CHAPTER 1 ELECTRIC CHARGES AND FIELDS

1. **Charge-** Charge is the property associated with matter due to which it produces and experiences electric and magnetic effect.

2. Conductors and Insulators Those substances which readily allow the passage of electricity through them are called conductors, e.g. metals, the earth and those substances which offer high resistance to the passage of electricity are called insulators, e.g. plastic rod and nylon.

3. Transference of electrons is the cause of frictional electricity.

4. Additivity of Charges Charges are scalars and they add up like real numbers. It means if a system consists of n charges q_1, q_2, q_3 , … ,q_n, then total charge of the system will be $q_1 + q_2 + \ldots + q_n$.

5. Conservation of Charge The total charge of an isolated system is always conserved, i.e. initial and final charge of the system will be same.

6. Quantisation of Charge Charge exists in discrete amount rather than continuous value and hence, quantised.

Mathematically, charge on an object, $q=\pm ne$

where, n is an integer and e is electronic charge. When any physical quantity exists in discrete packets rather than in continuous amount, the quantity is said to be quantised. Hence, charge is quantised.

7. Units of Charge

(i) SI unit coulomb (C)

(ii) CGS system

(a) electrostatic unit, esu of charge or stat-coulomb (stat-C)

(b) electromagnetic unit, emu of charge or ab-C (ab-coulomb)

1 ab-C = 10 C, 1 C = 3 x 10^9 stat-C

8. Coulomb's Law It states that the electrostatic force of interaction or repulsion acting between two stationary point charges is given by

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

where, q_1 and q_2 are the stationary point charges and r is the separation between them in air or vacuum.

Also, $$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N-m}^2/\text{C}^2$$

where, ε_0 = permittivity of free space = 8.85419×10^{-12} C^2/N-m^2

The force between two charges q_1 and q_2 located at a distance r in a medium other than free space may be expressed as

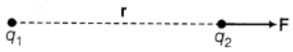
$$F = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2}$$

where, ε is **absolute permittivity** of the medium.

Now, $$\frac{F_{\text{vacuum}}}{F} = \frac{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}}{\frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2}} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$$

where, ε_r is called **relative permittivity** of the medium also called **dielectric constant** of the medium.

In vector form,



$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}|^2}\hat{\mathbf{r}} \quad \text{or} \quad |\mathbf{F}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

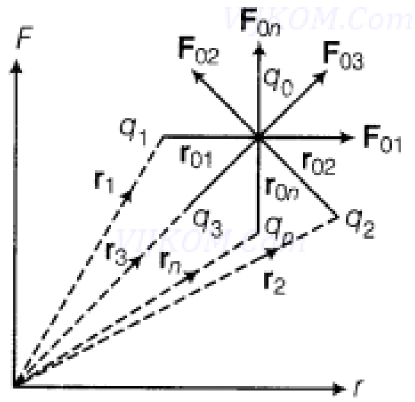
9. Electrostatic forces (Coulombian forces) are conservative forces.

10. Principle of Superposition of Electrostatic Forces This principle states that the net electric force experienced by a given charge particle q_0 due to a system of charged particles is equal to the vector sum of the forces exerted on it due to all the other charged particles of the system.

i.e. $\mathbf{F}_0 = \mathbf{F}_{01} + \mathbf{F}_{02} + \mathbf{F}_{03} + \ldots + \mathbf{F}_{0n}$

$$\mathbf{F}_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_0}{|\mathbf{r}_{01}|^3}\mathbf{r}_{01} + \frac{q_2 q_0}{|\mathbf{r}_{02}|^3}\mathbf{r}_{02} + \ldots + \frac{q_n q_0}{|\mathbf{r}_{0n}|^3}\mathbf{r}_{0n}\right]$$

where, $\mathbf{r}_{01} = \mathbf{r}_0 - \mathbf{r}_1$, $\mathbf{F}_{01}$ = force on q_0 due to q_1.



