Chapter 6
Ratio and Proportion

Exercise 6.1

Question 1.
Express the following ratios in simplest form:

(*i*) $\frac{1}{6}:\frac{1}{9}$ (*ii*) $4\frac{1}{2}:1\frac{1}{8}$

(*iii*) $\frac{1}{5}:\frac{1}{10}:\frac{1}{15}$

Solution:

(*i*) $\frac{1}{6}:\frac{1}{9}$

Given ratio : $\frac{1}{6}:\frac{1}{9}$

$= \frac{1}{6} \div \frac{1}{9}$

$= \frac{1}{6} \times \frac{9}{1}$

$= \frac{3}{2} = 3:2$

(*ii*) $4\frac{1}{2}:1\frac{1}{8}$

Given ratio $= \frac{9}{2}:\frac{9}{8} = \frac{9}{2} \div \frac{9}{8}$

$= \frac{9}{2} \times \frac{8}{9}$ [Dividing by 2]

$= \frac{4}{1} = 4:1$

(iii) $\frac{1}{5}:\frac{1}{10}:\frac{1}{15}$

Taking L.C.M. of 5, 10 and 15

L.C.M. of 5, 10 and 15

$$\begin{array}{r|l} 5 & 5 - 10 - 15 \\ \hline & 1 - 2 - 3 \end{array}$$

L.C.M. = 5 × 2 × 3 = 30

Question 2.
Find the ratio of each of the following in simplest form:
(i) ₹ 5 to 50 paise
(ii) 3 km to 300 m
(iii) 9 m to 27 cm
(iv) 15 kg to 210 g
(v) 25 minutes to 1.5 hours
(vi) 30 days to 36 hours
Solution:

(i) ₹ 5 to 50 paise

= 500 paise : 50 paise

= 10 : 1 (Dividing by 50)

(ii) 3 km to 300 m

= 3000 m to 300 m

= 10 : 1 (Dividing by 300)

(iii) 9 m to 27 cm

= 9 × 100 cm : 27 cm

= 100 : 3 (Dividing by 9)

(iv) 15 kg to 210 g

= 15 × 1000 g : 210 g

= 15000 : 210

= 500 : 7 (Dividing by 30)

(v) 25 minutes to 1.5 hours

= 25 minutes to $\frac{3}{2} \times 60$

= 25 : 90

= 5 : 18

(vi) 30 days to 36 hours

= 30 × 24 hours to 36 hours

= 720 : 36

= 20 : 1 (Dividing by 36)

Question 3.

If A : B = 3 : 4 and B : C = 8 : 9, then find A : C.

Solution:

A : B = 3 : 4 and B : C = 8 : 9

$\frac{A}{B} = \frac{3}{4}$

$$= \frac{1}{5} \times 30 : \frac{1}{10} \times 30 : \frac{1}{15} \times 30$$

$$= 6 : 3 : 2$$

Question 4.

If A : B = 5 : 8 and B : C = 18 : 25, then find A : B : C.

Solution:

A : B = 5 : 8 and B : C = 18 : 25

Here, In A : B, B = 8

and In B : C, B = 18

LCM of 8, 18 is 72

$$\frac{A}{B} = \frac{5}{8} = \frac{5 \times 9}{8 \times 9} = \frac{45}{72}$$

$$\frac{B}{C} = \frac{18}{25} = \frac{18 \times 4}{25 \times 4} = \frac{72}{100}$$

$$A : B : C = 45 : 72 : 100$$

Question 5.

If 3A = 2B = 5C, then find A : B : C.

Solution:

Let 3A = 2B = 5C = 1

$$\text{Then} = A = \frac{1}{3}, B = \frac{1}{2}, C = \frac{1}{5}$$

$$\therefore A : B : C = \frac{1}{3} : \frac{1}{2} : \frac{1}{5}$$

$$= \frac{10:15:6}{30} = 10 : 15 : 6$$

Question 6.
Out of daily income of ₹ 120, a labourer spends ₹ 90 on food and shelter and saves the rest. Find the ratio of his
(i) spending to income
(ii) saving to income
(iii) saving to spending.
Solution:

Daily income = ₹ 120

Expenditure = ₹ 90

Savings = ₹ 120 − ₹ 90 = ₹ 30

(i) Ratio between spending to income

= 90 : 120

= 3 : 4 (Dividing by 30)

(ii) Ratio between saving to income

= 30 : 120

= 1 : 4 (Dividing by 30)

(iii) Ratio between saving to spending

= 30 : 90

= 1 : 3 (Dividing by 30)

Question 7.
5 grams of an alloy contains $3\frac{3}{4}$ grams copper and the rest is nickel. Find the ratio by weight of nickel to copper.
Solution:

Total weight of an alloy = 5 gms

3 15

Weight of copper = $3\frac{3}{4}$ gms = $\frac{15}{4}$ gms

Weight of nickel = Total weight of alloy − weight of copper

$$= \frac{5}{1}\text{gms} - \frac{15}{4}\text{gms}$$

$$= \frac{(4 \times 5)\text{gms} - (1 \times 15)\text{gms}}{4}$$

$$= \frac{20\text{gms} - 15\text{gms}}{4}$$

$$\text{Weight of nickel} = \frac{5}{4}\text{gms}$$

Ratio of weight of nickel to weight of copper

Weight of nickel : Weight of copper

$$\frac{5}{4}\text{gms} : \frac{15}{4}\text{gms}$$

$$= \frac{5}{4} \div \frac{15}{4}$$

$$= \frac{5}{4} \times \frac{4}{15}$$ [Cancelling 4 by 4]

$$= \frac{5}{1} \times \frac{1}{15}$$ [Dividing both by 5]

$$= \frac{1}{3} = 1 : 3$$

Question 8.

A pole of height 3 meters is struck by a speeding car and breaks into two pieces such that the first piece is 1/2 of the second. Find the length of both pieces.

