Chapter

3

Statics

CONTENTS

Varignon

Historically, Mechanics was the earliest branch of Physics to be developed as an exact science. The Laws of levers and of fluids *were known to the Greeks in third century B.C. The fundamental theorem of statics, or rather another form of its, viz., the Triangle of Forces was first enunciated by Stevinus of Bruges in the year 1586. It was, however, left to Galileo (1564-1642) and Newton (1642-1727) to formulate the* laws of mechanics *and to place mechanics on a sound footing as an exact science. Newton was also the first to formulate correctly the* law of universal gravitation. *Following Newton's time, important contributions to mechanics were made by* Euler, D' Alembert, Lagrange, Laplace, Poinsot *and* Coriolis. *All these contributions were however within framework of Newton's laws of motion made.*

Statics

3.1 Introduction

Statics is that branch of mechanics which deals with the study of the system of forces in equilibrium.

Matter : Matter is anything which can be perceived by our senses of which can exert, or be acted on, by forces.

Force : Force is anything which changes, or tends to change, the state of rest, or uniform motion, of a body. To specify a force completely four things are necessary they are magnitude, direction, sense and point of application. Force is a vector quantity.

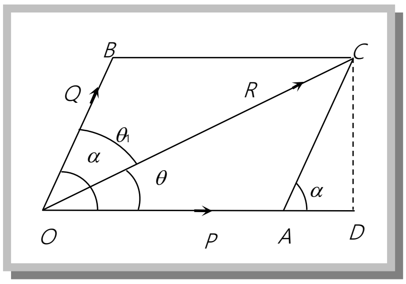
3.2 Parallelogram law of Forces

If two forces, acting at a point, be represented in magnitude and direction by the two sides of a parallelogram drawn from one of its angular points, their resultant is represented both in magnitude and direction of the parallelogram drawn through that point.

If *OA* and *OB* represent the forces *P* and *Q* acting at a point *O* and inclined to each other at an angle α. If *R* is the resultant of these forces represented by the diagonal *OC* of the parallelogram *OACB* and *R* makes an angle θ with *P* *i.e.* $\angle COA = \theta$, then $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$ and $\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$

The angle θ_1 which the resultant *R* makes with the direction of the force *Q* is given by

$$\theta_1 = \tan^{-1}\left(\frac{P\sin\alpha}{Q + P\cos\alpha}\right)$$



Case (i) : If $P = Q$

$\therefore R = 2P\cos(\alpha/2)$ and $\tan\theta = \tan(\alpha/2)$ or $\theta = \alpha/2$

Case (ii) : If $\alpha = 90°$, *i.e.* forces are perpendicular

$\therefore R = \sqrt{P^2 + Q^2}$ and $\tan\theta = \frac{Q}{P}$

Case (iii) : If $\alpha = 0°$, *i.e.* forces act in the same direction

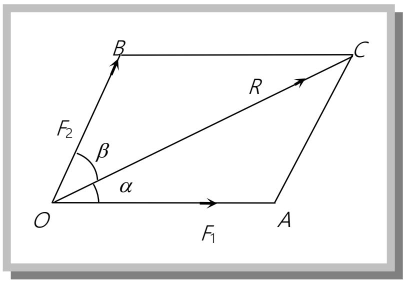
$\therefore R_{max} = P + Q$

Case (iv) : If $\alpha = 180°$, *i.e.* forces act in opposite direction

$\therefore R_{min} = P - Q$

Note : ❑ The resultant of two forces is closer to the larger force.

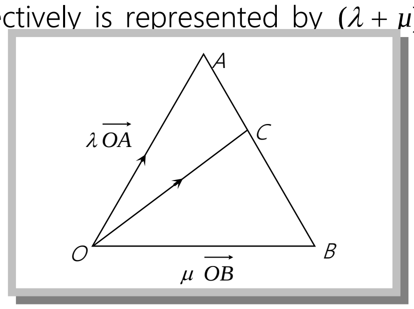
- ❑ The resultant of two equal forces of magnitude *P* acting at an angle α is $2P\cos\frac{\alpha}{2}$ and it bisects the angle between the forces.
- ❑ If the resultant *R* of two forces *P* and *Q* acting at an angle α makes an angle θ with the direction of *P*, then $\sin\theta = \frac{Q\sin\alpha}{R}$ and $\cos\theta = \frac{P + Q\cos\alpha}{R}$
- ❑ If the resultant *R* of the forces *P* and *Q* acting at an angle α makes an angle θ with the direction of the force *Q*, then $\sin\theta = \frac{P\sin\alpha}{R}$ and $\cos\theta = \frac{Q + P\sin\alpha}{R}$



- ❑ **Component of a force in two directions :** The component of a force *R* in two directions making angles α and β with the line of action of *R* on and opposite sides of it are

$$F_1 = \frac{OC.\sin\beta}{\sin(\alpha+\beta)} = \frac{R\sin\beta}{\sin(\alpha+\beta)} \text{ and } F_2 = \frac{OC.\sin\alpha}{\sin(\alpha+\beta)} = \frac{R.\sin\alpha}{\sin(\alpha+\beta)}$$

λ-μ theorem : The resultant of two forces acting at a point *O* in directions *OA* and *OB* represented in magnitudes by $\lambda.OA$ and $\mu.OB$ respectively is represented by $(\lambda+\mu)OC$, where *C* is a point in *AB* such that $\lambda.CA = \mu.CB$



Important Tips

☞ *The forces P, Q, R act along the sides BC, CA, AB of ΔABC.*

Their resultant passes through.

(a) Incentre, if $P+Q+R=0$

(b) Circumcentre, if $P\cos A+Q\cos B+R\cos C=0$

(c) Orthocentre, if $P\sec A+Q\sec B+R\sec C=0$

(d) Centroid, if $P\text{ cosec }A+Q\text{ cosec }B+R\text{ cosec }C=0$

or $\frac{P}{a}=\frac{Q}{b}=\frac{R}{c}$

Example: 1 Forces *M* and *N* acting at a point *O* make an angle 150°. Their resultant acts at *O* has magnitude 2 units and is perpendicular to *M*. Then, in the same unit, the magnitudes of *M* and *N* are [BIT Ranchi 1993]

(a) $2\sqrt{3},4$

(b) $\sqrt{\frac{3}{2}},2$

(c) 3, 4

(d) 4. 5

Solution: (a) We have, $2^2=M^2+N^2+2MN\cos 150^\circ \Rightarrow 4=M^2+N^2-\sqrt{3}MN$ .....(i)

and, $\tan\frac{\pi}{2}=\frac{M\sin 150^\circ}{M+N\cos 150^\circ}\Rightarrow M+N\cos 150^\circ=0$

$\Rightarrow M-N\frac{\sqrt{3}}{2}=0\Rightarrow M=\frac{N\sqrt{3}}{2}$ .....(ii)

Solving (i) and (ii), we get $M=2\sqrt{3}$ and $N=4$.

Example: 2 If the resultant of two forces of magnitude *P* and 2*P* is perpendicular to *P*, then the angle between the forces is [Roorkee 1997]

(a) 2π/3 (b) 3π/4 (c) 4π/5 (d) 5π/6

Solution: (a) Let the angle between the forces *P* and 2*P* be *α*. Since the resultant of *P* and 2*P* is perpendicular to *P*. Therefore,

$\tan\pi/2=\frac{2P\sin\alpha}{P+2P\cos\alpha}\Rightarrow P+2P\cos\alpha=0\Rightarrow \cos\alpha=\frac{-1}{2}\Rightarrow\alpha=\frac{2\pi}{3}$

Example: 3 If the line of action of the resultant of two forces *P* and *Q* divides the angle between them in the ratio 1 : 2, then the magnitude of the resultant is [Roorkee 1993]

(a) $\frac{P^2+Q^2}{P}$ (b) $\frac{P^2+Q^2}{Q}$ (c) $\frac{P^2-Q^2}{P}$ (d) $\frac{P^2-Q^2}{Q}$

Solution: (d) Let 3θ be the angle between the forces *P* and *Q*. It is given that the resultant *R* of *P* and *Q* divides the angle between them in the ratio 1 : 2. This means that the resultant makes an angle θ with the direction of *P* and angle 2θ with the direction of *Q*.

Therefore, $P = \frac{R \sin 2\theta}{\sin 3\theta}$ and $\theta = \frac{R \sin\theta}{\sin 3\theta}$

$\Rightarrow \frac{P}{Q} = \frac{\sin 2\theta}{\sin\theta} = 2\cos\theta$ .....(i)

Also $Q = \frac{R\sin\theta}{\sin 3\theta} \Rightarrow Q = \frac{R}{3 - 4\sin^2\theta}$

$\Rightarrow \frac{R}{Q} = 3 - 4\sin^2\theta \Rightarrow \frac{R}{Q} = -1 + 4\cos^2\theta \Rightarrow \frac{R}{Q} + 1 = (2\cos\theta)^2$ .....(ii)

From (i) and (ii), we get, $\left(\frac{P}{Q}\right)^2 = \frac{R}{Q} + 1 \Rightarrow \frac{R}{Q} = \frac{P^2 - Q^2}{Q^2} \Rightarrow R = \frac{P^2 - Q^2}{Q}$

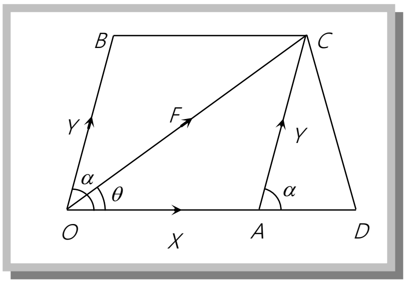
Example: 4 Two forces *X* and *Y* have a resultant *F* and the resolved part of *F* in the direction of *X* is of magnitude *Y*. Then the angle between the forces is

(a) $\sin^{-1}\sqrt{\frac{X}{2Y}}$ (b) $2\sin^{-1}\sqrt{\frac{X}{2Y}}$ (c) $4\sin^{-1}\sqrt{\frac{X}{2Y}}$ (d) None of these

Solution: (b) Let *OA* and *OB* represent two forces *X* and *Y* respectively. Let α be the angle between them and θ, the angle which the resultant *F* (represented by *OC*) makes with *OA*.

Now, resolved part of *F* along *OA*.

$F\cos\theta = OC \times \frac{OD}{OC} = OD = OA + AD = OA + AC\cos\alpha = X + Y\cos\alpha$



But resolved part of *F* along *OA* is given to by *Y*.

$\therefore Y = X + Y\cos\alpha$ or $Y(1-\cos\alpha) = X \Rightarrow Y.2\sin^2\frac{\alpha}{2} = X, \therefore \sin^2\alpha/2 = \frac{X}{2Y}$

i.e., $\sin\frac{\alpha}{2} = \sqrt{\frac{X}{2Y}}$ or $\frac{\alpha}{2} = \sin^{-1}\sqrt{\frac{X}{2Y}}$

Thus, $\alpha = 2\sin^{-1}\sqrt{\frac{X}{2Y}}$

Example: 5 The greatest and least magnitude of the resultant of two forces of constant magnitude are *F* and *G*. When the forces act an angle 2α, the resultant in magnitudes is equal to [UPSEAT 2001]

(a) $\sqrt{F^2\cos^2\alpha + G^2\sin^2\alpha}$ (b) $\sqrt{F^2\sin\alpha + G^2\cos^2\alpha}$ (c) $\sqrt{F^2 + G^2}$ (d) $\sqrt{F^2 - G^2}$

Solution: (a) Greatest resultant $= F = A + B$

Least resultant $= G = A - B$

On solving, we get $A = \frac{F+G}{2}$, $B = \frac{(F-G)}{2}$

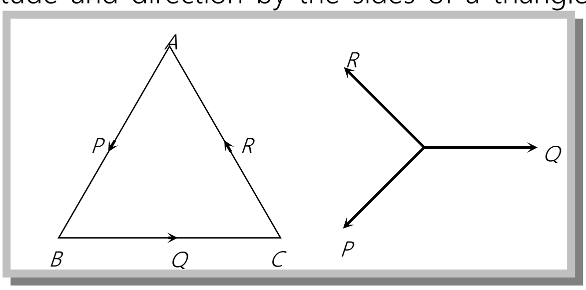
where *A* and *B* act an angle 2α, the resultant

$R = \sqrt{A^2 + B^2 + 2AB\cos 2\alpha} \Rightarrow R = \sqrt{F^2\cos^2\alpha + G^2\sin^2\alpha}$

3.3 Triangle law of Forces

If three forces, acting at a point, be represented in magnitude and direction by the sides of a triangle, taken in order, they will be in equilibrium.

Here $\overrightarrow{AB} = P$, $\overrightarrow{BC} = Q$, $\overrightarrow{CA} = R$



In triangle *ABC*, we have $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=0$

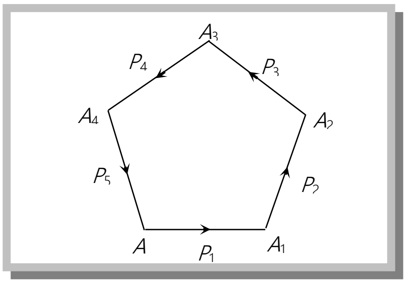
$\Rightarrow P+Q+R=0$

Hence the forces *P,Q,R* are in equilibrium.

Converse : If three forces acting at a point are in equilibrium, then they can be represented in magnitude and direction by the sides of a triangle, taken in order.

3.4 Polygon law of Forces

If any number of forces acting on a particle be represented in magnitude and direction by the sides of a polygon taken in order, the forces shall be in equilibrium.



Example: 6 *D* and *E* are the mid-points of the sides *AB* and *AC* respectively of a ΔABC. The resultant of the forces is represented by $\overrightarrow{BE}$ and $\overrightarrow{DC}$ is

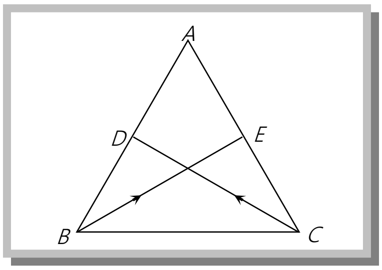
(a) $\frac{3}{2}\overrightarrow{AC}$ (b) $\frac{3}{2}\overrightarrow{CA}$ (c) $\frac{3}{2}\overrightarrow{AB}$ (d) $\frac{3}{2}\overrightarrow{BC}$

Solution: (d) We have,

$\overrightarrow{BE}+\overrightarrow{DC}=\left(\overrightarrow{BC}+\overrightarrow{CE}\right)+\left(\overrightarrow{DB}+\overrightarrow{BC}\right)$

$=2\overrightarrow{BC}+\frac{1}{2}\overrightarrow{CA}+\frac{1}{2}\overrightarrow{AB}=2\overrightarrow{BC}+\frac{1}{2}\left(\overrightarrow{CA}+\overrightarrow{AB}\right)$

$=2\overrightarrow{BC}+\frac{1}{2}\overrightarrow{CB}=2\overrightarrow{BC}-\frac{1}{2}\overrightarrow{BC}=\frac{3}{2}\overrightarrow{BC}$



Example: 7 *ABCDE* is pentagon. Forces acting on a particle are represented in magnitude and direction by $\overrightarrow{AB},\overrightarrow{BC},\overrightarrow{CD},2\overrightarrow{DE},\overrightarrow{AD}$, and $\overrightarrow{AE}$. Their resultant is given by

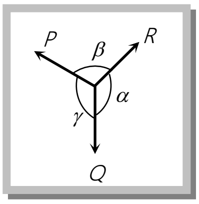
(a) $\overrightarrow{AE}$ (b) $2\overrightarrow{AB}$ (c) $3\overrightarrow{AE}$ (d) $4\overrightarrow{AE}$

Solution: (c) We have, $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+2\overrightarrow{DE}+\overrightarrow{AD}+\overrightarrow{AE}=\left(\overrightarrow{AB}+\overrightarrow{BC}\right)+\left(\overrightarrow{CD}+\overrightarrow{DE}\right)+\left(\overrightarrow{AD}+\overrightarrow{DE}\right)+\overrightarrow{AE}$

$=\left(\overrightarrow{AC}+\overrightarrow{CE}\right)+\overrightarrow{AE}+\overrightarrow{AE}=\overrightarrow{AE}+\overrightarrow{AE}+\overrightarrow{AE}=3\overrightarrow{AE}$.

3.5 Lami's Theorem

If three forces acting at a point be in equilibrium, each force is proportional to the sine of the angle between the other two. Thus if the forces are P, Q and R, α, β, γ be the angles between Q and R, R and P, P and Q respectively. If the forces are in equilibrium, we have, $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$.

