

$$\Rightarrow \frac{x^2}{5} - \frac{y^2 \sec^2 \alpha}{5} = 1$$

$$\Rightarrow \frac{x^2}{5} - \frac{y^2}{\frac{5}{\sec^2 \alpha}} = 1$$

$$\Rightarrow \frac{x^2}{(\sqrt{5})^2} - \frac{y^2}{\left(\frac{\sqrt{5}}{\sec \alpha}\right)^2} = 1$$

Eccentricity(e) of hyperbola is given by,

$$e_2 = \frac{c}{a} \text{ where } c = \sqrt{a^2 + b^2}$$

$$\text{Here } a = \sqrt{5} \text{ and } b = \frac{\sqrt{5}}{\sec\alpha}$$

$$c = \sqrt{\left(\sqrt{5}\right)^2 + \left(\frac{\sqrt{5}}{\sec\alpha}\right)^2}$$

$$\Rightarrow c = \sqrt{5 + \frac{5}{\sec^2\alpha}}$$

$$\Rightarrow c = \sqrt{\frac{5\sec^2\alpha + 5}{\sec^2\alpha}}$$

$$\Rightarrow c = \sqrt{\frac{5(\sec^2\alpha + 1)}{\sec^2\alpha}}$$

$$\Rightarrow c = \sqrt{\frac{5\left(\frac{1}{\cos^2\alpha} + 1\right)}{\frac{1}{\cos^2\alpha}}}$$

$$\Rightarrow c = \sqrt{\frac{5\left(\frac{1 + \cos^2\alpha}{\cos^2\alpha}\right)}{\frac{1}{\cos^2\alpha}}}$$

$$\Rightarrow c = \sqrt{5(1 + \cos^2\alpha)}$$

Therefore,

$$e_2 = \frac{\sqrt{5(1 + \cos^2\alpha)}}{\sqrt{5}}$$

$$\Rightarrow e_2 = \sqrt{1 + \cos^2\alpha} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

For ellipse:

$x^2 \sec^2\alpha + y^2 = 25$

$$\Rightarrow \frac{x^2\sec^2\alpha}{25} + \frac{y^2}{25} = 1$$

$$\Rightarrow \frac{x^2}{\frac{25}{\sec^2\alpha}} + \frac{y^2}{25} = 1$$

$$\Rightarrow \frac{x^2}{\left(\frac{5}{\sec\alpha}\right)^2} + \frac{y^2}{5^2} = 1$$

$$\Rightarrow \frac{x^2}{(5\cos\alpha)^2} + \frac{y^2}{5^2} = 1$$

Eccentricity(e) of ellipse is given by,

$$e_1 = \frac{c}{b} \text{ where } c = \sqrt{b^2 - a^2}$$

Here $a = 5\cos\alpha$ and $b = 5$

$c = \sqrt{5^2 - (5\cos\alpha)^2}$

$\Rightarrow c = \sqrt{25 - 25\cos^2\alpha}$

$\Rightarrow c = \sqrt{25(1 - \cos^2\alpha)}$

$\Rightarrow c = 5\sqrt{1 - \cos^2\alpha}$

Therefore,

$$e_1 = \frac{5\sqrt{1 - \cos^2\alpha}}{5}$$

$\Rightarrow e_1 = \sqrt{1 - \cos^2\alpha} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2)$

According to question:

Eccentricity of given hyperbola is $\sqrt{3}$ time eccentricity of given ellipse

$\Rightarrow\ e_2 = \sqrt{3}e_1$

From (1) and (2):

$\Rightarrow\ \sqrt{1 + \cos^2\alpha} = \sqrt{3}\sqrt{1 - \cos^2\alpha}$

Squaring both sides:

$\Rightarrow 1 + \cos^2 \alpha = 3(1 - \cos^2 \alpha)$

$\Rightarrow 1 + \cos^2 \alpha = 3 - 3\cos^2 \alpha$

$\Rightarrow 3\cos^2 \alpha + \cos^2 \alpha = 3 - 1$

$\Rightarrow 4\cos^2 \alpha = 2$

$$\Rightarrow \cos^2\alpha = \frac{1}{2}$$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \boldsymbol{\alpha = \frac{\pi}{4}}$$

