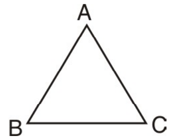
Chapter 22

TRIANGLES

KEY FACTS

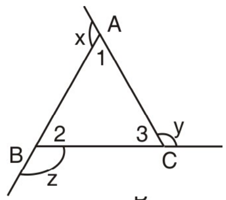
1. A **triangle** is a plane closed figure bounded by three line segments.



(*i*) Sum of the angles of a triangle is equal to 180°, *i.e.*,

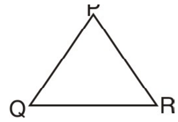
$$\angle A + \angle B + \angle C = 180°$$

(*ii*) Exterior angle of a triangle is equal to the sum of its interior opposite angles, *i.e.*,



$$\angle x = \angle 2 + \angle 3;\ \angle y = \angle 1 + \angle 2\ ;\ \angle z = \angle 1 + \angle 3$$

(*iii*) Sum of any two sides of a triangle is always, greater than the third side, *i.e.*,

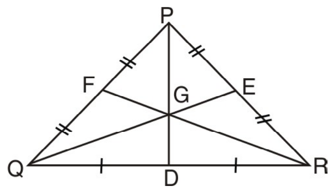


$$PQ + QR > PR$$
$$PQ + PR > QR$$
$$PR + QR > PQ$$

(iv) Side opposite to the greatest angle will be greatest in length and vice versa.

2. Important Terms of a Triangle

(i) **Median and centroid:** A line joining the mid-point of a side of a triangle to its opposite vertex is called the median. *D,E,F* are the mid-points of the sides *QR, PR* and *PQ* respectively of a ΔPQR. Then, *PD, QE* and *RF* are the medians of ΔPQR.



- The point of concurrency of the three medians of a triangle is called **centroid.**
- The centroid of a triangle divides each median in the ratio 2:1, *i.e.*,

$$PG : GD = QG : GE = RG : GF = 2:1$$

- A median divides a triangle into two parts of equal area.

(ii) **Perpendicular bisector and circumcentre:** Perpendicular bisector to any side is the line that is perpendicular to that side and passes through its mid-point. Perpendicular bisectors need not pass through the opposite vertex.

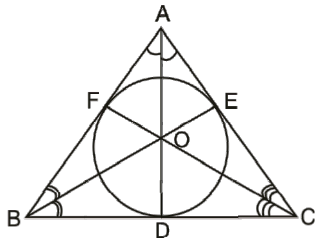
- The point of intersection of the three perpendicular bisectors of a triangle is called its **circumcentre**.
- The **circumcentre** of a triangle is **equidistant from its three vertices**.

If we draw a circle with circumcentre as the centre and the distance of any vertex from the circumcentre as radius, the circle passes through all the three vertices and the circle is called **circumcircle.**

Note. The circumcentre can be inside or outside the circle.

- Circumcentre of a right angled triangle is the mid-point of the hypotenuse.

(iii) **Angle bisector and in-centre:**



- The point of intersection of the three angle bisectors of a triangle is called its **in-centre.**
- The **in-centre** always lies inside the triangle.
- It is always **equidistant from the sides of a triangle.**
- The circle drawn with incentre as centre and touching all the three sides of a triangle is called **in-circle.**

(iv) **Altitude and ortho-centre:**

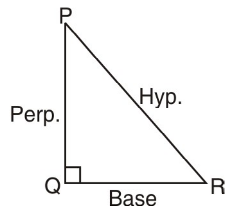
The perpendicular drawn from the vertex of a triangle to the opposite side is called an **altitude.**

- The point of intersection of the three altitudes of a triangle is called **ortho-centre**, which can lie inside or outside the triangle.

Note.
- For an isosceles triangle, the median drawn from a vertex to the opposite side is also the perpendicular bisector of that side.
- In an equilateral triangle, the median, angle bisector, altitude and perpendicular bisector of sides are all represented by the same straight line.
- The circumcentre, centroid, orthocentre and incentre all coincide in an equilateral triangle.

3. Pythagoras' theorem:

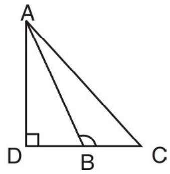
(i) In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



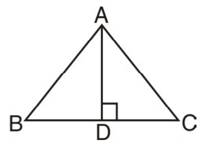
$\therefore$ $(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow$ $PQ^2 + QR^2 = PR^2$

(ii) In a ΔABC, obtuse angled at B, if $AD \perp CB$, then
$AC^2 = AB^2 + BC^2 + 2BC.BD$



(iii) $\angle B$ of ΔABC is acute and $AD \perp BC$, then
$AC^2 = AB^2 + BC^2 - 2BC.BD$



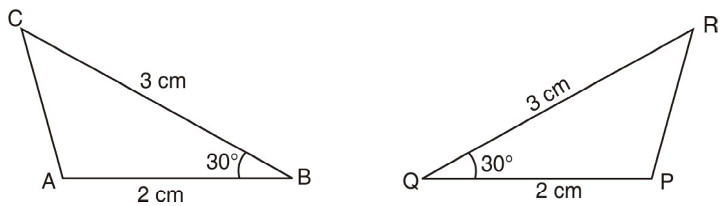
4. Congruent Figures

When two geometric figures have the same size and shape, they are said be congruent.

Conditions for congruency of triangles

(i) **SAS axiom (side-angle-side)**

If two triangles have two sides and the included angle of one respectively equal to two sides and the included angle of the other, the triangles are congruent.



In Δs ABC and PQR

$\angle B = \angle Q$

$AB = PQ$

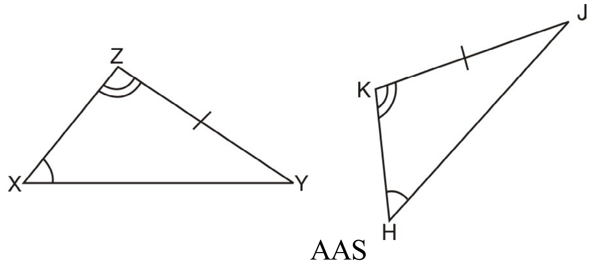
$BC = QR$

$\therefore$ $\Delta ABC \cong \Delta PQR$ (SAS)

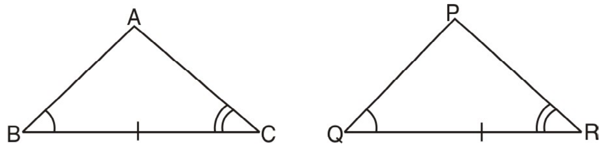
(ii) ***ASA* or *AAS* axiom (two angles - one side)**

If the triangles have two angles of one respectively equal to two angles of the other, and also a side of one triangle equal to the corresponding side of the other, the two triangles are congruent.

The side may be the included side of the two angles also.



AAS
(Angle, Angle, Side)



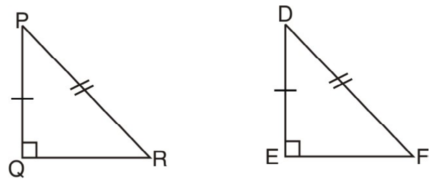
ASA
(Angle, Side, Angle)

(iii) **SSS axiom (three sides)**

If two triangles have three sides of one respectively equal to three sides of the other, the triangles are congruent.

(iv) **RHS axiom (right angle, hypotenuse, sides)**

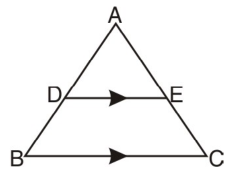
Two right triangles are congruent, if one side and hypotenuse of one are respectively equal to the corresponding side and hypotenuse of the other.



$$PQ = DE$$
$$\text{Hyp. } PR = \text{Hyp. } DF$$
$$\therefore \quad \Delta PQR \cong \Delta DEF$$

5. Basic Proportionality Theorem

If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided proportionally in the same ratio.



