

# Chapter I

## Power System Generation, Transmission and Distribution

### LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Basics of power system
- Structure of power systems
- Basic power generation concepts
- Thermal station
- Coal and ash handling plant
- Types of boilers
- Types of super heaters
- Types of condensers
- Types of hydro power stations
- Mass curve

### BASICS OF POWER SYSTEM

#### Complex Power

The total power in a single-phase AC system at any instant is the product of the instantaneous voltage and instantaneous current. For a single-phase AC circuit, the voltage and current phasors are conventionally written as

$$u_{an} = V_{\max} \cos \omega t$$

$$i_{an} = I_{\max} \cos(\omega t - \theta)$$

The instantaneous power is

$$S = u_{an} i_{an} = V_{\max} I_{\max} \cos \omega t \cos(\omega t - \theta)$$

The angle ' $\theta$ ' is positive for current lagging the voltage and negative for leading current.

$$S = \frac{V_{\max} I_{\max}}{2} \cos \theta (1 + \cos 2\omega t) + \frac{V_{\max} I_{\max}}{2} \sin \theta \sin^2 \omega t$$

The above equation shows that the term containing ' $\cos \theta$ ' is always positive and has an average value of  $P = \frac{V_{\max} I_{\max}}{2} \cos \theta$

RMS value of current and voltage will give the power expression  $P = |V| |I| \cos \theta$ ,

This average power ' $P$ ' is also called the real or active power. Unit for this power is watt.

The term containing  $\sin \theta$  component of the instantaneous power ' $S$ ' is called the 'instantaneous reactive power'. The maximum value of the pulsating power, designated  $Q$  is called reactive power or reactive volt ampere. The units for  $Q$  is VARS (Volt Ampere reactive), the reactive power is

$$Q = \frac{V_{\max} I_{\max}}{2} \sin \theta$$

$$Q = |V| |I| \sin \theta$$

The square root of the sum of the squares of  $P$  and  $Q$  is equal to the product of  $|V|$  and  $|I|$ .

$$\sqrt{P^2 + Q^2} = \sqrt{(|V| |I| \cos \theta)^2 + (|V| |I| \sin \theta)^2} = |V| |I|$$

In a simple series circuit, when ' $Z$ ' is equal to  $R + jX$

$$P = |I|^2 |Z| \cos \theta = |I|^2 R$$

$$Q = |I|^2 |Z| \sin \theta = |I|^2 X$$

$$\therefore |Z| \cos \theta = R$$

$$\therefore |Z| \sin \theta = X$$

The cosine of the phase angle ' $\theta$ ' between the voltage and the current is called the power factor. An inductive circuit is said to have a lagging power factor and a capacitive circuit is said to have a leading power factor. Power factor indicated whether the current

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is lagging or leading the applied voltage, the power factor is therefore

$$\cos \theta = \cos \left\{ \tan^{-1} \left( \frac{Q}{P} \right) \right\} \text{ or}$$

$$\cos \theta = \frac{P}{\sqrt{P^2 + Q^2}}$$

Phasor representation of the complex power is given as

$$|S| = \sqrt{P^2 + Q^2}$$

$$\theta = \tan^{-1} \left( \frac{Q}{P} \right)$$

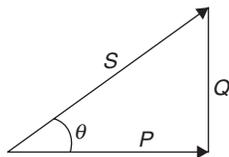


Figure 1 Power triangle of inductive load

#### Solved Examples

**Example 1:** A 15 HP, 440 V, three-phase motor is operating at full load, 85% efficiency and 90% power factor lagging, the real and reactive power drawn by the motor is

- (A) 13,169 W, 14,662 VAR
- (B) 13,169 W, 6365 VAR
- (C) 6378 W, 13169 VAR
- (D) 14,662 W, 13,169 VAR

**Solution:** (B)

$$\text{Current } (I) = \frac{15 \text{ HP} \times 746}{\sqrt{3} \times 440 \times 0.85 \times 0.9} = \mathbf{19.2 \text{ A}}$$

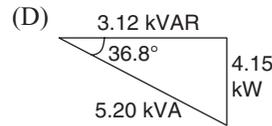
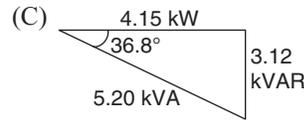
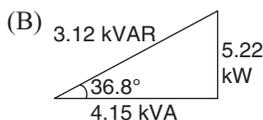
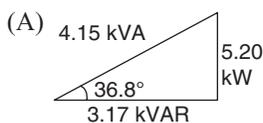
Real power drawn by the motor

$$\begin{aligned} &= \sqrt{3} \times V_L \cdot I_L \cos \theta = \sqrt{3} \times 440 \times 19.2 \times 0.9 \\ &= 13,169.12 \text{ W} \end{aligned}$$

Reactive power drawn by the motor  $= \sqrt{3} V_L I_L \sin \theta$

$$\begin{aligned} P &= \sqrt{3} \times 440 \times 19.2 \times 0.435 \\ &= 6365 \text{ VAR} \end{aligned}$$

**Example 2:** A 25 kVA, 200 V, 50 Hz, 3- $\phi$  alternator supplies a line current of 15 A per phase at 0.8 lagging power factor and at rated voltage under balance steady-state condition. For this operating condition, the power triangle is



**Solution:** (C)

Real power

$$\sqrt{3} V_L I_L \cos \theta = \sqrt{3} \times 200 \times 15 \times \cos \theta = 4.15 \text{ kW}$$

Reactive power

$$\sqrt{3} V_L I_L \sin \theta = \sqrt{3} \times 200 \times 15 \times \sin \theta = 3.12 \text{ kVAR}$$

$$\text{Complex power} = \sqrt{P^2 + Q^2} = 5.20 \text{ kVA}$$

**Example 3:** A single-phase load is supplied by a single-phase voltage source. If the current flowing from the load to the source is  $10 \angle -160^\circ \text{ A}$  and if the voltage at the load terminals is  $100 \angle 60^\circ \text{ V}$  then the

- (A) Load absorbs both real and reactive power
- (B) Load delivers both real and reactive power
- (C) Load absorbs real power and delivers reactive power
- (D) Load delivers real power and absorbs reactive power

**Solution:** (A)

$$\text{Voltage across load } (V) = 100 \angle 60^\circ \text{ V}$$

$$\text{Current entering load } (I) = -10 \angle -160^\circ \text{ V}$$

$$= 10 \angle 20^\circ$$

Complex power  $= VI$

$$= 100 \angle 60^\circ \times 10 \angle 20^\circ = 173.6 + j984.8$$

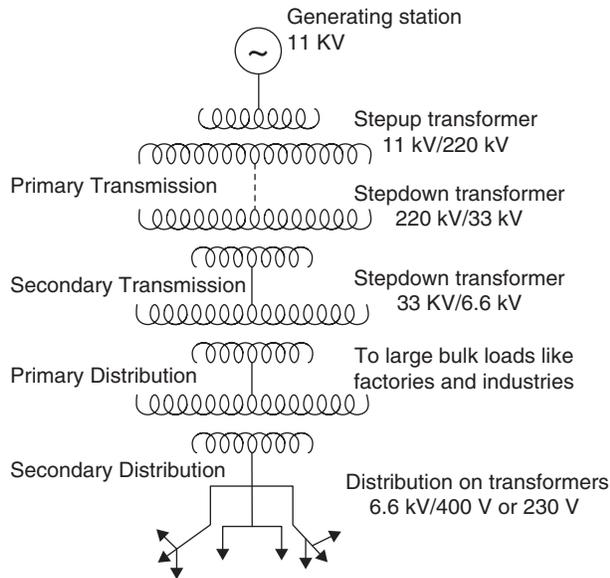
$P$  and  $Q$  are positive, so load absorbs both  $P$  and  $Q$ .

## STRUCTURE OF POWER SYSTEMS

The flow of the electric power from the generating station to the consumer is called an electric power system or electric supply systems. It consists of the following important components.

1. Generating station
2. Transmission network
3. Distribution network

The figure shows the line diagram of a typical transmission, distribution scheme and the complete flow of an electric power from generation to the consumer premises.



**Figure 2** Line diagram of typical transmission and distribution on system

At the generating stations, an electric power is generated with the help of three-phase alternators running in parallel. In the scheme shown in figure, the voltage level is 11 kV but the voltage level can be 6.6 kV, 22 kV or 33 kV depending upon the capacity of the generating station. After generating station, actual transmission and distribution starts, the overall scheme can be divided into four sections which are,

### Primary Transmission

It is basically with the help of overhead transmission lines. For the economic aspects, the voltage level is increased to 132 kV, 22 kV or more, with the help of step-up transformers. Hence, this is also called high voltage transmission.

### Secondary Transmission

The primary transmission line continuous transmission towers till the receiving stations. At the receiving station, the voltage level is reduced to 22 kV or 33 kV using the step-down transformers. This is secondary transmission; the conductors used for the secondary transmission are called feeders.

### Primary Distribution

At substations the voltage level is reduced to 6.6 kV, 3.3 kV or 11 kV with the help of step-down transformers. It uses three-phase three wire underground systems. And the power is further transmitted to the local distribution. For the large consumers like factories and industries, the power is directly transmitted to such loads from a substation.

### Secondary Distribution

At the local centres, there are step down distribution transformers. The voltage levels of 6.6 kV, 11 kV is further reduced to 400 V using distribution transformers and

service mains to the consumers. This is secondary distribution also called low voltage distribution.

## BASIC POWER GENERATION CONCEPTS

Electricity is not often generated at a power station by electromechanical generators, primarily driven by heat engines fuelled by chemical combustion or nuclear fission but also may be by other means such as kinetic energy of flowing water and wind. The types of electric generation concepts are as follows:

1. Thermal power plants
2. Hydroelectric power plants
3. Nuclear power plants
4. Diesel power plants
5. Wind plants
6. Solar power generation, etc.

## Thermal Station

### Definition

A generating station which converts heat energy of coal combustion into electrical energy is known as steam station (thermal station).

### Advantages

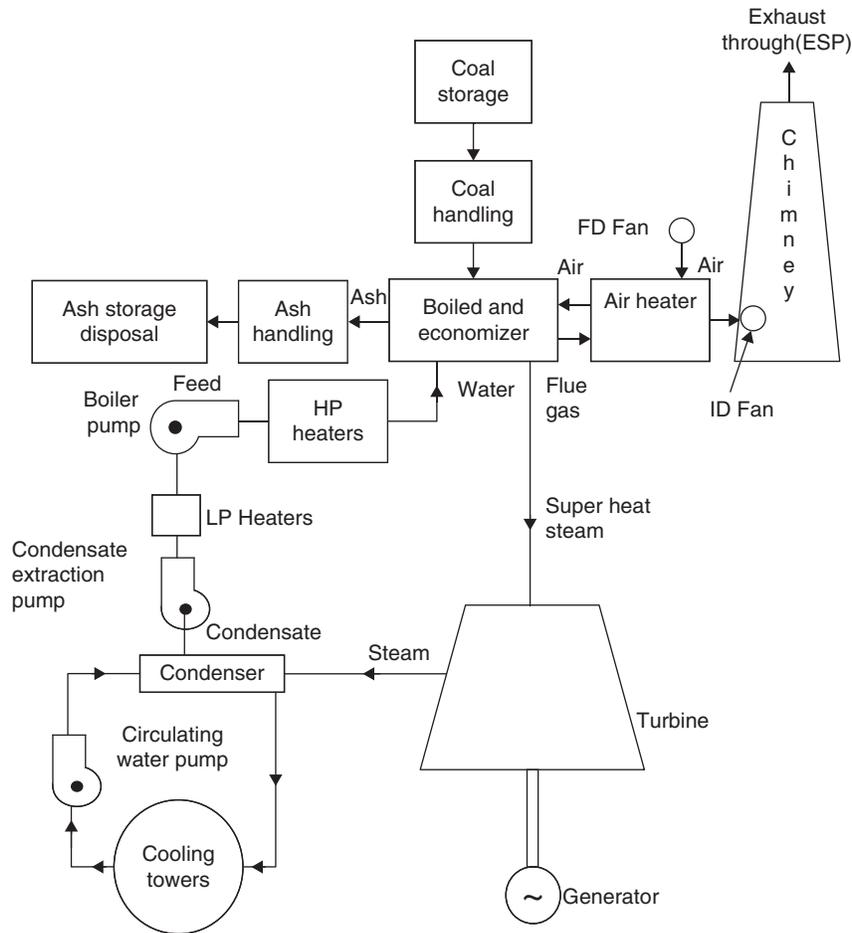
1. Less initial cost.
2. It requires less space as compared to hydroelectric power station.
3. Cost of generation is less than that of diesel power station.

### Disadvantages

1. It pollutes the atmosphere due to production of large amount of smoke and fumes.
2. Its running cost is more compared to hydro electric plant.

Points to decide the selection of site for thermal power station

1. Large quantity of cooling water should be available for the condensers, etc.
2. The fuel to be used in the power station should be cheap and available nearby or ample facilities for its transportation to the site should be available.
3. The soil should be such that, it should provide good and firm foundation to the building and the plants.
4. The site should not be surrounded by residential buildings.
5. The future expansion of power station should be possible.
6. Ample accommodation for the operational and maintenance staff should be available nearby at reasonable rates.
7. Facility for the disposal of ash, etc. should be available.
8. The cost of land should be reasonable.



### Schematic Arrangement of Steam Power Station

1. Coal and ash handling arrangement
2. Steam-generating plant
3. Steam turbine
4. Alternator
5. Feed water
6. Cooling arrangement

#### Coal and Ash Handling Plant

The coal is transported to the power station by road or rail and is stored in a coal storage plant. Storage of coal is primarily a matter of protection against coal strikes, failure of transportation system and general coal shortages.

From coal storage plant, coal is delivered to the coal handling plant where it is pulverized. The pulverized coal is fed to the boiler by conveyor belt. The coal is burnt in the boiler and the ash produced after complete combustion of coal is moved to the ash handling plant and then delivered to the ash storage plant for disposal.

#### Steam-generating Plant

1. **Boiler:** The heat of combustion of coal in the boiler is utilized to convert water into steam at high temperature and pressure.

2. **Superheater:** The steam produced in the boiler is wet and is passed through a superheater where it is superheated (i.e. steam temperature increased above that of boiling point of water).
  - Advantage is that efficiency is increased
3. **Economizer:** An economizer is a feed water heater, i.e. increases temperature of feed water from extracting heat of flue gases.
4. **Air Preheater:** It increases temperature of the air supplied for coal burning by delivering heat from flue gases.

#### Advantages

- Thermal efficiency increases.
- Steam capacity per square metre of boiler surface is increased.

#### Steam Turbine

The dry and superheated steam from the superheater is fed to the steam turbine and it converts into mechanical energy. After giving heat energy to the turbine, the steam is exhausted to the condenser, which condenses exhausted steam.

#### Alternator

The alternator converts mechanical energy into electrical energy. The electrical energy is delivered to busbars.

### Feed Water

The condensate from the condenser is used as feed water to the boiler. The feed water on its way to the boiler is heated by water heaters and economizer. This helps in raising the overall efficiency of the plant.

### Cooling Arrangement

Water is circulated through the condenser, which takes up heat of the exhausted steam. In case availability of water throughout the year is not assured, cooling tower is used.

### Types of Boilers

1. Water tube boiler
  2. Fire tube boiler
- In water tube boiler, water flows through the tubes and the hot gases of combustion flow over these tubes. Advantages are less space requirement, high pressure and less liable to explosion, etc.
  - In fire tube boiler, the hot products of combustion pass through the tubes surrounded by water. Disadvantage is low pressure.

### Types of Superheaters

1. Radiant superheater
  2. Convection superheater
- In Radiant superheater, it is placed in the furnace between water walls and receives heat from the burning fuel through radiation process. Disadvantage is that it may get over heated and temperature of superheater falls with increase in steam output.
  - In convection superheater, it is placed in the boiler tube bank and receives heat from flue gases entirely through the convection process.

Advantage is temperature of superheater increases with increase in steam output.

### Types of Air-pre Heaters

1. Recuperative type
  2. Regenerative type
- In recuperative type, air-heater consists of a group of steel tubes. The flue gases are passed through the tubes while air flows externally to the tubes.
  - In regenerative type, air-heater consists of slowly moving drum. The flue gases flow continuously on one side of the drum and air on the other side.

### Types of Condensers

1. Jet Condenser
  2. Surface condenser
- In a jet condenser, cooling water and exhausted steam are mixed together. Therefore, the temperature of

cooling water and condensate is the same when leaving the condenser.

#### Advantages

Low initial cost, less floor area required, less cooling water required and low maintenance charges.

#### Disadvantages

Condensate is wasted and high power is required for pumping water.

- In a surface condenser, there is no direct contact between cooling water and exhausted steam. It consists of a bank of horizontal tubes enclosed in a cast iron shell. The cooling water flows through the tubes and exhausted steam flows over the surface of the tubes.

#### Advantages

Condensate can be used as feed water, less pumping power required and creation of better vacuum at turbine exhaust.

#### Disadvantages

High initial cost, large floor area requirement and high maintenance charges.

### Types of Steam Turbines

1. Impulse turbine
  2. Reaction turbine
- In an impulse turbine, the steam expands completely in the stationary nozzles (or flexi blades), the pressure over the moving blades remaining constant. In doing so, the steam attains a high velocity and impinges against the moving blades.
  - In reaction turbine, the steam is partially expanded in the stationary nozzles; the remaining expansion takes place during its flow over the moving blades.

### Efficiency of Steam Power Station

#### Thermal Efficiency

The ratio of heat equivalent to the mechanical energy transmitted to the turbine shaft, to the heat of combustion of coal is known as thermal efficiency.

$$\eta_{\text{Thermal}} = \frac{\text{Heat equivalent to the mechanical energy transmitted to the turbine shaft}}{\text{Heat of coal combustion}}$$

- The thermal efficiency of a modern steam power station is about 30%

#### Overall Efficiency

The ratio of heat equivalent of the electrical output to the heat of combustion of coal is known as overall efficiency.

$$\eta_{\text{overall}} = \frac{\text{Heat equivalent of the electrical output}}{\text{Heat of combustion of coal}}$$

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- The overall efficiency of steam power station is about 29%
- Overall efficiency = Thermal efficiency  $\times$  Electrical efficiency

## Hydroelectric Power Station

### Definition

A generating station which utilizes the potential energy of water at a high level for the generation of electrical energy is known as a hydroelectric power station.

### Advantages

1. It is quiet and clean.
2. It requires less maintenance.
3. No fuel requirement.
4. It can be put into service instantly.
5. It has longer life.
6. In addition to the generation of electrical energy, they also help in irrigation and controlling floods.

### Disadvantages

1. It involves high capital cost.
2. Availability of huge amount of water.
3. Skilled and experienced hands are required to build the plant.
4. Increased cost of transmission lines because of remote location of plants.
5. Long dry season may affect the delivery of power.

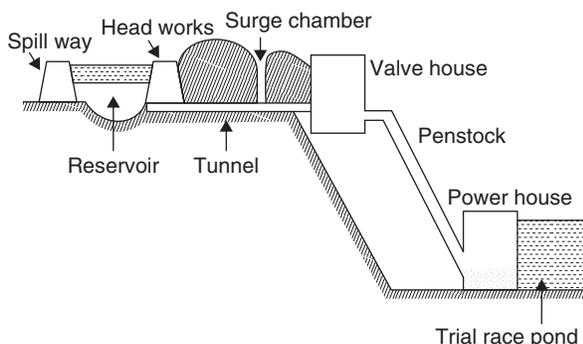


Figure 3 A typical layout of a Hydro plant

## Selection of Site for Hydroelectric Station

1. Sufficient quantity of water at reasonable level should be available.
2. The site should allow for strong foundation with minimum cost.
3. No possibility of future sources of leakage of water.
4. Reservoir should have large catchment area.
5. Local supplies of sand, gravel, etc., should be available.
6. The land for the construction of the plant should be available at reasonable price.

7. The site selected for hydroelectric plant should be accessible by rail and road so that necessary equipment and machinery could be easily transported.

## Components of Hydroelectric Plant

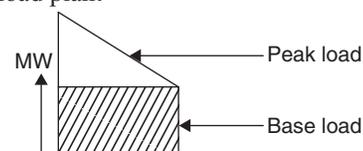
1. **Dam:** A dam is a barrier which stores water and creates water head.
2. **Spillways:** It discharges the surplus water to downstream side of the dam.
3. **Surge tank:** A surge tank is located near the beginning of the conduit. It overcomes the abnormal pressure in the conduit when load on the turbine falls. It acts as a reservoir during increase of load on the turbine.
4. **Penstocks:** Penstocks are open or closed conduits which carry water to the turbines.
  - Low-pressure penstocks  
E.g.: Cement canals
  - High pressure penstocks  
E.g.: Steel pipes
5. **Trash rack:** It is used to avoid the entry of debris into the pen stock.
6. **Forebay:** It is a small pondage where extra water is stored in it and regulates the load variations.
7. **Reservoir:** The area to store water.
8. **Turbines:** It converts energy of falling water into mechanical energy.
9. **Alternator:** It converts mechanical energy into electrical energy.

## Types of Turbines

1. Impulse turbine  
It has high power output and low efficiency.  
E.g.: Pelton wheel
2. Reaction turbines  
It has low power output and high efficiency turbines.  
E.g.: Kaplan, Francis and Propeller turbines.

## Types of Hydro Power Stations

- Based on the water availability
  1. Run off river plants
    - (a) Without pondage
    - (b) With pondage
- Based on the head available
  1. Low head (<30 m)
  2. Medium head (30–50 m)
  3. High head (>50 m)
- Based on the nature of load
  - (1) Base load plant
  - (2) Peak load plant



### Type of Alternator used for Hydroelectric Power Station

Air cooled salient pole construction, vertical configuration, large diameter and small axial length alternator is used.

Types of turbines used for hydroelectric power station based on head

1. **High head:** Pelton wheel turbines
2. **Medium head:** Francis, propeller turbines
3. **Low head:** Kaplan turbines.

Power

$$P = \frac{0.736 QWh\eta}{75} \text{ kW}$$

where

- $Q$  = Discharge in  $\text{m}^3/\text{s}$
- $W$  = Water density in  $1000 \text{ kg}/\text{m}^3$
- $h$  = Height of water in metre
- $\eta$  = Efficiency

### Specific Speed

With available head of 1 m to develop 1 HP power output, the speed at which the turbine will rotate is called specific speed.

$$N_s = \frac{N\sqrt{P}}{H^{\frac{5}{4}}}$$

where

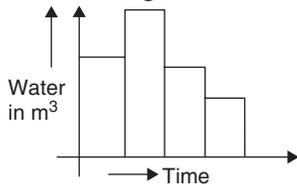
- $P$  = Power in HP
- $N$  = Actual speed in rpm
- $H$  = Head of water

$$N_s = \frac{1.145N\sqrt{P}}{H^{\frac{5}{4}}}$$

if  $P$  is in kW.

### Hydrograph

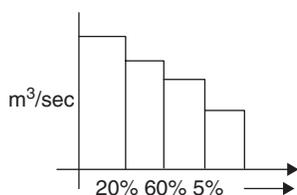
It is a graph between discharge of water in  $\text{m}^3$  to time.



- It gives the variation of water discharge from time to time.
- The area under this graph gives the total quantity of water available at a particular period.

### Flow Duration Curve

It is drawn from hydrograph.



It is drawn in descending order of water discharge.

### Mass Curve

A mass curve is a graph drawn between cumulative volume of water versus time.

- A mass curve gives the storage requirement.

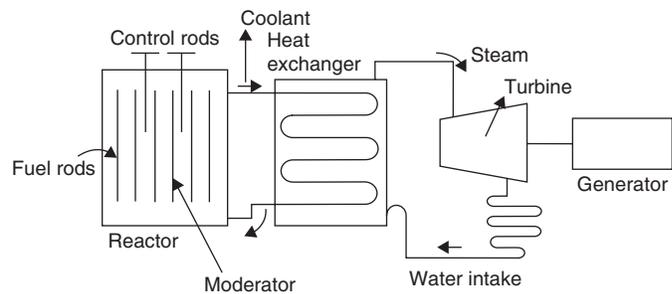
### Nuclear Power Station

#### Definition

A generating station in which nuclear energy is converted into electrical energy is known as nuclear power station.

#### Advantages

1. Fuel requirement is quite small.
2. It requires less space.
3. Running charges are low.
4. It can be located near the load centres because it does not require large quantities of water and it need not be near coal mines.
5. Such plants can ensure continued supply because of large deposits of fuel available.
6. Reliability operation.



#### Disadvantages

1. Fuel is expensive and difficult to recover.
2. Capital cost is very high.
3. Radioactive pollution problems.
4. It requires greater technical staff.
5. Not suited for varying loads.

#### Selection of site for nuclear power station

1. Plant should be located where ample quantity of water is available.
2. Plant should have adequate arrangement for disposal of radioactive waste.
3. It should be quite distant from populated areas as there is a danger of presence of radioactivity in the atmosphere near the plant.
4. Transportation facilities should be available.

### Schematic Arrangement of Nuclear Power Station

1. Nuclear reactor
2. Heat exchanger
3. Steam turbine
4. Alternator

### Components of Nuclear Reactor

1. Reactor core: It consists of fuel rods.
2. Moderator
  - The purpose of moderator is to slow down the fast neutrons.
  - It should have low molecular weight.
  - Materials that can be used as moderator are Beryllium, Graphite, H<sub>2</sub>O and Heavy water (D<sub>2</sub>O).
3. Control rods: The purpose of control rods is to absorb the fast neutrons and they control the value of K (multiplication factor).
  - Materials that can be used for control rods are cadmium, boron and Hafnium.
4. Reflector: The purpose of this reflector is to bounce back escaping neutrons into reactor core.  
Materials: Reactor grade graphite
5. Shielding: The purpose of shielding is to avoid a, b, g radiations outside the reactor core.  
Materials: Stainless steel, etc.
6. Coolant: The purpose of coolant is to absorb the heat from the reactor core and exchange with water in heat exchanger. Materials are  
Liquid coolants → H<sub>2</sub>O, D<sub>2</sub>O  
Liquid metals → Na and K  
Gases → CO<sub>2</sub>, H<sub>2</sub>

### Heat Exchanger

The coolant gives up heat to the heat exchanger which is utilized in rising the steam. After giving up heat, the coolant is again fed to the reactor.

### Steam Turbine

The steam produced in the heat exchanger is led to the steam turbine through a valve. After doing a useful work in the turbine, the steam is exhausted to the condenser. The condenser condenses the steam which is fed to the heat exchanger through feed water pump.

### Comparison of Various Power Plants

S. No.	Item	Steam Power station	Hydroelectric power plant	Diesel power plant	Nuclear power plant
1.	Site	Located at water and coal available places	Located at where reservoirs can be obtained by constructing dam Example: hill areas	Located at any place	Located away from thickly populated areas
2.	Initial cost	Lower than hydro and nuclear plants	Very high	Less compared to other plants	Highest
3.	Running cost	Higher than hydro and nuclear plants	Very less	Highest	Except hydro, it has minimum running cost
4.	Efficiency	Least about 25%	About 85%	About 35%	More efficient than steam power station
5.	Maintenance cost	Quite high	Quite low	Less	Very high
6.	Transmission and distribution	Quite low	Quite high	Least	Quite low
7.	Starting	Requires lot of time	Started instantly	Started quickly	Started easily

### Alternator

Alternator is a device which converts mechanical energy into electrical energy.

Multiplication factor ( $K$ )

$$K = \frac{\text{No. of neutrons released in fission}}{\text{No. of neutrons released in preceding fission}}$$

- The value of  $K$  can be controlled by control rods.
- $K > 1$ , for increasing generation
- $K = 1$ , for constant generation
- $K < 1$ , for decreasing generation
- $K = 0$ , for shut-down generation

### Fission Material

It is the material used as fuel in nuclear power station.

1. Natural Uranium

$$\left[ \begin{array}{l} 99.3\% \text{ U}^{238} \\ 0.7\% \text{ U}^{235} \end{array} \right.$$

2. Enriched Uranium
3. Thorium
4. Plutonium

- 'Enriched Uranium' is used in Indian nuclear power stations
- Fission materials can be divided into two types.
  1. Fissile materials
  2. Fertile materials
- Fissile materials are those with which chain reactions start directly.  
Ex: U<sub>235</sub>, U<sub>233</sub> and Pu<sub>239</sub>
- Fertile materials are those with which chain reactions start indirectly.  
Ex: U<sup>238</sup>, Th<sup>232</sup>

### Critical Mass

It is the mass to be added in the chain reaction to maintain the 'K' value at '1' only.

## ECONOMIC ASPECTS

Commonly used terms in system operation are discussed in the following sections.

### Connected Load

It is the sum of the continuous ratings of all load-consuming apparatus connected to the system.

### Maximum Demand

It is the highest load demand on the power system during a given period (day, month, and year).

### Firm Power

It is the power intended to be always available.

### Cold Reserve

It is the reserve generating capacity available for service but is not in operation.

### Hot Reserve

It is the reserve generating capacity in operation but not in service.

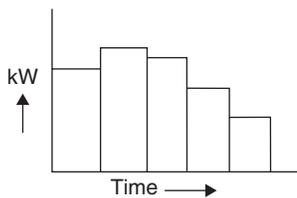
### Spinning Reserve

It is the generating capacity which is connected to the bus and is ready to take load.

## Factors Affecting the Cost of Generation

### Load Curve

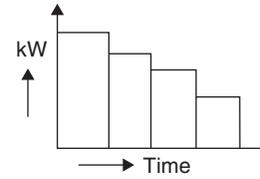
The curve showing the variation of load on power station with reference to time is known as a load curve.



- The variation of quantity w.r.t time, it is known as 'chronological curve'
- Load curve can have -ve or +ve slopes.
- The area under the curve gives the total number of units generated.
- Average demand =  $\frac{\text{Total no. of units generated in a period}}{\text{No. of hours in that period}}$

### Load Duration Curve

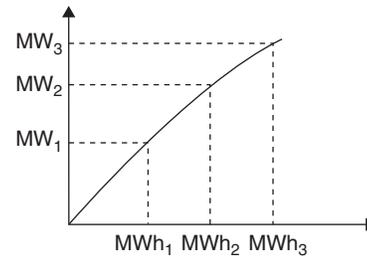
It is a graph drawn from load curve by rearranging load elements in descending order w.r.t their duration of occurrence.



- The curve will have only negative slope.
- The area under this curve gives the no. of units generated.

### Integrated Load Duration Curve

It is a graph drawn between MW versus MWh.



- It gives the cumulative value of energy when load is changing.
- By using integrated load curve, we can estimate the number of units generated when the load is changing from  $MW_1$  to  $MW_2$ .

### Load Factor (LF)

It is the ratio of average demand to maximum demand

$$\text{Load factor} = \frac{\text{Average demand}}{\text{Maximum demand}}$$

- Load factor is less than 1.
- Higher is the load factor, lesser will be the cost per unit.

### Demand Factor (DF)

It is the ratio of maximum demand to connected load

$$\text{Demand factor} = \frac{\text{Maximum demand}}{\text{Connected load}}$$

- It is usually less than 1.
- Low demand factor results in less capital cost.

