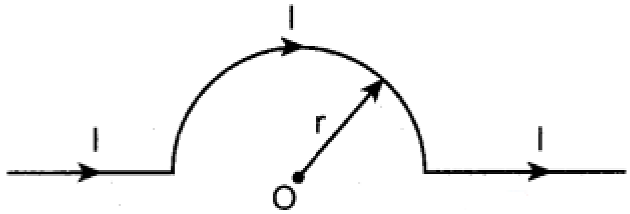
Chapter 3
Magnetism and Magnetic Effects of Electric Current

Question 1.
The magnetic field at the center O of the following current loop is-

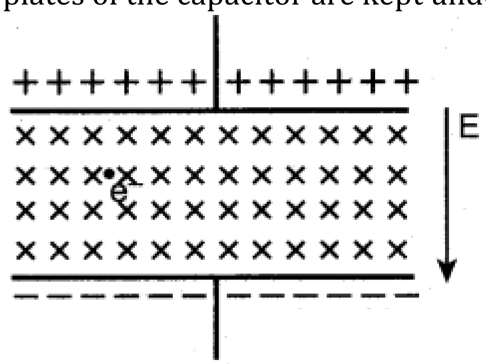


(a) $\frac{\mu_0 I}{4r} \otimes$ (b) $\frac{\mu_0 I}{4r} \odot$

(c) $\frac{\mu_0 I}{2r} \otimes$ (d) $\frac{\mu_0 I}{2r} \odot$

Answer:
(a) $\frac{\mu_0 I}{4r} \otimes$

Question 2.
An electron moves straight inside a charged parallel plate capacitor of uniform charge density σ The time taken by the electron to cross the parallel plate capacitor when the plates of the capacitor are kept under constant magnetic field of induction $\vec{B}$ is-



(a) $\varepsilon_0 \frac{elB}{\sigma}$ (b) $\varepsilon_0 \frac{lB}{\sigma l}$

(c) $\varepsilon_0 \frac{lB}{e\sigma}$ (d) $\varepsilon_0 \frac{lB}{\sigma}$

Answer:
(d) $\varepsilon_0 \frac{lB}{\sigma}$

Question 3.
The force experienced by a particle having mass m and charge q accelerated through a potential difference V when it is kept under perpendicular magnetic field $\vec{B}$ is-

(a) $\sqrt{\frac{2q^3BV}{m}}$ (b) $\sqrt{\frac{q^3B^2V}{2m}}$

(c) $\sqrt{\frac{2q^3B^2V}{m}}$ (d) $\sqrt{\frac{2q^3BV}{m^3}}$

Answer:
(c) $\sqrt{\frac{2q^3B^2V}{m}}$

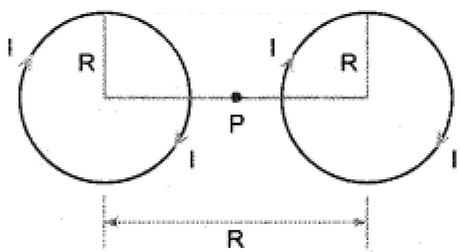
Question 4.
A circular coil of radius 5 cm and 50 turns carries a current of 3 ampere. The magnetic dipole moment of the coil is-
(a) 1.0 amp – m^2
(b) 1.2 amp – m^2
(c) 0.5 amp – m^2
(d) 0.8 amp – m^2
Answer:
(b) 1.2 amp – m^2

Question 5.
A thin insulated wire forms a plane spiral of N = 100 tight turns carrying a current 1 = 8 m A (milli ampere). The radii of inside and outside turns are a = 50 mm and b = 100 mm respectively. The magnetic induction at the center of the spiral is
(a) 5 μT
(b) 7 μT
(c) 8 μT
(d) 10 μT
Answer:
(b) 7 μT

Question 6.
Three wires of equal lengths are bent in the form of loops. One of the loops is circle, another is a semi-circle and the third one is a square. They are placed in a uniform magnetic field and same electric current is passed through them. Which of the following loop configuration will experience greater torque ?
(a) circle
(b) semi-circle
(c) square
(d) all of them

Answer:
(a) circle

Question 7.
Two identical coils, each with N turns and radius R are placed coaxially at a distance R as shown in the figure. If I is the current passing through the loops in the same direction, then the magnetic field at a point P which is at exactly at $\frac{R}{2}$ distance between two coils is-



(a) $\frac{8N\mu_0 I}{\sqrt{5}R}$ (b) $\frac{8N\mu_0 I}{5^{3/2}R}$

(c) $\frac{8N\mu_0 I}{5R}$ (d) $\frac{4N\mu_0 I}{\sqrt{5}R}$

Answer:
(b) $\frac{8N\mu_0 I}{5^{3/2}R}$

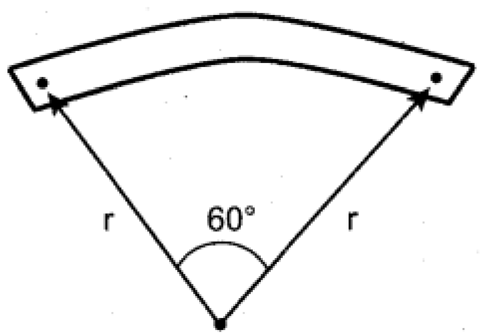
Question 8.
A wire of length l carries a current I along the Y direction and magnetic field is given by $\vec{B} = \frac{\beta}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$. The magnitude of Lorentz force acting on the wire is-

(a) $\sqrt{\frac{2}{\sqrt{3}}}\beta I l$ (b) $\sqrt{\frac{1}{\sqrt{3}}}\beta I l$

(c) $\sqrt{2}\beta I l$ (d) $\sqrt{\frac{1}{2}}\beta I l$

Answer:
(a) $\sqrt{\frac{2}{\sqrt{3}}}\beta I l$

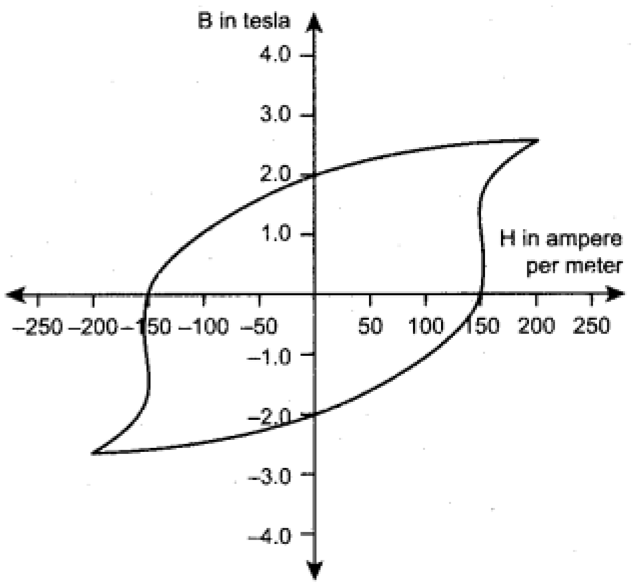
Question 9.
A bar magnet of length l and magnetic moment M is bent in the form of an arc as shown in figure. The new magnetic dipole moment will be- (NEET 2014)



(a) M
(b) $\frac{\pi}{3}$ M
(c) $\frac{2}{\pi}$ M
(d) $\frac{1}{2}$ M
Answer:
(b) $\frac{\pi}{3}$ M

Question 10.
A non-conducting charged ring of charge q, mass m and radius r is rotated with constant angular speed ω. Find the ratio of its magnetic moment with angular momentum is
(a) $\frac{q}{m}$ M
(b) $\frac{2q}{3}$ M
(c) $\frac{q}{2m}$ M
(d) $\frac{q}{4m}$ M
Answer:
(c) $\frac{q}{2m}$ M

Question 11.
The B_H curve for a ferromagnetic material is shown in the figure. The material is placed inside a long solenoid which contains 1000 turns/ cm. The current that should be passed in the solenoid to demagnetize the ferromagnet completely is-



(a) 1.00 m A(milli ampere)
(b) 1.25 mA
(c) 1.50 mA
(d) 1.75 mA
Answer:
(b) 1.25 mA

Question 12.
Two short bar magnets have magnetic moments 1.20 Am^2 and 1.00 Am^2respectively. They are kept on a horizontal table parallel to each other with their north poles pointing towards the south. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centers is (Horizontal components of Earth's magnetic induction is 3.6×10^{-5} Wb m^{-2}) (NSEP 2000-2001)
(a) 3.60×10^{-5} Wb m^{-1}
(b) 3.5×10^{-5} Wb m^{-1}
(c) 2.56×10^{-4} Wb m^{-1}
(d) 2.2×10^{-4} Wb m^{-1}
Answer:
(c) 2.56×10^{-4} Wb m^{-1}

