Chapter

2 Trigonometric Equation

Single Correct Option Type Questions

Q.1 In the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, the equation $\log_{\sin\theta}\cos 2\theta = 2$ has

(A) No solution (B) One solution (C) Two solution (D) Infinite solution

Q.2 Number of solution of the equation $\sin\frac{5x}{2} - \sin\frac{x}{2} = 2$ in the interval $[0, 2\pi]$, is -

(A) 1 (B) 2 (C) 0 (D) Infinite

Q.3 If θ_1, θ_2 and θ_3 are the three values of $\theta \in [0, 2\pi]$ for which $\tan\theta = \lambda$ then the value of $\tan\frac{\theta_1}{3}\tan\frac{\theta_2}{3} + \tan\frac{\theta_2}{3}\tan\frac{\theta_3}{3} + \tan\frac{\theta_3}{3}\tan\frac{\theta_1}{3}$ is equal to (λ is a constant)

(A) –3 (B) –2 (C) 2 (D) 3

Q.4 If $\log_{\cos x}\sin x \geq 2$ and $0 \leq x \leq 3\pi$ then $\sin x$ lies in the interval

(A) $\left[\frac{\sqrt{5}-1}{2},1\right]$ (B) $\left[0,\frac{\sqrt{5}-1}{2}\right]$ (C) $\left[\frac{1}{2},1\right]$ (D) none of these

Q.5 The number of solutions of the equation $4\sin^2x + \tan^2x + \cot^2x + \operatorname{cosec}^2x = 6$ in $[0, 2\pi]$

(A) 1 (B) 2 (C) 3 (D) 4

Q.6 The complete solution set of the equation $4\sin^2 x + \tan^2 x + \operatorname{cosec}^2 x + \cot^2 x = 6$ is

(A) $2n\pi \pm \pi/4$, $(n \in I)$ (B) $n\pi \pm \pi/3$, $(n \in I)$

(C) $n\pi \pm \pi/4$, $(n \in I)$ (D) $n\pi \pm \pi/6$, $(n \in I)$

Q.7 The number of solutions of the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$ which lie between 0 and 2π is

(A) 0 (B) 2 (C) 4 (D) 8

Q.8 The smallest positive value of p for which the equation $\cos(p\sin x) = \sin(p\cos x)$ has solution in $0 \leq x \leq 2\pi$ is

(A) $\frac{\pi}{\sqrt{2}}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{2\sqrt{2}}$ (D) $\frac{3\pi}{2\sqrt{2}}$

Q.9 The total number of ordered pairs (x, y) satisfying $|x| + |y| = 2$ and $\sin\left(\frac{\pi x^2}{3}\right) = 1$ is:

(A) 2 (B) 4 (C) 6 (D) 8

Q.10 The complete set of values of x, $x \in \left(-\frac{\pi}{2},\pi\right)$ satisfying the inequality $\cos 2x > |\sin x|$ is:

(A) $\left(-\frac{\pi}{6},\frac{\pi}{6}\right)$ (B) $\left(-\frac{\pi}{2},-\frac{\pi}{6}\right)\cup\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$

(C) $\left(-\frac{\pi}{2},-\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6},\pi\right)$ (D) $\left(-\frac{\pi}{6},\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6},\pi\right)$

Q.11 The total number of solution of the equation $\sin^4 x + \cos^4 x = \sin x \cos x$ in $[0, 2\pi]$ is:

(A) 2 (B) 4 (C) 6 (D) 8

Q.12 The sides of a triangle are $\sin\alpha$, $\cos\alpha$, $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$ then the greatest angle of the triangle is-

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

Q.13 The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

(A) infinite number of real roots (B) no real root

(C) exactly one real root (D) exactly four real roots

Q.14 The possible value(s) of 'θ' satisfying the equation $\sin^2\theta \tan\theta + \cos^2\theta \cot\theta - \sin 2\theta = 1 + \tan\theta + \cot\theta$ where $\theta \in [0, \pi]$ is/are