### Diversity Factor (Div.F)

Diversity factor =  $\frac{\text{Sum of individual maximum demands}}{\text{Maximum demand on power station}}$

- It is usually more than 1 only.
- Div.F ↑, Max. Demand ↓, Installed capacity ↓ cost ↓

### Utilization Factor (UF)

$$\text{U.F} = \frac{\text{Maximum Demand}}{\text{Installed capacity}}$$

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- It is always  $\leq 1$ .
- If UF is high, i.e. maximum demand is high, then the power station is having proper utilization.
- Reserve capacity = Installed capacity – Maximum demand.
- If reserve capacity is zero, utilization factor is one.

#### Capacity Factor (CF)

$$\text{Capacity factor} = \frac{\text{Average demand}}{\text{Installed capacity}}$$

$$= \frac{\text{Actual energy produced}}{\text{Max energy that could have been produced}}$$

- $CF = LF \times UF$

#### Plant use Factor

$$\text{Plant use factor} = \frac{\text{Station output in kWh}}{\text{Plant capacity} \times \text{Hours of use}}$$

- Plant use factor is more than or equal to plant capacity factor.

**Example 4:** A generating station has a connected load of 550 MW. The maximum demand is 380 MW. The number of units generated per annum is  $20 \times 10^8$ . The demand factor and load factor are

- (A) 0.69 and 0.60                      (B) 0.60 and 0.69  
(C) 1.45 and 0.60                      (D) 0.69 and 1.66

**Solution:** (A)

$$\text{Demand factor} = \frac{\text{Maximum demand}}{\text{Connected load}}$$

$$= \frac{380}{550} = 0.69$$

$$\text{Average demand} = \frac{\text{No. of units generated per annum}}{8760} \text{ kW}$$

$$= \frac{20 \times 10^8}{8760} = 228.31 \text{ MW}$$

$$\text{Load factor} = \frac{\text{Average demand}}{\text{Maximum demand}}$$

$$= \frac{228.31}{380} = 0.60$$

**Example 5:** A generating station has a maximum demand of 1000 MW. The annual load factor is 70% and plant capacity factor is 60%. The reserve capacity is

- (A) 700 MW                                  (B) 1.166 MW  
(C) 167 MW                                  (D) 1000 MW

**Solution:** (C)

$$\text{Average demand} = 0.7 \times 1000 = 700 \text{ MW}$$

$$\text{Plant capacity} = \frac{\text{Average demand}}{\text{Capacity factor}}$$

$$= \frac{700}{0.6} = 1166.6 \text{ MW}$$

$$\text{Reserve capacity} = \text{Plant capacity} - \text{Maximum demand}$$

$$= 1166.6 - 1000 = 166.6 \text{ MW} \approx 167 \text{ MW}$$

**Example 6:** Which of the following stations has minimum running cost?

- (A) Hydroelectric                                  (B) Nuclear  
(C) Diesel    (D) Coal

**Solution:** (A)

No fuel cost

**Example 7:** For a hydroelectric power plant with an available head of 60 m, which of the following turbines is most likely to be used?

- (A) Pelton wheel                                  (B) Francis  
(C) Kaplan    (D) None of the above

**Solution:** (A)

Pelton wheels are typically used for high heads.

## POWER SYSTEM REPRESENTATION

### Single Line Diagram

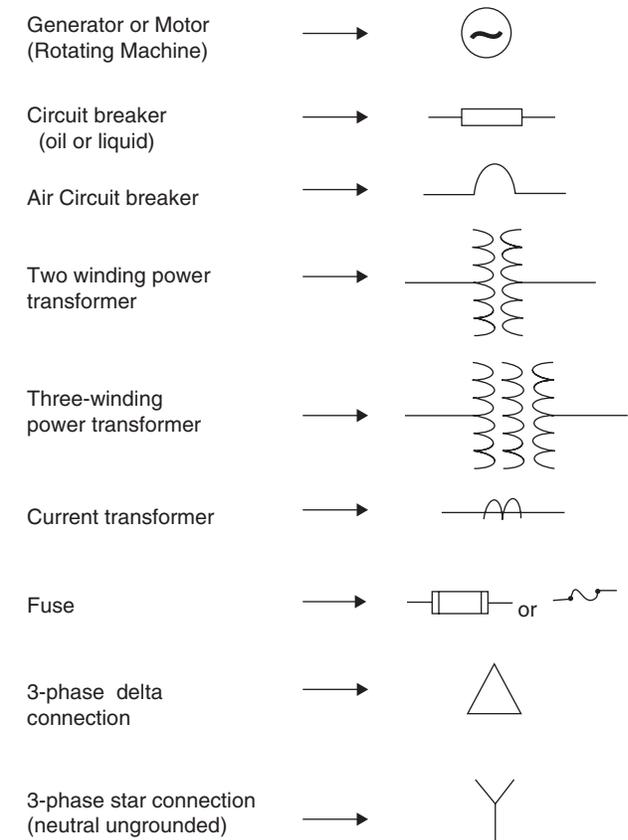


Figure 4 (Continued)

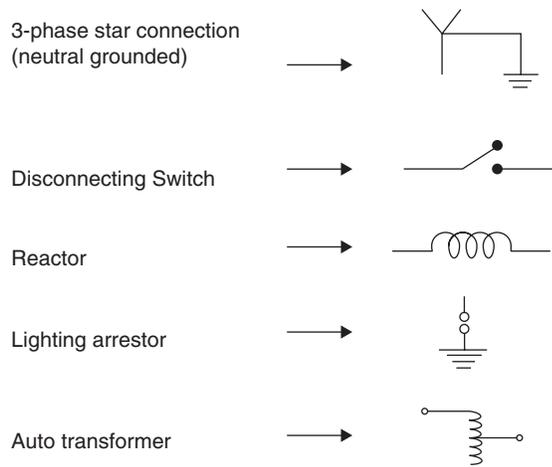


Figure 4 Symbols for representing single line diagram

Almost all modern power systems are three-phase systems with the phases of equal magnitude and equal phase difference (i.e.  $120^\circ$ ). These three-phase balanced systems are always solved as a single-phase circuit composed of one of the three lines and neutral return and this is sufficient to give a complete analysis. Such a simplified diagram of an electric system is called a 'one-line diagram' or 'single-line diagram'. Combined with a standard set of symbols for electric components, such one-line diagrams provide a compact way to represent information. The purpose of the one-line diagram is to supply in concise form, the significant information about the system, the importance of different features of a system varies with the problem under consideration, and the amount of information included on the diagram depends on the purpose for which the diagram is intended. Figure 3 below shows the symbols for representing the components of a three-phase power system.

Above Figure 4 shows the single line diagram of an electrical power system. Two generators grounded through reactors are connected to a bus and through a step-up transformer to a transmission line. Another generator, grounded through reactor, is connected to a bus and through a transformer to the opposite end of the transmission line. A load is connected at each bus.

### Impedance and Reactance Diagrams

The impedance diagram on single-phase basis for use under balanced operating conditions can be easily drawn from the one-line diagram. For the system of Figure 4, the impedance diagram is shown in Figure 5.

No currents flow in the ground under balanced conditions and the neutral of the generators are at the potential of the neutral of the system, so the impedance diagram does not include the current limiting impedances shown in the one line diagram between the neutral of the generator and ground.

Since the shunt current of a transformer is usually insignificant compared with the full load current, the shunt admittance is usually omitted in the equivalent circuit of the transformer.

The inductive reactance of a system is much larger than its resistance. So, the resistance is neglected in fault calculations.

Synchronous motor loads are always included in making fault calculations, since their generated emfs contribute to the short-circuit current. Induction motors are represented by a generated emf in series with an inductive reactance if the diagram is to be used to determine the current immediately after the occurrence of a fault. Induction motors are ignored in computing the current a few cycles after the fault occurrence because the current contributed by an induction

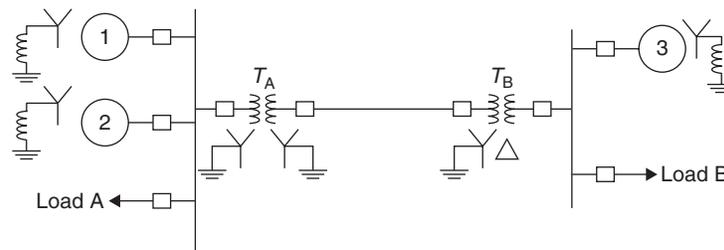


Figure 5 Single-line diagram of an electrical power system

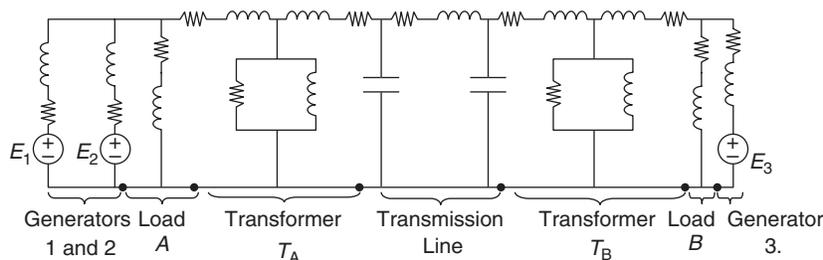


Figure 6 The per-phase impedance diagram

motor dies out very quickly after the induction motor is short circuited. Per-phase reactance diagram after neglecting all static loads, resistances, shunt admittances of each transformer and the capacitance of the transmission line is shown in Figure 6. The per-phase impedance and reactance diagrams are sometimes called the per-phase positive sequence diagram.

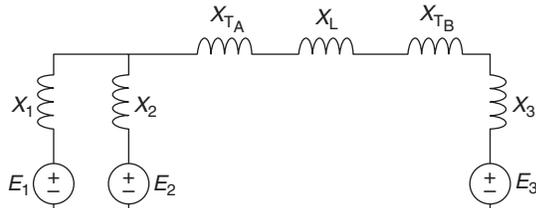


Figure 7 Simplified reactance diagram

### PER UNIT SYSTEM

The quantities in a power system (i.e., voltage, current, volt amperes and impedance) are often expressed as a per cent or per unit of a base or reference value specified for each. The per-unit value of any quantity is defined as the ratio of the actual quantity to its base quantity.

$$\text{Per Unit Value} = \frac{\text{The actual value in any units}}{\text{The base or reference value in the same units}}$$

The ratio in per cent is 100 times the value in per unit. Both the per cent and per-unit methods of calculation are similar and more informative than the use of actual quantities. The per-unit method has advantages over per cent calculation method because the product of two quantities expressed in per unit is expressed in per unit itself, but the product of two quantities expressed in per cent must be divided by 100 to obtain the result in per cent.

There are several reasons for using a per-unit system.

1. Similar apparatus (generators, transformers, lines) will have similar per-unit impedances and losses expressed on their own ratings, regardless of their absolute size.
2. Use of the constant  $\sqrt{3}$  is reduced in three-phase calculations.
3. Per-unit quantities are the same on either side of a transformer, independent of voltage level.
4. By normalizing quantities to a common base, the calculations are simplified.

A per unit system provides units for power, voltage, current and impedance. Only two of these are independent, usually power and voltage. Generally base values of power and voltage are chosen. Once the base power and the base voltage are chosen, the base current and the impedances are determined by the natural laws of electrical circuits.

The relationship between quantities in a per-unit system depends on whether the system is single phase or three phase.

### Single Phase

Assuming that the independent base values are power and voltage,

$$\text{Base volt amperes} = S_{\text{base}} = 1 \text{ p.u.}$$

$$\text{Base voltage} = V_{\text{base}} = 1 \text{ p.u.}$$

$$\text{Base active power} = P_{\text{base}} = S_{\text{base}} \cdot \cos \phi$$

$$\text{Base reactive power} = Q_{\text{base}} = S_{\text{base}} \cdot \sin \phi$$

$$\text{Base current} = I_{\text{base}} = \frac{(VA)_{\text{base}}}{V_{\text{base}}} = \frac{S_{\text{base}}}{V_{\text{base}}} = 1 \text{ p.u.}$$

$$\text{Base impedance} = Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} = \frac{V_{\text{base}}^2}{S_{\text{base}}}$$

$$\text{Base admittance} = Y_{\text{base}} = \frac{1}{Z_{\text{base}}}$$

If the actual impedance is  $Z$  (ohms), its per-unit value is given by

$$\begin{aligned} Z_{(\text{p.u.})} &= \frac{Z}{Z_{\text{base}}} = \frac{Z(\text{ohms}) \times (VA)_{\text{base}}}{V_{\text{base}}^2} \\ &= \frac{Z(\text{ohms}) \times S_{\text{base}}}{V_{\text{base}}^2} \end{aligned}$$

### Three Phase

Power and voltage are specified in the same way as single-phase system. But the difference is that the power is specified as total power (not per phase), and voltage is line-to-line voltage.

In three-phase systems the relations

$$P_{\text{base}} = S_{\text{base}} \cdot \cos \phi \text{ and } Q_{\text{base}} = S_{\text{base}} \sin \phi \text{ holds good.}$$

The apparent power  $S$  equals

$$S_{\text{base}} = \sqrt{3} V_{\text{base}} I_{\text{base}}$$

$$I_{\text{base}} = \frac{S_{\text{base}}}{\sqrt{3} V_{\text{base}}} = 1 \text{ p.u.}$$

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} = \frac{V_{\text{base}}^2}{S_{\text{base}}} = 1 \text{ p.u.}$$

$$Y_{\text{base}} = \frac{1}{Z_{\text{base}}} = \frac{S_{\text{base}}}{V_{\text{base}}^2} = 1 \text{ p.u.}$$

If the actual impedance is  $Z$  (ohms), its per-unit value is given by

$$Z_{\text{p.u.}} = \frac{Z}{Z_{\text{base}}} = \frac{Z(\text{ohms}) \times S_{\text{base}}}{V_{\text{base}}^2}$$

If the impedance has to be represented in a new base value denoted as  $Z_{\text{base new}}$  (Referred to  $S_{\text{base new}}$  and  $V_{\text{base new}}$ )

$$Z_{\text{p.u. new}} = Z_{\text{p.u. old}} \times \left( \frac{V_{\text{base old}}}{V_{\text{base new}}} \right)^2 \times \frac{S_{\text{base new}}}{S_{\text{base old}}}$$

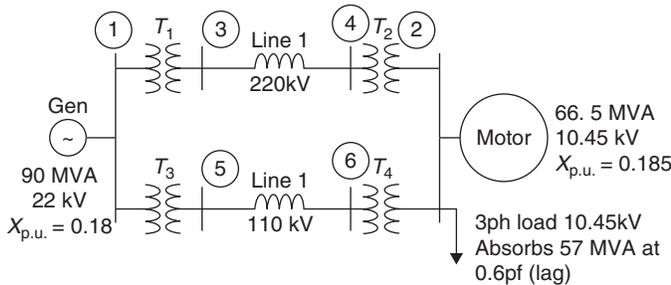
**Note:**

$$Z_{p.u.} \propto S_{base}$$

$$Z_{p.u.} \propto \frac{1}{V_{base}^2}$$

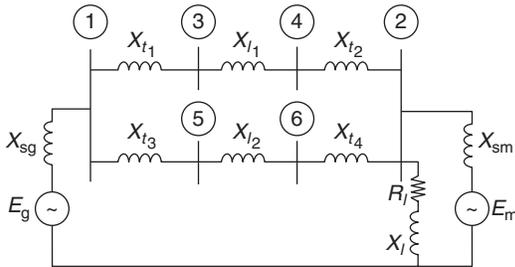
**Example:**

Single line diagram of a power system is shown in the figure. The system contains one generator, four transformers and two transmission lines. The system ratings and reactances are indicated in the figure. Draw the equivalent impedance diagram for the given system and per unit equivalent diagram with  $S_{base} = 100$  MVA and  $V_{base} = 22$  kV.

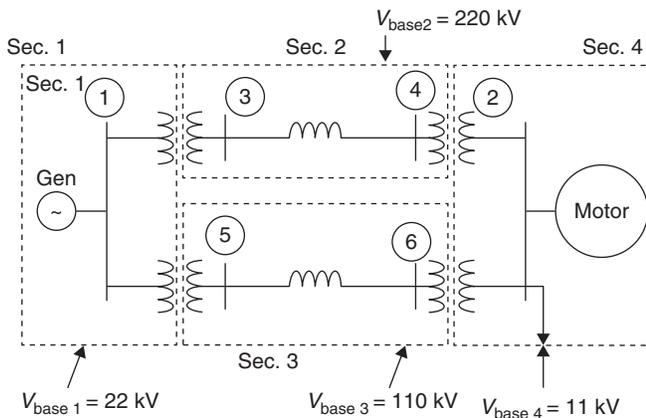


- $T_1$ : 50 MVA, 22/220 kV.  $X_{p.u.} = 0.1$ ,
- $T_2$ : 40 MVA, 220/11 kV.  $X_{p.u.} = 0.06$ ,
- $T_3$ : 40 MVA, 22/110 kV.  $X_{p.u.} = 0.064$ ,
- $T_4$ : 40 MVA, 110/11 kV.  $X_{p.u.} = 0.08$ ,
- Line 1 : 48.4  $\Omega$ , Line 2 : 65.43  $\Omega$

**Reactance Diagram**



**Per Unit Representation**



Base impedance with given base values

$$Z_{base} = \frac{V_{base}^2}{S_{base}}$$

For Section I  $Z_{base_1} = \frac{22^2 \times 10^6}{100 \times 10^6} = 4.84 \Omega$

For Section II  $Z_{base_2} = \frac{220^2 \times 10^6}{100 \times 10^6} = 484 \Omega$

For Section III  $Z_{base_3} = \frac{110^2 \times 10^6}{100 \times 10^6} = 121 \Omega$

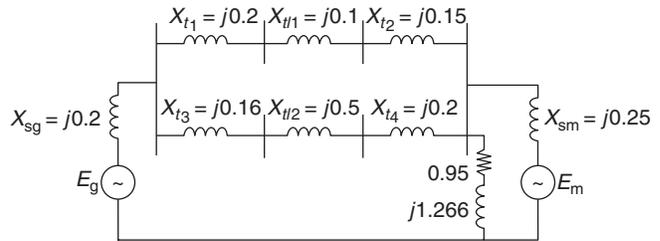
For Section IV  $Z_{base_4} = \frac{11^2 \times 10^6}{100 \times 10^6} = 1.21 \Omega$

Per unit impedance calculations  $Z_{p.u.} = \frac{Z_{actual}}{Z_{base}}$

$$Z_{p.u.,new} = Z_{p.u.,old} \times \frac{S_{base,new}}{S_{base,old}} \times \left( \frac{V_{base,old}}{V_{base,new}} \right)^2$$

For generator 1, new per unit reactance

$$X_{sg} = X_{sg,old} \times \frac{100}{90} \times \left( \frac{22}{22} \right)^2 = 0.2 \text{ p.u.}$$



**Figure 8** Per-unit impedance diagram

For Transmission line 1,  $X_{l_{p.u.}} = \frac{48.4}{484} = 0.1 \text{ p.u.}$

For Transmission line 2,  $X_{l_{p.u.}} = \frac{65.43}{121} = 0.5 \text{ p.u.}$

For Transformer 1,  $X_{t_1} = 0.1 \times \frac{100}{50} \times \left( \frac{22}{22} \right)^2 = 0.2 \text{ p.u.}$

For Transformer 2,

$$X_{t_2} = 0.06 \left( \frac{100}{40} \right) \left( \frac{22}{22} \right)^2 = 0.15 \text{ p.u.}$$

For transformer 3,

$$X_{t_3} = 0.064 \left( \frac{100}{40} \right) \left( \frac{220}{220} \right)^2 = 0.16 \text{ p.u.}$$

For transformer 4,

$$X_{t_4} = 0.08 \left( \frac{100}{40} \right) \left( \frac{220}{220} \right)^2 = 0.2 \text{ p.u.}$$

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For motor,  $X_{Sm} = 0.185 \left( \frac{100}{66.5} \right) \left( \frac{10.45}{11} \right)^2 = 0.25 \text{ p.u.}$

For three-phase load:

Power factor:  $\cos^{-1}(0.6) = 53.13^\circ$

Load  $S_{3\phi} = 57 \angle 53.13$

$$Z_{act} = \frac{V^2_{rated}}{S^*} = \frac{10.45^2}{57 \angle -53.13} = 1.1495 + j1.5326 \Omega$$

Per unit impedance of 3- $\phi$  load =  $\frac{1.1495 + j1.532}{1.21} = 0.95 + j1.2667 \text{ p.u.}$

**Example 8:** The base impedance and base voltage of a 345 kV system are chosen to be 3000 A and 300 kV, respectively, the base impedance of the system is

- (A) 115  $\Omega$  (B) 100  $\Omega$   
(C) 10  $\Omega$  (D) 0.01  $\Omega$

**Solution:** (B)

$$\text{Base impedance} = \frac{300 \times 10^3}{3000} = \frac{V_b}{I_b} = 100 \Omega$$

**Example 9:** Let a 10 kVA, 400/200-V transformer be approximately represented by a 4  $\Omega$  reactance referred to the low-voltage side, considering the rated values as base quantities, what is the transformer reactance as a per unit quantity.

- (A) 1 p.u. (B) 0.5 p.u.  
(C) 0.25 p.u. (D) 1.5 p.u.

**Solution:** (A)

$$\text{Base impedance } (Z_B) = \frac{V^2}{S_B} = \frac{200^2}{10,000} = 4 \Omega$$

The per unit reactance referred to the low-voltage side is

$$\text{Per unit reactance} = \frac{4}{4} = 1 \text{ p.u.}$$

**Example 10:** The per-unit impedance of an alternator corresponding to base values 13.2 kV and 25 MVA is 0.18 p.u. The per unit value of the impedance for base values are 13.8 kV and 60 MVA in p.u. will be

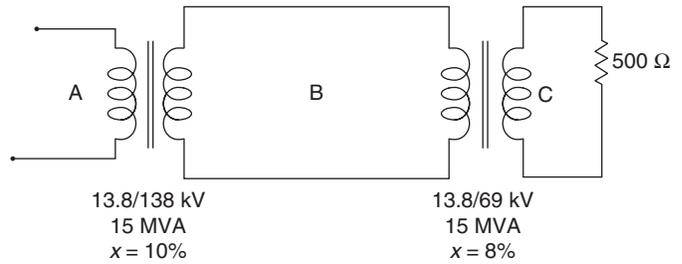
- (A) 0.395 p.u.  
(B) 0.472 p.u.  
(C) 0.329 p.u.  
(D) 0.082 p.u.

**Solution:** (A)

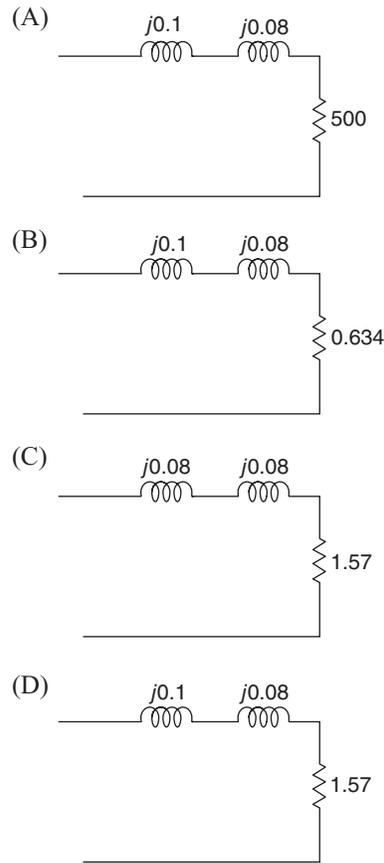
$$Z_{p.u.new} = Z_{p.u.old} \left( \frac{V_{base\ new}}{V_{base\ old}} \right)^2 \times \left( \frac{VA_{base\ old}}{VA_{base\ new}} \right)$$

$$= 0.18 \times \left( \frac{60}{25} \right) \left( \frac{13.2}{13.8} \right)^2 = 0.395 \text{ p.u.}$$

**Example 11:** A single-phase system is shown in figure below:



With the base in 'A' circuit chosen as 13.8 kV and 15 MVA, the impedance diagram is



**Solution:** (D)

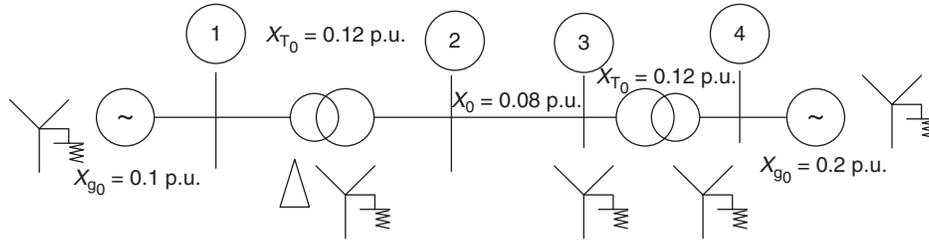
Chosen base values 13.8 kV and 15 MVA

Transformer 1  $X_{p.u.} = 0.1 \text{ p.u.}$  [old and new base values are same]

Transformer 2  $X_{p.u.} = 0.08 \text{ p.u.}$  [old and new values are same]

$$\text{Load } Z_{p.u.} = Z_{actual} \times \frac{S_{base}}{V^2_{base}} = 500 \times \frac{15 \times 10^6}{(69 \times 10^3)^2} = 1.57 \text{ p.u.}$$

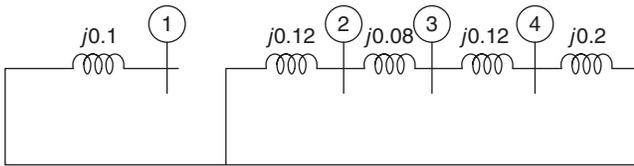
**Example 12:** The zero sequence reactance in p.u. are indicated in the network shown in the given below. The zero sequence driving point reactance of node-3 will be



- (A)  $j0.2$  p.u.                      (B)  $j0.32$  p.u.  
 (C)  $j0.123$  p.u.                  (D)  $j0.52$  p.u.

**Solution:** (C)

Reactance diagram is given by



Driving point reactance at node 3 =  $j(0.12 + 0.08) \parallel j(0.12 + 0.2) = j \frac{0.2 \times 0.32}{0.52} = 0.123$  p.u.

## MECHANICAL DESIGN OF TRANSMISSION LINES

### Introduction

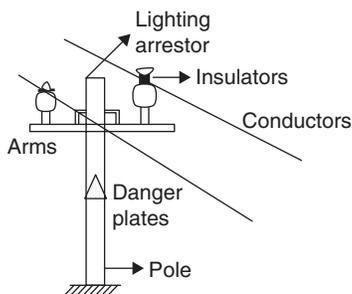
Electric power can be transmitted or distributed by means of overhead lines and underground cables, the underground cables are used rarely because of two main reasons. They are (i) high initial cost (ii) high Insulation cost. Hence, the power is transmitted by overhead lines by proper mechanical factors like strength weather conditions and other external interferences taken into consideration because of the continuity of operations in the line.

Components required for overhead lines

The successful operation of an overhead line depends to a great extent upon the mechanical design of the line.

The components of the OH lines are

1. Conductors
2. Poles
3. Insulators



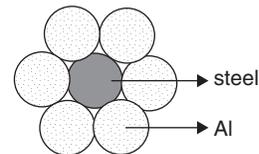
## Conductor Materials

The conductor is one of the important items as most of the capital outlay is invested for it. The proper choice of material and size of the conductor is of considerable importance. The conductors should have the following properties

1. High electrical conductivity
2. High tensile strength
3. Low cost
4. Low specific gravity

All above properties are not found in single material. So, combination of different materials will give above properties.

The common materials used for overhead lines are copper, aluminium steel-cored, aluminium galvanized steel and cadmium copper. Conductors are preferably stranded in order to maintain flexibility. In stranded conductors one central wire around this some no of conductors are used based on the voltage designed according to the 6, 12, 18, 24.....wires.



If there are 'n' layers, then,  $3n(n + 1) + 1$  is total no of individual layers. In the manufacture of stranded conductors, the layers of wires are twisted in opposite directions so that layers are bounded together.

### Copper

Copper is an ideal material for OH lines due to its high conductivity and tensile strength.

- High electrical conductivity
- Greater tensile strength
- High current density
- High cost
- Non-availability

Specific gravity of copper is 8.9 gm/cc. Due to disadvantage of copper, they are rarely used.

### Aluminium

- Aluminium is cheap and light in weight compared to copper.

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- Smaller conductivity and low tensile strength.
- The conductivity of aluminium is 60% that of copper.
- The diameter of Al conductor is 1.26 times the diameter of copper for equal resistances.
- The increased cross sections of Al give greater surface to wind pressure, so supporting towers should be in greater in strength.
- Specific gravity of Al is 2.71 gm/cc.

#### Galvanized Steel

- Galvanized steel conductors can be used for extremely long spans.
- They are suitable in rural areas where cheapness is the main consideration.
- It has poor conductivity due to high resistance of steel.

#### Cadmium Copper

Conductor material in this case is copper alloyed with cadmium. Addition of 2% cadmium to copper increases 50% tensile strength and 15% reduction of conductivity. Therefore, these conductors can be useful for exceptionally long spans.

#### ACSR (Aluminium Conductor Steel Reinforced)

- The aluminium conductor is reinforced with a core of galvanized steel wires.
- The diameter of both steel and Aluminium is same.
- The cross section of two metals is generally in the ratio of 1:6.
- They are represented by  $x/y$ .
- Steel-cored aluminium spans can be used for towers of smaller heights.
- Steel-cored aluminium conductor will produce smaller Sag, and hence longer spans can be used with smaller tower heights.

**Example 13:** An ACSR conductor consists of 30 Al and 7 steel strands

**Solution:**  $\frac{30}{7}$  Ans.

**Example 14:** ACSR conductors represented by 7/54

**Solution:** 7 steel and 54 Al

#### Diameter of Stranded Conductor

$$D = (2n - 1)d$$

$\therefore d$  = diameter of each strand

$n$  = no of layers

#### Poles

The supporting structure for overhead lines are poles and towers.

The poles or towers should have following properties

1. High mechanical strength
2. Cheap in cost

3. Longer life
4. Light in weight

Different types of poles

1. Wooden poles
2. Steel poles and steel towers
3. RCC (Reinforced concrete) poles

#### Insulators

The overhead line conductors are supported on the poles in such a way that the current should not flow to earth by poles, so insulators are properly fixed to conductor to provide necessary insulation between line conductors and support.

The Insulators should have following properties

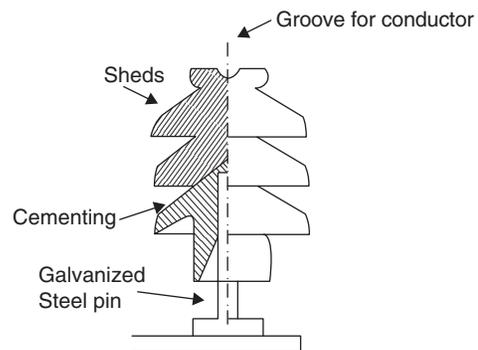
1. High mechanical strength
2. High relative permittivity
3. High electrical

Different types of Insulators

1. Pin type
2. Suspension type
3. Strain type
4. Shackle type

#### Pin-type Insulator

- The pin type is secured (fixed) to cross arm on the pole.
- Pin-type insulators are used for transmission and distribution of electrical power at voltage up to 33 kV.