Thus, in ΔABC, if $DE \parallel BC$, then $\frac{AD}{DB} = \frac{AE}{EC}$

Conversely, *if a straight line divides any two sides of a triangle in the same ratio, then the straight line is parallel to the third side of the triangle.*

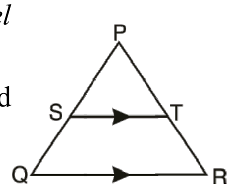
Thus, if in ΔABC, a line DE is drawn such that $\frac{AD}{DB} = \frac{AE}{EC}$, then $DE \parallel BC$.

6. Mid-point Theorem

A straight line drawn through the mid-point of one side, parallel to another side of a triangle bisects the third side.

In ΔPQR, a line ST drawn through the mid-point S of side PQ, $\parallel$ to QR, bisects PR, *i.e.*, T is the mid-point of PR.

Conversely, *the line joining the mid-points of any two sides of a triangle is always parallel to the third side and equal to half of it.*



If S and T are the mid-points of side PQ and PR respectively of ΔPQR, then $ST \parallel QR$ and $ST = \frac{1}{2} QR$.

7. Similar Triangles

Two triangles (figures) are said to be ***similar*** *if they have the same shape, but not necessarily the same size, i.e.,*

- *their corresponding angles are equal.*

or

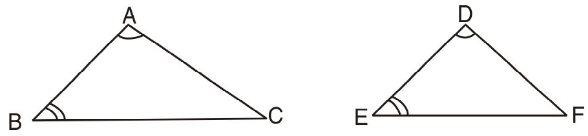
- *their corresponding sides are proportional.*

(i) **AAA-axiom of similarity**

If two triangles are equiangular, their corresponding sides are proportional.

If two triangles have two pairs of angles equal, their corresponding sides are proportional.

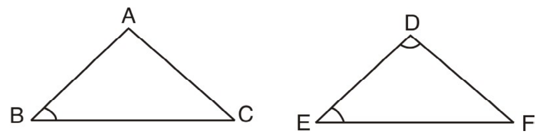
In Δs ABC and DEF, if $\angle A = \angle D$ and $\angle B = \angle E$, then $\Delta ABC \sim \Delta DEF$ and $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$



Note. It two pairs angles are given equal, the third pair becomes equal by angle sum property of a triangle.

(ii) **SAS-axiom of similarity**

If two triangles have a pair of corresponding angles equal and the sides including them proportional, then the triangles are similar.



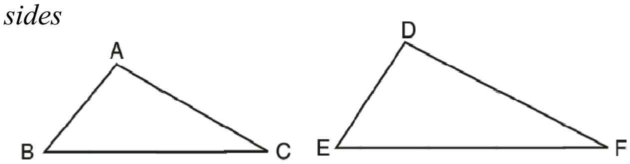
Thus, in Δs ABC and DEF, if $\angle B = \angle E$ and $\dfrac{AB}{DE} = \dfrac{BC}{EF}$, then $\Delta ABC \sim \Delta DEF$.

(iii) **SSS -axiom of similarity**

If two triangles have their three pairs of corresponding sides proportional, then the triangles are similar.

In Δs ABC and DEF,

if $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$, then $\Delta ABC \sim \Delta DEF$.

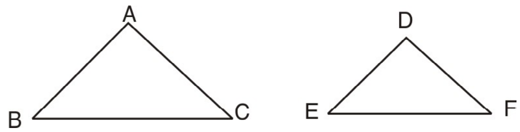


8. Theorems on Similar Triangles

(i) The areas of two similar triangles are proportional to the squares of corresponding sides.

Thus, if $\Delta ABC \sim \Delta DEF$, then

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$



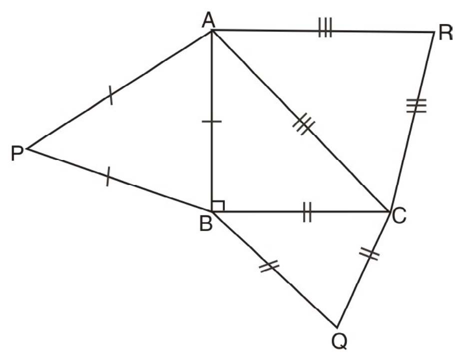
(ii) The ratio of the areas of similar triangles is equal to the ratio of the squares of the corresponding altitudes.

(iii) The ratio of the areas of similar triangles is equal to the ratio of the squares of the corresponding medians.

(iv) If the areas of two similar triangles are equal, then the triangles are congruent.

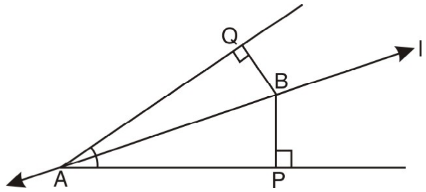
(v) The areas of similar or equilateral Δs described on two sides of a right angled triangle are together equal to the area of the similar or equilateral triangle on the hypotenuse.

ar (ΔPAB) + ar(ΔBQC) = ar(ΔARC)



Solved Examples

Ex. 1. ***In the given figure, line l is the bisector of an angle A and B is any point on l. BP and BQ are perpendiculars from B to the arms of ∠A. Show that B is equidistant from the arms of ∠A.***



Sol. In ∆s *APB* and *ABQ,* we have

$\angle APB = \angle AQB$ (Each = 90°)

$\angle PAB = \angle QAB$ (AB bisects $\angle PAQ$)

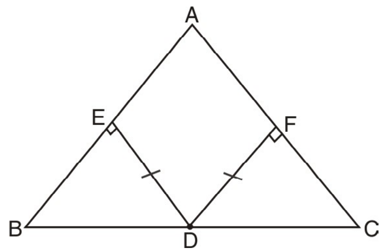
$AB = BA$ (common)

∴ $\Delta APB \cong \Delta ABQ$ (AAS)

⇒ $BP = BQ$ (cpct)

⇒ *B* is equidistant from the arms of ∠*A*.

Ex. 2. ***ABC is a triangle and D is the mid-point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.***



Sol. Let *DE* and *DF* be the perpendiculars from *D* on *AB* and *AC* respectively.

In ∆s *BDE* and *CDF,* $DE = DF$ (Given)

$\angle BED = \angle CFD = 90°$

$BD = DC$ (∵ *D* is the mid-point of *BC*)

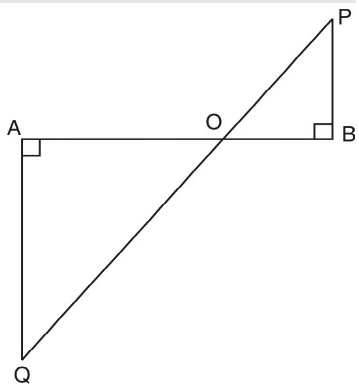
∴ $\Delta BDE \cong \Delta CDF$ (RHS)

⇒ $\angle B = \angle C$ (cpct)

⇒ $AC = AB$ (Sides opp. equal ∠ s are equal)

⇒ ∆*ABC* is isosceles.