14. Question

The equation of the hyperbola whose foci are (6, 4) and (-4, 4) and eccentricity 2, is

A. $\frac{(x-1)^2}{25/4} - \frac{(y-4)^2}{75/4} = 1$

B. $\frac{(x+1)^2}{25/4} - \frac{(y+4)^2}{75/4} = 1$

C. $\frac{(x-1)^2}{75/4} - \frac{(y-4)^2}{25/4} = 1$

D. none of these

Answer

Given: Foci are (6, 4) and (-4, 4) and eccentricity is 2

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$\frac{(x-x_1)^2}{a^2} - \frac{(y-y_1)^2}{b^2} = 1$ where center is (x_1, y_1)

Center is the mid-point of two foci.

Distance between the foci is 2ae and $b^2 = a^2(e^2 - 1)$

Distance between two points (m, n) and (a, b) is given by

$\sqrt{(m-a)^2 + (n-b)^2}$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$\left(\frac{m+a}{2}, \frac{n+b}{2}\right)$

Center of hyperbola having foci (6, 4) and (-4, 4) is given by

$= \left(\frac{6-4}{2}, \frac{4+4}{2}\right)$

$= \left(\frac{2}{2}, \frac{8}{2}\right)$

= (1, 4)

Distance between the foci is 2ae and Foci are (6, 4) and (-4, 4)

$\Rightarrow \sqrt{(6+4)^2 + (4-4)^2} = 2ae$

$\Rightarrow \sqrt{(10)^2 + (0)^2} = 2ae$

$\Rightarrow \sqrt{100} = 2ae$

$\Rightarrow 10 = 2ae$

$\Rightarrow \frac{10}{2} = ae$

$\Rightarrow ae = 5$

{∵ e = 2}

$\Rightarrow a \times 2 = 5$

$\Rightarrow a = \frac{5}{2}$

$\Rightarrow a^2 = \frac{25}{4}$

$b^2 = a^2(e^2 - 1)$

$\Rightarrow b^2 = \frac{25}{4}\{(2)^2 - 1\}$

$\Rightarrow b^2 = \frac{25}{4}(4 - 1)$

$\Rightarrow b^2 = \frac{25}{4}(3)$

$\Rightarrow b^2 = \frac{75}{4}$

Equation of hyperbola:

$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1$

$\Rightarrow \frac{(x - 1)^2}{\frac{25}{4}} - \frac{(y - 4)^2}{\frac{75}{4}} = 1$

Hence, required equation of hyperbola is $\frac{(x-1)^2}{\frac{25}{4}} - \frac{(y-4)^2}{\frac{75}{4}} = 1$

15. Question

The length of the straight line x – 3y = 1 intercepted by the hyperbola $x^2 - 4y^2 = 1$ is

A. $\frac{6}{\sqrt{5}}$

B. $3\sqrt{\frac{2}{5}}$

C. $6\sqrt{\frac{2}{5}}$

D. none of these

Answer

Given: A straight line x – 3y = 1 intercepts hyperbola $x^2 - 4y^2 = 1$

To find: Length of the intercepted line

Formula used:

Distance between two points (m, n) and (a, b) is given by

$\sqrt{(m - a)^2 + (n - b)^2}$

Firstly we will find point of intersections of given line and hyperbola

x – 3y = 1 ⇒ x = 1 + 3y

$x^2 - 4y^2 = 1$

$\Rightarrow (1 + 3y)^2 - 4y^2 = 1$

$\Rightarrow 1 + 9y^2 + 6y - 4y^2 = 1$

$\Rightarrow 5y^2 + 6y = 0$

⇒ y(5y + 6) = 0

⇒ y = 0 or 5y + 6 = 0

$\Rightarrow\ y = 0 \text{ or } -\frac{6}{5}$

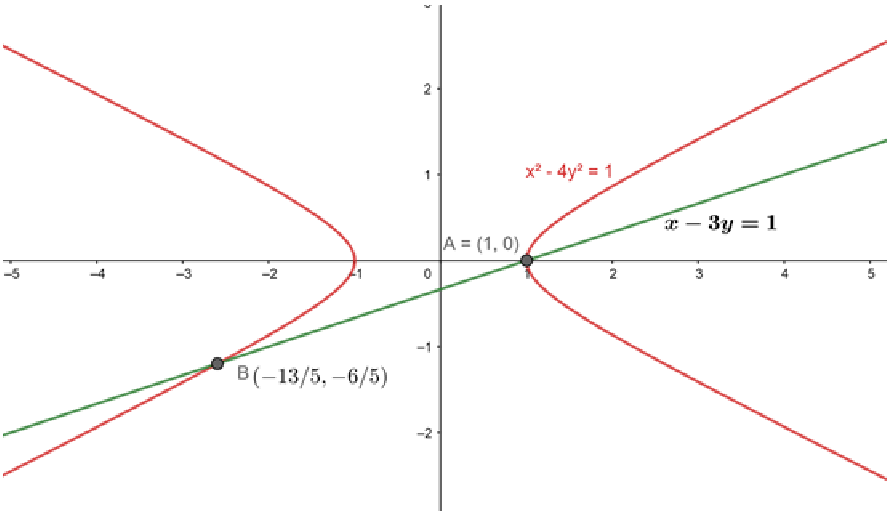
Now, x = 1 + 3y

$\Rightarrow\ x = 1 + 3(0) \text{ or } 1 + 3\left(-\frac{6}{5}\right)$

$\Rightarrow\ x = 1 \text{ or } 1 - \frac{18}{5}$

$\Rightarrow\ x = 1 \text{ or } -\frac{13}{5}$

So, Point of intersections are A(1, 0) and $B\left(-\frac{13}{5}, -\frac{6}{5}\right)$



Distance between point of intersections is

$= \sqrt{\left(1 - \left(-\frac{13}{5}\right)\right)^2 + \left(0 - \left(-\frac{6}{5}\right)\right)^2}$

$= \sqrt{\left(1 + \frac{13}{5}\right)^2 + \left(\frac{6}{5}\right)^2}$

$= \sqrt{\left(\frac{18}{5}\right)^2 + \left(\frac{6}{5}\right)^2}$

$= \sqrt{\frac{324}{25} + \frac{36}{25}}$

$= \sqrt{\frac{360}{25}}$

$= \sqrt{\frac{72}{5}}$

$= 6\sqrt{\frac{2}{5}}$

6. Question

The latus-rectum of the hyperbola $16x^2 - 9y^2 = 144$ is

A. 16/3

B. 32/3

C. 8/3

D. 4/3

Answer

Given: $16x^2 - 9y^2 = 144$

To find: length of latus-rectum of hyperbola.

$16x^2 - 9y^2 = 144$

$\Rightarrow \frac{16x^2}{144} - \frac{9y^2}{144} = 1$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$

$\Rightarrow \frac{x^2}{3^2} - \frac{y^2}{4^2} = 1$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Length of latus rectum is $\frac{2b^2}{a}$

Here, a = 3 and b = 4

Length of latus rectum,

$= \frac{2b^2}{a}$

$= \frac{2 \times (4)^2}{3}$

$= \frac{32}{3}$

17. Question

The foci of the hyperbola $2x^2 - 3y^2 = 5$ are

A. $(\pm 5\sqrt{6}, 0)$

B. (± 5/6, 0)

C. $(\pm\sqrt{5} / 6, 0)$

D. none of these

Answer

Given: $2x^2 - 3y^2 = 5$

To find: coordinates of the foci f(m,n)

$2x^2 - 3y^2 = 5$

$$\Rightarrow \frac{2x^2}{5} - \frac{3y^2}{5} = 1$$

$$\Rightarrow \frac{x^2}{\frac{5}{2}} - \frac{y^2}{\frac{5}{3}} = 1$$

$$\Rightarrow \frac{x^2}{\left(\sqrt{\frac{5}{2}}\right)^2} - \frac{y^2}{\left(\sqrt{\frac{5}{3}}\right)^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$e = \frac{c}{a}$, where $c = \sqrt{a^2 + b^2}$

Foci is given by (±ae, 0)