#### Causes of Insulator Failure

- Insulators are required to withstand both mechanical and electrical stress.
- The electrical breakdown of the insulator can occur either by flash – over or puncture.

Flashover: an arc occurs between the live conductor and insulator pin.

Puncture: the discharge occurs from the conductor to pin through the body of the insulator.

$$\text{Safety factor of insulator} = \frac{\text{Puncture strength}}{\text{Flash – Overvoltage}}$$

#### Mechanical–Electrical

1. Flashover: Arc occurs between line conductor and insulator or pin (i.e. Earth) and the discharge jumps over air gap, following the shortest path. After flashover,

insulator continues to work properly, unless destroyed by the heat produced during flashover.

2. Puncture: Discharge focus from pin to the body of insulator. This destroys the insulator and it must be replaced.

### Suspension-type Insulator

- Suspension-type insulator consists of a number of porcelain discs connected in series by metal links in the form of a string. The conductor is suspended at the bottom end of this string while the other end of the string is secured to the cross arm of the tower.
- Each disc is designed for low voltage 11 kV. For example, 6 discs → 66 kV.
- These insulators are cheaper than pin-type insulators for voltages beyond 33 kV.
- We can replace the damaged disc from the whole string and we can add the desired number of discs.
- These insulators are used with steel towers.

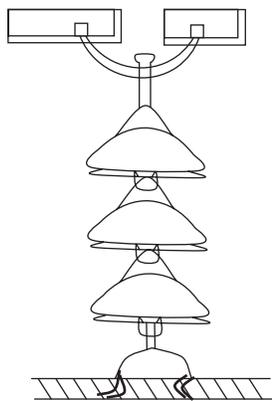
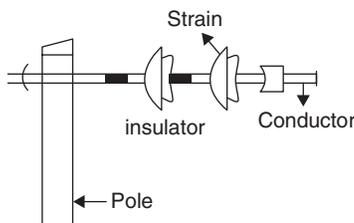


Figure 9 Suspension-type insulators

### Strain Insulators

When there is a dead end of the line or there is corner or sharp curve, the line is subjected to greater tension. Strain insulator is used for low voltages line (<11 kV).

- By assembling suspension insulators, we can use as the strain insulator.



### Potential Distribution Over a String of Insulators

A string of suspension insulators consist of a number of porcelain discs connected in series through metallic links.

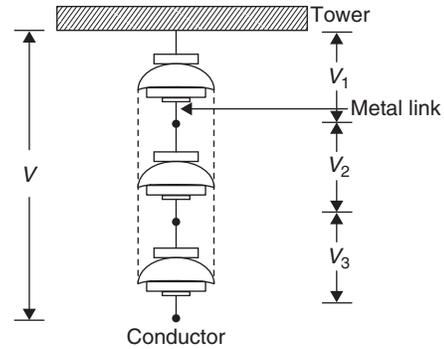


Figure 10 A typical suspension-type Insulator

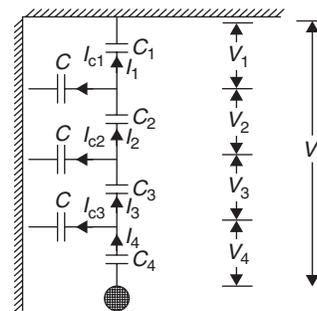


Figure 11 Potential distribution across a string of Insulators

Porcelain portion of each disc between two metallic links forms a capacitance. This capacitance is known as self-capacitance. If there were mutual capacitance alone, the potential distribution across each unit would have been the same. But in actual case, capacitance also exists between metal fitting and tower or earth. This capacitance is known as shunt capacitance. Due to the shunt capacitance, charging current is not the same through all the discs of the string. Therefore, voltage across each disc will be different. Presence of self-capacitance and shunt capacitance results in the

1. Non-uniform potential distribution of voltage across the individual discs of suspension-type insulators.
2. The disc nearest to the conductor has maximum voltage across it.
3. The disc nearest to the conductor is under maximum electrical stress and is likely to be punctured.
4. Since the insulator capacitances are ineffective for DC, the voltage across each disc would be same for DC voltage across the string.

The unequal potential distribution is generally expressed in terms of string efficiency:

### String Efficiency

The ratio of voltage across the whole string to the product of discs and the voltage across the disc nearest to the conductor is known as 'string efficiency'.

$$\text{String efficiency} = \frac{\text{Voltage across the string}}{n \times \text{Voltage across disc nearest to conductor}}$$

$n =$  no. of discs in the string.

$$\text{Let } K = \frac{\text{Capacitance per insulator}}{\text{Capacitance to ground}} = \frac{KC}{C}$$

where  $C_1 = C_2 = C_3 = C_4 = KC$

Let  $V$  be the operating voltage (Line to ground) and  $V_1, V_2, V_3$  and  $V_4$  the voltage drops across the units as shown in the figure.

$$V = V_1 + V_2 + V_3 + V_4.$$

From the figure, we can write

$$I_2 = I_1 + I_{c1}$$

$$\omega kcV_2 = \omega kcV_1 + \omega cV_1 (\because \omega = \text{supply frequency})$$

$$V_2 = V_1 \left( \frac{1+K}{K} \right) = V_1 \left( 1 + \frac{1}{K} \right).$$

$$V_3 = V_1 \left( 1 + \frac{3}{K} + \frac{1}{K^2} \right)$$

$$V_4 = V_1 \left( 1 + \frac{6}{K} + \frac{5}{K^2} + \frac{1}{K^3} \right).$$

$V_1$  can be expressed in terms of  $V$  and from this  $V_2, V_3$ , etc. can be obtained normally.

If  $k = 5$ , then  $V_2 = 1.2 V_1, V_3 = 1.64 V_1, V_4 = 2.40 V_1$

$$\therefore V_1 < V_2 < V_3 < V_4.$$

As ' $k$ ' increases, the division of voltage becomes uniform. The 100% string efficiency can be achieved by increasing ' $K$ ' value till the potential distribution across discs become uniform.

### Methods of Improving String Efficiency

1. Selection of  $K$
2. Grading of units (discs)
3. Static shielding

#### Selection of $K$

Performance of string (string efficiency) depends on the value of ' $K$ '. As  $K$  is increased the division of voltage becomes more equalized. To increase the ' $K$ ' value, longer cross arm for the tower to be used.

#### Grading of Units

Insulators of different dimensions are chosen so that each disc has different capacitance. The insulators are capacitance graded such a way that the top unit has the minimum capacitance and the capacitance increases progressively as the bottom unit is reached. This method has a disadvantage that a large number of different sized insulators are required.

#### Static Shielding

The string efficiency in this method is improved by providing grading in the form of a large metal ring surrounding

the bottom unit and connected to the metal work at the bottom of this unit and therefore to the line. The guard ring introduces capacitance between the metal fittings and the line conductor. The guard ring is connected in such a way that shunt capacitance currents  $i_{c1}, i_{c2}, i_{c3}$ , etc. are equal to metal fitting line Capacitance currents  $i_{m1}, i_{m2}, i_{m3}$ , etc. The result is that same charging current ' $I$ ' flows through each unit of string. Consequently, there will be uniform potential distribution across the units.

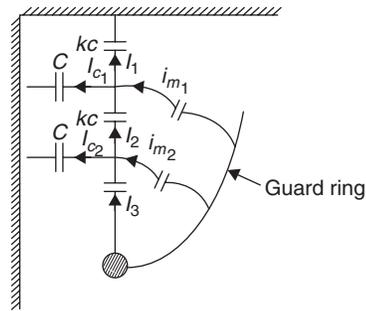


Figure 12 Static shielding

#### Common Data for Examples

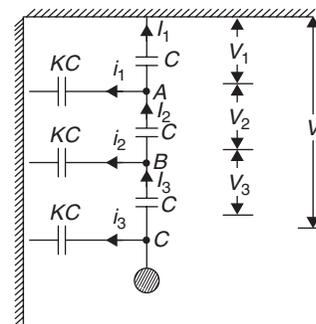
A 3- $\phi$  overhead transmission line is being supported by three disc insulators. The potential drop across top unit and middle unit are 10 kV and 12 kV, respectively.

**Example 15:** The ratio of capacitance between pin and earth to self-capacitance of each unit is

- (A) 0.2 (B) 5  
(C) 0.833 (D) 1.2

**Solution:** (A)

The equivalent circuit of string insulators is shown below



Conductor location

Let ' $K$ ' be the ratio of capacitance between pin and earth to self-capacitance.

' $C$ ' is self-capacitance of each unit.

Capacitance between pin to earth =  $KC$

Apply KCL at node A

$$I_2 = I_1 + i_1$$

$$V_2 \omega c = V_1 \omega c + V_1 k \omega c$$

$$V_2 = V_1(1 + k)$$

$$k = \frac{V_2 - V_1}{V_1} = \frac{12 - 10}{10} = 0.2$$

**Example 16:** The string efficiency of the system is  
 (A) 10.6% (B) 12%  
 (C) 78% (D) 92%

**Solution:** (C)

From the given data,

$$V_1 = 10 \text{ kV}, V_2 = 12 \text{ kV}$$

$$V_3 = V_2 + (V_1 + V_2)k$$

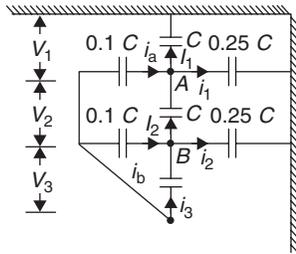
$$= 12 + (10 + 12) \times 0.2 = 16.4 \text{ kV}$$

Voltage between line and earth =  $V_1 + V_2 + V_3$   
 $= 38.4 \text{ kV}$

$$\text{String efficiency} = \frac{\text{Voltage across string}}{\text{No. of insulators} \times V_3} \times 100$$

$$= \frac{38.4}{3 \times 16.4} \times 100 = 78\%$$

**Example 17:** For the following figure, the voltage across middle insulator as a percentage of line voltage to earth



- (A) 29.85% (B) 40.65%  
 (C) 70.15% (D) 59.35%

**Solution:** (A)

Applying Kirchoff's law at node 'A'

$$I_2 + i_a = I_1 + i_1$$

$$V_2 \omega c + 0.1 \omega c (V_2 + V_3) = V_1 \omega c + 0.25 V_1 \omega c$$

$$V_2 + 0.1 V_2 + 0.1 V_3 = 1.25 V_1$$

$$1.1 V_2 + 0.1 V_3 = 1.25 V_1$$

At node 'B'  $I_3 + i_b = I_2 + i_2$   
 $0.25 V_2 + 1.25 V_2 = 1.1 V_3$

From the above two equations, we have

$$V_1 = 0.988 V_2$$

$$V_3 = 1.362 V_2$$

$$V = V_1 + V_2 + V_3$$

$$V = 0.988 V_2 + V_2 + 1.362 V_2 = 3.35 V_2$$

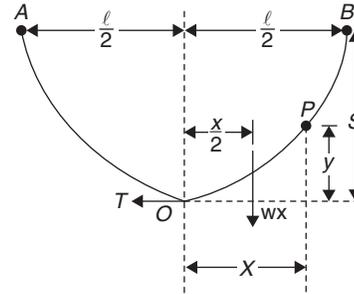
$$V_2 = \frac{1}{3.35} V = 0.2985 V$$

$$V_2 = 29.85\% \text{ of } V$$

### Sag Calculation in OH Lines

While erecting an overhead line, it is very important that conductors are under safe tension. In order to permit safe tension in the conductors, they are not fully stretched but are allowed to have a dip or sag.

The difference in level between points of supports and the lowest point on the conductor is called sag.

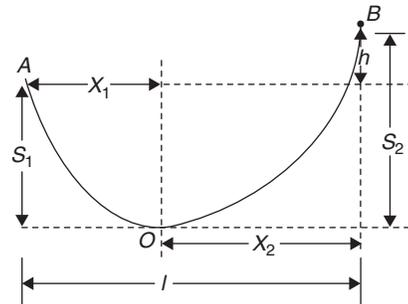


Consider a conductor between two equilateral supports A and B with 'O' as the lowest point as shown above.

$$\text{Sag } S = \frac{Wl^2}{8T}$$

where  $l$  = Length of span  
 $W$  = Weight per unit length of conductor  
 $T$  = Tension in the conductor

Let A and B be at different levels from the ground.



$$S_1 = \frac{Wx_1^2}{2T} \quad \text{where } x_1 = \frac{l}{2} - \frac{Th}{Wl}$$

$$S_2 = \frac{Wx_2^2}{2T} \quad \text{where } x_2 = \frac{l}{2} + \frac{Th}{Wl}$$

where  $h$  = Difference in levels between two supports.

$$S_2 - S_1 = \frac{Wl}{2T} [x_2 - x_1]$$

and

$$l = x_2 + x_1$$

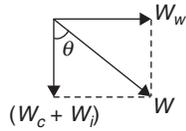
### Effect of Wind and Ice

The effect of ice covering ( $w_i$ ) is to increase the weight of the conductor ( $w_c$ ) and thus increases vertical sag. To guard against this, the tension on the conductors at the time of erection is suitably decreased.

If the wind pressure is also considered, the wind pressure acting on the projected area of the conductor per metre converted to local should be added to the weight. If  $W_w$  is wind pressure.

Resultant weight (per conductor)

$$W = \sqrt{(W_c + W_i)^2 + W_w^2}$$



**Example 18:** If the ultimate strength is 5750 kg, sag is 2 m and factor of safety is 2 and the overhead line has a span of 250 m. The weight of the conductor is

- (A) 0.736 kg
- (B) 184 kg
- (C) 92 kg
- (D) 23 kg

**Solution:** (B)

Allowable maximum tension

$$(T) = \frac{\text{Ultimate strength}}{\text{Safety factor}} = \frac{5750}{2} = 2875 \text{ kg}$$

Total sag(s) = 2 m

$$S = \frac{WL^2}{8T}$$

$$W = \frac{8ST}{L^2} = \frac{2 \times 8 \times 2875}{(250)^2}$$

$$W = 0.737 \text{ kg/m}$$

Half-span length =  $l + \frac{w^2 l^2}{6T^2}$

$$= \frac{250}{2} + \frac{(0.737)^2 \left(\frac{250}{2}\right)^2}{6 \times (2875)^2} = 125 \text{ m}$$

Total length = 250 m

$$\text{Weight of conductor} = (0.737 \times 250) \text{ kg} = 184 \text{ kg}$$

**Example 19:** An overhead line conductor is subjected to a horizontal wind load of 1.78 kg/m and vertical ice loading of 1.08 kg/m. If the maximum possible sag is 6 m. Safety factor is 2. Weight of conductor is 0.844 kg/m<sup>2</sup>. The tension is 7950 kg. The permissible span between two supports is

- (A) 135 m
- (B) 275 m
- (C) 13 m
- (D) 27 m

**Solution:** (B)

$$\begin{aligned} \text{Total weight } w &= \sqrt{(w_c + w_i)^2 + w_w^2} \\ &= \sqrt{(0.844 + 1.08)^2 + (1.78)^2} \\ &= 2.621 \text{ kg/m} \end{aligned}$$

Maximum possible sag  $D = \frac{wl^2}{2T}$

$$6 = \frac{2.621 \times l^2}{2 \times (7950/2)}$$

$$l = \sqrt{\frac{12 \times 3975}{2.621}} = 135 \text{ m}$$

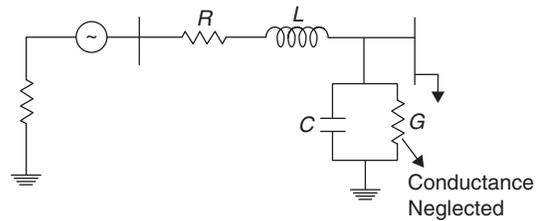
The span length =  $2 \times 135 = 270 \text{ m}$ .

## ELECTRICAL DESIGN OF OVERHEAD LINES

An AC transmission has resistance inductance and capacitance uniformly distributed along its length. These four are called parameters. These parameters will determine the efficiency of transmission system.

Parameters of Transmission line:

1. Resistance (R)
2. Inductance (L)
3. Capacitance (C)
4. Conductance (G)



The transmission line consists of a series combination of resistance, inductance and a parallel combination of capacitance, conductance. Conductance is normally neglected due to leakage over line insulation is almost always in overhead transmission line.

The series line parameters, i.e., inductance resistance are uniformly distributed along the line and they together form series impedance. Inductance is the most dominant line parameter for a power system. It is the inductive reactance which limits the transmission capacity of line.

### Resistance

It opposes the flow of current in a line conductor. It is the cause for power loss in the Transmission line

$$R = \frac{\rho l}{a}$$

- R = Resistance
- $\rho$  = Resistivity
- l = Length of conductor
- a = Area of conductor

The variation of resistance of metallic conductor with temperature is practically linear.

$$R_2 = R_1 [(1 + \alpha_1(t_2 - t_1))]$$

$$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1}$$

$\alpha_0$  = temperature coefficient

### Skin Effect

An alternating current flowing through the conductor does not distribute uniformly. Rather it has the tendency to concentrate near the surface of the conductor. This effect is known as skin effect.

The skin effect depends on

1. Nature of material
2. Diameter of wire
3. Frequency
4. shape of wire

When frequency < 50 Hz and conductor diameters < 1 cm, skin effect can be negligible.

### Flux Linkages

Inductance of a circuit is defined as the flux linkages per unit current.

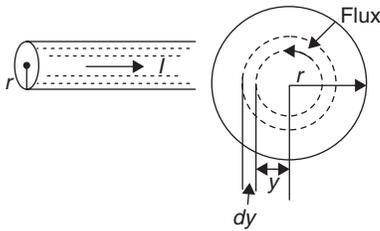
There are two important cases of flux linkages.

Flux linkages due to a single current carrying conductor

The magnetic lines of force will exist inside the conductor and outside of the conductor. Contribution of both fluxes will form the inductance

#### For example

A long straight cylindrical conductor of radius ‘r’ metre and carrying a current of ‘I’ Amperes



### Internal Flux

Figure shows the cross-sectional view of a long cylindrical conductor carrying current I

The mmf around a concentric closed circular path of radius (internal) of the conductor is

[∵  $H_y$  = magnetic field intensity] (AT/m)

$\oint H_y ds = I_y$  (i) (Amperes law), where

$I_y$  = current enclosed (A)

By symmetry,  $H_y$  is constant and is in direction of ds all along the circular path. Therefore from equation (1)

$$2\pi y H_y = I_y \tag{2}$$

Assuming uniform current density

$$I_y = \left( \frac{\pi y^2}{\pi r^2} \right) I = \left( \frac{y^2}{r^2} \right) I \tag{3}$$

From Equations (2) and (3), we obtain

$$H_y = \frac{yI}{2\pi r^2} \text{ AT/m} \tag{4}$$

The flux density ( $B_y$ ) at ‘y’ metre from the centre of conductor is

$$B_y = \mu H_y = \frac{\mu y I}{2\pi r^2} \text{ Wb/m}^2 \tag{5}$$

where  $\mu$  is the permeability of the conductor.

Consider now an infinitesimal tabular element of thickness dy and length 1 metre. The flux in the tabular element

( $d\phi = B_y dy$ ) weber links the fractional turn  $\frac{\ell y}{\ell r^2} = \frac{y}{r^2}$  resulting in flux linkages of

$$d\lambda = \left( \frac{y^2}{r^2} \right) d\phi = \left( \frac{y^2}{r^2} \right) \frac{\mu y I}{2\pi r^2} dy \text{ Weber-turns} \tag{6}$$

Integrating, we get the total internal flux linkages as

$$\lambda_{\text{int}} = \int_0^r \frac{\mu I}{2\pi r^4} y^3 dy = \frac{\mu I}{8\pi} \text{ Wb-T/m}$$

For relative permeability  $\mu_r = 1$  (non-magnetic conductor)

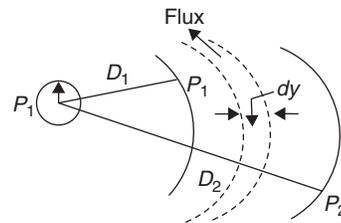
$$\mu = 4\pi \times 10^{-7} \text{ Wb-T/m}$$

and

$$L_{\text{int}} = \frac{1}{2} \times 10^{-7} \text{ H/m}$$

### Flux Linkage Due to Flux Between Two Points External to Conductor

Figure shows two points  $P_1$  and  $P_2$ , at distance  $D_1$  and  $D_2$  from a conductor which carries current of I amperes. The magnetic field external to the conductor is concentric circles around the conductor and therefore all the flux between  $P_1$  and  $P_2$  lies within the concentric cylindrical surfaces passing through  $P_1$  and  $P_2$ .



Magnetic field intensity at distance y from the conductor is

$$H_y = \frac{I}{2\pi y} \text{ AT/m}$$

The flux  $d\phi$  contained in the tabular element of thickness dy is

$$d\phi = \frac{\mu I}{2\pi y} dy \text{ Wb}$$

The flux  $d\phi$  links all the current in the conductor once and only once.

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Flux linkages,

$$d\lambda = 1 \times d\phi = \frac{\mu I}{2\pi y} dy$$

Total flux linkages of the conductor due to flux between points

$$\lambda_{12} = \int_{D_1}^{D_2} \frac{\mu I}{2\pi y} dy = \frac{\mu}{2\pi} \ln \frac{D_2}{D_1} \text{ Wb-turns/m}$$

where 'ln' stands for natural logarithm.

Since  $\mu_r = 1$   $\mu = 4\pi \times 10^{-7}$

$$\lambda_{12} = 2 \times 10^{-7} \ln \frac{D_2}{D_1} \text{ Wb-turns/m}$$

The inductance of the conductor by the flux included between points  $P_1$  and  $P_2$  is then

$$L_{12} = 2 \times 10^{-7} \ln \frac{D_2}{D_1} \text{ H/m}$$

Or

$$L_{12} = 0.461 \log \frac{D_2}{D_1} \text{ mH/km}$$

Flux linkages due to External flux:

Let the external point be at distance  $D$  from the centre of the conductor, due to external flux, these linkages can be obtained by substituting  $D_1 = r$  and  $D_2 = D$ , i.e.

$$\lambda_{\text{ext}} = 2 \times 10^{-7} \ln \frac{D}{r}$$

Total flux linkages of the conductor due to internal and external flux are

$$\begin{aligned} \lambda &= \lambda_{\text{int}} + \lambda_{\text{ext}} \\ &= \frac{I}{2} \times 10^{-7} + 2 \times 10^{-7} \ln \frac{D}{r} \\ &= 2 \times 10^{-7} \left( \frac{1}{4} + \ln \frac{D}{r} \right) = 2 \times 10^{-7} \ln \frac{D}{re^{-1/4}} \\ &= 2 \times 10^{-7} \ln \frac{D}{r'} \end{aligned}$$

where  $r' = re^{-1/4} = 0.77888 r$

$$\lambda = 2 \times 10^{-7} \ln \frac{D}{r'} \text{ Wb-T/m}$$

Inductance of the conductor due to flux up to an external point

$$L = 2 \times 10^{-7} \ln \left( \frac{D}{r'} \right) \text{ H/m}$$

### Inductance (L)

• It opposes sudden changes in currents. The voltage induced can be obtained by

$$e = \frac{d\phi}{dt} V$$

$\psi$  = flux linkages

$$e = \frac{d\phi}{di} \cdot \frac{di}{dt} V = L \frac{di}{dt}$$

where  $L = \frac{d\phi}{di}$  is defined as the inductance of circuit in Henry

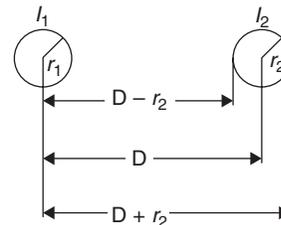
$$L = \frac{\phi}{i} \text{ Henry}$$

$$\psi = Li \text{ (Wb-T)}$$

If the current is alternative

$$\lambda = LI$$

Inductance of a single-phase two wire line



The flux linkage caused by current in Conductor 1 is given by

$$\lambda_1 = 2 \times 10^{-7} \ln \frac{D}{r'}$$

Inductance of the conductor due to current in conductor 1 only is

$$L_1 = 2 \times 10^{-7} \ln \frac{D}{r'_1}$$

Due to current in conductor 2 only is

$$L_2 = 2 \times 10^{-7} \ln \frac{D}{r'_2}$$

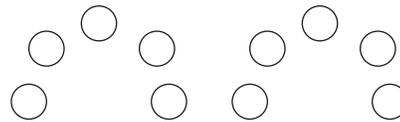
$$L = L_1 + L_2 = 4 \times 10^{-7} \ln \frac{D}{\sqrt{r'_1 r'_2}} \text{ H/m}$$

If  $r'_1 = r'_2 = r'$ ; then

$$L = 4 \times 10^{-7} \ln D/r' \text{ H/m}$$

$$L = 0.921 \ln D/r' \text{ mH/m.}$$

### Inductance of Composite Conductor Live



$$\lambda_i = 2 \times 10^{-7} \frac{I}{n} \left( \ln \frac{1}{D_{i1}} + \ln \frac{1}{D_{i2}} + \dots + \ln \frac{1}{D_{ii}} \dots \ln \frac{1}{D_{in}} \right)$$

$$= 2 \times 10^{-7} \frac{I}{m_1} \left( \ln \frac{1}{D_{i1}} + \ln \frac{1}{D_{i2}} + \dots \ln \frac{1}{D_{im_1}} \right)$$

$$= 2 \times 10^{-7} I \ln \frac{(D_{i1} \dots D_{i2} \dots D_{in})^{1/m'}}{(D_{i1} D_{i2} D_{ii'} D_{in})^{1/n}} \text{ Wb-T/m}$$

The inductance of filament is then

$$L_i = \frac{\lambda_i}{I/n} = 2n \times 10^{-7} L_n \frac{(D_{i1} \dots D_{ij} \dots D_{im'})}{(D_{i1} D_{i2} \dots D_{ii} \dots D_{in})} \text{H/m}$$

$$L_{\text{avg}} = \frac{L_1 + L_2 + L_3 + \dots + L_n}{n}$$

∴ Conductor A is composed of  $n$  filaments electrically in parallel

$$L_A = \frac{L_{\text{avg}}}{n} = \frac{L_1 + L_2 + \dots + L_n}{n^2}$$

$$L_A = 2 \times 10^{-7}$$

$$L_n \frac{[(D_{11'} \dots D_{1j'} \dots D_{1m'}) \dots (D_{ii'} \dots D_{ij'} \dots D_{im'}) \dots (D_{n1} \dots D_{nj'} \dots D_{nm'})]^{1/n^2}}{[(D_{i1}' \dots D_{1j} \dots D_{in}') \dots (D_{ii}' \dots D_{ij}' \dots D_{in}') \dots (D_{ni}' \dots D_{nj} \dots D_{nn})]^{1/n^2}}$$

$$L_A = 2 \times 10^{-7} \ln \times \left( \frac{\text{Products of all possible mutual inductances}}{\text{Geometric mean radius}} \right)$$

$$L_A = 2 \times 10^{-7} L_n \frac{\text{DM}}{\text{DSA}} \text{H/m} = 0.461 L_{\text{ong}} \frac{\text{DM}}{\text{DSA}} \text{mH/km}$$

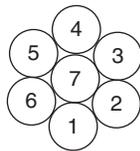
Total inductance of line

$$L = L_A + L_B$$

### Problems on Self-GMD

**Example 20:** A conductor is composed of seven (7) identical copper strands each having a radius  $r$ , as shown in Figure 1. Find the self-GMD of the conductor.

$$\begin{aligned} D_{14} &= 4r \\ D_{12} &= 2r \\ D_{26} &= 2\sqrt{3}r \end{aligned}$$



**Solution:** The self-GMD of seven strand conductor is the 49<sup>th</sup> root of the 49 distances. Thus,

$$D_s = ((r')^7 (D_{12}^2 D_{26}^2 D_{14} D_{17})^6 (2r)^6)^{1/49}$$

Substituting the value of various distances,

$$D_s = \left( (0.7788r)^7 (2^2 r^2 \times 3 \times 2^2 r^2 \times 2^2 r^2 \times 2^2 r \times 2r \times 2r)^6 (2r)^6 \right)^{1/49}$$

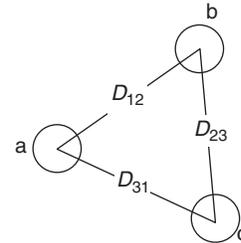
$$D_s = \frac{2r(3(0.7788))^{1/7}}{6^{1/49}}$$

$$D_s = 2.177r$$

### Inductance of Three-phase Lines

The basic equations developed can however be easily adapted to the calculation of the inductance of three-phase lines.

Figure shows conductor of a three-phase live Unsymmetrical spacing:



Practically transposition is difficult but mathematical it can be assumed.

$$L/\text{phase} = 2 \times 10^{-7} \ln \left( \frac{\text{GMD}}{\text{GMR}} \right) \text{H/M}$$

$$L/\text{phase} = 2 \times 10^{-7} \ln \left( \frac{\text{DM}}{\text{DSA}} \right) \text{H/M}$$

$$\text{GMD or DM} = 3\sqrt{XYZ} \text{ if } D_{12} = X, D_{23} = Y, D_{31} = Z$$

$$\text{GMR or DSA} = r'$$

Symmetrical lines

$$X = Y = Z = d \text{ (equilateral spacing)}$$

$$L/\text{phase} = 0.2 \text{Ln} \left( \frac{\text{GMD}}{\text{GMR}} \right) \text{mH/km}$$

$$\text{GMD} = d$$

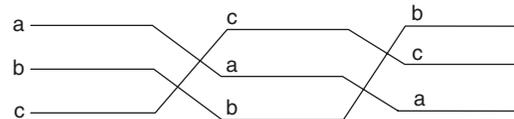
$$\text{GMR} = r'$$

Inductive reactance

$$X = 2\pi FL$$

### Unsymmetrical Spacing

When three-phase line conductors are not equidistant from each other, the conductor spacing is said to be unsymmetrical. So, flux linkages are different. Interchanging the positions of the conductor at regular intervals along the line such an exchange of positions is known as transposition.



$$I_a + I_b + I_c = 0$$

$$I_a = I(1 + j0)$$

$$I_b = I(-0.5 - j0.866)$$

$$I_c = I(-0.5 + j0.866)$$

$$\lambda_{a1} = 2 \times 10^{-7} \left( I_a \ln \frac{1}{r'_a} + I_b \ln \frac{1}{D_{12}} + I_c \ln \frac{1}{D_{31}} \right) \text{Wb-T/m}$$

$$\lambda_{a2} = 2 \times 10^{-7} \left( I_a \ln \frac{1}{r'_a} + I_b \ln \frac{1}{D_{23}} + I_c \ln \frac{1}{D_{12}} \right) \text{Wb-T/m}$$

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$$\lambda_{a3} = 2 \times 10^{-7} b \left( I_a \ln \frac{1}{r'_a} + I_b \ln \frac{1}{D_{31}} + I_c \ln \frac{1}{D_{23}} \right) \text{Wb-T/m}$$

Average flux linkages of conductor 'a' are

$$\lambda_a = \frac{\lambda_{a1} + \lambda_{a2} + \lambda_{a3}}{3}$$

$$= 2 \times 10^{-7} \left( I_a \ln \frac{1}{r'_a} + I_b \ln \frac{1}{(D_{12} D_{23} D_{31})^{1/3}} + I_c \ln \frac{1}{(D_{12} D_{23} D_{31})^{1/3}} \right)$$

$$I_b + I_c = -I_a$$

$$\lambda_0 = 2 \times 10^{-7} I_a \ln \frac{(D_{12} D_{23} D_{31})^{1/3}}{r'_a}$$

$$D_{eq} = (D_{12} D_{23} D_{31})^{1/3} = \text{equivalent equilateral spacing}$$

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{r'_a} = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s} \text{ H/m}$$

$$D_m = D_{eq}$$

$$r_a = r_b = r_c$$

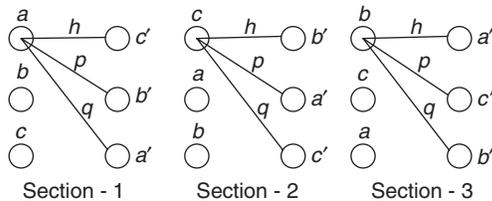
$$L_a = L_b = L_c$$

where  $D_s$  = self-GMD,  $D_m$  = Mutual GMD

### Double-circuit Three-phase Line

The double-circuit three-phase line is built so as to increase transmission reliability, at enhanced cost.