Superposition of electrostatic forces

Similarly, $\mathbf{r}_{0n} = \mathbf{r}_0 - \mathbf{r}_n$; $\mathbf{F}_{0n}$ = force on q_0 due to q_n

$$\therefore \quad \mathbf{F}_0 = \frac{q_0}{4\pi\varepsilon_0}\left[\sum_{i=1}^{n}\frac{q_i}{|\mathbf{r}_{0i}|^3}\mathbf{r}_{0i}\right]$$

Net force in terms of position vector,

$$\mathbf{F}_0 = \frac{q_0}{4\pi\varepsilon_0}\left[\sum_{i=1}^{n}\frac{q_i}{|\mathbf{r}_0 - \mathbf{r}_i|^3}(\mathbf{r}_0 - \mathbf{r}_i)\right]$$

11. Electrostatic Force due to Continuous Charge Distribution

The region in which charges are closely spaced is said to have continuous distribution of charge. It is of three types given as below:

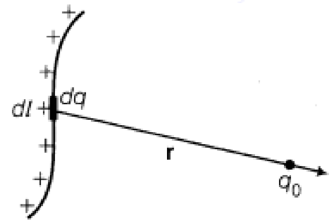
(i) Linear Charge Distribution

$$dq = \lambda\, dl$$

where, λ = linear charge density

$$d\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_0(dq)}{|\mathbf{r}|^2}\hat{\mathbf{r}} \Rightarrow d\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_0(\lambda\, dl)}{|\mathbf{r}|^2}\hat{\mathbf{r}}$$

Net force on charge q_0, $\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_l \frac{\lambda\, dl}{|\mathbf{r}|^2}\hat{\mathbf{r}}$

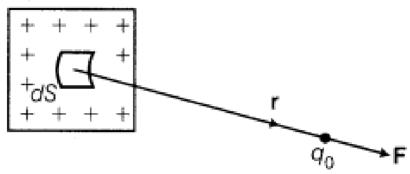


(ii) Surface Charge Distribution

$$dq = \sigma\, dS$$

where, σ = surface charge density

Net force on charge q_0, $\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_S \frac{\sigma\, dS}{|\mathbf{r}|^2}\hat{\mathbf{r}}$

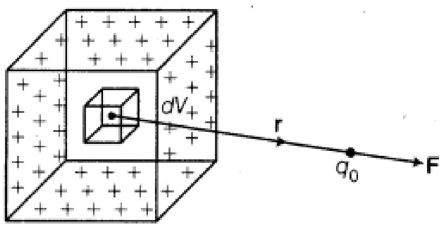


(iii) Volume Charge Distribution

$$dq = \rho\, dV$$

where, ρ = volume charge density

Net force on charge q_0, $\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_V \frac{\rho\, dV}{|\mathbf{r}|^2}\hat{\mathbf{r}}$



12. Electric Field Intensity The electric field intensity at any point due to source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge. It is expressed as

$$\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0}$$

Here, $q_0 \to 0$, i.e. the test charge q_0 must be small, so that it does not produce its own electric field.

SI unit of electric field intensity ($\mathbf{E}$) is N/C and it is a vector quantity.

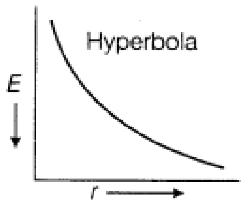
13. Electric Field Intensity (EFI) due to a Point Charge

Electric field intensity at P is, then

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{|\mathbf{r}|^2} \hat{\mathbf{r}}$$

The magnitude of the electric field at a point P is given by

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$



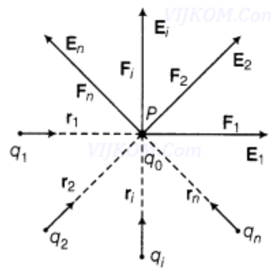
If $q > 0$, i.e. positive charge, then $\mathbf{E}$ is directed away from source charge. On the other hand if $q < 0$, i.e. negative charge, then $\mathbf{E}$ is directed towards the source charge.

$$\mathbf{E} \propto \frac{1}{r^2}$$

14. Electric Field due to a System of Charges Same as the case of electrostatic force, here we will apply principle of superposition, i.e.