Solution:

Total height of pole = 3 metres

Let length of 2nd piece = x

Length of 1st piece = $\frac{1}{2}$ x

Ratio of lengths of two parts = $\frac{1}{2}$ x : 1x

$$= \frac{1}{2} : \frac{1}{1}$$

Taking L.C.M. of 2 and 1

$$= \left(\frac{1}{2} \times 2\right) : \left(\frac{1}{1} \times 2\right)$$

$$= 1 : 2$$

Sum of terms of ratio = 1 + 2 = 3

$\therefore$ **Length of 1st part = $\frac{1}{3}$ of 3 m**

$$= \left(\frac{1}{3} \times 3\right) \text{m} = 1 \text{ m}$$

Length of 2nd part = $\frac{2}{3}$ of 3 m

$$= \left(\frac{2}{3} \times 3\right) \text{m} = 2 \text{ m}$$

Length of 1st part = 1 m

Length of 2nd part = 2m

Question 9.

Heights of Anshul and Dhruv are 1.04 m and 78 cm respectively. Divide 35 sweets between them in the ratio of their heights.

Solution:

Height of Anshul : Height of Dhruv

1.4 m : 78 cm

(1.04 × 100) cm : 78 cm

= 104 : 78

= $\frac{104}{78}$ (Dividing both by 2)

= $\frac{52}{39}$ (Dividing boht by 13)

= $\frac{4}{3}$

= 4 : 3

Ratio of heights of Anshul and Dhruv is 4 : 3

Thus, we are to divide 35 sweets in the ratio 4 : 3

Sum of the terms of the ratios = 4 + 3 = 7

Share of Anshul = $\frac{4}{7}$ of 35 sweets

= $\frac{4}{7}$ × 35

= 20 sweets

Share of Dhruv = $\frac{3}{7}$ of 35 sweets

= $\frac{3}{7}$ × 35

= 15 sweets

Question 10.

₹ 180 are to be divided among three children in the ratio 1/3:1/4:1/6 Find the share of each child.

Solution:

First we will simplify the given ratio

Given ratio $\frac{1}{3} : \frac{1}{4} : \frac{1}{6}$

Taking L.C.M. of 3, 4 and 6

L.C.M. of 3, 4 and 6 = 12

3	3 – 4 – 6
2	1 – 4 – 2
	1 – 2 – 1

L.C.M. = $3 \times 2 \times 2 = 12$

$\frac{1}{3} \times 12 : \frac{1}{4} \times 12 : \frac{1}{6} \times 12 = 4 : 3 : 2$

Thus, we are to divide ₹180 in the ratio 4 : 3 : 2.

Sum of the terms of the ratio

$= 4 + 3 + 2 = 9.$

∴ Share of first child $= \frac{4}{9}$ of ₹180

$= ₹\left(\frac{4}{9} \times 180\right)$ [Dividing both by 9]

$= ₹80$

Share of second child $= \frac{3}{9}$ of ₹180

$= ₹\left(\frac{3}{9} \times 180\right) = ₹60$

Share of third child $= \frac{2}{9}$ of ₹180

$= ₹\left(\frac{2}{9} \times 180\right) = ₹40$

Question 11.

A natural number has been divided into two parts in the ratio 7 : 11. If the difference between the two parts is 20, find the number and the two parts.

Solution:

Let the first part = 7x

Second part = 11x

According to given statement,

11x − 7x = 20

⇒ 4x = 20

⇒ x = 5

First part = 7x = 7 × 5 = 35

Second part = 11x = 11 × 5 = 55

and number will be 35 + 55 = 90

Question 12.

A certain sum of money has been divided into two parts in the ratio 9 : 13. If the second part is ₹ 260, find the total amount.

Solution:

Let the total amount = ₹ x

The amount has been divided into two parts in the ratio 9 : 13.

Sum of the terms of the ratio = 9 + 13 = 22

First part = $\frac{9}{22}$ of total amount

Second part = $\frac{13}{22}$ of total amount

According to given statement

$$\frac{13}{22} \text{ of } x = ₹260 \Rightarrow \frac{13}{22} \times x = ₹260$$

$$\Rightarrow x = ₹\frac{260 \times 22}{13} \Rightarrow x = ₹20 \times 22$$

$$\therefore x = ₹440$$

Question 13.

The ratio of the present ages of Anjali and Ashu is 2 : 3. Five years hence, the ratio of their ages will be 3 : 4. Find their present ages.

Solution:

Ratio of present ages of Anjali and Ashu = 2 : 3

Let age of Anjali = 2x

and age of Ashu = 3x

5 years hence,

Age of Anjali = 2x + 5

and age of Ashu = 3x + 5

$\frac{2x+5}{3x+5} = \frac{3}{4}$

$\frac{2x+5}{3x+5} = \frac{3}{4}$

9x + 15 = 8x + 20

9x − 8x = 20 − 15

x = 5

Present age of Anjali = 2x = 2 × 5 = 10 years

and age of Ashu = 3x = 3 × 5 = 15 years

Question 14.
The present ages of A and B are in the ratio 5 : 6. Three years ago, their ages were in the ratio 4 : 5. find their present ages.
Solution:

Ratio of the present age of A and B = 5 : 6

Let age of A = 5x

and age of b = 6x

3 years ago,

Age of A was = 5x − 3

and age of B was = 6x − 3

$\frac{5x-3}{6x-3} = \frac{4}{5}$

⇒ 25x − 15 = 24x − 12

⇒ 25x − 24x = -12 + 15

⇒ x = 3

Present age of A = 5x = 5 × 3 = 15 years

and age of B = 6x = 6 × 3 = 18 years

Question 15.
Two numbers are in the ratio 5 : 6. When 2 is added to first and 3 is added to second, they are in the ratio 4 : 5. Find the numbers.
Solution:

Ratio in two numbers = 5 : 6

Let first number = 5x

Then second number = 6x

Adding 2 in the first and 3 in the second

A = 5x + 2

B = 6x + 3

$\frac{5x+2}{6x+3} = \frac{4}{5}$

25x + 10 = 24x + 12

25x – 24x = 12- 10

x = 2

First number = 5x = 5 × 2 = 10

and second = 6x = 6 × 2 = 12

Question 16.
The ratio of number of boys to the number of girls in a school of 1430 students is 7 : 6. If 26 new girls are admitted in the school, find how many new boys may be admitted so that the ratio of number of boys to the number of girls may change to 8 : 7.
Solution:

