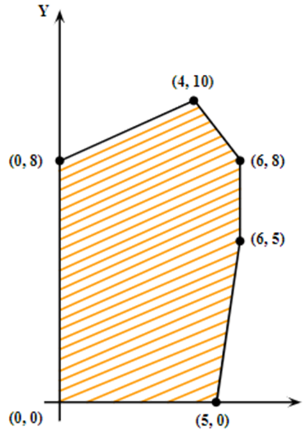
LINEAR PROGRAMMING

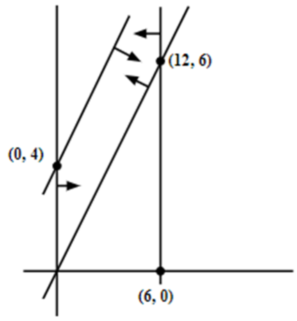
EXERCISE – 1: Basic Subjective Questions

Section–A (1 Mark Questions)

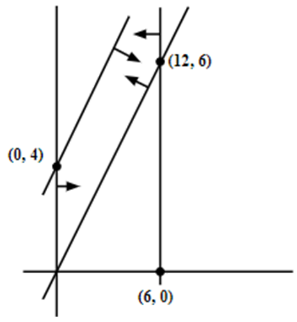
1. The feasible solution for a LPP is shown in following figure. Let $Z = 3x - 4y$ be the objective function. Find where minimum of Z occurs.



2. The feasible region for an LPP is shown in the following figure. Let $F = 3x - 4y$ be the objective function. Then find the Maximum value of F.



3. The feasible region for an LPP is shown in the following figure. Let $F = 3x - 4y$ be the objective function. Then find the Minimum value of F.



4. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let $F = 4x + 6y$ be the objective function. Then find the value of Maximum of F- minimum of F.

5. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px + qy$, where $pq > 0$. Then find the condition on p and q, so that the minimum of Z occurs at (3, 0) and (1, 1).

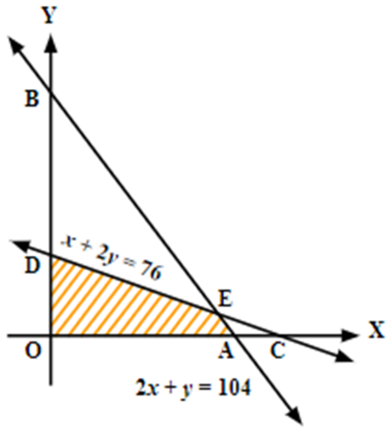
Section–B (2 Marks Questions)

6. Maximize $Z = 3x + 4y$, subject to the constraints $x + y \leq 1, x \geq 0, y \geq 0$.

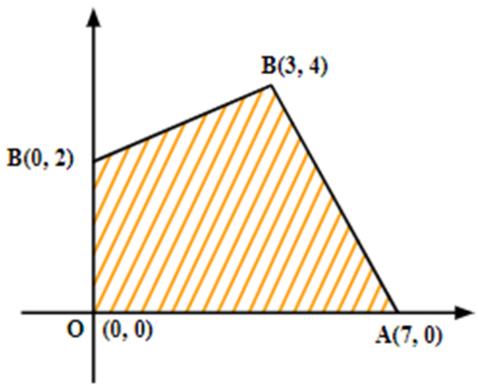
7. Maximize the function $Z = 11x + 7y$, subject to the constraints $x < 3, y \leq 2, x \geq 0$ and $y \geq 0$.

8. Minimize $Z = 13x - 15y$ subject to the constraints $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0$ and $y \geq 0$.

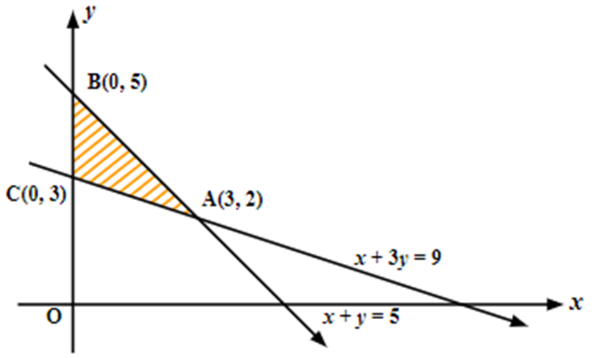
9. Determine the maximum value of $Z = 3x + 4y$, if the feasible region (shaded) for a LPP is shown in following figure.



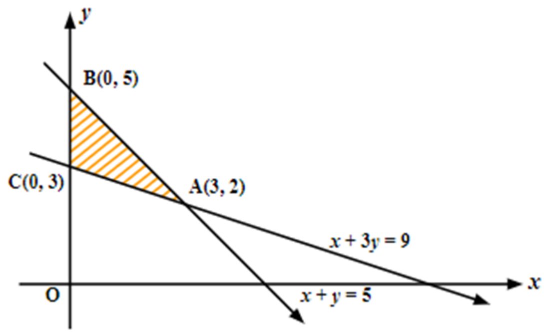
10. Feasible region (shaded) for a LPP is shown in following figure.
Maximize $Z = 5 + 7y$.