The three sections of the transposition cycle of two parallel circuit three-phase line with vertical spacing are shown below.



$$D_{eq} = (D_{ab} D_{bc} D_{ca})^{1/3}$$

$D_{ab}$  = mutual GMD between phases a and b in Section 1 of transposition cycle

$$= (D_p D_p)^{1/4} = (D_p)^{1/2}$$

$D_{bc}$  = mutual GMD between phases b and c in Section 1 of transposition cycle

$$= (D_p)^{1/2}$$

$D_{ca}$  = mutual GMD between phases a and c in

Section 1 of transposition cycle

$$= (2Dh)^{1/2}$$

$$D_{eq} = 2^{1/6} D^{1/2} p^{1/3} h^{1/6}$$

$$D_{sa} = (r'q r'q)^{1/4} = (r'q)^{1/2}$$

$$D_{sb} = (r'h r'h)^{1/4} = (r'h)^{1/2}$$

$$D_{sc} = (r'q r'q)^{1/4} = (r'q)^{1/2}$$

Equivalent self-GMD  $D_s = (D_{sa} D_{sb} D_{sc})^{1/3}$

$$= (r')^{1/2} q^{1/3} h^{1/6}$$

$$L = 2 \times 10^{-7} \ln \left[ 2^{1/6} \left( \frac{D}{r'} \right)^{1/2} \left( \frac{p}{q} \right)^{1/3} \right] \text{ H/phrase/m.}$$

Mutual inductance between the two circuits

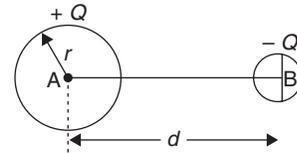
$$M = 2 \times 10^{-7} \ln \left( \frac{p}{q} \right)^{2/3}$$

$M \rightarrow 0$ .  $\therefore$  mutual impedance is zero.

$$L = 1 \times 10^{-7} \ln \frac{3\sqrt{2}D}{r'}$$

### Capacitance

The capacitance and conductance together forms the shunt admittance of a transmission line. When an alternating voltage is applied to the line, the line capacitance draws a leading sinusoidal current called the charging current which is drawn even when the line is open circuited at the far end.



$$V_A = \int_r^{\infty} \frac{Q}{2\pi x \epsilon_0} dx + \int_d^{\infty} \frac{-Q}{2\pi x \epsilon_0} dx$$

$$= \frac{Q}{2\pi \epsilon_0} \left( \log_e \frac{\infty}{r} - \log_e \frac{\infty}{d} \right) \text{ V}$$

$$= \frac{Q}{2\pi \epsilon_0} \log_e \frac{d}{r} \text{ V}$$

Potential difference between conductor B and central 'infinite'

$$V_B = \int_r^{\infty} \frac{-Q}{2\pi x \epsilon_0} dx + \int_d^{\infty} \frac{Q}{2\pi x \epsilon_0} dx$$

$$= \frac{-Q}{2\pi \epsilon_0} \left[ \log_e \frac{d}{r} \right] \text{ V}$$

$$= \frac{-Q}{2\pi\epsilon_0} \log_e \frac{d}{r} V$$

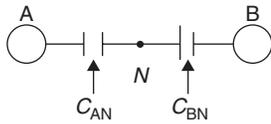
$$V_{AB} = 2V_A = \frac{2Q}{2\pi\epsilon_0} \log_e \frac{d}{r} V$$

$$C_{AB} = Q/V_{AB} = \frac{Q}{\frac{2Q}{2\pi\epsilon_0} \log_e \frac{d}{r}} \text{ F/m}$$

$$C_{AB} = \frac{\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$

### Capacitance to Neutral

The aim is to know the capacitance between one of the conductors and a neutral point between them. Since potential of the midpoint between the conductors is zero, the potential difference between each conductor and ground or neutral is half the potential differences between the conductors.



Capacitance of neutral

$$C_N = C_{AN} = C_{BN} = 2C_{AB}$$

$$C_N = \frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$

### Capacitance of a Three-phase

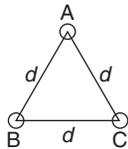
The capacitance of each conductor is considered instead of capacitance from conductor to conductor

Two cases:

1. Symmetrical spacing
2. Unsymmetrical spacing

#### Symmetrical Spacing

The three conductors A, B and C are having charges of  $Q_A$ ,  $Q_B$  and  $Q_C$  per metre (equidistant of  $d$  metre) from each other, the capacitance from line conductor to central in the symmetrical spaced line.



$$V_A = \int_r^{\infty} \frac{Q_A}{2\pi x \epsilon_0} dx + \int_d^{\infty} \frac{Q_B}{2\pi X \epsilon_0} + \int_d^{\infty} \frac{Q_C}{2\pi X \epsilon_0} dx$$

$$= \frac{1}{2\pi\epsilon_0} \left( Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d} + Q_C \log_e \frac{1}{d} \right)$$

$$= \frac{1}{2\pi\epsilon_0} \left[ Q_A \log_e \frac{1}{r} + (Q_B + Q_C) \log_e \frac{1}{d} \right]$$

Assume balanced supply, then

$$Q_A + Q_B + Q_C = 0$$

$$Q_B + Q_C = -Q_A$$

$$V_A = \frac{1}{2\pi\epsilon_0} \left( Q_A \log_e \frac{1}{r} - Q_A \log_e \frac{1}{d} \right) = \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{d}{r} V$$

Capacitance of conductor A with respect to neutral,

$$C_A = \frac{Q_A}{V_A} = \frac{Q_A}{\frac{Q_A}{2\pi\epsilon_0} \log_e \frac{d}{r}} \text{ F/M} = \frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$

#### Unsymmetrical Spacing

Three-phase transposed line having unsymmetrical spacing for balanced condition  $Q_A + Q_B + Q_C = 0$

Capacitance from conductor to neutral is

$$C_A = \frac{Q_A}{V_A} = \frac{2\pi\epsilon_0}{\log_e \frac{3\sqrt{d_1 d_2 d_3}}{r}} \text{ F/m}$$

**Example 21:** A single-phase transmission line has two parallel conductor 4 m apart, radius of each conductor being 2 cm. calculate the capacitance of line per km .given that  $\epsilon_0 = 8.854 \times 10^{-12}$  F/m.

**Solution:** Conductor radius = 2 cm

Spacing of conductors = 4 m = 400 cm

Capacitance of the line

$$= \frac{\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$

$$= \frac{\pi \times 8.85 \times 10^{-12}}{\log_e \frac{400}{2}} \text{ F/m}$$

$$= 5.247 \times 10^{-12} \text{ F/m}$$

$$= 5.247 \times 10^{-3} \mu\text{F/km}$$

**Example 22:** A three-phase overhead transmission line has its conductors arranged at the corners of an equilateral triangle of 8 m side. Calculate the capacitance of each line conductor per km. diameter of each conductor is 2.5 cm.

**Solution:** Conductor radius  $r = 2.5/2 = 1.25$  cm

Spacing of conductors  $d = 8$  m = 800 cm

Capacitance of each line conductor

$$= \frac{2\pi\epsilon_0}{\ln \frac{d}{r}} \text{ F/m} = \frac{2 \times \pi \times 8.854 \times 10^{-12}}{\ln \left( \frac{800}{1.25} \right)} \text{ F/m}$$

$$= 8.609703 \times 10^{-12} \text{ F/m}$$

$$= 8.6097 \times 10^{-3} \mu\text{F/km}$$

**Example 23:** Calculate the capacitance of a 400 km long three-phase 50 Hz overhead transmission line consisting of 3 conductors of each diameter 2 cm and spaced 4 m at the corner of an equilateral triangle.

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**Solution:** Equilateral spacing  $d = 4 \text{ m} = 400 \text{ cm}$   
 $R = 2/2 = 1 \text{ cm}$

Capacitance of each conductor to neutral

$$\frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} = \frac{2\pi\epsilon_0}{\log_e 400/1} \text{ F/m} = \frac{2\pi \times 8.854 \times 10^{-12}}{\log_e 400/1} \text{ F/m}$$

$$= 9.28 \times 10^{-9} \text{ F/km} = 9.28 \times 10^{-3} \mu\text{F/km}$$

**Example 24:** A long two-wire line composed of solid ground conductors is 0.5 cm and the distance between their centres is 2 m. If this distance is double, then the inductance per unit length.

- (A) Halves
- (B) Increases but does not double
- (C) Doubles
- (D) Decreases but does not halve

**Solution:** (B)

Inductance of conductors per phase  $= 0.2 \ln \left( \frac{d}{r'} \right) \text{ mH/km}$

$$L_1/\text{phase} = 0.2 \ln \left( \frac{2}{0.778 \times 0.5 \times 10^{-2}} \right) \text{ mH/km}$$

After distance is doubled

$$L_2/\text{phase} = 0.2 \ln \left( \frac{4}{0.778 \times 0.5 \times 10^{-2}} \right) \text{ mH/km}$$

$L_2/\text{phase} > L_1/\text{phase}$  but  $L_2/\text{phase} \neq 2L_1/\text{phase}$   
 Inductance per unit length has increased but not doubled

**Example 25:** The conductor of 15 km long, single-phase, two-wire line is separated by a distance of 1.5 m. The diameter of each conductor is 0.75 cm. If the conductors are of copper, the inductance of the circuit is

- (A) 2.22 mH
- (B) 22.8 mH
- (C) 1.10 mH
- (D) 11.0 mH

**Solution:** (B)

Inductance of 1- $\phi$  circuit  $= 2L$  per phase

$$= 2 \times 0.2 \ln \frac{d}{r'}$$

Distance ( $d$ ) = 1.5 m,

Radius ( $r'$ ) =  $0.778 \times 0.75 \times 10^{-2} \text{ m}$

$$L = 0.4 \ln \left( \frac{1.5}{0.778 \times 0.75 \times 10^{-2}} \right) \text{ mH/km}$$

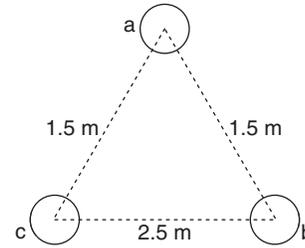
$$= 2.2L \text{ mH/km}$$

Length of the line is 10 km

Total inductance  $= L \times 10 \text{ km}$

$$L = 22.2 \text{ mH}$$

**Example 26:** A 100 km long three-phase transmission lines are shown in the figure. The diameter of each conductor is 0.5 cm. The value of the inductance per phase is



- (A) 1.36 mH
- (B) 136 mH
- (C) 6.817 mH
- (D) 681 mH

**Solution:** (B)

Inductance per phase  $= 2 \times 10^{-7} \ln \frac{\text{GMD}}{\text{GMR}}$

$$\text{GMD} = \sqrt[3]{D_{ab} D_{bc} D_{ca}} = \sqrt[3]{1.5 \times 1.5 \times 2.5} = 1.778 \text{ m}$$

$$\text{GMR} = 0.778 r = 0.778 \times 0.25 \text{ cm}$$

$$= 0.1945 \times 10^{-2} \text{ m}$$

Inductance per phase  $= 0.2 \ln \frac{\text{GMD}}{\text{GMR}} \text{ mH/km}$

$$= 0.2 \ln \left( \frac{1.778}{0.1945 \times 10^{-2}} \right)$$

$$= 1.36 \text{ mH/km}$$

Total inductance per phase  $= 1.36 \times 100 \text{ km} = 136 \text{ mH}$

**Example 27:** A 125 km long transmission line is loaded at 110 kV. If the loss of line is 20 MW and the load is 175 MVA the resistance of the line is

- (A) 7.9  $\Omega$ /phase
- (B) 4.56  $\Omega$ /phase
- (C) 13.68  $\Omega$ /phase
- (D) 45.6  $\Omega$ /phase

**Solution:** (A)

Given line loss expression is given by

$$P_{\text{loss}} = I_L^2 R$$

$$I_L = \frac{P_L}{V} = \frac{175 \times 10^6}{110 \times 10^3} = 1.6 \text{ kA}$$

$$P_{\text{loss}} = 20 \times 10^6 = I_L^2 R = (1.6 \times 10^3)^2 R$$

$$R = 7.9 \Omega/\text{phase}$$

### Common Data Questions

**Example 28:** A 100 km long, three-phase, 110 kV 50 Hz overhead transmission line has three conductors each diameter 1.5 cm. The conductors are spaced 2 m at the corners of equilateral triangle. The capacitance of the line is

- (A)  $1 \times 10^{-3} \mu\text{F}$
- (B) 2.29 F
- (C) 1.5 F
- (D) 1.5  $\mu\text{F}$

**Solution:** (A)

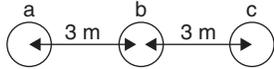
Capacitance of the line (C)  $= \frac{2\pi \epsilon_0}{\ln \frac{d}{r}}$

$$= \frac{2\pi \times 8.854 \times 10^{-12}}{\ln \left( \frac{200}{0.75} \right)} = 10 \times 10^{-12} \text{ F/m}$$

Capacitance of 100 km line =  $10 \times 100 \times 10^{-12}$   
 $C = 1 \times 10^{-9}$

**Example 29:** The capacitance to neutral is  
 (A) 8.4  $\mu\text{F}/\text{km}$  (B) 0.0119  $\mu\text{F}/\text{km}$   
 (C) 8.4 F/km (D) 0.0084 F/km

**Solution:** (B)



$$C_n = \frac{0.242}{\ln \frac{D_{eq}}{r}}$$

$$D_{eq} = (3 \times 3 \times 6)^{1/3} = 3.78 \text{ m}$$

$$C_n = \frac{0.0242}{\ln \frac{3.78}{0.5}} = 0.0119 \text{ } \mu\text{F}/\text{km}$$

**Example 30:** The charging current per kilometre line is  
 (A) 0.379 A/km (B) 0.167 A/m  
 (C) 0.379 A/m (D) 0.23 A/km

**Solution:** (D)

$$X_n = \frac{1}{\omega C_n} = \frac{1}{2 \times \pi \times 50 \times 0.0119 \times 10^{-6}}$$

$$= 267.48 \times 10^3 \text{ } \Omega/\text{km}$$

$$\text{Charging current} = \frac{V_n}{X_n} = \frac{\left(\frac{100}{\sqrt{3}}\right) \times 1000}{267.4 \times 10^3}$$

$$= 0.23 \text{ A/km}$$

## PERFORMANCE OF LINES

Performance of transmission lines is to find the efficiency and Regulation.

The efficiency of the transmission line is %

$$\text{Efficiency} = \frac{\text{Power delivered at the receiving end}}{\text{Power sent from sending end}} \times 100$$

$$\frac{\text{Power delivered at the receiving end}}{\text{Power sent from sending end}} \times 100$$

Receiving end in a transmission line is where the load is connected and the supply is from the sending end.

## Regulation

The regulation of the line is defined as the change in the receiving end voltage, expressed as the percentage of full load voltage from no - load to full load, keeping the sending end voltage and frequency constant.

$$\% \text{ Regulation} = \frac{V_{R_1} - V_{R_2}}{V_{R_2}} \times 100$$

where  $V_{R_1}$  is the receiving end voltage under No-load condition.

$V_{R_2}$  is the receiving end voltage under full load condition. Note that  $V_{R_1}$  and  $V_{R_2}$  are magnitudes only Regulation is calculated on

1. Speed
2. Voltage

Performance of rotating equipment is evaluated based on efficiency and speed regulation.

Performance of static equipment is evaluated based on Efficiency and voltage regulation.

Since transmission line comes under static equipment, performance is evaluated when the system reaches steady state rather than in transient state.

- The loss in a transmission line is due to real power but not due to reactive power. Hence, the efficiency is calculated in terms of real power.

## Classification of Overhead Transmission Lines

- In steady-state condition, for a line whose lengths is less than 160 km, the voltage or current does not vary much and for the line lengths of about 160 km, the parameters can be assumed as lumped but not distributed. These lines are called as electrically short transmission lines. These electrically short transmission lines are again divided as short and medium transmission lines.

### Short Transmission Lines

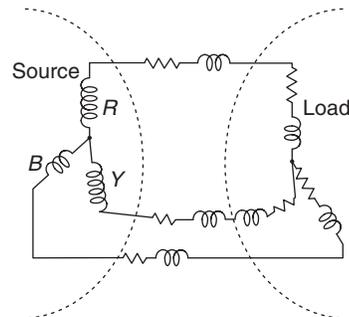
The lines of lengths up to 80 km are known as short transmission lines in which the effect of shunt capacitance is neglected.

### Medium Transmission Lines

The lines of lengths above 80 km and below 160 km are known as medium transmission lines in which the shunt capacitance is assumed to be lumped at the centre (or) half of the capacitance is lumped at each end of the line

### Long Transmission Lines

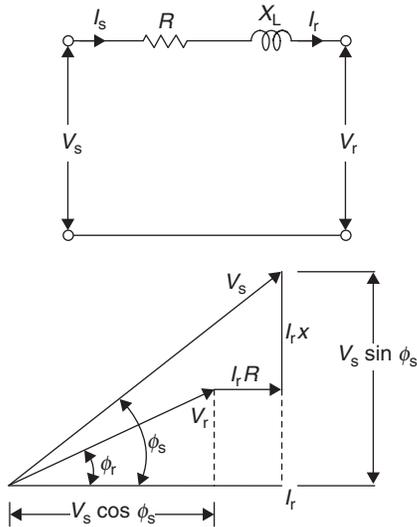
For line lengths more than 160 km, all the parameters are distributed and in some cases where the line length is less than 250 km, the analysis is done using nominal- $\pi$  and T.



- The above figure shows a balanced 3- $\phi$  source which is connected to a balanced star load. As the sum of all the current in a balanced polyphase network is zero, hence the current through the wire connected between the star point of the load and neutral of the system is zero. Hence, the star point of the load and the neutral point of the source are at the source potential. Hence, for all purposes, a 3- $\phi$  balanced system can therefore be analysed on a single-phase basis.

### Short Transmission Line

- The parameters considered here are the lumped parameters. Lumped parameters are those which can be placed at one point (or) two points to the maximum, and these parameters have physical and electrical separation. These parameters are used to study the steady-state behaviour of the long transmission lines.
- The inductance will be high and capacitance will be less in short transmission lines. Hence the capacitance is neglected. Therefore the parameters present are only resistance and inductance.
- The equivalent circuit and the vector diagram for a short transmission line are shown in the below figure.



The vector diagram is drawn taking  $I_r$ , the receiving end current, as the reference

From the vector diagram,

$$V_s \cos \phi_s = V_r \cos \phi_r + I_r R \tag{7}$$

$$V_s \sin \phi_s = V_r \sin \phi_r + I_r X \tag{8}$$

Squaring and adding equations (7) and (8),

$$V_s^2 = V_r^2 + 2 I_r V_r \cos \phi_r + 2 I_r X V_r \sin \phi_r + I_r^2 (R^2 + X^2)$$

$$\Rightarrow V_s = V_r \sqrt{1 + \frac{2 I_r R \cos \phi_r}{V_r} + \frac{2 I_r X \sin \phi_r}{V_r} + \frac{I_r^2}{V_r^2} (R^2 + X^2)}$$

The last term in the square root can be neglected.

$$V_s = V_r \left\{ 1 + \left( \frac{2 I_r R}{V_r} \right) \cos \phi_r + \frac{2 I_r X}{V_r} \sin \phi_r \right\}^{1/2}$$

Using Binomial expansion, and limiting only to two terms,

$$V_s \approx V_r + I_r R \cos \phi_r + I_r X \sin \phi_r$$

Here,  $V_s$  is the sending end voltage corresponding to a particular load current and power factor condition. From the equivalent circuit of short line, we can see that the receiving end voltage under no load  $V_r'$  is same as the sending end voltage under full-load condition, i.e.

$$V_r' = V_s$$

$$\therefore \% \text{ Regulation} = \frac{V_s - V_r}{V_r} \times 100$$

$$= \left( \frac{I_r R}{V_r} \cos \phi_r + \frac{I_r X}{V_r} \sin \phi_r \right) \times 100$$

$$\text{Per unit Regulation} = \frac{I_r R}{V_r} \cos \phi_r + \frac{I_r X}{V_r} \sin \phi_r$$

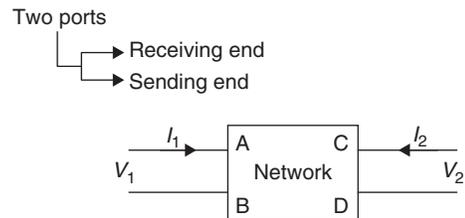
$$= u_r \cos \phi_r + u_x \sin \phi_r$$

where  $u_r$  and  $u_x$  are the per unit values of the resistance and reactance of the line, respectively.

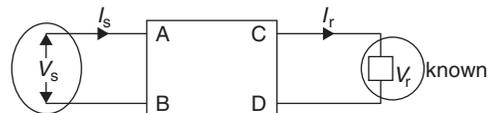
- The performance of transmission line can be analysed at a faster rate with the help of less no. of equations by representing the transmission line in a 2-port network model rather R-L-C model.

### Two-port Network Model

**Port:** pair of terminals (phase and Neutral) is known as ports.



As we want  $V_1, I_1$  from  $V_2, I_2$ , we take the transmission line as A B C D networks.



The unknown sending end electrical quantities are expressed in terms of known receiving end electrical quantities along with certain parameters.

Those parameters are called A B C D parameters.

$$V_s = A V_r + B I_r \text{ kV/phase}$$

$$I_s = C V_r + D I_r \text{ kA/phase}$$

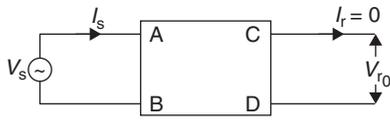
$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

- If  $A = D \rightarrow$  Symmetrical network  
 If  $AD - BC = 1$   
 (or)  $-AD + BC = 1$  }  $\rightarrow$  Reciprocal network

- Electrical network may be symmetrical (or) unsymmetrical but always reciprocal network

### Units for A, B, C, D

#### Open Circuit at the Receiving End



$$V_s = A V_{r0}$$

$$I_s = C V_{r0}$$

$$A = \frac{V_s}{V_{r0}} \Big|_{I_r=0} \text{ (unitless)}$$

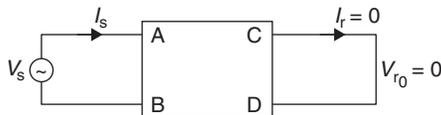
$$C = \frac{I_s}{V_{r0}} \Big|_{I_r=0}$$

'C' open circuit admittance in Mho's

$$\% \text{ Regulation} = \frac{|V_{r0}| - |V_r|}{|V_r|} \times 100$$

$$\% \varepsilon = \frac{\left| \frac{V_s}{A} \right| - |V_r|}{|V_r|} \times 100$$

#### Short Circuit at Receiving End



When the receiving end is short circuited  $V_r = 0$ . Then ABCD parameters are given by

$$V_s = B I_r$$

$$I_s = D I_r$$

$$\Rightarrow \frac{V_s}{I_s} = \frac{B}{D}$$

$$B = \frac{V_s}{I_r} \Big|_{V_r=0}$$

(Short-circuit impedance in ohms)

$$D = \frac{I_s}{I_r} \Big|_{V_r=0}$$

- For short transmission lines, regulation is equal to voltage drop but this is not possible in medium and long transmission lines, as shunt capacitance is present.
- In case of short transmission line, if load is open, then  $|V_r| = |V_s|$  and  $I_s = 0$ . But this is not true in case of medium and long transmission lines as shunt capacitance is present and hence  $I_s \neq 0$  but  $I_r = 0$  and  $V_s = V_r$ .

### Condition for Maximum Regulation

The expression is given by

$$\% \varepsilon = \% V_r \cos \phi_r + \% V_x \sin \phi_r$$

The condition for maximum regulation can be obtained from

$$\frac{d\% \varepsilon}{d\phi_r} = 0$$

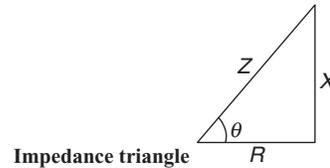
$$\Rightarrow -\% V_r \sin \phi_r + \% V_x \cos \phi_r = 0$$

$$\Rightarrow \tan \phi_r = \frac{\% V_x}{\% V_r} = \frac{\frac{I_r X}{V_r} \times 100}{\frac{I_r R}{V_r} \times 100} = \frac{X}{R}$$

$$\Rightarrow \phi_r = \tan^{-1} \left( \frac{X}{R} \right)$$

X – Line reactance Impedance triangle

R – Line resistance



Impedance triangle

$$\tan \theta = \frac{X}{R}$$

When the phase angle of load is equal to the impedance angle, the regulation is maximum.

$$\Rightarrow \tan \phi_r = \frac{X}{R} = \tan \theta$$

$$\Rightarrow \phi_r = \theta$$

### Condition for Zero Regulation

$$\% \varepsilon = \% V_r \cos \phi_r - \% V_x \sin \phi_r = 0$$

$$\Rightarrow \tan \phi_r = \frac{\% V_r}{\% V_x} \frac{\frac{I_r R}{V_r} \times 100}{\frac{I_r X}{V_r} \times 100} = \frac{R}{X}$$

$$\Rightarrow \tan \phi_r = \frac{R}{x} = \cot \theta = \tan \left( \frac{\pi}{2} - \theta \right)$$

$$\Rightarrow \phi_r = \frac{\pi}{2} - \theta$$

$$\Rightarrow \phi_r + \theta = \frac{\pi}{2}$$

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- Max regulation occurs for lagging load,

$$\% \varepsilon = \% V_r \cos \phi_r + \% V_x \sin \phi_r$$

- Min regulation occurs for leading load,

$$\% \varepsilon = \% V_r \cos \phi_r - \% V_x \sin \phi_r$$

(negative value)

- We get % regulation as '0' only in the leading loads provided

$$(i) \frac{X}{R} = 1.0$$

(ii) 0.707 lead p.f.

**Example 31:** A single-phase overhead transmission line delivers 1100 kW at 33 kV at 0.8 p.f. lagging. The total resistance and inductive reactance of the line are 15 Ω and 20 Ω, respectively. Find the (i) sending end voltage (ii) sending end power factor and (iii) transmission efficiency.

**Solution:** Load power factor,  $\cos \phi_r = 0.8$  lag

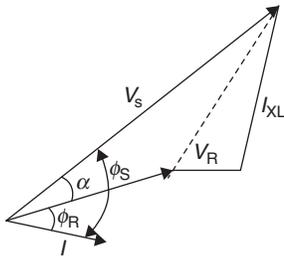
Total line impedance  $\rightarrow Z = R + jX_L$   
 $= 15 + j20$

Receiving end voltage,  $V_R = 33$  kV

$$\begin{aligned} \therefore \text{Line current, } I &= \frac{\text{kW} \times 10^3}{V_R \times \cos \phi_r} \\ &= \frac{1100 \times 10^3}{33,000 \times 0.8} = 41.66 \end{aligned}$$

$$\cos \phi_r = 0.8$$

$$\Rightarrow \sin \phi_r = 0.6$$



$$\vec{V}_R = V_R + j0 = 33,000 \text{ V}$$

$$\begin{aligned} I &= I(\cos \phi_r - j \sin \phi_r) \\ &= 41.67(0.8 - j0.6) \\ &= 33.33 - j25 = 41.67 \angle -36.86^\circ \end{aligned}$$

$$\begin{aligned} (i) \text{ Sending end voltage, } \vec{V}_s &= \vec{V}_R + \vec{I} * \vec{Z} \\ &= 33,000 + (33.33 - j25.0)(15 + j20) \\ &= 33,999.95 + 291.6j \end{aligned}$$

$$\begin{aligned} \therefore \text{ Magnitude of } V_s &= \sqrt{(33,999.95)^2 + (291.6)^2} \\ &= 34,001.20 \text{ V} \end{aligned}$$

- (ii) Angle between  $\vec{V}_s$  and  $\vec{V}_r$

$$\alpha = \tan^{-1} \left( \frac{291.6}{33,999.95} \right) = 0.49^\circ$$

- $\therefore$  Sending power factor angle,

$$\phi_s = \phi_r + \alpha = 36^\circ 86' + 0^\circ 49' = 37^\circ 35'$$

- $\therefore$  Sending end power factor,  $\cos \phi_s = \cos(37^\circ 35') = 0.79$  lagging

- (iii) Line losses =  $I^2 R = (41.67)^2 \times 15 = 26.045$  kW

$$\text{Output} = 1100 \text{ kW}$$

$$\begin{aligned} \text{Power sent} &= 1100 + 26.045 \text{ kW} \\ &= 1126.045 \text{ kW} \end{aligned}$$

- $\therefore$  Transmission efficiency

$$\begin{aligned} &= \frac{\text{Power delivered}}{\text{Power sent}} \times 100 \\ &= \frac{1100}{1126.045} \times 100 = 97.68\% \end{aligned}$$

**Example 32:** What is the maximum length in km for a single-phase transmission line having copper conductor of 0.775 cm<sup>2</sup> cross-section over which 200 kW at unity power factor and at 3300 V are to be delivered? The efficiency of transmission is 90%. Take specific resistance as 1.725 μΩ cm.

**Solution:** Receiving end power = 200 kW

Transmission efficiency = 0.9

$$\begin{aligned} \therefore \text{ Sending end power} &= \frac{2,00,000}{0.9} \\ &= 2,22,222 \text{ W} \end{aligned}$$

$$\begin{aligned} \therefore \text{ Line losses} &= 2,22,222 - 2,00,000 \\ &= 22,222 \text{ W} \end{aligned}$$

$$\text{Line current } I = \frac{200 \times 10^3}{3300 \times 1} = 60.6 \text{ A}$$

Let  $R$  Ω be the resistance of the 2 conductors, hence line losses =  $2 I^2 R$

$$22,222 = 2 I^2 R$$

$$\Rightarrow R = \frac{22,222}{2 \times (60.6)^2} = 3.025 \text{ } \Omega$$

$$\begin{aligned} \therefore R &= \frac{\rho I}{A} \Rightarrow l = \frac{R \times A}{\rho} \\ &= \frac{3.025 \times 0.775}{1.725 \times 10^{-6}} = 1.36 \times 10^6 \text{ cm} \\ &= 13.6 \text{ km} \end{aligned}$$

### Medium Transmission Lines

We know that the transmission lines with lengths between 80 km and 160 km are categorized as medium length lines, in which the parameters are assumed to be lumped.

