Session 2

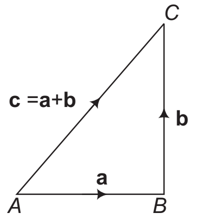
Addition & Subtraction of Vectors, Multiplication of Vector by Scalar, Section Formula

Addition of Vectors

(Resultant of Vectors)

1. Triangle Law of Addition

If two vectors are represented by two consecutive sides of a triangle, then their sum is represented by the third side of the triangle, but in opposite direction. This is known as the triangle law of addition of vectors. Thus, if $\mathbf{AB} = \mathbf{a}$, $\mathbf{BC} = \mathbf{b}$ and $\mathbf{AC} = \mathbf{c}$, then $\mathbf{AB} + \mathbf{BC} = \mathbf{AC}$ i.e. $\mathbf{a} + \mathbf{b} = \mathbf{c}$.

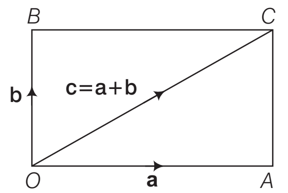


2. Parallelogram Law of Addition

If two vectors are represented by two adjacent sides of a parallelogram, then their sum is represented by the diagonal of the parallelogram whose initial point is the same as the initial point of the given vectors. This is known as parallelogram law of vector addition.

Thus, if $\mathbf{OA} = \mathbf{a}, \mathbf{OB} = \mathbf{b}$ and $\mathbf{OC} = \mathbf{c}$

Then, $\mathbf{OA} + \mathbf{OB} = \mathbf{OC}$ i.e. $\mathbf{a} + \mathbf{b} = \mathbf{c}$, where OC is a diagonal of the parallelogram $OACB$.

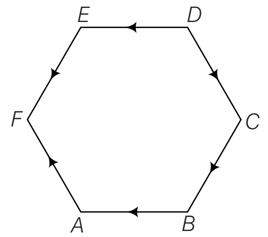


Remarks

1. The magnitude of $\mathbf{a} + \mathbf{b}$ is not equal to the sum of the magnitudes of $\mathbf{a}$ and $\mathbf{b}$.
2. From the figure, we have $\mathbf{OA} + \mathbf{AC} = \mathbf{OC}$ (By triangle law of vector addition)
   or $\mathbf{OA} + \mathbf{OB} = \mathbf{OC}$ ($\because \mathbf{AC} = \mathbf{OB}$), which is the parallelogram law. Thus, we may say that the two laws of vector addition are equivalent to each other.

3. Polygon law of addition

If the number of vectors are represented by the sides of a polygon taken in order, the resultant is represented by the closing side of the polygon taken in the reverse order.



In the figure, $\mathbf{AB} + \mathbf{BC} + \mathbf{CD} + \mathbf{DE} + \mathbf{EF} = \mathbf{AF}$

4. Addition in Component Form

If the vectors are defined in terms of $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$, i.e. if $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$ and $\mathbf{b} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$, then their sum is defined as

$\mathbf{a} + \mathbf{b} = (a_1 + b_1)\hat{\mathbf{i}} + (a_2 + b_2)\hat{\mathbf{j}} + (a_3 + b_3)\hat{\mathbf{k}}$.

Properties of Vector Addition

Vector addition has the following properties

(i) **Closure** The sum of two vectors is always a vector.

(ii) **Commutativity** For any two vectors $\mathbf{a}$ and $\mathbf{b}$,

$$\Rightarrow \quad \mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$$

(iii) **Associativity** For any three vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$,

$$\Rightarrow \quad \mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$$

(iv) **Identity** Zero vector is the identity for addition. For any vector $\mathbf{a}$.

$$\Rightarrow \quad \mathbf{0} + \mathbf{a} = \mathbf{a} = \mathbf{a} + \mathbf{0}$$

(v) **Additive inverse** For every vector $\mathbf{a}$ its negative vector $-\mathbf{a}$ exists such that $\mathbf{a} + (-\mathbf{a}) = (-\mathbf{a}) + \mathbf{a} = \mathbf{0}$ i.e. $(-\mathbf{a})$ is the additive inverse of the vector $\mathbf{a}$.

Example 11. ***Find the unit vector parallel to the resultant vector of*** $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ ***and*** $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$**.**

Sol. Resultant vector, $\mathbf{r} = (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}) + (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$$= 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

Unit vector parallel to $\mathbf{r} = \dfrac{1}{|\mathbf{r}|}\mathbf{r}$

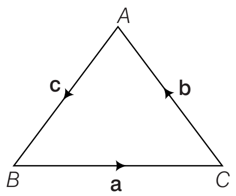
$$= \frac{1}{\sqrt{3^2 + 6^2 + (-2)^2}}(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$

$$= \frac{1}{7}(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$

Example 12. ***If*** $\mathbf{a}, \mathbf{b}$ ***and*** $\mathbf{c}$ ***are the vectors represented by the sides of a triangle, taken in order, then prove that*** $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$.

Sol. Let ABC be a triangle such that

$$\mathbf{BC} = \mathbf{a},\ \mathbf{CA} = \mathbf{b} \text{ and } \mathbf{AB} = \mathbf{c}$$



Then, $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{BC} + \mathbf{CA} + \mathbf{AB}$

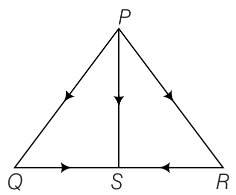
$= \mathbf{BA} + \mathbf{AB}$ $\quad (\because \mathbf{BC} + \mathbf{CA} = \mathbf{BA})$

$= -\mathbf{AB} + \mathbf{AB}$

$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$ **Hence proved**.

Example 13. ***If*** S ***is the mid-point of side*** QR ***of a*** ΔPQR***, then prove that*** $\mathbf{PQ} + \mathbf{PR} = 2\mathbf{PS}$.