11. The feasible region for a LPP is shown in following figure. Find the minimum value of $Z = 11x + 7y$.



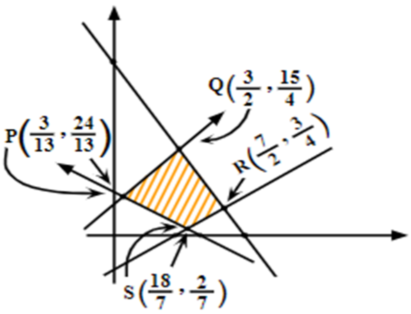
12. The feasible region for a LPP is shown in following figure. Find the maximum value of $Z = 11x + 7y$.



13. The feasible region for a LPP is shown in the following figure. Evaluate $Z = 4x + y$ at each of the corner points of this region. Find the minimum value of Z, if it exists.

Section–C (3 Marks Questions)

14. In following figure, the feasible region (shaded) for a LPP is shown. Determine The maximum and minimum value of $Z = x + 2y$.



15. A manufacturer of electronic circuits has a stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits A and B. Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B requires 10 resistors, 20 transistors and 30 capacitors. If the profit on type A circuit is Rs 50 and that on type B circuit is Rs 60, formulate this problem as a LPP, so that the manufacturer can maximize his profit.

16. A firm has to transport 1200 packages using large vans which can carry 200 Packages each and small vans which can take 80 packages each. The cost for engaging each large van is Rs 400 and each small van is Rs 200. Not more than Rs 3000 is to be spent on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize cost.

17. A company manufactures two types of screws A and B. All the screws have to pass through a threading machine and a slotting machine. A box of type A screws requires 2 min on the threading machine and 3 min on the slotting machine. A box of type B screws requires 8 min on the threading machine and 2 min on the slotting machine. In a week, each machine is available for 60 h. On selling these screws, the company gets a profit of Rs 100 per box on type A screws and Rs 170 per box on type B screws. Formulate this problem as a LPP given that the objective is to maximize profit.

18. A company manufactures two types of sweaters type A and type B. It costs Rs 360 to make a type A sweater and Rs 120 to make a type B sweater. The company can make atmost 300 sweaters and spend atmost Rs 72000 a day. The number of sweaters of type B cannot exceed the number of sweaters of type A by more than 100. The company makes a profit of Rs 200 for each sweater of type A and Rs 120 for every sweater of type B. Formulate this problem as a LPP to maximize the profit to the company.

19. A man rides his motorcycle at the speed of 50 km/h. He has to spend Rs 2 per km on petrol. If he rides it at a faster speed of 80 km/h, the petrol cost increases to Rs 3 per km. He has almost Rs 120 to spend on petrol and one hour's time. He wishes to find the maximum distance that he can travel. Express this problem as a linear programming problem.

20. Solve the following LPP

Maximize $Z = 5x + 3y$, subject to constraints

$3x + 5y \le 15, 5x + 2y \le 10$ and $x \ge 0, y \ge 0$

21. Solve the following linear programming problem

Minimize $Z = 200x + 500y$

Subject to the constraints $x + 2y \ge 10$

$3x + 4y \ge 24$ and $x \ge 0, y \ge 0$

22. Solve the following linear programming problem graphically:

Maximize $Z = 2x + 3y$, Subject to constraints

$4x + 6y \le 60$, $2x + y \le 20$, $x \ge 0, y \ge 0$.

23. Minimize $Z = 3x + 5y$

Subject to the constraints

$x + 3y \ge 3$, $x + y \ge 2$ and $x \ge 0, y \ge 0$.

24. A human requires definite amount of two type of vitamin (Vitamin A and Vitamin B) for balanced food. These vitamins find in two different food product $(F_1$ and $F_2)$. Vitamin contained in one unit of each food product, minimum requirement for balanced food and prices of per unit food product is given in table.

Vitamin	Food Product		Daily Requirement
	F_1	F_2	
A	2	4	40
B	3	2	50
Price per units (In Rs)	3	2.5	

How much unit of both produce is used so that the minimum requirement for balanced food is fulfilled.

Section–D (5 Marks Questions)

25. A firm manufacturing two types of electric items, A and B. Can make a profit of Rs. 20 per unit of A and Rs. 30 per unit of B. Each unit of A requires 3 motors and 2 transformers and each unit of B requires 2 motors and 4 transformers. The total supply of these per month is restricted to 210 motors and 300 transformers. Type B is an expert model requiring a voltage stabilizer which has a supply restricted to 65 units per month. Formulate the LPP for maximum profit and solve it graphically.

26. There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories of P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

From	Cost (In Rs.)		
To	A	B	C
P	16	10	15
Q	10	12	10

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. What will be the minimum transportation cost ?

27. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs. 5 kg to purchase food 'I' and 7/kg to purchase food 'II'. Formulate this problem as a LPP to minimize the cost of such a mixture and solve it graphically.

28. A housewife wishes to mix together two kinds of food, X and Y in such a way that the mixtures contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C.

The vitamin contents of one kg of food are given below:

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	1

One kg of food X costs Rs. 6 and one kg of food Y cost Rs. 10. Find the least cost of the mixture which will produce the diet.

29. One kind of cake requires 300 kg of flour and 15g of fat and another kind of cake requires 150 g of flour and 30 g of fat, find the maximum number of cakes which can be made from 7.5 kg of flour and 600 g of fat assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the LPP solve the problem by graphical method.

30. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hour on machine B to produce a package of nuts, it takes 3 hour of work on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs. 2.50 per package on nuts and Rs. 1 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day? Translate this problem mathematically and then solve it.

EXERCISE – 2: Basic Objective Questions

Section–A (Single Choice Questions)

1. The solution set of the inequation $2x+y>5$ is

(a) half plane that contains the origin

(b) open half plane not containing the origin

(c) whole xy-plane except the points lying on the line $2x+y=5$

(d) None of these

2. Objective function of a LPP is

(a) a constraint
(b) a function to be optimized
(c) a relation between the variable
(d) none of these

3. Which of these following sets are convex?

(a) $\{(x,y): x^2+y^2 \ge 1\}$
(b) $\{(x,y): y^2 \ge x\}$
(c) $\{(x,y): 3x^2+4y^2 \ge 5\}$
(d) $\{(x,y): y \ge 2, y \le 4\}$

4. Let X_1 and X_2 are optimal solutions of a LPP, then

(a) $X=\lambda X_1+(1-\lambda)X_2 \quad \lambda \in R$ is also an optimal solution
(b) $X=\lambda X_1+(1-\lambda)X_2, 0 \le \lambda \le 1$ gives an optimal solution
(c) $X=\lambda X_1+(1+\lambda)X_2, 0 \le \lambda \le 1$ gives an optimal solution
(d) $X=\lambda X_1+(1+\lambda)X_2, \lambda \in R$ given an optimal solution

5. The maximum value of the objective function is attained at the points
(a) given by intersection of inequations with the axes only
(b) given by intersection of inequations with x-axis only
(c) given by corner points of the feasible region
(d) none of these

6. The objective function $Z=4x+3y$ can be maximized subjected to the constraints $3x+4y \le 24, 8x+6y \le 48, x \le 5, y \le 6; x, y \ge 0$

(a) At only one point

(b) At two points only

(c) At an infinite number of points

(d) None of these

7. If the constraints in a linear programming problem are changed

(a) the problem is to be re-evaluated

(b) solution is not defined

(c) the objective function has to be modified

(d) the change in constraints is ignored

8. Which of the following statement is correct?

(a) Every LPP admits an optimal solution

(b) A LPP admits unique optimal solution

(c) If a LPP admits two optimal solution it has an infinite number of optimal solutions

(d) The set of all feasible solutions of a LPP is not a converse set

9. Which of the following is a convex set?

(a) $\{(x,y): 2x+5y<7\}$

(b) $\{(x,y): x^2+y^2 \le 4\}$

(c) $\{x: |x|=5\}$

(d) $\{(x,y): 3x^2+2y^2 \le 6\}$

10. By graphical method, the solution of linear programming problem Maximize $Z=3x_1+5x_2$ Subject to $3x_1+2x_2=18$ $x_1 \le 4$, $x_2 \le 6$, $x_1 \ge 0$, $x_2 \ge 0$, is

(a) $x_1=2, x_2=0, Z=6$
(b) $x_1=2, x_2=6, Z=36$
(c) $x_1=4, x_2=3, Z=27$
(d) $x_1=4, x_2=6, Z=42$

11. The value of objective function is maximum under linear constraints

(a) at the centre of feasible region

(b) at (0, 0)

(c) at any vertex of feasible region

(d) the vertex which is maximum distance from (0, 0)

12. The corner points of the feasible region determined by the following system of linear inequalities:

$2x + y \leq 10,\ x + 3y \leq 15, x, y \geq 0$ are

$(0,0),(5,0),(3,4)$ and $(0,5)$.

Let $Z = px + qy$, where $p, q > 0$.