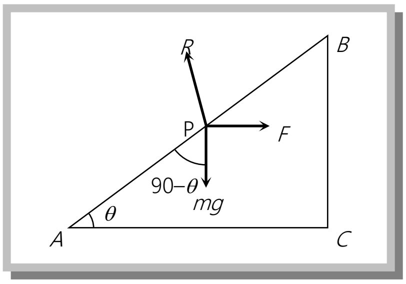


The converse of this theorem is also true.

Example: 8 A horizontal force F is applied to a small object P of mass m on a smooth plane inclined to the horizon at an angle θ. If F is just enough to keep P in equilibrium, then $F =$ [BIT Ranchi 1993]

(a) $mg\cos^2\theta$ (b) $mg\sin^2\theta$ (c) $mg\cos\theta$ (d) $mg\tan\theta$

Solution: (d) By applying Lami's theorem at P, we have



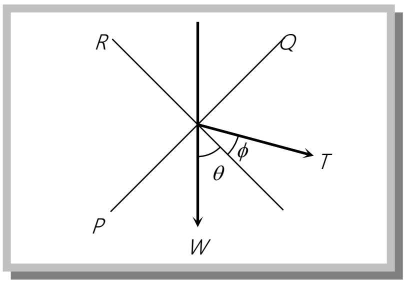
$$\frac{R}{\sin 90^\circ} = \frac{F}{\sin(180^\circ - \theta)} = \frac{mg}{\sin(90^\circ + \theta)}$$

$$\Rightarrow \frac{R}{1} = \frac{F}{\sin\theta} = \frac{mg}{\cos\theta} \Rightarrow F = mg\tan\theta$$

Example: 9 A kite of weight W is flying with its string along a straight line. If the ratios of the resultant air pressure R to the tension T in the string and to the weight of the kite are $\sqrt{2}$ and $(\sqrt{3}+1)$ respectively, then [Roorkee 1990]

(a) $T = (\sqrt{6}+\sqrt{2})W$ (b) $R = (\sqrt{3}+1)W$ (c) $T = \frac{1}{2}(\sqrt{6}-\sqrt{2})W$ (d) $R = (\sqrt{3}-1)W$

Solution: (b) From Lami's theorem,



$$\frac{R}{\sin(\theta+\phi)} = \frac{T}{\sin(180^o - \theta)} = \frac{W}{\sin(180^o - \phi)}$$

$$\Rightarrow \frac{R}{\sin(\theta+\phi)} = \frac{T}{\sin\theta} = \frac{W}{\sin\phi} \quad .....(i)$$

Given, $\frac{R}{T} = \sqrt{2}$ .....(ii) and $\frac{R}{W} = \sqrt{3}+1$ .....(iii)

Dividing (iii) by (ii), we get $\frac{\frac{R}{W}}{\frac{R}{T}} = \frac{\sqrt{3}+1}{\sqrt{2}}$

$$\Rightarrow \frac{T}{W} = \frac{\sqrt{3}+1}{\sqrt{2}} \Rightarrow T = \frac{\sqrt{3}+1}{\sqrt{2}}W = \frac{1}{2}(\sqrt{6}+\sqrt{2})W \Rightarrow R = T\sqrt{2} = \frac{\sqrt{2}}{\sqrt{2}}(\sqrt{3}+1)W = (\sqrt{3}+1)W$$

Example: 10 Three forces $\vec{P}, \vec{Q}\, \vec{R}$ are acting at a point in a plane. The angles between $\vec{P}$ and $\vec{Q}$ and $\vec{Q}$ and $\vec{R}$ are 150° and 120° respectively, then for equilibrium, forces P, Q, R are in the ratio [MNR 1991; UPSEAT 2000]

(a) $1:2:\sqrt{3}$ (b) $1:2:3$ (c) 3:2:1 (d) $\sqrt{3}:2:1$

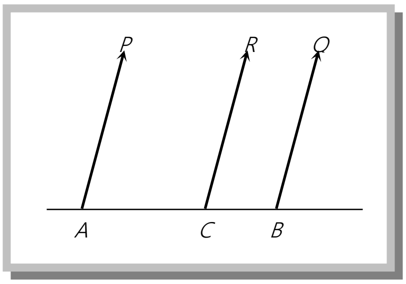
Solution: (d) Clearly, the angle between P and R is $360^\circ - (150^\circ + 120^\circ) = 90^\circ$. By Lami's theorem,

$$\frac{P}{\sin 120°} = \frac{Q}{\sin 90°} = \frac{R}{\sin 150°} \Rightarrow \frac{P}{\sqrt{3}/2} = \frac{Q}{1} = \frac{R}{1/2} \Rightarrow \frac{P}{\sqrt{3}} = \frac{Q}{2} = \frac{R}{1}$$

3.6 Parallel Forces

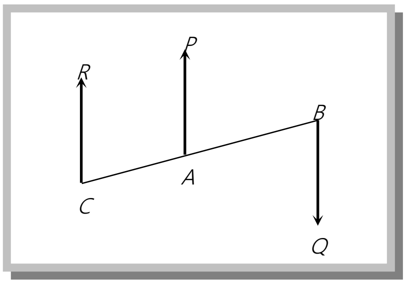
(1) **Like parallel forces :** Two parallel forces are said to be like parallel forces when they act in the same direction.

The resultant R of two like parallel forces P and Q is equal in magnitude of the sum of the magnitude of forces and R acts in the same direction as the forces P and Q and at the point on the line segment joining the point of action P and Q, which divides it in the ratio $Q : P$ internally.



(2) **Two unlike parallel forces :** Two parallel forces are said to be unlike if they act in opposite directions.

If P and Q be two unlike parallel force acting at A and B and P is greater in magnitude than Q. Then their resultant R acts in the same direction as P and acts at a point C on BA produced. Such that $R = P - Q$ and $P.CA = Q.CB$



Then in this case C divides BA externally in the inverse ratio of the forces,

$$\frac{P}{CB} = \frac{Q}{CA} = \frac{P-Q}{CB-CA} = \frac{R}{AB}$$

Important Tips

☞ *If three like parallel forces P, Q, R act at the vertices A, B, C repectively of a triangle ABC, then their resultant act at the*

(i) Incentre of ΔABC, if $\frac{P}{a} = \frac{Q}{b} = \frac{R}{c}$

(ii) Circumcentre of ΔABC, if $\frac{P}{\sin 2A} = \frac{Q}{\sin 2B} = \frac{R}{\sin 2C}$

(iii) Orthocentre of ΔABC, if $\frac{P}{\tan A} = \frac{Q}{\tan B} = \frac{R}{\tan C}$

(iv) Centroid of ΔABC, if $P = Q = R$.

Example: 11 Three like parallel forces *P, Q, R* act at the corner points of a triangle *ABC*. Their resultant passes through the circumcentre,if [Rookee 1995]

(a) $\frac{P}{a}=\frac{Q}{b}=\frac{R}{c}$ (b) $P=Q=R$ (c) $P+Q+R=0$ (d) None of these

Solution: (c) Since the resultant passes through the circumcentre of *ΔABC*, therefore, the algebraic sum of the moments about it, is zero.

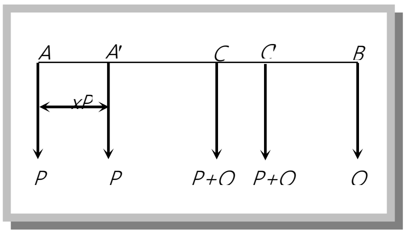
Hence, $P+Q+R=0$.

Example: 12 *P* and *Q* are like parallel forces. If *P* is moved parallel to itself through a distance *x*, then the resultant of *P* and *Q* moves through a distance. [Rookee 1995]

(a) $\frac{Px}{P+Q}$ (b) $\frac{Px}{P-Q}$ (c) $\frac{Px}{P+2Q}$ (d) None of these

Solution: (a) Let the parallel forces *P* and *Q* act at *A* and *B* respectively. Suppose the resultant *P* + *Q* acts at *C*.

Then, $AC=\left(\frac{AB}{P+Q}\right)Q$ .....(i)



If *P* is moved parallel to itself through a distance *x i.e.* at *A'*.

Suppose the resultant now acts at *C'*. Then ,

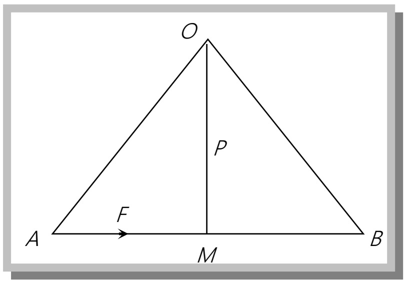
$A'C'=\left(\frac{A'B}{P+Q}\right)Q \Rightarrow A'C'=\left(\frac{AB-x}{P+Q}\right)Q$ .....(ii)

Now $CC'=AC'-AC=AA'+A'C'-AC$

$\Rightarrow CC'=x+\left(\frac{AB-x}{P+Q}\right)Q-\left(\frac{AB}{P+Q}\right)Q \Rightarrow CC'=x-\frac{Qx}{P+Q} \Rightarrow CC'=\frac{Px}{P+Q}$

3.7 Moment

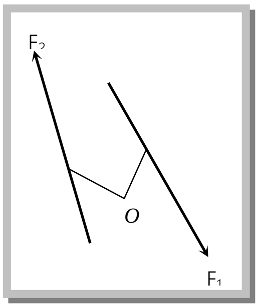
The moment of a force about a point *O* is given in magnitude by the product of the forces and the perpendicular distance of *O* from the line of action of the force.



If *F* be a force acting a point *A* of a rigid body along the line *AB* and *OM* (= *p*) be the perpendicular distance of the fixed point *O* from *AB*, then the moment of force about $O=F.p=AB\times OM=2\left[\frac{1}{2}(AB\times OM)\right]=2(\text{area of }\Delta AOB)$

The S.I. unit of moment is *Newton-meter* (*N-m*).

(1) **Sign of the moment :** The moment of a force about a point measures the tendency of the force to cause rotation about that point. The tendency of the force F_1 is to turn the lamina in the clockwise direction and of the force F_2 is in the anticlockwise direction.



The usual convention is to regard the moment which is anticlockwise direction as positive and that in the clockwise direction as negative.

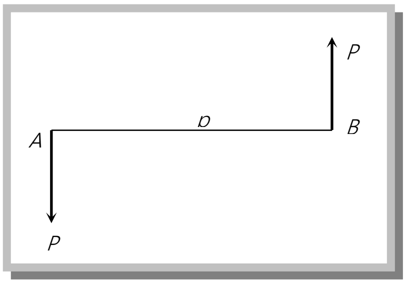
(2) **Varignon's theorem :** The algebraic sum of the moments of any two coplanar forces about any point in their plane is equal to the moment of their resultant about the same point.

Note :
- Thy algebraic sum of the moments of any two forces about any point on the line of action of their resultant is zero.
- Conversely, if the algebraic sum of the moments of any two coplanar forces, which are not in equilibrium, about any point in their plane is zero, their resultant passes through the point.
- If a body, having one point fixed, is acted upon by two forces and is at rest. Then the moments of the two forces about the fixed point are equal and opposite.

3.8 Couples

Two equal unlike parallel forces which do not have the same line of action, are said to form a couple.

Example : Couples have to be applied in order to wind a watch, to drive a gimlet, to push a cork screw in a cork or to draw circles by means of pair of compasses.

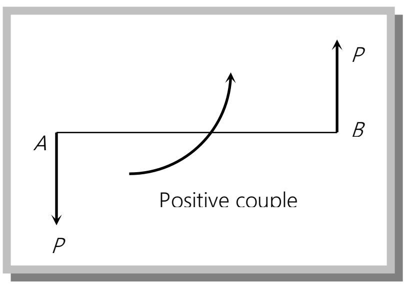


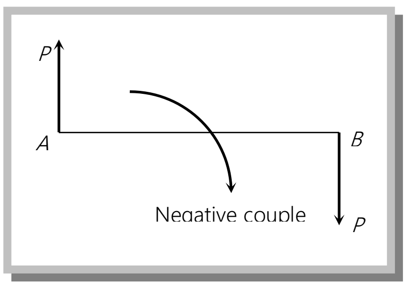
(1) **Arm of the couple :** The perpendicular distance between the lines of action of the forces forming the couple is known as the arm of the couple.

(2) **Moment of couple :** The moment of a couple is obtained in magnitude by multiplying the magnitude of one of the forces forming the couple and perpendicular distance between the lines of action of the force. The perpendicular distance between the forces is called the arm of the couple. The moment of the couple is regarded as positive or negative according as it has a tendency to turn the body in the anticlockwise or clockwise direction.

Moment of a couple = Force × Arm of the couple = $P.p$

(3) **Sign of the moment of a couple :** The moment of a couple is taken with positive or negative sign according as it has a tendency to turn the body in the anticlockwise or clockwise direction.



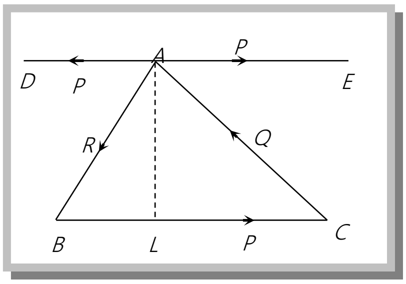


Note : ❑ A couple can not be balanced by a single force, but can be balanced by a couple of opposite sign.

3.9 Triangle theorem of Couples

If three forces acting on a body be represented in magnitude, direction and line of action by the sides of triangle taken in order, then they are equivalent to a couple whose moment is represented by twice the area of triangle.

Consider the force P along AE, Q along CA and R along AB. These forces are three concurrent forces acting at A and represented in magnitude and direction by the sides BC, CA and AB of ΔABC. So, by the triangle law of forces, they are in equilibrium.



The remaining two forces P along AD and P along BC form a couple, whose moment is $m = P.AL = BC.AL$

Since $\frac{1}{2}(BC.AL) = 2\left(\frac{1}{2}\text{area of the } \Delta ABC\right)$

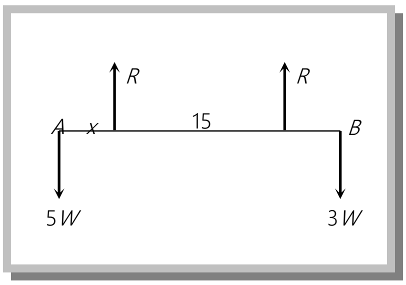
$\therefore$ Moment = $BC.AL$ = 2 (Area of ΔABC)

Example: 13 A light rod AB of length 30 *cm.* rests on two pegs 15 *cm.* apart. At what distance from the end A the pegs should be placed so that the reaction of pegs may be equal when weight $5W$ and $3W$ are suspended from A and B respectively

[Roorkee 1995, UPSEAT 2001]

(a) 1.75 *cm.*, 15.75 *cm.* (b) 2.75 *cm.*, 17.75 *cm.* (c) 3.75 *cm.*, 18.75 *cm.* (d) None of these

Solution: (c) Let R, R be the reactions at the pegs P and Q such that $AP = x$



Resolving all forces vertically, we get

$R + R = 8W \Rightarrow R = 4W$

Take moment of forces about A, we get

$R.AP + R.AQ = 3W.AB$

$\Rightarrow 4W.x + 4W.(x+15) = 3W.30$

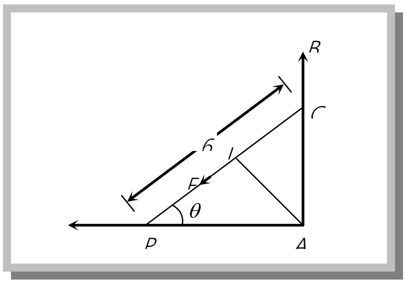
$\Rightarrow x = 3.75cm$

$\therefore AP = x = 3.75cm$ and $AQ = 18.75cm$

Example: 14 At what height from the base of a vertical pillar, a string of length 6 *metres* be tied, so that a man sitting on the ground and pulling the other end of the string has to apply minimum force to overturn the pillar

(a) 1.5 *metres* (b) $3\sqrt{2}$ *metres* (c) $3\sqrt{3}$ *metres* (d) $4\sqrt{2}$ *metres*

Solution: (b) Let the string be tied at the point *C* of the vertical pillar, so that *AC* = *x*



Now moment of *F* about *A* = *F. AL*

= *F. AP* sinθ

= *F.*6 cosθ sin θ

= 3 *F* sin 2θ

To overturn the pillar with maximum (fixed) force *F*, moment is maximum if

sin 2θ = 1 (max.)