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \dots + \mathbf{E}_n$$

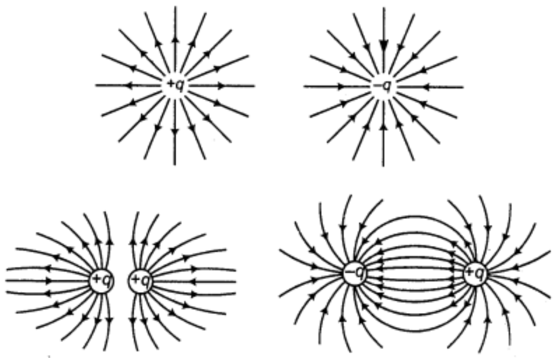
$$\Rightarrow \qquad \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\mathbf{r}_i|^2} \hat{\mathbf{r}}_i$$



A system of charges

15. Electric Field Lines Electric field lines are a way of pictorially mapping the electric field around a configuration of charge(s). These lines start on positive charge and end on negative charge. The tangent on these lines at any point gives the direction of field at that point.

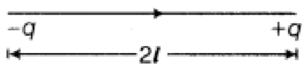
16. Electric field lines due **to positive and negative charge** and their combinations are shown as below:



Different electric field lines

17. Electric Dipole Two-point charges of same magnitude and opposite nature separated by a small distance altogether form an electric dipole.

18. Electric Dipole Moment The strength of an electric dipole is measured by a vector quantity known as electric dipole moment (p) which is the product of the charge (q) and separation between the charges (2l).



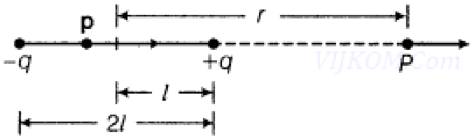
$$\mathbf{p} = q \times 2\mathbf{l}$$

$$\Rightarrow \quad |\mathbf{p}| = q\,(2l)$$

Direction Its direction is from negative charge $(-q)$ to positive charge $(+q)$.
SI unit Its SI unit is C-m.

NOTE The line joining the two charges $-q$ and $+q$ is called the dipole axis.

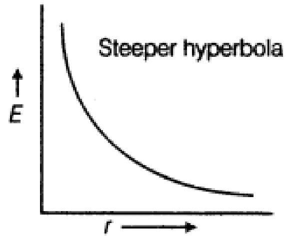
(i) Electric Field at any Point on the Axial Line/End-on Position of Electric Dipole



$$\mathbf{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\mathbf{p}r}{(r^2 - l^2)^2}$$

When $l << r$, $\mathbf{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\mathbf{p}}{r^3} \Rightarrow |\mathbf{E}_{\text{axial}}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2|\mathbf{p}|}{r^3}$

$$\mathbf{E} \propto \frac{1}{r^3}$$



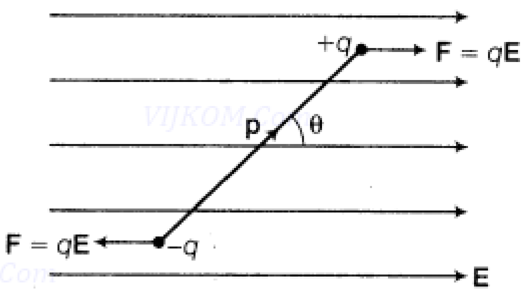
The direction of electric field at any point on axial line is along the direction of electric dipole moment.

19. Electric Field due to a Dipole Electric field of an electric dipole is the space around the dipole in which the electric effect of the dipole can be experienced.

