CHAPTER 09

Traverse Computations and Adjustments

9.1 Need for Traverse Adjustment

- In a closed traverse, there normally occurs the error of closure. This error of closure is given by:

$$e = \sqrt{e_x^2 + e_y^2} \qquad ...(9.1)$$

where $e_x = \Sigma D$ = Algebraic sum of departures

and $e_y = \Sigma L$ = Algebraic sum of latitudes

- If the error of closure is within the acceptable/permissible limits, the traverse can be adjusted. The main objective of traverse adjustment is to reduce the error of closure to zero. Thus the total error in traverse is distributed among the various sides of the traverse so that the traverse figure actually closes geometrically. In that case, the algebraic sum of latitudes and departures comes to zero. The sign of correction is opposite to that of error.

9.2 Methods of Traverse Adjustments

The following methods are in general used for adjusting a traverse with an accuracy of say up to 0.005%.

(a) Arbitrary Method (b) Bowditch's Rule (c) Transit Rule (d) Axis Method

9.2.1 Arbitrary Method

- In this method of traverse adjustment, the linear error of mis-closure is distributed arbitrarily depending on surveyor's judgment. The surveyor may decide to have large corrections in some of the sides of traverse as compared to other traverse sides.
- This judgment depends on the level of difficulty encountered while taking the observations.
- This method of traverse adjustment is quite simple but the personal judgment of surveyor varies from person to person. Moreover, the surveyor making the traverse adjustment in office might not be aware of the actual field situations.

9.2.2 Bowditch's Rule

- This method of traverse adjustment is suitable where linear and angular measurements are made with equal precision.
- This method is usually used for balancing a compass traverse but can be used for theodolite traverse also provided angular and linear measurements are done with same precision.
- This method assumes that the errors/mistakes are accidental in nature and the probable error in a traverse line is proportional the square root of its length.
- As per Bowditch's rule,

  Error in latitude or departure of a traverse line

$$= \text{Total error in latitude or departure of traverse} \times \frac{\text{Length of traverse line}}{\text{Perimeter of traverse}} \qquad ...(9.2)$$

- The required correction will be numerically equal to the value of error but its sign will be opposite to that of error.

NOTE : When a traverse is adjusted by Bowditch's rule, then both the lengths and bearings of the traverse get affected but here **lengths get changed less and angles get changed more.**

Advantages of Bowditch's Rule

(a) Bowditch's rule is quite easy to apply.

(b) The altered bearings of the traverse lines do not significantly affect the plotted position of the traverse points.

(c) This method is backed up by a logical mathematical reason and is not an empirical one.

9.2.3 Transit Rule

- This method of traverse adjustment is used in situations where angular measurements are made with more precision as compared to linear measurements.
- In theodolite traverse, angular measurements are more precise as compared to linear measurements and thus this method is quite suitable for theodolite traverse.
- As per Transit rule,

Error in latitude or departure of a traverse line

$$= \text{Total error in latitude or departure of traverse} \times \frac{\text{Numerical value of latitude or departure of traverse line}}{\text{Arithematic sum of latitudes or departures of traverse}} \qquad ...(9.3)$$

NOTE : In transit rule, **angles are changed less but the lengths are changed more.**

9.2.4 Axis Method

- This method is used to balance a traverse where angles are measured more precisely than the lengths and thus this axis method is used for correction of lengths only.
- Since the angles are measured more precisely and therefore the direction of lines do not change much while applying the correction and thus the general shape of the traverse do not change much.
- In this method, the correction applied to any traverse line is expressed as

$$\frac{\text{One half of closing error} \times \text{Length of traverse line}}{\text{Axis length}}$$

9.3 Gale's Traverse Table

All the traverse computations are done in a tabular form which is referred to as **Gale's Traverse Table.** A typical Gale's traverse table is shown in **Fig. 9.1.**

9.4 Inversing

- In Gale's traverse table the independent co-ordinates of the traverse points are determined from the lengths and bearings of the traverse lines.
- But inversing is just opposite of that. Here, from the given independent co-ordinates, the lengths and bearings of traverse lines are determined.