The shunt capacitance is assumed to be either at the middle of the line or half of the total capacitance is concentrated at each end of the line. These two configurations are classified as—nominal-T and nominal- $\pi$ , respectively. The circuits are shown below.

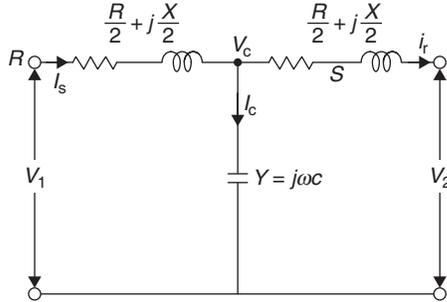


Figure 13 Nominal-T network

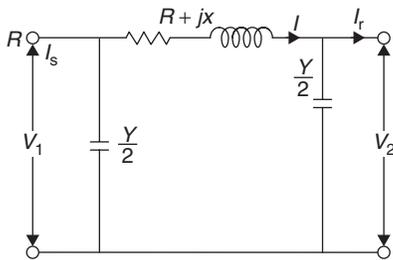


Figure 14 Nominal- $\pi$  network

### Nominal-T Network

To analyse the medium length lines by this network, it would be convenient to take receiving end current as the reference vector rather than taking the receiving end voltage ( $V_r$ ) as the reference. The vector diagram for lagging power factor load is given below.

From the network diagram, in order to calculate the regulation, we have to first calculate the sending end voltage  $V_s$ .

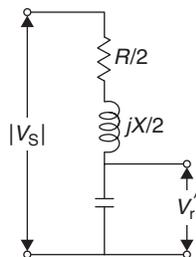
$$V_c = (|V_r| \cos \phi_r + j|V_r| \sin \phi_r) + I_r \left( \frac{R}{2} + j \frac{x}{2} \right)$$

$$I_c = j\omega c V_c$$

$$I_s = I_c + I_r = I_r + j\omega c V_c$$

$$V_s = V_c + I_s \left( \frac{R}{2} + j \frac{x}{2} \right)$$

$$= (|V_r| \cos \phi_r + j|V_r| \sin \phi_r) + I_r \left( \frac{R}{2} + j \frac{x}{2} \right) + I_s \left( \frac{R}{2} + j \frac{x}{2} \right)$$



To calculate the regulation, we have to find  $V_r'$ , the receiving end no load voltage keeping the sending end voltage  $V_s$  fixed in magnitude. The above nominal-T network reduces to the network beside.

$$V_r' = \frac{|V_s| \left( \frac{-j}{\omega c} \right)}{\frac{R}{2} + j \frac{x}{2} - \frac{j}{\omega c}}$$

∴ The regulations for the nominal-T network is obtained as

$$\% \text{ regulations} = \frac{V_r' - V_r}{V_r} \times 100$$

The efficiency is obtained as

$$\% \eta = \frac{\text{Power at receiving end}}{\text{Power at receiving + Loss}} \times 100$$

In order to determine A, B, C, D constants for nominal-T,

$$V_c = V_r + I_r \left( \frac{Z}{2} \right)$$

$$I_c = V_c Y$$

$$I_s = I_r + I_c = I_r + V_c Y \Rightarrow I_r + \left( V_r + I_r \frac{Z}{2} \right) Y$$

$$V_s = V_c + I_s \frac{Z}{2} = V_r + I_r \frac{Z}{2} + \left\{ I_r + \left( V_r + I_r \frac{Z}{2} \right) Y \right\} \frac{Z}{2}$$

$$= V_r \left( 1 + \frac{YZ}{2} \right) + I_r \left( \frac{Z}{2} + \frac{Z}{2} + \frac{YZ^2}{4} \right)$$

$$= V_r \left( 1 + \frac{YZ}{2} \right) + I_r \left( Z + \frac{YZ^2}{4} \right)$$

$$I_s = I_r \left( 1 + \frac{YZ}{2} \right) + V_r Y = YV_r + \left( 1 + \frac{YZ}{2} \right) I_r$$

By comparing the above equations with the standard ones,

$$V_s = AV_r + BI_r$$

$$I_s = CV_r + DI_r$$

$$\Rightarrow A = 1 + \frac{YZ}{2}, B = Z \left( 1 + \frac{YZ}{4} \right), C = Y, D = \left( 1 + \frac{YZ}{2} \right)$$

We can see that  $A = D$ ,

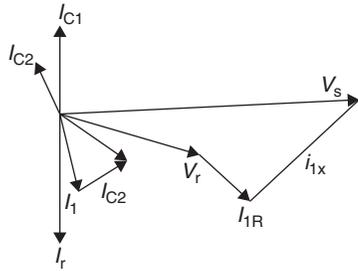
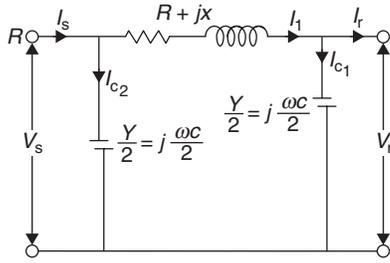
$$\text{and } AD - BC = \left( 1 + \frac{YZ}{2} \right)^2 - YZ \left( 1 + \frac{YZ}{4} \right)$$

$$= 1 + \frac{Y^2 Z^2}{4} - YZ - YZ - \frac{Y^2 Z^2}{4} = 1$$

### Nominal- $\pi$ Representation

Nominal- $\pi$  representation of medium length transmission lines and their phasor diagram are shown below.

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From the above representation,

$$I_{c_1} = jV_r \frac{\omega C}{2}, I_1 = |I_1| (\cos \phi_r - j \sin \phi_r) + jV_r \frac{\omega C}{2}$$

$$V_s = V_r + I_1 Z$$

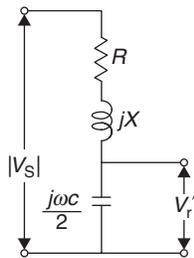
$$= V_r + \left\{ |I_1| (\cos \phi_r - j \sin \phi_r) + j \frac{V_r \omega C}{2} \right\} (R + jX)$$

and  $I_s = I_1 + I_{c_2} = I_1 + j \frac{V_s \omega C}{2}$

$$I_s = |I_1| (\cos \phi_r - j \sin \phi_r) + \frac{jV_r \omega C}{2} + \frac{jV_s \omega C}{2}$$

$$V_s = V_r + \left\{ |I_1| (\cos \phi_r - j \sin \phi_r) + \frac{jV_r \omega C}{2} \right\} (R + jX)$$

To calculate the regulation, we have to find no load receiving end voltage by keeping the sending end voltage constant. The nominal- $\pi$  circuit for no-load reduces as follows.



$$V_r' = \frac{|V_s| \left( \frac{-2j}{\omega C} \right)}{R + jX - \frac{2j}{\omega C}}$$

$\therefore$  % Regulation =  $\frac{V_r' - V_r}{V_r} \times 100$

and %  $\eta = \frac{P}{P + 3I_1^2 R} \times 100$

To find the A, B, C, D constants for nominal- $\pi$  network,

$$I_c = V_r \frac{Y}{2}$$

$$I_1 = I_r + I_c = I_r + \frac{V_r Y}{2}$$

$$V_s = V_r + I_1 Z = V_r + \left( I_r + \frac{V_r Y}{2} \right) Z = \left( 1 + \frac{YZ}{2} \right) V_r + ZI_r$$

$$I_s = I_1 + I_{c_2} = I_1 + \frac{V_s Y}{2} = I_r + \frac{V_r Y}{2} + \left\{ V_r \left( 1 + \frac{YZ}{2} \right) + ZI_r \right\} \frac{Y}{2}$$

$$= V_r \left( Y + \frac{Y^2 Z}{4} \right) + \left( 1 + \frac{YZ}{2} \right) I_r$$

Comparing the coefficients of equations with the standard ones,

$$A = 1 + \frac{YZ}{2},$$

$$B = Z$$

$$C = Y \left( 1 + \frac{YZ}{4} \right)$$

$$D = \left( 1 + \frac{YZ}{2} \right)$$

It is seen that  $A = D$ ,

and  $AD - BC = \left( 1 + \frac{YZ}{2} \right)^2 - ZY \left( 1 + \frac{YZ}{4} \right) = 1$

**Example 33:** A three-phase 50 Hz overhead transmission line 100 km long has the following constants:

Resistance/km/phase = 0.1  $\Omega$

Inductive reactance/km/phase = 0.2  $\Omega$

Capacitive susceptance/km/ phase =  $0.04 \times 10^{-4}$  mho

Determine (i) sending end current (ii) sending end voltage (iii) sending end power factor and (iv) transmission efficiency when supplying a balanced load of 10,000 kW at 66 kV, p.f. 0.8 lagging, use nominal-T method.

**Solution:** Total resistance/phase,  $R = 0.1 \times 100 = 10 \Omega$

Total resistance/phase,  $X_L = 0.2 \times 100 = 20 \Omega$

Capacitive susceptance,

$$Y = 0.04 \times 10^{-4} \times 100 = 4 \times 10^{-4} \Omega$$

Receiving end voltage/phase

$$V_R = \frac{66,000}{\sqrt{3}} = 38,105 \text{ V}$$

Load current,  $I_R = \frac{10,000 \times 10^3}{\sqrt{3} \times 66 \times 0.8} = 109 \text{ A}$

$$\cos \phi_r = 0.8, \Rightarrow \sin \phi_r = 0.6$$

Impedance per phase,  $\bar{Z} = R + jX_L = 10 + j20$

Taking receiving end voltage as the reference vector, we have,

Receiving voltage,  $\bar{V}_R = V_R + j0 = 38,105 \text{ V}$

Load current,  $\bar{I}_R = I_R (\cos\phi_r - j\sin\phi_r)$   
 $= 109 (0.8 - j0.6) = 87.2 - j65.4$

Voltage across C,  $\bar{V}_1 = \bar{V}_R + \bar{I}_R \frac{\bar{Z}}{2}$   
 $= 38,105 + (87.2 - j65.4) (5 + j10)$   
 $= 38,105 + 436 + j872 - j327 + 654$   
 $= 39,195 - j545$

Charging current,  $\bar{I}_c = jY\bar{V}_1$   
 $= j4 \times 10^{-4} (39,195 - j545)$   
 $= 0.218 + j15.6$

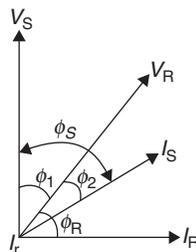
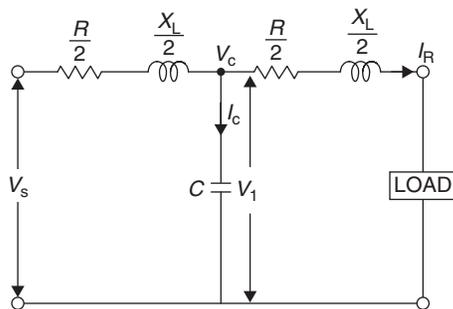
Sending end current

$\bar{I}_s = \bar{I}_R + \bar{I}_c$   
 $= (87.2 - j65.4) + (0.218 + j15.6)$   
 $= 87.4 - j49.8 = 100 \angle -29^\circ 40' = 100 \text{ A}$

Sending end voltage,

$\bar{V}_s = \bar{V}_1 + \bar{I}_s \frac{\bar{Z}}{2}$   
 $= (39,195 - j545) + (87.4 - j49.8) (5 + j10)$   
 $= 40130 + j80$

$\therefore$  Line-to-line sending end voltage =  $40130 \times \sqrt{3}$   
 $= 69,507 \text{ V} = 69.507 \text{ kV}$



$\phi_s = \theta_1 + \theta_2 = 6' + 29^\circ 40' = 29^\circ 46'$

$\therefore$  Sending end power factor,  $\cos\phi_s$   
 $= \cos 29^\circ 46' = 0.868$

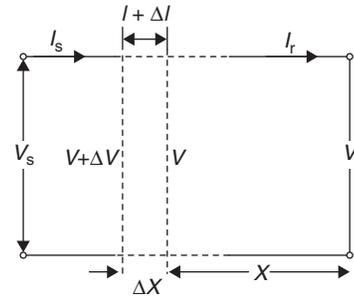
$\therefore$  Sending end power =  $3 V_s I_s \cos\phi_s$   
 $= 3 \times 40,130 \times 100 \times 0.868 = 10,449.85 \text{ kW}$

Power delivered = 10,000 kW

$\therefore$  Transmission efficiency =  $\frac{10000}{10,449.855} \times 100 = 95.7\%$

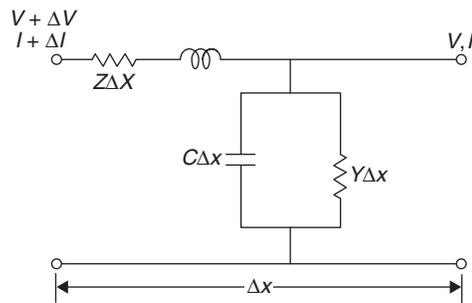
### Long Transmission Lines

If the lines are more than 160 km in length, for the analysis to be accurate, the parameters must be taken as distributed rather than lumped for the voltage and currents.



Let  $Z$  = Series impedance per unit length  
 $y$  = Shunt admittance per unit length  
 $l$  = Length of the line  
 $Z = zl$  = Total series impedance  
 $Y = yl$  = Total shunt admittance

For analysis, the elemental length  $dx$  is redrawn



We take the receiving end as the reference for measuring distances. Take an elemental length  $dx$  of the line at a distance ' $x$ ' from the receiving end. Let the voltage and current at a distance ' $x$ ' are  $V$  and  $I$  and at a distance  $x + dx$ ,  $V + \Delta V$  and  $I + \Delta I$ , respectively.

$$\Delta V = Iz \Delta x \Rightarrow \Delta I = Vy \Delta x$$

$$\Rightarrow \frac{\Delta V}{\Delta x} Iz \Rightarrow \frac{\Delta I}{\Delta x} = Vy$$

When  $\Delta x \rightarrow 0$ , the equations become,

$$\frac{dV}{dx} = Iz \tag{9}$$

$$\frac{dI}{dx} = Vy \tag{10}$$

Differentiating Equation (9), we get

$$\Rightarrow \frac{d^2V}{dx^2} = Z \frac{dI}{dx} = Z \cdot y \cdot V \Rightarrow \frac{d^2V}{dx^2} = Z y V = 0$$

The solution of the above equation is

$$V = A \exp(\sqrt{yz} \cdot x) + B \exp(-\sqrt{yz} \cdot x)$$

Let  $Z_c = \sqrt{\frac{z}{y}}$  and  $\gamma = \sqrt{yz} = \alpha + j\beta$

where  $Z_c$  is known as characteristic impedance and ' $\gamma$ ' is the propagation constant.

The above equations can be rewritten as

$$V = A e^{\gamma x} + B e^{-\gamma x}$$

$$I = \frac{1}{Z_c} (A e^{\gamma x} - B e^{-\gamma x})$$

The constants can be determined by the boundary conditions, at  $x = 0$ ,

$$V = V_r, I = I_r \Rightarrow V_r = A + B$$

$$I_r = \frac{1}{Z_c} (A - B) \Rightarrow A = \frac{V_r + I_r Z_c}{2} \text{ and } B = \frac{V_r - I_r Z_c}{2}$$

Substituting 'A' and 'B' in above equations

$$V = \frac{V_r + I_r Z_c}{2} e^{\gamma x} + \frac{V_r - I_r Z_c}{2} e^{-\gamma x}$$

and  $I = \frac{1}{Z_c} \left[ \frac{V_r + I_r Z_c}{2} e^{\gamma x} - \frac{V_r - I_r Z_c}{2} e^{-\gamma x} \right]$

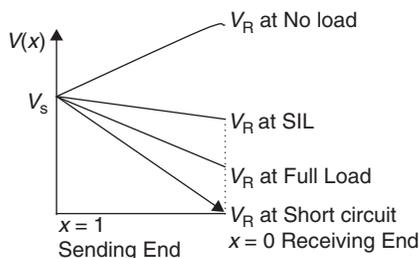
We can observe that  $V$  and  $I$  are the functions of the distance  $x$ , receiving end voltage  $V_r$  and current  $I_r$  and the parameters of the line, which means they will vary as we move from receiving end towards the sending end.

$$Z_c \text{ and } \gamma \text{ are complex} \Rightarrow Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{r + j\omega L}{g + j\omega c}}$$

$$\text{For a loss-less line, } r = 0, g = 0, \Rightarrow Z_c = \sqrt{\frac{L}{c}}$$

The Surge impedance ( $Z_c$ ) is purely resistive and is known as surge impedance of the line.

### Voltage Profiles



During high frequencies (or) surges, normally the losses are neglected and hence, the characteristic impedance becomes surge impedance. '**Surge impedance loading**' of a line is the power transmitted when the line is terminated through a resistance equal to surge impedance. A line terminated in its characteristic impedance is called a flat line or an infinite line.

The lower value of surge impedance in case of cables is due to relatively large capacitance and low inductance of the cables.

The propagation constant  $\gamma = \alpha + j\beta$ , the real part is known as attenuation constant ( $\alpha$ ) and the quadrature component  $\beta$  is known as the phase constant and is measured in radians per unit length.

In the above equation, the first part,  $\frac{V_r + I_r Z_c}{2} e^{\alpha x} e^{j\beta x}$  is called the incident voltage wave as its value increases as ' $x$ ' is increased, this means that the magnitude of voltage wave decreases in magnitude as it travels towards the receiving end. Similarly, the second part of the equation  $\frac{V_r - I_r Z_c}{2} e^{-\alpha x} e^{-j\beta x}$  is called reflected voltage. At any point on the line, the voltage wave is the sum of the incidence and reflected waves. The above result is true even for the current equation.

The above equations for voltage and current can be re-arranged as follows,

$$V = V_r \frac{e^{\gamma x} + e^{-\gamma x}}{2} + I_r Z_c \frac{e^{\gamma x} - e^{-\gamma x}}{2}$$

$$= V_r \cosh \gamma x + I_r Z_c \sinh \gamma x$$

and  $I = \frac{1}{Z_c} \left[ V_r \frac{e^{\gamma x} + e^{-\gamma x}}{2} + I_r Z_c \frac{e^{\gamma x} + e^{-\gamma x}}{2} \right]$

$$= \frac{1}{Z_c} [V_r \sinh \gamma x + I_r Z_c \cosh \gamma x]$$

$$= \frac{V_r}{Z_c} \sinh \gamma x + I_r \cosh \gamma x$$

The above equations can be re-written for  $x = l$ , where  $V = V_s$  and  $I = I_s$

$$V_s = V_r \cosh \gamma l + I_r Z_c \sinh \gamma l$$

$$I_s = V_r \frac{\sinh \gamma l}{Z_c} + I_r \cosh \gamma l$$

Comparing the above equations with standard equations,

$$A = \cosh \gamma l$$

$$B = Z_c \sinh \gamma l$$

$$C = \frac{\sinh \gamma l}{Z_c}, D = \cosh \gamma l.$$

We can see that  $A = D$  and

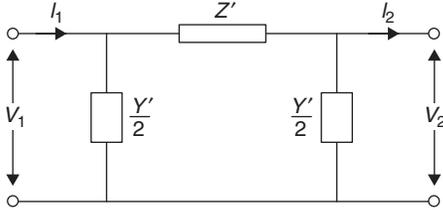
$$AD - BC = \cosh^2 \gamma l - \sinh^2 \gamma l = 1$$

### Equivalent Representation of a Long Line

For Line lengths more than 160 km, the parameters are assumed to be distributed. The equivalent-T and Equivalent- $\pi$  networks are represented. For long lines, nominal-T and nominal- $\pi$  networks may not be used as these networks consider the lines with lumped parameters.

### Equivalent- $\pi$ Representation

The sending end and receiving end voltages and currents are found from this equivalent network.



$V_1$  – Sending end voltage  
 $V_2$  – Receiving end voltage  
 $I_1$  – Sending end current  
 $I_2$  – Receiving end current.

$$V_1 = \left(1 + \frac{Y'Z'}{2}\right)V_r + Z'I_r \quad (11)$$

$$I_1 = y' \left(1 + \frac{Y'Z'}{4}\right)V_r + \left(1 + \frac{Y'Z'}{2}\right)I_r \quad (12)$$

Comparing the above two equations with the standard ones, i.e.,

$$V_1 = V_2 \cosh \gamma l + \frac{T}{2} Z_c \sinh \gamma l$$

$$I_1 = V_2 \frac{\sinh \gamma l}{Z_c} + \frac{T}{2} \cosh \gamma l$$

$$\Rightarrow 1 + \frac{Y'Z'}{2} = \cosh \gamma l, Z' = Z_c \sinh \gamma l,$$

$$Y' \left(1 + \frac{Y'Z'}{4}\right) = \frac{\sinh \gamma l}{Z_c} \text{ and } 1 + \frac{Y'Z'}{2} = \cosh \gamma l$$

$$\text{Consider } Z' = Z_c \sinh \gamma l$$

$$= \sqrt{\frac{z}{y}} \times \sinh \gamma l \times \frac{l}{\sqrt{yz}} \times \sqrt{yz}$$

$$= Zl \frac{\sinh \gamma l}{\gamma l} = Z \frac{\sinh \gamma l}{\gamma l}.$$

From the above expression, we can observe that the equivalent series impedance, the lumped impedance must be multiplied with  $\frac{\sinh \gamma l}{\gamma l}$ .

In order to get the shunt parameters of the equivalent- $\pi$  network, substitute  $Z'$  from the above equation, i.e.,

$$1 + \frac{Y'Z'}{2} = \cosh \gamma l \Rightarrow 1 + \frac{Y'Z'}{2} \times Z_c \sinh \gamma l = \cosh \gamma l$$

(as  $Z = zl$  and  $\gamma = \sqrt{yz}$ )

$$\Rightarrow \frac{Y'}{2} Z_c \sinh \gamma l = \cosh \gamma l - 1$$

$$= \cosh^2 \frac{\gamma l}{2} + \sinh^2 \frac{\gamma l}{2} - \cosh^2 \frac{\gamma l}{2} + \sinh^2 \frac{\gamma l}{2}$$

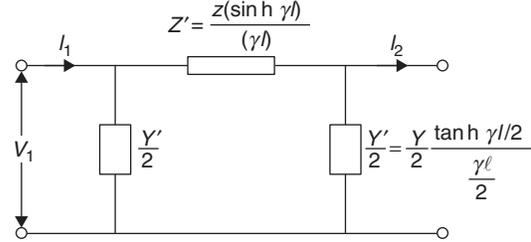
$$\Rightarrow Y' Z_c \sinh \frac{\gamma l}{2} \cosh \frac{\gamma l}{2} = 2 \sinh^2 \frac{\gamma l}{2}$$

$$\Rightarrow \frac{Y'}{2} = \frac{1}{Z_c} \tanh \frac{\gamma l}{2} = \sqrt{\frac{y}{z}} \times \frac{\sqrt{yzl}}{2} \times \frac{\tanh \gamma l / 2}{\sqrt{yz} l / 2}$$

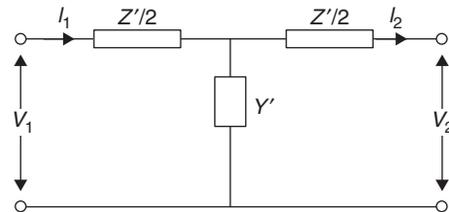
$$= \frac{Y \tanh \gamma l / 2}{2 \gamma l / 2}.$$

where ' $Y$ ' is the total shunt admittance of the nominal- $\pi$  network. Hence, we can conclude that the total shunt admittance of the nominal- $\pi$  network should be multiplied with  $\frac{\tanh \gamma L / 2}{\gamma L / 2}$

Hence, the equivalent circuit can be redrawn as



### Equivalent T-Representation of Long Line



$V_1$  and  $V_2$  are the sending end and receiving end voltages, respectively, whereas  $I_1$  and  $I_2$  are the sending end and receiving end currents, respectively.

From the network of nominal-T,

We have,

$$V_s = V_r \left(1 + \frac{YZ}{2}\right) + I_r Z \left(1 + \frac{YZ}{4}\right)$$

$$I_s = Y V_r + \left(1 + \frac{YZ}{2}\right) I_r$$

As the above network is similar to nominal-T network, we can use the above equations to analyse the equivalent-T network.

$$V_1 = \left(1 + \frac{Y'Z'}{2}\right)V_2 + Z' \left(1 + \frac{Y'Z'}{4}\right)I_2$$

$$I_1 = \left(1 + \frac{Y'Z'}{2}\right)I_2 + Y'V_2$$

Comparing the above equations with the equations of long transmission lines, i.e.

$$V_1 = V_s \cosh \gamma l + I_2 Z_c \sinh \gamma l$$

$$I_1 = V_2 \frac{\sinh \gamma l}{Z_c} + I_s \cosh \gamma l$$

$$\left(1 + \frac{YZ}{2}\right) = \cosh \gamma l, \quad Z' \left(1 + \frac{Y'Z'}{4}\right) = Z_c \sinh \gamma l$$

$$1 + \frac{Y'Z'}{2} = \cosh \gamma l, \quad Y' = \frac{\sinh \gamma l}{Z_c}$$

To find the shunt branch parameters of the equivalent-T circuit,

$$Y' = \frac{1}{Z_c} \sinh \gamma l = \sqrt{\frac{y}{z}} \times \sinh \gamma l \times \frac{\sqrt{yz} \times l}{\sqrt{yz} \times l}$$

$$= Y \frac{\sinh \gamma l}{\gamma l} \quad (\text{where } \gamma = \sqrt{yz}).$$

Hence, to find the equivalent shunt admittance, the lumped shunt admittance should be multiplied with  $\frac{\sinh \gamma l}{\gamma l}$ .

To find the series impedance, consider the equation.

$$\left(1 + \frac{Y'Z'}{2}\right) = \cosh \gamma l$$

Substitute  $Y' = \frac{\sinh \gamma l}{Z_c}$  in the above equation,

$$\Rightarrow 1 + \frac{Z'}{2} \times \frac{\sinh \gamma l}{Z_c} = \cosh \gamma l$$

$$\Rightarrow \frac{Z'}{2} \times \frac{2 \sinh \frac{\gamma l}{2} \cosh \frac{\gamma l}{2}}{Z_c} = \cosh \gamma l - 1 = 2 \sin^2 \frac{\gamma l}{2}$$

$$\Rightarrow \frac{Z'}{2} = Z_c \tan h \frac{\gamma l}{2} = \sqrt{\frac{z}{y}} \tan h \frac{\gamma l}{2}$$

Multiply and divide by  $\frac{\sqrt{yz} \cdot l}{2}$

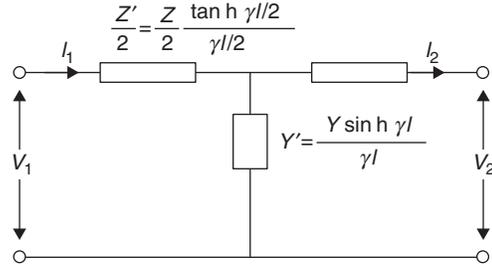
$$\Rightarrow \frac{Z'}{2} = \sqrt{\frac{z}{y}} \times \frac{\sqrt{yz} \cdot l}{2} \times \frac{\tan h (\gamma l/2)}{\frac{\sqrt{yz} \cdot l}{2}}$$

$$= \frac{Z \tan h \frac{\gamma l}{2}}{\frac{\gamma l}{2}} \quad (\text{where } z = x \cdot l \text{ and } \gamma = \sqrt{zy})$$

Therefore to get the series equivalent impedance, the lumped series impedance of the nominal-T network should

be multiplied with  $\frac{\tan h \frac{\gamma l}{2}}{\frac{\gamma l}{2}}$ .

The equivalent T-representation is as follows.



### Calculation of Receiving End Voltages and Currents

We know that

$$V_s = AV_r + B I_r \quad (13)$$

$$I_s = CV_r + D I_r \quad (14)$$

where  $V_s, I_s$  are sending end voltage and current, respectively, while  $V_r, I_r$  are receiving end voltage and current, respectively.

Multiply (13) with C and (14) with A

$$\Rightarrow CV_s = CA V_r + CB I_r \quad (15)$$

$$AI_s = AC V_r + AD I_r \quad (16)$$

Subtract (15) from (16),

$$AI_s - CV_s = AC V_r + AD I_r - CA V_r - CB I_r$$

$$\Rightarrow AI_s - CV_s = I_r(AD - BC)$$

We know that  $AD - BC = 1$  and  $A = D$ , receiving end current is expressed as

$$\Rightarrow I_r = -CV_s + DI_s \quad (17)$$

Multiply (13) with 'D' and (14) with 'B'

$$DV_s = AD V_r + DB I_r \quad (18)$$

$$BI_s = BC V_r + BD I_r \quad (19)$$

Subtract (18) from (19),

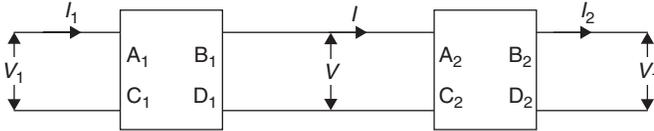
$$DV_s - BI_s = (AD - BC) V_r$$

We know that  $AD - BC = 1$ , receiving end voltage is expressed as

$$\Rightarrow V_r = DV_s - BI_s \quad (20)$$

### Constants of Two Networks in Series

Two networks are said to be connected in series when the output of one network is connected to the input of the other network. Let the constants of the, network be  $A_1, B_1, C_1, D_1$  and the constants of the networks be  $A_2, B_2, C_2, D_2$ , provided there are two networks connected is series as shown in the below figure.



From the figure,

$$V = D_1 V_1 - B_1 I_1$$

$$I = -C_1 V_1 + A_1 I_1$$

And,  $V = A_2 V_2 + B_2 I_2$

$$I = C_2 V_2 + D_2 I_2$$

From the above equations, we can see that

$$D_1 V_1 - B_1 I_1 = A_2 V_2 + B_2 I_2 \quad (21)$$

$$-C_1 V_1 + A_1 I_1 = C_2 V_2 + D_2 I_2 \quad (22)$$

Multiply Equation (21) with  $A_1$  and (22) with  $B_1$  and adding the equations,

$$\begin{aligned} & A_1 D_1 V_1 - A_1 B_1 I_1 = A_1 A_2 V_2 + A_1 B_2 I_2 \\ + & -B_1 C_1 V_1 + A_1 B_1 I_1 = B_1 C_2 V_2 + B_1 D_2 I_2 \\ \hline V_1(A_1 D_1 - B_1 C_1) &= (A_1 A_2 + B_1 C_2)V_2 + (A_1 B_2 + B_1 D_2)I_2 \quad (23) \end{aligned}$$

Multiply (21) with  $C_1$  and (22) by  $D_1$  and the equations,

$$\begin{aligned} & C_1 D_1 V_1 - B_1 C_1 I_1 = A_2 C_1 V_2 + B_2 C_1 I_2 \\ + & -C_1 D_1 V_1 + A_1 D_1 I_1 = C_2 D_1 V_2 + D_1 D_2 I_2 \\ \hline I_1(A_1 D_1 - B_1 C_1) &= V_2(A_2 C_1 + C_2 D_1) + I_2(B_2 C_1 + D_1 D_2) \quad (24) \end{aligned}$$

We know that  $A_1 D_1 - B_1 C_1 = 1$   
Equations (23) and (24) represent the combination of two network and representing as a single network with a single constants.