Number of students = 1430

Ratio in number of boys and girls = 7 : 6

Let number of boys = 7x and of girls = 6x

7x + 6x = 1430

⇒ 13x = 1430

⇒ x = 110

Number of boys = 7x = 7 × 110 = 770

and number of girls = 6x = 6 × 110 = 660

Now adding 26 new girls, the number of girls will be = 660 + 26 = 686

Let new boys be added = y

The number of boys = 770 + y

Now new ratio = 8 : 7

$\frac{770+y}{686} = \frac{8}{7}$

5390 + 7y = 5488

7y = 5488 – 5390 = 98

y = 14

Number of new boys admitted = 14

Question 17.
Which ratio is greater?
(i) 5 : 6 or 6 : 7

(ii) 13 : 24 or 17 : 32

Solution:

(i) 5 : 6 or 6 : 7

5 : 6 = $\frac{5}{6}$ and 6 : 7 = $\frac{6}{7}$

Converting them into equivalent fraction by taking L.C.M. of 6 and 7 = 42

$$\frac{5}{6} = \frac{5 \times 7}{6 \times 7} = \frac{35}{42} \text{ and}$$

$$\frac{6}{7} = \frac{6 \times 6}{7 \times 6} = \frac{36}{42}$$

$$\text{As } 36 > 35 \therefore \frac{36}{42} > \frac{35}{42}$$

$$\Rightarrow \frac{6}{7} > \frac{5}{6}$$

Hence 6 : 7 is the greater ratio.

(*ii*) 13 : 24 or 17 : 32

$$13 : 24 = \frac{13}{24} \text{ and } 17 : 32 = \frac{17}{32}$$

Converting them into equivalent fraction by taking L.C.M. of 24 and 32 = 96

$$\frac{13}{24} = \frac{13 \times 4}{24 \times 4} = \frac{52}{96} \text{ and}$$

$$\frac{17}{32} = \frac{17 \times 3}{32 \times 3} = \frac{51}{96}$$

2	24 – 32
2	12 – 16
2	6 – 8
2	3 – 4
	3 – 2

L.C.M. = 2 × 2 × 2 × 2 × 3 × 2 = 96

As 52 > 51

$\therefore \quad \frac{52}{96} > \frac{51}{96}$

$\Rightarrow \quad \frac{13}{24} > \frac{17}{32}$

$\therefore$ 13 : 24 is the greater ratio.

Exercise 6.2

Question 1.
Which of the following statements are true?
(i) 2.5 : 1.5 :: 7.0 : 4.2
(ii) 1/2:1/3=1/3:1/4
(iii) 24 men : 16 men = 33 horses : 22 horses.
Solution:

(i) 2.5 : 1.5 :: 7.0 : 4.2

Product of extremes = 2.5 × 4.2 = 10.50

Product of means = 1.5 × 7.0 = 10.50

By cross product rule

Product of extremes = Product of means

2.5 : 1.5 :: 7.0 : 4.2 is true statement

(ii) $\frac{1}{2} : \frac{1}{3} = \frac{1}{3} : \frac{1}{4}$

Product of extremes = $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

Product of means = $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

By cross product rule

Product of extremes ≠ Product of means

$\frac{1}{2} : \frac{1}{3} = \frac{1}{3} : \frac{1}{4}$ is not a true statement.

(in) 24 men : 16 men = 33 horses : 22 horses

Product of extremes = 24 × 22 = 528

Product of means = 16 × 33 = 528

By cross product rule

Product of extremes = Product of means

24 men : 16 men = 33 horses : 22 horses is a true statement.

Question 2.
Check whether the following numbers are in proportion or not:
(i) 18, 10, 9, 5
(ii) 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$
(iii) 0.1, 0.2, 0.3, 0.6
Solution:

(i) 18, 10, 9, 5

Product of extremes = 18 × 5 = 90

Product of means = 10 × 9 = 90

By cross product rule

Product of extremes = Product of means

The numbers 18, 10, 9, 5 are in proportion.

(*ii*) $3, 3\frac{1}{2}, 4, 4\frac{1}{2}$

Product of extremes

$$= 3 \times 4\frac{1}{2} = 3 \times \frac{9}{2} = \frac{27}{2}$$

Product of means

$$= 3\frac{1}{2} \times 4 = \frac{7}{2} \times 4 = \frac{28}{2} = 14$$

By cross product rule

Product of extremes ≠ Product of means

The numbers 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$ are not in proportion.

(iii) 0.1, 0.2, 0.3, 0.6

Product of extremes = 0.1 × 0.6 = 0.06

Product of means = 0.2 × 0.3 = 0.06

By cross product rule

Product of extremes = Product of means

The numbers 0.1, 0.2, 0.3, 0.6 are in proportion.

Question 3.

Find x in the following proportions:

(i) x : 4 = 9 : 12

(ii) 1/13 : x :: 1/2 : 1/5

(iii) 3.6 : 0.4 = x : 0.5

Solution:

(i) x : 4 = 9 : 12

By cross product rule

Product of extremes = Product of means

x × 12 = 4 × 9

$$x = \frac{4 \times 9}{12}$$

$$x = \frac{36}{12}$$

$$x = 3$$

(*ii*) $\frac{1}{13} : x :: \frac{1}{2} : \frac{1}{5}$

By cross product rule

Product of means = Product of extremes

$$x \times \frac{1}{2} = \frac{1}{13} \times \frac{1}{5}$$

$$x \times \frac{1}{2} = \frac{1}{65}$$

$$x = \frac{1}{65} \div \frac{1}{2}$$

$$x = \frac{1}{65} \times 2$$

$$x = \frac{2}{65}$$

(*iii*) 3.6 : 0.4 = x : 0.5

By cross product rule

Product of means = Product of extremes

$0.4 \times x = 3.6 \times 0.5$

$$x = \frac{3.6 \times 0.5}{0.4}$$

$$x = \frac{1.80}{0.4}$$

$$x = 4.5$$

Question 4.
Find the fourth proportional to
(i) 42, 12, 7
(ii) 1/3, 1/4, 1/5
(iii) 3 kg, 12 kg, 15 kg
Solution:

(i) 42, 12, 7

Let the fourth proportional be x.