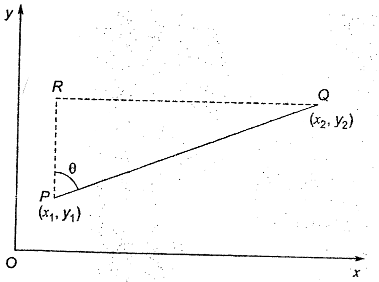


Fig. 9.2 *Inversing*

As shown in **Fig. 9.2**, independent co-ordinates of point P is (x_1, y_1) and that of point Q is (x_2, y_2). The length of line PQ is,

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad ...(9.4)$$

The angle θ as shown is given by,

$$\tan\theta = \frac{QR}{PR} = \frac{(x_2 - x_1)}{(y_2 - y_1)} \qquad ...(9.5)$$

It is always helpful to draw a rough sketch in order to have an idea of the quadrant in which a point lies.

9.5 Omitted Measurements

- Ideally all the lengths of the sides and all the angles of a closed traverse should be measured in the field so as to have proper checks and the traverse can be adjusted.
- However this is not always possible or at most not economically feasible to measure all the sides and angles of the traverse.
- Thus some measurements are omitted which are called as **omitted measurements.**

Station	Line	Length	Interior angle	Corrections	Corrected angles	WCB	Reduced bearing	Quadrant	Consecutive co-ordinate				Corrections				Corrected consecutive co-ordinate				Independent co-ordinate	
									Latitude		Departure		Latitude		Departure		Latitude		Departure			
			(+)						N (+)	S (–)	E (+)	W (–)	N (+)	S (–)	E (+)	W (–)	N (+)	S (–)	E (+)	W (–)	N	E
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)

Fig. 9.1 Gale's Traverse Table

- The other wise missing quantities (i.e. bearings and lengths) can be determined from other measurements.

  For a closed traverse, the algebraic sum of all the latitudes and departures must be zero i.e.

$$\Sigma L = 0$$

$$l_1 \cos\theta_1 + l_2 \cos\theta_2 + ... = 0 \qquad ...(9.6)$$

and

$$\Sigma D = 0$$

$$l_1 \sin\theta_1 + l_2 \sin\theta_2 + ... = 0 \qquad ...(9.7)$$

where $l_1, l_2 ...$ are the lengths of the sides of the traverse and $\theta_1, \theta_2 ...$ are the bearings of the corresponding sides of the traverse.

- If the missing quantities are not more than two then these can be determined from the above equations provided there is no error in the measurement of quantities. But since errors are always present and thus the assumption of zero error is not justified.
- In case of omitted measurements there is no check on the field work. That is to say that the traverse cannot be balanced. Indirectly the errors in the measurements of various quantities are distributed among the computed values of the omitted measurements.
- Depending on the missing quantities, there are a total of five cases which can occur. In general, trigonometric solutions are preferred.

Case-I Bearing or length or both of one side of traverse missing

Let there be a traverse whose length or bearing or the both i.e. the length and bearing are missing (say of side DE).

Let $\Sigma L'$ = Algebraic sum of latitudes of sides AB, BC, CD and EA.

L_{DE} = Latitude of side DE

Thus, $\Sigma L' + L_{DE} = 0$

$$L_{DE} = -\Sigma L' \qquad ...(9.8)$$

Similarly let $\Sigma D'$ = Algebraic sum of departures of sides AB, BC, CD and EA.

D_{DE} = Departure of side DE

Then, $\Sigma D' + D_{DE} = 0$

$$D_{DE} = -\Sigma D' \qquad ...(9.9)$$

But $L_{DE} = l \cos\theta$

$D_{DE} = l \sin\theta$

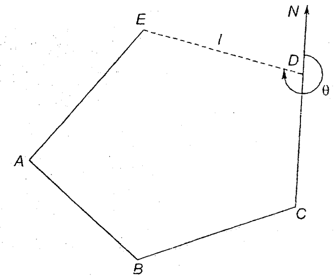
where l = Length of side DE

θ = bearing of side DE

Thus,

$$l = \sqrt{L_{DE}^2 + D_{DE}^2} \qquad ...(9.10)$$

$$\tan\theta = \frac{D_{DE}}{L_{DE}} \qquad ...(9.11)$$



l or θ or both unknown

Fig. 9.3 *Case-I of omitted measurements*

The signs of the latitude and departure of side DE will decide the quadrant in which the side DE will lie.

Case-II Length of one side and bearing of adjacent side missing

Let bearing of one side say AB and length of adjacent side EA is missing.

Join points B and E so that $BCDE$ becomes a closed traverse with lengths and bearings of all the sides known except that of side BE. The length and bearing of side BE can be determined just like in **Case-I** above.