Condition on p and q so that the maximum of Z occurs at both $(3,4)$ and $(0,5)$ is

(a) $p = q$ (b) $p = 2q$
(c) $p = 3q$ (d) $q = 3p$

13. Which of the following is not a vertex of the positive region bounded by the inequalities $2x + 3y \leq 6$, $5x + 3y \leq 15$ and $x,\ y \geq 0$

(a) $(0,2)$ (b) $(0,0)$
(c) $(3,0)$ (d) None of these

14. For the constraints of a L.P. problem given by $x_1 + 2x_2 \leq 2000, x_1 + x_2 \leq 1500, x_2 \leq 600$, and $x_1, x_2 \geq 0$, which one of the following points does not lie in the positive bounded region

(a) $(1000,\ 0)$ (b) $(0,\ 500)$
(c) $(2,\ 0)$ (d) $(2000,\ 0)$

15. The solution of set of constraints

$x + 2y \geq 11, 3x + 4y \leq 30, 2x + 5y \leq 30,\ x \geq 0, y \geq 0$ includes the point

(a) $(2,3)$ (b) $(3,2)$
(c) $(3,4)$ (d) $(4,3)$

16. The graph of $x \leq 2$ and $y \geq 2$ will be situated in the.

(a) First and second quadrant

(b) Second and third quadrant

(c) First and third quadrant

(d) Third and fourth quadrant

17. The position of points $O(0,0)$ and $P(2,-2)$ in the region of graph of inequation $2x - 3y < 5$, will be

(a) O inside and P outside

(b) O and P both inside

(c) O and P both outside

(d) O outside and P inside

18. A firm produces two types of products A and B. The profit on both is *Rs*. 2 per item. Every product requires processing on machines M_1 and M_2. For A, machines M_1 and M_2 takes 1 minute and 2 minutes respectively and for B, machines M_1 and M_2 takes the time 1 minute each. The machines M_1, M_2 are not available more than 8 hours and 10 hours, any of day, respectively. If the products made x of A and y of B, then the objective function is.

(a) $2x + y$ (b) $x + 2y$
(c) $2x + 2y$ (d) $8x + 10y$

19. In a test of Mathematics, there are two types of questions to be answered–short answered and long answered. The relevant data is given below

Type of ques.	Time taken to solve	Marks	No. of ques
Short answered	5 minutes	3	10
Long answered	10 minutes	5	14

The total marks is 100. Students can solve all the questions. To secure maximum marks, a student solves x short answered and y long answered questions in three hours, then the linear constraints except $x \geq 0, y \geq 0$, are

(a) $5x + 10y \leq 180, x \leq 10, y \leq 14$

(b) $x + 10y \geq 180, x \leq 10,\ y \leq 14$

(c), $5x + 10y \geq 180, x \geq 10,\ y \geq 14$

(d) $5x + 10y \leq 180, x \geq 10,\ y \geq 14$

20. In a test of Mathematics, there are two types of questions to be answered–short answered and long answered. The relevant data is given below

Type of ques.	Time taken to solve	Marks	No. of ques
Short answered	5 minutes	3	10
Long answered	10 minutes	5	14

The total marks is 100. Students can solve all the questions. To secure maximum marks, a student solves x short answered and y long answered questions in three hours, then the objective function is.

(a) $10x+14y$ (b) $5x+10y$

(c) $3x+5y$ (d) $5y+3x$

21. A factory produces two products A and B. In the manufacturing of product A, the machine and the carpenter requires 3 *hour* each and in manufacturing of product B, the machine and carpenter requires 5 *hour* and 3 *hour* respectively. The machine and carpenter work at most 80 *hour* and 50 *hour* per week respectively. The profit on A and B is *Rs*. 6 and 8 respectively. If profit is maximum by manufacturing x and y units of A and B type product respectively, then for the function $6x+8y$ the constraints are.

(a) $x\geq 0, y\geq 0, 5x+3y\leq 80, 3x+2y\leq 50$

(b) $x\geq 0, y\geq 0, 3x+5y\leq 80, 3x+3y\leq 50$

(c) $x\geq 0, y\geq 0, 3x+5y\geq 80, 2x+3y\geq 50$

(d) $x\geq 0, y\geq 0, 5x+3y\geq 80, 3x+2y\geq 50$

22. The sum of two positive integers is at most 5. The difference between two times of second number and first number is at most 4. If the first number is x and second number y, then for maximizing the product of these two numbers, the mathematical formulation is.

(a) $x+y\geq 5, 2y-x\geq 4, x\geq 0, y\geq 0$

(b) $x+y\geq 5, -2x+y\geq 4, x\geq 0, y\geq 0$

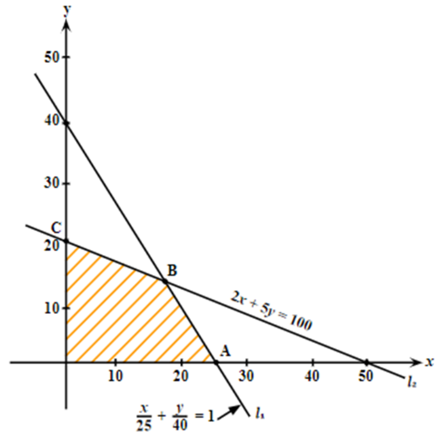
(c) $x+y\leq 5, 2y-x\leq 4, \ x\geq 0, y\geq 0$

(d) None of these

Section–B (Case Study Questions)

Case Study–1

23. Deepa rides her car at 25 km/hr. she has to spend ₹2 per km on diesel and if she rides it at a faster speed of 40 km/hr, the diesel cost increases to ₹5 per km. She has ₹100 to spend on diesel. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr. The feasible region for the LPP is shown below:



(i) Based on the above information, answer the following questions.