Then 42, 12, 7, x are in proportion

Using the cross product rule

Product of extremes = Product of means

42 × x = 12 × 7

$$x = \frac{12 \times 7}{42}$$

$$x = \frac{84}{42}$$

$$x = 2$$

(*ii*) $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

Let x be the fourth proportional, then

$\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, x$ are in proportion

Using cross product rule

Product of extremes = Product of means

$$\frac{1}{3} \times x = \frac{1}{4} \times \frac{1}{5}$$

$$\frac{1}{3} \times x = \frac{1}{20}$$

$$x = \frac{1}{20} \div \frac{1}{3}$$

$$x = \frac{1}{20} \times 3$$

$$x = \frac{3}{20}$$

(iii) 3 kg, 12 kg, 15 kg

Let the fourth proportional be x kg, then

3 kg, 12 kg, 15 kg, x kg are in proportion

Using cross product rule

Product of extremes = Product of means

3 × x = (12 × 15) kg

3x = 180 kg

x = 60 kg

Question 5.

Check whether 7, 49, 343 are in continued proportion or not.

Solution:

Three quantities are said to be in continued proportion if a : b = b : c i.e., if $\frac{a}{b} = \frac{b}{c}$ i.e., if $b^2 = ac$

Here, a = 7, b = 49, c = 343

$b^2 = ac$

$(49)^2 = 7 \times 343$

$49 \times 49 = 7 \times 343$

2401 = 2401

Yes, the number 7,49, 343 are in continued proportion.

Question 6.

Find the third proportional to

(i) 36, 18

(ii) $5\frac{1}{4}, 7$

(iii) 3.2, 0.8

Solution:

(i) 36, 18

Let the third proportional to 36, 18 be x.

Then 36, 18 and x are in continued proportion

36 : 18 :: 18 : x

Using the cross product rule

Product of extremes = Product of means

36 × x = 18 × 18

x = 9

Hence, the third proportion is 9

(*ii*) $5\frac{1}{4}, 7$

Let the third proportional to $5\frac{1}{4}, 7$ be x

Then $5\frac{1}{4}$, 7 and x are in continued proportion

i.e. $5\frac{!}{4} : 7 :: 7 : x$

$\frac{21}{4} : 7 :: 7 : x$

Using cross product rule

Product of extremes = Product of means

$$\frac{21}{4} \times x = 7 \times 7$$

$$\frac{21}{4} \times x = 49$$

$$x = 49 \div \frac{21}{4}$$

$$x = 49 \times \frac{4}{21}$$

$$x = \frac{196}{21} = \frac{28}{3}$$

$$x = 9\frac{1}{3}$$

Hence, third proportion is $9\frac{1}{3}$

(iii) 3.2, 0.8

Let the third proportional to 3.2, 0.8 be x.

Then 3.2, 0.8 and x are in continued proportion

i.e., 3.2 : 0.8 :: 0.8 : x

Products of extremes = 3.2 × x

Product of means = 0.8 × 0.8

3.2 × x = 0.8 × 8

x = 0.2

Hence, third proportion is 0.2

Question 7.

The ratio between the length and width of a rectangular sheet of paper is 7 : 5. If the width of the sheet is 20.5 cm, find its length.

Solution:

Let the length of the sheet be x.

Then the ratio of length to width is x : 20.5 cm.

According to given statement,

x : 20.5 cm = 7 : 5

Using cross product rule

Product of extremes = Product of means

x × 5 = 20.5 cm × 7

x × 5 = 143.5 cm

x = 28.7 cm

Hence, length of the sheet = 28.7 cm

Question 8.

The ages of Amit and Archana are in the ratio 4 : 5. If Amit is 4 years 8 months old, find the age of Archana.

Solution:

Let the age of Archana be x.

Then the ratio of ages of Amit and Archana be 4 years 8 months : x.

1 year = 12 months

4 years = 4 × 12 months = 48 months

4 years 8 months = (48 + 8) months = 56 months

According to given statement,

56 months : x :: 4 : 5

Using cross product rule

Product of means = Product of extremes

x × 4 = 56 months × 5

x = 70 months

Converting months in years

$$\begin{array}{r} 12\overline{)70}(5 \text{ years} \\ \underline{60} \\ 10 \text{ months} \end{array}$$

70 months = 5 years 10 months

Hence, the age of Archana is 5 years 10 months.

Exercise 6.3

Question 1.
6 bowls cost ₹ 90. What would be the cost of 10 such bowls?
Solution:

Cost of 6 bowls = ₹ 90

Let cost of 10 bowls = ₹ x

6 : 10 = 90 : x

⇒ 6 × x = 10 × 90 (ad = bc)

x = ₹ 150

Cost of 10 bowls = ₹ 150

Question 2.
Ten pencils cost ₹ 15. How many pencils can be bought with ₹ 72?
Solution:

Cost of 10 pencils = ₹ 15

Let number of pencils will be bought in ₹ 72 = x

10 : x = 15 : 72

x × 15 = 10 × 72 (bc = ad)

x = 48

Number of pencils = 48

Question 3.
400 grams cake costs 80 rupees. How much would a 1.5 kg cake cost?
Solution:

Cost of 400 g cake = ₹ 80

Let cost of 1.5 kg cake = ₹ x

1.5 kg = 1500 gm

400 : 1500 = 80 : x

400 × x – 1500 × 80 (ad = bc)

x = ₹ 300

Cost of 1.5 kg = ₹ 300

Question 4.
A man earns ₹ 18000 in 3 months.

(i) How much time would he take to earn ₹ 30000?
(ii) How much money will he earn in 7 months?
Solution:

A man earns in 3 months = ₹ 18000

Let ₹ 30000 be earned in = x months

18000 : 30000 = 3 : x (ad = bc)

18000 × x = 30000 × 3

x = 5

₹ 30000 will be earn in 5 months

(ii) Let money will be earned in 7 months = ₹ x

3 : 7 = 18000 : x (ad = bc)

3 × x = 7 × 18000

x = 42000

A money of ₹ 42000 will be earned.