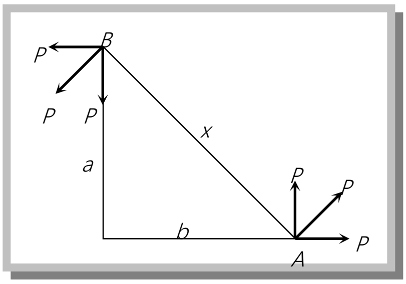
$\Rightarrow 2\theta = 90°$, *i.e.* $\theta = 45°$

$\therefore AC = PC \sin 45° = 6.\frac{1}{\sqrt{2}} = 3\sqrt{2}$

Example: 15 Two unlike parallel forces acting at points *A* and *B* form a couple of moment *G*. If their lines of action are turned through a right angle, they form a couple of moment *H*. Show that when both act at right angles to *AB*, they form a couple of moment.

(a) *GH* (b) $G^2 + H^2$ (c) $\sqrt{G^2 + H^2}$ (d) None of these

Solution: (c) We have, $Pa = G$ and $Pb = H$ .....(i)



Clearly, $a^2 + b^2 = x^2$

$\Rightarrow x = \sqrt{\frac{G^2}{P^2} + \frac{H^2}{P^2}}$ [from (i)]

$\Rightarrow Px = \sqrt{G^2 + H^2}$

Hence, required moment = $\sqrt{G^2 + H^2}$

Example: 16 The resultant of three forces represented in magnitude and direction by the sides of a triangle *ABC* taken in order with *BC* = 5 *cm*, *CA* = 5 *cm*, and *AB* = 8 *cm*, is a couple of moment

(a) 12 units (b) 24 units (c) 36 units (d) 16 units

Solution: (b) Resultant of three forces represented in magnitude and direction by the sides of a triangle taken in order is a couple of moment equal to twice the area of triangle.

$\therefore$ the resultant is a couple of moment = 2 × (area of ΔABC)

Here, *a* = 5 *cm*, *b* = 5 *cm* and c = 8 *cm*

$\therefore$ 2*S* = 5 + 5 + 8 $\Rightarrow$ *S* = 9.

Area = $\sqrt{S(S-a)(S-b)(S-c)} = \sqrt{9(9-5)(9-5)(9-8)} = 12$

$\therefore$ Required moment = 2 (12) = 24 units.

3.10 Equilibrium of Coplanar Forces

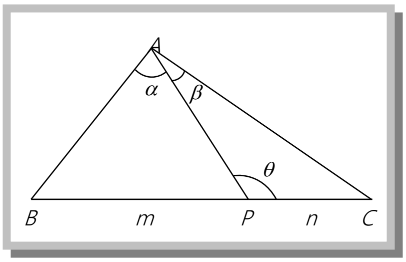
(1) If three forces keep a body in equilibrium, they must be coplanar.

(2) If three forces acting in one plane upon a rigid body keep it in equilibrium, they must either meet in a point or be parallel.

(3) When more than three forces acting on a rigid body, keep it in equilibrium, then it is not necessary that they meet at a point. The system of forces will be in equilibrium if there is neither translatory motion nor rotatory motion.

i.e. $X = 0$, $Y = 0$, $G = 0$ or $R = 0$, $G = 0$.

(4) A system of coplanar forces acting upon a rigid body will be in equilibrium if the algebraic sum of their resolved parts in any two mutually perpendicular directions vanish separately, and if the algebraic sum of their moments about any point in their plane is zero.



(5) A system of coplanar forces acting upon a rigid body will be in equilibrium if the algebraic sum of the moments of the forces about each of three non-collinear points is zero.

(6) Trigonometrical theorem : If P is any point on the base BC of ΔABC such that $BP : CP = m : n$.

Then, (i) $(m+n)\cot\theta = m\cot\alpha - n\cot\beta$ where $\angle BAP = \alpha, \angle CAP = \beta$

(ii) $(n+n)\cot\theta = n\cot B - m\cot C$

Example: 17 Two smooth beads A and B, free to move on a vertical smooth circular wire, are connected by a string. Weights W_1, W_2 and W are suspended from A, B and a point C of the string respectively.

In equilibrium, A and B are in a horizontal line. If $\angle BAC = \alpha$ and $\angle ABC = \beta$, then the ratio $\tan\alpha : \tan\beta$ is

[Roorkee 1996, UPSEAT 2001]

(a) $\frac{\tan\alpha}{\tan\beta} = \frac{W - W_1 + W_2}{W + W_1 - W_2}$ (b) $\frac{\tan\alpha}{\tan\beta} = \frac{W + W_1 - W_2}{W - W_1 + W_2}$ (c) $\frac{\tan\alpha}{\tan\beta} = \frac{W + W_1 + W_2}{W + W_1 - W_2}$ (d) None of these

Solution: (a) Resolving forces horizontally and vertically at the points A, B and C respectively, we get

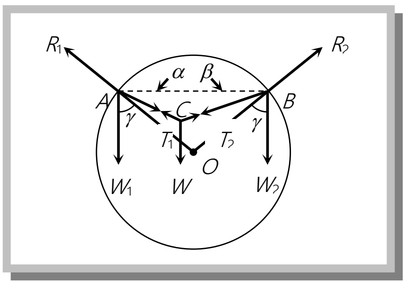
$T\cos\alpha = R_1\sin\gamma$ .....(i)

$T_1\sin\alpha + W_1 = R_1\cos\gamma$ .....(ii)

$T_1\cos\beta = R_2\sin\gamma$ .....(iii)

$T_2\sin\beta + W_2 = R_2\cos\gamma$ .....(iv)

$T_1\cos\alpha = T_2\cos\beta$ .....(v)



and $T_1 \sin\alpha + T_2 \sin\beta = W$ .....(vi)

Using (v), from (i)and (ii), we get, $R_1 = R_2$

$\therefore$ From (ii) and (vi), we have

$T_1 \sin\alpha + W_1 = T_2 \sin\beta + W_2$

or $T_1 \sin\alpha - T_2 \sin\beta = W_2 - W_1$ .....(vii)

Adding and subtracting (vi) and (vii), we get

$2T_1 \sin\alpha = W + W_2 - W_1$ ......(viii)

$2T_2 \sin\beta = W - W_2 + W_1$ ......(ix)

Dividing (viii) by (ix), we get

$\frac{T_1}{T_2} \cdot \frac{\sin\alpha}{\sin\beta} = \frac{W - W_1 + W_2}{W + W_1 - W_2}$ or $\frac{\cos\beta}{\cos\alpha} \cdot \frac{\sin\alpha}{\sin\beta} = \frac{W - W_1 + W_2}{W + W_1 - W_2}$ (from (v)) or $\frac{\tan\alpha}{\tan\beta} = \frac{W - W_1 + W_2}{W + W_1 - W_2}$

Example: 18 A uniform beam of length 2*a* rests in equilibrium against a smooth vertical plane and over a smooth peg at a distance *h* from the plane. If θ be the inclination of the beam to the vertical, then $\sin^3\theta$ is [MNR 1996]

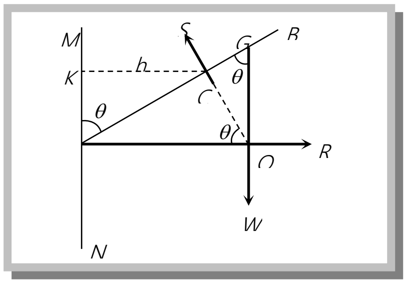
(a) $\frac{h}{a}$ (b) $\frac{h^2}{a^2}$ (c) $\frac{a}{h}$ (d) $\frac{a^2}{h^2}$

Solution: (a) Let *AB* be a rod of length 2*a* and weight *W*. It rests against a smooth vertical wall at *A* and over peg *C*, at a distance *h* from the wall. The rod is in equilibrium under the following forces :

(i) The weight *W* at *G*

(ii) The reaction *R* at *A*

(iii) The reaction *S* at *C* perpendicular to *AB*.



Since the rod is in equilibrium. So, the three force are concurrent at *O*.

In ΔACK, we have, $\sin\theta = \frac{h}{AC}$

In ΔACO, we have, $\sin\theta = \frac{AO}{a}$

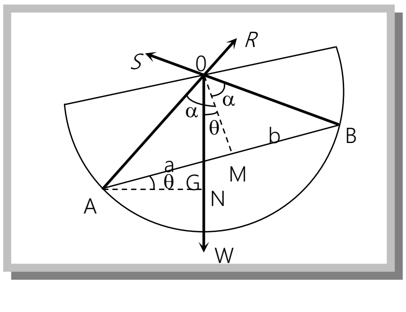
In ΔAGO, we have $\sin\theta = \frac{AO}{a}$; $\therefore \sin^3\theta = \frac{h}{AC} \cdot \frac{AC}{AO} \cdot \frac{AO}{a} = \frac{h}{a}$

Example: 19 A beam whose centre of gravity divides it into two portions *a* and *b*, is placed inside a smooth horizontal sphere. If θ be its inclination to the horizon in the position of equilibrium and 2α be the angle subtended by the beam at the centre of the sphere, then [Roorkee 1994]

(a) $\tan\theta = (b-a)(b+a)\tan\alpha$ (b) $\tan\theta = \frac{(b-a)}{(b+a)}\tan\alpha$ (c) $\tan\theta = \frac{(b+a)}{(b-a)}\tan\alpha$ (d) $\tan\theta = \frac{1}{(b-a)(b+a)}\tan\alpha$

Solution: (b) Applying *m* – *n* theorem in ΔABC, we get

$(AG + GB)\cot\angle OGB = GB\cot\angle OAB - AG\cot\angle OBG$

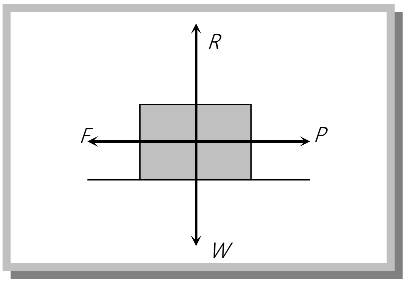


$$\Rightarrow (a+b)\cot(90° - \theta) = b\cot\left(\frac{\pi}{2} - \alpha\right) - a\cot\left(\frac{\pi}{2} - \alpha\right)$$

$$\Rightarrow (a+b)\tan\theta = b\tan\alpha - a\tan\alpha \;\; \Rightarrow \tan\theta = \left(\frac{b-a}{a+b}\right)\tan\alpha$$

3.11 Friction

Friction is a retarding force which prevent one body from sliding on another.



It is, therefore a reaction.

When two bodies are in contact with each other, then the property of roughness of the bodies by virtue of which a force is exerted between them to resist the motion of one body upon the other is called friction and the force exerted is called force of friction.

(1) **Friction is a self adjusting force :** Let a horizontal force *P* pull a heavy body of weight *W* resting on a smooth horizontal table. It will be noticed that up to a certain value of *P*, the body does not move. The reaction *R* of the table and the weight *W* of the body do not have any effect on the horizontal pull as they are vertical. It is the force of friction *F*, acting in the horizontal direction, which balances *P* and prevents the body from moving.

As *P* is increased, *F* also increases so as to balance *P*. Thus *F* increases with *P*. A stage comes when *P* just begins to move the body. At this stage *F* reaches its maximum value and is equal to the value of *P* at that instant. After that, if *P* is increased further, *F* does not increase any more and body begins to move.

This shows that friction is self adjusting, *i.e.* amount of friction exerted is not constant, but increases gradually from zero to a certain maximum limit.

(2) **Statical friction :** When one body tends to slide over the surface of another body and is not on the verge of motion then the friction called into play is called statical friction.

(3) **Limiting friction :** When one body is on the verge of sliding over the surface of another body then the friction called into play is called limiting friction.

(4) **Dynamical friction :** When one body is actually sliding over the surface of another body the friction called into play is called dynamical friction.

(5) **Laws of limiting friction/statical friction/Dynamical friction :**

(i) Limiting friction acts in the direction opposite to that in which the body is about to move.

(ii) The magnitude of the limiting friction between two bodies bears a constant ratio depends only on the nature of the materials of which these bodies are made.

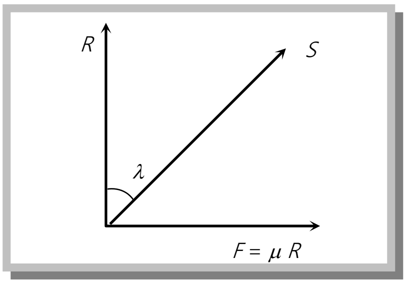
(iii) Limiting friction is independent of the shape and the area of the surfaces in contact, so long as the normal reaction between them is same, if the normal reaction is constant.

(iv) Limiting friction f_s is directly proportional to the normal reaction R, *i.e.* $f_s \propto R$

$f_s = \mu_s . R;\ \mu_s = f_s / R$, where μ_s is a constant which is called coefficient of statical friction.

In case of dynamic friction, $\mu_k = f_k/R$, where μ_k is the coefficient of dynamic friction.

(6) **Angle of friction :** The angle which the resultant force makes with the direction of the normal reaction is called the angle of friction and it is generally denoted by λ.



Thus λ is the limiting value of α, when the force of friction F attains its maximum value.

$$\therefore \tan \lambda = \frac{\text{Maximum force of friction}}{\text{Normal reaction}}$$

Since R and μ R are the components of S, we have, $S \cos \lambda = R$, $S \sin\lambda = \mu R$.

Hence by squaring and adding, we get $S = R\sqrt{1+\mu^2}$ and on dividing them, we get $\tan \lambda = \mu$. Hence we see that the coefficient of friction is equal to the tangent of the angle of friction.

3.12 Coefficient of Friction

When one body is in limiting equilibrium in contact with another body, the constant ratio which the limiting force of friction bears to normal reaction at their point of contact, is called the coefficient of friction and it is generally denoted by μ.

Thus, μ is the ratio of the limiting friction and normal reaction.

Hence, $\mu = \tan \lambda = \dfrac{\text{Maximum force of friction}}{\text{Normal reaction}}$

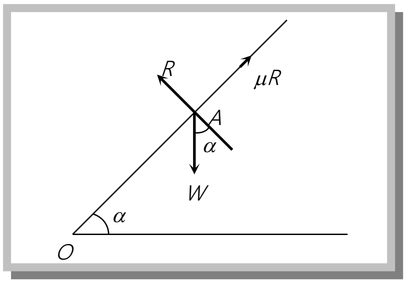
$\Rightarrow \mu = \dfrac{F}{R} \Rightarrow F = \mu R,$ where F is the limiting friction and R is the normal reaction.

Note : ❑ The value of μ depends on the substance of which the bodies are made and so it differs from one body to the other. Also, the value of μ always lies between 0 and 1. Its value is zero for a perfectly smooth body.

❑ Cone of friction : A cone whose vertex is at the point of contact of two rough bodies and whose axis lies along the common normal and whose semi-vertical angle is equal to the angle of friction is called cone of friction.

3.13 Limiting equilibrium on an Inclined Plane

Let a body of weight *W* be on the point of sliding down a plane which is inclined at an angle α to the horizon. Let *R* be the normal reaction and μR be the limiting friction acting up the plane.



Thus, the body is in limiting equilibrium under the action of three forces : *R*, μR and *W*.

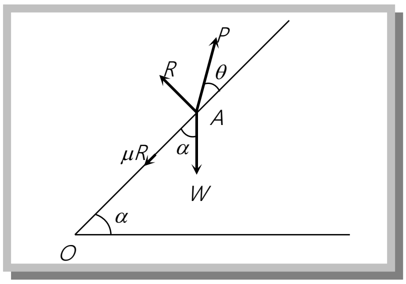
Resolving the forces along and perpendicular to the plane, we have

$\mu R = W\sin\alpha$ and $R = W\cos\alpha$

$$\Rightarrow \frac{\mu R}{R} = \frac{W\sin\alpha}{\cos\alpha} \Rightarrow \mu = \tan\alpha \Rightarrow \tan\lambda = \tan\alpha \Rightarrow \alpha = \lambda$$

Thus, if a body be on the point of sliding down an inclined plane under its own weight, the inclination of the plane is equal to the angle of the friction.