Sol. Clearly, by triangle law of addition, we have



$$\mathbf{PQ} + \mathbf{QS} = \mathbf{PS} \quad \text{...(i)}$$

and $$\mathbf{PR} + \mathbf{RS} = \mathbf{PS} \quad \text{... (ii)}$$

On adding Eqs. (i) and (ii), we get

$$(\mathbf{PQ} + \mathbf{QS}) + (\mathbf{PR} + \mathbf{RS}) = 2\mathbf{PS}$$

$$\Rightarrow \quad (\mathbf{PQ} + \mathbf{PR}) + (\mathbf{QS} + \mathbf{RS}) = 2\mathbf{PS}$$

$$\Rightarrow \quad \mathbf{PQ} + \mathbf{PR} + 0 = 2\mathbf{PS}$$

$[\because S$ is the mid-point of $\mathbf{QR}$ $\therefore$ $\mathbf{QS} = -\mathbf{RS}]$

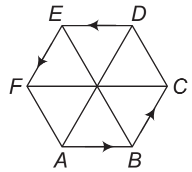
Hence, $\mathbf{PQ} + \mathbf{PR} = 2\mathbf{PS}$ **Hence proved**.

Example 14. ***If*** $ABCDEF$ ***is a regular hexagon, prove that*** $\mathbf{AD} + \mathbf{EB} + \mathbf{FC} = 4\mathbf{AB}$.

Sol. We have,

$\mathbf{AD} + \mathbf{EB} + \mathbf{FC} = (\mathbf{AB} + \mathbf{BC} + \mathbf{CD}) + (\mathbf{ED} + \mathbf{DC} + \mathbf{CB}) + \mathbf{FC}$

$= \mathbf{AB} + (\mathbf{BC} + \mathbf{CB}) + (\mathbf{CD} + \mathbf{DC}) + \mathbf{ED} + \mathbf{FC}$



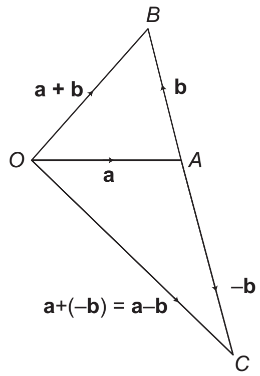
$= \mathbf{AB} + \mathbf{O} + \mathbf{O} + \mathbf{AB} + 2\mathbf{AB} = 4\mathbf{AB}$

$(\because \mathbf{ED} = \mathbf{AB}, \mathbf{FC} = 2\mathbf{AB})$

Hence proved.

Subtraction of Vectors

If $\mathbf{a}$ and $\mathbf{b}$ are two vectors, then their subtraction $\mathbf{a} - \mathbf{b}$ is defined as $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$, where $-\mathbf{b}$ is the negative of $\mathbf{b}$ having magnitude equal to that of $\mathbf{b}$ and direction opposite to $\mathbf{b}$.



If $\quad \mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

and $\quad \mathbf{b} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$

Then, $\quad \mathbf{a} - \mathbf{b} = (a_1 - b_1)\hat{\mathbf{i}} + (a_2 - b_2)\hat{\mathbf{j}} + (a_3 - b_3)\hat{\mathbf{k}}$

Properties of Vector Subtraction

(i) $\mathbf{a} - \mathbf{b} \neq \mathbf{b} - \mathbf{a}$

(ii) $(\mathbf{a} - \mathbf{b}) - \mathbf{c} \neq \mathbf{a} - (\mathbf{b} - \mathbf{c})$

(iii) Since, any one side of a triangle is less than the sum and greater than the difference of the other two sides, so for any two vectors a and b, we have

(a) $|\mathbf{a} + \mathbf{b}| \le |\mathbf{a}| + |\mathbf{b}|$ (b) $|\mathbf{a} + \mathbf{b}| \ge |\mathbf{a}| - |\mathbf{b}|$

(c) $|\mathbf{a} - \mathbf{b}| \le |\mathbf{a}| + |\mathbf{b}|$ (d) $|\mathbf{a} - \mathbf{b}| \ge |\mathbf{a}| - |\mathbf{b}|$

Remark

If A and B are two points in space having coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2), then

$\mathbf{AB}$ = Position Vector of B – Position Vector of A

$= (x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}}) - (x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}})$

$= (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\hat{\mathbf{k}}$

Example 15. ***If*** $A=(0,1), B=(1,0)$, $C=(1,2)$ ***and*** $D=(2,1)$, ***prove that vector*** $\mathbf{AB}$ ***and*** $\mathbf{CD}$ ***are equal.***

Sol. Here, $\mathbf{AB}=(1-0)\hat{\mathbf{i}}+(0-1)\hat{\mathbf{j}}=\hat{\mathbf{i}}-\hat{\mathbf{j}}$

and $\mathbf{CD}=(2-1)\hat{\mathbf{i}}+(1-2)\hat{\mathbf{j}}=\hat{\mathbf{i}}-\hat{\mathbf{j}}$

Clearly, $\mathbf{AB}=\mathbf{CD}$ **Hence proved.**

Example 16. ***If the position vectors of*** A ***and*** B ***respectively*** $\hat{\mathbf{i}}+3\hat{\mathbf{j}}-7\hat{\mathbf{k}}$ ***and*** $5\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$, ***then find*** $\mathbf{AB}$.

Sol. Let O be the origin, then we have

$$\mathbf{OA}=\hat{\mathbf{i}}+3\hat{\mathbf{j}}-7\hat{\mathbf{k}}$$

and $$\mathbf{OB}=5\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$$

Now, $$\mathbf{AB}=\mathbf{OB}-\mathbf{OA}=(5\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}})-(\hat{\mathbf{i}}+3\hat{\mathbf{j}}-7\hat{\mathbf{k}})$$
$$=4\hat{\mathbf{i}}-5\hat{\mathbf{j}}+11\hat{\mathbf{k}}$$

Example 17. ***Vectors drawn from the origin*** O ***to the points*** A, B ***and*** C ***are respectively*** $\mathbf{a}, \mathbf{b}$ ***and*** $4\mathbf{a}-3\mathbf{b}$. ***Find*** $\mathbf{AC}$ ***and*** $\mathbf{BC}$.