Ex. 3. ***In the given figure, QA and PB are perpendiculars to AB. If AO = 10 cm, BO = 6 cm and PB= 9 cm, find AQ.***



Sol. In ∆s *AOQ* and *BOP*, we have

$\angle OAQ = \angle OBP$ (Each equal to 90°)

$\angle AOQ = \angle POB$ (Vertically opp. ∠s)

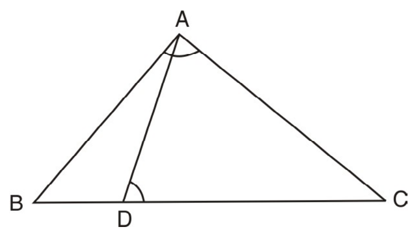
∴ By AA–similarity,

$\Delta AOQ \sim \Delta BOP$

⇒ $\frac{AO}{BO} = \frac{OQ}{OP} = \frac{AQ}{BP}$

⇒ $\frac{AO}{BO} = \frac{AQ}{BP} \Rightarrow \frac{10}{6} = \frac{AQ}{9} \Rightarrow AQ = \frac{10 \times 9}{6} = \mathbf{15\ cm}$.

Ex. 4. ***D is a point on the side BC of ∆ABC such that ∠ADC = ∠BAC. Prove that*** $\frac{CA}{CD} = \frac{CB}{CA}$ ***or*** $CA^2 = CB \times CD$



Sol. In ∆ *ABC* and ∆ *DAC*, we have

$\angle ADC = \angle BAC$ and $\angle C = \angle C$

∴ By AA - axiom of similarity

$\Delta ABC \sim \Delta DAC$

⇒ $\frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$

⇒ $\frac{CB}{CA} = \frac{CA}{CD}$

Ex. 5. ***If ΔABC is similar to ΔDEF such that BC = 3 cm, EF = 4 cm and area of ΔABC = 54 cm². Find the area of ΔDEF.***

Sol. Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

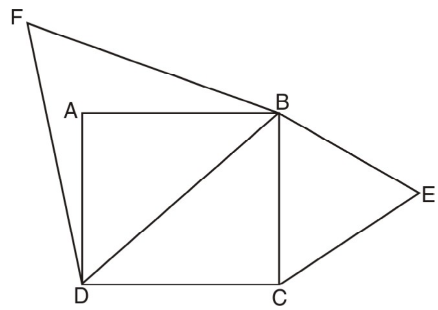
$$\Rightarrow \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{BC^2}{EF^2} \quad \Rightarrow \frac{54}{\text{ar}(\Delta DEF)} = \frac{3^2}{4^2} \Rightarrow \text{ar}(\Delta DEF) = \frac{54 \times 16}{9} = \mathbf{96\ cm^2}.$$

Ex. 6. ***Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.***

Sol. Given: A square *ABCD,*

An equilateral ΔBCE described on side *BC* of the square.

An equilateral ΔBDF described on the diagonal *BD* of the square.



$\Delta\, BCE \sim \Delta\, BDF$ (∵ both are equiangular, each angle = 60°)

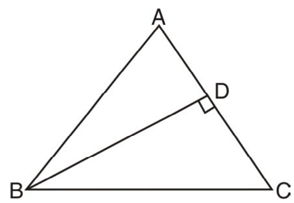
$$\therefore \frac{ar(\Delta BCE)}{ar(\Delta BDF)} = \frac{BC^2}{BD^2} = \frac{BC^2}{(\sqrt{2}BC)^2}$$

(∵ diagonal of a square = $\sqrt{2}$ side)

$$= \frac{BC^2}{2\,BC^2} = \mathbf{\frac{1}{2}}.$$

Ex. 7. ***In an isosceles triangle ABC with AB = AC, BD is perpendicular from B to side AC. Prove that $BD^2 - CD^2 = 2\,CD.AD$***

Sol. Since Δ *ADB* is right angled at *D,*



$$AB^2 = AD^2 + BD^2$$

$\Rightarrow \quad AC^2 = AD^2 + BD^2 \quad (\because AB = AC)$

$\Rightarrow \quad (AD+CD)^2 = AD^2 + BD^2$

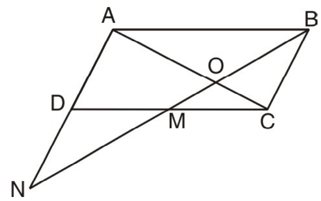
$\Rightarrow \quad AD^2 + CD^2 + 2AD.CD = AD^2 + BD^2$

$\Rightarrow \quad BD^2 - CD^2 = \mathbf{2AD.CD}$

Ex. 8. ***In the given figure, M is the mid-point of the side CD of the parallelogram ABCD. What is ON:OB?***

(a) 3 : 2 ***(b) 2 : 1*** ***(c) 3 : 1*** ***(d) 5 : 2***

Sol. From similar Δs *ABN* and *DMN*



$$\frac{AB}{DM} = \frac{AN}{DN} = \frac{BN}{MN} \Rightarrow \frac{2}{1} = \frac{AN}{DN} \quad ...(1)$$

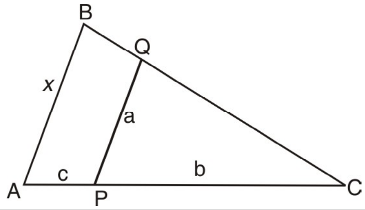
From similar Δs *AOB* and *COM*

$$\frac{AB}{MC} = \frac{AO}{OC} = \frac{OB}{OM} \Rightarrow \frac{2}{1} = \frac{AO}{OC} = \frac{OB}{OM} \quad ...(2)$$

Again from similar Δs *AON* and *BOC*

$$\frac{AO}{OC} = \frac{ON}{OB} \Rightarrow \frac{ON}{OB} = 2 \Rightarrow ON : OB = \mathbf{2:1}.$$

Ex. 9. ***In the given triangle, AB is parallel to PQ. AP = c, PC = b, PQ = a, AB = x. What is the value of x?***

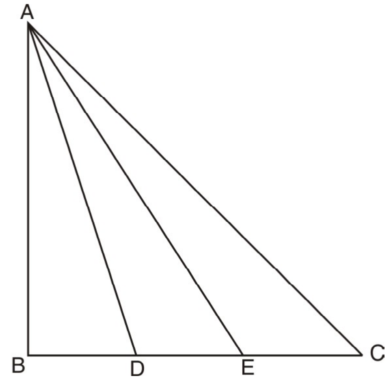


Sol. In Δs ABC and PQC

$$\frac{AB}{PQ}=\frac{AC}{PC}=\frac{AP+PC}{PC}=\frac{AP}{PC}+1$$

$$\Rightarrow \quad \frac{x}{a}=\frac{c}{b}+1 \Rightarrow x=a+\frac{ac}{b}$$

Ex. 10. ***In the given figure, D and E trisect the side BC of a right triangle ABC. Prove that $8AE^2 = 3AC^2 + 5AD^2$***



Sol. Since D and E are the points of trisection of BC, therefore

$BD = DE = CE$

Let $BD = DE = CE = x$. Then $BE = 2x$ and $BC = 3x$

In rt $\Delta\, ABD$,

$AD^2 = AB^2 + BD^2 = AB^2 + x^2$ ...(*i*)

In rt $\Delta\, ABE$,

$AE^2 = AB^2 + BE^2 = AB^2 + 4x^2$ ...(*ii*)

In rt $\Delta\, ABC$,

$AC^2 = AB^2 + BC^2 = AB^2 + 9x^2$ ...(*iii*)

Now, $8AE^2 - 3AC^2 - 5AD^2$

$= 8(AB^2 + 4x^2) - 3(AB^2 + 9x^2) - 5(AB^2 + x^2)$

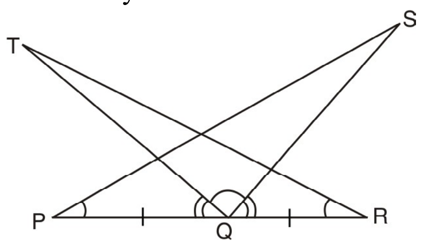
$= 0$

$\Rightarrow$ $\mathbf{8AE^2 = 3AC^2 + 5AD^2}$

Question Bank–22

Problems on Congruency

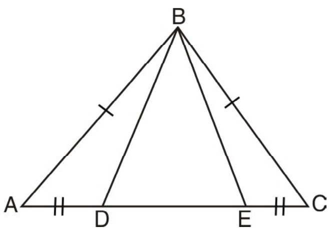
1. In the given figure, $\Delta RTQ \cong \Delta PSQ$ by ASA congruency condition. Which of the following pairs does not satisfy the condition.