Question 13.
The vertical component of Earth's magnetic field at a place is equal to the horizontal component. What is the value of angle of dip at this place?
(a) 30°
(b) 45°
(c) 60°
(d) 90°
Answer:
(b) 45°

Question 14.
A flat dielectric disc of radius R carries an excess charge on its surface. The surface charge density is σ. The disc rotates about an a × is perpendicular to its plane passing through the center with angular velocity ω. Find the magnitude of the torque on the disc if it is placed in a uniform magnetic field whose strength is B which is directed perpendicular to the a × is of rotation
(a) $\frac{1}{4}$ σωπ BR
(b) $\frac{1}{4}$ σωπ BR^2
(c) $\frac{1}{4}$ σωπ BR^3
(d) $\frac{1}{4}$ σωπ BR^4
Answer:
(d) 14 σωπ BR^4

Question 15.
A simple pendulum with charged bob is oscillating with time period T and let θ be the angular displacement. If the uniform magnetic field is switched ON in a direction perpendicular to the plane of oscillation then-
(a) time period will decrease but θ will remain constant
(b) time period remain constant but θ will decrease
(c) both T and θ will remain the same
(d) both T and θ will decrease
Answer:
(c) both T and θ will remain the same

Short Answer Questions

Question 1.
What is meant by magnetic induction?
Answer:

1. The process by which an object or material is magnetized by an external magnetic field.
2. S.I unit of magnetic induction is the tesla (T) or Wbm^{-2}
3. Dimensional Formula is MT^2A^{-1}

Question 2.
Define magnetic flux.
Answer:
The number of magnetic field lines crossing per unit area is called magnetic flux B.

$\Phi_B = \vec{B} . \vec{A} = BA \cos\theta = B \perp A$

Question 3.
Define magnetic dipole moment.
Answer:
1. The product of the pole strength and magnetic length.
2. It is a vector quantity, denoted by $\vec{p}_m$;
$\vec{p}_m = \vec{q}_m \vec{d}$
3. The magnitude of magnetic dipole moment $P_m = 2q_m l$

Question 4.
State Coulomb's inverse law.
Answer:
The force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.

$$\vec{F} \propto \frac{q_{m_A} q_{m_b}}{r^2} \hat{r}$$

Question 5.
What is magnetic susceptibility?
Answer:
1. The ratio of the intensity of magnetisation (M→) induced in the material due to the magnetising field (H→)
2. $X_m = \frac{|\vec{M}|}{|\vec{H}|}$
3. It is a dimensionless quantity.
4. Isotropic medium – Scalar, Non-isotropic medium – vector.

Question 6.
State Biot-Savart's law.
Answer:
The magnitude of magnetic field $d\vec{B}$ at a point P at a distance r from the small elemental length taken on a conductor carrying current varies-

- directly as the strength of the current I
- directly as the magnitude of the length element $\vec{dl}$

- directly as the sine of the angle (say θ) between $\vec{dl}$ and $\hat{r}$.
- inversely as the square of the distance between the point P and length element $\vec{dl}$.
- This is expressed as $dB \propto \frac{Idl}{r^2} \sin\theta$

Question 7.
What is magnetic permeability?
Answer:

1. The measure of the ability of the material to allow the passage of magnetic field lines through it. (or)
2. The measure of the capacity of the substance to take magnetisation. (or)
3. The degree of penetration of the magnetic field through the substance.

Question 8.
State Ampere's circuital law.
Answer:
The line integral of magnetic field over a closed-loop is μ_0 times net current enclosed by the loop.

$$\oint_C \vec{B}.\vec{dl} = \mu_0 I_{enclosed}$$

Question 9.
Compare dia, para and ferromagnetism.
Answer:

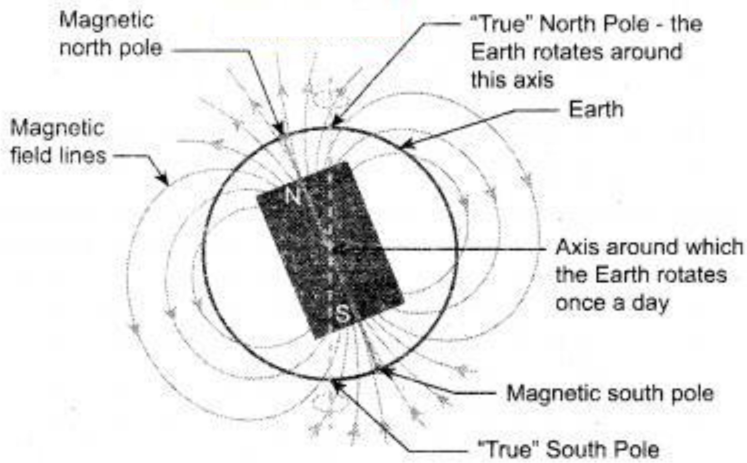
Dia	Para	Ferro
(*i*) Magnetic susceptibility is negative.	(*i*) Magnetic susceptibility is positive and small.	(*i*) Magnetic susceptibility is positive and large.
(*ii*) Relative permeability is slightly less than unity.	(*ii*) Relative permeability is greater than unity.	(*ii*) Relative permeability is large.
(*iii*) The magnetic field lines are repelled or expelled by diamagnetic materials when placed in a magnetic field.	(*iii*) The magnetic field lines are attracted into the paramagnetic materials when placed in a magnetic field.	(*iii*) The magnetic field lines are strongly attracted into the ferromagnetic materials when placed in a magnetic field.
(*iv*) Susceptibility is nearly temperature independent.	(*iv*) Susceptibility is inversely proportional to temperature.	(*iv*) Susceptibility is inversely proportional to temperature.
Examples: Bismuth, Copper and Water etc.	**Examples:** Aluminium, Platinum and chromium etc.	**Examples:** Iron, Nickel and Cobalt.

Question 10.
What is meant by hysteresis?
Answer:

1. Lagging of magnetic induction behind the magnetising field.
2. Hysteresis means lagging behind.

Long Answer Questions

Question 1.
Discuss Earth's magnetic field in detail.
Answer:
Gover suggested that the Earth's magnetic field is due to hot rays coming out from the Sun. These rays will heat up the air near-equatorial region. Once air becomes hotter, it rises above and will move towards northern and southern hemispheres and get electrified. This may be responsible to magnetize the ferromagnetic materials near the Earth's surface.



The north pole of magnetic compass needle is attracted towards the magnetic south pole of the Earth which is near the geographic north pole. Similarly, the south pole of magnetic compass needle is attracted towards the geographic north pole of the Earth which is near magnetic north pole. There are three quantities required to specify the magnetic field of the Earth on its surface, which are often called as the elements of the Earth's magnetic field. They are:

(a) Magnetic declination (D): The angle between magnetic meridian at a point and geographical meridian is called the declination or magnetic declination (D).

(b) Magnetic dip or inclination (I): The angle subtended by the Earth's total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called dip or magnetic inclination (I) at that point.

(c) The horizontal component of the Earth's magnetic field (B_H): The component of Earth's magnetic field along the horizontal direction in the magnetic meridian is called horizontal component of Earth's magnetic field, denoted by B_H.
Let B_E be the net Earth's magnetic field at a point P on the surface of the Earth. B_E can be resolved into two perpendicular components.
Horizontal component $B_H = B_E \cos I$ …… (1)
Vertical component $B_V = B_E \sin I$ …… (2)
Dividing the equation, we get $\tan I = \frac{BV}{BH}$ …….(3)

(i) At magnetic equator The Earth's magnetic field is parallel to the surface of the Earth (i.e., horizontal) which implies that the needle of magnetic compass rests horizontally at an angle of dip, I = 0°.
B $B_E = B_E$
$Bv = 0$
This implies that the horizontal component is maximum at equator and vertical component is zero at equator.

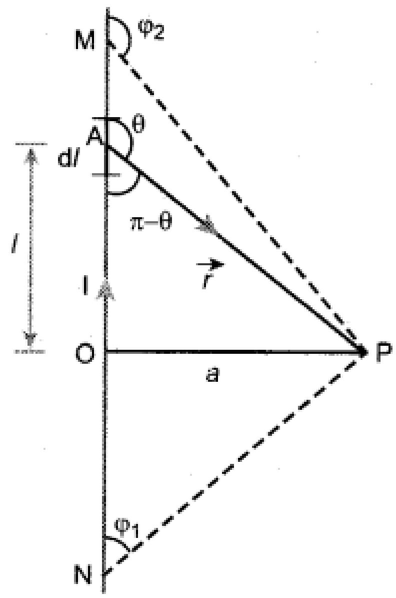
(ii) At magnetic poles. The Earth's magnetic field is perpendicular to the surface of the Earth (i.e., vertical) which implies that the needle of magnetic compass rests vertically at an angle of dip, I = 90°
Hence, $B_H = 0$
$Bv = B_E$
This implies that the vertical component is maximum at poles and horizontal component is zero at poles.