Sol. We have, $\mathbf{OA}=\mathbf{a}$, $\mathbf{OB}=\mathbf{b}$ and $\mathbf{OC}=4\mathbf{a}-3\mathbf{b}$

Clearly, $$\mathbf{AC}=\mathbf{OC}-\mathbf{OA}=(4\mathbf{a}-3\mathbf{b})-(\mathbf{a})$$
$$=3\mathbf{a}-3\mathbf{b}$$

and $$\mathbf{BC}=\mathbf{OC}-\mathbf{OB}=(4\mathbf{a}-3\mathbf{b})-(\mathbf{b})=4\mathbf{a}-4\mathbf{b}$$

Example 18. ***Find the direction cosines of the vector joining the points*** $A(1,2,-3)$ ***and*** $B(-1,-2,1)$, ***directed from*** A ***to*** B.

Sol. Clearly,

$$\mathbf{AB}=(-1-1)\hat{\mathbf{i}}+(-2-2)\hat{\mathbf{j}}+(1+3)\hat{\mathbf{k}}=-2\hat{\mathbf{i}}-4\hat{\mathbf{j}}+4\hat{\mathbf{k}}$$

Now, $$|\mathbf{AB}|=\sqrt{(-2)^2+(-4)^2+(4)^2}=\sqrt{36}=6$$

$\therefore$ Unit vector along $$\mathbf{AB}=\frac{\mathbf{AB}}{|\mathbf{AB}|}=\frac{-2\hat{\mathbf{i}}-4\hat{\mathbf{j}}+4\hat{\mathbf{k}}}{6}$$
$$=-\frac{1}{3}\hat{\mathbf{i}}-\frac{2}{3}\hat{\mathbf{j}}+\frac{2}{3}\hat{\mathbf{k}}$$

Example 19. ***Let*** α, β ***and*** γ ***be distinct real numbers. The points with position vectors*** $\alpha\hat{\mathbf{i}}+\beta\hat{\mathbf{j}}+\gamma\hat{\mathbf{k}}$, $\beta\hat{\mathbf{i}}+\gamma\hat{\mathbf{j}}+\alpha\hat{\mathbf{k}}$ and $\gamma\hat{\mathbf{i}}+\alpha\hat{\mathbf{j}}+\beta\hat{\mathbf{k}}$

(a) are collinear

(b) form an equilateral triangle

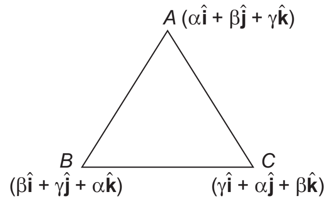
(c) form a scalene triangle

(d) form a right angled triangle

Sol. (b) Let the given points be A, B and C with position vectors $\alpha\hat{\mathbf{i}}+\beta\hat{\mathbf{j}}+\gamma\hat{\mathbf{k}}$, $\beta\hat{\mathbf{i}}+\gamma\hat{\mathbf{j}}+\alpha\hat{\mathbf{k}}$ and $\gamma\hat{\mathbf{i}}+\alpha\hat{\mathbf{j}}+\beta\hat{\mathbf{k}}$.

As, α, β and γ are distinct real numbers, therefore ABC form a triangle.

Clearly, $$\mathbf{AB}=\mathbf{OB}-\mathbf{OA}=(\beta\hat{\mathbf{i}}+\gamma\hat{\mathbf{j}}+\alpha\hat{\mathbf{k}})-(\alpha\hat{\mathbf{i}}+\beta\hat{\mathbf{j}}+\gamma\hat{\mathbf{k}})$$
$$=(\beta-\alpha)\hat{\mathbf{i}}+(\gamma-\beta)\hat{\mathbf{j}}+(\alpha-\gamma)\hat{\mathbf{k}}$$



Now, $$|\mathbf{AB}|=\sqrt{(\beta-\alpha)^2+(\gamma-\beta)^2+(\alpha-\gamma)^2}$$

Similarly, $$\mathbf{BC}=\mathbf{CA}=\sqrt{(\beta-\alpha)^2+(\gamma-\beta)^2+(\alpha-\gamma)^2}$$

$\therefore \Delta ABC$ is an equilateral triangle.

Example 20. ***If the position vectors of the vertices of a triangle be*** $2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}$, $4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\hat{\mathbf{k}}$ ***and*** $3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-3\hat{\mathbf{k}}$, ***then the triangle is***

(a) right angled (b) isosceles

(c) equilateral (d) None of these

Sol. (a, b) Let A, B, C be the vertices of given triangle with position vectors, $2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}$, $4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ respectively.

Then, we have

$$\mathbf{OA}=2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}},\ \mathbf{OB}=4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\hat{\mathbf{k}}$$

and $$\mathbf{OC}=3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-3\hat{\mathbf{k}}$$

Clearly, $$\mathbf{AB}=\mathbf{OB}-\mathbf{OA}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$$
$$\mathbf{BC}=-\hat{\mathbf{i}}+\hat{\mathbf{j}}-4\hat{\mathbf{k}}$$

and $$\mathbf{AC}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-2\hat{\mathbf{k}}$$

Now, $$AB=|\mathbf{AB}|=\sqrt{2^2+1^2+2^2}=3$$
$$BC=|\mathbf{BC}|=\sqrt{(-1)^2+(1)^2+(-4)^2}=3\sqrt{2}$$

and $$AC=|\mathbf{AC}|=\sqrt{1^2+2^2+(-2)^2}=3$$

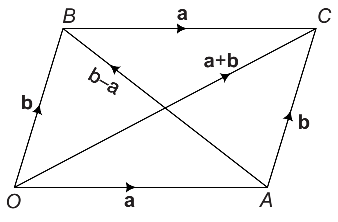
$\because$ $AB=AC$ and $BC^2=AB^2+AC^2$

$\therefore$ The triangle is isosceles and right angled.

Example 21. ***The two adjacent sides of a parallelogram are*** $2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ ***and*** $\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$. ***Find the unit vectors along the diagonals of the parallelogram.***

Sol. Let $OABC$ be the given parallelogram and let the adjacent sides OA and OB be represented by $a=2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ and $b=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ respectively.