(a) $PQ = QR$ (b) $\angle P = \angle R$

(c) $\angle TQP = \angle SQR$ (d) None of these

2. It is given that $AB = BC$ and $AD = EC$. The $\Delta\, ABE \cong \Delta\, CBD$ by ——— congruency.



(a) SSS (b) ASA

(c) SAS (b) AAS

3. $ABCD$ is a quadrilateral. AM and CN are perpendiculars to BD, $AM = CN$ and diagonals AC and BD intersect at O, then which one of the following is correct?

(a) $AO = OC$ (b) $BO = OD$

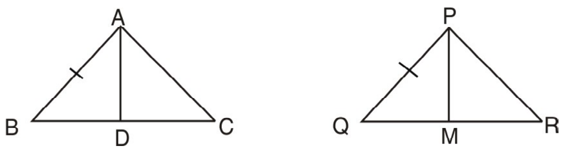
(c) $AO = BO$ (d) $CO = DO$

4. Squares $ABDE$ and $ACFH$ are drawn externally on the sides AB and AC respectively of a scalene ΔABC. Which one of the following is correct?

(a) $BH = CE$ (b) $AD = AF$

(c) $BF = CD$ (d) $DF = EH$

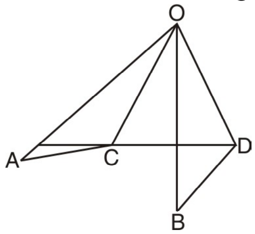
5. In the given figure, two sides AB and BC and the median AD drawn to side BC of $\Delta\, ABC$ are equal to the two sides PQ and QR and the corresponding median PM of the other $\Delta\, PQR$. Which of the following is not correct?



(a) $\Delta ABD \cong \Delta PQM$ (b) $\Delta ABC \cong \Delta PQR$

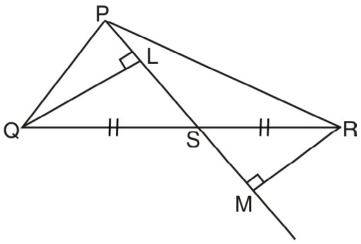
(c) $\Delta ABD \cong \Delta PMR$ (d) $\Delta ADC \cong \Delta PMR$

6. In the given figure, $OA = OB$, $OC = OD$, $\angle AOB = \angle COD$. Which of the following statements is true?



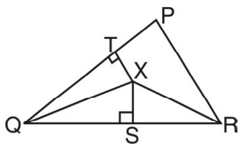
(a) $AC = CD$ (b) $OA = OD$
(c) $AC = BD$ (d) $\angle OCA = \angle ODC$

7. PS is a median and QL and RM are perpendiculars drawn from Q and R respectively on PS and PS produced. Then which of the following statements is correct?



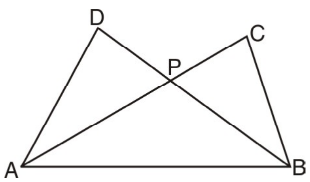
(a) $PQ = RM$ (b) $QL = RM$
(c) $PL = SR$ (d) $PS = SM$

8. In the figure, QX and RX are the bisectors of angles Q and R respectively of ΔPQR. If $XS \perp QR$ and $XT \perp PQ,$ then $\Delta XTQ \cong \Delta XSQ$ by — congruency.



(a) SAS (b) RHS
(c) AAS (d) ASA

9. In the given figure, $AD= BC$, $AC= BD$. Then ΔPAB is



(a) equilateral (b) right angled
(c) scalene (d) isosceles

10. In a right angled triangle, one acute angle is double the other. The hypotenuse is —— the smallest side.

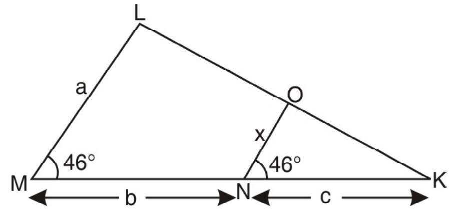
(a) $\sqrt{2}$ times (b) three times
(c) double (d) 4 times

Problems on Similar Triangles

11. If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to $\Delta DEF,$ then which of the following is not true?

(a) $BC.EF = AC.FD$ (b) $AB.EF = AC.DE$
(c) $BC.DE = AB.EF$ (d) $BC.DE = AB.FD$

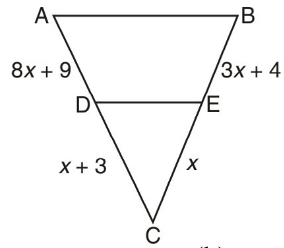
12. In the given figure, x equals



(a) $\dfrac{ab}{a+c}$ (b) $\dfrac{ac}{a+b}$

(c) $\dfrac{ac}{b+c}$ (d) $\dfrac{ab}{b+c}$

13. What value of x will make $DE \parallel AB$ in the given figure?



(a) $x = 3$ (b) $x = 2$
(c) $x = 1$ (d) $x = 5$

14. If the medians of two equilateral triangles are in the ratio 3 : 2, then what is ratio of the sides?

(a) 1 : 1 (b) 2 : 3
(c) 3 : 2 (d) $\sqrt{3} : \sqrt{2}$

15. The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm, then the corresponding median of the other is :

(a) 6.4 cm (b) 10 cm
(c) 8.8 cm (d) 3.2 cm

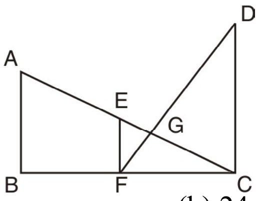
16. In the given figure, DE is parallel to BC and the ratio of the areas of ΔADE and trapezium $BDEC$ is 4 : 5. What is $DE : BC$?

(a) 1 : 2 (b) 2 : 3
(c) 4 : 5 (d) None of these

17. $ABCD$ is a trapezium in which $AB \parallel DC$ and $AB = 2\ DC$. O is the point of intersection of the diagonals. The ratio of the areas of ΔAOB and ΔCOD is:

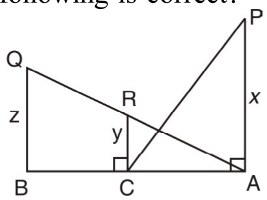
(a) 1 : 2 (b) 2 : 1
(c) 4 : 1 (d) 1 : 4

18. $AB,$ EF and CD are parallel lines. Given that $EG = 5$cm, $GC = 10$ cm, $AB = 15$ cm and $DC = 18$ cm. What is the value of AC?



(a) 20 cm (b) 24 cm
(c) 25 cm (d) 28 cm

19. In the given figure, if $PA = x$, $RC = y$ and $QB = z$, then which one of the following is correct?



(a) $2y = x + z$
(b) $4y = x + z$
(c) $xy + yz = xz$
(d) $xy + xz = yz$

20. In ΔPQR, $QR = 10$, $RP = 11$ and $PQ = 12$. D is the midpoint of PR, DE is drawn parallel to PQ meeting QR in E. EF is drawn parallel to RP meeting PQ in F. What is the length of DF?

