12. SOUND WAVES

1. INTRODUCTION

This chapter discusses the nature of sound waves. We will apply concepts learned in the chapter on waves on a string are applied to understand the phenomena related to sound waves. We will learn about what all parameters the speed of sound in a medium depends. Reflection, transmission and interference are important phenomena associated with sound. The study of sound waves enables us to design musical instruments and auditoriums. We will understand the properties of sound waves in air columns and the phenomena of echo. Phenomena of beats and doppler effect have been discussed.

2. NATURE AND PROPAGATION OF SOUND WAVES

Sound is a mechanical wave that results from the back and forth vibration of the particles of a medium through which the sound wave is traveling. Further, if a sound wave is traveling from left to right in air, then particles in air will be displaced in both rightward and leftward directions due to the energy of the sound wave passing through it. However, the motion of the particles is parallel (and antiparallel) to the direction of the energy transport. This unique property characterizes sound waves in air as longitudinal waves.

A typical case of propagation of sound waves in air is shown in the Fig. 12.1.

We know that as the prong vibrates in simple harmonic motion, the pressure variations in the layer close to the prong also change in a simple harmonic fashion. Thus, the increase in pressure above its normal value may, therefore, be written as $\delta P = P - P_0 = \delta P_0 \sin \omega t$,

where δP_0 is the maximum increase in pressure above its normal value. As this disturbance, due to the traveling of the sound wave, moves towards right with the speed u (the above speed and not the particle speed), the equation for the excess pressure at any point x at any time t is given by $\delta P = \delta P_0 \sin \omega \left(t - x / v\right)$.

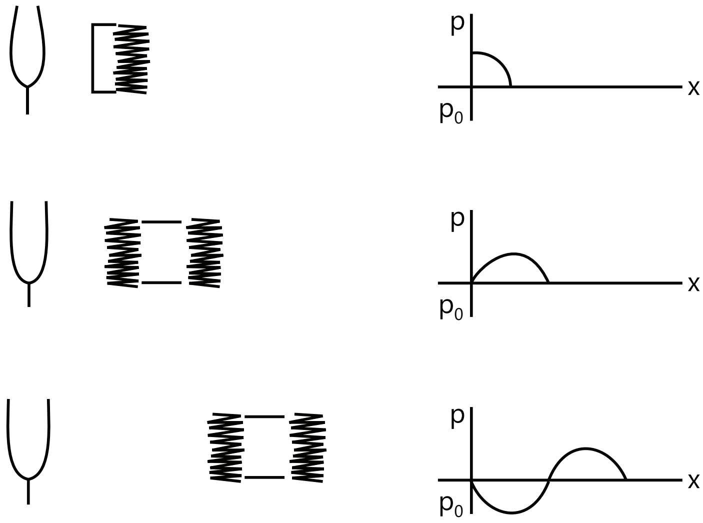
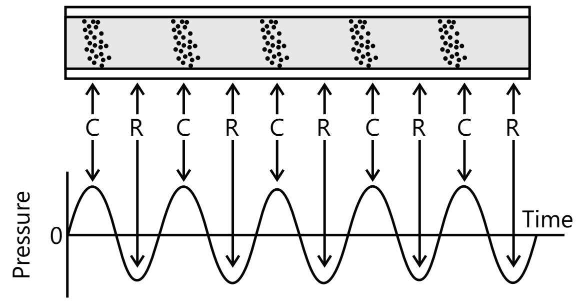


Figure 12.1

2.1 Compression and Rarefaction

Due to the phenomenon of longitudinal motion of the air particles, we observe that there are regions in the air where the air particles are compressed together and other regions where the is air spread apart. These regions are respectively known as compression and rarefaction. The formation of these regions is due to back and forth motion of the particles of the medium.

Compression is the result of increase in density and pressure in a medium, such as air, due to the passage of a sound wave. However, rarefaction is quite just the opposite of compression, i.e., decrease in density and pressure in a medium due to the passage of a sound wave.



Note: "C" stands for compression and "R" stands for rarefaction

Figure 12.2

2.2 Wavelength

The wavelength of a wave is just the distance that a disturbance is carried along the medium when one wave cycle is completed. A longitudinal wave typically consists of a repeating pattern of compressions and rarefactions.

Hence, the wavelength is commonly measured either as the distance from one compression point to the next adjacent compression or the distance from one rarefaction point to the next adjacent rarefaction.

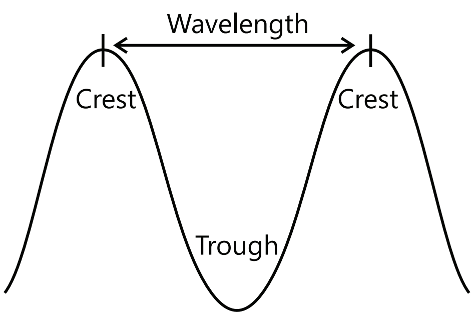


Figure 12.3

2.3 Polarization of Sound Waves

It is to be noted that all directions perpendicular to the propagation of sound waves are equivalent and therefore sound waves cannot be polarized.

2.4 Wave Front

A wave front usually is the locus of point that is having the same phase, i.e., a line or curve in 2d, or a surface for a wave propagating in 3d. Further, the sound observed at some point by a vibrating source travels virtually in all directions of the medium only if the medium is extended. However, for a homogenous and isotropic medium, the wave fronts are usually normal to the direction of propagation.