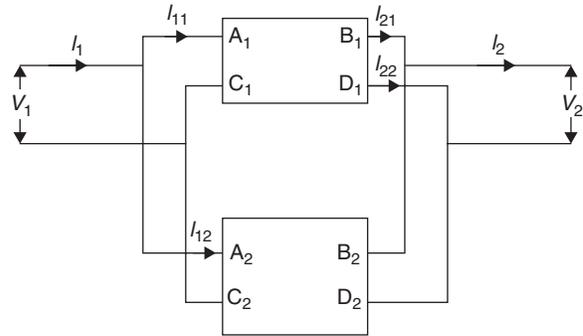
$$\begin{aligned} \therefore A &= A_1 A_2 + B_1 C_2 \\ B &= A_1 B_2 + B_1 D_2 \\ C &= A_2 C_1 + C_2 D_1 \\ D &= B_2 C_1 + D_1 D_2 \end{aligned}$$

Therefore the above relations represented in matrix form as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

### Constants for Two Networks in Parallel

The two networks connected in parallel are as shown below and the constants for the combination of two networks are calculated as follows.



From the above network, we can write that

$$V_1 = A_1 V_2 + B_1 I_{21} \quad (25)$$

$$V_1 = A_2 V_2 + B_2 I_{22} \quad (26)$$

And the combination of the two networks as a single network can be written is the form of equations as

$$V_1 = A V_2 + B I_2$$

where

$$I_2 = I_{21} + I_{22}$$

Multiply Equation (25) with  $B_2$  and (26) with  $B_1$  and adding these results in

$$\begin{aligned} & V_1 B_2 = A_1 B_2 V_2 + B_2 B_1 I_{21} \\ + & V_1 B_1 = A_2 B_1 V_2 + B_1 B_2 I_{22} \\ \hline V_1(B_1 + B_2) &= (A_1 B_2 + A_2 B_1)V_2 + B_1 B_2 (I_{21} + I_{22}) \\ \Rightarrow V_1 &= \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} V_2 + \frac{B_1 B_2}{B_1 + B_2} I_2 \quad (27) \end{aligned}$$

From Equation (27), we can infer that

$$A = \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} \text{ and } B = \frac{B_1 B_2}{B_1 + B_2}$$

As the transmission line in a symmetric network,

$$A = D = \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} = \frac{D_1 B_2 + D_2 B_1}{B_1 + B_2}$$

And  $AD - BC = 1$  and substituting  $A, B, D$  in this equation, we get

$$\begin{aligned} \Rightarrow & \left( \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} \right) \left( \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} \right) - \frac{B_1 B_2}{B_1 + B_2} \times C \\ \Rightarrow & C = C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2} \end{aligned}$$

**Example 34:** A 220 kV, 20 km long, 3- $\phi$  transmission line has the following  $A, B, C, D$  constants.

$$A = D = 0.96 \angle 3^\circ$$

$$B = 55 \angle 65^\circ \Omega/\text{phase}$$

$$C = 0.5 \times 10^{-4} \angle 90^\circ \text{ S/phase}$$

Its charging current per phase is

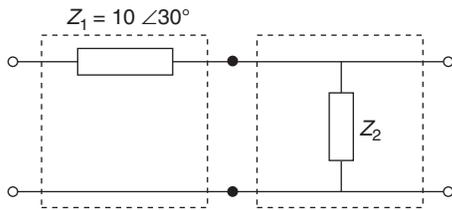
- (A) 6.3 A
- (B) 11 A
- (C) 127 A
- (D) 220 A

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**Solution:** (A)

$$\begin{aligned} \text{Admittance per phase } (Y) &= C \\ &= 0.5 \times 10^{-4} \angle 90^\circ \\ \text{Charging current per phase} &= V_{pn} \times Y \\ &= \frac{220}{\sqrt{3}} \times 10^3 \times 0.5 \times 10^{-4} \\ &= \frac{11}{\sqrt{3}} \text{ A} = 6.35 \text{ A} \end{aligned}$$

**Example 35:** Two networks are connected in cascade as shown in figure with the usual notation the equivalents  $A$ ,  $B$ ,  $C$  and  $D$  constants are obtained. Given that  $C = 0.025 \angle 40^\circ$ , the value of  $Z_2$  is



- (A)  $40 \angle 40^\circ$                       (B)  $20 \angle 40^\circ$   
(C)  $10 \angle 20^\circ$                       (D)  $20 \angle 20^\circ$

**Solution:** (A)

For the given two part network

$$\begin{aligned} \text{The open-circuit admittance} &= \frac{1}{Z_2} \\ \therefore C &= \frac{1}{Z_2} \\ Z_2 &= \frac{1}{C} = 40 \angle -40^\circ \Omega \end{aligned}$$

**Example 36:** The total reactance and total susceptance of a lossless overhead EHV line operating at 50 Hz are given by 0.04 p.u. and 1.1 p.u., respectively. If the velocity of wave propagation is  $30 \times 10^5$  km/s, then the approximate length of line is

- (A) 200 km                      (B) 2000 km  
(C) 20 km                      (D) 2 km

**Solution:** (B)

$$\text{Velocity of wave propagation } (v) = \frac{1}{\sqrt{(L/\text{km}) / (C/\text{km})}}$$

Total reactance of line ( $X_L$ ) = 0.04 p.u.

$$L/\text{km} = \frac{X_L}{2\pi fl} = \frac{0.04}{2\pi \times 50 \times l}$$

Total susceptance of line = 1.1 p.u.

$$\frac{1}{X_c} = 2\pi fC = 1.1 \text{ p.u.}$$

$$C = \frac{1.1}{2\pi \times 50}$$

$$C/\text{km} = \frac{1.1}{2\pi \times 50 \times l}$$

$$\text{Velocity of propagation} = \frac{1}{\sqrt{LC}}$$

$$30 \times 10^5 = \frac{l}{\sqrt{\frac{0.04}{2\pi \times 50} \times \frac{1.1}{2\pi \times 50}}}$$

$$l = 2124.5 \text{ km}$$

**Example 37:** A single-phase 50 Hz generator supplies an inductive load of 5 MW at a power factor of 0.7 lagging by means of an overhead transmission line 20 km long. The line resistance and inductance are 0.019  $\Omega$  and 0.62 mH per km. The voltage at receiving end is required to be kept at 10 kV.

Calculate

- The sending end voltage and voltage regulation of the line.
- The value of capacitors to be placed in parallel with the load such that the regulation is reduced to 60% of that obtained in the previous problem.
- Calculate the transmission efficiency in case (i) and (ii).

**Solution:** Given transmission line is 20 km, so it is considered as a short line.

$$\therefore I = I_R = I_S \text{ given by}$$

$$I_R = \frac{5 \times 10^3}{10 \times 0.7} = \frac{P}{V \cos \phi} = 714 \text{ A}$$

Source voltage

$$\begin{aligned} |V_s| &= |V_R| + |I|(R \cos \phi_R + X \sin \phi_R) \\ &= 10,000 + 714(0.38 \times 0.7 + 3.89 \times 0.714) \end{aligned}$$

Sending end voltage = 12.173 kV

$$\text{Voltage regulation} = \frac{12.173 - 10}{10} \times 100$$

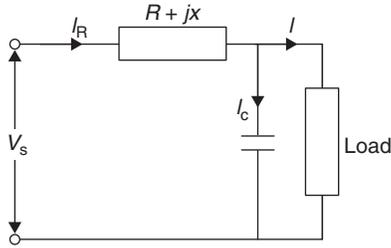
$$= 21.73\%$$

(ii) Desired voltage regulation = 60% of 21.73

$$= 0.6 \times 21.73 = 13.03\%$$

$$\text{Regulation} \frac{V_s - 10}{10} = 0.13$$

$$V_s = 11.3 \text{ kV}$$



Source voltage in the above circuit after placing capacitor to improve voltage regulation

$$V_s = V_r + I_R(R \cos \phi_r + X \sin \phi_r)$$

$$V_s - V_r = 0.13 \text{ kV} = I_R(R \cos \phi_r + X \sin \phi_r)$$

Since capacitance does not draw any real power

$$|I_R| = \frac{5000}{10 \times \cos \phi_r}$$

From the above two equations,  $\cos \phi_r = 0.8685$ ,  $I_R = 575.7 \text{ A}$

Now capacitor current  $I_c = I_R - I$

$$= 575(0.8685 - j0.495) - 714(0.7 - j0.714)$$

$$= 0.195 + j225.17$$

Capacitor current  $I_c = j225.17 \text{ A}$

$$X_c = \frac{1}{\omega c} = \frac{|V_R|}{|I_c|} = \frac{10 \times 1000}{225.17}$$

$$C = \frac{225.17}{10^4 \times \omega} = \frac{225.17}{10^4 \times 314} \Rightarrow C = 71.71 \mu\text{F}$$

(iii) Efficiency of transmission

$$\eta = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

$$\text{Case (i): } \eta = \frac{5000}{5000 + 714^2 \times 0.38 \times 10^{-3}} = 0.962\%$$

$$\text{Case (ii): } \eta = \frac{5000}{500 + 575^2 \times 0.38 \times 10^{-3}} = 97.5\%$$

**Example 38:** A 200 km, 3- $\phi$ , 50 Hz transmission line delivering 25 MVA at 0.85 lagging power factor to a balanced load at 132 kV. The line conductors are spaced equilaterally 2.5 m apart. The conductor resistance is 0.1  $\Omega$ /km and its effective diameter is 1.5 cm. Calculate the sending end voltage and voltage regulation.

**Solution:** Transmission line parameters

$$L = 0.461 \log \frac{D}{r'} = 0.461 \cdot \log \frac{250}{0.778 \times 0.75} = 1.21 \text{ mH/km}$$

$$C = \frac{0.0242}{\log \frac{D}{r}} = \frac{0.0242}{\log \frac{250}{0.75}} = 0.0096 \mu\text{F/km}$$

$$R = 0.1 \times 200 = 20 \Omega$$

$$X_L = 2\pi fL = 2\pi \times 50 \times 1.21 \times 10^{-3} \times 200 = 76.02 \Omega$$

$$Z = R + jX_L = 20 + j76.02 = 78.6 \angle 75.2^\circ$$

$$Y = j\omega c l = j.314 \times 0.0096 \times 10^{-6} \times 200$$

$$Y = 6.02 \times 10^{-4} \angle 90^\circ$$

$$I_R = \frac{25 \times 1000}{\sqrt{3} \times 132} \angle 31.78 = 109.3 \angle -31.78$$

$$V_R (\text{per phase}) = \frac{132}{\sqrt{2}} \angle 0^\circ = 76.2 \angle 0^\circ \text{ kV}$$

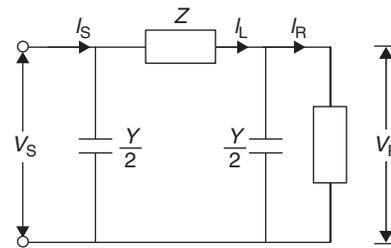


Figure 15 Medium line  $\pi$ -representation

Source voltage

$$\begin{aligned} V_s &= \left(1 + \frac{1}{2}YZ\right)V_R + ZI_R \\ &= \left[1 + \frac{1}{2} \times 6.02 \times 10^{-4} \angle 90^\circ \times 78.6 \angle 75.2^\circ\right] \times 76.2 \\ &\quad + 78.6 \angle 75.2^\circ \times 109.3 \angle -31.78 \times 10^{-3} \\ &= 76.2 + 1.8 \angle 166.2 + 8.59 \angle 43.42 \\ V_s &= 80.69 + 6.3j = 80.93 \angle 4.48^\circ \end{aligned}$$

$$|V_s|(\text{line}) = 80.93 \times \sqrt{3} = 140.17 \text{ kV}$$

$$\begin{aligned} 1 + \frac{1}{2}YZ &= 1 + \frac{1}{2} \times 6.02 \times 10^{-4} \angle 90^\circ \times 78.6 \angle 75.2^\circ \\ &= 1 + 0.023 \angle 165.2 = 0.977 + j0.006 \end{aligned}$$

$$|V_{R_0}|(\text{line no load}) = \frac{140.17}{0.977} = 143.47 \text{ kV}$$

$$\text{Voltage regulation} = \frac{143.47 - 140.17}{140.17} = 2.4\%$$

**Example 39:** A 50 Hz, 250 km long transmission line has a total series impedance of  $30 + j100 \Omega$  and a total shunt admittance of  $10^{-3} \Omega$ . The receiving end load is 40 MW at 220 kV with 0.8 lagging power factor. Calculate the sending end voltage, current, power and power factor using.

- Short line approximation
- Nominal  $\pi$ -method

**Solution:** Impedance of the line  $Z = 30 + j100 \Omega$   
 $= 104.4 \angle 73.3$   
 $Y = 10^{-3} \angle 90^\circ \therefore$

The receiving end load is 40 MW at 220 kV, 0.8 p.f. lagging

$$\therefore I_R = \frac{40}{\sqrt{3} \times 220 \times 0.8} \angle -36.9^\circ$$

$$= 0.1312 \angle -36.9^\circ \text{ kA} \quad V_R = \frac{220}{\sqrt{3}} \angle 0^\circ = 127 \angle 0^\circ \text{ kV}$$

(i) Short line approximation

$$V_s = V_R + I_R Z = 127 + (0.164 \angle -36.9^\circ \times 104.4 \angle 73.3)$$

$$= 127 + 17.12 \angle 36.4$$

$$= 141.14 \angle 4.127^\circ$$

$$= 140.7 + 10.15j$$

$$|V_s|_{\text{line}} = 244.46^\circ \text{ kV}$$

$$I_s = I_R = 0.1312 \angle -36.9$$

Sending end power factor

$$= \cos(4.127 + 36.9) = 0.754 \text{ lagging}$$

$$\text{Sending end power} = \sqrt{3} \times 244.4 \times 0.1312 \times 0.754$$

$$= 41.87 \text{ MW}$$

(ii) Nominal- $\pi$  method

$$A = D = 1 + \frac{1}{2} YZ = 1 + \frac{1}{2} \times 10^{-3} \angle 90^\circ \times 104.4 \angle 73.3^\circ$$

$$= 1 + 0.0522 \angle 163.3 = 0.95 \angle 0.90^\circ$$

$$B = Z = 104.4 \angle 73.3^\circ$$

$$C = Y \left( 1 + \frac{1}{4} YZ \right) = Y \left( 1 + \frac{1}{4} \times 10^{-3} \angle 90^\circ \times 104.4 \angle 73.3 \right)$$

$$= 9.75 \times 10^{-4} \angle 90.4^\circ$$

$$V_s = AV_R + BI_R = 0.95 \angle 0.90^\circ \times 127 +$$

$$104.4 \angle 73.3^\circ \times 0.13 \angle -36.9^\circ$$

$$= 120.6 \angle 0.9 + 13.57 \angle 36.4$$

$$V_s = 131.88 \angle 4.32$$

$$|V_s|_{\text{line}} = 228.4 \text{ kV}$$

$$I_s = CV_R + DI_R$$

$$= 9.75 \times 10^{-4} \angle 90.4^\circ \times 127 + 0.95 \angle 90^\circ$$

$$\times 0.13 \angle -36.9^\circ$$

$$= 0.123 \angle 90.4 + 0.1235 \angle -36$$

$$I_s = 0.111 \angle 26.96^\circ$$

$$\text{Sending power factor} = \cos(26.96 - 4.32) = \cos 22.64 = 0.922 \text{ leading}$$

Sending end power

$$= \sqrt{3} \times 228.4 \times 0.111 \times 0.922 = 40.5 \text{ MW}$$

## UNDERGROUND CABLES

Underground transmission has more advantages over overhead transmission like low maintenance, cost, good looking, free from damage due to lightning and thunder storms and has no interference with communication circuit. The underground transmission is done by underground cables.

The cable used in underground transmission generally has three parts.

1. The core (or) conductor.
2. The insulating material.
3. A metallic sheath of lead is provided around the insulation to provide external protection.

The characteristics of insulating material used in cables are

- (i) High dielectric strength.
- (ii) Good mechanical properties and the insulating material should not be affected by chemicals.
- (iii) High insulation resistance.

Depend upon the operating voltage and temperature, different types of insulating materials used in cables those are listed as follows.

- (i) Polyvinyl chloride (PVC)
- (ii) Impregnated paper
- (iii) Vulcanized rubber
- (iv) Polythene

## Classification of Cables

The underground cables are classified depending upon the operating voltages are

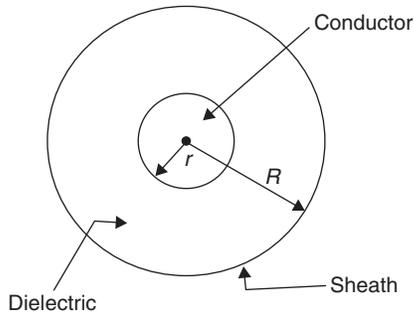
1. Low-tension cable (LV-cable): These cables are generally used for an operating voltage below 1 kV.
2. High-tension cable (HV-cable): These cables are used for an operating voltage below 11 kV.
3. Super tension cable (ST-cable): These cables are used for an operating voltage up to 33 kV.
4. Extra high-tension cable (EHT-cable): These cables are used for an operating voltage of 66 kV.
5. Extra super tension cable: These cables are used for an operating voltage of above 132 kV.

## Dielectric Stress in Cable

In a cable the current carries in a core (or) conductor of the cable. So, the stress is more on the surface of the core and it decreases towards the surface of the cable (or) sheath of the cable.

Maximum gradient at the surface of the conductor is

$$g_{\text{max}} = \frac{V}{r \ln \left( \frac{R}{r} \right)}$$



Minimum gradient at the sheath of the cable

$$g_{\min} = \frac{V}{R \ln \left( \frac{R}{r} \right)}$$

Here,  $V$  is the voltage applied to the cable  
 $r$  – Radius of the conductor (or) core  
 $R$  – Radius of the cable

### Grading of Cable

Grading of cable is done according to the stress on the dielectric material of the cable. There are two methods of grading.

1. Capacitance grading
2. Inter sheath grading

### Capacitance Grading

In this method, different dielectric materials having different permittivities are used and these materials are used in cables by two ways.

1. The factor of safety for all the materials is same, i.e., the material which has highest product of dielectric strength and permittivity should be placed near to the core of the cable.
2. The same working stress for different materials but the material which has highest permittivity placed near to the conductor of the cable.

### Intersheath Grading

In this method, different metallic sheaths are used in between the core and sheath of the cable for the same dielectric material by applying different voltages to different metallic sheath from operating voltage to earth potential. So, the stress on the same material also changed corresponding to the voltage.

### Laying and Heating of Cables

Generally there are three methods of laying of cables

1. Draw-in-system
2. Direct laying
3. Solid system

In cable if the rate of heat generation is more than the rate of heat dissipation, then the temperature of the cable increases. The sources of heat generation in cables are

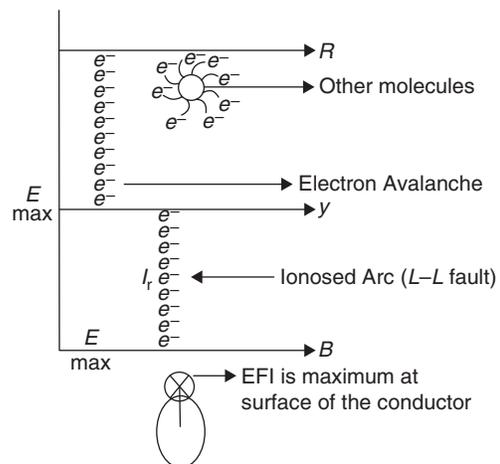
1. Dielectric loss
2. Copper loss in the core
3. Sheath losses

### CORONA

The ionization of air surrounding the power conductor is corona. There are free electrons normally present in the free space due to radioactivity and cosmic rays. The gradient around the surface of the conductor increases with increases in the potential between the conductors. The free electrons present in the air will move with certain velocity depending on the field strength. These moving electrons collide with the molecules of the air and some of the fast moving electrons may also dislodge electrons from the air molecules, thereby increasing the number of electrons. Hence the process of ionization is thus cumulative and may lead to electron avalanche. Hence the air surrounding the conductor may be ionized.

When a voltage higher than the critical voltage is applied between two parallel polished wires, the glow is quite obvious. After it is operated for some time, reddish heads or tufts form along the wire, bluish white glow is appeared around the surface of the wire.

It is noticed that the reddish tufts or beads are formed when the conductor is negative, and a smoother bluish white glow is formed when the conductor is positive. The corona due to AC is same as the corona due to DC when viewed through stroboscope. Due to corona, a hissing noise is observed and ozone gas is formed which is detected by its characteristic odour.

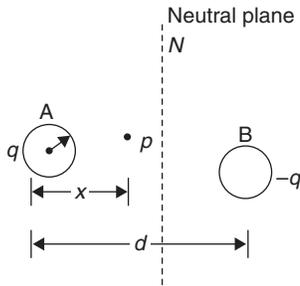


Overhead transmission lines are used for all economic reasons than underground cables. If line-to-line operating voltage  $\geq 275$  kV, then there will be abnormal electric field intensity (EFI) at the surface of the conductor.

EFI (increase or decrease a potential difference a mobility of electrons).

### Critical Disruptive Voltage

Consider a single-phase transmission line. Let 'r' be the radius of each conductor and 'd' be the distance of separation such that 'd' is far greater than 'r'. Let 'q' be the charge per unit length of the conductor on one of the conductors and hence '-q' is on the other. If the operating voltage is 'V', the potential of conductor 'A' with respect to neutral plane 'N' will be  $\frac{V}{2}$  and that of 'B' will be ' $\frac{-V}{2}$ '. Consider a point 'p' at a distance 'x' where we want to find the electric field intensity.



The field due to 'A' will be repulsive and due to 'B' would be attractive by bringing a unit positive charge from infinity to that point 'p'. Hence the EFI's due to A and B at 'p' would be additive

$$E = \frac{q}{2\pi\epsilon_0 x} + \frac{q}{2\pi\epsilon_0 (d-x)} = \frac{q}{2\pi\epsilon_0} \left[ \frac{1}{x} + \frac{1}{d-x} \right]$$

The potential difference between the conductors,

$$\begin{aligned} V &= - \int_{d-r}^r E dx = - \int_{d-r}^r \frac{q}{2\pi\epsilon_0} \left[ \frac{1}{x} + \frac{1}{d-x} \right] dx \\ &= \frac{q}{2\pi\epsilon_0} [\ln x - \ln(d-x)] r^{d-r} \\ &= \frac{q}{2\pi\epsilon_0} [\ln(d-r) - \ln(d-(d-r)) - \ln r + \ln(d-r)] \\ &= \frac{q}{2\pi\epsilon_0} \times 2 \ln \left[ \frac{d-r}{r} \right] = \frac{q}{\pi\epsilon_0} \ln \left[ \frac{d-r}{r} \right] \end{aligned}$$

as  $d \gg r$ ,  $d-r \cong d$ ,

$$\Rightarrow V = \frac{q}{\pi\epsilon_0} \ln \frac{d}{r}$$

Now the EFI at the point, which is 'x' distance from the centre of the conductor 'A' is given by,

$$E = \frac{q}{2\pi\epsilon_0} \left[ \frac{1}{x} + \frac{1}{d-x} \right] = \frac{q}{2\pi\epsilon_0} \cdot \frac{d}{x(d-x)}$$

From (1), substitute 'q' in above equation.

$$q = \frac{\pi\epsilon_0 V}{\ln \frac{d}{r}}$$

$$\Rightarrow E = \frac{\pi\epsilon_0 V}{\ln \frac{d}{r}} \times \frac{1}{2\pi\epsilon_0} \times \frac{d}{x(d-x)} = \frac{V}{2 \ln \frac{d}{r}} \times \frac{d}{x(d-x)}$$

Let  $V' = \frac{V}{2}$  which is line to neutral voltage. In case of three-phase system,  $V' = \frac{V}{\sqrt{3}}$ , where 'V' is line-to-line voltage

$$\Rightarrow E = \frac{V'd}{x(d-x) \ln \frac{d}{r}}$$

From the above expression, it is clear the EFI is inversely proportional to 'x', i.e. as 'x' decreases, EFI increases and it is maximum when  $x = r$ , i.e., at the surface of the conductor and this value  $i$  given by

$$E_{\max} = \frac{V'd}{(d-r)r \ln \frac{d}{r}} \approx \frac{V'}{r \ln \frac{d}{r}} \Rightarrow V' = E_{\max} r \ln \frac{d}{r}$$

Critical disruptive voltage is hence defined as the voltage at which the complete disruption of dielectric occurs. This voltage gives the gradient at the surface of the conductor equal to the breakdown strength of the air. This dielectric strength is normally denoted by  $g_0$  and is equal to 30 kV/cm peak at normal temperature and pressure, i.e. 25°C and 76 cm of Hg. At any other temperature and pressure,

$$g'_0 = g_0 \times \delta$$

where 'δ' is the air density correction and is given by

$$\delta = \frac{3.92 b}{273 + t}$$

where 'b' is the barometric pressure in cm of Hg and 't' the temperature in °c.

Hence, the critical disruptive voltage is given by

$$V' = r g_0 \delta \ln \frac{d}{r} \text{ kV.}$$

where the gradient 'g' is nothing but electric field intensity (EFI).

The above expression is derived based on the consideration that the conductor is solid and the surface is smooth. ACSR conductors are used for higher voltages. The potential gradient for ACSR conductors will be greater than the potential gradient of an equivalent smooth conductor, and hence the breakdown voltage of a stranded conductor will be comparatively less than a smooth and solid conductor. The irregularities on the surface of such a conductor are increased further due to the deposition of dust and dirt on its surface and hence the breakdown voltage is further reduced.

Hence, the final expression for the critical disruptive voltage after considering atmospheric conditions and the surface of the conductor is

$$V' = r g_0 \delta m_0 \ln \frac{d}{r} \text{ kV.}$$

When the voltage applied corresponds to the critical disruptive voltage, corona creates but may not be visible as the charged ions in the air must receive some finite energy to cause further ionization by collision. For a radial field, it must reach a gradient  $g_v$  at the surface of the conductor to cause a gradient  $g_0$ , a finite distance away from the surface of the conductor. ' $g_v$ ' is not constant as ' $g_0$ ' is, and is a function of the size of the conductor.

$$g_v = g_0 \delta \left( 1 - \frac{0.3}{\sqrt{r\delta}} \right) \text{ kV/cm}$$

If ' $V_v$ ' is the critical visual disruptive voltage, then

$$V_v = g_v r \ln \frac{d}{r}$$

$$\Rightarrow g_v = \frac{V_v}{r \ln \frac{d}{r}} = g_0 \delta \left( 1 + \frac{0.3}{\sqrt{r\delta}} \right)$$

$$\Rightarrow V_v = r g_0 \delta \left[ 1 + \frac{0.3}{\sqrt{r\delta}} \right] \ln \frac{d}{r} \text{ kV}$$

In case the irregularity factor is taken into account,

$$V_v = g_0 m_v \delta r \left[ 1 + \frac{0.3}{\sqrt{r\delta}} \right] \ln \frac{d}{r}$$

$$= 21.1 m_v \delta r \left[ 1 + \frac{0.3}{\sqrt{r\delta}} \right] \ln \frac{d}{r} \text{ kV RMS}$$

where ' $r$ ' is the radius in cm. The irregularity factor  $m_v$  has the following values:

$m_v = 1.0$  for polished wires

= 0.98 to 0.93 for rough conductor exposed to atmospheric conditions

= 0.72 for local corona on stranded conductors

Since the surface of the conductor is irregular, the corona does not start immediately on the whole surface but it takes place at difference points of the conductor which are pointed and this is known as local corona.

**Example 40:** A three-phase line has conductors 4 cm in diameter spaced equally 2 m apart. If the dielectric strength of the air is 30 kV (max) per cm, find the disruptive critical voltage for the line. Take air density factor,  $d = 0.951$  and irregularity factor = 0.9.

**Solution:** Conductor radius =  $\frac{4}{2} = 2$  cm

Conductor spacing = 2 m = 200 cm

Dielectric strength of the air,

$$g_0 = 30 \text{ kV/cm (max)}$$

$$= 21.2 \text{ kV (RMS) per cm}$$

Disruptive critical voltage

$$V' = r g_0 \delta m_0 \ln \left( \frac{d}{r} \right) \text{ kV.ph}$$

$$= 2 \times 21.2 \times 0.951 \times 0.9 \times \ln \left( \frac{200}{2} \right)$$

$$= 167.28 \text{ kV/ph}$$

$$\therefore \text{Line voltage (RMS)} = \sqrt{3} \times 167.28$$

$$= 289.72 \text{ kV}$$

**Example 41:** A 132 kV line with 2 cm diameter conductors is built so that corona takes place if the line voltage exceeds 220 kV (RMS). If the value of potential gradient at which ionization occurs can be taken as 30 kV per cm, find the spacing between the conductors.

**Solution:** If the line is 3- $\phi$ , Radius of the conductor,  $r = \frac{2}{2} = 1$  cm. Dielectric strength of the air,  $g_0 = \frac{30}{\sqrt{2}} = 21.2$  kV (RMS). Disruptive voltage/phase =  $\frac{220 \text{ kV}}{\sqrt{3}} = 127.020$

kV. If the conductors are assumed to be smooth conductors, then irregularity factor  $m_0 = 1$ . At standard temperature and pressure, air density factor  $d = 1$

$$\therefore V' = r g_0 \delta m_0 \ln \left( \frac{d}{r} \right) \text{ kV}$$

where ' $d$ ' is spacing between the conductors.

$$127.020 = 1 \times 21.2 \times 1 \times 1 \times \ln \left( \frac{d}{1} \right)$$

$$\Rightarrow \ln \left( \frac{d}{1} \right) = 5.9915 \Rightarrow d = 400.014 \text{ cm}$$

### Corona Loss

The electric field produces ions in the space which further leads to the production of space charge which move around the conductor. If there is a continuous transmission of energy, there is a continuous motion for these charges. Hence, it can indirectly be said that the energy required for the charges to remain in motion is derived from the supply system. In order to maintain the flow of energy over the conductor in the field where in this additional energy would have been otherwise absent, it is necessary to supply this additional loss from the supply system. This additional power is referred as corona loss.

An empirical relation was derived by Peek by conducting a number of experiments on corona loss.

$$P = 241 \times \frac{(f + 25)}{\delta} \sqrt{\frac{r}{d}} (V_p - V_c)^2 \times 10^{-5} \text{ kW/km/phase}$$

where ' $f$ ' is the supply of frequency,

' $\delta$ ' - Air density correction factor,

$V_p$  - The operating voltage in kV

$V_c$  - Critical disruptive voltage

$$\text{and } \delta = \frac{3.92 b}{273 + t},$$

where  $b$  = Atmospheric pressure

$t$  = Temperature.