Thus the included $\angle AEB$ can be determined from the bearings of line EA and EB.

In triangle ABE, the lengths AB and BE and the $\angle AEB$ are now become known. From sine law,

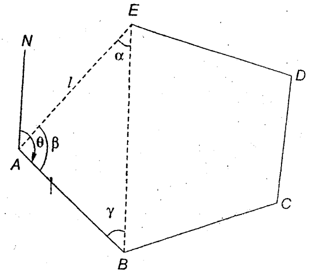


Fig. 9.4 *Case-II of omitted measurement*

$$\frac{\sin\angle AEB}{AB} = \frac{\sin\angle EAB}{BE} = \frac{\sin\angle ABE}{AE}$$

$$\frac{\sin\alpha}{AB} = \frac{\sin\beta}{BE} = \frac{\sin\gamma}{AE}$$

$$\sin\beta = \left(\frac{BE}{AB}\right)\sin\alpha \qquad ...(9.12)$$

This accuracy is quite poor when angle β is close to 90° since the value of sine near 90° changes very slowly and a small error in the calculated values may cause a large error in the angle β.

Knowing the bearing of line AE and the angle β, the bearing of line AB i.e., θ can be determined.

Now
$$\gamma = 180° - (\alpha + \beta)$$

Thus
$$AE = AB \times \frac{\sin\gamma}{\sin\alpha} \qquad ...(9.13)$$

Case-III Lengths of two adjacent sides are omitted

Join points B and E and determine the length and bearing of line EB from the so formed closed traverse $BCDE$ which is same as **Case-I**.

In triangle ABE, the bearings of all the three sides are known and thus $\angle ABE$, $\angle AEB$ and $\angle BAE$ can be found out. Now the length of side BE is also known and from this the lengths of sides AB and AE can be determined as:

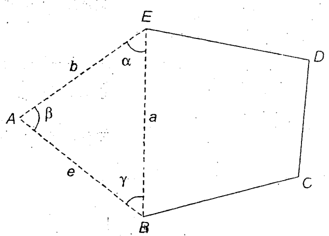


Fig. 9.5 *Case-III of omitted measurements*

From sine law in triangle ABE

$$\frac{\sin\angle BEA}{AB} = \frac{\sin\angle BAE}{BE} = \frac{\sin\angle ABE}{AE}$$

$$\frac{\sin\alpha}{AB} = \frac{\sin\beta}{BE} = \frac{\sin\gamma}{AE}$$

Thus
$$AB = BE \times \frac{\sin\alpha}{\sin\beta} \qquad ...(9.14)$$

and
$$AE = BE \times \frac{\sin\gamma}{\sin\beta} \qquad ...(9.15)$$

Case-IV Bearings of two adjacent sides are omitted

Let bearings of sides AB and EA are unknown.

As in earlier case, join points B and E. Determine the length and bearing of side BE from the so formed closed traverse $BCDE$ which is similar to **Case-I** above.

In triangle ABE, the lengths of sides AB, BE and AE are known. From Hero's formula for area of triangle,

$$A = \sqrt{s(s-a)(s-b)(s-e)} \qquad ...(9.16)$$

where
a = Length of side BE
b = Length of side AE
e = Length of side AB
$s = (a + b + e)/2$
= Semi-perimeter of triangle ABE ...(9.17)

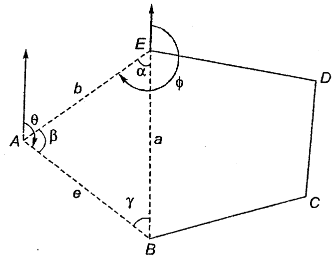


Fig. 9.6 *Case-IV of omitted measurement*

Also the area of triangle ABE is,

$$\left.\begin{aligned} A &= (1/2)\, eb\sin\beta \\ A &= (1/2)\, ea\sin\gamma \\ A &= (1/2)\, ab\sin\alpha \end{aligned}\right\} \qquad ...(9.18)$$

From the known bearing of side BE and angles α, β and γ, the bearings of sides AE and AB can be arrived at.

Case-V When the two affected sides are not adjacent

Let the two affected sides CD and FA are not adjacent to each other.

In order to bring the two affected sides adjacent to each other, a geometrical construction is needed. The underlying fact is that a line can be moved from one location to another location parallel to the first location without any change in its latitude and departure.