What is the point of intersection of line l_1 and l_2.

(a) $\left(\frac{40}{3},\frac{50}{3}\right)$ (b) $\left(\frac{50}{3},\frac{40}{3}\right)$

(c) $\left(\frac{-50}{3},\frac{40}{3}\right)$ (d) $\left(\frac{-50}{3},\frac{-40}{3}\right)$

(ii) The corner points of the feasible region shown in above graph are.

(a) $(0,25),(20,0),\left(\frac{40}{3},\frac{50}{3}\right)$

(b) $(0,0),(25,0),(0,20)$

(c) $(0,0),\left(\frac{40}{3},\frac{50}{3}\right),(0,20)$

(d) $(0,0),(25,0),\left(\frac{50}{3},\frac{40}{3}\right)(0,20)$

(iii) If $Z=x+y$ be the objective function and max $Z=30$. The maximum value occurs at point.

(a) $\left(\frac{50}{3},\frac{40}{3}\right)$ (b) $(0,0)$

(c) $(25,0)$ (d) $(0,20)$

(iv) If $Z=6x-9y$ be the objective function, the maximum value of Z is.

(a) – 20 (b) 150

(c) 180 (d) 20

Case Study–2

24. Suppose a dealer in rural area wishes to purpose a number of sewing machines. He has only ₹5760 to invest and has space for at most 20 items for storage. An electronic sewing machine costs him ₹360 and a manually operated sewing machine ₹240. He can sell an electronic sewing machine at a profit of ₹22 and a manually operated sewing machine at a profit of ₹18.

Based on the above information, answer the following questions.

(i) Let x and y denotes the number of electronic sewing machines and manually operated sewing machines purchased by the dealer. If it is assumed that the dealer purchased at least one of the given machines, then.

(a) $x+y\geq 0$ (b) $x+y<0$

(c) $x+y>0$ (d) $x+y\leq 0$

(ii) Let the constraints in the given problem is represented by the following inequalities.

$x+y\leq 20$

$360x+240y\leq 5760$

$x,y\geq 0$

Then which of the following point lie in its feasible region.

(a) $(0,24)$ (b) $(8,12)$

(c) $(20,2)$ (d) None of these

(iii) If the objective function of the given problem is maximise $z=22x+18y$, then its optimal value occur at.

(a) $(0,0)$ (b) $(16,0)$

(c) $(8,12)$ (d) $(0,20)$

(iv) If an LPP admits optimal solution at two consecutive vertices of a feasible region, then.

(a) the required optimal solution is at the midpoint of the line joining two points.

(b) the optimal solution occurs at every point on the line joining these two points.

(c) the LPP under consideration is not solvable.

(d) the LPP under consideration must be reconstructed.

Case Study-3

25. Let R be the feasible region (convex polygon) for a linear programming problem and let $Z=ax+by$ be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

Based on the above information, answer the following questions.

(i) Objective function of a L.P.P. is.

(a) a constant

(b) a function to be optimized

(c) a relation between the variables

(d) none of these

(ii) Which of the following statement is correct.

(a) Every LPP has at least one optimal solution.

(b) Every LPP has a unique optimal solution.

(c) If an LPP has two optimal solutions, then it has infinitely many solutions.

(d) None of these

(iii) In solving the LPP: "minimize $f=6x+10y$ subject to constraints $x\geq 6, y\geq 2, 2x+y\geq 10, x\geq 0, y\geq 0$" redundant constraints are.

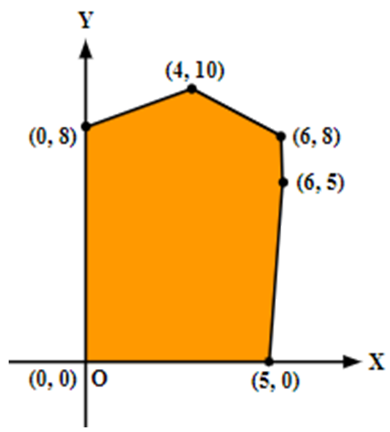
(a) $x\geq 6, y\geq 2$

(b) $2x+y\geq 10, x\geq 0, y\geq 0$

(c) $x\geq 6$

(d) none of these

(iv) The feasible region for a LPP is shown shaded in the figure. Let $Z=3x-4y$ be the objective function. Minimum of Z occurs at.



(a) $(0,0)$ (b) $(0,8)$

(c) $(5,0)$ (d) $(4,10)$

Section–C (Assertion & Reason Type Questions)

26. **Assertion:** The maximum value of $Z = 5x + 2y$ subjected to the constraints $x + y \le 2, 3x + 3y \ge 12; x, y \ge 0$ can't be determined.