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The above equation is for a fair weather condition. For a foul weather condition, the power loss can be obtained by taking  $V_0$  as 0.8 times the fair weather value. The above empirical formula has certain limitations and is applicable only if the frequency is in between 25 and 120 Hz, the conductor radius is greater than 0.25 cm and the ratio of  $\frac{V_p}{V_0} > 1.8$ .

**Example 42:** A 3- $\phi$  220 kV, 50 Hz transmission line consists of 3 cm diameter conductor spaced 4 m apart in an equilateral triangle form. If the temperature is 40°C, atmospheric pressure is 76 cm, calculate the corona loss per km of the line. Take  $m_0 = 0.9$ .

**Solution:** We know that

$$P = 241 \times 10^{-5} \frac{(f+25)}{\delta} \sqrt{\frac{r}{d}} (V_p - V_0)^2 \text{ kW/km/phase}$$

$$\delta = \frac{3.92 \times b}{273 + t} = \frac{3.92 \times 76}{273 + 40} = 0.952$$

$$g_0 = 21.2 \text{ kV/cm (RMS)}$$

Critical disruptive voltage per phase

$$V_0 = m_0 g_0 \delta r \log_e \frac{d}{r} \text{ kV}$$

$$V_0 = 0.9 \times 21.2 \times 0.952 \times 1.5 \times \log_e \frac{200}{1.5} \\ = 27.24 \times 4.80 = 133.281 \text{ kV}$$

Supply voltage per phase,

$$V_p = \frac{220}{\sqrt{3}} = 127 \text{ kV}$$

$$\therefore P = 241 \times 10^{-5} \times \frac{(50+25)}{0.952} \times \sqrt{\frac{1.5}{200}} \times (127 - 133.281)^2 \\ \text{ kW/km/phase}$$

$$= 0.01644 \times 39.450 = 0.648 \text{ kW/km/phase}$$

$\therefore$  Total corona loss per km for the three phases

$$= 3 \times 0.648 = 1.944 \text{ kW.}$$

**Example 43:** Which of the following will minimize the corona loss?

- Reduction in conductor size
- Reduction in smoothness of the conductor
- Sharp points are provided in the line hardware
- Current density in conductor is reduced.

**Solution:** (D)

**Example 44:** The corona loss on a particular system at 50 Hz is 1.2 kW/km/phase. The corona loss at 60 Hz would be

- 1.2 kW/km/ph
- 1.36 kW/km/ph
- 1.14 kW/km/ph
- 1.42 kW/km/ph

**Solution:** (B)

Corona loss  $\alpha(f+25)$

$$\frac{P_2}{P_1} = \frac{f_2 + 25}{f_1 + 25}$$

$$P_2 = \frac{60 + 25}{50 + 25} \times P_1 = 1.36 \text{ kW/km/ph}$$

### Factors Affecting Corona Loss

The factors that affect corona loss on overhead transmission lines:

- Electrical factors
- Atmospheric factors and
- Factors with respect to conductors

#### Electrical Factors

According to the above power loss expressions corona loss is a function of frequency. Hence if the frequency increases, the corona loss also increases proportionately. Hence we can infer that DC corona loss is comparatively less than AC corona loss. Hence if in case of AC supply, there may be harmonic which may further increase the corona loss.

#### Field Around the Conductor

The field around the conductor is a function of the voltage. Along with voltage, field also depends on the configuration of the conductors. The height of the conductors from the ground also effects the corona loss. The smaller the height, the greater will be the corona loss.

#### Atmospheric Factors

##### Pressure and Temperature Effect

From the expression for power loss, we can observe that it is a function of air density correction factor ' $\delta$ ' and even critical disruptive voltage is a function of ' $\delta$ '. The lower the value of ' $\delta$ ', the higher is the loss. For lower values of ' $\delta$ ', the pressure should be low and the temperature should be high. Hence the corona loss is low in plain areas and high in hilly areas.

##### Dust, Rain, Snow and Storm Effect

The particles of dust deposit on the conductor, thereby reducing the critical disruptive voltage for local corona and also the rains, snow and hail storm reduce the critical disruptive voltage and the corona loss increases.

#### Factors with Respect to Conductor

##### Diameter of the Conductor

From the expression, we can see that the corona loss is directly proportional to the radius of the conductor and also we know that loss  $\alpha (V_p - V_0)^2$ , where  $V_0$  is the critical disruptive voltage which is inversely proportional to the radius of the conductors. Hence, the larger the size of the conductor, the lower is the corona loss.

### Number of Conductors/phase

If the operating voltage  $\geq 380$  kV, it is found that using one conductor per phase produces large coroner loss and also large radio interference in the communication lines running parallel to power lines. Hence, using '2' or more conductors per phase reduces the corona loss, i.e. using bundled conductors. By using bundled conductors, the self-GMD of the conductor reduces increases the critical disruptive voltage and thereby the corona loss reduces.

### Heating of the Conductor by Load Current

The current passing through the conductor produces  $I^2R$  losses which would help to reduce the corona loss. If there is no such heat produced, then the conductor's temperature would be slightly less than the surrounding temperature. In the absence of heating, due to high humidity at times, additional corona is introduced.

### Methods of Reducing Corona Loss

The losses can be reduced by

1. Large diameter conductors
2. Hollow conductors
3. Bundled conductors
4. By increasing the conductor's spacing

### Advantages and Disadvantages of Corona

#### Advantages

1. Due to corona formation, the air surrounding the conductor also becomes conducting and therefore virtually the diameter of the conductor increases. This increased diameter reduces electrostatic stresses between the conductors.
2. Corona reduces the effects of transients produced by surges by dissipating them as corona loss.

#### Disadvantages

1. Corona reduces the energy to be transmitted thereby reducing the transmission efficiency of the transmission line.
2. The ozone produced during the corona may cause corrosion of the conductor.
3. The current drawn by the conductor during the time of corona is non-sinusoidal and produces non-sinusoidal voltage drop which may cause inductive interference with neighbouring communication lines.
4. Due to corona, the effective capacitance of the conductors is increased as the effective diameter of the conductor is increases. The charging current flowing also increases.

**Example 45:** A 3- $\phi$  line is operating at 220 kV and 50 Hz. The line has a conductor of 1.3 cm diameter arranged in a 3 m delta connection. Assume air density factor of 1.05 and

the dielectric strength of air to be 21.1 kV/cm, the corona losses will be ( $m = 0.75$ )

- (A) 5 kW/km (B) 10 kW/km  
(C) 15 kW/km (D) 20 kW/km

**Solution:** (C)

$$\text{Phase voltage} = \frac{220}{\sqrt{3}} \text{ kV} = 127 \text{ kV}$$

Critical disruptive voltage

$$\begin{aligned} (E_0) &= mg_0 \delta \left( \ln \frac{d}{r} \right) \text{ kV/phase} \\ &= 0.75 \times 1.05 \times 21.1 \ln \left( \frac{300}{0.65} \right) \\ &= 101.93 \approx 102 \text{ kV} \end{aligned}$$

Critical disruptive voltage  $E_0(L - L)$

$$= \sqrt{3} \times 102 = 176.6 \text{ kV}$$

Corona loss in the system is given by

$$\begin{aligned} P &= 241 \times 10^{-5} \left( \frac{f+25}{8} \right) \sqrt{\frac{r}{D}} (E - E_0)^2 \text{ kW/phase/km} \\ &= 241 \times 10^{-5} \times \frac{75}{8} \times 29.09 \\ &= 5.007 \text{ kW/phase/km} \\ &= 15.02 \text{ kW/km} \end{aligned}$$

### SKIN EFFECT

When a current-carrying conductor is carrying direct current (DC), the current is uniformly distributed over the whole cross section of the conductor. But, if an alternating current is flowing through the conductor, the current does not distribute uniformly, instead, it is concentrated more on the surface of the conductor compared to the current flowing through the centre of the conductor. This is known as skin effect.

The tendency of the alternating current to concentrate near the surface of a conductor is known as skin effect.

Due to skin effect, the effective area of cross section of the conductor through which the current flows is reduced. Hence, the resistance of the conductor increases while carrying an alternating current. The conductor consisting of a large number of strands each carrying part of the total current. The inner strands carrying currents give rise to flux which link only the inner strands whereas the strands outside produce flux which links both inner and outside strands. The flux linkages per ampere are more for the inner strands than for the outer strands. Thus the inductance as well as impedance of the inner strands is more than that of

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the outer strands, resulting in more current to flow in outer strands. Skin effect depends on the following factors.

1. Nature of the material
2. Radius of the conductor
3. Frequency of supply
4. Shape of wire

Skin effect is more if the frequency of supply is more and is proportional to the size of the conductor.

### Proximity Effect

The alternating magnetic flux produced in the conductor due to current flowing through the neighbouring conductor gives rise to circulating currents which apparently increases the resistance of the conductor. This phenomenon is called proximity effect. Consider a two-wire system in which the lines of flux link with inner and outer elements of the conductor but most of them link with the elements farther apart compared to the inner elements. Hence the inductance of the farthest elements are more compared to the nearer elements, thereby impedance of the farther elements are more compared to nearer elements. This gives rise to non-uniform distribution of current. The effective resistance is thereby increased due to non-uniform distribution of current. The proximity effect is more in case of underground cables, where the distance between overhead line transmission lines is more. Proximity effect depends on conductor's size, frequency, distance between conductors and permeability of conductor material.

### Ferranti Effect

A long-transmission line operated under no load or lightly loaded condition, the receiving end voltage is more than the sending end voltage. This phenomenon is known as Ferranti effect. This is explained as follows.

We know the equation of voltage

$$V = \frac{V_r + I_r Z_c}{2} e^{\alpha x} e^{j\beta x} + \frac{V_r + I_r Z_c}{2} e^{-\alpha x} e^{-j\beta x}$$

under no-load condition,

The above equation reduces to

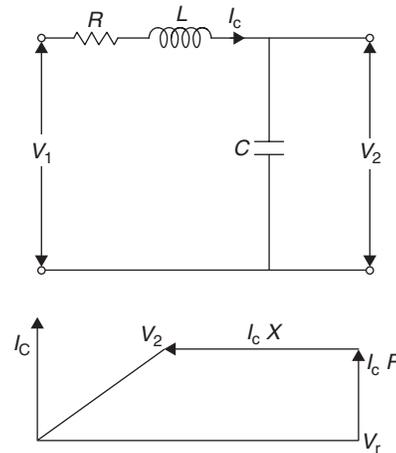
When  $x = L, I_r = 0$

$$V_s = \frac{V_r}{2} e^{\alpha L} e^{j\beta L} + \frac{V_r}{2} e^{-\alpha L} e^{-j\beta L}$$

At  $L = 0, V_r = \frac{V_r}{2} + \frac{V_r}{2}$

As 'L' increases, the incident component of sending end voltage increases exponentially and turns the vector anticlockwise through an angle  $\beta L$  whereas the reflected sending end voltage decreases by the same amount and is rotated clockwise through the same angle  $\beta L$ . The sum of these two components of sending end voltage gives a voltage which is smaller than the receiving end voltage  $V_r$ .

Ferranti effect can be explained by approximating the distributed parameters of the long transmission line by lumped impedance as shown below.



As the capacitive reactance of the line is more than the inductive reactance, under no load condition the line current is of leading power factor. The charging current produces drop in the reactance of the line which is in phase opposition to the receiving end voltage and therefore the sending end voltage becomes smaller than the receiving end voltage. This is shown in the above phasor diagram.

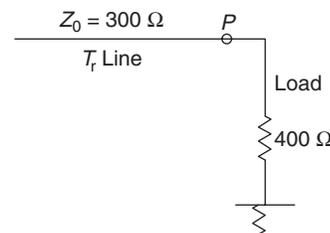
**Example 46:** The surge impedance of a 200 km long overhead line is 200  $\Omega$ . For a 100 km length of the same line, the surge impedance will be

- (A) 200  $\Omega$  (B) 100  $\Omega$   
(C) 400  $\Omega$  (D) 150  $\Omega$

**Solution:** (A)

Surge impedance is independent of length of the line

**Example 47:** The reflection co-efficient for the transmission line shown in figure at 'P' is



- (A) 0.25 (B) 0.142  
(C) 0.33 (D) 7

**Solution:** (B)

$$\begin{aligned} \text{Reflection co-efficient} &= \frac{Z_L - Z_s}{Z_L + Z_s} \\ &= \frac{400 - 300}{400 + 300} = 0.142 \end{aligned}$$

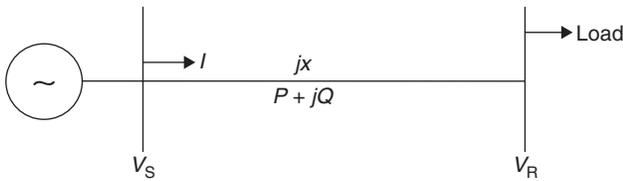
**Example 48:** An overhead line having a surge impedance of  $400 \Omega$  is connected in series with an underground cable having a surge impedance of  $80 \Omega$ . If surge of  $40 \text{ kV}$  travels from the line towards the cable junction, the value of the transmitted voltage wave at the junction is

- (A)  $160 \text{ kV}$  (B)  $40 \text{ kV}$   
 (C)  $80 \text{ kV}$  (D)  $13.3 \text{ kV}$

**Solution:** (D)

$$\begin{aligned} \text{The transmitted voltage} &= \frac{2Z_L}{Z_L + Z_S} \times V \\ &= \frac{2 \times 80}{80 + 400} \times 40 \times 10^3 \\ &= 13.33 \text{ kV} \end{aligned}$$

Consider a power system network as shown in the figure.



**Figure 16** Power system single-line diagram

Generally the load is Inductive load. The complex power at the sending end is

$$S = V_s^* I = P - jQ$$

$$I = \frac{P - jQ}{V_s^*}$$

The receiving end voltage  $V_R = V_s - j \left( \frac{P - jQ}{V_s} \right) X$   
 ( $\because V_s$  - reference voltage)

$$V_R = \left[ V_s - \frac{Q}{V_s} X \right] - j \frac{P}{V_s} X$$

From the above equation  $V_R$  is not considerable affect by the variation of real power then by the variation reactive power of the load, Bus voltage is varied. So, by controlling the reactive power the voltage is maintained within the specified limits.

## METHODS OF VOLTAGE CONTROL

The different voltage control methods are

1. Shunt reactors
2. Shunt capacitors
3. Tap-changing transformers
4. Synchronous condensers
5. Series capacitors
6. Static VAR system

### Shunt Reactors

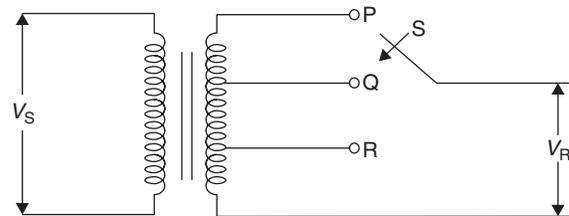
In this method, inductors are connected across the load. Shunt reactors generally used when the bus voltage more than the specified voltage, i.e., under light load condition ((or) Ferranti effect).

### Shunt Capacitor

In this method, capacitors connected in parallel with the load. Shunt capacitors are generally used when the bus voltage less than the specified voltage, i.e., under voltage condition. By connecting shunt capacitors, the VARS power required by the load decreases so that the reactive power drawn by the load from the source decreases and the power factor, voltage of the bus Increases.

### Tap-changing Transformer

In this method, the voltage of the line changes by changing the position of the winding connection as shown in the figure.

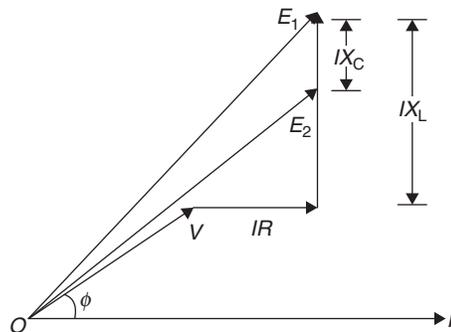


The above figure shows that the secondary voltage of the transformer changes by changing the corresponding switch position.

### Series Capacitors

In this method, a capacitor is connected in series with the line. So, the resultant reactance of the line decreases then the corresponding voltage drop of the line also decreases.

The change in voltage by connecting series capacitor is  $\Delta V = IX_c \sin \phi$ .



In the above phasor diagram  $E_1$  represent the sending end voltage before connecting the series capacitor and  $E_2$  represent the sending end voltage after connecting the series capacitor.

Therefore by connecting series capacitor the voltage drop decreases by  $IX_c$  factor.

## POWER FACTOR IMPROVEMENT

As the electrical energy is generated, transmitted and distributed in the form of alternating wave, hence power factor is considered. As most of the loads are inductive in nature, therefore they have low lagging power factor. Low power factor causes the increase active power loss. To ensure the most favourable conditions for a supply system from engineering and economical standpoint, it is important to have power factor close to unity.

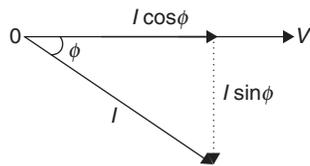
### Power Factor

The cosine of angle between voltage and current in an AC circuit is known as power factor.

In an AC circuit, there is generally phase difference ' $\phi$ ' between voltage and current. The term  $\cos\phi$  is called the power factor of the circuit.

If the circuit is inductive, the current lags the voltage and power factor is said to be lagging. In case of capacitive circuit, current leads the voltage and the power factor is said to be leading.

Consider an inductive circuit taking a lagging current ' $I$ ' from the supply voltage ' $V$ ' the lagging angle being equal to ' $\phi$ '. The vector diagram for the above circuit is shown below.



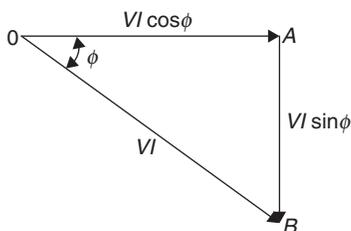
The current ' $I$ ' is resolved into two perpendicular components,

1.  $I\cos\phi$  in phase with ' $V$ '
2.  $I\sin\phi$ ,  $90^\circ$  out of phase with  $V$ . (for  $L$  and  $C$ )

$I\cos\phi$  is known as active (or) wattful component while the component  $I\sin\phi$  is called Reactive (or) wattless component. If the reactive component is small, then the phase angle ' $\phi$ ' is small and hence the power factor  $\cos\phi$  will be high. Hence the circuit having small reactive current ( $I\sin\phi$ ) will have high power factor and vice versa. The value of power factor can never be more than unity.

### Power Triangle

Power factor can also be analysed by the power drawn by the AC circuit. If each side of the current triangle is multiplied by voltage  $V$ , then we get the power triangle.



$OA = VI \cos\phi$  which represents the active power in watts (or) kW.

$AB = VI \sin\phi$  represents the reactive power in VAR or kVAR.  
 $OB = VI$  represents the apparent power in VA or kVA

From the power triangle, we can infer that,

1. The apparent power in AC circuit has two components, i.e. active and reactive power at right angles to each other.

$$OB^2 = OA^2 + AB^2$$

$$\Rightarrow (\text{Apparent power})^2 = (\text{Active power})^2 + (\text{Reactive power})^2$$

2. Power factor  $\cos\phi = \frac{OA}{OB} = \frac{\text{Active power}}{\text{Apparent power}}$
3. For leading currents, the power triangle should be reversed. To improve the power factor of the load, the device taking leading reactive power (e.g. capacitor) is connected in parallel with the load, neutralizing partly the lagging reactive power.

### Disadvantages of Low Power Factor

We know that  $P_L = V_L I_L \cos\phi$

$$\Rightarrow I_L = \frac{P}{V_L \cos\phi} \quad (\text{for single-phase})$$

$$P = \sqrt{3} V_L I_L \cos\phi$$

$$\Rightarrow I_L = \frac{P}{\sqrt{3} V_L \cos\phi} \quad (\text{for three-phase})$$

From the above expression, we can conclude that for the fixed power and voltage, the load current is inversely proportional to the power factor.

Due to the low power factor, the current drawn from the source is large resulting in the following disadvantages.

1. The electrical machinery is always rated in kVA. While designing the machine, the power factor of the load is not known.

$$\text{kVA} = \frac{\text{kW}}{\cos\phi}$$

From the above formula, kVA rating of the Equipment is inversely proportional to the power factor. Hence, lower the power factor, the kVA of the machinery should be increased.

2. The large current drawn at low power factor causes more copper losses ( $I^2R$ ) in all the elements of the supply system. Hence the efficiency is reduced.

### Economics of Power Factor Improvement

By using power factor improvement devices the power factor of the load increases. Therefore net maximum demand power drawn from the supply decreases then the corresponding total current and transmission losses decreases

which leads to the net saving on the economic cost. But by considering the cost and maintain of power factor correction equipment the total saving is decreases. Generally  $P$  is the expenditure per kVA per annum of the power factor correction equipment and  $q$  is the rate per kVA of maximum demand per annum. Therefore the corresponding economic power factor improvement is  $\cos \phi$ , i.e.,

$$\cos \phi = \sqrt{1 - \left(\frac{P}{q}\right)^2}$$

The power factor is improved by different methods.

1. Static VAR compensator
2. Synchronous condenser
3. STATCOM
4. Static capacitor

### Static VAR Compensator

In this method, a capacitor bank fixed or switched (or) fixed, capacitor and switch reactor bank are used in parallel. These type of compensations are generally used when reactive power leading and lagging by corresponding switching ON and OFF of the capacitors and reactors into the circuit. By changing the reactive power, the power factor of the circuit also changes.

### Static Capacitor

In the above method both capacitors and reactors are connected but In these method only capacitors are used in parallel with the load then there is net improvement of the power factor and also the reactive power drawn from the supply also reduces.

### Synchronous Condenser

It is a synchronous motor operating under no-load and over excitation condition. When these motor is connected in parallel with the load then the motor supplying reactive power to the load. There for the corresponding power factor of the load Increases.

**Example 49:** A shunt reactor of 100 MVAR is operated at 98% of its rated voltage and 95% of its rated frequency the reactive power absorbed by the reactor is

- (A) 98.92 MVAR (B) 101.09 MVAR  
(C) 95.32 MVAR (D) 104.32 MVAR

**Solution:** (B)

$$\text{Reactive power absorbed by reactor} = \frac{V^2}{X_L}$$

$$Q_1 = \frac{V_1^2}{2\pi f_1 L} = 100 \text{ MVAR}$$

$$V_2 = 0.98 V_1$$

$$f_2 = 0.95 f_1$$

$$\text{New case reactive power absorbed} = \frac{V_2^2}{2\pi f_2 L}$$

$$Q_2 = \frac{(0.98V_1)^2}{2\pi(0.95f_1)L}$$

$$Q_2 = \frac{0.98^2}{0.95} Q_1 = 1.0109 Q_1$$

$$Q_2 = 101.09 \text{ MVAR}$$

**Example 50:** A balanced delta connected load of  $(8 + j6) \Omega$  per phase is connected to a 400 V, 60 Hz, 3- $\phi$  supply lines. If the input power factor is to be improved to 0.9 by connecting a bank of star connected capacitor, the required kVAR of the bank is.

- (A) 28.8 kVAR (B) 18.5 kVAR  
(C) 10.7 kVAR (D) 20.4 kVAR

**Solution:** (C)

$$\text{Per-phase load current} = \frac{400}{8 + j6} = 40 \angle -36.86^\circ$$

$$\text{Load power factor} = \cos 36.86 = 0.8 \text{ lagging}$$

Capacitor placement affects the reactive component of load but real power component of the load remains unchanged

$$\therefore I_1 \cos \phi_1 = I_2 \cos \phi_2$$

$$40 \times 0.8 = I_2 \times 0.9 \Rightarrow I_2 = 35.55 \text{ A}$$

$$I_2 = I_1 + I_c \text{ from the system configuration}$$

$$I_c = I_2 - I_1 [\because \phi_2 = \cos^{-1} 0.9]$$

$$= 35.55 \angle -25.84 - 40 \angle -36.86$$

$$I_c = 8.5 \angle 90^\circ$$

Reactive power of capacitor bank

$$= Q \text{ at } 0.8 \text{ power factor} - Q \text{ at } 0.9 \text{ power factor}$$

$$= \sqrt{3} V_L I_L \sin \phi_1 - \sqrt{3} V_L I_L \sin \phi_2$$

$$= \sqrt{3} \times 400 \times \sqrt{3} \times 40 \times 0.6 - \sqrt{3} \times 400 \times \sqrt{3} \times 35.5 \times \sin 25$$

$$= 10.7 \text{ kVAR}$$

**Example 51:** At an industrial substation with a 4 MW load, a capacitor of 2.5 MVAR is installed to maintain the load factor at 0.97 lagging. If the capacitor goes out of service the load power factor is

- (A) 0.9 (B) 0.75  
(C) 0.8 (D) 0.842

**Solution:** (B)

$$\text{Initial phase difference } \phi_1 = \cos^{-1} 0.97$$

$$= 14.07^\circ$$

$$\text{kVAR supplied by capacitor bank} = P(\tan \phi_1 - \tan \phi_2)$$

$$4 \times 10^6 (\tan \phi_1 - \tan 14.07) = 2.5 \times 10^6$$

$$\phi_1 = 41.2^\circ$$

Power factor before adding capacitor bank

$$\cos\phi_1 = 0.752$$

**Example 52:** A loss less transmission line having surge impedance loading of 2000 MW. A series capacitive compensation of 30% the surge impedance loading of the compensated transmission line will be

- (A) 1673 MW (B) 2390 MW  
(C) 600 MW (D) 2600 MW

**Solution:** (B)

$$\text{Surge impedance loading (SIL)} = \frac{V^2}{Z_C}$$

$$\text{Surge impedance loading} \propto \frac{1}{Z_C}$$

When the capacitive compensation of 30% is provided, the series impedance of the line is reduced to 70% of its original value which in turn reduced the characteristic impedance.

Characteristic impedance  $\propto \sqrt{Z_{\text{series}}}$

$$\frac{Z_{C_1}}{Z_{C_2}} = \frac{\sqrt{Z_{\text{series}}}}{\sqrt{0.7 \times Z_{\text{series}}}}$$

Characteristic impedance after compensation

$$Z_{C_2} = 0.836 Z_{C_1}$$

Since surge impedance loading  $\alpha = \frac{1}{Z_C}$

$$\frac{\text{SIL}_1}{\text{SIL}_2} = \frac{Z_{C_2}}{Z_{C_1}}$$

$$\begin{aligned} \text{SIL}_2 &= \text{SIL}_1 \times \frac{Z_{C_1}}{Z_{C_2}} \\ &= 1.195 \times 2000 = 2390 \text{ MW} \end{aligned}$$

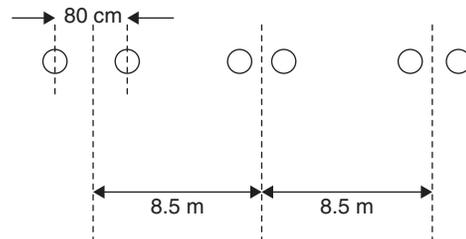
## EXERCISES

### Practice Problems I

**Directions for questions 1 to 27:** Select the correct alternative from the given choices.

- A generating station has a maximum demand of 60 MW. The plant capacity factor is 0.5 and the load factor is 0.6. The plant running as per schedule is fully loaded. The daily energy produced will be  
(A) 720 MWh (B) 964 MWh  
(C) 864 MWh (D) 820 MWh
- The inductance in mH/km of a single-phase overhead transmission line of diameter 1 cm and distance of separation 60 cm is  
(A)  $0.05 + 0.2 \ln 60$   
(B)  $0.05 + 0.2 \ln \left( \frac{60}{0.5} \right)$   
(C)  $0.2 \ln \left( \frac{60}{0.5} \right)$   
(D)  $0.2 \ln 60$
- A 50 km transmission line when excited with a source of frequency 500 Hz, it will be modelled as  
(A) Short line (B) Medium line  
(C) Long line (D) Data insufficient
- An existing AC system has a string efficiency of 80%. The same line, if used for DC transmission, then string efficiency will be  
(A) 100% (B) >80%  
(C) <80% (D) 0%

- The figure shows a single circuit 460 kV line using two bundled conductors per phase. If diameter of each conductor is 5 cm, the inductance/km/phase is



- (A) 0.24 mH/km/phase  
(B) 0.76 mH/km/phase  
(C) 0.89 mH/km/phase  
(D) 0.90 mH/km/phase
- A conductor is composed of seven identical strands of copper, each having a radius  $r$ . The self-GMD of the conductor will be  
(A)  $2.17 r$  (B)  $2.645 r$   
(C)  $2.141 r$  (D)  $1.21 r$
  - Determine the capacitance for a 2-conductor 1- $\phi$  line operating at 50 Hz, the conductors being of 2 cm in diameter and spaced 3 m apart.  
(A)  $9.74 \times 10^{-9}$  F/km (B)  $4.87 \times 10^{-9}$  F/km  
(C)  $3.05 \times 10^{-9}$  F/km (D) None of these
  - A 1- $\phi$  transmission line having  $R = 1.4 \Omega$  and  $X = 0.8 \Omega$  is delivering a load of 200 kVA at 2500 V. The voltage at sending end when p.f. of the load is 0.8 p.f. lag is  
(A) 2.069 kV (B) 2.840 kV  
(C) 2.627 kV (D) 3.54 kV

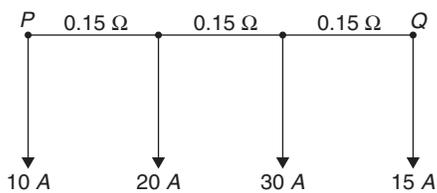
9. The parameters of a single circuit 3- $\phi$ , 50 Hz transmission line are

$$R = 0.2 \Omega/\text{km}, L = 1.3 \text{ mH}/\text{km}$$

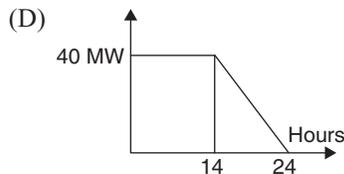
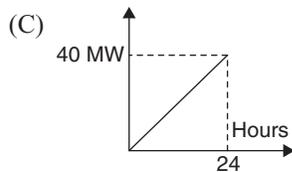
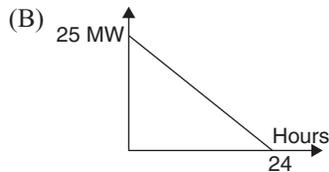
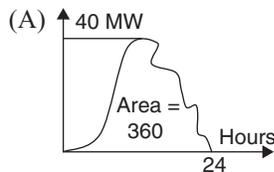
$$C = 0.01 \mu\text{F}/\text{km}, L = 120 \text{ km}$$

The efficiency of the line if it delivers 40 MW at 132 kV and 0.8 p.f. lag is

- (A) 85.4% (B) 90.0%  
(C) 98% (D) 92.6%
10. A DC distribution system is shown with load currents as marked. The two ends of the feeder  $P$  and  $Q$  are fed such that  $V_P - V_Q = 3 \text{ V}$ . The value of voltage  $V_P$  for a minimum voltage of 220 V at any point along the feeder is



- (A) 220 V (B) 223 V  
(C) 226 V (D) 229 V
11. An underground cable with a surge impedance of 100  $\Omega$  is connected in series with an overhead line having a surge impedance of 400  $\Omega$ . If a source of 50 kV travels from the line towards the line cable junctions, the value of the transmitted voltage wave at the junction is
- (A) 20 kV (B) 30 kV  
(C) -30 kV (D) 80 kV
12. For the figures given below, which curve has the least load factor?