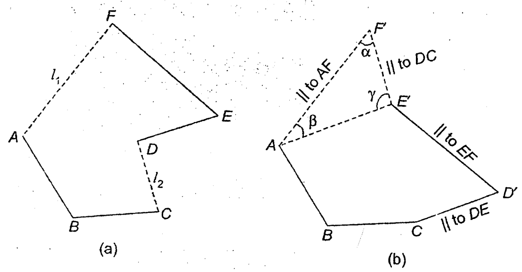


Fig. 9.7 *Case-V of omitted measurements*

The following three cases arise in this:

Case-V (a) Lengths of both sides $E'F'$ and $F'A$ are omitted

In triangle $AE'F'$, the bearings of all the three sides are known and thus the included angles α, β and γ are known.

From sine law,
$$\frac{\sin\alpha}{AE'} = \frac{\sin\beta}{E'F'} = \frac{\sin\gamma}{AF'}$$

Thus,
$$AF' = AE'\left(\frac{\sin\gamma}{\sin\alpha}\right) \qquad ...(9.19)$$

and
$$E'F' = AE'\left(\frac{\sin\beta}{\sin\alpha}\right) \qquad ...(9.20)$$

Thus the lengths of sides $E'F'$ and AF' are respectively equal to sides CD and AF.

Case-V (b) Length of one side $E'F'$ and bearing of line AF' are omitted

In triangle $AE'F'$, the lengths of the two sides AE' and AF' are known. From the bearings of sides AE' and $F'E'$, angle γ can be determined.

From sine law,

$$\frac{\sin\alpha}{AE'} = \frac{\sin\beta}{E'F'} = \frac{\sin\gamma}{AF'}$$

$$\sin\alpha = \sin\gamma\left(\frac{AE'}{AF'}\right)$$

Now $$\beta = 180° - (\alpha + \gamma)$$

Thus, $$E'F' = AF'\left(\frac{\sin\beta}{\sin\gamma}\right) \quad ...(9.21)$$

From the angles α and β, the bearing of line AF' can be arrived at.

Case-V(c) Length of one side AF' and bearing of one side $E'F'$ are omitted

This is very similar to **Case-V(b)** above. Here the angle β and lengths AE' and $E'F'$ are known.

From sine law,

$$\frac{\sin\alpha}{AE'} = \frac{\sin\beta}{E'F'} = \frac{\sin\gamma}{AF'}$$

$$\sin\alpha = \sin\beta\left(\frac{AE'}{E'F'}\right)$$

$$\gamma = 180° - (\alpha + \beta)$$

Thus, $$AF' = E'F'\left(\frac{\sin\gamma}{\sin\beta}\right) \quad ...(9.22)$$

After knowing the value of angle α the bearing of line $E'F'$ can be determined. It is to be noted that the sides $E'F'$ and AF' represent the affected sides CD and AF respectively.

Illustrative Examples

Example 9.1 In a traverse, the latitudes and departures were observed to be:

Σ latitude = 1.45 m

Σ departure = − 2.16 m

What are the length and bearing of closing error?

Solution:

$$\text{Closing error} = \sqrt{(\Sigma\text{latitude})^2 + (\Sigma\text{departure})^2}$$

$$= \sqrt{(1.45)^2 + (-2.16)^2} = 2.602 \text{ m}$$

$$\text{Bearing of closing error} = \tan^{-1}\frac{2.16}{1.45} = 56.13°$$

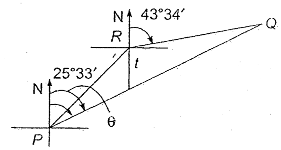
Closing error lies in fourth gradient. Since latitude is positive and departure is negative.

$\therefore$ Bearing of closing error = $N 56.13° W (QB) = 303.87° (WCB)$

Example 9.2 Co-ordinates of two points P and Q are given below. A third point R is so selected that bearings of PR and RQ are 25° 33′ and 43°34′ respectively. Find the lengths of PR and RQ.