Question 2.
Deduce the relation for the magnetic induction at a point due to an infinitely long straight conductor carrying current.
Answer:
Magnetic field due to a long straight conductor carrying current:

Consider a long straight wire NM with current I flowing from N to M. Let P be the point at a distance a from point O. Consider an element of length dl of the wire at a distance l from point O and $\vec{r}$ be the vector joining the element dl with the point P. Let θ be the angle between $\overrightarrow{dl}$ and $\vec{r}$. Then, the magnetic field at P due to the element is $d\vec{B} = \frac{\mu_0 I \overrightarrow{dl}}{4\pi r^2}$ sinθ (unit vector perpendicular to $\overrightarrow{dl}$ and $\vec{r}$) …… (1)

The direction of the field is perpendicular to the plane of the paper and going into it. This can be determined by

taking the cross product between two vectors $\overrightarrow{dl}$ and $\vec{r}$ (let it be $\hat{n}$). The net magnetic field can be determined by integrating equation with proper limits.



Magnetic field due to a long straight current carrying conductor

$\vec{B} = \int d\vec{B}$

From the figure, in a right angle triangle PAO,

tan (π – θ) = al

$l = \frac{a}{\tan\theta}$ (since tan (π – θ) = -tan θ) ⇒ $\frac{1}{\tan\theta}$

l = a cot θ and r = a cosec θ

differentiating,

dl = a cosec² θdθ

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{\left(a\cos ec^2\theta d\theta\right)}{\left(a\cos ec\theta\right)^2}\sin\theta d\theta\hat{n}$$

$$= \frac{\mu_0 I}{4\pi}\frac{\left(a\cos ec^2\theta d\theta\right)}{a^2\cos ec^2\theta}\sin\theta d\theta\hat{n}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi a}\sin\theta d\theta\hat{n} \qquad ...(2)$$

This is the magnetic field at a point P due to the current in small elemental length. Note that we have expressed the magnetic field OP in terms of angular coordinate i.e. θ. Therefore, the net magnetic field at the point P which can be obtained by integrating $d\vec{B}$ by varying the angle from θ = (φ_1 to θ = φ_1 is

$$\vec{B} = \frac{\mu_0 I}{4\pi a}\int_{\varphi_1}^{\varphi_2}\sin\theta d\theta\hat{n} = \frac{\mu_0 I}{4\pi a}(\cos\varphi_1 - \cos\varphi_2)\hat{n}$$

For a an infinitely long straight wire, 1 = 0 and 2 = , the magnetic field is

$\vec{B} = \frac{\mu_0}{2\pi a}\ \hat{n}$ ...... (3)

Note that here $\hat{n}$ represents the unit vector from the point O to P.

Question 3.
Obtain a relation for the magnetic induction at a point along the axis of a circular coil carrying current.
Answer:

Magnetic field produced along the axis of the current-carrying circular coil:

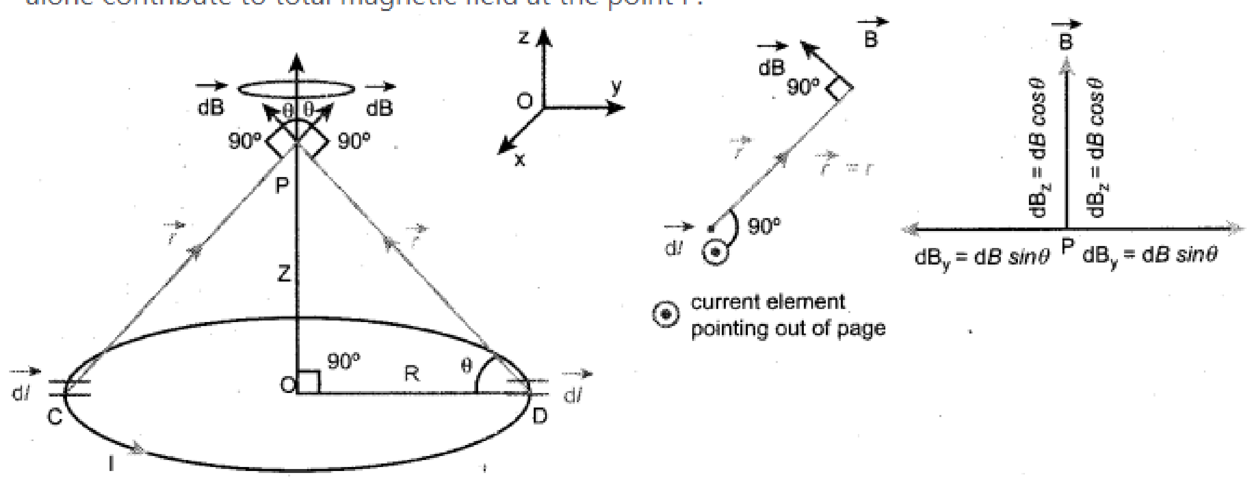
Consider a current-carrying circular loop of radius R and let I be the current flowing through the wire in the direction. The magnetic field at a point P on the axis of the circular coil at a distance z from its center of the coil O. It is computed by taking two diametrically opposite line elements of the coil each of length $\vec{dl}$ at C and D. Let $\vec{r}$ be the vector joining the current element (1 $\vec{dl}$) at C to the point P.

PC = PD = T = $\sqrt{R^2 + Z^2}$ and angle ∠CPO = ∠DPO = θ

According to Biot-Savart's law, the magnetic field at P due to the current element I $\vec{dl}$ is

$d\vec{B} = \frac{\mu_0}{4\pi}\ \frac{Id\vec{l} \times \hat{r}}{r^2}$ ..... (1)

The magnitude of magnetic field due to current element I dl at C and D are equal because of equal distance from the coil. The magnetic field dB due to each current element I $\vec{dl}$ is resolved into two components; dB sin θ along y-direction and dB cos θ along z-direction. Horizontal components of each current element cancels out while the vertical components (dB cos θ $\hat{k}$) alone contribute to total magnetic field at the point P.



Current carrying circular loop using Biot-Savart's law

If we integrate $\overrightarrow{dl}$ around the loop, $d\vec{B}$ sweeps out a cone, then the net magnetic field $\vec{B}$ at point P is

$$\vec{B} = \int d\vec{B} = \int dB\cos\theta\hat{k} \quad ...(2)$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{dl}{r^2}\cos\theta\hat{k} \quad ...(3)$$

$$\text{But } \cos\theta = \frac{R}{\left(R^2+Z^2\right)^{\frac{1}{2}}}$$

Using Pythagorous theorem $r^2 = R^2 + Z^2$ and integrating line element from 0 to $2\pi R$, we get

$$\vec{B} = \frac{\mu_0 I}{2\pi}\frac{R^2}{\left(R^2+Z^2\right)^{\frac{3}{2}}}\hat{k} \quad ...(4)$$

Note that the magnetic field $\vec{B}$ points along the direction from the point O to P. Suppose if the current flows in clockwise direction, then magnetic field points in the direction from the point P to O.

Question 4.
Compute the torque experienced by a magnetic needle in a uniform magnetic field.
Answer:

Torque Acting on a Bar Magnet in Uniform Magnetic Field:

Consider a magnet of length 21 of pole strength q_m kept in a uniform magnetic field $\vec{B}$ Each pole experiences a force of magnitude q_mB but acts in opposite direction.

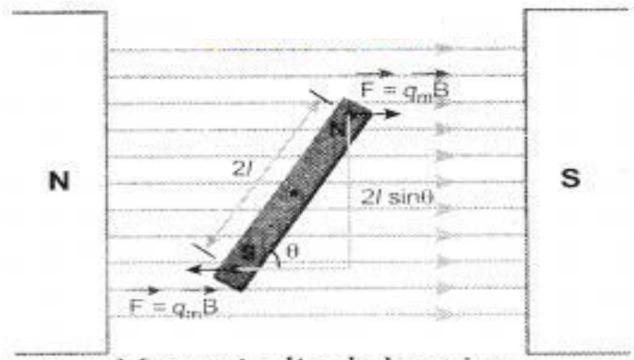
Therefore, the net force exerted on the magnet is zero, so that there is no translatory motion. These two forces constitute a couple (about midpoint of bar magnet) which will rotate and try to align in the direction of the magnetic field $\vec{B}$.