Here, $a = \sqrt{\frac{5}{2}}$ and $b = \sqrt{\frac{5}{3}}$

$$c = \sqrt{\left(\sqrt{\frac{5}{2}}\right)^2 + \left(\sqrt{\frac{5}{3}}\right)^2}$$

$$\Rightarrow c = \sqrt{\frac{5}{2} + \frac{5}{3}}$$

$$\Rightarrow c = \sqrt{\frac{15 + 10}{6}}$$

$$\Rightarrow c = \sqrt{\frac{25}{6}}$$

$$\Rightarrow c = \frac{5}{\sqrt{6}}$$

Therefore,

$$e = \frac{\frac{5}{\sqrt{6}}}{\sqrt{\frac{5}{2}}}$$

$$\Rightarrow e = \sqrt{\frac{5}{3}}$$

$$\Rightarrow ae = \sqrt{\frac{5}{2}} \times \sqrt{\frac{5}{3}} = \frac{5}{\sqrt{6}}$$

Foci: $\left(\pm\frac{5}{\sqrt{6}}, 0\right)$

18. Question

The eccentricity the hyperbola $x = \frac{a}{2}\left(t + \frac{1}{t}\right), y = \frac{a}{2}\left(t - \frac{1}{t}\right)$ is

A. $\sqrt{2}$

B. $\sqrt{3}$

C. $2\sqrt{3}$

D. $3\sqrt{2}$

Answer

Given: Equation of hyperbola $x = \frac{a}{2}\left(t + \frac{1}{t}\right), y = \frac{a}{2}\left(t - \frac{1}{t}\right)$

To find: Eccentricity of the hyperbola

$x = \frac{a}{2}\left(t + \frac{1}{t}\right)$

$\Rightarrow \frac{2x}{a} = t + \frac{1}{t}$

Squaring both sides:

$\Rightarrow \left(\frac{2x}{a}\right)^2 = \left(t + \frac{1}{t}\right)^2$

$\Rightarrow \frac{4x^2}{a^2} = t^2 + \frac{1}{t^2} + 2(t)\left(\frac{1}{t}\right)$

$\Rightarrow \frac{4x^2}{a^2} = t^2 + \frac{1}{t^2} + 2$

$\Rightarrow t^2 + \frac{1}{t^2} = \frac{4x^2}{a^2} - 2 \ldots\ldots\ldots\ldots\ldots..(1)$

$y = \frac{a}{2}\left(t - \frac{1}{t}\right)$

$\Rightarrow \frac{2y}{a} = t - \frac{1}{t}$

Squaring both sides:

$\Rightarrow \left(\frac{2y}{a}\right)^2 = \left(t - \frac{1}{t}\right)^2$

$\Rightarrow \frac{4y^2}{a^2} = t^2 + \frac{1}{t^2} - 2(t)\left(\frac{1}{t}\right)$

$\Rightarrow \frac{4y^2}{a^2} = t^2 + \frac{1}{t^2} - 2$

$\Rightarrow t^2 + \frac{1}{t^2} = \frac{4y^2}{a^2} + 2 \ldots\ldots\ldots\ldots\ldots..(2)$

From (1) and (2):

$$\frac{4x^2}{a^2} - 2 = \frac{4y^2}{a^2} + 2$$

$$\Rightarrow \frac{4x^2}{a^2} - \frac{4y^2}{a^2} = 4$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$$

Formula used:

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{where } c = \sqrt{a^2 + b^2}$$

Here a = a, b = a

$$c = \sqrt{(a)^2 + (a)^2}$$

$$\Rightarrow c = \sqrt{2a^2}$$

$$\Rightarrow c = \sqrt{2}a$$

Therefore,

$$e = \frac{\sqrt{2}a}{a}$$

$$\Rightarrow e = \sqrt{2}$$

Hence, the **eccentricity of the hyperbola is $\sqrt{2}$**

19. Question

The equation of the hyperbola whose centre is (6, 2) one focus is (4, 2) and of eccentricity 2 is