20. When $l << r$, $\dfrac{|\mathbf{E}_{\text{axial}}|}{|\mathbf{E}_{\text{equatorial}}|} = 2$

21. Torque on an electric dipole placed in a uniform electric field (E) is given by

$$\tau = \mathbf{p} \times \mathbf{E} \Rightarrow |\tau| = pE\sin\theta$$



22. **Minimum torque** experienced by electric dipole in electric field, when $\theta = 0°$ or π

$$\tau = \tau_{\min} = 0$$

23. **Maximum torque** $\tau = \tau_{\max}$, when $\sin\theta = 1 \Rightarrow \theta = \pi/2$

$$\tau_{\max} = pE$$

24. Dipole is in stable equilibrium in uniform electric field when angle between p and E is 0° and in unstable equilibrium when angle θ= 180°.
25. Net force on electric dipole placed in a uniform electric field is zero.
26. There exists a net force and torque on electric dipole when placed in non-uniform electric field.
27. Work done in rotating the electric dipole from θ_1 to θ_2 is $W = pE(\cos\theta_1 - \cos\theta_2)$

28. Potential energy of electric dipole when it rotates from $\theta_1 = 90°$ to $\theta_2 = 0$
U = pE (cos 90° – cosθ) = -pE cos θ = – p .E

29. Work done in rotating the dipole from the position of stable equilibrium to unstable equilibrium, i.e. when $\theta_1 = 0°$ and $\theta_2 = \pi$.
W = 2 pE

30. Work done in rotating the dipole from the position of stable equilibrium to the position in which dipole experiences maximum torque, i.e. when $\theta_1 = 0°$ and $\theta_2 = 90°$.

W = pE

31. **Electric flux**. The electric flux through a small surface ts defined as the electric lines of force passing through that are when held normally to the lines of force.
Mathematically-- **φ=** $\vec{E}.\overrightarrow{\Delta S}$

where E is the electric field and AS is the area vector representing the elementary surface area.

Unit. In SI, unit of electric flux is newton metre^2 coulomb^{-2} (N m^2 C^{-2}).

32. Gauss' theorem. It states that the total outward electric flux through a closed surface is $\frac{1}{\epsilon_o}$ times the charge enclosed |by the closed surface.

Mathematically**:** $\oint \vec{E}.\overrightarrow{dS} = \frac{q}{\epsilon_0}$

where q is charge enclosed by the closed surface.

33. Gaussian surface. Any closed surface around the charge distribution (may be a point charge, a line charge, a surface charge or a volume charge) so that Gauss' theorem can be conveniently applied to find electrical field due to it is called the gaussian surface.

34.Electric field due to infinitely long straight wire of linear charge density λ

$$E = \frac{\lambda}{2\pi\epsilon_0\ r}$$

where r is perpendicular distance of the observation point from the wire.

35. Electric field due to an infinite plane sheet of charge of surface charge density σ

Electric field between two infinite plane parallel sheets of charge of surface charge density 0 and -o:

$$E = \frac{\sigma}{2\epsilon_0}$$

36. Electric field due to spherical shell of surface charge density o and radius R:

$E= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ **for r>R (outside the shell)**

E=0, for r<R (inside the shell)

$E= \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$**' for r=R (at the surface)**

Here, $q = 4\pi R^2$ **σ**

37. Electric field due to a solid sphere of volume charge density ρ and radius R:

$E= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ **for r>R (outside the sphere)**

$E= \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3}$ **for r<R (inside the sphere)**

$E= \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$**' for r=R (at the surface)** Here, $q = \frac{4\pi}{3} R^3\rho$

QUESTIONS WITH ANSWERS

Q. 1 When a polythene piece is rubbed with wool, it acquires negative charge. Is there a transfer of mass from wool to polythene?

Ans. The polythene piece acquires negative charge due to transfer of electrons from wool to it. Since electrons are material particles, there is a transfer of mass from wool to polythene.

Q. 2 A glass rod, when rubbed with silk cloth, acquires a charge 1.6×10^{-8} coulomb. What is the charge on the silk cloth?