The force experienced by north pole, $\vec{F}_N = q_m \vec{B}$ .... (1)

The force experienced by south pole, $\vec{F}_S = q_m \vec{B}$ .... (2)

Adding equations (1) and (2), we get the net force acting on the dipole as

$\vec{F} = \vec{F}_N + \vec{F}_S = \vec{O}$



Magnetic dipole kept in a uniform magnetic field

This implies, that the net force acting on the dipole is zero, but forms a couple which tends to rotate the bar magnet clockwise (here) in order to align it along $\vec{B}$.

The moment of force or torque experienced by the north and south pole about point O is

$$\vec{\tau} = \overrightarrow{ON} \times \vec{F}_N + \overrightarrow{OS} \times \vec{F}_S$$

$$\vec{\tau} = \overrightarrow{ON} \times q_m \vec{B} + \overrightarrow{OS} \times \left(-q_m \vec{B}\right)$$

By using right hand cork screw rule, we conclude that the total torque is pointing into the paper. Since the magnitudes

$$\left|\overrightarrow{ON}\right| = \left|\overrightarrow{OS}\right| = l \; and \left|q_m \vec{B}\right| = \left|-q_m \vec{B}\right|,$$

The magnitude of total torque about point O

$\tau = l \times q_m B \sin\theta + l \times q_m B \sin\theta$

$\tau = 2l \, x q_m B \sin\theta$

$\tau = P_m B \sin\theta$

$(\therefore q_m \times 2l = P_m)$

In vector notation, $\tau = p_m \times \vec{B}$

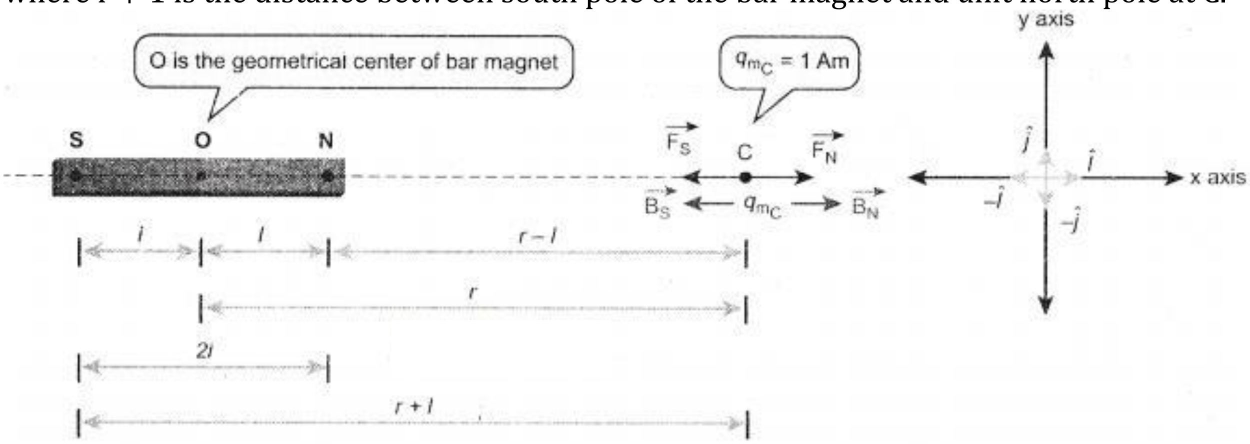
Question 5.
Calculate the magnetic induction at a point on the a×ial line of a bar magnet.
Answer:
The magnetic field at a point along the axial line of the magnetic dipole (bar magnet):
Consider a bar magnet NS. Let N be the North Pole and S be the south pole of the bar magnet, each of pole strength q_m and separated by a distance of 2l. The magnetic field at a point C (lies along the axis of the magnet) at a distance from the geometrical center O of the bar magnet can be computed by keeping unit north pole (q_{MC} = 1 A m) at C. The force experienced by the unit north pole at C due to pole strength can be computed using Coulomb's law of magnetism as follows:
The force of repulsion between the north pole of the bar magnet and the unit north pole at point C (in free space) is

$$\vec{F}_N = \frac{\mu_0}{4\pi} \frac{q_m}{(r-l)^2} \hat{i} \qquad ...(1)$$

where r – l is the distance between north pole of the bar magnet and unit north pole at C. The force of attraction between the South Pole of the bar magnet and unit North Pole at point C (in free space) is

$$\vec{F}_S = -\frac{\mu_0}{4\pi}\frac{q_m}{(r+l)^2}\hat{i} \qquad ...(2)$$

where r + 1 is the distance between south pole of the bar magnet and unit north pole at C.



From equation (1) and (2), the net force at point C is $\vec{F} = \vec{F}_N + \vec{F}_S$.

From definition, this net force is the magnetic field due to magnetic dipole at a point C($\vec{F} = \vec{B}$)

$$\vec{B} = \frac{\mu_0}{4\pi(r-l)^2}\hat{i} + \left(\frac{\mu_0}{4\pi}\frac{q_m}{(4+l)^2}\hat{i}\right) = \frac{\mu_0 q_m}{4\pi}\left(\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right)\hat{i}$$

$$\vec{B} = \frac{\mu_0 2r}{4\pi}\left(\frac{q_m.(2l)}{(r^2-l^2)^2}\right)\hat{i} \qquad ...(3)$$

Since, magnitude of magnetic dipole moment is $|\vec{P}_m|$ $p_m = q_m. 2l$ the magnetic field point C equation (3) can be written as

$$\vec{B}_{axial} = \frac{\mu_0}{4\pi}\left(\frac{2rp_m}{(r^2-l^2)^2}\right)\hat{i} \qquad ...(4)$$

If the distance between two poles in a bar magnet is small (looks like a short magnet) compared to the distance between geometrical centre O of a bar magnet and the location of point C i.e.,

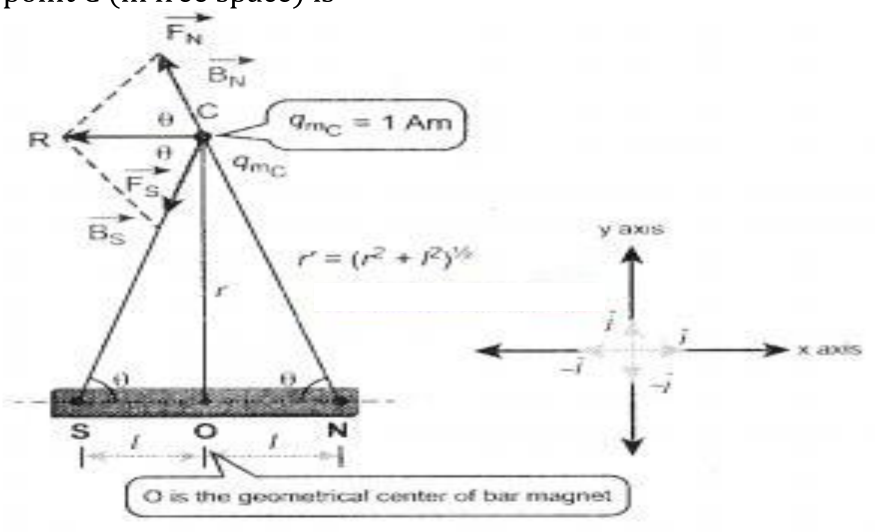
r >>1 then, $(r^2 - l^2)^2 \approx r^4$ ..... (5)

Therefore, using equation (5) in equation (4), we get

$$\vec{B}_{axial} = \frac{\mu_0}{4\pi}\left(\frac{2rp_m}{(r^3}\right)\hat{i} = \frac{\mu_0}{4\pi}\frac{2}{r^3}\vec{p}_m$$

Where $\vec{p}_m = p_m\hat{i}$. ...(6)

Question 6.
Obtain the magnetic induction at a point on the equatorial line of a bar magnet. The magnetic field at a point along the equatorial line due to a magnetic dipole (bar magnet)
Answer:
Consider a bar magnet NS. Let N be the north pole and S be the south pole of the bar magnet, each with pole strength q_m and separated by a distance of 2l. The magnetic field at a point C (lies along the equatorial line) at a distance r from the geometrical center O of the bar magnet can be computed by keeping unit north pole (q_mC = 1 A m) at C. The force experienced by the unit north pole at C due to pole strength N-S can be computed using Coulomb's law of magnetism as follow's:
The force of repulsion between the North Pole of the bar magnet and the unit north pole at point C (in free space) is



Magnetic field at a point along the equatorial line due to a magnetic dipole

$\vec{F}_N = -F_N \cos\theta\ \hat{i} + F_N \sin\theta\ \hat{j}$ ...... (1)

Where $F_N = \frac{\mu_0}{4\pi} \frac{q_m}{r'^2}$

The force of attraction (in free space) between south pole of the bar magnet and unit north pole at point C is