A. $3(x - 6)^2 - (y - 2)^2 = 3$

B. $(x - 6)^2 - 3(y - 2)^2 = 1$

C. $(x - 6)^2 - 2(y - 2)^2 = 1$

D. $2(x - 6)^2 - (y - 2)^2 = 1$

Answer

Given: Foci is (4, 2), e = 2 and center at (6, 2)

To find: equation of the hyperbola

Formula used:

Standard form of the equation of hyperbola is,

$$\frac{(x - x_1)^2}{a^2} - \frac{(y - y_1)^2}{b^2} = 1 \text{ where center is } (x_1, y_1)$$

Center is the mid-point of two vertices

The distance between two vertices is 2a

The distance between the foci and vertex is ae – a and $b^2 = a^2(e^2 - 1)$

The distance between two points (m, n) and (a, b) is given by

$$\sqrt{(m-a)^2+(n-b)^2}$$

Mid-point theorem:

Mid-point of two points (m, n) and (a, b) is given by

$$\left(\frac{m+a}{2}, \frac{n+b}{2}\right)$$

Therefore

Let one of the two foci is (m, n) and the other one is (4, 2)

Since, Centre(6, 2)

$$\left(\frac{m+4}{2}=6, \frac{n+2}{2}=2\right)$$

$$\Rightarrow (m+4=12, n+2=4)$$

$$\Rightarrow (m=8, n=2)$$

Foci are (4, 2) and (8, 2)

The distance between the foci is 2ae and Foci are (4, 2) and (8, 2)

$$\Rightarrow \sqrt{(4-8)^2+(2-2)^2}=2ae$$

$$\Rightarrow \sqrt{(-4)^2+(0)^2}=2ae$$

$$\Rightarrow \sqrt{16}=2ae$$

$$\Rightarrow 4=2ae$$

$$\Rightarrow \frac{4}{2}=ae$$

$$\Rightarrow ae=2$$

$$\{\because\ e\ =2\}$$

$$\Rightarrow a\times 2=2$$

$$\Rightarrow a=\frac{2}{2}=1$$

$\Rightarrow a^2 = 1$

$b^2 = a^2(e^2 - 1)$

$$\Rightarrow b^2=1\{(2)^2-1\}$$

$\Rightarrow b^2 = 4 - 1$

$\Rightarrow b^2 = 3$

Equation of hyperbola:

$$\frac{(x-x_1)^2}{a^2}-\frac{(y-y_1)^2}{b^2}=1$$

$\Rightarrow \frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$

$\Rightarrow \frac{3(x-6)^2 - (y-2)^2}{3} = 1$

⇒ 3(x - 6)^2 - (y - 2)^2 = 3

Hence, required equation of hyperbola is **3(x - 6)^2 - (y - 2)^2 = 3**

20. Question

The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}\lambda = 0$ and $\sqrt{3}\lambda x + \lambda y - 4\sqrt{3} = 0$ is a hyperbola of eccentricity

A. 1

B. 2

C. 3

D. 4

Answer

Given: A hyperbola is formed by the locus of the point of intersection of lines $\sqrt{3}x - y - 4\sqrt{3}\lambda = 0$ and $\sqrt{3}\lambda x + \lambda y - 4\sqrt{3} = 0$

To find: Eccentricity of the hyperbola

$\sqrt{3}\lambda x + \lambda y - 4\sqrt{3} = 0 \dots\dots\dots\dots\dots\dots.(1)$

$\sqrt{3}x - y - 4\sqrt{3}\lambda = 0$

Multiply by λ:

$\Rightarrow \sqrt{3}\lambda x - \lambda y - 4\sqrt{3}\lambda^2 = 0 \dots\dots\dots\dots\dots\dots(2)$

Adding (1) and (2):

$\sqrt{3}\lambda x + \lambda y - 4\sqrt{3} + \sqrt{3}\lambda x - \lambda y - 4\sqrt{3}\lambda^2 = 0 + 0$