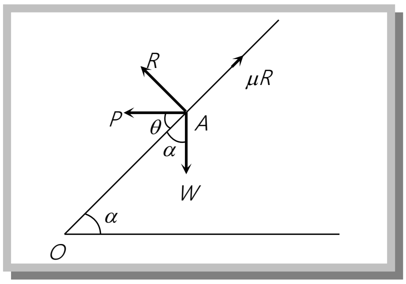
(1) **Least force required to pull a body up an inclined rough plane :** Let a body of weight *W* be at point *A*, α be the inclination of rough inclined plane to the horizontal and λ be the angle of friction. Let *P* be the force acting at an angle θ with the plane required just to move body up the plane.



$$P = W\frac{\sin(\alpha+\lambda)}{\cos(\theta-\lambda)} \qquad \{\because \mu = \tan\lambda\}$$

Clearly, the force *P* is least when $\cos(\theta-\lambda)$ is maximum, *i.e.* when $\cos(\theta-\lambda) = 1$, *i.e.* $\theta - \lambda = 0$ or $\theta = \lambda$. The least value of *P* is $W\sin(\alpha+\lambda)$

(2) **Least force required to pull a body down an inclined plane :** Let a body of weight *W* be at the point *A*, α be the inclination of rough inclined plane to the horizontal and λ be the angle of friction. Let *P* be the force acting an angle θ with the plane, required just to move the body up the plane.



$$P = \frac{W\sin(\lambda-\alpha)}{\cos(\theta-\lambda)} \qquad [\because \mu = \tan\lambda]$$

Clearly, *P* is least when $\cos(\theta-\lambda)$ is maximum, *i.e.* when $\theta - \lambda = 0$ or $\theta = \lambda$. The least value of *P* is $W\sin(\lambda-\alpha)$.

Note : ❑ If $\alpha = \lambda$, then the body is in limiting equilibrium and is just on the point of moving downwards.

❑ If $\alpha < \lambda$, then the least force required to move the body down the plane is $W \sin(\lambda - \alpha)$.

❑ If $\alpha = \lambda, \alpha > \lambda$ or $\alpha < \lambda$, then the least force required to move the body up the plane is $W \sin(\alpha + \lambda)$.

❑ If $\alpha > \lambda,$ then the body will move down the plane under the action of its weight and normal reaction.

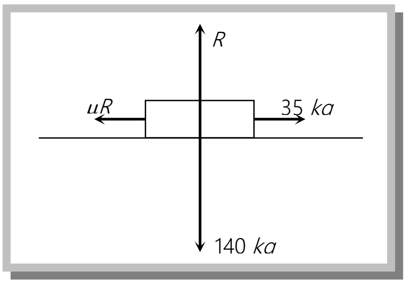
Important Tips

☞ ***Least force on the horizontal plane :*** *Least force required to move the body with weight W on the rough horizontal plane is W sin λ .*

Example: 20 A force of 35 *Kg* is required to pull a block of wood weighing 140 *Kg* on a rough horizontal surface. The coefficient of friction is [BIT Ranchi 1995]

(a) 1 (b) 0 (c) 4 (d) $\frac{1}{4}$

Solution: (d) In the position of limiting equilibrium, we have $\mu R = 35$ and $R = 140$ $\Rightarrow \mu = \frac{35}{140} = \frac{1}{4}$



Example: 21 A uniform ladder rests in limiting equilibrium, its lower end on a rough horizontal plane and its upper end against a smooth vertical wall. If θ is the angle of inclination of the ladder to the vertical wall and μ is the coefficient of friction, then tanθ is equal to [MNR 1991; UPSEAT 2000]

(a) μ (b) 2μ (c) $\frac{3\mu}{2}$ (d) $\mu + 1$

Solution: (b) Resolving the forces horizontally and vertically, we get

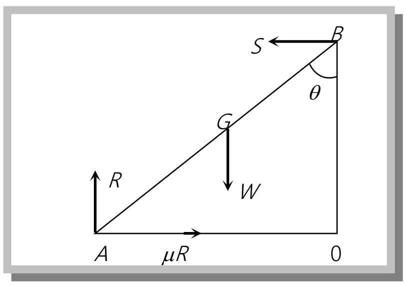
$S = \mu R$ and $R = W$

$\Rightarrow S = \mu W$ .....(i)

Taking moments about *A*, we get

$-W.AG \sin\theta + S.AB \cos\theta = 0$

$\Rightarrow W.AG \sin\theta = S.AB \cos\theta \Rightarrow W.\frac{AB}{2}\sin\theta = S.AB\cos\theta$ $\left[\because AG = \frac{AB}{2}\right]$



$\Rightarrow \frac{W}{2}.AB\sin\theta = \mu W.AB\cos\theta$ [from (i)]

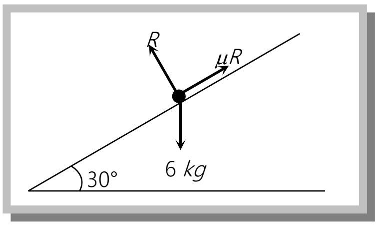
$\Rightarrow \tan\theta = 2\mu$.

Example: 22 A body of 6 *Kg*. rests in limiting equilibrium on an inclined plane whose slope is 30°. If the plane is raised to slope of 60°, the force in *Kg*. weight along the plane required to support it is

(a) 3 (b) $2\sqrt{3}$

(c) $\sqrt{3}$ (d) $3\sqrt{3}$

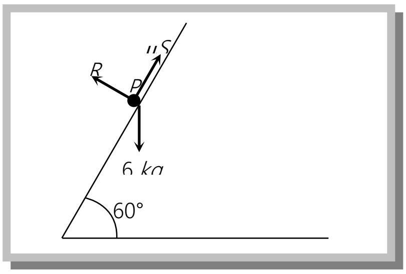
Solution: (b) In case (i),



$R = 6\cos 30°$, $\mu R = 6\sin 30°$.

$\therefore \mu = \tan 30° = \frac{1}{\sqrt{3}}$

In case (ii),



$S = 6\cos 60°$

$P + \mu S = 6\sin 60°$

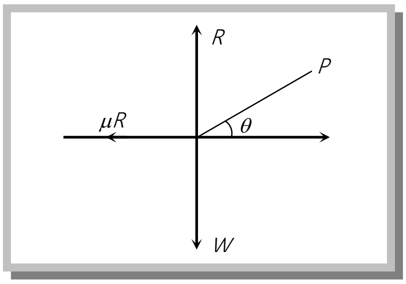
$\therefore P + \frac{1}{\sqrt{3}}(6\cos 60°) = 6\sin 60° = 6\frac{\sqrt{3}}{2} = 3\sqrt{3}$.

$\therefore P = 3\sqrt{3} - \frac{1}{\sqrt{3}}6\times\frac{1}{2} = 3\sqrt{3} - \frac{3}{\sqrt{3}} = 3\sqrt{3} - \sqrt{3} = 2\sqrt{3}$.

Example: 23 The coefficient of friction between the floor and a box weighing 1 ton if a minimum force of 600 *Kgf* is required to start the box moving is **[SCRA 1995]**

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) 1

Solution: (b) Resolving horizontally and vertically



$P\cos\theta = \mu R;\ P\sin\theta + R = W$

$\therefore P\cos\theta = \mu[W - P\sin\theta]$

or $P[\cos\theta + \mu\sin\theta] = \mu W$

or $P = \frac{\mu W}{\cos\theta + \frac{\sin\lambda}{\cos\lambda}.\sin\theta} = \frac{\mu W\cos\lambda}{\cos(\theta-\lambda)} = \frac{W\sin\lambda}{\cos(\theta-\lambda)}$

Now *P* is minimum when $\cos(\theta-\lambda)$ is maximum, *i.e.* when $\cos(\theta-\lambda) = 1$

$\therefore \text{Min} P = W\sin\lambda$

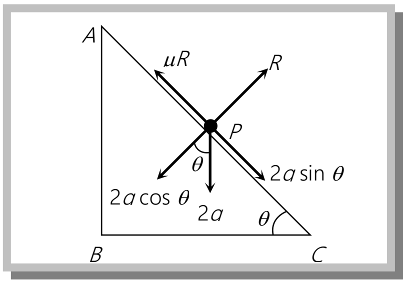
But $W = 1\ ton\ wt. = 1000\ Kg.$ and $P = 600\ kg$

$\therefore \sin\lambda = \frac{P}{W} = \frac{600}{1000} = \frac{3}{5}$; $\therefore \tan\lambda = \frac{3}{4}, \therefore \mu = \frac{3}{4}$

Example: 24 A block of mass 2 *Kg.* slides down a rough inclined plane starting from rest at the top. If the inclination of the plane to the horizontal is θ with $\tan\theta = \frac{4}{5}$, the coefficient of friction is 0.3 and the acceleration due to gravity is $g = 9.8$. The velocity of the block when it reaches the bottom is

(a) 6.3 (b) 5.2 (c) 7 (d) 8.1

Solution: (c) Let *P* be the position of the man at any time.



Clearly, $R = 2g\cos\theta$

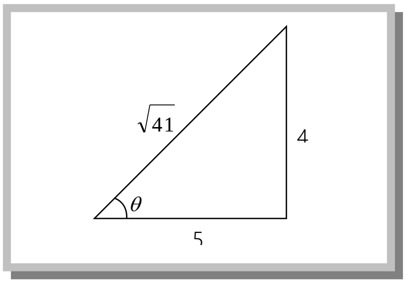
Let *f* be acceleration down the plane.

Equation of motion is $2f = 2g\sin\theta - \mu R$

$2f = 2g\sin\theta - \mu(2g\cos\theta)$

$2f = 2g(\sin\theta - \mu\cos\theta)$

Here, $\tan\theta = \frac{4}{5}$, $\sin\theta = \frac{4}{\sqrt{41}}$, $\cos\theta = \frac{5}{\sqrt{41}}$



Now, $2f = 2g\left(\frac{4}{\sqrt{41}} - \frac{3}{10}.\frac{5}{\sqrt{41}}\right)$

$2f = \frac{2g}{\sqrt{41}}\left(4 - \frac{3}{2}\right) = \frac{2g}{\sqrt{41}}.\frac{5}{2} = \frac{5g}{\sqrt{41}}$, $\therefore f = \frac{5g}{2\sqrt{41}}$

Let *v* be the velocity at *C*.

Then, $v^2 = u^2 + 2fS = 0 + 2\frac{5g}{2\sqrt{41}}AC$

$v^2 = \frac{5g}{\sqrt{41}}.\sqrt{41}$ $\left\{\text{we can take } AC = \sqrt{41}, \text{since } \tan\theta = \frac{4}{5}\right\}$

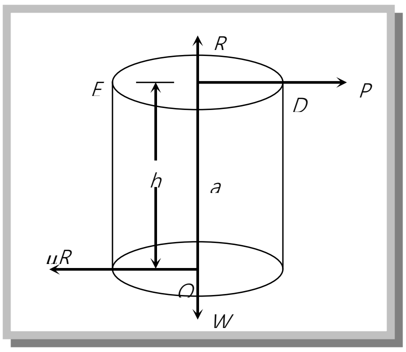
$v^2 = 5g = 5\times 9.8 = 49.0$, *i.e.*, $v^2 = 7m/\sec$

Example: 25 A circular cylinder of radius *r* and height *h* rests on a rough horizontal plane with one of its flat ends on the plane. A gradually increasing horizontal force is applied through the centre of the upper end. If the coefficient of friction is μ. The cylinder will topple before sliding of [UPSEAT 1994]

(a) $r < \mu h$ (b) $r \geq \mu h$ (c) $r \geq 2\mu h$ (d) $r = 2\mu h$

Solution: (b) Let base of cylinder is *AB*.

$\therefore BC = r$



Let force *P* is applied at *O*.

Let reaction of plane is *R* and force of friction is μR. Let weight of cylinder is *W*.

In equilibrium condition,

$R = W$ .....(i) and $P = \mu R$ .....(ii)

From (i) and (ii), we have $P = \mu W$

Taking moment about the point *O*,

We have $W \times BC - P \times OC = 0 \Rightarrow P = \frac{W \times BC}{OC} = \frac{W \times r}{h}$

If $\frac{W \times r}{h} \geq \mu W$ or $r \geq \mu h$

The cylinder will be topple before sliding.

3.14 Centre of Gravity

The centre of gravity of a body or a system of particles rigidly connected together, is that point through which the line of action of the weight of the body always passes in whatever position the body is placed and this point is called centroid. A body can have one and only one centre of gravity.

If $w_1, w_2, ..., w_n$ are the weights of the particles placed at the points $A_1(x_1, y_1), A_2(x_2, y_2), ..., A_n(x_n, y_{n)}$ respectively, then the centre of gravity $G(\bar{x}, \bar{y})$ is given by

$\bar{x} = \frac{\sum w_1 x_1}{\sum w_1}, \bar{y} = \frac{\sum w_1 y_1}{\sum w_1}$.

(1) **Centre of gravity of a number of bodies of different shape :**

(i) **C.G. of a uniform rod :** The C.G. of a uniform rod lies at its mid-point.

(ii) **C.G. of a uniform parallelogram :** The C.G. of a uniform parallelogram is the point of inter-section of the diagonals.

(iii) **C.G. of a uniform triangular lamina :** The C.G. of a triangle lies on a median at a distance from the base equal to one third of the medians.

(2) **Some Important points to remember :**

(i) The C.G. of a uniform tetrahedron lies on the line joining a vertex to the C.G. of the opposite face, dividing this line in the ratio 3 : 1.

(ii) The C.G. of a right circular solid cone lies at a distance *h*/4 from the base on the axis and divides it in the ratio 3 : 1.

(iii) The C.G. of the curved surface of a right circular hollow cone lies at a distance *h*/3 from the base on the axis and divides it in the ratio 2 : 1

(iv) The C.G. of a hemispherical shell at a distance *a*/2 from the centre on the symmetrical radius.

(v) The C.G. of a solid hemisphere lies on the central radius at a distance 3*a*/8 from the centre where *a* is the radius.

(vi) The C.G. of a circular arc subtending an angle 2α at the centre is at a distance $\frac{a\sin\alpha}{\alpha}$ from the centre on the symmetrical radius, *a* being the radius, and α in radians.

(vii) The C.G. of a sector of a circle subtending an angle 2α at the centre is at a distance $\frac{2a}{3}\frac{\sin\alpha}{\alpha}$ from the centre on the symmetrical radius, *a* being the radius and α in radians.

(viii) The C.G. of the semi circular arc lies on the central radius at a distance of $\frac{2a}{\pi}$ from the boundry diameter, where *a* is the radius of the arc.

Important Tips

☞ *Let there be a body of weight w and x be its C.G. If a portion of weight w_1 is removed from it and x_1 be the C.G. of the removed portion. Then, the C.G. of the remaining portion is given by* $x_2 = \frac{wx - w_1x_1}{w - w_1}$

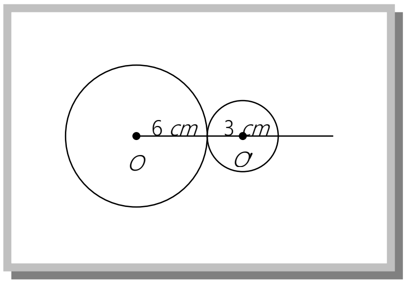
☞ *Let x be the C.G. of a body of weight w. If x_1, x_2, x_3 are the C.G. of portions of weights w_1, w_2, w_3 respectively, which are removed from the body, then the C.G. of the remaining body is given by* $x_4 = \frac{wx - w_1x_1 - w_2x_2 - w_3x_3}{w - w_1 - w_2 - w_3}$

Example: 26 Two uniform solid spheres composed of the same material and having their radii 6 *cm* and 3 *cm* respectively are firmly united. The distance of the centre of gravity of the whole body from the centre of the larger sphere is **[MNR 1980]**

(a) 1 *cm.* (b) 3 *cm.* (c) 2 *cm.* (d) 4 *cm.*

Solution: (a) Weights of the spheres are proportional to their volumes.

Let *P* be the density of the material, then



w_1 = Weight of the sphere of radius $6cm = \frac{4}{3}\pi(6^3)\rho = 288\pi\rho$

w_2 = Weight of the sphere of radius $3cm = \frac{4}{3}\pi(3^3)\rho = 36\pi\rho$

x_1 = Distance of the *C.G.* of the larger sphere from its centre *O* = 0

x_2 = Distance of the *C.G.* of smallar sphere from *O* = 9 *cm.*

$\bar{x}$ = Distance of the *C.G.* of the whole body from *O*

Now $\bar{x} = \frac{w_1x_1 + w_2x_2}{w_1 + w_2} = \frac{288\pi\rho \times 0 + 36\pi\rho \times 9}{288\pi\rho + 36\pi\rho}$

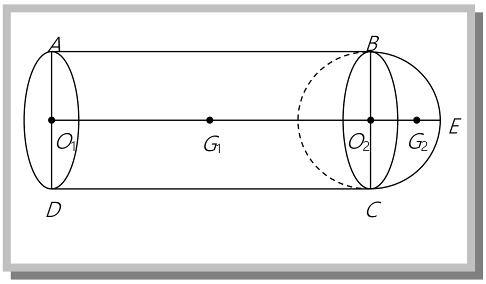
$\bar{x} = \frac{36 \times 9}{324} = 1$

Example: 27 A solid right circular cylinder is attached to a hemisphere of equal base. It the C.G. of combined solid is at the centre of the base, then the ratio of the radius and height of cylinder is

(a) 1 : 2 (b) $\sqrt{2}:1$ (c) 1 : 3 (d) None of these

Solution: (b) Let *a* be the radius of the base of the cylinder and *h* be the height of the cylinder. Let w_1 and w_2 be the weight of the cylinder and hemisphere respectively. These weights act at their centres of gravity G_1 and G_2 respectively.