(a) $\frac{11}{2}$ (b) 6
(c) $\frac{33}{4}$ (d) 5

Problems on Pythagoras' Theorem

21. The hypotenuse of a right triangle is 6 m more than twice the shortest side. If the third side is 2 m less than the hypotenuse, find the hypotenuse of the triangle.

(a) 24 m (b) 34 m
(c) 26 m (d) 10 m

22. If the distance from the vertex to the centroid of an equilateral triangle is 6 cm, then what is the area of the triangle?

(a) 24 cm^2 (b) $27\sqrt{3}$ cm^2
(c) 12 cm^2 (d) $12\sqrt{3}$ cm^2

23. ΔABC is an equilateral triangle such that $AD \perp BC$, then $AD^2 =$

(a) $\frac{3}{2}DC^2$ (b) $2\,DC^2$
(c) $3\,CD^2$ (d) $4\,DC^2$

24. P and Q are points on the sides CA and CB respectively of ΔABC right angled at C. $AQ^2 + BP^2$ equals

(a) $BC^2 + PQ^2$ (b) $AB^2 + PC^2$
(c) $AB^2 + PQ^2$ (d) $BC^2 + AC^2$

25. ABC is a right-angled triangle, right angled at A and AD is the altitude on BC. If $AB : AC = 3 : 4$, what is the ratio $BD : DC$?

(a) 3 : 4 (b) 9 : 16
(c) 2 : 3 (d) 1 : 2

26. ABC is a right angled triangle, right angled at A. A circle is inscribed in it. The lengths of two sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle?

(a) 3 cm (b) 2 cm
(c) 5 cm (d) 4 cm

27. ΔABC is right angled at A and $AD \perp BC$. Then $\frac{BD}{DC} =$

(a) $\left(\frac{AB}{AC}\right)^2$ (b) $\frac{AB}{AC}$
(c) $\left(\frac{AB}{AD}\right)^2$ (d) $\frac{AB}{AD}$

28. If ΔABC is right angled at B and M, N are the mid-points of AB and BC respectively, then $4(AN^2 + CM^2) =$

(a) $4AC^2$ (b) $5AC^2$
(c) $\frac{5}{4}AC^2$ (d) $6AC^2$

29. In ΔPQR, $PD \perp QR$ such that D lies on QR. If $PQ = a$, $PR = b$, $QD = c$ and $DR = d$, then

(a) $(a - d)(a + d) = (b - c)(b + c)$
(b) $(a - c)(b - d) = (a + c)(b + d)$
(c) $(a - b)(a + b) = (c + d)(c - d)$
(d) $(a - b)(c - d) = (a + b)(c + d)$

30. ABC is a triangle right- angled at B and D is a point on BC produced ($BD > BC$), such that $BD = 2DC$. Which one of the following is correct?

(a) $AC^2 = AD^2 - 3CD^2$
(b) $AC^2 = AD^2 - 2CD^2$
(c) $AC^2 = AD^2 - 4CD^2$
(d) $AC^2 = AD^2 - 5CD^2$

Answers

1. (d)	**2.** (c)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (c)
11. (c)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (c)	**19.** (c)	**20.** (d)
21. (c)	**22.** (b)	**23.** (c)	**24.** (c)	**25.** (b)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (c)	**30.** (a)

Hints and Solutions

1. (c) In Δs RTQ and PSQ,

$QR = PQ$ (Given)

$\angle P = \angle R$ (Given)

$\angle TQR\ (\angle SQR + \angle SQT) = \angle PQS\ (\angle TQP + \angle SQT)$

$\therefore \Delta RTQ \cong \Delta PSQ$ (ASA)

2. (c) Given, $AD = EC$

$\Rightarrow AD + DE = DE + EC$

$\Rightarrow AE = DC$

Also, $AB = BC$

$\Rightarrow \angle BCA = \angle BAC$ (isos. Δ property)

$\Rightarrow \angle BCD = \angle BAE$

$\therefore$ In Δs ABE and CBD,

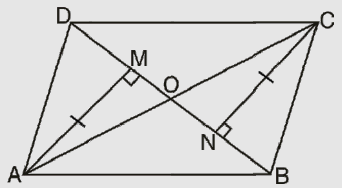
$AB = CB$ (Given)

$AE = DC$ (Proved above)

$\angle BAE = \angle BCD$ (Proved above)

$\therefore \Delta\ ABE \cong \Delta\ CBD$ (SAS)

3. (a) In $\Delta\ AMO$ and $\Delta\ CNO$



$AM = CN$ (Given)

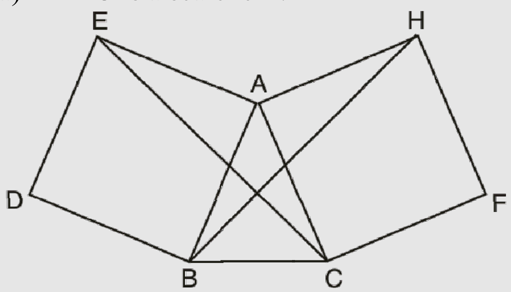
$\angle AMO = \angle CNO = 90°$

$\angle AOM = \angle CON$ (vert. opp. $\angle s$)

$\therefore \Delta\ AMO \cong \Delta\ CNO$ (AAS)

$\Rightarrow AO = OC$ (cpct)

4. (a) $\Delta\ ABC$ is a scalene Δ.



$ACFH$ and $ABDE$ are squares drawn on sides AC and AB respectively.

$\angle\ BAE = \angle CAH = 90°$ (Angle of a square)

$\therefore \angle\ BAE + \angle BAC = \angle CAH + \angle BAC$

$\Rightarrow \angle CAE = \angle BAH$

In Δs EAC and HAB

$EA = AB$ { sides of same square}

$AC = AH$

$\angle\ CAE = \angle BAH$ (Proved)

$\therefore \Delta EAC \cong \Delta HAB$ (SAS)

$\Rightarrow EC = BH$ (cpct)

5. (c) In Δs ABD and PQM

$AB = PQ$
AD $= PM$
{Given}

$BD\left(\frac{1}{2}BC\right) = QM\left(\frac{1}{2}QR\right)$

$\therefore$ **$\Delta ABD \cong \Delta PQM$** (SSS)

$\angle\ B = \angle\ Q$ (cpct)

In Δs ABC and PQR

$AB = PQ$ (Given)

$\angle\ B = \angle\ Q$ (Proved above)

$BC = QR$ (Given)

$\therefore$ **$\Delta ABC \cong \Delta PQR$** (SAS)

$AC = PR$ (cpct)

In Δs ADC and PMR

$AD = PM$ (Given)

$DC\left(\frac{1}{2}BC\right) = MR\left(\frac{1}{2}QR\right)$ (Given)

$AC = PR$ (Proved above)

$\therefore$ **$\Delta ADC \cong \Delta PMR$** (SSS)

$\therefore \Delta ABD \ncong \Delta PMR$

6. (c) In Δs AOC and BOD

$OA = OB$ (Given)

$OC = OD$ (Given)

$\angle AOB - \angle COB = \angle COD - \angle COB$,

i.e., $\angle AOC = \angle BOD$

$\therefore \Delta AOC \cong \Delta BOD$ (SAS)

$\Rightarrow AC = BD$ (cpct)

7. (b) In Δs QLS and RMS,

$\angle QLS = \angle RMS = 90°$

$\angle QSL = \angle RSM$ (Vert. opp. $\angle s$)

$QS = SR$ (PS is the median)

$\therefore \Delta\ QLS \cong \Delta RMS$ (AAS)

$\Rightarrow QL = RM$ (cpct)

8. (c) In Δs XTQ and XSQ.