Question 5.
12 mangoes weigh 2.4 kg. What is the weight of 8 mangoes?
Solution:

Weight of 12 mangoes = 2.4 kg

Let weight of 8 mangoes = x kg

12 : 8 = 2.4 : x

12 × x = 8 × 2.4

x = 1.6

Weight of 8 mangoes = 1.6 kg

Question 6.
If the weight of 12 sheets of thick paper is 40 grams, how many sheets of the same paper would weigh $2\frac{1}{2}$ kilograms?

Solution:

Weight of 12 sheets of paper = 40 grams

Let $2\frac{1}{2}$ kg = $\frac{5}{2}$ × 1000 = 2500 gram,

sheets of thick paper = x sheets

12 : x = 40 : 2500

12 × 2500 = x × 40 (ad = bc)

x = 750

Sheets are 750

Question 7.

A bus consumes 25 litres of diesel in covering a distance of 90 kilometres. How much diesel is needed to cover 288 kilometres?

Solution:

A bus consum 251 of diesel to cover = 90km

Let it will consum x litres of diesel for 288 km

90 : 288 = 25 : x

90 × x = 288 × 25

x = 80

Required diesel = 80 litre

Question 8.

If 4/5 metre cloth costs ₹ 36, find the cost of $2\frac{1}{5}$ metres of cloth.

Solution:

$$\text{Cost of } \frac{4}{5}\text{ m} = ₹36$$

$$\text{Let cost of } 2\frac{1}{5} = \frac{11}{5}\text{ m of cloth} = ₹\, x$$

$$\therefore \ \frac{4}{5} : \frac{11}{5} = 36 : x$$

$$\Rightarrow \ \frac{4}{5} \times x = \frac{11}{5} \times 36 \qquad (ad = bc)$$

$$\Rightarrow x = \frac{11}{5} \times 36 \times \frac{5}{4} = 99$$

$$\therefore \text{ Cost of } \frac{11}{5} \text{ litre of diesel} = ₹99$$

Question 9.
If 15 men can pack 540 parcels per day, how many men are needed to pack 396 parcels per day?
Solution:

15 men can pack = 540 parcels per day

Let 396 parcles be packed by x men

540 : 396 = 15 : x

540 × x = 396 × 5 (ad = bc)

x = 11

Required men = 11

Question 10.
Which is a better buy 12 kg potatoes for ₹ 132 or 16 kg potatoes for ₹ 168?
Solution:

12 kg potatoes for ₹ 132

Cost of 1 kg = $\frac{132}{12}$ = ₹ 11

and cost of 16 kg = ₹ 168

Cost of 1 kg = $\frac{168}{16}$ = ₹ 10.50

Second deal is better.

i.e., 16 kg of potatoes for ₹ 168 is better.

Exercise 6.4

Question 1.
Convert the following speeds into m/sec:
(i) 72 km/h
(ii) 9 km/h
(iii) 1.2 km/minutes
(iv) 600 m/hour
Solution:

(i) 72 km/h

1 km = 1000 m

and 1 h = 3600 sec.

$$\frac{1\text{ km}}{h} = \frac{1000}{3600} = \frac{5}{18}\text{ m/s}$$

$$\therefore\ 72\text{ km/h} = \frac{5}{18} \times 72\text{ m/sec}$$

$$= 20\text{ m/sec}$$

(*ii*) 9 km/h

1 hour = 3600 sec.

1 km = 1000 m

$$\frac{1\text{ km}}{m} = \frac{1000}{3600} = \frac{5}{18}\text{ m/s}$$

$$\therefore\ 9\text{ km/h} = \frac{5}{18} \times 9\text{ m/sec}$$

$$= \frac{5}{2}\text{ or } 2.5\text{ m/sec}$$

(*iii*) 1.2 km/minutes

1 hour = 60 minutes

1.2 km/min. = 1.2 × 60 km/hr

Now, $\frac{1\text{ km}}{h} = \frac{5}{18}$ m/s

1.2×60 km/h $= 1.2 \times 60 \times \frac{5}{18}$ m/sec

$= 72 \times \frac{5}{18}$ m/sec $= 20$ m/sec

(*iv*) 600 m/hour

$= \frac{600}{1000}$ km/h $= \frac{600 \times 5}{1000 \times 18}$ m/sec

$= \frac{1}{6}$ m/sec

Question 2.
Convert the following speeds into km/h:
(i) 15 m/sec
(ii) 1.5 m/sec
Solution:

(i) 15 m/sec

$1\text{ m} = \frac{1}{1000}$ km

$1\text{ sec.} = \frac{1}{3600}$ hours

$\frac{1\text{ m}}{\text{s}} = \frac{1}{1000} \div \frac{1}{3600} = \frac{3600}{1000}$

$= \frac{18}{5}$ km/h

(*ii*) 1.5 m/sec

$$1 \text{ m} = \frac{1}{1000} \text{ km}$$

$$1 \text{ sec.} = \frac{1}{3600} \text{ hours}$$

$$\frac{1 \text{ m}}{\text{s}} = \frac{1}{1000} \div \frac{1}{3600} = \frac{3600}{1000}$$

$$= \frac{18}{5} \text{ km/h}$$

$$\therefore \ 1.5 \text{ m/s} = 1.5 \times \frac{18}{5} \text{ km/h} = 5.4 \text{ km/h}$$

Question 3.
Which is greater- the speed of 30 m/sec or 30 km/h?
Solution:

Which is greater

m/s = $\frac{18}{5}$ km/h

30 km/h = $\frac{18}{5} \times 30 = 108$ km/h

That means 108 km cover in 1 hour 30 m/sec is greater

Question 4.
An aeroplane is flying at a speed of 720 km/h.
(i) If the aerial distance between two cities is 1800 km, how much time will the aeroplane take in crossing these cities?
(ii) How much distance does the aeroplane cover in 40 minutes?
(iii) How far will it fly in 15 seconds?
Solution:

Speed of an aeroplane = 720 km/h

(i) Distance between two cities = 1800 km

Time taken = $\frac{D}{S}$

= $\frac{1800}{720}$ hours

= $\frac{5}{2}$ hours

= $2\frac{1}{2}$ hours

(ii) Time = 40 min

= $\frac{40}{60}$

= $\frac{2}{3}$ h

Distance travelled = Time × Speed

= $\frac{2}{3}$ × 720

= 480 km

(iii) Distance travelled in 15 sec = $\frac{15}{3600}$ = $\frac{1}{240}$ h

= $\frac{1}{240}$ × 720

= 3 km

Question 5.
A dog is walking at a speed of 6 km/h.
(i) How much distance does it cover in 5 minutes?
(ii) How much time would it take to cover 200 metres?
Solution:

Speed of a dog = 6 km/h

(i) Distance travelled in 5 min. = $\frac{5}{60}$ × 6 km

= $\frac{1}{2}$ km

= 500 m

(ii) Time taken to cover 200 m

= $\frac{200}{1000}$ km × $\frac{1}{6}$ h

= $\frac{1}{30}$ h

= $\frac{1}{30}$ × 60

= 2 minutes

Question 6.
A swimming pool is 50 metres long. A boy can swim across the length and then return to his starting position in 5 minutes. What is his swimming speed in km/h?
Solution:

Length of a swimming pool = 50 m

A boy crosses it and then come back

Total distance covered = 50 × 2 = 100 m

Time taken = 5 minutes

$$\therefore \text{ Speed in km/h} = \frac{D}{T} = \frac{100 \times 60}{1000 \times 5}$$

$$= 1\frac{1}{5} \text{ km/h} = 1.2 \text{ km/h}$$

Question 7.
A bus takes 48 minutes to cover a certain distance when travelling at a speed of 50 km/h. How much time will it take to cover the same distance when travelling at a speed of 30 km/h?
Solution:

Speed of a bus = 50 km/h

Time taken = 48 minutes

Distance = Speed × Time

$= 50 \times \frac{48}{60}$ km = 40 km

In second time speed = 30 km/h

Time taken = $\frac{D}{S}$

$= \frac{40}{30}$

$= \frac{4}{3}$ h

= 1 hours $\frac{1}{3}$ min.

= 1 hours $\frac{1}{3} \times 60$ mm.

= 1 hour 20 minutes

Objective Type Questions

Question 1.
Fill in the blanks:
(i) The simplest form of the ratio 1/6 : 1/4 is .....
(ii) 75 cm : 1.25 m = ...
(iii) If two ratios are equivalent, then the four quantities are said to be in .....
(iv) If 8, x, 48 and 18 are in proportion then the value of x is .....
(v) If the cost of 10 pencils is ₹ 15, then the cost of 6 pencils is .....
(vi) If a cyclist is travelling at a speed of 15 km/h, then the distance covered by him in 20 minutes is .....
Solution:

(i) The simplest form of the ratio $\frac{1}{6} : \frac{1}{4}$ is 2 : 3.

(ii) 75 cm : 1.25 m = 3 : 5

(iii) If two ratios are equivalent,

then the four quantities are said to be in proportion.

(iv) If 8, x, 48 and 18 are in proportion then the value of x is

8 : x :: 48 : 18

x = 3

(v) If the cost of 10 pencils is ₹ 15, then the cost of 6 pencils is

Cost of 10 pencils = ₹ 15

Let cot of 6 pencils = ₹ x

10 : 6 15 : x

10x = 6 × 5

x = 3

Cost of 6 pencils = ₹ 3

(vi) If a cyclist is travelling at a speed of 15 km/ h,

then the distance covered by him in 20 minutes is .....

Speed of a cyclist = 15 km/h

Distance travelled in 20 minutes = $\frac{20}{60}$ × 15 km = 5 km

Question 2.
State whether the following statements are true (T) or false (F):
(i) A ratio is always greater than 1.
(ii) Ratio of half an hour to 20 seconds is 30 : 20.
(iii) The ratio 5 : 7 is greater than the ratio 5 : 6.
(iv) If the numbers 3, 5, 12 and x are in proportion then the value of x is 20.

(v) The ratio 3 : 2 and 4 : 5 are equivalent.

Solution:

(i) A ratio is always greater than 1. (False)

Correct:

Ratio can be less than 1 or equal to 1.

It is not necessary that it is greater than 1.

(ii) Ratio of half an hour to 20 seconds is 30 : 20. (False)

Correct:

= 30 × 60 : 20 = 1800 : 20 = 90 : 1

(iii) The ratio 5 : 7 is greater than the ratio 5 : 6. (False)

Correct:

$\frac{5}{7}, \frac{5}{6}$

$\frac{30:35}{42}$

30 < 35

5 : 7 is not greater than the ratio 5 : 6

(iv) If the numbers 3, 5, 12 and x are in proportion

then the value of x is 20. (True)

3, 5, 12 and x are in proportion then

x × 3 = 5 × 12

x = 20

(v) The ratio 3 : 2 and 4 : 5 are equivalent. (False)

Correct:

$\frac{3}{2} \neq \frac{4}{5}$ as 15 ≠ 8

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 14):

Question 3.

A ratio equivalent to 6 : 10 is

(a) 3 : 4

(b) 18 : 30

(c) 12 : 40

(d) 5 : 3

Solution:

6 : 10 = 3 : 5

18 : 30 = 3 : 5

6 : 40 is equivalent to 18 : 30 (b)

Question 4.

A ratio equivalent to the ratio 2/3 : 3/4 is

(a) 4 : 6

(b) 8 : 9

(c) 6 : 8

(d) 9 : 8

Solution:

$\frac{2}{3} : \frac{3}{4}$

$\frac{8:9}{12}$

= 8 : 9 (b)

Question 5.

The ratio of 75 mL to 3 litres is

(a) 25 : 1

(b) 40 : 1

(c) 1 : 40

(d) 3 : 200

Solution:

75 mL to 3 litres

= 75 : 3000

= 1 : 40 (c)

Question 6.