Now, w_1 = weight of the cylinder = $\pi a^2 h\rho g$



w_2 = weight of the hemisphere = $\frac{2}{3}\pi a^3 \rho g$

$O_1G_1 = \frac{h}{2}$ and $O_1G_2 = h + \frac{3a}{8}$

Since the combined C.G. is at O_2. Therefore

$$O_1O_2 = \frac{w_1 \times O_1G + w_2 + O_1G_2}{w_1 + w_2}$$

$$\Rightarrow h = \frac{(\pi a^2 h\rho g)\times\frac{h}{2} + \left(\frac{2}{3}\pi a^3\rho g\right)\times\left(h+\frac{3a}{8}\right)}{\pi a^2 h\rho g + \frac{2}{3}\pi a^3 \rho g} \Rightarrow h = \frac{\frac{h^2}{2} + \frac{2}{3}a\left(h+\frac{3a}{8}\right)}{h+\frac{2}{3}a} \Rightarrow h^2 + \frac{2ah}{3} = \frac{h^2}{2} + \frac{2ah}{3} + \frac{a^2}{4}$$

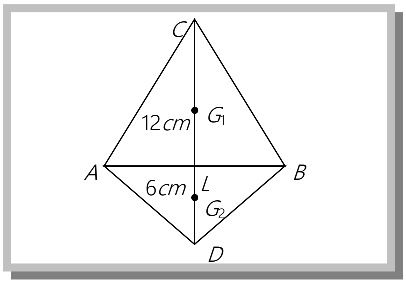
$$\Rightarrow 2h^2 = a^2 \Rightarrow \frac{a}{h} = \sqrt{2} \Rightarrow a : h = \sqrt{2} : 1$$

Example: 28 On the same base *AB* and on opposite side of it, isosceles triangles *CAB* and *DAB* are described whose altitudes are 12 *cm* and 6 *cm* respectively. The distance of the centre of gravity of the quadrilateral *CADB* from *AB*, is

(a) 0.5 *cm* (b) 1 *cm* (c) 1.5 *cm* (d) 2 *cm*

Solution: (b) Let *L* be the midpoint of *AB*. Then $CL \perp AB$ and $DL \perp AB$.

Let G_1 and G_2 be the centres of gravity of triangular lamina *CAB* and *DAB* respectively.



Then, $LG_1 = \frac{1}{3}CL = 4cm.$ and $LG_2 = \frac{1}{3}DL = 2cm.$

The C.G. of the quadrilateral *ABCD* is at *G*, the mid point of G_1 G_2.

$\therefore G_1G_2 = GG_1 = 3cm.$

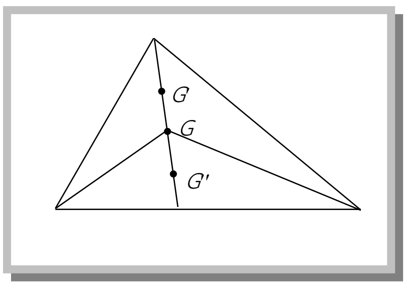
$\Rightarrow GL = G_1L - GG_1 = (4-3)cm = 1cm.$

Example: 29 *ABC* is a uniform triangular lamina with centre of gravity at *G*. If the portion *GBC* is removed, the centre of gravity of the remaining portion is at *G*. Then *GG* is equal to [UPSEAT 1994]

(a) $\frac{1}{3}AG$ (b) $\frac{1}{4}AG$ (c) $\frac{1}{5}AG$ (d) $\frac{1}{6}AG$

Solution: (d) Since *G* and *G* are the centroids of ΔABC and *GBD* respectively. Therefore $AG = \frac{2}{3}AD$,

$GD = \frac{1}{3}AD$ and $GG" = \frac{2}{3}GD = \frac{2}{3}\left(\frac{1}{3}AD\right) = \frac{2}{9}AD$



Now, $AG = \frac{2}{3}AD$ and $GD = \frac{1}{3}AD$

$\Rightarrow$ Area of $\Delta GBC = \frac{1}{3}$ Area of ΔABC

$\Rightarrow$ Weight of triangular lamina *GBC* = $\frac{1}{3}$ (weight of triangle lamina *ABC*)

Thus, if W is the weight of lamina *GBC*, then the weight of lamina *ABC* is $3W$.

Now, *G* is the *C.G.* of the remaining portion *ABGC*.

Therefore,

$$AG' = \frac{3W(AG) - W(AG'')}{3W - W}$$

$$= \frac{1}{2}(3AG - AG'') \qquad \left\{\because AG'' = AG + GG'' = \frac{2}{3}AD + \frac{2}{9}AD = \frac{8}{9}AD\right\}$$

$$= \frac{1}{2}\left(3 \times \frac{2}{3}AD - \frac{8}{9}AD\right) = \frac{5}{9}AD$$

$$\therefore GG' = AG - AG' = \frac{2}{3}AD - \frac{5}{9}AD = \frac{1}{9}AD = \frac{1}{9}\left(\frac{3}{2}AG\right) = \frac{1}{6}AG .$$

Assignment

Composition and resolution of forces and condition of equilibrium of

Basic Level

1. The resultant of two forces $3P$ and $2P$ is R, if the first force is doubled, the resultant is also doubled. The angle between the forces is

[MNR 1985; UPSEAT 2000]

(a) $\pi/3$ (b) $2\pi/3$ (c) $\pi/6$ (d) $5\pi/6$

2. The resultant of two forces $\vec{P}$ and $\vec{Q}$ is of magnitude P. If the force $\vec{P}$ is doubled, $\vec{Q}$ remaining unaltered, the new resultant will be

[MNR 1995]

(a) Along $\vec{P}$ (b) Along $\vec{Q}$ (c) At 60^o to $\vec{Q}$ (d) At right angle to $\vec{Q}$

3. If the resultant of two forces $2P$ and $\sqrt{2}P$ is $\sqrt{10}P$, then the angle between them will be

(a) π (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$

4. The maximum resultant of two forces is P and the minimum resultant is Q, the two forces are at right angles, the resultant is

[Roorkee 1990]

(a) $P+Q$ (b) $P-Q$ (c) $\frac{1}{2}\sqrt{P^2+Q^2}$ (d) $\sqrt{\frac{P^2+Q^2}{2}}$

5. Two equal forces act at a point. If the square of the magnitude of their resultant is three times the product of their magnitudes, the angle between the forces is

(a) 30^o (b) 45^o (c) 90^o (d) 60^o

6. A force is resolved into components P and Q equally inclined to it. Then

(a) $P=2Q$ (b) $2P=Q$ (c) $P=Q$ (d) None of these

7. If the square of the resultant of two equal forces is equal to $(2-\sqrt{3})$ times their product, then the angle between the forces is

(a) 60^o (b) 150^o (c) 120^o (d) 30^o

8. The resultant of two equal forces is equal to either of these forces. The angle between them is

(a) $\pi/4$ (b) $\pi/3$ (c) $\pi/2$ (d) $2\pi/3$

9. When two equal forces are inclined at an angle 2α, their resultant is twice as great as when they act at an angle 2β, then

[UPSEAT 1999]

(a) $\cos\alpha = 2\sin\beta$ (b) $\cos\alpha = 2\cos\beta$ (c) $\cos\beta = 2\cos\alpha$ (d) $\sin\beta = 2\cos\alpha$

10. Two forces of 13 N and $3\sqrt{3}$ N act on a particle at an angle θ and are equal to a resultant force of 14N, the angle between the forces is

(a) 30^o (b) 60^o (c) 45^o (d) 90^o

11. If two forces $P+Q$ and $P-Q$ make an angle 2α with each other and their resultant makes an angle θ with the bisector of the angle between the two forces, then $\frac{P}{Q}$ is equal to

(a) $\frac{\tan\theta}{\tan\alpha}$ (b) $\frac{\tan\alpha}{\tan\theta}$ (c) $\frac{\sin\theta}{\sin\alpha}$ (d) $\frac{\sin\alpha}{\sin\theta}$

12. A force *F* is resolved into two components *P* and *Q*. If *P* be at right angles to *F* and has the same magnitude as that of *F*, then the magnitude of *Q* is

(a) $\frac{F}{2}$ (b) $\frac{F}{\sqrt{2}}$ (c) $2F$ (d) $\sqrt{2}F$

13. The direction of three forces 1*N*, 2*N* and 3*N* acting at a point are parallel to the sides of an equilateral triangle taken in order, The magnitude of their resultant is

(a) $\frac{\sqrt{3}}{2}N$ (b) 3*N* (c) $\sqrt{3}N$ (d) $\frac{3}{2}N$

14. Forces of magnitudes 5, 10, 15 and 20 *Newton* act on a particle in the directions of North, South, East and West respectively. The magnitude of their resultant is

(a) $15\sqrt{2}N$ (b) $10N$ (c) $25\sqrt{2}N$ (d) $5\sqrt{2}N$

15. Forces of magnitudes $P-Q, P$ and $P+Q$ act at a point parallel to the sides of an equilateral triangle taken in order. The resultant of these forces, is

(a) $\sqrt{3}P$ (b) $\sqrt{3}Q$ (c) $3\sqrt{3}P$ (d) 3*P*

16. Two forces acting in opposite directions on a particle have a resultant of 34 *Newton*; if they acted at right angles to one another, their resultant would have a magnitude of 50 *Newton*. The magnitude of the forces are

(a) 48, 14 (b) 42, 8 (c) 40, 6 (d) 36, 2

17. Three forces of magnitude 30, 60 and *P* acting at a point are in equilibrium. If the angle between the first two is 60^o, the value of *P* is

[Roorkee 1991]

(a) $30\sqrt{7}$ (b) $30\sqrt{3}$ (c) $20\sqrt{6}$ (d) $25\sqrt{2}$

18. The resultant of two forces *P* and *Q* acting at an angle θ is equal to $(2m+1)\sqrt{P^2+Q^2}$; when they act at an angle $90^o-\theta$, the resultant is $(2m-1)\sqrt{P^2+Q^2}$; then $\tan\theta =$ **[UPSEAT 2000; SCRA 1995]**

(a) $\frac{1}{m}$ (b) $\frac{m+1}{m-1}$ (c) $\frac{m-1}{m+1}$ (d) $\sqrt{1+m^2}$

19. If forces of magnitude *P*, *Q* and *R* act at a point parallel to the sides *BC*, *CA* and *AB* respectively of a ΔABC, then the magnitude of their resultant is

(a) $\sqrt{P^2+Q^2+R^2}$ (b) $\sqrt{P^2+Q^2+R^2-2PQ\cos C-2QR\cos A-2PR\cos B}$

(c) $P+Q+R$ (d) None of these

20. Two forces of magnitudes $P+Q$ and $P-Q$ *Newton* are acting at an angle of 135^o. If their resultant is a force of 2 *Newton* perpendicular to the line of action of the second force, then

(a) $P=(\sqrt{2}+1), Q=(\sqrt{2}-1)$ (b) $P=(\sqrt{2}-1), Q=(\sqrt{2}+1)$ (c) $P=(\sqrt{3}+1), Q=(\sqrt{3}-1)$ (d) $P=(\sqrt{3}-1), Q=(\sqrt{3}+1)$

21. Let *R* be the resultant of *P* and *Q* and if $\frac{P}{3}=\frac{Q}{7}=\frac{R}{5}$, then the angle between *P* and *R* is

(a) $\cos^{-1}\left(\frac{11}{14}\right)$ (b) $\cos^{-1}\left(\frac{-11}{14}\right)$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

22. The resultant of two forces *P* and *Q* is at right angles to *P*, the resultant of *P* and Q' acting at the same angle α is at right angles to Q'. Then,

(a) P, Q, Q' are in GP (b) Q, P, Q' are in GP (c) P, Q', Q are in GP (d) None of these

23. The resultant *R* of two forces *P* and *Q* act at right angles to *P*. Then the angle between the forces is

(a) $\cos^{-1}\left(\frac{P}{Q}\right)$ (b) $\cos^{-1}\left(-\frac{P}{Q}\right)$ (c) $\sin^{-1}\left(\frac{P}{Q}\right)$ (d) $\sin^{-1}\left(-\frac{P}{Q}\right)$

24. The sum of the two forces is 18 and their resultant perpendicular to the lesser of the forces is 12, then the lesser force is

[MNR 1987, 1989; UPSEAT 2000]

(a) 5 (b) 3 (c) 7 (d) 15

25. The magnitudes of two forces are 3, 5 and the direction of the resultant is at right angles to that of the smaller force. The ratio of the magnitude of the larger force and of the resultant is

(a) 5 : 3 (b) 5 : 4 (c) 4 : 5 (d) 4 : 3

26. If the resultant of two forces P and Q is $\sqrt{3}Q$ and makes an angle 30° with the direction of P, then

(a) $P = 2Q'$ (b) $Q = 2P$ (c) $P = 3Q$ (d) None of these

27. The resolved part of a force of 16 *Newton* in a direction is $8\sqrt{3}$ *Newton*. The inclination of the direction of the resolved part with the direction of the force is

(a) 30^o (b) 60^o (c) 120^o (d) 150^o

28. Let P, $2P$ and $3P$ be the forces acting along AB, BC, CA of an equilateral ΔABC. Suppose R is the magnitude of their resultant and θ the angle made by the resultant with the side BC, then

(a) $R = P\sqrt{3}, \theta = \pi/2$ (b) $R = 2P\sqrt{3}, \theta = \pi/2$ (c) $R = P\sqrt{3}, \theta = \pi/6$ (d) $R = 2P\sqrt{3}, \theta = \pi/6$

29. When a particle be kept at rest under the action of the following forces

(a) $\uparrow 8N, \uparrow 5N, 13N \downarrow$ (b) $\uparrow 7N, \uparrow 4N, \downarrow 12N$ (c) $\uparrow 5N, \uparrow 8N, \downarrow 10N$ (d) $\uparrow 4N, \uparrow 2\sqrt{5}N, \downarrow 6N$

30. In a triangle ABC three forces of magnitudes $3\overrightarrow{AB}, 2\overrightarrow{AC}$ and $6\overrightarrow{CB}$ are acting along the sides AB, AC and CB respectively. If the resultant meets AC at D_1, then the ratio $DC : AD$ will be equal to

(a) 1 : 1 (b) 1 : 2 (c) 1: 3 (d) 1: 4

31. ABC is a triangle. Forces P, Q, R act along the lines OA, OB and OC and are in equilibrium. If O is incentre of ΔABC, then

[UPSEAT 1998]

(a) $\frac{P}{\cos A/2} = \frac{Q}{\cos B/2} = \frac{R}{\cos C/2}$ (b) $\frac{P}{OA} = \frac{Q}{OB} = \frac{R}{OC}$

(c) $\frac{P}{\sin A/2} = \frac{Q}{\sin B/2} = \frac{R}{\sin C/2}$ (d) None of these

32. If the forces of 12, 5 and 13 units weight balance at a point, two of them are inclined at

(a) 30^o (b) 45^o (c) 90^o (d) 60^o

33. Forces of 1, 2 units act along the lines $x = 0$ and $y = 0$. The equation of the line of action of the resultant is

[MNR 1981; UPSEAT 2000]

(a) $y - 2x = 0$ (b) $2y - x = 0$ (c) $y + x = 0$ (d) $y - x = 0$

34. If N is resolved in two components such that first is twice of other, the components are

(a) $5N, 5\sqrt{2}N$ (b) $10N, 10\sqrt{2}N$ (c) $\frac{N}{\sqrt{5}}, \frac{2N}{\sqrt{5}}$ (d) None of these

35. O is the circumcentre of ΔABC. If the forces P, Q and R acting along OA, OB, and OC are in equilibrium then P : Q : R is

(a) $\sin A : \sin B : \sin C$ (b) $\cos A : \cos B : \cos C$ (c) $a\cos A : b\cos B : c\cos C$ (d) $a\sec A : b\sec B : c\sec C$

36. Three forces P, Q and R acting on a particle are in equilibrium. If the angle between P and Q is double the angle between P and R, then P is equal to

(a) $\frac{Q^2 + R^2}{R}$ (b) $\frac{Q^2 - R^2}{Q}$ (c) $\frac{Q^2 - R^2}{R}$ (d) $\frac{Q^2 + R^2}{Q}$