$XQ = QX$ (Common)

$\angle XQT = \angle XQS$ (QX bisects $\angle Q$)

$\angle XTQ = \angle XSQ = 90°$

$\therefore \Delta XTQ \cong \Delta XSQ$ (AAS)

9. (d) In Δs *ADB* and *ACB*

$AD = BC$ (Given)

$AC = BD$

$AB = BA$ (Common)

$\therefore \Delta ADB \cong \Delta ACB$ (SSS)

$\Rightarrow \angle ABD = \angle CAB$ (cpct)

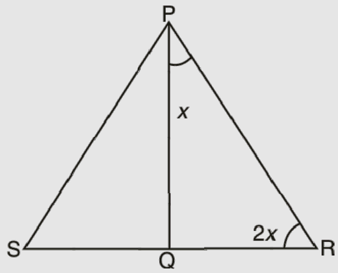
$\Rightarrow \angle ABP = \angle PAB$

$\Rightarrow PA = PB$ (Sides opp. equal angles are equal)

$\Rightarrow \Delta PAB$ is isosceles.

10. (c) **Given :** A ΔPRQ in which

$\angle Q = 90°$ and $\angle PRQ = 2\angle QPR$



Const. Produce *RQ* to *S* such that

$RQ = QS$. Join *PS*.

In Δs *PQS* and *PQR*

$QS = QR$ (By construction)

$PQ = PQ$ (Common)

$\angle PQS = \angle PQR$ (Each = 90°)

$\therefore \Delta PQS \cong \Delta PQR$ (SAS)

$\Rightarrow PS = PR$ and $\angle SPQ = \angle RPQ$ (cpct)

Let $\angle SPQ = x$.Then $\angle PRQ = 2x$ (Given)

Then $\angle SPR = \angle SPQ + \angle RPQ$

$= x + x = 2x$

$\Rightarrow \angle SPR = \angle PRQ \Rightarrow SR = PS$

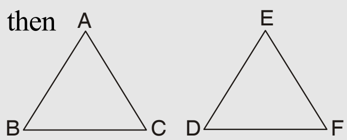
(Sides opp. equal $\angle s$ are equal)

$\Rightarrow 2QR = PS$ $\{\because SQ = QR \therefore SR = 2QR\}$

$\Rightarrow 2QR = PR$ $(\because PS = PR)$

$\Rightarrow$ The hypotenuse *PR* is **double** the smallest side *QR*.

11. (c) If $\Delta ABC \sim \Delta EDF$, then



$$\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$$

$$\therefore \frac{AB}{ED} = \frac{BC}{DF} \Rightarrow AB \times DF = BC \times ED$$

$$\frac{BC}{DF} = \frac{AC}{EF} \Rightarrow BC \times EF = AC \times DF$$

$$\frac{AB}{ED} = \frac{AC}{EF} \Rightarrow AB \times EF = AC \times ED$$

$\therefore BC \times DE \neq AB \times EF$

12. (c) In Δs *LMK* and *ONK*,

$\angle KML = \angle ONK = 46°$

$\angle K = \angle K$ (Common)

$\therefore \Delta LMK \sim \Delta ONK$ (*AA* similarity)

$$\Rightarrow \frac{KM}{KN} = \frac{LM}{ON} \Rightarrow \frac{b+c}{c} = \frac{a}{x} \Rightarrow x = \frac{ac}{b+c}$$

13. (b) By the converse of basic proportionality theorem,

if $\frac{CD}{DA} = \frac{CE}{EB}$, then $DE \parallel AB$

$$\Rightarrow \frac{x+3}{8x+9} = \frac{x}{3x+4}$$

$\Rightarrow (3x + 4)(x + 3) = x(8x + 9)$

$\Rightarrow 3x^2 + 13x + 12 = 8x^2 + 9x$

$\Rightarrow 5x^2 - 4x - 12 = 0$

$\Rightarrow 5x^2 - 10x + 6x - 12 = 0$

$\Rightarrow 5x(x - 2) + 6(x - 2) = 0$

$\Rightarrow (5x + 6)(x - 2) = 0$

$\Rightarrow x = \frac{-6}{5}$ or 2

Since, x cannot be negative, $x = 2$.

14. (c) Equilateral triangles are similar triangles.

In similar triangles, the ratio of their corresponding sides is the same as the ratio of their medians.

15. (c) The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding medians. Therefore,

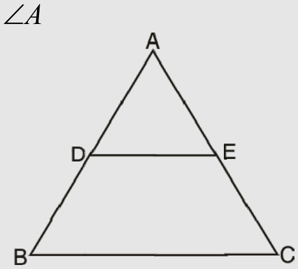
$\frac{121}{64} = \frac{(12.1)^2}{x^2}$, where x is the median of the other Δ.

$$\Rightarrow x^2 = \frac{(12.1)^2 \times 64}{121} \Rightarrow x = \sqrt{\frac{121}{100} \times 64}$$

$$= \frac{11}{10} \times 8 = \mathbf{8.8\ cm}.$$

16. (b) In Δs *ADE* and *ABC*,

$\angle A = \angle A$



$\angle ADE = \angle ABC$ ($\angle DE \parallel BC$, corr $\angle s$ are equal)

$\therefore \Delta ADE \sim \Delta ABC$ (*AA* similarity)

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{BC^2}{DE^2}$$

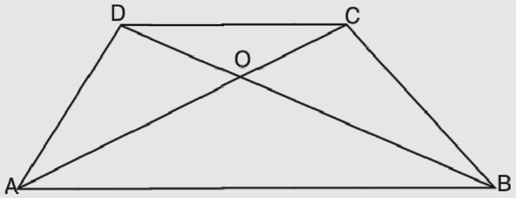
$$\Rightarrow \frac{ar(\Delta ADE) + ar(\text{trap.} DEBC)}{ar(\Delta ADE)} = \frac{BC^2}{DE^2}$$

$\Rightarrow 1+\dfrac{ar\text{ (trap. }DEBC)}{ar(\Delta ADE)}=\dfrac{BC^2}{DE^2}$

$\Rightarrow\ 1+\dfrac{5}{4}=\dfrac{BC^2}{DE^2}\Rightarrow\dfrac{9}{4}=\dfrac{BC^2}{DE^2}\Rightarrow\dfrac{BC}{DE}=\dfrac{3}{2}$

$\therefore\ DE : BC = 2 : 3.$

17. (c) In Δs AOB and COD



$\angle AOB = \angle COD$ (vert. opp. $\angle s$)
$\angle OAB = \angle DCO$ ($DC \parallel AB$, alt.$\angle s$ are equal)
$\therefore\ \Delta AOB \sim \Delta COD$ (AA similarity)

$\Rightarrow\ \dfrac{ar\ (\Delta AOB)}{ar(\Delta COD)}=\dfrac{AB^2}{CD^2}$

{ Ratio of areas of two similar Δs is equal to the ratio of the squares of the corresponding sides}

$=\dfrac{(2CD)^2}{CD^2}=\dfrac{4CD^2}{CD^2}=\dfrac{4}{1}.$

18. (c) In ΔEFG and ΔGCD,
$\angle EFG = \angle GDC$ ($EF \parallel CD$, alt. $\angle s$ are equal)
$\angle EGF = \angle CGD$ (vert. opp. $\angle s$)
$\therefore\ \Delta EFG \sim \Delta GCD$ (By AA similarity)

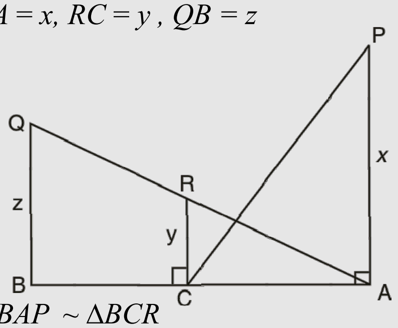
$\therefore\ \dfrac{EG}{GC}=\dfrac{EF}{DC}\Rightarrow\dfrac{EF}{18}=\dfrac{5}{10}\Rightarrow EF=9$ cm