(A) $\frac{\pi}{4}$ (B) π (C) $\frac{7\pi}{12}$ (D) None of these

Q.15 Consider a trigonometric equation ($x \in [0, 2\pi]$), $3\cot^2 x + 8\cot x + 3 = 0$ then the sum of all solutions is -

(A) π (B) 3π (C) 5π (D) None of these

Multiple Correct Option Type Questions

Q.16 If $\sin\theta + \sqrt{3}\cos\theta = 6x - x^2 - 11$, $0 \le \theta \le 4\pi$, $x \in R$ holds for

(A) No value of x and θ

(B) One value of x and two values of θ

(C) Two values of x and two values of θ

(D) Two pairs of values of (x, θ)

Q.17 Which of the following sets can be the subset of the general solution of the equation : $1 + \cos 3x = 2\cos 2x$?

(A) $n\pi + \frac{\pi}{3}$ (B) $n\pi + \frac{\pi}{6}$ (C) $n\pi - \frac{\pi}{6}$ (D) $2n\pi$

(where $n \in I$)

Q.18 If $0 \le \theta \le \pi$ and $\sin\frac{\theta}{2} = \sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}$, then possible values of $\tan\theta$, is -

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{3}{4}$ (D) $-\frac{4}{3}$

Q.19 If $\cot^3\alpha + \cot^2\alpha + \cot\alpha = 1$ then which of the following is/are correct

(A) $\cos 2\alpha \tan\alpha = 1$ (B) $\cos 2\alpha \cdot \tan\alpha = -1$

(C) $\cos 2\alpha - \tan 2\alpha = -1$ (D) $\cos 2\alpha - \tan 2\alpha = 1$

Q.20 The equation $\cos^2 x - \sin x + \lambda = 0$, $x \in (0, \pi/2)$ has roots then value(s) of λ can be equal to

(A) 0 (B) –1 (C) $\frac{1}{2}$ (D) 1

Passage Based Questions

Passage # 1 (Q.21 to 23)

Let $f(x) = \sin^2 x - (a-1)\sin x + 2(a-3)$

Q.21 If $x \in [0, \pi]$ and $f(x) = 0$ has exactly one real root, then 'a' lies in

(A) (3, 5) (B) (2, 4) (C) (4, 5) (D) None of these

Q.22 If $f(x) = 0$ have two real roots in $(0, \pi)$, then $a \in$

(A) (1, 2) (B) (3, 4) (C) $(3, 4) \cup \{5\}$ (D) (3, 5)

Q.23 If $f(x) \ge 0\ \forall\ x \in R$ then range of 'a' is

(A) $[2, \infty)$ (B) $[4, \infty)$ (C) $(4, \infty)$ (D) None of these

Passage # 2 (Q.24 & 25)

Let $f(x, y, z) = \cos x + \cos y + \cos z$

Q.24 If x, y, z are in A.P. then f(x, y, z) is

(A) $\frac{\cos\left(\frac{x+z}{2}\right)\sin\left(\frac{3(y-x)}{2}\right)}{\sin\left(\frac{(z-y)}{2}\right)}$ (B) $\frac{\cos\left(\frac{z-x}{2}\right)\sin\left(\frac{3(z-y)}{2}\right)}{\sin\left(\frac{(y-x)}{2}\right)}$

(C) $\frac{\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{3(x+z)}{2}\right)}{\sin\left(\frac{(x+z)}{2}\right)}$ (D) $\frac{\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{3(x+y)}{2}\right)}{\sin\left(\frac{(x+y)}{2}\right)}$

Q.25 The general solution of $f\left(x, \frac{2\pi}{3}-x, \frac{2\pi}{3}+x\right) = f(x, 2x, 3x)$ and $f(x, x, x) = \frac{3}{\sqrt{2}}$ is :