$\Rightarrow 2\sqrt{3}\lambda x - 4\sqrt{3} - 4\sqrt{3}\lambda^2 = 0$

$\Rightarrow 2\sqrt{3}\lambda x = 4\sqrt{3} + 4\sqrt{3}\lambda^2$

$\Rightarrow 2\sqrt{3}\lambda x = 4\sqrt{3}(1 + \lambda^2)$

$\Rightarrow x = \frac{4\sqrt{3}(1+\lambda^2)}{2\sqrt{3}\lambda}$

$\Rightarrow x = \frac{2(1+\lambda^2)}{\lambda}$

$\Rightarrow x = 2\left(\lambda + \frac{1}{\lambda}\right)$

Now, From (1):

$\sqrt{3}\lambda\left(2\left(\lambda + \frac{1}{\lambda}\right)\right) + \lambda y - 4\sqrt{3} = 0$

$\Rightarrow 2\sqrt{3}\lambda\left(\lambda + \frac{1}{\lambda}\right) + \lambda y = 4\sqrt{3}$

$$\Rightarrow 2\sqrt{3}\left(\lambda+\frac{1}{\lambda}\right)+y=\frac{4\sqrt{3}}{\lambda}$$

$$\Rightarrow y=\frac{4\sqrt{3}}{\lambda}-2\sqrt{3}\left(\lambda+\frac{1}{\lambda}\right)$$

$$\Rightarrow y=\frac{4\sqrt{3}}{\lambda}-2\sqrt{3}\lambda-\frac{2\sqrt{3}}{\lambda}$$

$$\Rightarrow y=\frac{2\sqrt{3}}{\lambda}-2\sqrt{3}\lambda$$

$$\Rightarrow y=2\sqrt{3}\left(\frac{1}{\lambda}-\lambda\right)$$

$$x=2\left(\lambda+\frac{1}{\lambda}\right) \text{ and } y=2\sqrt{3}\left(\frac{1}{\lambda}-\lambda\right)$$

$$\Rightarrow \frac{x}{2}=\left(\lambda+\frac{1}{\lambda}\right) \text{ and } \frac{y}{2\sqrt{3}}=\left(\frac{1}{\lambda}-\lambda\right)$$

Squaring both sides:

$$\Rightarrow \left(\frac{x}{2}\right)^2=\left(\lambda+\frac{1}{\lambda}\right)^2 \text{ and } \left(\frac{y}{2\sqrt{3}}\right)^2=\left(\frac{1}{\lambda}-\lambda\right)^2$$

$$\Rightarrow \left(\frac{x}{2}\right)^2=\lambda^2+\frac{1}{\lambda^2}+2(\lambda)\left(\frac{1}{\lambda}\right)$$

$$\Rightarrow \frac{x^2}{4}=\lambda^2+\frac{1}{\lambda^2}+2$$

$$\Rightarrow \lambda^2+\frac{1}{\lambda^2}=\frac{x^2}{4}-2 \ldots\ldots\ldots..(3)$$

$$\Rightarrow \frac{y^2}{12}=\lambda^2+\frac{1}{\lambda^2}-2(\lambda)\left(\frac{1}{\lambda}\right)$$

$$\Rightarrow \frac{y^2}{12}=\lambda^2+\frac{1}{\lambda^2}-2$$

$$\Rightarrow \lambda^2+\frac{1}{\lambda^2}=\frac{y^2}{12}+2 \ldots\ldots\ldots.(4)$$

From (3) and (4):

$$\frac{x^2}{4}-2=\frac{y^2}{12}+2$$

$$\Rightarrow \frac{x^2}{4}-\frac{y^2}{12}=4$$

$$\Rightarrow \frac{x^2}{16}-\frac{y^2}{48}=1$$

$$\Rightarrow \frac{x^2}{4^2}-\frac{y^2}{\left(4\sqrt{3}\right)^2}=1$$

Formula used:

For hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$:

Eccentricity(e) is given by,

$$e = \frac{c}{a}, \text{where } c = \sqrt{a^2 + b^2}$$

$$\text{Here } a = 4 \text{ and } b = 4\sqrt{3}$$

$$c = \sqrt{(4)^2 + \left(4\sqrt{3}\right)^2}$$

$$\Rightarrow c = \sqrt{16 + 48}$$

$$\Rightarrow c = \sqrt{64}$$

⇒ c = 8

Therefore,

$$e = \frac{8}{4}$$

⇒ e = 2

Hence, **eccentricity of hyperbola is 2**