Now, the vectors along the two diagonals are



$$\mathbf{d}_1=\mathbf{a}+\mathbf{b}=3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-2\hat{\mathbf{k}}$$

The required unit vectors are

$$\hat{\mathbf{n}}_1 = \frac{\mathbf{d}_1}{|\mathbf{d}_1|} = \frac{3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{\sqrt{3^2 + 6^2 + (-2)^2}}$$

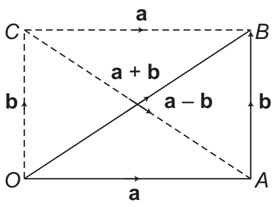
$$= \frac{3}{7}\hat{\mathbf{i}} + \frac{6}{7}\hat{\mathbf{j}} - \frac{2}{7}\hat{\mathbf{k}}$$

and $$\hat{\mathbf{n}}_2 = \frac{d_2}{|d_2|} = \frac{-\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 8\hat{\mathbf{k}}}{\sqrt{(-1)^2 + (-2)^2 + 8^2}}$$

$$= \frac{-1}{\sqrt{69}}\hat{\mathbf{i}} - \frac{2}{\sqrt{69}}\hat{\mathbf{j}} + \frac{8}{\sqrt{69}}\hat{\mathbf{k}}$$

Example 22. ***If* a *and* b *are any two vectors, then give the geometrical interpretation of the relation*** $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$.

Sol. Let $\mathbf{OA} = \mathbf{a}$ and $\mathbf{AB} = \mathbf{b}$. Completing the parallelogram *OABC*.



Then, $\mathbf{OC} = \mathbf{b}$ and $\mathbf{CB} = \mathbf{a}$

From ΔOAB, we have

$$\mathbf{OA} + \mathbf{AB} = \mathbf{OB} \Rightarrow \mathbf{a} + \mathbf{b} = \mathbf{OB} \quad ...(i)$$

From ΔOCA, we have

$$\mathbf{OC} + \mathbf{CA} = \mathbf{OA}$$

$$\Rightarrow \quad \mathbf{b} + \mathbf{CA} = \mathbf{a} \Rightarrow \mathbf{CA} = \mathbf{a} - \mathbf{b} \quad ...(ii)$$

Clearly, $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}| \Rightarrow |\mathbf{OB}| = |\mathbf{CA}|$

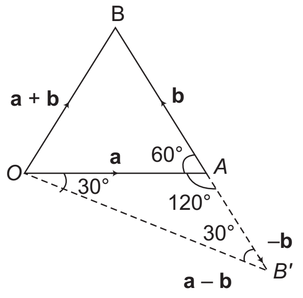
Diagonals of parallelogram *OABC* are equal.

OABC is a rectangle.

$\Rightarrow \quad \mathbf{OA} \perp \mathbf{OC} \Rightarrow \mathbf{a} \perp \mathbf{b}$

Example 23. ***If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is*** $\sqrt{3}$.

Sol. Let $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ be two unit vectors represented by sides *OA* and *AB* of a ΔOAB.



Then, $\mathbf{OA} = \hat{\mathbf{a}}$, $\mathbf{AB} = \hat{\mathbf{b}}$

$$\mathbf{OB} = \mathbf{OA} + \mathbf{AB} = \hat{\mathbf{a}} + \hat{\mathbf{b}}$$

(using triangle law of vector addition)

It is given that, $|\hat{\mathbf{a}}| = |\hat{\mathbf{b}}| = |\hat{\mathbf{a}} + \hat{\mathbf{b}}| = 1$

$\Rightarrow \quad |\mathbf{OA}| + |\mathbf{AB}| = |\mathbf{OB}| = 1$

ΔOAB is equilateral triangle.

Since, $|\mathbf{OA}| = |\hat{\mathbf{a}}| = 1 = |-\hat{\mathbf{b}}| = |\mathbf{AB'}|$

Therefore, $\Delta OAB'$ is an isosceles triangle.

$\Rightarrow \quad \angle AB'O = \angle AOB' = 30°$

$\Rightarrow \quad \angle BOB' = \angle BOA + \angle AOB' = 60° + 30° = 90°$

(since, $\Delta BOB'$ is right angled)

$\therefore$ In $\Delta BOB'$, we have

$$|BB'|^2 = |OB|^2 + |OB'|^2$$

$$= \left|\hat{\mathbf{a}} + \hat{\mathbf{b}}\right|^2 + \left|\hat{\mathbf{a}} - \hat{\mathbf{b}}\right|^2$$

$$2^2 = 1^2 + |\hat{\mathbf{a}} - \hat{\mathbf{b}}|^2$$

$$|\hat{\mathbf{a}} - \hat{\mathbf{b}}| = \sqrt{3}$$

Hence proved.

Multiplication of a Vector by a Scalar

If **a** is a vector and m is a scalar (i.e. a real number), then $m\,\mathbf{a}$ is a vector whose magnitude is m times that of **a** and whose direction is the same as that of **a**, if m is positive and opposite to that of **a**, if m is negative.