Now in Δs ABC and EFC,
$\angle ACB = \angle ECF$ (common)
$\angle ABC = \angle EFC$ ($AB \parallel EF$, corr. $\angle$s are equal)
$\therefore\ \Delta ABC \sim \Delta EFC$ (By AA similarity)

$\Rightarrow\ \dfrac{AC}{EC}=\dfrac{AB}{EF}\Rightarrow\dfrac{AC}{(EG+GC)}=\dfrac{AB}{EF}$

$\Rightarrow\ \dfrac{AC}{(5+10)}=\dfrac{15}{9}\Rightarrow AC=\mathbf{25\ cm}.$

19. (c) $PA = x$, $RC = y$, $QB = z$



$\Delta BAP \sim \Delta BCR$
($\therefore \angle B$ is common and $\angle BAP = \angle BCR = 90°$)

$\therefore\ \dfrac{RC}{PA}=\dfrac{BC}{AB}\Rightarrow\dfrac{BC}{AB}=\dfrac{y}{x}$ ...(i)

Also, $\Delta ABQ \sim \Delta ACR$
($\angle A$ common, $\angle ABQ = \angle ACR = 90°$)

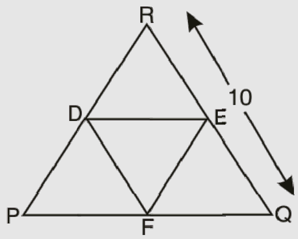
$\therefore\ \dfrac{RC}{BQ}=\dfrac{AC}{AB}\Rightarrow\dfrac{AB-BC}{AB}=\dfrac{y}{z}\Rightarrow 1-\dfrac{BC}{AB}=\dfrac{y}{z}$ ...(ii)

$\therefore$ From (i) and (ii)

$1-\dfrac{y}{x}=\dfrac{y}{z}\Rightarrow\dfrac{x-y}{x}=\dfrac{y}{z}\Rightarrow xy=xz-yz$

$\Rightarrow \boldsymbol{xy + yz = xz}$

20. (d) D is the mid-point of PR and $DE \parallel PQ$.

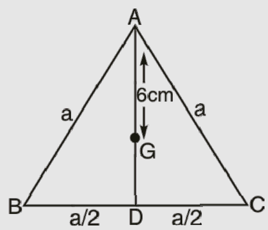


$\therefore$ E is the mid-point of QR (Mid-point Theorem)
$\because$ E is the mid-point of QR and $EF \parallel PR$,
F is the mid-point of PQ (Mid-point Theorem)

$\therefore\ DF=\dfrac{1}{2}\times QR=\dfrac{1}{2}\times 10=\mathbf{5}.$

21. (c) Let the shortest side of the triangle be x m.
Then, hypotenuse = $(2x + 6)$ m
Third side = $(2x + 6) - 2 = (2x + 4)$ m
By Pythagoras' Theorem,
$(2x + 6)^2 = (2x + 4)^2 + x^2$
$\Rightarrow 4x^2 + 24x + 36 = 4x^2 + 16x + 16 + x^2$
$\Rightarrow x^2 - 8x - 20 = 0$
$\Rightarrow x^2 - 10x + 2x - 20 = 0$
$\Rightarrow (x - 10)(x + 2) = 0$
$\Rightarrow x = 10$ or -2
$\therefore$ x cannot be negative, $x = 10$
$\therefore$ Hypotenuse = $(2 \times 10 + 6)$ m = **26 m**

22. (b) Let ABC be the equilateral triangle whose centroid G is at a distance 6 cm from vertex A.



Let each side of ΔABC be a cm.
The median AD is also the perpendicular bisector in case of an equilateral Δ so, $\angle ADB = 90°$ and $BD = DC = a/2$

Now $AG : GD = 2 : 1$

(Centroid divides a median in the ratio 2:1)

$\therefore \dfrac{6}{GD} = \dfrac{2}{1} \Rightarrow GD = 3\text{cm}$

$\therefore AD = AG + GD = 6\text{ cm} + 3\text{ cm} = 9\text{ cm}$

Now, $AB^2 = AD^2 + BD^2$ (Pythagoras' Theorem)

$\Rightarrow AB^2 - BD^2 = AD^2$

$\Rightarrow a^2 - \left(\dfrac{a}{2}\right)^2 = 81 \Rightarrow \dfrac{3a^2}{4} = 81 \Rightarrow a^2 = \dfrac{81 \times 4}{3}$

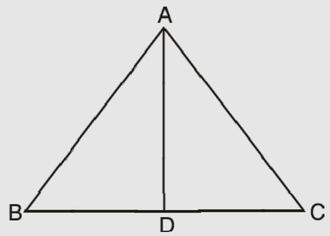
$\Rightarrow a = \sqrt{27 \times 4} = 6\sqrt{3}$ cm

$\therefore$ Area of the equilateral triangle

$= \dfrac{\sqrt{3}}{4} a^2 = \dfrac{\sqrt{3}}{4} \times 6\sqrt{3} \times 6\sqrt{3}$ cm^2

$=$ **$27\sqrt{3}$ cm^2**.

23. (c) In equilateral ΔABC, $AD \perp BC$

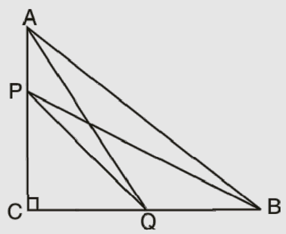


$\Rightarrow AD$ bisects the base BC

$\Rightarrow BD = DC = \dfrac{1}{2}BC = \dfrac{1}{2}AB = \dfrac{1}{2}AC$

Now use Pythagoras' theorem in ΔADB.

24. (c)



In rt. $\angle d\, \Delta ACQ$,

$\Rightarrow AC^2 + CQ^2 = AQ^2$ ...(i)

In rt. $\angle d\, PCB$,

$PC^2 + CB^2 = PB^2$ ...(ii)

Adding eqn (i) and (ii)

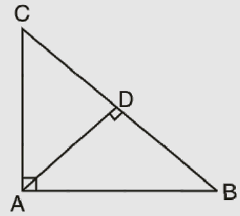
$AC^2 + CQ^2 + PC^2 + CB^2 = AQ^2 + PB^2$

$\Rightarrow (AC^2 + CB^2) + (CQ^2 + PC^2) = AQ^2 + PB^2$

$\Rightarrow \underset{(\text{rt}\angle d\Delta ABC)}{AB^2} + \underset{(\text{rt}\angle d\Delta PQC)}{PQ^2} = AQ^2 + PB^2$

(Pythagoras' Theorem)

25. (b) In ΔABC and ΔDBA,



$\angle B$ is common

$\angle CAB = \angle BDA = 90°$

$\Rightarrow \Delta ABC \sim \Delta DBA$

$\Rightarrow \dfrac{AB}{AC} = \dfrac{DB}{DA} \Rightarrow \dfrac{DB}{DA} = \dfrac{3}{4}$

$\Rightarrow AD = \dfrac{4}{3} DB$ ...(i)

In ΔABD and ΔADC,

$\angle DAB = \angle ACD$

(Third angles of similar Δs ABC and DBA)

$\angle ADB = \angle ADC = 90°$

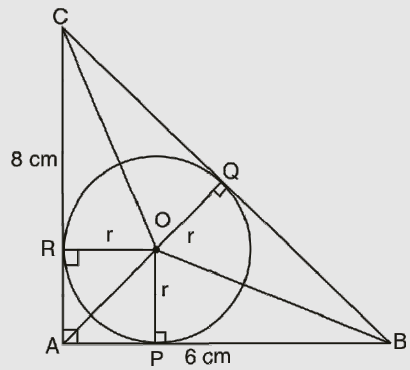
$\therefore \dfrac{AB}{AC} = \dfrac{AD}{CD} = \dfrac{3}{4} \Rightarrow AD = \dfrac{3}{4} CD$ ...(ii)

From (i) and (ii)

$\dfrac{4}{3}DB = \dfrac{3}{4}CD \Rightarrow \mathbf{\dfrac{BD}{CD}} = \dfrac{3}{4} \times \dfrac{3}{4} = \mathbf{\dfrac{9}{16}}$.