Point	Northing	Easting
P	135 m	150 m
Q	1200 m	1500 m

Solution:



Latitude of line PQ = Latitude of Q − latitude of P

= 1200 − 135 = 1065 m

Departure of line PQ = Departure of Q − departure of P

= 1500 − 150 = 1350 m

$$\therefore \quad \tan\theta = \frac{\text{Departure of line } PQ}{\text{Latitude of line } PQ} = \frac{1350}{1065}$$

$\therefore \quad \theta = 51.73°$

$\therefore$ Bearing of line PQ = N 51.73° E (QB)

= 51.73° (WCB)

$$\text{Length of line } PQ = \sqrt{(1065)^2 + (1350)^2} = 1719.51 \text{ m}$$

Now in the closed traverse PQR,

$$\Sigma_{\text{Latitude}} = 0$$

$$\Rightarrow \quad L_{PR}\cos 25°33' + L_{RQ}\cos 45°34' - L_{PQ}\cos 51.73° = 0$$

$$\Rightarrow \quad 0.9022\,L_{PR} + 0.7\,L_{RQ} - 1065 = 0 \quad ...(i)$$

$$\Sigma_{\text{Departure}} = 0$$

$$\Rightarrow \quad L_{PR}\sin 25°33' + L_{RQ}\sin 45°34' - L_{PQ}\sin 51.73° = 0$$

$$\Rightarrow \quad 0.4313\,L_{PR} + 0.714\,L_{RQ} - 1350 = 0 \quad ...(ii)$$

Solving (i) and (ii)

L_{PR} = 539.32 m (Ignoring −ve sign)

L_{RQ} = 2216.5 m

Example 9.3 For theodolite traverse $PQRS$, the consecutive coordinates are as shown in table below:

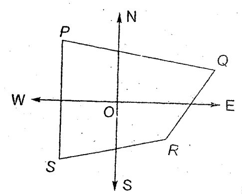
Station	N	S	E	W
P	305.50	—	—	210.35
Q	200.75	—	280.45	—
R	—	280.80	175.65	—
S	—	302.50	—	305.86

Determine the magnitude and direction of closing error. If independent coordinates of station P are (105, 110) then determine the corrected independent coordinates of station Q.

Solution:

The latitudes and departures of traverse lines are as given in table below:

Line	*Latitude*	*Departure*
PQ	200.75	280.45
QR	−280.80	175.65
RS	−302.50	−305.86
SP	305.5	−210.35
	$\Sigma L = -77.05$	$\Sigma D = -60.11$



∴ Error in x-direction $(e_x) = \Sigma D = -60.11$

Error in y-direction $(e_y) = \Sigma L = -77.05$

∴ Closing error $(e) = \sqrt{e_x^2 + e_y^2} = \sqrt{(-60.11)^2 + (-77.05)^2} = 97.72$

$$\tan\theta = \frac{\Sigma D}{\Sigma L} = \frac{-60.11}{-77.05} = 0.7801$$

⇒ $\theta = 37.96°$

This angle is in the III quadrant.

∴ WCB of closing error = $180° + \theta = 180° + 37.96° = 217.96°$

∴ Correction in latitude of $PQ = \dfrac{-(-77.05)200.75}{(200.75 + 280.80 + 302.50 + 305.5)} = 14.2$

Correction in departure of $PQ = -\dfrac{(-60.11)\times 280.45}{(280.45 + 175.65 + 305.86 + 210.35)} = 17.338$

∴ Correct latitude of $PQ = 200.75 + 14.2 = 214.95$

Correct departure of $PQ = 280.45 + 17.338 = 297.788$

Independent coordinates of *P* are (105, 110)

∴ Independent coordinates of *Q* are {(105 + 297.788), (110 + 214.95)} = (402.788, 324.95)

Example 9.4 The lengths and bearings of all sides except one are given in table below for a traverse *PQRST*. Find the length and bearing of missing side.

Side	Length (m)	WCB
PQ	89.31	45°10′
QR	220.76	73°35′
RS	150.28	159°40′
ST	162.20	229°37′
TP	—	—

Solution:

Let length and bearing of *TP* be x and θ respectively.

Side	Length (m)	WCB	Latitude (m)	Departure (m)
PQ	89.31	45°10′	62.97	63.34
QR	220.76	73°35′	62.39	211.76
RS	150.28	159°40′	−140.92	52.22
ST	162.20	229°37′	−105.09	−123.55
TP	x	θ	$x\cos\theta$	$x\sin\theta$
			$\Sigma L = (x\cos\theta - 120.65)$	$\Sigma D = (x\sin\theta + 203.77)$

For a closed traverse, $\Sigma L = 0$ and $\Sigma D = 0$

∴ $x\cos\theta = 120.65$

and $x\sin\theta = -203.77$

After squaring and adding

$x^2 = 56078.64$

⇒ $x = 236.81$ m

Also $\tan\theta = -\dfrac{203.77}{120.65} = -1.68893$

∴ $\theta = 300.63°$

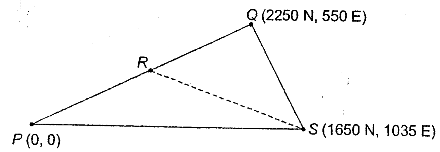
(∵ Departure is positive and latitude is negative and thus side lies is fourth quadrant).