Reason: There is no common region in the graph of constraints $x + y \le 2, 3x + 3y \ge 12;\ x, y \ge 0$.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

27. **Assertion:** The coordinates of the point at which the Maximum value of $z = 0.25x_1 + 0.45x_2$ subjected to the constraints $x_1 + 2x_2 \le 300$, $3x_1 + 2x_2 \le 480$ $x_1 \ge 0, x_2 \ge 0$ is attained is $(90,105)$.

Reason: Let R be the feasible region for a linear programming problem and $Z = ax + by$ be the objective function, if R is bounded, then the objective function Z has no solution.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

28. **Assertion:** The objective function $Z = 20x + 10y$ Subject to $x + 2y \le 40$, $3x + y \ge 30$, $4x + 3y \ge 60$ and $x, y \ge 0$ has minimum value 240 at point $(6,12)$.

Reason: The linear inequalities or equation or restrictions on the variables of a linear programming problem is called objective function.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

29. **Assertion:** A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760.00 to invest and has space for at most 20 items. A fan costs him Rs. 360.00 and a sewing machine Rs. 240.00. His expectation is that he can sell a fan at a profit of Rs. 22.00 and a sewing machine at a profit Rs. 18.00. Assuming that he can sell all the items that he can buy, the maximum profit he can make is Rs. 360.

Reason: The total profit is $Z = 22x + 18y$.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

30. **Assertion:** m is the minimum value of Z, If the open half of plane determined by $ax + by > m$ has no point in common with the feasible region.

Reason: In every LPP, we set up the non-negative condition to the decision variables.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

EXERCISE – 3: Previous Year Questions

1. Find graphically, the maximum value of $z = 2x + 5y$, subject to constraints given below: $2x + 4y \leq 8, 3x + y \leq 6, x + y \leq 4; x \geq 0, y \geq 0$

(Delhi 2015)

2. Maximise $z = 8x + 9y$ subject to the constraints given below:

$2x + 3y \leq 6, 3x - 2y \leq 6, y \leq 1; x, y \geq 0$.

(Foreign 2015)

3. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at Rs 7 profit and that of B at a profit of Rs 4. Find the production level per day for maximum profit graphically. **(Delhi 2016)**

4. There are two types of fertilisers 'A' and 'B'. 'A' consists of 12% nitrogen and 5% phosphoric acid whereas 'B' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If 'A' costs Rs 10 per kg and 'B' costs Rs 8 per kg, then graphically determine how much of each type of fertiliser should be used so that the nutrient requirements are met at a minimum cost.

(AI 2016)

5. In order to supplement daily diet, a person wishes to take X and Y tablets.

The contents (in milligrams per tablet) of iron, calcium and vitamins in X and Y are given as below:

Tablets	Iron	Calcium	Vitamin
X	6	3	2
Y	2	3	4

The person needs to supplement at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligrams of vitamins. The price of each tablet of X and Y is Rs 2 and Rs 1 respectively. How many tablets of each type should the person take in order to satisfy the above requirement at the minimum cost? Make an LPP and solve graphically.

(Foreign 2016)

6. A company manufactures three kinds of calculators : A, B and C in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kinds A, 4000 of kind B and 4800 of kind C. The daily output of factory I is of 50 calculators of kind A, 50 calculators of kind B and 30 calculators of kind C. The Daily output of factory II is of 40 calculators of kind A, 20 of kind B and 40 of kind C. The cost per day to run factory I is Rs 12,000 and of factory II is Rs 15,000. How many days do the two factories have to be in operation to produce the order with the minimum cost ? Formulate this problem as an LPP and solve graphically. **(AI 2015)**

7. One kind of cake requires 200 g of flour and 25 g of fat, another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it an LPP and solve it graphically.

(Delhi 2015C, AI 2014C, 2011C)

8. A manufacturer produces nuts and bolts. It take 2 hours work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 2 hours on machine B to produce a package of bolts. He earns a profit of Rs 24 per package on nuts and Rs 18 per package on bolts. How many package of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 10 hours a day. Make an LPP from above and solve it graphically.

(AI 2015C)

9. A dealer in rural area wishes to purchase a number of sewing machines. He has only Rs 5760 to invest and has space for at most 20 items for storage. An electronic sewing machine costs him Rs 360 and a manually operated sewing machine Rs 240. He can sell an electronic sewing machine at a profit of Rs 22 and a manually operated sewing machine at a profit of Rs 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximise his profit ? Make it as an LPP and solve graphically.

(Delhi 2014C, AI 2007)

10. A manufacturing company makes two types of teaching aids A and B of Mathematics for class XII. Each type of A requires 9 labour hours of fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ` 80 on each piece of type A and ` 120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week.

(AI 2014)

11. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/ cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 25 and that from a shade is Rs 15. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit ? Formulate an LPP and solve it graphically.

(Foreign 2014)

12. If a young man rides his motorcycle at 25 km per hour, he had to spend Rs 2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km per hour, the petrol cost increases to Rs 5 per km and rate of pollution also increases. He has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this problem as an LPP. Solve it graphically to find the distance to be covered with different speeds. What value is indicated in this question.