26. (b) Let ΔABC be right angled at A.

Since the incentre is equidistant from the sides, let the radius of the incircle be r.



$\therefore \quad OP = OQ = OR = r$ cm

By Pythagoras' Theorem

$AC^2 + AB^2 = BC^2$

$\Rightarrow BC^2 = 6^2 + 8^2 = 36 + 64 = 100$

$\Rightarrow BC = 10$ cm. Now,

Area of ΔABC = Area of ΔOAB + Area of ΔOBC + Area of ΔOCA

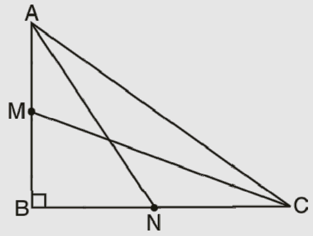
$\Rightarrow \dfrac{1}{2} \times AB \times AC = \dfrac{1}{2} \times r \times AB + \dfrac{1}{2} \times r \times BC + \dfrac{1}{2} \times r \times CA$

$\Rightarrow \dfrac{1}{2} \times 6 \times 8 = \dfrac{1}{2} \times r \times 6 + \dfrac{1}{2} \times r \times 10 + \dfrac{1}{2} \times r \times 8$

$\Rightarrow 12\, r = 24 \Rightarrow$ **$r = 2$ cm.**

27. (b) Similar to Q. 25.

28. (b) Similar to Q. 24.



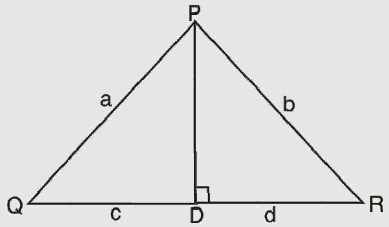
$AN^2 = AB^2 + BN^2$

$= AB^2 + \left(\frac{1}{2}BC^2\right)$

$CM^2 = \left(\frac{1}{2}AB\right)^2 + BC^2$

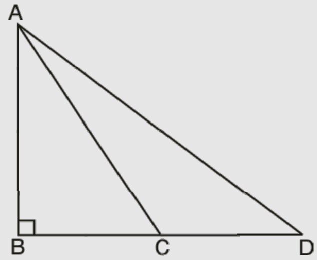
Now solve.

29. (c) Use Pythagoras' Theorem to find PD from ΔPQD and ΔPDR.



Now equate the two values.

30. (a) Given $BD = 2DC$



$\Rightarrow BC + CD = 2DC \Rightarrow BC = DC$

In ΔABC,

$AC^2 = AB^2 + BC^2$ ... (*i*)

In ΔABD,

$AD^2 = AB^2 + BD^2$ ... (*ii*)

$AC^2 - AD^2 = BC^2 - BD^2$

$\Rightarrow AC^2 - AD^2 = CD^2 - (2\,CD)^2$

$= CD^2 - 4\,CD^2 = -3\,CD^2$

$\Rightarrow \boldsymbol{AC^2 = AD^2 - 3CD^2}$

Self Assessment Sheet–22

1. In a right triangle ABC, $\angle C = 90°$. M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B. $\Delta AMC \cong \Delta BMD$ by

(a) ASA (b) RHS

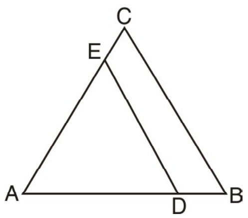
(c) SSS (d) SAS

2. AD is angular bisector of ΔABC such that $BD : DC = 2 : 3$. If $AB = 7$cm, what is $AC : BC$?

(a) 2 : 3 (b) 3 : 2

(c) 21 : 10 (d) None of these

3. In the given figure, $DE \parallel BC$.



$AD = x$, $DB = x - 2$

$AE = x + 2$, $EC = x - 1$

What is the value of x?

(a) 3 (b) 4

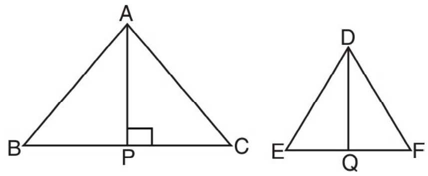
(c) 5 (d) 6

4. If in Δs ABC and DEF, $\angle A = \angle E = 37°$, $AB : ED = AC : EF$ and $\angle F = 69°$, then what is the value of $\angle B$?

(a) 69° (b) 74°

(c) 84° (d) 94°

5. Triangles ABC and DEF are similar. If the length of the perpendicular AP from A on the opposite side BC is 2 cm and the length of the perpendicular DQ from D on the opposite side EF is 1 cm, then what is the area of ΔABC?



(a) One and half times the area of the triangle DEF.

(b) Four times the area of triangle DEF.

(c) Twice the area of the triangle DEF.

(d) Three times the area of triangle DEF.

6. If $ABCD$ is a parallelogram and E and F are the centroids of triangles ABD and BCD respectively, then EF equals

(a) AE (b) BE

(c) CE (d) DE

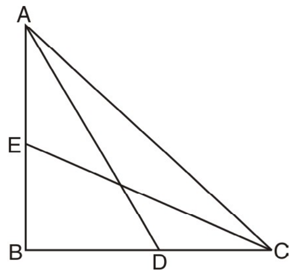
7. In a ΔABC, perpendicular AD from A on BC meets BC at D. If $BD = 8$ cm, $DC = 2$ cm and $AD = 4$ cm, then

(a) ΔABC is isosceles
(b) ΔABC is equilateral
(c) $AC = 2AB$
(d) ΔABC is right angled at A

8. If E is a point on the side CA of an equilateral triangle ABC, such that $BE \perp CA$, then $AB^2 + BC^2 + CA^2 =$
(a) $2\ BE^2$ (b) $3BE^2$
(c) $4\ BE^2$ (d) $6\ BE^2$

9. In a right triangle ABC right angled at C, P and Q are points on the sides CA and CB respectively, which divide these sides in the ratio 2:1. Then, which of the following statements is true?
(a) $9AQ^2 = 9\ BC^2 + 4\ AC^2$
(b) $9AQ^2 = 9\ AC^2 + 4\ BC^2$
(c) $9AQ^2 = 9\ BC^2 + 4\ AC^2$
(d) $9AQ^2 = 9\ AB^2 - 4\ BP^2$

10. In the given figure ΔABC is a right-angle at B. AD and CE are the two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm, then CE equals.



(a) 2 cm (b) $2\sqrt{5}$ cm
(c) $5\sqrt{2}$ cm (d) $3\sqrt{2}$ cm

Answers

1. (d) **2.** (c) **3.** (b) **4.** (b) **5.** (b)

6. (a) **[Hint.** Centroid divides the median in the ratio 2:1. $OE = \frac{1}{3}OA$, $OF = \frac{1}{3}OC$] **7.** (d) **8.** (c)

9. (b) **[Hint.** See Q. No. 24 of Question Bank] **10.** (b)