Ans. Silk cloth will also acquire a charge 1.6×10^{-8} coulomb. However, it will be negative in nature.

Q. 3 How does the mass of a body changes after charging?

Ans. When a body is charged, either electrons get removed (becomes positively charged) or get added (becomes negatively charged) to it. Since electron is a material particle, the mass of a body decreases on getting positively charged and increases on getting negatively charged.

Q. 4 Ordinary rubber is an insulator. But the special rubber tyres of aircrafts are made slightly conducting. Why is this necessary?

Ans. During landing, the tyres of a space-craft get charged due to friction between the tyres and the ground. In case, the tyres are slightly conducting, the charge developed on the tyres will not stay on them and it will find its way (leak) to the earth.

Q. 5. **'Automobile ignition failure occurs in damp weather.'** Explain, why.

Ans. The insulating porcelain of the spark plugs accumulates a film of dirt. The surface dirt is hygroscopic and picks up moisture from the air. Therefore, in humid weather, the insulating porcelain of the plugs becomes quasi-conductor. This allows an appreciable proportion of the spark to leak across the surface of the plug instead of discharging across the gap.

Q. 6 A bird perches on a bare high-powerline and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why?

Ans. When a bird is perched on a bare high-power line, the circuit does not get completed between the bird and the earth. Therefore, nothing happens to the bird. When a man standing on ground touches the same line, the circuit between the man and the earth gets completed. As a result, he gets a fatal shock.

Q.7 Can a charged body attract another uncharged body? Explain.

Or

Why does a charged glass rod attract a piece of paper?

Ans. Yes, a charged body can attract another uncharged body. It is because, when the charged body is placed in front of an uncharged body, the induced charges of opposite kind are produced on the uncharged body. Due to this, the charged body attracts the uncharged body.

Q. 8 The test charge used to measure electric field at a point should be vanishingly small. Why?

Ans. In case, test charge is not vanishingly small, it will produce its own electric field and the measured value of electric field will be different from the actual value of electric field at that point.

Q. 9 Why do the electric field lines never cross each other?
Ans. The tangent at a point on the line of force gives the direction of electric field at that point. If two lines of force intersect each other at a point, then electric field at that point will have two directions. As the same cannot be true, two lines of force can never intersect each other.
Q. 10 Why do the electrostatic field lines not form closed loops?
Ans. The electrostatic field lines originate from positive charge and end at the negative charge. As the isolated positive and negative charges do exist, the electrostatic field lines do not form closed loops.
Q. 11 Does an electric dipole always experience a torque, when placed in a uniform electric field?
Ans. No. It does not experience a torque, when it is placed along the direction of electric field.
Q. 12. What is the net force on an electric dipole placed in a uniform electric field?
Ans. An electric dipole does not experience any net force in a uniform electric field.
Q. 13 When is the torque acting on an electric dipole maximum, when placed in uniform electric-field?
Ans. The torque is maximum, when the electric dipole is placed perpendicular to the direction of electric field.
Q. 14 What is the angle between the directions of electric dipole moment and electric field at any,
(i) axial point and (ii) equatorial point due to an electric dipole?
Ans. (i) The electric field at a point on the axial line of an electric dipole is same as that of electric dipole moment and hence angle between them is zero.
(ii) The electric field at a point on the equatorial line of an electric dipole is opposite to that of electric dipole moment and hence angle between them is 180°.
Q. 15 An electric dipole of dipole moment 20 x 10-© C mis enclosed by a closed surface. What is the net flux coming out of the surface?
Ans. Since an electric dipole consists of two equal and opposite charges, the net charge on the dipole is zero. Hence, the net electric flux coming out of the closed surface is Zero.
Q.16 Does the strength of electric field due to an infinitely long line charge depend upon the distance of the observation points from the line charge?
Ans. Yes, the electric field due to an infinitely long line charge depends upon the distance of the observation point from the line charge.
Q. 17 Does the strength of electric field due to an infinite plane sheet of charge depend upon the distance of the observation point from the sheet of charge?
Ans. No, the electric field due to an infinite plane sheet of charge does not depend upon the distance of the observation point from the plane sheet of charge.
Q. 18 What is the difference between a sheet of charge and a plane conductor having charge?
Ans. On a sheet of charge, the same charge shows up on its two sides; whereas in case of a charged plane conductor, the charges showing up on the two surfaces are not the same.
Q. 19 How does electric field at a point change with distance r from an infinitely long charged wire?