Example 9.5 A sewer is to be laid between two points *P* and *Q* with coordinate as follows:

Point	Northing	Easting
P	0	0
Q	2250 m	550 m

A man hole is to be constructed at mid-point *R* of *P* and *Q* but it is at present not accessible. So another point *S* was selected with coordinates 1650 m northing and 1035 m easting. Compute the co-ordinates of point *R*, length and bearing of line *RS*.

Solution:



R is the mid-point of *PQ*

∴ Co-ordinate of $R = \left(\dfrac{0+2250}{2}\ \text{N},\ \dfrac{0+550}{2}\ \text{E}\right) = (1125\ \text{N},\ 275\ \text{E})$

Now, Northing of S = Northing of R + $L_{RS}\cos\theta$ where θ = WCB of RS

⇒ $1650 = 1125 + L_{RS}\cos\theta$

⇒ $L_{RS}\cos\theta = 525$ ...(i)

Similarly Easting of S = Easting of R + $L_{RS} \sin\theta$

$\Rightarrow$ $1035 = 275 + L_{RS} \sin\theta$

$\Rightarrow$ $L_{RS} \sin\theta = 760$ ...(ii)

From eq. (i) and (ii)

$$\tan\theta = \frac{760}{525} = 1.42857$$

$\therefore$ $\theta = 55°$ (θ lies in I quadrant)

Also, $L_{RS} = \sqrt{525^2 + 760^2} = 923.70179$ m

Now, $\theta = 55°$

$\Rightarrow$ WCB of RS = 55°

Objective Brain Teasers

Q.1 In triangulation, the log sine correction is made to meet the:
(a) Side condition
(b) Apex condition
(c) Opposite angles
(d) All of the above

Q.2 In order to balance a traverse in which angular measurements are more precise than the linear measurements, the most suitable method is:
(a) Transit rule (b) Axis correction
(c) Bowditch's rule (d) Arbitrary method

Q.3 Omitted measurements can be computed if they do not exceed ______ in number.
(a) Three (b) Two
(c) Six (d) One

Q.4 The sum of interior angles of a closed traverse of sides n is:
(a) $(n+2)180°$ (b) $(n+3)90°$
(c) $(n-1)90°$ (d) $(n-2)180°$

Q.5 Which of the following statement is correct for balancing a traverse by Bowditch's rule?
(a) Error in angular measurement in inversely proportional to $\sqrt{l}$
(b) Error in linear measurement is inversely proportional to $\sqrt{l}$
(c) Error in linear measurement is proportional to l
(d) Error in angular measurement in proportional to l

Q.6 The latitude and departure of a line are 80 m are −60 m respectively. The WCB of the line is ____.
(a) 330° 33′ (b) 243° 45′
(c) 323° 7.8′ (d) 263° 8.7′

Q.7 Which of the following statement(s) is (are) correct?
1. In Bowditch's rule, lengths get change less and angles get change more.
2. In Transit rule, angles get change less but lengths get change more.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Q.8 Computations of independent co-ordintes in Gale's traverse table and method of inversing ______.
(a) are opposite
(b) are same
(c) may be same or different
(d) data insufficient

Answers

1. (a) 2. (a) 3. (b) 4. (d) 5. (a) 6. (c) 7. (c) 8. (a)

Student's Assignments

Ex.1 The bearings of two stations P and Q take from a station R were 250° 00′ and 153° 2 respectively. The co-ordinates of station P a Q are (300 easting, 200 northing) and (4 easting, 150 northing) respectively. Compute the independent co-ordinates of station R.

Ans. (363.5, 223.1)

Ex.2 Two points A and B has following co-ordinates:

Point	N	E
A	788.35 m	630.45 m
B	1306.37 m	418.41 m

Determine the length and bearing of line AB.

Ans. 559.74 m, 337.74°

Ex.3 Compute the missing data for the traverse $ABCD$.

Line	Length (m)	WC Bearing
AB	100	314° 30′
BC	605	05° 30′
CD	95	88° 20′
DA	—	—

Ans. 679.97 m, 186.89°

■■■■