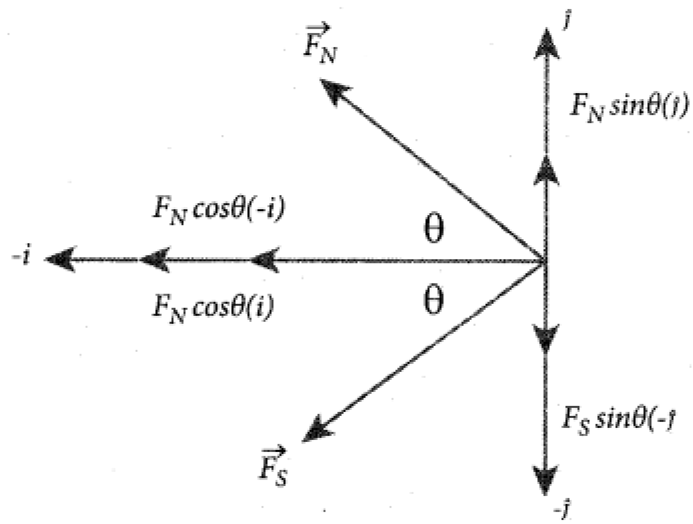
$\vec{F}_S = -F_S \cos\theta\ \hat{i} + F_S \sin\theta$ j ...... (2)

Where $\vec{F}_S = \frac{\mu_0}{4\pi} \frac{q_m}{r'^2}$

From equation (1) and equation (2), the net force at point C is $\vec{F} = F_N + F_S$ This net force is equal to the magnetic field at the point C.

$\vec{B} = -(F_N + F_S) \cos\theta\ \hat{i}$

Since, $F_N = F_S$



Components of force

$$\vec{B} = -\frac{2\mu_0}{4\pi}\frac{q_m}{r'^2}\cos\theta\hat{i} = -\frac{2\mu_0}{4\pi}\frac{q_m}{(r^2+l^2)}\cos\theta\hat{i} \quad ...(3)$$

In a right angle triangle NOC as shown in the Figure 1

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{l}{r'} = \frac{1}{\left(r^2+l^2\right)^{\frac{1}{2}}} \quad ...(4)$$

Substituting equation 4 in equation 3 We get

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{q_m \times (2l)}{\left(r^2+l^2\right)^{\frac{3}{2}}} \quad ...(5)$$

Since, magnitude of magnetic dipole moment is $|\vec{P}_m|$ = P_m = q_m. 2l and substituting in equation (5), the magnetic field at a point C is

$$\vec{B}_{\text{equatorial}} = -\frac{\mu_0}{4\pi}\frac{P_m}{\left(r^2+l^2\right)^{\frac{3}{2}}}\hat{i} \quad ...(6)$$

If the distance between two poles in a bar magnet are small (looks like a short magnet) when compared to the distance between geometrical center O of a bar magnet and the location of point C i.e., r>> l, then,

$(r^2 + l^2)^{\frac{3}{2}}$

Therefore, using equation (7) in equation (6), we get ≈ r^3 ……… (7)

$$\vec{B}_{\text{equatorial}} = -\frac{\mu_0}{4\pi}\frac{p_m}{r^3}\hat{i}$$

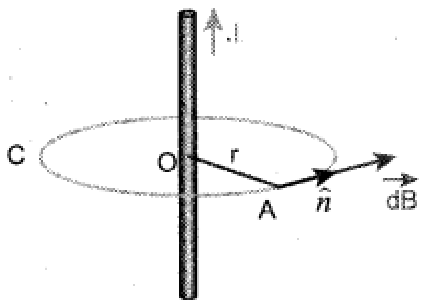
Since P_m i^ = ||P→ m||m, in general, the magnetic field at equatorial point is given by

$$\vec{B}_{\text{equatorial}} = -\frac{\mu_0}{4\pi}\frac{p_m}{r^3} \quad ...(8)$$

Note that magnitude of B_{axial} is twice that of magnitude of $B_{equatorial}$ and the direction of are opposite.

Question 7.
Find the magnetic induction due to a long straight conductor using Ampere's circuital law. Magnetic field due to the current-carrying wire of infinite

Answer:
length using Ampere's law:
Consider a straight conductor of infinite length carrying current I and the direction of magnetic field lines. Since the wire is geometrically cylindrical in shape C and symmetrical about its axis, we construct an Amperian loop in the form of a circular shape at a distance r from the centre of the conductor. From the Ampere's law, we get



Ampèrian loop for current carrying straight wire

$$\oint_C \vec{B}.d\vec{l} = \mu_0 I$$

Where dl is the line element along the amperian loop (tangent to the circular loop). Hence, the angle between the magnetic field vector and the line element is zero. Therefore,

$$\oint_C B.dl = \mu_0 I$$

where I is the current enclosed by the Amperian loop. Due to the symmetry, the magnitude of the magnetic field is uniform over the Amperian loop, we can take B out of the integration.

$$B\oint_C dl = \mu_0 I$$

For a circular loop, the circumference is 2πr, which implies,

$$B \int_{n}^{2\pi r} dl = \mu_0 I$$

$$\vec{B}.\, 2\pi r = \mu_0 I \quad \Rightarrow \quad \vec{B} = \frac{\mu_0 I}{2\pi r}$$

In vector form, the magnetic field is

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{n}$$

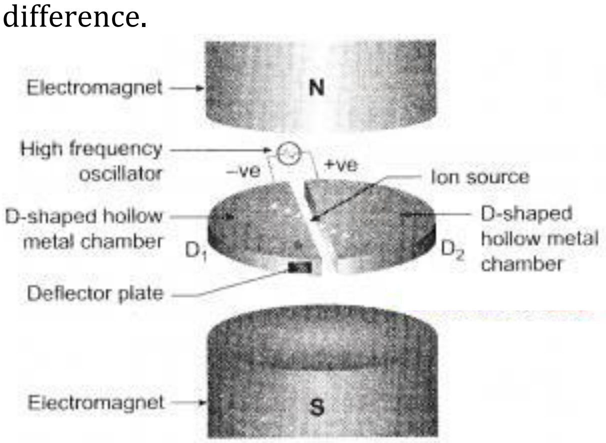
Where $\hat{n}$ is the unit vector along the tangent to the Amperian loop. This perfectly agrees with the result obtained from Biot-Savarf s law as given in equation

$$\vec{B} = \frac{\mu_0 I}{2\pi a}\hat{n}$$

Question 8.
Discuss the working of the cyclotron in detail.
Answer:
Cyclotron:
A cyclotron is a device used to accelerate the charged particles to gain large kinetic energy. It is also called as high energy accelerator. It was invented by Lawrence and Livingston in 1934.

Principle:
When a charged particle moves normal to the magnetic field, it experiences magnetic Lorentz force.

Construction:
The particles are allowed to move in between two semicircular metal containers called Dees (hollow D – shaped objects). Dees are enclosed in an evacuated chamber and it is kept in a region with uniform magnetic field controlled by an electromagnet. The direction of the magnetic field is normal to the plane of the Dees. The two Dees are kept separated with a gap and the source S (which ejects the particle to be accelerated) is placed at the center in the gap between the Dees. Dees are connected to high frequency alternating potential difference.



Working:
Let us assume that the ion ejected from source S is positively charged. As soon as ion is ejected, it is accelerated towards a Dee (say, Dee – 1) which has negative potential at that time. Since the magnetic field is normal to the plane of the Dees, the ion undergoes circular

path. After one semi-circular path in Dee-1, the ion reaches the gap between Dees. At this time, the polarities of the Dees are reversed so that the ion is now accelerated towards Dee-2 with a greater velocity. For this circular motion, the centripetal force of the charged particle q is provided by Lorentz force.

$$\frac{mv^2}{r} \text{ qvB} \Rightarrow r = \frac{m}{qb} v \Rightarrow r \propto v$$