37. A smooth sphere is supported in contact with a smooth vertical wall by a string fastened to a point on its surface, the other end being attached to a point in the wall. If the length of the string is equal to the radius of the sphere, the tension of the string is

(a) $\frac{2W}{\sqrt{3}}$ (b) $\frac{2W}{3}$ (c) $\frac{W}{2}$ (d) None of these

38. Three forces P, Q, R are acting at a point in a plane. The angle between P, Q and Q, R are 150^o and 120^o respectively, then for equilibrium; forces P, Q, R are in the ratio

(a) 1 : 2 : 3 (b) 1 : 2: $3^{1/2}$ (c) 3 : 2 : 1 (d) $(3)^{1/2}$:2 : 1

39. If *A*, *B* ,*C* are three forces in equilibrium acting at a point and if 60^o, 150^o and 150^o respectively denote the angles between *A* and *B*, *B* and *C* and *C* and *A*, then the forces are in proportion of

(a) $\sqrt{3}:1:1$ (b) $1:1:\sqrt{3}$ (c) $1:\sqrt{3}:1$ (d) $1:2.5:2.5$

40. The resultant of two forces *P* and *Q* is *R*. If Q is doubled, *R* is doubled and if *Q* is reversed, R is again doubled. If the ratio $P^2:Q^2:R^2=2:3:x$, then *x* is equal to **[MNR 1993; UPSEAT 2001; AIEEE 2003]**

(a) 5 (b) 4 (c) 3 (d) 2

41. If the angle α between two forces of equal magnitude is reduced to $\alpha-\pi/3$, then the magnitude of their resultant becomes $\sqrt{3}$ times of the earlier one. The angle α is

(a) $\pi/2$ (b) $2\pi/3$ (c) $\pi/4$ (d) $4\pi/5$

42. The resultant of two forces *P* and *Q* is *R*. If one of the forces is reversed in direction, the resultant becomes R', then

(a) $R'^2=P^2+Q^2+2PQ\cos\alpha$ (b) $R'^2=P^2-Q^2-2PQ\cos\alpha$

(c) $R'^2+R^2=2(P^2+Q^2)$ (d) $R'^2+R^2=2(P^2-Q^2)$

43. Forces proportional to *AB*, *BC* and 2*CA* act along the sides of triangle *ABC* in order, their resultants represented in magnitude and direction is

(a) *CA* (b) *AC* (c) *BC* (d) *CB*

Advance Level

44. The resultant of *P* and *Q* is *R*. If *P* is reversed, *Q* remaining the same, the resultant becomes R'. If *R* is perpendicular to R', then

(a) $2P=Q$ (b) $P=Q$ (c) $P=2Q$ (d) None of these

45. *ABCD* is a parallelogram, a particle *P* is attracted towards *A* and *C* by forces proportional to *PA* and *PC* respectively and repelled from *B* and *D* by forces proportional to *PB* and *PD*. The resultant of these forces is

(a) $2\overrightarrow{PA}$ (b) $2\overrightarrow{PB}$ (c) $2\overrightarrow{PC}$ (d) None of these

46. A particle is acted upon by three forces *P*, *Q* and *R*. It cannot be in equilibrium, if $P:Q:R=$

(a) $1:3:5$ (b) $3:5:7$ (c) $5:7:9$ (d) $7:9:11$

47. Forces of 7 *N*, 5*N* and 3*N* acting on a particle are in equilibrium, the angle between the pair of forces 5 and 3 is

(a) 30^o (b) 60^o (c) 90^o (d) 120^o

48. *ABCD* is a quadrilateral. Forces represented by $\overrightarrow{DA},\overrightarrow{DB},\overrightarrow{AC}$ and $\overrightarrow{BC}$ act on a particle. The resultant of these forces is

(a) $\overrightarrow{DC}$ (b) $2\overrightarrow{DC}$ (c) $\overrightarrow{CD}$ (d) $2\overrightarrow{CD}$

49. With two forces acting at a point, the maximum effect is obtained when their resultant is 4*N*. If they act at right angles, then their resultant is 3N. Then the forces are

(a) $\left(2+\frac{1}{2}\sqrt{3}\right)N$ and $\left(2-\frac{1}{2}\sqrt{3}\right)N$ (b) $(2+\sqrt{3})N$ and $(2-\sqrt{3})N$

(c) $\left(2+\frac{1}{2}\sqrt{2}\right)N$ and $\left(2-\frac{1}{2}\sqrt{2}\right)N$ (d) $(2+\sqrt{2})N$ and $(2-\sqrt{2})N$

50. The resultant of two forces *P* and *Q* is equal to $\sqrt{3}Q$ and makes an angle of 30^o with the direction of *P*, then $\frac{P}{Q}=$

(a) 1 or 2 (b) 3 or 5 (c) 3 or 4 (d) 4 or 5

51. Two men carry a weight of 240 *Newton* between them by means of two ropes fixed to the weight. One rope is inclined at 60^o to the vertical and the other at 30^o. The tensions in the ropes are

(a) $120\,N, 120\,N$ (b) $120\,N, 120\sqrt{3}\,N$ (c) $120\sqrt{3}N, 120\sqrt{3}N$ (d) None of these

52. Three forces keep a particle in equilibrium. One acts towards west, another acts towards north-east and the third towards south. If the first be 5*N*, then other two are

(a) $5\sqrt{2}N, 5\sqrt{2}N$ (b) $5\sqrt{2}N, 5N$ (c) $5N, 5N$ (d) None of these

53. A particle is attracted to three points *A*, *B* and *C* by forces equal to $\overrightarrow{PA}, \overrightarrow{PB}$ and $\overrightarrow{PC}$ respectively such that their resultant is $\lambda\overrightarrow{PG}$, where G is the centroid of ΔABC. Then $\lambda =$

(a) 1 (b) 2 (c) 3 (d) None of these

54. Three forces of magnitudes 8 Newton, 5N and 4N acting at a point are in equilibrium, then the angle between the two smaller forces is

(a) $\cos^{-1}\left(\frac{23}{40}\right)$ (b) $\cos^{-1}\left(\frac{-23}{40}\right)$ (c) $\sin^{-1}\left(\frac{23}{40}\right)$ (d) None of these

55. *ABC* is an equilateral triangle. *E* and *F* are the middle- points of the sides *CA* and *AB* respectively. Forces of magnitudes 4*N*, P*N*, 2*N*, P *N* and Q *N* act at a point and are along the lines *BC*, *BE*, *CA*, *CF* and *AB* respectively. If the system is in equilibrium, then

(a) $P = 2\sqrt{3}N, Q = 6N$ (b) $P = 6N, Q = 2\sqrt{3}N$ (c) $P = \sqrt{3}N, Q = 6N$ (d) $P = 2\sqrt{3}N, Q = 3N$

56. The resultant of forces *P* and *Q* acting at a point including a certain angle α is *R*, that of the forces $2P$ and *Q* acting at the same angle is $2R$ and that of P and $2Q$ acting at the supplementary angle is $2R$. Then $P : Q : R =$

(a) 1 : 2 : 3 (b) $\sqrt{6} : \sqrt{2} : \sqrt{5}$ (c) $\sqrt{2} : \sqrt{3} : \sqrt{5}$ (d) None of these

57. The resultant of *P* and *Q* is *R*. If *Q* is doubled, *R* is also doubled and if *Q* is reversed, *R* is again doubled. Then $P : Q : R$ is given by

(a) 1 : 1: 1 (b) $\sqrt{2} : \sqrt{2} : \sqrt{3}$ (c) $\sqrt{2} : \sqrt{3} : \sqrt{2}$ (d) $\sqrt{3} : \sqrt{2} : \sqrt{2}$

58. The resultant of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the magnitude of the other. The ratio of the larger force to the smaller is

(a) $3 : 2\sqrt{2}$ (b) $3\sqrt{3} : 2$ (c) $3 : 2$ (d) $4 : 3$

59. *ABCD* is a rigid square, on which forces 2, 3 and 5 *kg. wt*; act along *AB*, *AD* and *CA* respectively. Then the magnitude of the resultant correct to one decimal place in *kg. wt.* is

(a) 1 (b) 2 (c) 16 (d) None of these

60. A uniform rod of weight *W* rests with its ends in contact with two smooth planes, inclined at angles α and β respectively to the horizon, and intersecting in a horizontal line. The inclination θ of the rod to the vertical is given by

(a) $2\cot\theta = \cot\beta - \cot\alpha$ (b) $\tan\theta = 2\tan\alpha\tan\beta/(\tan\alpha - \tan\beta)$

(c) $\cot\theta = \sin(\alpha - \beta)/2\sin\alpha\sin\beta$ (d) All of these

61. Two forces $P + Q, P - Q$ make an angle 2α with one another and their resultant make an angle θ with the bisector of the angle between them. Then

(a) $P\tan\theta = Q\tan\alpha$ (b) $P\cot\alpha = Q\cot\theta$ (c) $P\tan\alpha = Q\tan\theta$ (d) None of these

62. A heavy rod 15 cm long is suspended from a fixed point by strings fastened to its ends, their lengths being 9 and 12 *cm*. If θ be the angle at which the rod is inclined to the vertical, then

(a) $\cos\theta = 7/25$ (b) $\sin\theta = 8/9$ (c) $\sin\theta = 19/20$ (d) $\sin\theta = 24/25$

63. A uniform triangular lamina whose sides are of lengths 3 *cm*, 4 *cm* and 5 *cm*, is suspended by a string tied at the middle point of the largest side. In equilibrium position the inclination of this side to the vertical is

(a) $\sin^{-1}(24/25)$ (b) $\sin^{-1}(12/25)$ (c) $\cos^{-1}(7/25)$ (d) None of these

64. Three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ acting along IA, IB and IC, where I is the incentre of a ΔABC, are in equilibrium . Then $\vec{P} : \vec{Q} : \vec{R}$ is

[AIEEE 2004]

(a) $\text{cosec}\frac{A}{2} : \text{cosec}\frac{B}{2} : \text{cosec}\frac{C}{2}$ (b) $\sec\frac{A}{2} : \sec\frac{B}{2} : \sec\frac{C}{2}$

(c) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$ (d) $\cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$

65. If two forces *P* and *Q* act on such an angle that their resultant force *R* is equal to force *P*, then if *P* is doubled then the angle between new resultant force and *Q* will be

(a) 30^o (b) 60^o (c) 45^o (d) 90^o

66. What will be that force when applying along any inclined plane will stop 10 kilogram weight, it is given that force, reaction of plane and weight of body are in arithmetic series

(a) 4 kilogram weight (b) 6 kilogram weight (c) 8 kilogram weight (d) 7 kilogram weight

67. A bead of weight W can slide on a smooth circular wire in a vertical plane, the bead is attached by a light thread to the highest point of the wire, and in equilibrium the thread is taut. Then the tension of the thread and the reaction of the wire on the bead, if the length of the string is equal to the radius of the wire, are

(a) $W, 2W$ (b) W, W (c) $W, 3W$ (d) None of these

Parallel Forces, Moment and Couples

Basic Level

68. Like parallel forces act at the vertices *A*, *B* and *C* of a triangle *ABC* and are proportional to the lengths *BC*, *AC* and *AB* respectively. The centre of the force is at the

(a) Centroid (b) Circum- centre (c) Incentre (d) None of these

69. Three forces *P*, *Q*, *R* act along the sides *BC*, *CA*, *AB* of a ΔABC taken in order, if their resultant passes through the centroid of ΔABC, then

(a) $P+Q+R=0$ (b) $\frac{P}{a}+\frac{Q}{b}+\frac{R}{c}=0$ (c) $\frac{P}{\cos A}+\frac{Q}{\cos B}+\frac{R}{\cos C}=0$ (d) None of these

70. *P*, *Q*, *R* are the points on the sides *BC*, *CA*, *AB* of the triangle *ABC* such that $BP:PC=CQ:QA=AR:RB=m:n$. If Δ denotes the area of the ΔABC, then the forces $\overrightarrow{AP}, \overrightarrow{BQ}, \overrightarrow{CR}$ reduce to a couple whose moment is

(a) $2\frac{m+n}{m-n}$ (b) $2\frac{n-m}{n+m}$ (c) $2(m^2-n^2)\Delta$ (d) $2(m^2+n^2)\Delta$

71. If the resultant of forces *P,Q,R* acting along the sides *BC, CA, AB* of a ΔABC passes through its circumcentre, then

(a) $P\sin A+Q\sin B+R\sin C=0$ (b) $P\cos A+Q\cos B+R\cos C=0$

(c) $P\sec A+Q\sec B+R\sec C=0$ (d) $P\tan A+Q\tan B+R\tan C=0$

72. The resultant of two unlike parallel forces of magnitude *P* each acting at a distance of *p* is a

(a) Force *P* (b) Couple of moment *p.P* (c) Force 2*P* (d) Force $\frac{P}{2}$

73. The moment of a system of coplanar forces (not in equilibrium) about three collinear points *A,B,C* in the plane are G_1, G_2, G_3 then,

(a) $G_1.AB+G_2.BC+G_3.AC=0$ (b) $G_1.BC+G_2.CA+G_3.AB=0$

(c) $G_1.CA+G_2.AB+G_3.BC=0$ (d) None of these

74. A rod can turn freely about one of its ends which is fixed. At the other end a horizontal force equal to half the weight of the body is acting. In the position of equilibrium, the rod is inclined to the vertical at an angle

(a) 30^o (b) 45^o (c) 60^o (d) None of these

75. The resultant of two like parallel forces *P*, *Q* passes through a point *O*. If the resultant also passes through *O* when *Q* and *R* replace *P* and *Q* respectively, then

(a) P, Q, R are in G.P. (b) Q, P, R are in G.P. (c) R, P, Q are in G.P. (d) P, Q, R are in A.P.

76. Any two coplanar couples of equal moments

(a) Balance each other (b) Are equivalent (c) Need not be equivalent (d) None of these

77. Two like parallel forces *P* and 3*P* act on a rigid body at points *A* and *B* respectively. If the forces are interchanged in position, the resultant will be displaced through a distance of

(a) $\frac{1}{2}AB$ (b) $\frac{1}{3}AB$ (c) $\frac{1}{4}AB$ (d) $\frac{3}{4}AB$

78. Three like parallel forces *P,Q, R* act at the corners *A,B,C* of a ΔABC. If their resultant passes through the incentre of ΔABC, then

(a) $\frac{P}{a}+\frac{Q}{b}+\frac{R}{c}=0$ (b) $Pa+Qb+Rc=0$ (c) $\frac{P}{a}=\frac{Q}{b}=\frac{R}{c}$ (d) $Pa=Qb=Rc$

79. If the sum of the resolved parts of a system of coplanar forces along two mutually perpendicular direction is zero, then the sum of the moment of the forces about a given point

(a) Is zero always (b) Is positive always (c) Is negative always (d) May have any value

80. Three forces *P*, *Q*, *R* act along the sides *BC*, *CA*, *AB* of triangle *ABC*, taken in order. If their resultant passes through the incentre of ΔABC , then

(a) $P+Q+R=0$ (b) $\frac{P}{a}+\frac{Q}{b}+\frac{R}{c}$ (c) $aP+bQ+cR=0$ (d) None of these

81. If the resultant of two unlike parallel forces of magnitudes 10 *N* and 16 *N* act along a line at a distance of 24 *cm* from the line of action of the smaller force, then the distance between the lines of action of the forces is

(a) 12 *cm* (b) 8 *cm* (c) 9 *cm* (d) 18 *cm*

82. If the position of the resultant of two like parallel forces *P* and *Q* is unaltered, when the positions of *P* and *Q* are interchanged, then

(a) $P=Q$ (b) $P=2Q$ (c) $2P=Q$ (d) None of these

83. Three parallel forces *P,Q, R* act at three points *A,B, C* of a rod at distances of 2*m*, 8*m* and 6*m* respectively from one end. If the rod be in equilibrium, then $P:Q:R=$

(a) $1:2:3$ (b) $2:3:1$ (c) $3:2:1$ (d) None of these

84. The magnitude of the moment of a force $\vec{F}$ about a point is

(a) $|\vec{F}|$ (b) $|\vec{r}\times\vec{F}|$ (c) $\frac{|\vec{r}\times\vec{F}|}{|\vec{F}|}$ (d) $\frac{\vec{r}\times\vec{F}}{|\vec{r}|}$

85. The resultant of two like parallel forces is 12*N*. The distance between the forces is 18*m*. If one of the force is 4*N*, then the distance of the resultant from the smaller force is