$\therefore$ Magnitude of $m a = |m\mathbf{a}| \Rightarrow m$ (magnitude of $\mathbf{a}$) $= m\,|\mathbf{a}|$

Again, if $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$,

then $\quad m\,\mathbf{a} = (ma_1)\,\hat{\mathbf{i}} + (ma_2)\,\hat{\mathbf{j}} + (ma_3)\,\hat{\mathbf{k}}$

Properties of Multiplication of Vectors by a Scalar

The following are properties of multiplication of vectors by scalars, for vectors **a, b** and scalars m, n

(i) $m(-\mathbf{a}) = (-m)\,\mathbf{a} = -(m\mathbf{a})$

(ii) $(-m)(-\mathbf{a}) = m\,\mathbf{a}$

(iii) $m(n\mathbf{a}) = (mn)\,\mathbf{a} = n(m\mathbf{a})$

(iv) $(m+n)\,\mathbf{a} = m\mathbf{a} + n\mathbf{a}$

(v) $m(\mathbf{a}+\mathbf{b}) = m\,\mathbf{a} + m\,\mathbf{b}$

Example 24. ***If* a *is a non-zero vector of modulus* a *and* m, *is a non-zero scalar, then* ma *is a unit vector, if***

(a) $m = \pm 1$ (b) $m = |\mathbf{a}|$

(c) $m = \dfrac{1}{|\mathbf{a}|}$ (d) $m = \pm 2$

Sol. (c) Since, $m\mathbf{a}$ is a unit vector, $|m\mathbf{a}| = 1$

$\Rightarrow \quad |m|\,|\mathbf{a}| = 1$

$\Rightarrow \quad |m| = \dfrac{1}{|\mathbf{a}|} \Rightarrow m = \pm\dfrac{1}{|\mathbf{a}|}$

Example 25. ***For a non-zero vector* a, *the set of real numbers, satisfying* $|(5-x)\mathbf{a}|<|2\mathbf{a}|$ *consists of all* x *such that***

(a) $0 < x < 3$

(b) $3 < x < 7$

(c) $-7 < x < -3$

(d) $-7 < x < 3$

Sol. (b) We have, $|(5-x)\,\mathbf{a}|<|2\mathbf{a}|$

$$|5-x|\,|\mathbf{a}|<2|\,\mathbf{a}|$$

$$\Rightarrow \quad |5-x|<2$$

$$\Rightarrow \quad -2<5-x<2$$

$$\Rightarrow \quad 3<x<7$$

Example 26. ***Find a vector of magnitude (5/2) units which is parallel to the vector*** $3\hat{\mathbf{i}}+4\hat{\mathbf{j}}$.

Sol. Here, $\mathbf{a} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

Then, $\quad |\mathbf{a}| = \sqrt{3^2 + 4^2} = 5$

$\therefore$ A unit vector parallel to

$$\mathbf{a} = \hat{\mathbf{a}} = \frac{a}{|\mathbf{a}|} \cdot = \frac{1}{5}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \qquad \text{...(i)}$$

Hence, the required vector of magnitude (5/2) units and parallel to **a**

$$= \frac{5}{2}\cdot\hat{\mathbf{a}} = \frac{5}{2}\cdot\frac{1}{5}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$$

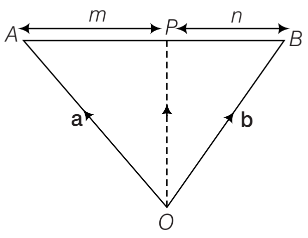
$$= \frac{1}{2}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$$

Section Formula

Let A and B be two points with position vectors **a** and **b** respectively. Let P be a point on AB dividing it is the ratio $m:n$.

Internal Division

If P divides AB internally in the ratio $m:n$. Then the position vector of P is given by



$$\mathbf{OP} = \frac{m\mathbf{b} + n\mathbf{a}}{m+n}$$

Proof

Let O be the origin. Then $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$. Let r be the position vector of P which divides AB internally is the ratio $m:n$. Then

$$\frac{AP}{PB} = \frac{m}{n}$$

or $\quad n\mathbf{AP} = m\mathbf{PB}$

or $\quad n(\text{PV of } P - \text{PV of } A) = m\,(\text{PV of } B - \text{PV of } P)$

or $\quad n(\mathbf{r} - \mathbf{a}) = m(\mathbf{b} - \mathbf{r})$

or $\quad n\mathbf{r} - n\mathbf{a} = m\mathbf{b} - m\mathbf{r}$

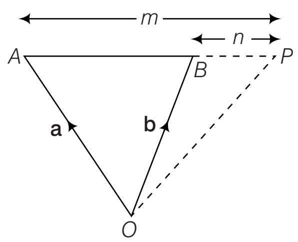
or $\quad \mathbf{r}(n+m) = m\mathbf{b} + n\mathbf{a}$

or $\quad r = \dfrac{m\mathbf{b} + n\mathbf{a}}{m+n}$

or $\quad \mathbf{OP} = \dfrac{m\mathbf{b} + n\mathbf{a}}{m+n}$

External Division

If P divides AB externally in the ratio $m:n$. Then, the position vector of P is given by



$$\mathbf{OP} = \frac{m\mathbf{b} - n\mathbf{a}}{m-n}$$

Proof

Let O be the origin. Then $\mathbf{OA} = \mathbf{a}$, $\mathbf{OB} = \mathbf{b}$. Let $\mathbf{r}$ be the position vector of point P dividing AB externally in the ratio $m:n$.

Then, $\quad \dfrac{AP}{BP} = \dfrac{m}{n}$

or $\quad nAP = mBP$

or $\quad n\mathbf{AP} = m\mathbf{BP}$

or $\quad n\,(\text{PV of } P - \text{PV of } A) = m\,(\text{PV of } P - \text{PV of } B)$

or $\quad n(\mathbf{r} - \mathbf{a}) = m(\mathbf{r} - \mathbf{b})$

or $\quad n\mathbf{r} - n\mathbf{a} = m\mathbf{r} - m\mathbf{b}$

or $\quad r(m-n) = m\mathbf{b} - n\mathbf{a}$

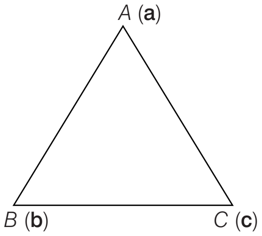
or $\quad \mathbf{r} = \dfrac{m\mathbf{b} - n\mathbf{a}}{m-n}$

or $\quad \mathbf{OP} = \dfrac{m\mathbf{b} - n\mathbf{a}}{m-n}$

Remarks

1. Position vector of mid-point of AB is $\dfrac{\mathbf{a}+\mathbf{b}}{2}$.

2. In ΔABC, having vertices $A(\mathbf{a})$, $B(\mathbf{b})$ and $C(\mathbf{c})$



(i) Position vector of centroid is $\dfrac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$.