From the equation, the increase in velocity increases the radius of circular path. This process continues and hence the particle undergoes spiral path of increasing radius. Once it reaches near the edge, it is taken out with the help of deflector plate and allowed to hit the target T. Very important condition in cyclotron operation is the resonance condition. It happens when the frequency ƒ at which the positive ion circulates in the magnetic field must be equal to the constant frequency of the electrical oscillator f_{osc} From equation

$$f_{osc} = \frac{qB}{2\pi m} \quad T = \frac{1}{f_{osc}}$$

The time period of oscillation is

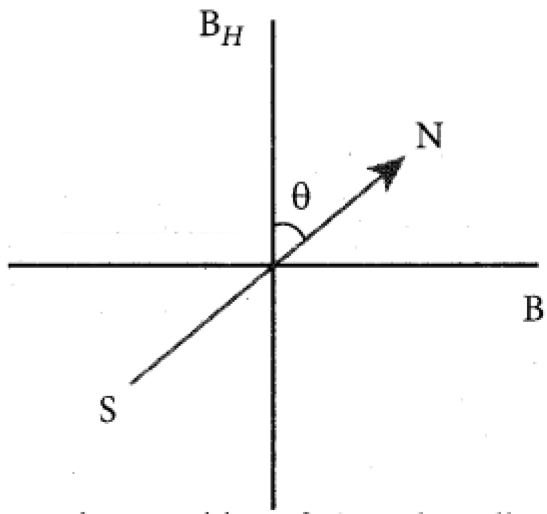
$$T = \frac{2\pi m}{qB}$$

The kinetic energy of the charged paricle is

$$K\,E = \frac{1}{2} mv^2 = \frac{q^2}{B^2} r^2 2m$$

Limitations of cyclotron:
(a) the speed of the ion is limited
(b) electron cannot be accelerated
(c) uncharged particles cannot be accelerated

Question 9.
What is tangent law? Discuss in detail.
Answer:
Tangent law:
Statement:
When a magnetic needle or magnet is freely suspended in two mutually perpendicular uniform magnetic fields, it will come to rest in the direction of the resultant of the two fields.
Explanation:
Let B be the magnetic field produced by passing current through the coil of the tangent Galvanometer and B_H be the horizontal component of earth's magnetic field. Under the action of two magnetic fields, the needle comes to rest making angle θ with B_H, such that



resultant position of pivoted needle

$\tan\theta = \frac{B}{B_H}$

$B = B_H \tan\theta$ .... (1)

When no current is passed through the coil, the small magnetic needle lies along the horizontal component of Earth's magnetic field. When the circuit is switched ON, the electric current will pass through the circular coil and produce magnetic field. Now there are two fields which are acting mutually perpendicular to each other. They are:

- the magnetic field (B) due to the electric current in the coil acting normal to the plane of the coil.
- the horizontal component of Earth's magnetic field (B_H)

Because of these crossed fields, the pivoted magnetic needle deflects through an angle θ. From tangent law, $B = B_H \tan\theta$ When an electric current is passed through a circular coil of radius R having N turns, the magnitude of the magnetic field at the center is

$B = \mu_0 \frac{NI}{2R}$ .... (2)

From equation (1) and equation (2), we get

$\mu_0 \frac{NI}{2R} = B_H \tan\theta$

The horizontal component of Earth's magnetic field can be determined as

$B = \mu_0 \frac{NI}{2R\tan\theta}$ in tesla .... (3)

Question 10.

Explain the principle and working of a moving coil galvanometer.

Answer:

Moving coil galvanometer:

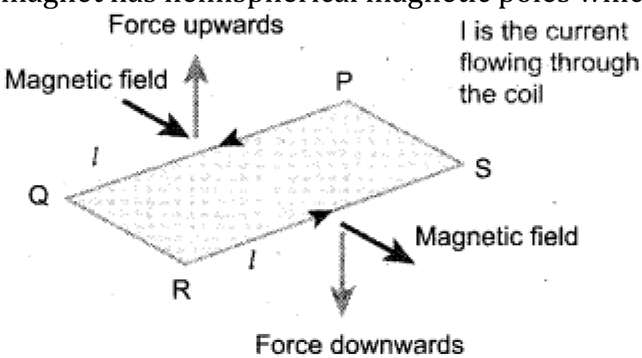
Moving coil galvanometer is a device which is used to indicate the flow of current in an electrical circuit.

Principle:
When a current-carrying loop is placed in a uniform magnetic field it experiences a torque.

Construction:
A moving coil galvanometer consists of a rectangular coil PQRS of insulated thin copper wire. The coil contains a large number of turns wound over a light metallic frame. A cylindrical soft-iron core is placed symmetrically inside the coil. The rectangular coil is suspended freely between two pole pieces of a horse-shoe magnet.

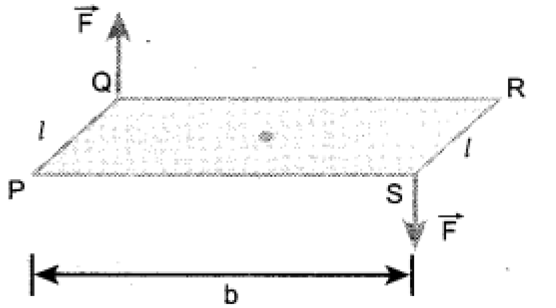
The upper end of the rectangular coil is attached to one end of fine strip of phosphor bronze and the lower end of the coil is connected to a hair spring which is also made up of phosphor bronze. deflection of the coil with the help of lamp and scale arrangement. The other end of the mirror is connected to a torsion head T. In order to pass electric current through the galvanometer, the suspension strip W and the spring S are connected to terminals.

Working:
Consider a single turn of the rectangular coil PQRS whose length be l and breadth b. PQ = RS = l and QR = SP = b.
Let I be the electric current flowing through the rectangular coil PQRS. The horse-shoe magnet has hemispherical magnetic poles which produce a radial magnetic field.



Force acting on current carrying coil

Due to this radial field, the sides QR and SP are always parallel to to the B-field (magnetic field) and experience no force. The sides PQ and RS are always parallel to the B-field and experience force and due to this, torque is produced.
For a single turn, the deflection couple.
$\tau = bF = bBIl = (lb)\,BI = ABI$ since, area of the coil
$A = lb$ For a coil with N turns, we get $r = NABI$ …… (1)
Due to this deflecting torque, the coil gets twisted and restoring torque (also known as restoring couple) is developed. Hence the magnitude of restoring the couple is proportional to the amount of twist θ. Thus $\tau = K\theta$ ……. (2)
where K is the restoring couple per unit twist or torsional constant of the spring. At equilibrium, the deflection couple is equal to the restoring couple. Therefore by comparing equation (1) and (2), we get



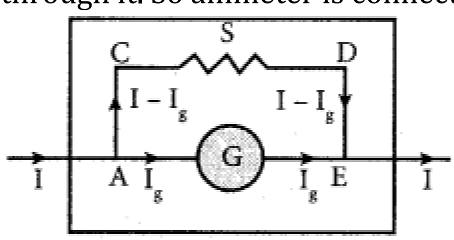
Deflection couple

$$NABI = K\theta \Rightarrow I = \frac{K}{NAB}\theta \text{ .... (3)}$$

$$(or)\ I = G\,\theta$$

Where, $G = \frac{K}{NAB}$ is called is called galvanometer constant or current reduction factor of the galvanometer. Since, suspended moving coil galvanometer is very sensitive, we have to handle with high care while doing experiments. For most of the galvanometer, we use arc pointer type moving coil galvanometer.

Question 11.
Discuss the conversion of a galvanometer into an ammeter and also a voltmeter.
Answer:
Conversion of galvanometer into ammeter and voltmeter:
A galvanometer is a very sensitive instrument to detect the current. It can be easily converted into an ammeter and voltmeter.
(i) Galvanometer to an Ammeter:
An ammeter is an instrument used to measure the current flowing in the electrical circuit. The ammeter must offer low resistance such that it will not change the current passing through it. So ammeter is connected in series to measure the circuit current.



Ammeter

A galvanometer is converted into an ammeter by connecting a low resistance in parallel with the galvanometer. This low resistance is called shunt resistance S. The scale is now calibrated in ampere and the range of ammeter depends on the values of the shunt resistance.