(a) 4*m* (b) 8*m* (c) 12*m* (d) None of these

86. Force forming a couple are of magnitude 12*N* each and the arm of the couple is 8*m*. The force of an equivalent couple whose arm is 6*m* is of magnitude

(a) 8*N* (b) 16*N* (c) 12*N* (d) 4*N*

87. The resultant of three equal like parallel forces acting at the vertices of a triangle act at its

(a) Incentre (b) Circumcentre (c) Orthocentre (d) Centroid

88. If the force acting along the sides of a triangle, taken in order, are equivalent to a couple, then the forces are

(a) Equal (b) Proportional to sides of triangle (c) In equilibrium (d)

89. If two like parallel forces of $\frac{P}{Q}$ *Newton* and $\frac{Q}{P}$ *Newton* have a resultant of 2 *Newton*, then

(a) $P=Q$ (b) $P=2Q$ (c) $2P=Q$ (d) None of these

90. Two parallel forces not having the same line of action form a couple if they are **[MNR 1978]**

(a) Like and unlike (b) Like and equal (c) Unequal and unlike (d) Equal and unlike

91. The resultant of non parallel forces and a couple in a plane always reduces to

(a) A single force (b) A couple (c) Two forces (d) None of these

92. Two like parallel forces *P* and 3*P* are 40 *cm* apart. If the direction of *P* is reversed, then their resultant shifts through a distance of

[Roorkee Screening 1998]

(a) 30 *cm* (b) 40 *cm* (c) 50 *cm* (d) 60 *cm*

93. Let a force *P* be represented by the straight line *AB* and *O* is any point. Then the moment of *P* about *O* is represented in magnitude by

(a) ΔAOB (b) $2\Delta AOB$ (c) $3\Delta AOB$ (d) $(1/2)\Delta AOB$

94. The resultant of two parallel forces *P,Q* acting at *A* and *B* respectively acts at *C* when like and at *D* when unlike. If *P>Q*, then *CD* =

(a) $\frac{PQ}{P^2-Q^2}AB$ (b) $\frac{2PQ}{P^2-Q^2}AB$ (c) $\frac{2PQ}{P^2+Q^2}AB$ (d) None of these

95. If the resultant of two like parallel forces of magnitudes 6*N* and 4*N* acts at a distance of 12 *cm* from the line of action of the smaller force, then the distance between the lines of action of the forces is

(a) 18 *cm* (b) 24 *cm* (c) 20 *cm* (d) None of these

96. Two like parallel forces of 5*N* and 15 *N*, act on a light rod at two points *A* and *B* respectively 6*m* apart. The resultant force and the distance of its point of a application from the point *A* are **[Roorkee Screning 1993]**

(a) $10\,N, 4.5m$ (b) $20\,N, 4.5m$ (c) $20\,N, 1.5m$ (d) $10\,N, 15m$

97. Two weights of 10*gms* and 2 *gms* hang from the ends of a uniform lever one meter long and weighting 4*gms*. The point in the lever about which it will balance is from the weight of 10*gms* at a distance of

(a) 5 *cm* (b) 25 *cm* (c) 45 *cm* (d) 65 *cm*

98. In a right angle ΔABC, $\angle A = 90^o$ and sides *a,b,c* are respectively 5 *cm*, 4 *cm* and 3 *cm*. If a force $\vec{F}$ has moments 0, 9 and 16 in *N*-cm. units respectively about vertices *A*, *B* and *C*, then magnitude of $\vec{F}$ is

(a) 9 (b) 4 (c) 5 (d) 3

99. If the forces 6*W*, 5*W* acting at a point (2, 3) in cartesian rectangular coordinates are parallel to the positive *x* and *y* axis respectively, then the moment of the resultant force about the origin is

(a) 8*W* (b) – 3*W* (c) 3*W* (d) – 8*W*

100. A man carries a hammer on his shoulder and holds it at the other end of its light handle in his hand. If he changes the point of support of the handle at the shoulder and if *x* is the distance between his hand and the point of support, then the pressure on his shoulder is proportional to

(a) x (b) x^2 (c) $1/x$ (d) $1/x^2$

101. If the force represented by $3\hat{j} + 2\hat{k}$ is acting through the point $5\hat{i} + 4\hat{j} - 3\hat{k}$, then its moment about the point (1, 3, 1) is **[UPSEAT 2002]**

(a) $14\hat{i} - 8\hat{j} + 12\hat{k}$ (b) $-14\hat{i} + 8\hat{j} - 12\hat{k}$ (c) $-6\hat{i} - \hat{j} + 9\hat{k}$ (d) $6\hat{i} + \hat{j} - 9\hat{k}$

102. If a couple is acting on 2 particles of mass 1 *kg* attached with a rigid rod of length 4*m*, fixed at centre, acting at the end and the angular acceleration of system about centre is 1 rad/s^2, then magnitude of force is

(a) 2*N* (b) 4*N* (c) 1*N* (d) None of these

Advance Level

103. Forces *P*, 3*P*, 2*P* and 5*P* act along the sides *AB*, *BC*, *CD* and *DA* of the square *ABCD*. If the resultant meets *AD* produced at the point *E*, then *AD* : *DE* is

(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) 1 : 5

104. If R and R' are the resultants of two forces $\frac{P}{Q}$ and $\frac{Q}{P}(P > Q)$ according as they are like or unlike such that $R : R' = 25 : 7$, then $P : Q$ =

(a) 2 :1 (b) 3 :4 (c) 4 :3 (d) 1: 2

105. Two like parallel forces *P* and *Q* act on a rigid body at *A* and *B* respectively. If *P* and *Q* be interchanged in positions, show that the point of application of the resultant will be displaced through a distance along *AB*, where *d* =

(a) $\frac{P+Q}{P-Q}AB$ (b) $\frac{2P+Q}{2P-Q}AB$ (c) $\frac{P-Q}{P+Q}AB$ (d) $\frac{P-Q}{2P+Q}AB$

106. A rigid wire, without weight, in the form of the arc of a circle subtending an angle α at its centre and having two weights *P* and *Q* at its extremities rests with its convexity downwards upon a horizontal plane. If θ be the inclination to the vertical of the radius to the end at which *P* is suspended, then $\tan\theta$ =

(a) $\frac{Q\sin\alpha}{P+Q\cos\alpha}$ (b) $\frac{P\sin\alpha}{Q+P\cos\alpha}$ (c) $\frac{Q\cos\alpha}{P+Q\sin\alpha}$ (d) $\frac{P\cos\alpha}{Q+P\sin\alpha}$

107. *ABCD* is a rectangle such that $AB = CD = a$ and $BC = DA = b$. Forces *P*, *P* act along *AD* and *CB*, and forces *Q*,*Q* act along *AB* and *CD*. The perpendicular distance between the resultant of forces *P*, *Q* at *A* and the resultant of forces *P*,*Q* at *C* is

(a) $\frac{Pa+Qb}{\sqrt{P^2+Q^2}}$ (b) $\frac{Pa-Qb}{\sqrt{P^2+Q^2}}$ (c) $\frac{Pb+Qa}{\sqrt{P^2+Q^2}}$ (d) $\frac{Pb-Qa}{\sqrt{P^2+Q^2}}$

108. A horizontal rod AB is suspended at its ends by two vertical strings. The rod is of length 0.6m and weighs 3 *Newton*. Its centre of gravity G is at a distance 0.4m from A. Then the tension of the string at A is

(a) 0.2 N (b) 1.4 N (c) 0.8 N (d) 1 N

109. A horizontal rod of length 5m and weight 4 N is suspended at the ends by two strings. The weights of 8N, 12N, 16N and 20N are placed on the rod at distance 1m, 2m, 3m and 4 m from one end of the rod. The tension in the strings are

(a) $26\,N, 34\,N$ (b) $20\,N, 30\,N$ (c) $10\,N, 40\,N$ (d) None of these

110. Two unlike parallel forces P and Q act at points 5m apart. If the resultant force is 9N and acts at a distance of 10m from the greater force P, then

(a) $P=16\,N, Q=7\,N$ (b) $P=15\,N, Q=6\,N$ (c) $P=27\,N, Q=18\,N$ (d) $P=18\,N, Q=9\,N$

111. A force $\sqrt{5}$ units act along the line $(x-3)/2=(y-4)/(-1)$, the moment of the force about the point (4,1) along z-axis is

[UPSEAT 2000]

(a) 0 (b) $5\sqrt{5}$ (c) $-\sqrt{5}$ (d) 5

112. The height from the base of a pillar must be end B of a rope AB of given length be fixed so that a man standing on the ground and pulling at its other end with a given force may have the greatest tendency to make the pillar overturn is

(a) AB (b) $AB/2$ (c) $AB/\sqrt{2}$ (d) None of these

113. In a triangle ABC right angled at C, the lengths of sides AC and BC are 3 cm and 4 cm respectively. Parallel forces each equal to P act at the vertices A,B,C and parallel forces each equal to 2P act at the middle points of all the sides of the triangle. The distance of C.G. from the vertex C is

(a) 2/3 cm (b) 4/3 cm (c) 5/3 cm (d) None of these

114. A uniform rod BC of length 6 cm and weight 2 kg can turn freely about the fixed point B. The rod is attached to the point A by a string AC of length 8 cm. The points A and B are in a horizontal line at a distance 10 cm. The tension in the string is

(a) 3/5 kg wt (b) 1/5 kg wt (c) 2/5 kg wt (d) None of these

115. If each of the two unlike parallel forces P and Q ($P>Q$) acting at a distance d apart be increased by S, then the point of application of the resultant is moved through a distance

(a) $\frac{d}{P-Q}$ (b) $\frac{S}{P-Q}$ (c) $\frac{Sd}{P-Q}$ (d) $\frac{S}{d(P-Q)}$

116. Three forces P, Q and R act along the sides BC, AC and BA of an equilateral triangle ABC. If their resultant is a force parallel to BC through the centroid of the triangle ABC, then

(a) $P=Q=R$ (b) $P=2Q=2R$ (c) $2P=Q+2R$ (d) $2P=2Q=R$

117. Two equal heavy rods, of weight W and length 2a are freely hinged together and placed symmetrically over a smooth fixed sphere of radius r. The inclination θ of each rod to the horizontal is given by

(a) $r\tan\theta\sec^2\theta=a$ (b) $r(\tan^3\theta+\tan\theta)=a$ (c) $r\sin\theta=a\cos^3\theta$ (d) None of these

118. A couple is of moment $\vec{G}$ and the force forming the couple is $\vec{P}$. If $\vec{P}$ is turned through a right angle, the moment of the couple thus formed is $\vec{H}$. If instead, the force $\vec{P}$ are turned an angle α, then the moment of couple becomes **[AIEEE 2003]**

(a) $\vec{G}\sin\alpha-\vec{H}\cos\alpha$ (b) $\vec{H}\cos\alpha+\vec{G}\sin\alpha$ (c) $\vec{G}\cos\alpha+\vec{H}\sin\alpha$ (d) $\vec{H}\sin\alpha-\vec{G}\cos\alpha$

Equilibrium of Coplanar Forces

Basic Level

119. A heavy uniform rod, 15 *cm* long, is suspended from a fixed point by strings fastened to its ends, their lengths being 9 and 12 *cm*. If θ be the angle at which the rod is inclined to the vertical, then $\sin\theta =$

(a) $\frac{4}{5}$ (b) $\frac{8}{9}$ (c) $\frac{19}{20}$ (d) $\frac{24}{25}$

120. A light string of length l is fastened to two points A and B at the same level at a distance 'a' apart. A ring of weight W can slide on the string, and a horizontal force P is applied to it such that the ring is in equilibrium vertically below B. The tension in the string is equal to

(a) $\frac{aW}{l}$ (b) *law* (c) $\frac{W(l^2+a^2)}{2l^2}$ (d) $\frac{2W(l^2+a^2)}{2a^2}$

121. Two forces P and Q acting parallel to the length and base of an inclined plane respectively would each of them singly support a weight W, on the plane , then $\frac{1}{P^2}-\frac{1}{Q^2}=$

(a) $1/W^2$ (b) $2/W^2$ (c) $3/W^2$ (d) None of these

122. The resultant of the forces 4, 3, 4 and 3 units acting along the lines AB, BC, CD and DA of a square $ABCD$ of side 'a' respectively is

[MNR 1988]

(a) A force $5\sqrt{2}$ through the centre of the square (b) A couple of moment 7*a*

(c) A null force (d) None of these

123. A body of 6.5 kg is suspended by two strings of lengths 5 and 12 metres attached to two points in the same horizontal line whose distance apart is 13 *m*. The tension of the strings in *kg wt*. are

(a) 3,5 (b) 2.5, 6 (c) 4, 5 (d) 3, 4

124. A body of mass 10 *kg* is suspended by two strings 7 *cm* and 24 *cm* long, their other ends being fastened to the extremities of a rod of length 25 *cm*. If the rod be so held that the body hangs immediately below its middle point, then the tension of the strings in *kg* wt. are

(a) 7/5, 24/5 (b) 14/5, 48/5 (c) 3/5, 7/5 (d) None of these

125. A sphere of radius r and weight W rests against a smooth vertical wall, to which is attached a string of length l where one end is fastened to a point on the wall and the other to the surface of the sphere. Then the tension in the string is

(a) $\frac{W(l-r)}{\sqrt{(l^2+2lr)}}$ (b) $\frac{W(l-r)}{l+r}$ (c) $\frac{W(l+r)}{\sqrt{(l^2+2lr)}}$ (d) None of these

126. A system of five forces whose directions and non-zero magnitudes can be chosen arbitrarily, will never be in equilibrium if n of the forces are concurrent, where

(a) $n=2$ (b) $n=3$ (c) $n=4$ (d) $n=5$

127. A string ABC has its extremities tied to two fixed points A and B in the same horizontal line. If a weight W is knotted at a given point C, then the tension in the portion CA is (where a, b, c and the sides and Δ is the area of triangle ABC)

(a) $\frac{Wb}{4c\Delta}(a^2+b^2+c^2)$ (b) $\frac{Wb}{4c\Delta}(b^2+c^2-a^2)$ (c) $\frac{Wb}{4c\Delta}(c^2+a^2-b^2)$ (d) $\frac{Wb}{4c\Delta}(a^2+b^2-c^2)$

128. A uniform rod of weight W and length $2l$ is resting in a smooth spherical bowl of radius r. The rod is inclined to the horizontal at an angle of

(a) 0° (b) $\pi/4$ (c) $\tan^{-1}(l/r)$ (d) $l/\sqrt{(r^2-l^2)}$

129. There are three coplanar forces acting on a rigid body. If these are in equilibrium, then they are

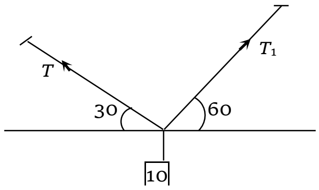
(a) Parallel (b) Concurrent (c) Concurrent or parallel (d) All of these

130. There is a system of coplaner forces acting on a rigid body represented in magnitude, direction and sense by the sides of a polygon taken in order, then the system is equivalent to

(a) A single non-zero force (b) A zero force

(c) A couple, where moment is equal to the area of polygon (d) A couple, where moment is twice the area of polygon

131. A weight of 10 *N* is hanged by two ropes as shown in fig., find tensions T_1 and T_2. **[UPSEAT 2002]**



(a) $5N, 5\sqrt{3}N$ (b) $5\sqrt{3}N, 5N$ (c) $5N, 5N$ (d) $5\sqrt{3}N, 5\sqrt{3}N$

132. Three coplanar forces each equal to *P*, act at a point. The middle one makes an angle of 60^o with each one of the remaining two forces. If by applying force *Q* at that point in a direction opposite to that of the middle force, equilibrium is achieved, then

(a) $P = Q$ (b) $P = 2Q$ (c) $2P = Q$ (d) None of these

133. A 2*m* long uniform rod *ABC* is resting against a smooth vertical wall at the end *A* and on a smooth peg at a point *B*. If distance of B from the wall is 0.3*m*, then

(a) $AB < 0.3m$ (b) $AB < 1.0m$ (c) $AB > 0.3m$ (d) $AB > 1.0m$

Advance Level

134. A uniform rod *AB* movable about a hinge at *A* rests with one end in contact with a smooth wall. If α be the inclination of the rod to the horizontal, then reaction at the hinge is

(a) $\frac{W}{2}\sqrt{3 + \text{cosec}^2\alpha}$ (b) $\frac{W}{2}\sqrt{3 + \sin^2\alpha}$ (c) $W\sqrt{3 + \text{cosec}^2\alpha}$ (d) None of these

135. A uniform rod *AB*, 17*m* long whose mass in 120*kg* rests with one end against a smooth vertical wall and the other end on a smooth horizontal floor, this end being tied by a chord 8*m* long, to a peg at the bottom of the wall, then the tension of the chord is