(ii) Position vector of incentre is

$$\frac{BC\mathbf{a}+AC\mathbf{b}+AB\mathbf{c}}{AB+BC+AC}.$$

(iii) Position vector of orthocentre is

$$\frac{\tan A\,\mathbf{a}+\tan B\,\mathbf{b}+\tan C\,\mathbf{c}}{\tan A+\tan B+\tan C}.$$

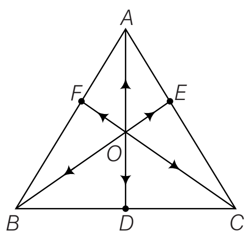
(iv) Position vector of circumcentre is

$$\frac{\sin 2A\,\mathbf{a}+\sin 2B\,\mathbf{b}+\sin 2C\,\mathbf{c}}{\sin 2A+\sin 2B+\sin 2C}.$$

Example 27. ***If D, E and F are the mid-points of the sides BC, CA and AB respectively of the ΔABC and O be any point, then prove that***

$$\mathbf{OA}+\mathbf{OB}+\mathbf{OC}=\mathbf{OD}+\mathbf{OE}+\mathbf{OF}$$

Sol. Since, D is the mid-point of BC, therefore by section formula, we have



$$\mathbf{OD}=\frac{\mathbf{OB}+\mathbf{OC}}{2}$$

$\Rightarrow \quad \mathbf{OB}+\mathbf{OC}=2\mathbf{OD}$ ...(i)

Similarly, $\quad \mathbf{OC}+\mathbf{OA}=2\mathbf{OE}$ ...(ii)

and $\quad \mathbf{OB}+\mathbf{OA}=2\mathbf{OF}$ ...(iii)

On adding Eqs. (i), (ii) and (iii), we get

$$2(\mathbf{OA}+\mathbf{OB}+\mathbf{OC})=2(\mathbf{OD}+\mathbf{OE}+\mathbf{OF})$$

$\Rightarrow \quad \mathbf{OA}+\mathbf{OB}+\mathbf{OC}=\mathbf{OD}+\mathbf{OE}+\mathbf{OF}$

Hence proved.

Example 28. ***Find the position vectors of the points which divide the join of points $A(2\mathbf{a}-3\mathbf{b})$ and $B(3\mathbf{a}-2\mathbf{b})$ internally and externally in the ratio $2:3$.***

Sol. Let P be a point which divide AB internally in the ratio $2:3$. Then, by section formula, position vector of P is given by

$$\mathbf{OP}=\frac{2(3\mathbf{a}-2\mathbf{b})+3(2\mathbf{a}-3\mathbf{b})}{2+3}$$

$$=\frac{6\mathbf{a}-4\mathbf{b}+6\mathbf{a}-9\mathbf{b}}{5}=\frac{12}{5}\mathbf{a}-\frac{13}{5}\mathbf{b}$$

Similarly, the position vector of the point (P') which divides AB externally in the ratio $2:3$ is given by

$$\mathbf{OP}'=\frac{2(3\mathbf{a}-2\mathbf{b})-3(2\mathbf{a}-3\mathbf{b})}{2-3}$$

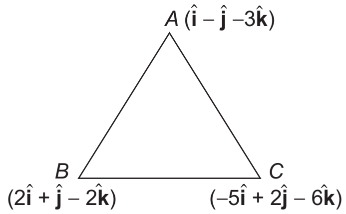
$$=\frac{6\mathbf{a}-4\mathbf{b}-6\mathbf{a}+9\mathbf{b}}{-1}=\frac{5\mathbf{b}}{-1}=-5\mathbf{b}$$

Example 29. ***The position vectors of the vertices A, B and C of a triangle are $\hat{\mathbf{i}}-\hat{\mathbf{j}}-3\hat{\mathbf{k}}$, $2\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and $-5\hat{\mathbf{i}}+2\hat{\mathbf{j}}-6\hat{\mathbf{k}}$, respectively. The length of the bisector AD of the $\angle BAC$, where D is on the segment BC, is***

(a) $\dfrac{3}{4}\sqrt{10}$ (b) $\dfrac{1}{4}$

(c) $\dfrac{11}{2}$ (d) None of these

Sol. (b)



(a) $|\mathbf{AB}|=|(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}})-(\hat{\mathbf{i}}-\hat{\mathbf{j}}-3\hat{\mathbf{k}})|$

$=|\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}|$

$=\sqrt{1^2+2^2+1^2}=\sqrt{6}$

$|\mathbf{AC}|=|(-5\hat{\mathbf{i}}+2\hat{\mathbf{j}}-6\hat{\mathbf{k}})-(\hat{\mathbf{i}}-\hat{\mathbf{j}}-3\hat{\mathbf{k}})|$

$=|-6\hat{\mathbf{i}}+3\hat{\mathbf{j}}-3\hat{\mathbf{k}}|$

$=\sqrt{(-6)^2+3^2+(-3)^2}=\sqrt{54}=3\sqrt{6}$

$$\mathbf{BD}:\mathbf{DC}=\mathbf{AB}:\mathbf{AC}=\frac{\sqrt{6}}{3\sqrt{6}}=\frac{1}{3}$$

$\therefore$ Position vector of $D=\dfrac{1(-5\hat{\mathbf{i}}+2\hat{\mathbf{j}}-6\hat{\mathbf{k}})+3(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}})}{1+3}$

$$=\frac{1}{4}(\hat{\mathbf{i}}+5\hat{\mathbf{j}}-12\hat{\mathbf{k}})$$

$\therefore$ $\quad \mathbf{AD}$ = Position vector of D – Position vector of A

$$\mathbf{AD} = \frac{1}{4}(\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 12\hat{\mathbf{k}}) - (\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = \frac{1}{4}(-3\hat{\mathbf{i}} + 9\hat{\mathbf{j}})$$

$$= \frac{3}{4}(-\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$$

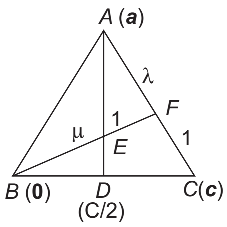
$$|\mathbf{AD}| = \frac{3}{4}\sqrt{(-1)^2 + 3^2} = \frac{3}{4}\sqrt{10}$$

Example 30. ***The median*** *AD* ***of the*** ΔABC ***is bisected at*** *E.BE* ***meets*** *AC* ***in*** *F*. ***Then,*** *AF* : *AC* ***is equal to***

(a) 3/4 (b) 1/3

(c) 1/2 (d) 1/4

Sol. (b) Let position vector of A w.r.t. B is $\mathbf{a}$ and that of C w.r.t. B is $\mathbf{c}$.