Let I be the current passing through the circuit. When current I reach junction A, it divides into two components. Let I_g be the current passing through the galvanometer of resistance

R_g through a path AGE and the remaining current $(I - I_g)$ passes along the path ACDE through shunt resistance S.
The value of shunt resistance is so adjusted that current I produce full-scale deflection in the galvanometer. The potential difference across galvanometer is the same as the potential difference across shunt resistance.

$$V_{galvanometer} = V_{shunt} \Rightarrow I_g R_g = (I - I_g)S$$

$$S = \frac{I_s}{(I - I_g)} R_g \quad \text{(or)} \quad I_g = \frac{S}{S + R_g} I \Rightarrow I_g \propto I$$

Since, the deflection in the galvanometer is proportional to the current passing through it.

$$\theta = \frac{1}{g} I_g \Rightarrow \theta \, I_g \Rightarrow \theta \propto I$$

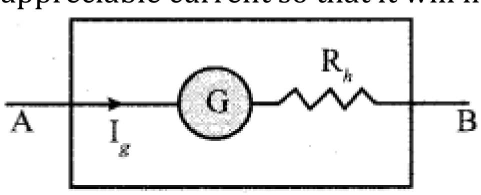
So, the deflection in the galvanometer measures the current I passing through the circuit (ammeter). Shunt resistance is connected in parallel to a galvanometer. Therefore, the resistance of the ammeter can be determined by computing the effective resistance, which is

$$\frac{1}{R_{eff}} = \frac{1}{R_g} + \frac{1}{S} \Rightarrow R_{eff} = \frac{R_g S}{R_g + S} = R_a$$

Since, the shunt resistance is a very low resistance and the ratio $\frac{S}{Rg}$ is also small. This means, R_g is also small, i.e., the resistance offered by the ammeter is small. So, when we connect ammeter in series, the ammeter will not change the resistance appreciably and also the current in the circuit. For an ideal ammeter, the resistance must be equal to zero. Hence, the reading in the ammeter is always lesser than the actual current in the circuit. Let I_{ideal} be current measured from ideal ammeter and I_{actual} be the actual current measured in the circuit by the ammeter. Then, the percentage error in measuring a current through an ammeter is

$$\frac{\Delta I}{I} \times 100\% = \frac{I_{ideal} - I_{actual}}{I_{actual}} \times 100\%$$

(ii) Galvanometer to a voltmeter: A voltmeter is an instrument used to measure potential difference across any two points in the electrical circuits. It should not draw any current from the circuit otherwise the value of potential difference to be measured will change. Voltmeter must have high resistance and when it is connected in parallel, it will not draw appreciable current so that it will indicate the true potential difference.



Voltmeter

A galvanometer is converted into a voltmeter by connecting high resistance R_h in series with a galvanometer. The scale is now calibrated in volt and the range of voltmeter

depends on the values of the resistance connected in series i.e. the value of resistance is so adjusted that only current I_g produces full scale deflection in the galvanometer.
Let Rg be the resistance of galvanometer and 'g be the current with which the galvanometer produces full scale deflection. Since the galvanometer is connected in series with high resistance, the current in the electrical circuit is same as the current passing through the galvanometer.
Potential difference
$I = I_g$

$$I = I_g \Rightarrow I_g = \frac{\text{Potential difference}}{\text{total resistance}}$$

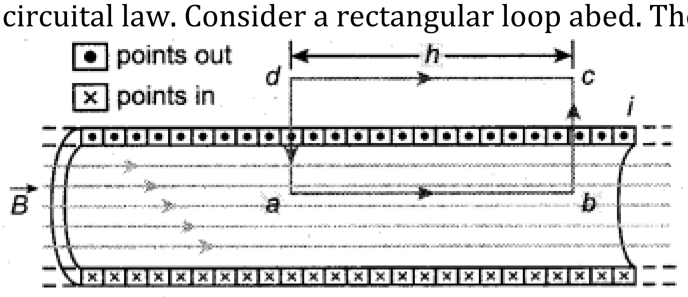
Since the galvanometer and high resistance are connected in series, the total resistance or effective resistance gives the resistance of the voltmeter. The voltmeter resistance is
$R_v \; R_g + R_h$

$$\text{Therefore, } I_g = \frac{V}{R_g + R_v} \Rightarrow R_h = \frac{V}{I_g} - R_g$$

Note that $I_g \propto V$
The deflection in the galvanometer is proportional to the current I. But current is proportional to the potential difference. Hence the deflection in the galvanometer is proportional to a potential difference. Since the resistance of voltmeter is very large, a voltmeter connected in an electrical circuit will draw least current in the circuit. An ideal voltmeter is one which has infinite resistance.

Question 12.
Calculate the magnetic field inside and outside of the long solenoid using Ampere's circuital law.
Answer:
Magnetic field due to a long current-carrying solenoid:
Consider a solenoid of length L having N turns. The diameter of the solenoid is assumed to be much smaller when compared to its length and the coil is wound very closely.

In order to calculate the magnetic field at any point inside the solenoid, we use Ampere's circuital law. Consider a rectangular loop abed. Then from Ampere's circuital law,



Magnetic field of a solenoid
Amperian loop for solenoid

$$\oint_C \vec{B}.d\vec{l} = \mu_0 I_{enclosed}$$

$= \mu_0$ x (total current enclosed by Amperian loop)
The left-hand side of the equation is

$$\oint_C \vec{B}.d\vec{l} = \int_a^b \vec{B}.d\vec{l} + \int_b^c \vec{B}.d\vec{l} + \int_c^d \vec{B}.d\vec{l} + \int_d^a \vec{B}.d\vec{l}$$

Since the elemental lengths along bc and da are perpendicular to the magnetic field which is along the axis of the solenoid, the integrals

$$\int_b^c \vec{B}.d\vec{l} = \int_b^c |\vec{B}||d\vec{l}|\cos 90^\circ = 0$$

$$\int_b^a \vec{B}.d\vec{l} = 0$$

Since the magnetic field outside the solenoid is zero, the integral

$$\int_c^d \vec{B}.d\vec{l} = 0$$

For the path along ab, the integral is

$$\int_a^b \vec{B}.d\vec{l} = B\int_a^b dl\cos 0^\circ = B\int_a^b dl$$

where the length of the loop ab is h. But the choice of length of the loop ab is arbitrary. We can take a very large loop such that it is equal to the length of the solenoid L. Therefore the integral is

$$\int_a^b \vec{B}.d\vec{l} = BL$$

let N I be the current passing through the solenoid of N turns, then

$$\int_a^b \vec{B}.d\vec{l} = BL = \mu_\circ NI \Rightarrow B = \mu_\circ \frac{NI}{L}$$

The number of turns per unit length is given by $\frac{NI}{L}$ = n, then

$B = \mu_0$ nLIL $= \mu_0 nI$

Since n is a constant for a given solenoid and p0 is also constant. For a fixed current I, the magnetic field inside the solenoid is also a constant.

Current Numerical problems

Question 1.

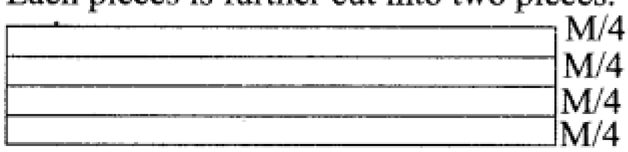
A bar magnet having a magnetic moment $\vec{M}$ is cut into four pieces i.e., first cut in two pieces along the axis of the magnet and each piece is further cut into two pieces. Compute the magnetic moment of each piece.

Solution:

Consider a bar magnet of magnetic moment $\vec{M}$. When a bar magnet first cut in two pieces

along the axis, their magnetic moment is $\frac{\vec{M}}{2}$

Each pieces is further cut into two pieces.

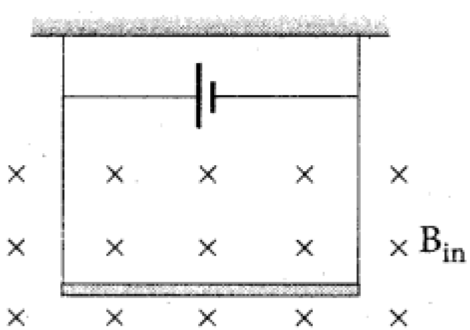


Their magnetic moment of each pieces $\frac{\vec{M}}{4}$

Their magnetic moment of each pieces $\vec{M}_{new}$ $\frac{1}{4}\vec{M}$

Question 2.

A conductor of linear mass density 0.2 g m^{-1} suspended by two flexible wires as shown in the figure. Suppose the tension in the supporting wires ¡s zero when it is kept inside the magnetic field of I T whose direction is into the page. Compute the current inside the conductor and also the direction of the current. Assume g = 10 m s^{-2}.