(a) 32 *kg wt* (b) 16 *kg wt* (c) 64 *kg wt* (d) 8 *kg wt*

136. Forces of magnitudes 3, *P*, 5, 10 and *Q Newton* are respectively acting along the sides *AB*, *BC*, *CD*, *AD* and the diagonal *CA* of a rectangle *ABCD*, where *AB*= 4 *m* and *BC* =3*m*. If the resultant is a single force along the other diogonal *BD* then *P*,*Q* and the resultant are

(a) $4, 10\frac{5}{12}, 12\frac{11}{12}$ (b) 5, 6, 7 (c) $3\frac{1}{2}, 8, 9\frac{1}{2}$ (d) None of these

137. A uniform rod *AB* of length *a* hangs with one end against a smooth vertical wall, being supported by a string of length *l*, attached to the other end of the rod and to a point of the rod vertically above *B*. If the rod rests inclined to the wall at an angle θ, then $\cos^2\theta =$

(a) $(l^2 - a^2)/a^2$ (b) $(l^2 - a^2)/2a^2$ (c) $(l^2 - a^2)/3a^2$ (d) None of these

138. The resultant of two forces sec *B* and sec *C* along sides *AB*, *AC* of a triangle *ABC* is a force acting along *AD*, where *D* is **[MNR 1995]**

(a) Middle point of *BC*
(b) Foot of perpendicular from *A* on *BC*
(c) *D* divides *BC* in the ratio cos *B*: cos *C*
(d) *D* divides *BC* in the ratio cos *C*: cos *B*

139. Three coplanar forces each of weight 10 kilogram are acting at a particle. If their line of actions make same angle, then their resultant force will be

(a) Zero (b) $5\sqrt{2}$ (c) $10\sqrt{2}$ (d) 20

Friction

Basic Level

140. A rough plane is inclined at an angle α to the horizon. A body is just to slide due to its own weight. The angle of friction would be

[BIT Ranchi 1994]

(a) $\tan^{-1}\alpha$ (b) α (c) $\tan\alpha$ (d) 2α

141. A particle is resting on a rough inclined plane with inclination α . The angle of friction is λ , the particle will be at rest if and only if,

[UPSEAT 2000; MNR 1991]

(a) $\alpha > \lambda$ (b) $\alpha \geq \lambda$ (c) $\alpha \leq \lambda$ (d) $\alpha < \lambda$

142. The relation between the coefficient of friction (μ) and the angle of friction (λ) is given by

(a) $\mu = \cos\lambda$ (b) $\mu = \sin\lambda$ (c) $\mu = \tan\lambda$ (d) $\mu = \cot\lambda$

143. A rough inclined plane has its angle of inclination equal to 45^o and $\mu = 0.5$ .The magnitude of the least force in *kg wt*, parallel to the plane required to move a body of 4*kg* up the plane is

(a) $3\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

144. A body of weight *W* rests on a rough plane, whose coefficient of friction is $\mu (= \tan\lambda)$ which is inclined at an angle α with the horizon. The least force required to pull the body up the plane is

(a) $W\sin\lambda$ (b) $W\cos\lambda$ (c) $W\tan\lambda$ (d) $W\cot\lambda$

145. The minimum force required to move a body of weight *W* placed on a rough horizontal plane surface is

(a) $W\sin\lambda$ (b) $W\cos\lambda$ (c) $W\tan\lambda$ (d) $W\cos\lambda$

146. A body of weight 4 *kg* is kept in a plane inclined at an angle of 30^o to the horizontal. It is in limiting equilibrium. The coeffiecient of friction is then equal to

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $\frac{1}{4\sqrt{3}}$ (d) $\frac{\sqrt{3}}{4}$

147. A cubical block rests on an inclined plane with four edges horizontal. The coefficient of friction is $\frac{1}{\sqrt{3}}$. The block just slides when the angle of inclination of the plane is

(a) 0^o (b) 30^o (c) 60^o (d) 45^o

148. A weight *W* can be just supported on a rough inclined plane by a force *P* either acting along the plane or horizontally. The ratio $\frac{P}{W}$, for the angle of friction ϕ , is

(a) $\tan\phi$ (b) $\sec\phi$ (c) $\sin\phi$ (d) None of these

149. A ball *AB* of weight *W* rests like a ladder, with upper end *A* against a smooth vertical wall and the lower end *B* on a rough horizontal plane. If the bar is just on the point of sliding, then the reaction at *A* is equal to (μ is the coeffcient of friction)

(a) μW (b) *W* (c) Normal reaction at *B* (d) W/μ

150. A body is in equilibrium on a rough inclined plane of which the coeffcient of friction is $(1/\sqrt{3})$. The angle of inclination of the plane is gradually increased. The body will be on the point of sliding downwards, when the inclinician of the plane reaches **[MNR 1995]**

(a) 15^o (b) 30^o (c) 45^o (d) 60^o

151. A body of weight 40 *kg* rests on a rough horizontal plane, whose coefficient of friction is 0.25. The least force which acting horizontally would move the body is

(a) 10 *kg* wt (b) 20 *kg* wt (c) 30 *kg* wt (d) 40 *kg* wt

152. The least force required to pull a body of weight *W* up an inclined rough plane is **[BIT Ranchi 1992]**

(a) $W\sin(\alpha + \lambda)$ (b) $2W\sin(\alpha - \lambda)$ (c) $W\sin(\alpha - \lambda)$ (d) $2W\sin(\alpha + \lambda)$

153. The foot of a uniform ladder is on a rough horizontal ground and the top rests against a smooth vertical wall. The weight of the ladder is 400 units. A man weighing 800 units stands on the ladder at one quarter of its length from the bottom. If the inclination of the ladder to the horizontal is 30^o , the reaction at the wall is

(a) 0 (b) $1200\sqrt{3}$ (c) $800\sqrt{3}$ (d) $400\sqrt{3}$

154. A hemi spherical shell rests on a rough inclined plane, whose angle of friction is λ, the inclination of the plane base of the rim to the horizon cannot be greater than

(a) $\sin^{-1}(2\sin\lambda)$ (b) $\cos^{-1}(2\cos\lambda)$ (c) $\tan^{-1}(2\tan\lambda)$ (d) $\cot^{-1}(2\cot\lambda)$

155. A uniform ladder of length 70*m* and weight *W* rests against a vertical wall at an angle of 45^o with the wall. The coefficient of friction of the ladder with the ground and the wall are 1/2 and 1/3 respectively. A man of weight *W*/2 climbs the ladder without slipping. The height in metres to which he can climb is

(a) 30 (b) 40 (c) 50 (d) 60

156. A body is on the point of sliding down an inclined plane under its own weight. If the inclination of the plane to the horizon be 30^o, the angle of friction will be

(a) 30^o (b) 60^o (c) 45^o (d) 15^o

157. A ladder 20 *ft* long is resting against a smooth vertical wall at an angle of 30^o with it. Its foot lies on a rough ground with coefficient of friction 0.3. If the weight of the ladder is 30 kg, then a man of 60kg weight can climb on the ladder upto the height of

(a) $(9\sqrt{3}-5)ft$ (b) $(9\sqrt{3}+5)ft$ (c) $9\sqrt{3}\,ft$ (d) None of these

Advance Level

158. The end of a heavy uniform rod *AB* can slide along a rough horizontal rod *AC* to which it is attached by a ring. *B* and *C* are joined by a string. If $\angle ABC$ be a right angle, when the rod is on the point of sliding, μ the coefficient of friction and α the angle between *AB* and the vertical, then

(a) $\mu = 2\tan\alpha/(2+\tan^2\alpha)$ (b) $\mu = \tan\alpha/(2+\tan^2\alpha)$ (c) $\mu = 2\cot\alpha/(1+\cot^2\alpha)$ (d) $\mu = \cot\alpha/(2+\cot^2\alpha)$

159. A solid cone of semi- vertical angle θ is placed on a rough inclined plane. If the inclination of the plane is increased slowly and $\mu < 4\tan\theta$, then

(a) Cone will slide down before toppling (b) Cone will topple before sliding down

(c) Cone will slide and topple simultaneously (d) Cone will rest in limiting equilibrium

160. A circular cylinder of radius *r* and height *h* rests on a rough horizontal plane with one of its flat ends on the plane. A gradually increasing horizontal force is applied through the centre of the upper end. If the coefficient of friction is μ, the cylinder will topple before sliding, if

(a) $r < \mu h$ (b) $r \geq \mu h$ (c) $r \geq 2\mu h$ (d) $r = 2\mu h$

161. A uniform beam *AB* of weight *W* is standing with the end *B* on a horizontal floor and end A leaning against a vertical wall. The beam stands in a vertical plane perpendicular to the wall inclined at 45^o to the vertical, and is in the position of limiting equilibrium. If the two points of contact are equally rough, then the coefficient of friction at each of them is **[Roorkee 1970]**

(a) $\sqrt{2}-1$ (b) $1/\sqrt{2}$ (c) $1/\sqrt{3}$ (d) None of these

162. A ladder, 10 metre long, rests with one end against a smooth vertical wall and the other end on the ground which is rough; the coefficient of friction being $\frac{1}{2}$. The foot of the ladder being 2 metre from the wall. A man whose weight is 4 times that of the ladder can ascend before it begins to slip a distance (in metre), is

(a) $\frac{3}{4}(10\sqrt{6}-1)$ (b) $\frac{5}{4}(10\sqrt{6}-1)$ (c) $\frac{2}{3}(5\sqrt{2}-1)$ (d) None of these

163. A body is pulled up an inclined rough plane. Let λ be the angle of friction. The required force is least when it makes an angle $k\lambda$ with the inclined plane, where $k=$

(a) 1/3 (b) 1/2 (c) 1 (d) 2

164. A body of 6*kg* rests in limiting equilibrium on an inclined plane whose slope is 30^o. If the plane is raised to a slope of 60^o, the force in *kg* along the plane required to support it, is

(a) 3 *kg* (b) $2\sqrt{3}\ kg$ (c) $\sqrt{3}\ kg$ (d) $3\sqrt{3}\ kg$

Centre of Gravity

Basic Level

165. The C.G. of three particles placed at the vertices of a triangle is at its

(a) Incentre (b) Centroid (c) Circumcentre (d) Orthocentre

166. In a circular disc of uniform metal sheet of radius 10 *cm* and centre *O*, two circular holes of radii 5*cm* and 2.5*cm* are punched. The centre G_1 and G_2 of the wholes are on the same diameter of the circular disc. If *G* is the centre of gravity of the punched disc, then *OG*=

(a) $\frac{22}{25}cm$ (b) $\frac{55}{22}cm$ (c) $\frac{25}{22}cm$ (d) None of these

167. The centre of mass of a rod of length '*a*' *cm* whose density varies as the square of the distance from one end, will be at a distance

(a) $\frac{a}{2}$ from this end (b) $\frac{a}{3}$ from this end (c) $\frac{2a}{3}$ from this end (d) $\frac{3a}{4}$ from this end

168. *AB* is a straight line of length 150*cm*. Two particles of masses 1*kg* and 3*kg* are placed at a distance of 15*cm* form *A* and 50 *cm* from *B* respectively. The distance of the third particle of mass 2*kg* from *A*, so that the C.G. of the system is at the middle point of *AB* is

(a) 40 *cm* (b) 50 *cm* (c) 67.5 *cm* (d) None of these

169. A solid right circular cylinder is attached to a hemisphere of equal base. If the C.G of combined solid is at the centre of the base, then the ratio of the radius and height of cylinder is

(a) 1:2 (b) $\sqrt{2}:1$ (c) 1: 3 (d) None of these

170. In a right angled triangle one side is thrice the other side in length. The triangle is suspended by a string attached at the right angle. The angle that the hypotenuse of the triangle will make with the vertical is

(a) $\sin^{-1}(3/5)$ (b) $\sin^{-1}(4/5)$ (c) 60^o (d) None of these

171. A square hole is punched out of a circular lamina of diameter 4*cm,* the diagonal of the square being a radius of the circle. Centroid of the remainder from the centre of the circle is at a distance

(a) $\frac{1}{2\pi+1}$ (b) $\frac{1}{2\pi-1}$ (c) $\frac{1}{\pi+1}$ (d) $\frac{1}{\pi-1}$

172. The centre of gravity of the surface of a hollow cone lies on the axis and divides it in the ratio

(a) 1 : 2 (b) 1 : 3 (c) 2 : 3 (d) 1 : 1

173. A body consists of a solid cylinder with radius *a* and height *a* together with a solid hemisphere of radius *a* placed on the base of the cylinder. The centre of gravity of the complete body is

(a) Inside the cylinder (b) Inside the hemisphere

(c) On the interface between the two (d) Outside both

174. The centre of gravity *G* of three particles of equal mass placed at the three vertices of a right angled isosceles triangle whose hypotenuse is equal to 8 units is on the median through A such that *AG* is

(a) 4/3 (b) 5/3 (c) 8/3 (d) 10/3

175. Weights 2, 3, 4 and 5 *lbs* are suspended from a uniform lever 6 *ft* long at distances of 1, 2, 3 and 4 *ft* from one end. If the weight of the lever is 11 *lbs*, then the distance of the point at which it will balance from this end is

(a) 53/25 (b) 63/25 (c) 73/25 (d) None of these

Advance Level

176. *ABC* is a uniform triangular lamina with centre of gravity at *G*. If the portion *GBC* is removed, the centre of gravity of the remaining portion is at G'. Then GG' is equal to

(a) $\frac{1}{3}AG$ (b) $\frac{1}{4}AG$ (c) $\frac{1}{2}AG$ (d) $\frac{1}{6}AG$

177. On the same base *AB*, and on opposite side of it, isosceles triangles *CAB* and *DAB* are described whose altitudes are 12 *cm* and 6 *cm* respectively. The distance of the centre of gravity of the quadrilateral *CADB* from *AB*, is

(a) 0.5 *cm* (b) 1*cm* (c) 1.5*cm* (d) 2*cm*

178. A straight rod AB of length 1*ft* balances about a point 5 inches from A when masses of 9 and 6 *lbs* are suspended from A and B respectively. It balances about a point $3\frac{1}{2}$ inches from B when the mass of 6 *lbs* is replaced by one of 23 *lbs*. The distance of C.G. of the rod from the end B is

(a) $3\frac{1}{2}$ inches (b) $5\frac{1}{2}$ inches (c) $2\frac{1}{2}$ inches (d) None of these

179. A uniform rod of length $2l$ and weight W is lying across two pegs on the same level 'a' *ft* apart. If neither peg can bear a pressure greater than P, then the greatest length of the rod which may be projected beyond either peg is

(a) $l-\frac{a(W+P)}{W}$ (b) $l-\frac{a(W-P)}{W}$ (c) $l+\frac{a(W-P)}{W}$ (d) None of these

180. A rod $2\frac{1}{2}ft$ long rests on two pegs 10 inches apart with its centre mid way between them. The greatest masses that can be suspended in succession from the two ends without disturbing equilibrium are 4 and 6 *lbs*. respectively. The weight of the rod is

(a) 2 *lbs* (b) 4 *lbs* (c) 3 *lbs* (d) None of these

181. A heavy rod $ACDB$, where $AC = a$ and $DB = b$ rests horizontally upon two smooth pegs C and D. If a load P were applied at A, it would just disturb the equilibrium. Similar would do the load Q applied to B. If $CD = c$, then the weight of the rod is

(a) $\frac{Pa+Qb}{c}$ (b) $\frac{Pa-Qb}{c}$ (c) $\frac{Pa+Qb}{2c}$ (d) None of these

* * *

Answer Sheet

Statics — *Assignment (Basic and Advance Level)*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	d	d	d	d	c	b	d	b	d	b	d	c	d	b	a	a	c	b	a
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
c	b	b	a	b	a	a	a	a	b	a	c	b	c	c	b	a	d	b	d
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
b	c	a	b	d	a	b	b	c	a	b	b	c	a	a	b	c	a	c	d
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
a,b	a,d	a,c	b	d	b	b	c	b	b	b	b	b	b	a	b	a	c	d	a
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
c	a	a	b	c	b	d	b	a	d	a	a	b	b	c	b	b	c	d	c
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
a	a	c	c	c	a	b	d	a	c	d	c	c	a	c	b	a,b,c	c	d	c
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
a	b	b	b	c	c	c	a	d	b	b	c	b,c	a	a	a	c	b	a	b
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160

c	c	a	b	a	a	b	a	a	b	a	a	d	a	c	a	a	b	a	a
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
a	b	c	b	b	c	d	c	b	a	b	a	a	c	c	d	d	b	b	b
181																			
a																			