(A) $x = (2n\pi+1)\frac{\pi}{4}, n \in Z$

(B) $x = \frac{n\pi}{4}, n \in Z$

(C) $x = 2n\pi \pm \frac{\pi}{4}, n \in Z$

(D) $x = 2n\pi \pm \frac{3\pi}{4}, n \in Z$

Column Matching Type Questions

Q.26 Match the following :

Column-I	Column-II
(A) If $2a\sin x - a\sin 3x \le 6\ \forall\ x \in R$ then the number of non zero integral value(s) of 'a' is	(P) 0
(B) Tangents are drawn to $x^2 + y^2 = 16$ from the point P(0, h). These tangents meet the x-axis at A and B. If the area of triangle PAB is minimum, then $\frac{h^2}{8} =$	(Q) 1
(C) If 'k' denotes the number of ways in which 3 squares can be selected on chess board which lie on same diagonal line and lie below the main diagonal and 'ℓ' denotes the coefficient of x^3 in $(1+x)^3 + (1+x)^4 + (1+x)^5 + (1+x)^6 + (1+x)^7$, then $k - \ell$ is equal to (where the main diagonal on chess board is a diagonal along which the north-west corner lie).	(R) 2
(D) Sum of all real solutions of equation $\frac{1}{2} + \lvert\sin x\rvert = \cos x$ in $[0, 2\pi]$ is $k\pi$, then the value of k is	(S) 3
	(T) 4

Numeric Response Type Questions

Q.27 If the sum of all values of θ, $0 \le \theta \le 2\pi$ satisfying the equation.
$(8\cos 4\theta - 3)(\cot\theta + \tan\theta - 2)(\cot\theta + \tan\theta + 2) = 12$ is $k\pi$, then k is equal to

Q.28 Find the number of solutions of the equation $2\sin^2 x + \sin^2 2x = 2$, $\sin 2x + \cos 2x = \tan x$ in $[0, 4\pi]$ satisfying the condition $2\cos^2 x + \sin x \le 2$.

Q.29 If the sum of all the solutions of the equation $3\cot^2\theta + 10\cot\theta + 3 = 0$ in $[0, 2\pi]$ is $k\pi$ where $k \in I$, then find the value of k

Q.30 Let the inequality $\sin^2 x + a\cos x + a^2 \ge 1 + \cos x$ is satisfied $\forall\ x \in R$, for $a \in (-\infty, k_1] \cup [k_2, \infty)$, then $|k_1| + |k_2| =$

Q.31 The sum of all integral values of 'a' for which the equation $2x^2 - (1+2a)x + 1 + a = 0$ has a integral root.

Q.32 Let f(x) be a polynomial of degree 8 such that $F(r) = \frac{1}{r}$, $r = 1, 2, 3, \ldots, 8., 9$, then $\frac{1}{F(10)} =$

Q.33 Sum of all solutions of the equation $\frac{1}{2} + |\sin x| = \cos x$, where $x \in [0, 4\pi]$ is $k\pi$, then the value of k is

Q.34 The number of distinct solutions $x \in [0, \pi]$ which satisfy the equation $8\cos x \cos 4x \cos 5x = 1$ is k then $\frac{k}{2}$ is equal to

ANSWER KEY

Single Correct Option type Questions

1. (B) **2.** (C) **3.** (A) **4.** (B) **5.** (D) **6.** (C) **7.** (A)

8. (C) **9.** (B) **10.** (D) **11.** (A) **12.** (C) **13.** (B) **14.** (C)

15. (C)

Multiple Correct Option type Questions

16. (B,D) **17.** (B,C,D) **18.** (B,D) **19.** (B,D) **20.** (A,C)

Passage Based Questions

21. (A) **22.** (B) **23.** (B) **24.** (A) **25.** (C)

Column Matching Type Questions

26. $A \rightarrow T$; $B \rightarrow R$; $C \rightarrow P$; $D \rightarrow R$

Numeric Response Type Questions

27. 8 **28.** 4 **29.** 5 **30.** 3 **31.** 1 **32.** 5 **33.** 8

34. 5