(Delhi 2014C, 2013C)

13. A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profits from crops A and B per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds a liquid herbicide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximise the total profit? Form an LPP from the above information and solve it graphically. Do you agree with the message that the protection of wildlife is utmost necessary to preserve the balance in environment.

(Delhi 2013)

14. A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively; which he uses to produce two types of goods A and B. To produce one unit of A, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of B. If A and B are priced at Rs 100 and Rs 120 per unit respectively, how should he use his resources to maximise the total revenue? Form the above as an LPP and solve graphically. Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate.

(AI 2013)

15. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 5 per kg to purchase food I and Rs 7 per kg to purchase food II. Determine the minimum cost of such a mixture. Formulate the above as an LPP and solve it graphically. **(AI 2012)**

16. A decorative item dealer deals in two items A and B. He has Rs 15,000 to invest and a space to store at the most 80 pieces. Item A costs him Rs 300 and item B costs him Rs 150. He can sell items A and B at respective profits of Rs 50 and Rs 28. Assuming he can sell all he buys, formulate the linear programming problem in order to maximise his profit and solve it graphically. **(Delhi 2012C)**

17. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. If the profit on a racket and on a bat are Rs 20 and Rs 10 respectively, then find the number of tennis rackets and cricket bats that the factory must manufacture to earn maximum profit. Form it as an LPP and solve it graphically. **(Delhi 2011)**

18. A merchant plans to sell two types of personal computers - a desktop model and a portable model that will cost Rs 25,000 and Rs 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit, if he does not want to invest more than Rs 70 lakhs and his profit on the desktop model is Rs 4500 and on the portable model is Rs 5,000. Form it as an LPP and solve it graphically. **(AI 2011)**

19. A library has to accommodate two different types of books on a shelf. The books are 6 cm and 4 cm thick and weigh 1 kg and $1\frac{1}{2}$ kg each respectively. The shelf is 96 cm long and at most can support a weight of 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books ? Form it as an LPP and solve it graphically. **(AI 2010C)**

20. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs 4 per unit and F_2 costs Rs 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consists of mixture of these foods and also meets the minimal nutritional requirements.

(Delhi 2009)

21. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is Rs.15 and on an item of model B is Rs.10. How many of items of each model should be made per day in order to maximize daily profit? Formulate the above LPP and solve it graphically and find the maximum profit. **(Delhi 2019)**

22. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. It costs Rs.50 per kg to produce food 1. Food II contains I unit/kg of vitamin A and 2 units/kg of vitamin C and it costs Rs.70 per kg to produce food II. Formulate this problem as a LPP to minimize the cost of a mixture that will produce the required diet. Also find the minimum cost. **(AI 2019)**

23. A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws 'A' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws 'A' at a profit of Rs 7 and screws 'B' at a profit of Rs.10. Assuming that he can sell all the screws be manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit? Also, find the maximum profit. **(Delhi 2018)**

24. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and and 8 minutes each for assembling. Given that total time for cutting is 3 hours 20 minutes and for assembling 4 hours. The profit for type A souvenir is Rs.100 each and for type B souvenir, profit is Rs.120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit? Formulate the problem as an LPP and solve it graphically. **(AI 2020, Delhi 2020)**

25. Solve the following LPP graphically:

Minimize $z = 5x + 7y$

Subject to the constraints

$2x + y \geq 8$

$x + 2y \geq 10$

$x, y \geq 0$

(Delhi 2019)

Answer Key

EXERCISE-1:

Basic Subjective Questions

1. at (0, 8) **2.** 12 **3.** –16

4. 60 **5.** $p = \frac{q}{2}$

6. The maximum value of z is 4 at (0, 1).
7. The maximum value of z is 47 at (3, 2).
8. The minimum value of z is –30 at (0, 2).
9. The maximum value of z is 196 at (44, 16).
10. The maximum value of z is 43 at (3, 4).
11. The minimum value of z is 21 at (0, 3).
12. The maximum value of z is 47 at (3,3).
13. The minimum value of z is 3 at (0, 3).

14. The maximum and minimum value of z are 9 and $3\frac{1}{7}$ respectively.

15. Maximize $z = 50x + 60y$ subject to the constraints
$2x + y \le 20,\ x + 2y \le 12,\ x + 3y \le 15,\ x \ge 0, y \ge 0$

16. Minimize $z = 400x + 200y$ subject to the constraints
$5x + 2y \ge 30, 2x + y \le 15,\ x \le y, x \ge 0,\ y \ge 0$

17. Maximize $z = 100x + 170y$ subject to the constraints
$x + 4y \le 1800,\ 3x + 2y \le 3600,\ x \ge 0,\ y \ge 0$

18. Maximize $z = 200x + 120y$ subject to the constraints
$3x + y \le 600, x + y \le 300,\ x - y \ge -100,\ x \ge 0,\ y \ge 0$