Solution:

Downward force, F = mg

Linear mass density, ml = 0.2 gm^{-1}

ml = 0.2 x 10^{-3} kg m^{-1}

m = (0.2 x 10^{-3} x l) kg m^{-1}

F = (0.2 x 10^{-3} x l x 10) N
The upward magnetic force acting on the wire
F = BIl
0.2 x 10^{-3} x l x 10 = 1 x I x l
I = 2 x 10^{-3}
I = 2 m A

Question 3.
A circular coil with cross-sectional area 0.1 cm^2 is kept in a uniform magnetic field of strength 0.2 T. If the current passing in the coil is 3 A and plane of the loop is perpendicular to the direction of magnetic field. Calculate
(a) total torque on the coil
(b) total force on the coil
(c) average force on each electron in the coil due to the magnetic field of the free electron density for the material of the wire is 10^{28} m^{-3}
Solution:
Cross-sectional area of coil, A = 0.1 cm^2
A = 0.1 x $10^{-4}m^2$
Uniform magnetic field of strength, B = 0.2T
Current passing in the coil, I = 3A
The angle between the magnetic field and normal to the coil, θ = 0°
(a) Total torque on the coil,
τ = ABI sin θ = 0.1 x 10^{-4} x 0.2 x 3 sin0° sin0° = 0
τ = 0
(b) Total force on the coil
F = BIl sin θ = 0.2 x 3 x l x sin 0°
F = 0

(c) Average force:

$F = qV_dB$

drift velocity, $V_d = \frac{1}{neA}$

[∵ q = e]

$F = e\left(\frac{1}{neA}\right)B$

[∵n = 10^{28} m^{-3}

$\frac{IB}{nA} = \frac{3\times0.2}{10^{28}\times0.1\times10^{-4}} = 6 \times 10^{-24}$

$F_{av} = 0.6 \times 10^{-23}$ N

Question 4.
A bar magnet is placed in a uniform magnetic field whose strength is 0.8 T. Suppose the bar magnet orient at an angle 30° with the external field experiences a torque of 0.2 N m. Calculate:
(i) the magnetic moment of the magnet
(ii) the work done by an applied force in moving it from most stable configuration to the most unstable configuration and also compute the work done by the applied magnetic field in this case.
Solution:
Uniform magnetic field strength B = 0.8T
Bar magnet orient an angle with magnetic field θ = 30°
Torque τ = 0.2 Nm

(i) Magnetic moment of the magnet,

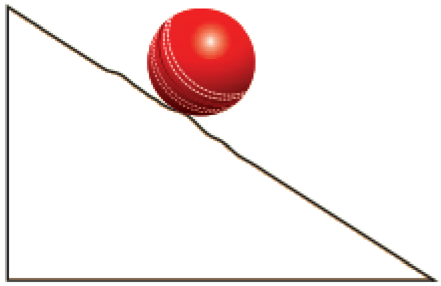
Torque $\tau = P_m B \sin\theta$

$\therefore$ Magnetic moment, $P_m = \frac{\tau}{B\sin\theta} = \frac{0.2}{0.8 \times \sin 30^\circ} = \frac{0.2}{0.4}$

$P_m = 0.5\ Am^2$

(ii) Work done by external torque is stored in the magnet as potential energy.
$W = U = -P_m B \sin\theta$
Here, the applied force acting on the magnet is moving from most stable θ' to most unstable θ.
θ'= 0° and θ = 180°
So, workdone $W = U = -P_m B (\cos\theta - \cos\theta')$
$= -P_m B (\cos 180^\circ - \cos 0^\circ) = -0.5 \times 0.8 ((-1) - 1) = -0.4 (-2)$
W = U = 0.8
W= 0.8 J

Question 5.
A non – conducting sphere has a mass of 100 g and a radius 20 cm. A flat compact coil of wire with turns 5 is wrapped tightly around it with each turn concentric with the sphere. This sphere is placed on an inclined plane such that plane of coil is parallel to the inclined plane. A uniform magnetic field of 0.5 T exists in the region in the vertically upward direction. Compute the current 1 required to rest the sphere in equilibrium.



Solution:

At equilibrium

f_s R-p_m B sin θ = 0

mgR = NBAI

$I = \frac{mgR}{NBA} = \frac{mgR}{NB\pi R^2}$

$I = \frac{mg}{\pi RNB}$

Mass of the sphere, m= 100 g = 100 x 10^{-3} kg

Radius of the sphere R = 20 cm = 20 x 10^{-2} m

Number of turns n = 5

Uniform magnetic field B = 0.5 T

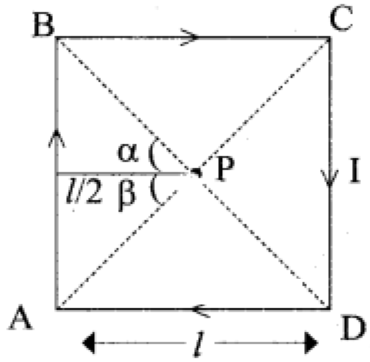
$I = \frac{100\times10^{-3}\times10}{\pi\times20\times10^{-2}\times5\times0.5} = \frac{1000\times10^{-3}}{\pi\times50\times10^{-2}} = \frac{20\times10^{-1}}{\pi}$

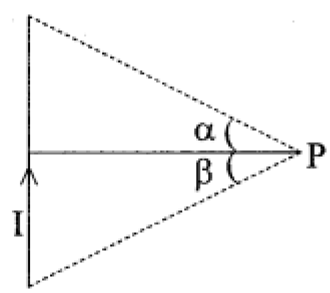
$I = \frac{2}{\pi}$ A.

Question 6.

Calculate the magnetic field at the center of a square loop which carries a current of 1.5 A, length of each loop is 50 cm.

Solution:





Current through the square loop, I = 1.5A

Length of each loop, l = 50cm 50 x 10^{-2} m
According to Biot-Savart Law.
Magnetic field due to a current-carrying straight wire

$$B = \frac{\mu_0 I}{4\pi a}(\text{Sin } \alpha + \text{Sin } \beta) = \frac{4\pi \times 10^{-7} \times 1.5}{4\pi \times \left(\frac{l}{2}\right)} (\text{Sin } 45^\circ + \text{Sin } 45^\circ)$$

$$= \frac{2 \times 1.5 \times 10^{-7}}{l}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{2 \times 1.5 \times 10^{-7}}{50 \times 10^{-2}}\left(\frac{2}{\sqrt{2}}\right)$$

B =0.084866 x 10^{-5} T
Magnetic field at a point p? of centre of current carrying square ioop
B' =4 sides x B
= 4 x 0.08487 x 10^{-5} = 0.33948 x 10^{-5}
B' =3.4 x 10^{-6} T

Question 7.
Show that the magnetic field at any point on the axis of the solenoid having n turns per unit length is $B = \frac{1}{2}\mu_0 nI(\cos\theta_1 - \cos\theta_2)$
Solution:
A solenoid is a cylindrical coil having a number of circular turns. Consider a solenoid having radius R consists of n number of turns per unit length.
Let 'P' be the point at a distance 'x' from the origin of the solenoid. The current carrying element dx at a distance x from origin and the distance r from point 'P'.

$r = \sqrt{R^2 + (x_0 - x)^2}$

The magnetic field due to current carrying circular coil at any axis is

$dB = \frac{\mu_0 IR^2}{2r^3} \times N$

Where N = ndx, then

$dB = \frac{\mu_0}{2}\frac{nIR^2dx}{r^3}$ .... (1)

$\sin\theta = \frac{R}{r}$

$r = R \times \frac{1}{\sin\theta} = R \text{ cosec } \theta$ .... (2)

$\tan\theta = \frac{R}{x_0 - x}$

$x_0 - x = R \times \frac{1}{\tan\theta} = R \cot\theta$

$\frac{dx}{d\theta} = R \text{ cosec}^2 \theta$

$\Rightarrow dx = R \text{ cosec}^2 \theta \, d\theta$ .... (3)

From above three equations, wer get

$dB = \frac{\mu_0}{2}\frac{nIR^2(R\csc^2\theta d\theta)}{R^3\csc^3\theta}$

$dB = \frac{\mu_0}{2} n I \sin\theta \, d\theta$

Now total magnetic field can be obtained by integrating from θ_1 to θ_2, we get

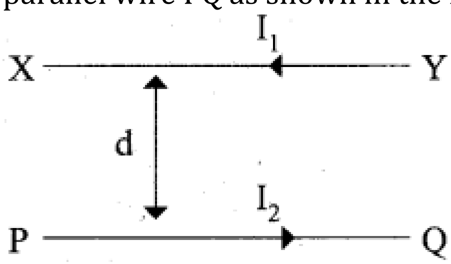
$$B = \frac{\mu_0 n I}{2} \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = \frac{\mu_0 n I}{2} [-\cos\theta]_{\theta_1}^{\theta_2}$$

$$B = -\frac{\mu_0 n I}{2} [\cos\theta_2 - \cos\theta_1]$$

$$B = \frac{\mu_0 n I}{2} [\cos\theta_1 - \cos\theta_2]$$

Question 8.

Let I_1 and I_2 be the steady currents passing through a long horizontal wire XY and PQ respectively. The wire PQ is fixed in the horizontal plane and the wire XY be is allowed to move freely in a vertical plane. Let the wire XY is in equilibrium at a height d over the parallel wire PQ as shown in the figure.



Solution:

If the wire XY is slightly displaced and released, it executes simple harmonic motion due to the force of repulsion produced between the current-carrying wire.

Acceleration of the wire, $a = -\omega^2 y$

Time period of oscillation of the wire,

$$T = 2\pi \sqrt{\frac{d}{g}}$$