Position vector of D w.r.t.

$$B = \frac{0 + \mathbf{c}}{2} = \frac{\mathbf{c}}{2}$$

Position vector of

$$E = \frac{\mathbf{a} + \frac{\mathbf{c}}{2}}{2} = \frac{\mathbf{a}}{2} + \frac{\mathbf{c}}{4} \qquad \text{...(i)}$$

Let $AF : FC = \lambda : 1$ and $BE : EF = \mu : 1$

Position vector of $F = \dfrac{\lambda\mathbf{c} + \mathbf{a}}{1 + \lambda}$

Now, position vector of

$$E = \frac{\mu\left(\frac{\lambda\mathbf{c} + \mathbf{a}}{1 + \lambda}\right) + 1 \cdot 0}{\mu + 1} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\mathbf{a}}{2} + \frac{\mathbf{c}}{4} = \frac{\mu}{(1 + \lambda)(1 + \mu)}\mathbf{a} + \frac{\lambda\mu}{(1 + \lambda)(1 + \mu)}\mathbf{c}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{\mu}{(1 + \lambda)(1 + \mu)}$$

and

$$\frac{1}{4} = \frac{\lambda\mu}{(1 + \lambda)(1 + \mu)}$$

$$\Rightarrow \quad \lambda = \frac{1}{2}$$

$$\therefore \quad \frac{AF}{AC} = \frac{AF}{AF + FC} = \frac{\lambda}{1 + \lambda} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$$

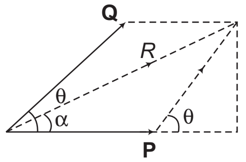
Magnitude of Resultant of Two Vectors

Let $\mathbf{R}$ be the resultant of two vectors $\mathbf{P}$ and $\mathbf{Q}$. Then,

$\mathbf{R} = \mathbf{P} + \mathbf{Q}$

$$|\mathbf{R}| = R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

where, $|\mathbf{P}| = P, |\mathbf{Q}| = Q, \ \tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$



Deduction When $|\mathbf{P}| = |\mathbf{Q}|$, i.e. $P = Q$

$$\tan\alpha = \frac{P\sin\theta}{P + P\cos\theta}$$

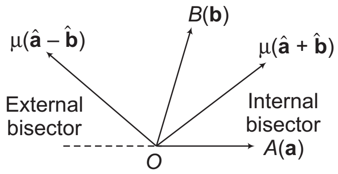
$$= \frac{\sin\theta}{1 + \cos\theta} = \tan\frac{\theta}{2}$$

$$\therefore \quad \alpha = \frac{\theta}{2}$$

Hence, the angular bisector of two unit vectors $\mathbf{a}$ and $\mathbf{b}$ is along the vector sum $\mathbf{a} + \mathbf{b}$.

Remarks

1. The internal bisector of the angle between any two vectors is along the vector sum of the corresponding unit vectors.
2. The external bisector of the angle between two vectors is along the vector difference of the corresponding unit vectors.



Example 31. ***The sum of two forces is 18 N and resultant whose direction is at right angles to the smaller force is 12 N. The magnitude of the two forces are***

(a) 13, 5 (b) 12, 6

(c) 14, 4 (d) 11, 7

Sol. (a) We have, $|\mathbf{P}| + |\mathbf{Q}| = 18\text{N}; |\mathbf{R}| = |\mathbf{P} + \mathbf{Q}| + 12\,\text{N}$

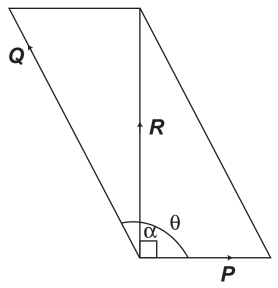
$$\alpha = 90°$$

$$P + Q\cos\theta = 0$$

$$\Rightarrow \quad Q\cos\theta = -P$$

Now. $\quad R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$$\Rightarrow \quad R^2 = P^2 + Q^2 + 2P(-P) = Q^2 - P^2$$



$\Rightarrow \quad 12^2 = (P+Q)(Q-P) = 18(Q-P)$

$\Rightarrow \quad Q - P = 8$ and $Q + P = 18$

$\Rightarrow \quad Q = 13, P = 5$

∴ Magnitude of two forces are 5 N and 13 N.

Example 32. ***The length of longer diagonal of the parallelogram constructed on $5\mathbf{a}+2\mathbf{b}$ and $\mathbf{a}-3\mathbf{b}$, when it is given that $|\mathbf{a}|=2\sqrt{2}, |\mathbf{b}|=3$ and angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{4}$, is***

(a) 15 (b) $\sqrt{113}$

(c) $\sqrt{593}$ (d) $\sqrt{369}$

Sol. (c) Length of the two diagonals will be

$$d_1 = |(5\mathbf{a}+2\mathbf{b}) + (\mathbf{a}-3\mathbf{b})|$$

and $d_2 = |(5\mathbf{a}+2\mathbf{b}) - (\mathbf{a}-3\mathbf{b})|$

$\Rightarrow \quad d_1 = |6\mathbf{a} - \mathbf{b}|, d_2 = |4\mathbf{a}+5\mathbf{b}|$