Ans. The electric field due to a line charge falls off with distance as $\frac{1}{r^2}$.

Q. 20 What is the importance of Gauss's theorem?

Ans. Gauss' theorem is of great importance. Those situations, in which the calculation of electric field by applying Coulomb's law or the principle of superposition of electric fields becomes very difficult, the results can be obtained by applying Gauss' theorem with great ease.

Q.21 A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

(a) Estimate the number of electrons transferred (from which to which?).

(b) Is there a transfer of mass from wool to polythene?

Ans. When two neutral bodies are rubbed together, electrons of one body are transferred to the other. The body which gains electrons is negatively charged and the body which loses electrons is positively charged.

(a) From quantisation of charge q = ne

Here, $q = 3 \times 10^{-7}$C and $e = 1.6 \times 10^{-19}$C

∴ Number of electrons transferred, $n = \frac{q}{e} = \frac{3 \times 10^{-7}}{1.6 \times 10^{-19}} = 1.875 \times 10^{12}$

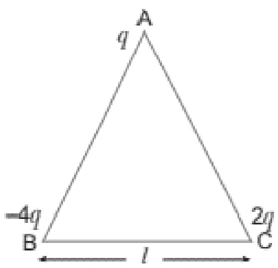
When polythene is rubbed with wool, the polythene becomes negatively charged and wool becomes positively charged. This implies that the electrons are transferred from wool to polythene.

(b) Yes, as electrons have finite mass, the mass is transferred from wool to polythene.

$M = n \times m = 1.875\ 10^{12} \times \times 9.1 \times 10^{-31} = 1.7 \times 10^{-18}$ kg

Q.22 Three-point charges **q, – 4q and 2q** are placed at the vertices of an equilateral triangle ABC of side 'l' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q.

Ans-



Force on charge q due to the charge $-4q$

$$F_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{l^2}\right), \text{ along } AB$$

Force on the charge q, due to the charge $2q$

$$F_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q^2}{l^2}\right), \text{ along CA}$$

The forces F_1 and F_2 are inclined to each other at an angle of 120°

Hence, resultant electric force on charge q

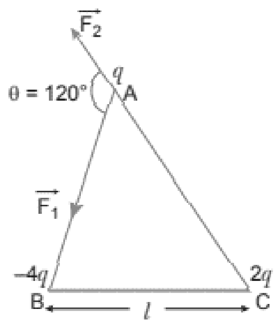
$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°}$$

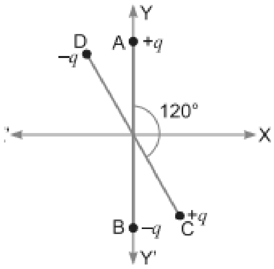
$$= \sqrt{F_1^2 + F_2^2 - F_1F_2}$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}\right)\sqrt{16+4-8}$$

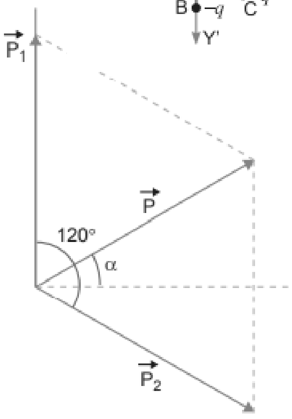
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{2\sqrt{3}q^2}{l^2}\right)$$



Q.23 Two small identical electrical dipoles AB and CD, each of dipole moment 'p' are kept at an angle of 120° as shown in the figure. What is the resultant dipole moment of this combination? If this system is subjected to electric field $(\overrightarrow{E})$ directed along + X direction, what will be the magnitude and direction of the torque acting on this?