19. Maximize $z = x + y$ subject to the constraints
$2x + 3y \le 120,\ 8x + 5y \le 400,\ x \ge 0, y \ge 0$

20. Maximum of $z = \frac{235}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$

21. Minimum of $z = 2300$ at (4.3)

22. Maximum of $z = 30$ at $\left(\frac{15}{2}, 5\right)$ and (0, 10)

23. Minimum of $z = 7$ at $\left(\frac{3}{2}, \frac{1}{2}\right)$

24. 15 unit of F_1 and $\frac{3}{2}$ of F_2 ; Minimum value is Rs. 51.25

25. 30 unit of A and 60 unit of B ; Maximum profit is Rs. 2400

26. The minimum transportation cost is Rs. 155

27. Minimum $z = 5x + 7y$ subject to constraints
$2x + y \ge 8$; $x + 2y \ge 10,\ x \ge 0, y \ge 0$
The minimum cost = Rs. 38

28. The least cost of the mixture is Rs. 52

29. Maximise $z = x + y$ subject to constraints
$300x + 150y \le 7500$
$15x - 30y \le 600$
$x \ge 0,\ y \ge 0$

30. Manufacture would make 9 packets of each nut and bolts daily to get maximum profit of Rs. 31.50.

EXERCISE-2:

Basic Objective Questions

1. (b)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (c)
6. (c)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (b)
11. (c)	**12.** (d)	**13.** (d)	**14.** (d)	**15.** (c)
16. (a)	**17.** (a)	**18.** (c)	**19.** (a)	**20.** (c)
21. (b)	**22.** (c)			

23. **(i)** (b) **(ii)** (d) **(iii)** (a) **(iv)** (b)

24. **(i)** (c) **(ii)** (b) **(iii)** (c) **(iv)** (b)

25. **(i)** (b) **(ii)** (c) **(iii)** (a) **(iv)** (b)

26. (c)	**27.** (c)	**28.** (c)	**29.** (d)	**30.** (d)

EXERCISE-3:

Previous Year Questions

1. Maximum value of z is 10 at $(0,2)$.

2. $z_{\max} = 22.6$ at $\left(\frac{30}{13}, \frac{6}{13}\right)$.

3. 2 units of product A and 3 units of product B.

4. The minimum requirement of fertilizer of type A will be 30kg and that of type B will be 210 kg.

5. 1 tablet of Type X and 6 tablets of type Y ; Minimum cost= Rs. 8

6. Minimise $z = 12000x + 15000y$ subject to constraints

$5x + 4y \geq 640,\ 5x + 2y \geq 400,\ 3x + 4y \geq 480,\ x, y \geq 0$

7. Maximise $z = x + y$ subjected to constraints

$2x + y \leq 50,\ x + 2y \leq 40,\ x \geq 0, y \geq 0$

8. 2 packages of nuts and 2 packages of bolts ; maximum profit = Rs.84

9. Maximize $z = 22x + 18y$ subject to constraints $x + y \leq 20,\ 3x + 2y \leq 48,\ x, y \geq 0$; 8 electronic and 12 manually operated sewing machine ; Maximum profit = Rs.392

10. 12 pieces of type A and 6 pieces of type B respectively Maximum profit = Rs. 1680 per weeks

11. 4 Pedestal lamps and 4 wooden shades:

Maximum profit = Rs. 160

12. Maximum distance = 30 km if he rides $\frac{50}{3}$ km at 25 km/hr and $\frac{40}{3}$ km at 40 km/hr.

13. 30 hectares of land to crop A and 20 hectares of land to crop B. Maximum profit = Rs. 495000

14. 3 units of good A and 8 units of B: maximum revenue : 1260 Rs.

15. Minimize $z = 5x + 7y$ subject to constraints

$2x + y \geq 8,\ x + 2y \geq 10,\ x, y \geq 0$; Minimum cost = Rs. 38

16. Maximise $z = 50x + 28y$

$x + y \leq 80$, $2x + y \leq 100$, $x, y \geq 0$

Maximum profit is Rs. 2680 at (20, 60)

17. 4 tennis rackets and 12 cricket bats; Maximum profit is Rs. 200

18. 200 units of desktop model and 50 units of portable model; maximums profit= Rs. 1150000.

19. 12 books of type I and 6 books of type II; maximum number of books = 18

20. Minimize $z = 4x + 6y$ subject to constraints

$3x + 6y \geq 80,\ 4x + 3y \geq 100,\ x, y \geq 0$; Minimum cost = Rs. 104.

21. 10 terms of model A and 20 terms of model 13; maximum daily profit = Rs. 350

22. Minimise $z = 50x + 70y$ subject to constraints

$2x + y \geq 8,\ x + 2y \geq 10,\ x, y \geq 0;$

2 units of food I and 4 units of food II; maximum cost = Rs. 380

23. 30 packets of screw A and 20 packets of screw B; maximum profit = Rs. 410

24. 8 souvenirs of type A and 20 souvenirs of type B; maximum profit = Rs 32,000

25. $z_{\min} = 38$ at (2.4.)