Thus,

$$d_1 = \sqrt{|6\mathbf{a}|^2 + |-\mathbf{b}|^2 + 2|6\mathbf{a}||-\mathbf{b}|\cos(\pi - \pi/4)}$$

$$= \sqrt{36(2\sqrt{2})^2 + 9 + 12\cdot 2\sqrt{2}\cdot 3\cdot\left(-\frac{1}{\sqrt{2}}\right)} = 15$$

$$d_2 = \sqrt{|4\mathbf{a}|^2 + |5\mathbf{b}|^2 + 2|4\mathbf{a}||5\mathbf{b}|\cos\frac{\pi}{4}}$$

$$= \sqrt{16\times 8 + 25\times 9 + 40\times 2\sqrt{2}\times 3\times\frac{1}{\sqrt{2}}}$$

$$= \sqrt{593}$$

∴ Length of the longer diagonal $= \sqrt{593}$

Example 33. ***The vector $\mathbf{c}$, directed along the internal bisector of the angle between the vectors $\mathbf{a} = 7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ and $\mathbf{b} = -2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ with $|\mathbf{c}| = 5\sqrt{6}$, is***

(a) $\frac{5}{3}(\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ (b) $\frac{5}{3}(5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

(c) $\frac{5}{3}(\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ (d) $\frac{5}{3}(-5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

Sol. (a) Let $\mathbf{a} = 7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

and $\mathbf{b} = -2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

Now, required vector $\mathbf{c} = \lambda\left(\frac{\mathbf{a}}{|\mathbf{a}|} + \frac{\mathbf{b}}{|\mathbf{b}|}\right)$

$$= \lambda\left(\frac{7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}}{9} + \frac{-2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{3}\right)$$

$$= \frac{\lambda}{9}(\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$$

$$|\mathbf{c}|^2 = \frac{\lambda^2}{81}\times 54 = 150$$

$\Rightarrow \quad \lambda = \pm 15$

$\Rightarrow \quad \mathbf{c} = \pm\frac{5}{3}(\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

Exercise for Session 2

1. If $\mathbf{a} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{b} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, then find $\mathbf{a} + \mathbf{b}$. Also, find a unit vector along $\mathbf{a} + \mathbf{b}$.
2. Find a unit vector in the direction of the resultant of the vectors $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} + \hat{\mathbf{j}}$.
3. Find the direction cosines of the resultant of the vectors $(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}), (-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}), (\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$ and $(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$.
4. In a regular hexagon *ABCDEF*, show that $\mathbf{AE}$ is equal to $\mathbf{AC} + \mathbf{AF} - \mathbf{AB}$
5. Prove that $3\mathbf{OD} + \mathbf{DA} + \mathbf{DB} + \mathbf{DC}$ is equal to $\mathbf{OA} + \mathbf{OB} - \mathbf{OC}$.
6. In a regular hexagon *ABCDEF*, prove that $\mathbf{AB} + \mathbf{AC} + \mathbf{AD} + \mathbf{AE} + \mathbf{AF} = 3\mathbf{AD}$.
7. *ABCDE* is a pentagon, prove that $\mathbf{AB} + \mathbf{BC} + \mathbf{CD} + \mathbf{DE} + \mathbf{EA} = 0$.
8. The position vectors of A, B, C, D are $\mathbf{a}$, $\mathbf{b}$, $2\mathbf{a} + 3\mathbf{b}$ and $\mathbf{a} - 2\mathbf{b}$, respectively. Show that $\mathbf{DB} = 3\mathbf{b} - \mathbf{a}$ and $\mathbf{AC} = \mathbf{a} + 3\mathbf{b}$.
9. If $P(-1, 2)$ and $Q(3, -7)$ are two points, express the vector $\mathbf{PQ}$ in terms of unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$. Also, find distance between point P and Q. What is the unit vector in the direction of $\mathbf{PQ}$?
10. If $\mathbf{OP} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{OQ} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, find the modulus and direction cosines of $\mathbf{PQ}$.
11. Show that the points A, B and C with position vectors $\mathbf{a} = 3\hat{\mathbf{j}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, $\mathbf{b} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ respectively, form the vertices of a right angled triangle.
12. If $\mathbf{a} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $|x\mathbf{a}| = 1$, then find x.
13. If $\mathbf{p} = 7\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{q} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, then find the magnitude of $\mathbf{p} - 2\mathbf{q}$.
14. Find a vector in the direction of $5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, which has magnitude 8 units.
15. If $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{b} = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, then find a vector in the direction of $\mathbf{a}$ and having magnitude as $|\mathbf{b}|$.
16. Find the position vector of a point P which divides the line joining two points A and B whose position vectors are $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ respectively, in the ratio $2 : 1$.
    (i) internally (ii) externally
17. If the position vector of one end of the line segment AB be $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and the position vector of its middle point be $3(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$, then find the position vector of the other end

Answers

Exercise for Session 2

1. $\hat{\mathbf{i}} + \hat{\mathbf{k}}; \frac{1}{\sqrt{2}}\hat{\mathbf{i}} + \frac{1}{\sqrt{2}}\hat{\mathbf{k}}$

2. $\frac{3}{5\sqrt{2}}\hat{\mathbf{i}} + \frac{5}{5\sqrt{2}}\hat{\mathbf{j}} + \frac{4}{5\sqrt{2}}\hat{\mathbf{k}}$

3. $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

9. $4\hat{\mathbf{i}} - 9\hat{\mathbf{j}}, \sqrt{97}, \frac{4}{\sqrt{97}}\hat{\mathbf{i}} - \frac{9}{\sqrt{97}}\hat{\mathbf{j}}$

10. $\sqrt{59}; \frac{1}{\sqrt{59}}\hat{\mathbf{i}} - \frac{7}{\sqrt{59}}\hat{\mathbf{j}} + \frac{3}{\sqrt{59}}\hat{\mathbf{k}}$

12. $\pm\frac{1}{3}$

13. $\sqrt{66}$

14. $\frac{8}{\sqrt{30}}(5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

15. $7/3, (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

16. (i) $-\frac{1}{3}\hat{\mathbf{i}} + \frac{4}{3}\hat{\mathbf{j}} + \frac{1}{3}\hat{\mathbf{k}}$ (ii) $-3\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$

17. $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$