The ratio of the number of sides of a rectangle to the number of edges of a cuboid is

(a) 1 : 2

(b) 1 : 3

(c) 2 : 3

(d) none of these

Solution:

Ratio of number of sides of a rectangle

to the number of edges of a cuboid = 4 : 12 = 1 : 3 (b)

Question 7.
In a class of 35 students, the number of girls is 20. The ratio of number of boys to the number of girls in the class is
(a) 3 : 4
(b) 4 : 3
(c) 7 : 4
(d) 7 : 3
Solution:

Total number of students = 35

No. of girls = 20

No. of boys = 35 − 20 = 15

Ratio in boys and girls = 15 : 20 = 3 : 4 (a)

Question 8.
The ratio of number of girls to the number of boys in a class is 6 : 7. If there are 21 boys in the class, then the number of girls in the class is
(a) 39
(b) 24
(c) 18
(d) 13
Solution:

Girls : boys = 6 : 7

No. of boys = 21

Let no. of girls = x

6 : 7 :: x : 21

$7 \times x = 6 \times 21$

x = 18

No. of boys = 18 (c)

Question 9.
Two numbers are in the ratio 3 : 5. If the sum of the numbers is 144, then the largest number is
(a) 48
(b) 54
(c) 72
(d) 90

Solution:

Ratio between two numbers = 3 : 5

Sum of numbers 144

Then larger number = $\frac{144}{3+5} \times 5$

= $\frac{144}{8} \times 5 = 90$ (d)

Question 10.
If x, 12, 8 and 32 are in proportion, then x is
(a) 6
(b) 4
(c) 3
(d) 2
Solution:

x, 12, 8 and 32 are in proportion, then

$x \times 32 = 12 \times 8$

x = 3 (c)

Question 11.
If 3, 12 and x are in continued proportion, then x is
(a) 4
(b) 6
(c) 16
(d) 48
Solution:

3, 12 and x are in continued proportion, then

3 : 12 :: 12 : x

$3x = 12 \times 12$

x = 48 (d)

Question 12.
If the weight of 5 bags of sugar is 27 kg, then the weight of one bag of sugar is
(a) 5.4 kg
(b) 5.2 kg
(c) 5.4 kg
(d) 5.6 kg

Solution:

Weight of 5 bags of sugar = 27 kg

Weight of one bag of sugar

= 5 : 1 :: 27 : x

5x = 1 × 27

$x = \frac{27}{5}$ = 5.4 kg (c)

Question 13.

Sonali bought one dozen notebooks for ₹ 66. What did she pay for one notebook?

(a) ₹ 6.50

(b) ₹ 6.60

(c) ₹ 5.60

(d) ₹ 5.50

Solution:

Cost of 12 books = ₹ 66

Let cost of one book = x

12 : 1 = 66 : x

12 × x = 1 × 66

x = 5.5

Cost of 1 book = ₹ 5.50 (d)

Question 14.

The speed of 90 km/h is equal to

(a) 10 m/sec

(b) 18 m/sec

(c) 25 m/sec

(d) none of these

Solution:

Speed of 90 km/h = $\frac{90 \times 5}{18}$ m/sec

= 25 m/sec (c)

Value Based Questions

Question 1.

Sudhanshu divided his property into two parts in the ratio 8 : 5. If the first part is ₹ 1,60,000 and second part is donated to an orphanage, find the amount donated to

the orphanage. What values are being promoted?
Solution:

Ratio in two parts of a property = 8 : 5

First part = ₹ 1,60,000

Second part = $\frac{160000 \times 5}{8}$ = ₹ 10,00,000

It is a good to donate the needy people and support them.

Higher Order Thinking Skills (HOTS)

Question 1.
Present ages of Rohit and Mayank are in the ratio 11 : 8. 8 years hence the ratio of their ages will be 5 : 4. Find their present ages.
Solution:

Ratio in the present ages of Rohit and Mayank = 11 : 8

Let age of Rohit = 11x, then that of Mayank = 8x

8 years hence, their ages will be

Age of Rohit = 11x + 8 years

and age of Mayank = 8x + 8 years

and the ratio of their ages after 8 years = 5 : 4

$\frac{11x+8}{8x+8} = \frac{5}{4}$

⇒ 44x + 32 = 40x + 40

⇒ 44x − 40x = 40 − 32

⇒ 4x = 8

⇒ x = 2

Present age of Rohit = 11x = 11 × 2 = 22 years

and age of Mayank = 8x = 8 × 2 = 16 years

Question 2.
Ratio of length and breadth of a rectangle is 3 : 2. If the length of the rectangle is 5 m more than the breadth, find the perimeter of the rectangle.

Solution:

Ratio in length and breadth of a rectangle = 3 : 2

Let length = 3x m and Breadth = 2x

Also, l of rectangle is 5 m more than the breadth

i.e. $3x = 2x + 5$

$\Rightarrow 3x - 2x = 5$

$\Rightarrow x = 5$

Length = $3x = 3 \times 5 = 15m$

Breadth = $2x = 2 \times 5 = 10m$

Perimeter = 2(Length + Breadth) = 2 (15 + 10) m = $2 \times 25 = 50$ m

Check Your Progress

Question 1.
A rectangular park is 120 m long and 75 m wide. Find the ratio of:
(i) breadth to its length
(ii) length to its perimeter
Solution:

Length of park (l) = 120 m and width (b) = 75 m

Perimeter = 2(l + b) = 2(120 + 75) m = 2 × 195 = 390 m

(i) Ratio between breadth to length = 75 : 120 = 5 : 8 (Dividing by 15)

(ii) Ratio between length to perimeter = 120 : 390 = 4 : 13 (Dividing by 30)

Question 2.
Divide the angles of a triangle in the ratio 2 : 3 : 4.
Solution:

Given ratio = 2 : 3 : 4

Sum of the terms of ratio = 2 + 3 + 4 = 9

Sum of angles of a triangle = 180°

Thus, we are to divide 180° in the ratio 2 : 3 : 4

$$\therefore \quad m\angle 1 = \frac{2}{9} \times 180^\circ$$

$$= \frac{360^\circ}{9} = 40^\circ$$

$$m\angle 2 = \frac{3}{9} \times 180^\circ$$

$$= \frac{540^\circ}{9} = 60^\circ$$

$$m\angle 3 = \frac{4}{9} \times 180^\circ$$

$$= \frac{720^\circ}{9} = 80^\circ$$

$\therefore \quad m\angle 1 = 40^\circ, m\angle 2 = 60^\circ,$
$m\angle 3 = 80^\circ$

Question 3.
Heights of Anshul, Ankita and Dhruv are 1.04 m, 1.30 m and 91 cm respectively. Divide 100 sweets among them in the ratio of their heights.
Solution:

Height of Anshul = 1.04 m = (1.04 × 100) cm = 104 cm

Height of Ankita = 1.30 m = (1.30 × 100) cm = 130 cm

Height of Dhruv = 91 cm

Ratio of their heights = 104 cm : 130 cm : 91 cm

= 104 : 130 : 91

= 8 : 10 : 7 (Dividing each by 13)

We are to divide 100 sweets in the ratio 8 : 10 : 7.

Sum of the terms of the ratio = 8 + 10 + 7 = 25

$\therefore$ Anshul will get sweets $= \frac{8}{25}$ of 100

$= \frac{8}{25} \times 100 = \frac{800}{25} = 32 = 32$ sweets

Ankita will get $= \frac{10}{25}$ of 100

$= \frac{10}{25} \times 100 = \frac{1000}{25} = 40$ sweets

Dhruv will get $= \frac{7}{25}$ of 100

$= \frac{7}{25} \times 100 = \frac{700}{25} = 28$ sweets

Question 4.
The weights of Divya and Himanshu are in the ratio 5 : 7. If Himanshu weighs 28 kg, find the weight of Divya.

Solution:

Let the weight of Divya = x

Then the ratio of the weight of Divya and Himanshu will be x : 28 kg

According to the given statement

x : 28 :: 5 : 7

By cross product rule

Product of extremes = Product of means

x × 7 = (28 × 5) kg

x = 20 kg

Weight of Divya = 20 kg

Question 5.

Which ratio is smaller- 9 : 13 or 7 : 11 ?

Solution:

9 : 13 = $\frac{9}{11}$ and 7 : 11 = $\frac{7}{11}$

Converting them into equivalent like fraction

by taking L.C.M. of 13 and 11 = 143

$$\frac{9}{13} = \frac{9 \times 11}{13 \times 11} = \frac{99}{143} \text{ and}$$

$$\frac{7}{11} = \frac{7 \times 13}{11 \times 13} = \frac{91}{143}$$

As 91 < 99,

$$\frac{91}{143} < \frac{99}{143}$$

$$\Rightarrow \frac{7}{11} < \frac{9}{13}$$

Hence, $\frac{7}{11}$ is the smaller ratio.

∴ 7 : 11 is the smaller ratio.

Question 6.

Find the fourth proportional to

(i) 4, 7, 20

(ii) $2\frac{1}{2}, 1\frac{1}{4}, 2.2$

Solution:

(i) 4, 7, 20

Let the fourth proportional be x, then

4, 7, 20, x are in proportion

4 : 7 :: 20 : x

Using cross product rule

Product of extremes = Product of means

4 × x = 7 × 20

x = 35

(ii) $2\frac{1}{2}, 1\frac{1}{4}, 2.2$

$= \frac{5}{2}, \frac{5}{4}, 2.2$

Let the fourth proportional be x, then

$\frac{5}{2}, \frac{5}{4}, 2.2$, x are in proportion

$\frac{5}{2} : \frac{5}{4} :: 2.2 : x$

Using cross product rule

Product of extremes = Product of means

$$\frac{5}{2} \times x = \frac{5}{4} \times 2{\cdot}2$$

$$\frac{5}{2} \times x = \frac{11{\cdot}0}{4}$$

$$x = \frac{11{\cdot}0}{4} \div \frac{5}{2}$$

$$x = \frac{11{\cdot}0}{4} \times \frac{2}{5}$$

$$x = \frac{22}{20}$$

$$x = 1{\cdot}1$$

Question 7.

A typist types 70 pages in 3 hours 30 minutes. How long will she take to type 300 pages?

Solution:

A typist types 70 pages in 3 hours 30 minutes

$$3 \text{ hours } 30 \text{ minutes} = 3\frac{30}{60} \text{ hours}$$

$$= 3\frac{1}{2} = \frac{7}{2} \text{ hours}$$

Let she will type 300 pages = x hours

$$\therefore\ 70 : 300 = \frac{7}{2} : x \qquad (ad = bc)$$

$$70 \times x = \frac{300 \times 7}{2} \Rightarrow x = \frac{300 \times 7}{70 \times 2} = 15$$

$\therefore$ He will type in 15 hours.

Question 8.
12 looms weave 210 m cloth per day. How many metres of cloth will 8 looms weave per day?
Solution:

12 looms weave 210 m cloth per day

Let 8 looms weave per day = x m

12 : 8 = 210 : x

12 × x = 8 × 210 (ad = bc)

x = 140 m

They will weave 140 m of cloth.

Question 9.
A journey takes 4 hours 30 minutes at a speed of 60 km/h. How long will the same journey take at a speed of 15 m/sec?

Solution:

A journey takes 4 hours 30 minutes

$= 4\frac{1}{2} = \frac{9}{2}$ h at a speed of 60 km/h

If speed is 15 m/sec

Convert m/s into km/h

$$= 15 \times \frac{18}{5} = 54 \text{ km/h}$$

Let time taken = x h

$$\therefore 60 : 54 = \frac{9}{2} : x$$

Less speed, more time

$$\therefore 54 : 60 = \frac{9}{2} : x$$

$$\Rightarrow 54 \times x = 60 \times \frac{9}{2} \Rightarrow x = \frac{60 \times 9}{54 \times 2}$$

$$\Rightarrow x = 5$$

$\therefore$ **Time taken = 5 hours**