Ans-



Resultant dipole moment

$$\vec{p}_r = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos 120^\circ}$$

$$= \sqrt{2p^2 + 2p^2\cos 120^\circ} \qquad (\because p_1 = p_2 = p)$$

$$= \sqrt{2p^2 + (2p^2)\times\left(-\frac{1}{2}\right)} = \sqrt{2p^2 - p^2} = p,$$

Using law of addition of vectors, we can see that the resultant dipole makes an angle of 60° with the y axis or 30° with x - axis.

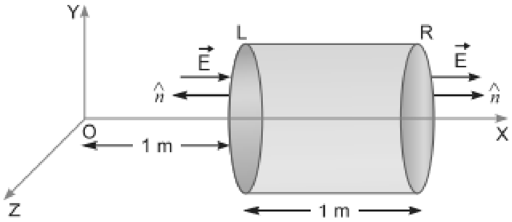
Torque, $\vec{\tau} = \vec{p}\times\vec{E}$ ($\vec{\tau}$ is perpendicular to both $\vec{p}$ and $\vec{E}$)

$$= pE\sin 30^\circ = \frac{1}{2}pE.$$

Direction of torque is into the plane of paper or along positive Z-direction.

Q.24 A hollow cylindrical box of length 1m and area of cross-section 25 cm2 is placed in a three-dimensional coordinate system as shown in the figure. The electric field in the region is given by $\vec{E} = 50xi$, t where E is in NC^{-1} and x is in metres. Find (i) net flux through the cylinder.

(ii) charge enclosed by the cylinder



Ans-

(i) Electric flux through a surface, $\phi = \vec{E}.\vec{S}$

Flux through the left surface, $\phi_L = ES\cos 180^\circ = -ES = (-50x)S$

Since $x = 1$ m, $\phi_L = -50 \times 1 \times 25 \times 10^{-4}$

$= -1250 \times 10^{-4} = -0.125$ N m^2 C^{-1}

Flux through the right surface,

$\phi_R = ES\cos 0^\circ$

$= ES = (50x)S$

Since $x = 2$ m, $\phi_R = 50 \times 2 \times 25 \times 10^{-4} = 2500 \times 10^{-4} = 0.250$ N m^2 C^{-1}

Net flux through the cylinder, $\phi_{net} = \phi_R + \phi_L$

$= 0.250 - 0.125 =$ **0.125 N m² C⁻¹**

(ii) Charge inside the cylinder, by Gauss's Theorem

$$\phi_{net} = \frac{q}{\varepsilon_0} \Rightarrow q = \varepsilon_0\,\phi_{Net}$$

$$= 8.854\times10^{-12}\times0.125 = 8.854\times10^{-12}\times\frac{1}{8} = \mathbf{1.107\times10^{-12}C}$$

Q. 25 Two charges q and –3q are placed fixed on x-axis separated by distance 'd'. Where should a third charge 2q be placed such that, it will not experience any force?

Ans-

Let the charge $2q$ be placed at point P as shown. The force due to q is to the left and that du $-3q$ is to the right.

$$\therefore \qquad \frac{2q^2}{4\pi\varepsilon_0 x^2} = \frac{6q^2}{4\pi\varepsilon_0(d+x)^2} \Rightarrow (d+x)^2 = 3x^2$$

$$\therefore \qquad 2x^2 - 2dx - d^2 = 0 \quad \Rightarrow \quad x = \frac{d}{2} \pm \frac{\sqrt{3}\,d}{2}$$

(–ve sign shows charge $2q$ at p would be lie between q and $-3q$ and hence is unacceptable.)

$$\Rightarrow \qquad x = \frac{d}{2} + \frac{\sqrt{3}\,d}{2} = \frac{d}{2}(1+\sqrt{3}) \text{ to the left of } q.$$