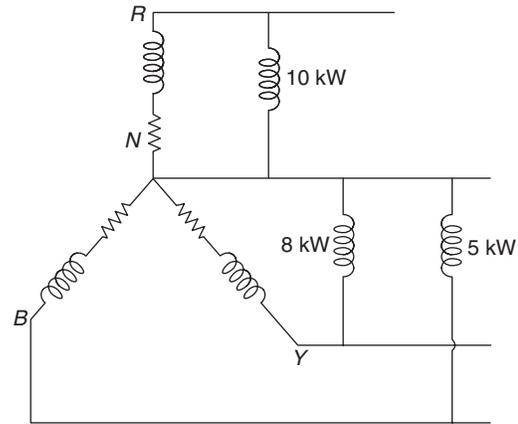


- (A) Curve A (B) Curve B  
(C) Curve C (D) Curve D

13. A 275 kV transmission line has the following line constants.  $A = 0.85 \angle 5^\circ$ ,  $B = 200 \angle 75^\circ$ . The power that can be received at unity p.f. if voltage profile at each end is to be maintained at 275 kV is

- (A) 118 MW (B) 218 MW  
(C) 98 MW (D) None of these

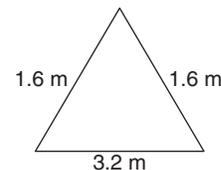
14. In a 3- $\phi$ , 4-wire system, the line voltage is 400 V and now inductive loads of 10, 8 and 5 kW are connected between the three line conductors and the neutral as shown in figure. Calculate the current in the neutral conductor.



- (A) 15.8 A (B) 16.8 A  
(C) 17.8 A (D) 18.8 A
15. A 3- $\phi$  transmission line consists of 1 cm radius conductors spaced symmetrically 4 m apart. Dielectric strength of air is 30 kV/cm. The line voltage for corona to commence is
- (A) 110 kV (B) 220 kV  
(C) 330 kV (D) 400 kV

**Common Data for Questions 16 and 17:**

The conductors of a 3- $\phi$  transmission line are arranged as shown, radius of each line = 0.4 cm



The operating voltage is 132 kV.

16. The capacitance per phase is
- (A)  $8.9 \times 10^{-9} \text{ F}/\text{km}$   
(B)  $9.2 \times 10^{-6} \text{ F}/\text{km}$   
(C)  $11.5 \times 10^{-9} \text{ F}/\text{km}$   
(D)  $7.5 \times 10^{-7} \text{ F}/\text{km}$
17. The charging current when the transmission line is operated at 264 kV is

- (A) 0.312 A/km  
 (B) 0.257 A/km  
 (C) 0.427 A/km  
 (D) 0.214 A/km

**Common Data for Questions 18 and 19:**

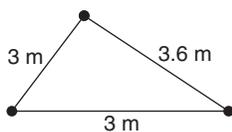
A 3- $\phi$  50 Hz transmission line is 400 km long. The voltage at the sending end is 220 kV. The line parameters are  $r = 0.125 \Omega/\text{km}$ ,  $x = 0.4 \Omega/\text{km}$ ,  $y = 2.8 \times 10^{-6} \text{ S}/\text{km}$

18. The sending end current and receiving end voltage are.  
 (A) 76 A, 121 kV (B) 152 A, 242 kV  
 (C) 152 A, 226 kV (D) 76 A, 242 kV
19. The maximum permissible line length of receiving end no load voltage is not to exceed 235 kV  
 (A) 138 km (B) 238 km  
 (C) 338 km (D) 438 km
20. A 100 MVA, 11 kV alternator has synchronous reactance 0.3 p.u. The p.u. synchronous reactance on a 50 MVA, 33 kV base is  
 (A) 1.67 p.u. (B) 0.45 p.u.  
 (C) 0.016 p.u. (D) 0.167 p.u.
21. A 3- $\phi$  induction motor rated at 400 V, 50 Hz is running at a p.f. of 0.707 lag. The input to the motor is 8 kVA. It is proposed to improve the p.f. of the system to 0.9 by a  $\Delta$  connected bank of capacitors. The value of capacitance/phase required is  
 (A) 0.58  $\mu\text{F}$  (B) 58  $\mu\text{F}$   
 (C) 0.19  $\mu\text{F}$  (D) 19  $\mu\text{F}$
22. A shunt capacitor rated 100 MVAR is operated at 99% of rated voltage and 98% of rated frequency. The reactive power absorbed would be approximately  
 (A) 100 MVAR (B) 98 MVAR  
 (C) 96 MVAR (D) 95 MVAR
23. The per-unit impedance of a circuit element is 0.30. If the base kV and base MVA are halved, then the new value of per unit impedance is  
 (A) 0.9 (B) 0.6  
 (C) 0.3 (D) 0.15
24. Two buses 1 and 2 are connected by an impedance  $(0 + j10) \Omega$ . The bus 1 voltage is  $400\angle 45^\circ$  and bus 2 voltage is  $400\angle 0^\circ$  V. The real and reactive powers supplied by bus 1, respectively, are  
 (A) +11314 W, +4686 VAR  
 (B) +11314 W, -4686 VAR  
 (C) -11314 W, -2687 VAR  
 (D) -5656 W, +2687 VAR
25. A load is drawing 500 kW at 0.707 lag. A synchronous motor improves the p.f. of the load to 0.96. The motor carries a load of 300 kW. The leading reactive power supplied by the motor is  
 (A) 500 kVAR (B) 233.33 kVAR  
 (C) 266.67 kVAR (D) 250 kVAR
26. The voltage phasor of a circuit is  $20\angle 15^\circ$  V and current phasor is  $5\angle -45^\circ$  A. The active and reactive powers in the circuit are  
 (A) 50 W and -86.67 VAR  
 (B) 50 W and 86.67 VAR  
 (C) 86.67 W and 50 VAR  
 (D) 86.67 W and -50 VAR
27. A 240 V single-phase AC source is connected to a load with an impedance of  $20\angle 45^\circ$ . A capacitor is connected in parallel with the load. If the capacitor supplies 2400 VAR, the real power supplied by source is  
 (A) 3394 W (B) 2400 W  
 (C) 994 W (D) None of these

**Practice Problems 2**

**Directions for questions 1 to 40:** Select the correct alternative from the given choices.

1. A 1- $\phi$  two conductor circuit has a spacing of 3 m between them and each conductor has a diameter of 10 mm. The inductance of each circuit per km is  
 (A) 1.24 mH/km (B) 1.19 mH/km  
 (C) 2.38 mH/km (D) 0.59 mH/km
2. The conductors of a 3- $\phi$  50 Hz circuit are each of diameter 21 mm. The spacing is as shown. The inductance is



- (A)  $10.5 \times 10^{-4}$  H/km (B)  $8.2 \times 10^{-4}$  H/km  
 (C)  $5.6 \times 10^{-3}$  H/km (D) None of these

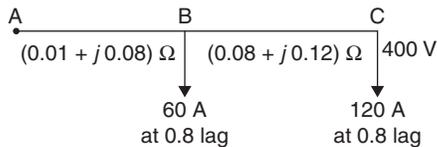
3. A two conductor 1- $\phi$  line operating at 50 Hz has a diameter of 20 mm and spacing between the conductors is 3 m. Height of conductor above the ground is 6 m. The capacitance of line to neutral will be  
 (A) 9.7 pF/km (B) 10.2 pF/km  
 (C) 8.7  $\mu\text{F}/\text{km}$  (D) 2.4  $\mu\text{F}/\text{km}$
4. The power losses in a 3- $\phi$  20 km line delivering 10 MW at 11 kV, 50 Hz, 0.8 p.f. lagging is 10% of power delivered. The resistance of each conductor is  
 (A) 0.774  $\Omega$  (B) 2.14  $\Omega$   
 (C) 1.54  $\Omega$  (D) 0.86  $\Omega$
5. If bundled conductors are used instead of single conductors, then  
 (A) Inductance and capacitance increase  
 (B) Inductance and capacitance decrease  
 (C) Inductance increases and capacitance decreases  
 (D) Inductance decreases and capacitance increases

6. A 400 V, 3- $\phi$  4-wire system supplies a Y-connected load. Resistance of each line is  $0.2 \Omega$  and of neutral is  $0.4 \Omega$ . The load impedances are

$$Z_R = 6 + j10, Z_Y = 10 \Omega$$

$$Z_B = 8 - j10 \Omega. \text{ Current in neutral is}$$

- (A) 16.56 A (B) 17.56 A  
(C) 18.56 A (D) 19.56 A
7. The non-uniform distribution of voltage across the units in a string of suspension-type insulator is due to  
(A) Non-uniform distance between the cross arm and the units  
(B) Existence of the stray capacitance between the metallic junctions of the units and tower body  
(C) Non-uniform distance of separation of units from tower body  
(D) Unequal self-capacitance of the units
8. A long overhead transmission line is terminated by its characteristic impedance. The ratio of voltage to current at different points along the line  
(A) Progressively increases from receiving end to the sending end  
(B) Progressively decreases from receiving end to the sending end  
(C) Remains the same at the two ends being maximum at the centre of the line  
(D) Remains the same at all points
9. A 1- $\phi$ , AC distributor supplies two single-phase loads as shown in the figure.



The voltage drop from A to C is

- (A) 21 V (B) 26.7 V  
(C) 41 V (D) 51 V
10. The use of hollow conductors reduce corona loss as  
(A) Provides better ventilation of the conductor  
(B) Eddy currents in the conductor are eliminated  
(C) Current density is reduced  
(D) For a given cross section radius of conductor it increases
11. An insulator string has three units. The capacitance of the link between two units to earth is one eighth of the capacitance of each insulator. Each unit has a safe working voltage of 18.5 kV. The system voltage is  
(A) 55.5 kV (B) 46.7 kV  
(C) 37 kV (D) 36.7 kV
12. The increase in resistance of a conductor carrying alternating current, due to non-uniform distribution of current is called  
(A) Hall effect (B) Proximity effect  
(C) Skin effect (D) Ferranti effect

13. The transmission capacity of a line is related to its inductance varies in  
(A) Direct proportion  
(B) Inverse proportion  
(C) Square  
(D) Independent of inductance
14. A lossless radial transmission line with surge impedance loading  
(A) Has sending end voltage higher than receiving end voltage  
(B) Has flat voltage profile and unity p.f. at all points along it  
(C) Takes positive VAR at sending end and zero VAR at receiving end  
(D) Takes negative VAR at sending end and zero VAR at receiving end
15. The surge impedance of a 400 km line is  $500 \Omega$ . For a 200 km length of same line, the impedance will be  
(A)  $250 \Omega$  (B)  $1000 \Omega$   
(C)  $500 \Omega$  (D)  $750 \Omega$
16. In a string of suspension insulators, the voltage distribution across the different units could be made uniform by using grading ring because it  
(A) Decreases the capacitances of upper insulators units causing equal voltage drop  
(B) Increases capacitances of lower insulator units to cause equal voltage drop  
(C) Forms capacitances with link pins which helps to cancel the charging current from link pins  
(D) Forms capacitances with link pins to carry the charging current from the link pins
17. The cause of Ferranti effect in transmission lines is  
(A) Line inductance  
(B) Line capacitance  
(C) Line capacitance and inductance  
(D) Line resistances and capacitance
18. Series capacitors are used in a transmission line to  
(A) To increase the phase angles at sending end and receiving ends under heavy load conditions  
(B) Reduce the phase angle between sending end and receiving ends under heavy load conditions  
(C) To reduce the capacitive current  
(D) None of these
19. The load factor and capacity factor of a plant are equal if  
(A) Peak load is equal to capacity of plant  
(B) Average load is half the capacity of the plant  
(C) Average load is same as peak load  
(D) Group diversity factor is equal to peak diversity factor
20. The voltage across the units in a two unit suspension insulator is 40% and 60%. The ratio of capacitance of insulator to that of earth will be

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- (A) 0.4 (B) 0.45  
(C) 0.5 (D) 0.55
21. The receiving end voltage of a long transmission line at no load condition is  
(A) Less than sending end voltage  
(B) Greater than sending end voltage  
(C) Equal to sending end voltage  
(D) Zero voltage
22. The level of insulation of a 400 kV extra high voltage line is decided based on  
(A) Lightning over voltage  
(B) Switching over voltage  
(C) Corona inception voltage  
(D) Radio and TV interference
23. The transmission line feeding power on either side of main transmission line is  
(A) Primary distribution  
(B) Primary transmission  
(C) Secondary transmission  
(D) Secondary distribution
24. Suppose the conductors are dead ended and there is a change in the direction of transmission line, insulators used are  
(A) Strain type (B) Pin type  
(C) Shackle type (D) Suspension type
25. For a fixed inductive shunt compensation, series capacitive compensation  
(A) Increases virtual surge impedance loading  
(B) Decreases vertical surge impedance loading  
(C) Increases effective length of line  
(D) None of these

**Common Data for Questions 26 and 27:**

A 50 Hz, 3- $\phi$ , transmission line 30 km long has a total series impedance of  $(40 + j125) \Omega$  and shunt admittance of  $10^{-3}$  mho. The load is 50 MW at 220 kV with 0.8 lagging p.f.

26. Sending end voltage using nominal  $\pi$  is  
(A) 216 kV (B) 204 kV  
(C) 232 kV (D) 238 kV
27. The sending end power factor is  
(A) 0.8 lag (B) 0.9 lag  
(C) 0.98 lag (D) 1
28. A pure inductance absorbs reactive power from the AC line when  
(A) Both applied voltage and current decreases  
(B) Both applied voltage and current increases  
(C) Applied voltage increases and current decreases  
(D) Applied voltage decreases and current increases
29. The power factor of the load is corrected by connecting a static capacitor in parallel with the load. The power taken by the load  
(A) Increases (B) Decreases  
(C) Remains same (D) Is uncertain

30. Shunt reactors are connected in transmission lines to  
(A) Limit the fault current  
(B) Improve stability limit  
(C) Absorb the excess reactive power  
(D) Generate requisite reactive power
31. The excitation of a synchronous generator connected to an infinite bus bar delivering power at a lagging power factor is increased. Then  
(A) Current delivered increases  
(B) Terminal voltage increases  
(C) Voltage angle  $\delta$  increases  
(D) Both A and C
32. The voltage of an infinite bus bar is  $1\angle 0^\circ$  and that of an alternator connected to it is  $0.95\angle 0^\circ$ . The alternator acts as a  
(A) Shunt coil  
(B) Shunt capacitor  
(C) Shunt resistance  
(D) None of these
33. The power dispatch through a transmission line can be increased by installing  
(A) Shunt capacitance  
(B) Series capacitance  
(C) Shunt reactance  
(D) Series reactance
34. The combined frequency regulation of machines in an area of capacity 3000 MW and operating at a normal frequency of 60 Hz is 0.15 p.u. The regulation in Hz/MW is  
(A)  $\frac{3}{1000}$  (B)  $\frac{0.3}{1000}$   
(C)  $\frac{1}{200}$  (D)  $\frac{1}{100}$

35. Which amongst the following is not true?  
(A) Synchronous phase modifiers are installed at sending end  
(B) Synchronous phase modifiers are installed at load end  
(C) Synchronous phase modifiers are specially designed synchronous motors  
(D) Synchronous phase modifiers do not carry load
36. In EHV lines, series capacitance compensation is used to  
(A) Improve the stability of the system  
(B) Reduce the loading on the line  
(C) Improve the protection of the line  
(D) Reduce the voltage profile

**II. Question Nos.: 37 and 38**

A line operating at 60 Hz is 81% series compensated.

37. The resonating frequency is  
(A) 60 Hz (B) 66.6 Hz  
(C) 50 Hz (D) 54 Hz

38. Now if the line is 100% series compensated. Resonant frequency is  
 (A) 60 Hz (B) 66.6 Hz  
 (C) 50 Hz (D) 54 Hz.

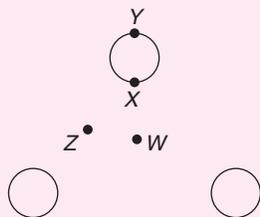
**III. Question Nos.: 39 and 40**

A20 kVA, 2000/200 V single-phase transformer has an equivalent impedance of  $Z = 8 + j15$  when referred to the HV side.

39. The p.u. value of impedance when referred to the LV side is  
 (A)  $2.13 \angle 32^\circ$  (B)  $0.0085 \angle 62^\circ$   
 (C)  $0.085 \angle 62^\circ$  (D)  $0.213 \angle 32^\circ$
40. If the transformer has an exciting current of 10 A on HV side, p.u. value of current referred to LV side is  
 (A) 1 p.u. (B) 2 p.u.  
 (C) 0.5 p.u. (D) 0.25 p.u.

**PREVIOUS YEARS' QUESTIONS**

1. The concept of an electrically short, medium, and long line is primarily based on the [2006]  
 (A) nominal voltage of the line  
 (B) physical length of the line  
 (C) wavelength of the line  
 (D) power transmitted over the line
2. A 400 V, 50 Hz, three-phase balanced source supplies power to a star connected load whose rating is  $12\sqrt{3}$  kVA, 0.8 p.f. (lag). The rating (in kVAR) of the delta connected (capacitive) reactive power bank necessary to bring the p.f. to unity is [2006]  
 (A) 28.78 (B) 21.60  
 (C) 16.60 (D) 12.47
3. The  $A, B, C, D$  constants of a 220 kV line are:  
 $A = D = 0.94 \angle 1^\circ, B = 130 \angle 73^\circ, C = 0.001 \angle 90^\circ$ . If the sending end voltage of the line for a given load delivered at nominal voltage is 240 kV, then % voltage regulation of the line is [2006]  
 (A) 5 (B) 9  
 (C) 16 (D) 21
4. Consider a bundled conductor of an overhead line, consisting of three identical sub-conductors placed at the corners of an equilateral triangle as shown in the figure. If we neglect the charges on the other phase conductors and ground, and assume that spacing between sub-conductors is much larger than their radius, the maximum electric field intensity is experienced at [2007]



- (A) Point X (B) Point Y  
 (C) Point Z (D) Point W
5. The total reactance and total susceptance of a lossless overhead EHV line, operating at 50 Hz, are given by 0.045 p.u. and 1.2 p.u., respectively. If the velocity of wave propagation is  $3 \times 10^5$  km/s, then the approximate length of the line is [2007]

- (A) 122 km (B) 172 km  
 (C) 222 km (D) 272 km
6. Out of the following plant categories  
 (i) Nuclear (ii) Run-of-river  
 (iii) Pump storage (iv) Diesel  
 The base load power plants are [2009]  
 (A) (i) and (ii) (B) (ii) and (iii)  
 (C) (i), (ii) and (iv) (D) (i), (iii) and (iv)
7. For a fixed value of complex power flow in a transmission line having a sending end voltage  $V$ , the real power loss will be proportional to [2009]  
 (A)  $V$  (B)  $V^2$   
 (C)  $\frac{1}{V}$  (D)  $1/V$
8. Match the items in List-I with the items in List-II and select the correct answer using the codes given below the lists. [2009]

List I To	List II Use
a. Improve power factor	1. Shunt reactor
b. Reduce the current ripples	2. Shunt capacitor
c. Increase the power flow in line	3. Series capacitor line
d. Reduce the Ferranti effect	4. Series reactor

- (A)  $a \rightarrow 2, b \rightarrow 3, c \rightarrow 4, d \rightarrow 1$   
 (B)  $a \rightarrow 2, b \rightarrow 4, c \rightarrow 3, d \rightarrow 1$   
 (C)  $a \rightarrow 4, b \rightarrow 3, c \rightarrow 1, d \rightarrow 2$   
 (D)  $a \rightarrow 4, b \rightarrow 1, c \rightarrow 3, d \rightarrow 2$

9. Match the items in List-I with the items in List-II and select the correct answer using the codes given below the lists. [2009]

List I Type of transmission line	List II Type of distance relay preferred
a. Short line	1. Ohm relay
b. Medium line	2. Reactance relay
c. Long line	3. Mho relay

- (A)  $a \rightarrow 2 \ b \rightarrow 1 \ c \rightarrow 3$
- (B)  $a \rightarrow 3 \ b \rightarrow 2 \ c \rightarrow 1$
- (C)  $a \rightarrow 1 \ b \rightarrow 2 \ c \rightarrow 3$
- (D)  $a \rightarrow 1 \ b \rightarrow 3 \ c \rightarrow 2$

10. Three generators are feeding a load of 100 MW. The details of the generators are [2009]

	Rating (MW)	Efficiency (%)	Regulation (p.u.) on 100 MVA base
Generator-1	100	20	0.02
Generator-2	100	30	0.04
Generator-3	100	40	0.03

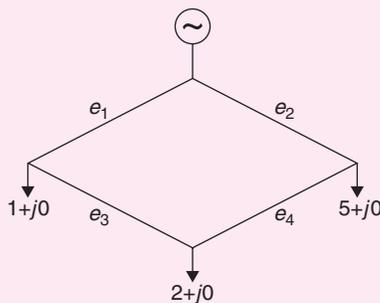
In the event of increased load power demand, which of the following will happen?

- (A) All the generators will share equal power
- (B) Generator-3 will share more power compared to Generator-1
- (C) Generator-1 will share more power compared to Generator-2
- (D) Generator-2 will share more power compared to Generator-3

11. An extra high voltage transmission line of length 300 km can be approximated by a lossless line having propagation constant  $\beta = 0.00127$  radians per km. Then the percentage ratio of line length to wavelength will be given by [2008]

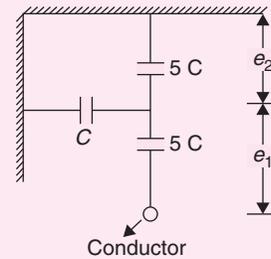
- (A) 24.24%
- (B) 12.12%
- (C) 19.05%
- (D) 6.06%

12. Single line diagram of a 4-bus single source distribution system is shown below, branches  $e_1, e_2, e_3$  and  $e_4$  have equal impedances. The load current values indicated in the figure are in per unit. Distribution company's policy requires radial system operation with minimum loss. This can be achieved by opening of the branch [2008]



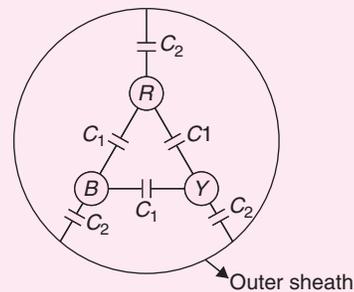
- (A)  $e_1$
- (B)  $e_2$
- (C)  $e_3$
- (D)  $e_4$

13. Consider a three-phase, 50 Hz, 11 kV distribution system. Each of the conductors is suspended by an insulator string having two identical porcelain insulators. The self-capacitance of the insulator is 5 times the shunt capacitance between the link and the ground, as shown in the figure. The voltage across the two insulators is [2010]



- (A)  $e_1 = 3.74 \text{ kV}, e_2 = 2.61 \text{ kV}$
- (B)  $e_1 = 3.46 \text{ kV}, e_2 = 2.89 \text{ kV}$
- (C)  $e_1 = 6.0 \text{ kV}, e_2 = 4.23 \text{ kV}$
- (D)  $e_1 = 5.5 \text{ kV}, e_2 = 5.5 \text{ kV}$

14. Consider a three-core, three-phase, 50 Hz, 11 kV cable whose conductors are denoted as R, Y and B in the figure. The inter-phase capacitance ( $C_1$ ) between each pair of conductors is 0.2 mF and the capacitance between each line conductor and the sheath is 0.4 mF. The per-phase charging current is [2010]

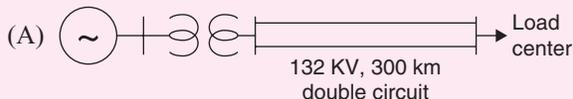
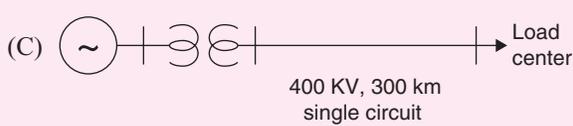
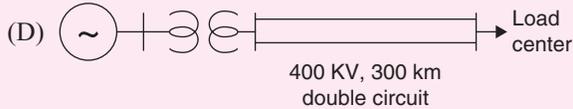


- (A) 2.0 A
- (B) 2.4 A
- (C) 2.7 A
- (D) 3.5 A

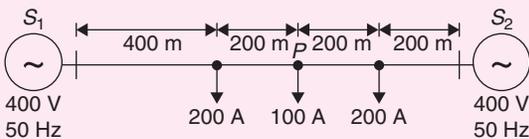
15. For enhancing the power transmission in a long EHV transmission line, the most preferred method is to connect a [2011]

- (A) Series inductive compensator in the line.
- (B) Shunt inductive compensator at the receiving end.
- (C) Series capacitive compensator in the line.
- (D) Shunt capacitive compensator at the sending end.

16. A nuclear power station of 500 MW capacity is located at 300 km away from a load centre. So the most suitable power evacuation transmission configuration among the following options. [2011]

- (A)  132 KV, 300 km double circuit
- (B)  132 KV, 300 km single circuit with 40% series capacitor compensation
- (C)  400 KV, 300 km single circuit
- (D)  400 KV, 300 km double circuit

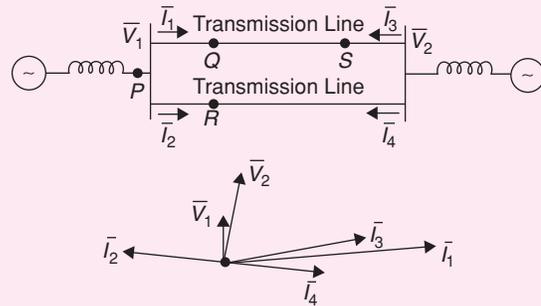
17. A distribution feeder of 1 km length having resistance, but negligible reactance, is fed from both the ends by 400 V, 50 Hz balanced sources. Both voltage sources  $S_1$  and  $S_2$  are in phase. The feeder supplies concentrated loads of unity power factor as shown in the figure. [2014]



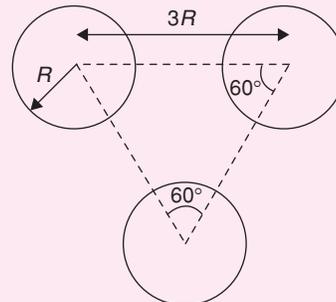
The contributions of  $S_1$  and  $S_2$  in 100 A current supplied at location P, respectively, are

- (A) 75 A and 25 A  
 (B) 50 A and 50 A  
 (C) 25 A and 75 A  
 (D) 0 A and 100 A
18. Shunt reactors are sometimes used in high-voltage transmission systems to [2004]  
 (A) limit the short-circuit current through the line  
 (B) compensate for the series reactance of the line under heavily loaded condition.  
 (C) limit overvoltages at the load side under lightly loaded condition.  
 (D) compensate for the voltage drop in the line under heavily loaded condition.
19. Base load power plants are [2015]  
 P: wind farms.  
 Q: run-of-river plants.  
 R: nuclear power plants.  
 S: diesel power plants.  
 (A) P, Q and S only  
 (B) P, R and S only  
 (C) P, Q and R only  
 (D) Q and R only
20. A sustained three-phase fault occurs in the power system shown in the figure. The current and voltage

phasors during the fault (on a common reference), after the natural transients have died down, are also shown. Where is the fault located? [2015]



- (A) Location P  
 (B) Location Q  
 (C) Location R  
 (D) Location S
21. A composite conductor consists of three conductors of radius  $R$  each. The conductors are arranged as shown below. The geometric mean radius (GMR) (in cm) of the composite conductor is  $kR$ . The value of  $k$  is \_\_\_\_\_. [2015]



22. A three-phase cable is supplying 800 kW and 600 kVAr to an inductive load. It is intended to supply an additional resistive load of 100 kW through the same cable without increasing the heat dissipation in the cable, by providing a three-phase bank of capacitors connected in star across the load. Given the line voltage is 3.3 kV, 50 Hz, the capacitance per phase of the bank, expressed in microfarads, is \_\_\_\_\_. [2016]
23. Single-phase transmission line has two conductors each of 10 mm radius. These are fixed at a center-to-center distance of 1 m in a horizontal plane. This is now converted to a three-phase transmission line by introducing a third conductor of the same radius. This conductor is fixed at an equal distance  $D$  from the two single-phase conductors. The three-phase line is fully transposed. The positive sequence inductance per phase of the three-phase system is to be 5% more than that of the inductance per conductor of the single-phase system. The distance  $D$ , in meters, is \_\_\_\_\_. [2016]

24. The inductance and capacitance of a 400kV, three-phases 50HZ lossless transmission line are 1.6mH/km/phase and 10nF/km/phase respectively. The sending end voltage is maintained at 400kV. To maintain a voltage of 400kV at the receiving end, when the line is delivering 300MW load, the shunt compensation required is [2016]  
 (A) Capacitive (B) Inductive  
 (C) Resistive (D) Zero
25. At no load condition, a 3-phase, 50Hz, lossless power transmission line has sending-end and receiving-end voltages of 400KV and 420KV respectively. Assuming the velocity of travelling wave to be the velocity of light, the length of the line, in km, is \_\_\_\_\_ [2016]
26. The power consumption of an industry is 500kVA, at 0.8 p.f lagging. A synchronous motor is added to raise the power factor of the industry to unity. If the power intake of the motor is 100kW, the p. f. of the motor is \_\_\_\_\_ [2016]

## ANSWER KEYS

### EXERCISES

#### Practice Problems 1

- |       |       |       |       |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1. C  | 2. B  | 3. C  | 4. A  | 5. C  | 6. A  | 7. B  | 8. C  | 9. D  | 10. C |
| 11. A | 12. A | 13. A | 14. D | 15. B | 16. A | 17. C | 18. B | 19. C | 20. C |
| 21. D | 22. C | 23. B | 24. A | 25. C | 26. B | 27. A |       |       |       |

#### Practice Problems 2

- |       |       |       |       |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1. B  | 2. A  | 3. A  | 4. A  | 5. D  | 6. C  | 7. B  | 8. D  | 9. B  | 10. D |
| 11. B | 12. C | 13. B | 14. B | 15. C | 16. C | 17. C | 18. B | 19. A | 20. C |
| 21. B | 22. B | 23. C | 24. A | 25. D | 26. D | 27. C | 28. A | 29. C | 30. C |
| 31. D | 32. A | 33. B | 34. A | 35. A | 36. A | 37. D | 38. A | 39. C | 40. A |

#### Previous Years' Questions

- |                  |           |          |       |               |           |       |       |       |       |
|------------------|-----------|----------|-------|---------------|-----------|-------|-------|-------|-------|
| 1. B             | 2. D      | 3. C     | 4. B  | 5. C          | 6. A      | 7. C  | 8. B  | 9. A  | 10. C |
| 11. D            | 12. D     | 13. B    | 14. A | 15. C         | 16. D     | 17. D | 18. C | 19. C | 20. B |
| 21. 1.85 to 1.95 | 22. 48.23 | 23. 1.42 | 24. B | 25. 294.847Km | 26. 0.316 |       |       |       |       |