2.5 Infrasonic and Ultrasonic Sound Waves

Sound waves are audible only if the frequency of alternation of pressure is in the range of 20 Hz and 20,000Hz. In other words, beyond this upper limit they are not audible. The waves are classified based on their frequency range, i.e., below the audible range they are called infrasonic waves, whereas those with frequency greater than the audible range are termed ultrasonic waves.

Illustration 1: A wave of wavelength of 0.60 cm is produced in air and it travels at a speed of 300 ms^{-1}. Will it be audible? **(JEE MAIN)**

Sol: The frequency of the sound wave is given as $\nu = \frac{V}{\lambda}$. The audible range is 20 Hz to 20 KHz.

From the relation $V = \nu\lambda$, we calculate the frequency of the wave as $\nu = \frac{V}{\lambda}$

$$= \frac{300\ \text{ms}^{-1}}{0.60 \times 10^{-2}\ \text{m}} = 50000\ \text{Hz}$$

This is clearly very much above the audible range. Therefore, it is an ultrasonic wave and hence will not be audible.

2.6 Displacement Wave and Pressure Wave

A longitudinal wave can be described either in terms of longitudinal displacement of the particle of the medium or in terms of excess pressure generated due to phenomena of compression and rarefaction.

3. EQUATION OF SOUND WAVE

As we have already noted above, a longitudinal wave in a fluid (liquid or gas) can be described either in terms of the longitudinal displacement suffered by the particles of the medium or in terms of the excess pressure generated due to the compression or rarefaction. Let us now verify how these two representations are related to each other.

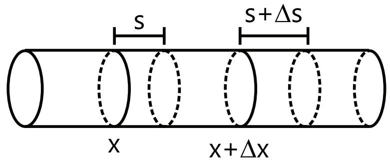


Figure 12.4

Consider a wave traveling in the x-direction in a fluid. Further, now suppose that at time t, the particle at the undisturbed position x suffers a displacement s in the x-direction. The wave can then be described by the equation

$$s = s_0 \sin\omega\left(t - x/v\right) \qquad \text{... (i)}$$

Now, consider the element of the material which is within x and $x + \Delta x$ (see the fig. 12.4) in the undisturbed state. Therefore, by considering a cross-sectional area A, the volume of the element in the undisturbed state is $A\Delta x$ and its mass is $\rho A\Delta x$. As the wave passes through, the ends at x and $x + \Delta x$ are displaced by amount s and $s + \Delta s$ according to Eq. (i). Thus, the increase in volume of the element at time t is given as

$$\Delta V = A\,\Delta s = A\frac{\delta s}{\delta x}\Delta x = As_0\left(-\omega/v\right)\cos\omega\left(t - x/v\right)\Delta x$$

(where Δs has been obtained by differentiating Eq. (i) with respect to x. The element is, therefore, under a volume strain. $\frac{\Delta v}{v} = \frac{-As_0\omega\cos\left(t - x/v\right)}{vA\Delta x} = \frac{-s_0\omega}{v}\cos\omega\left(t - x/v\right)$)

However, the corresponding stress, i.e., the excess pressure developed in the element at x at time t is $p = B\left(\frac{-\Delta v}{v}\right)$, where B is the bulk modulus of the material.

Thus, $p = B\frac{s_0\omega}{v}\cos\omega\left(t - x/v\right)$ ... (ii)

Comparing with standard wave equation, we see that the amplitude p_0 and the displacement amplitude s_0 are related as $p_0 = \frac{B\omega}{v}s_0 = Bks_0$ (where k is the wave number)

Also, we observe from (i) and (ii) that the pressure wave differs in phase by $\pi/2$ from the displacement wave. Further, the pressure maxima observed is at the point where the displacement is zero and displacement maxima occur where the pressure is at its normal level.

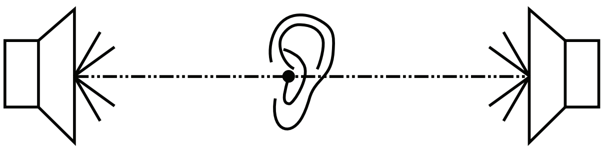


Figure 12.5

The assertion here being that displacement is zero where the pressure change is maximum and vice versa, and therefore sets the two descriptions on different footings. Naturally, the human ear or an electronic detector responds only to the change in pressure and not to the displacement. Let us suppose that two audio speakers are driven by the same amplifier and are placed facing each other. Further, a detector is placed midway between them. Now, it is clear that the displacement of the air particles near the detector will be zero as the two sources drive these particles in opposite directions. However, both the sources send compression waves and rarefaction waves together.

PLANCESS CONCEPTS

The human ear or an electronic detector responds to the pressure change and not the displacement in a straightforward way.

Vaibhav Krishnan (JEE 2009, AIR 22)

Illustration 2: Suppose that a sound wave of wavelength 40 cm travels in air. If the difference between the maximum and minimum pressures at a given a point is $1.0\times10^{-3}\ \text{Nm}^{-2}$, then find the amplitude of vibration of the particles of the medium. The bulk modulus of air is $1.4\times10^{5}\ \text{Nm}^{-2}$. **(JEE MAIN)**

Sol: The amplitude of pressure at a point is given by $P_o = \frac{P_{max} - P_{min}}{2}$. As the bulk modulus of the air is given, the amplitude of the vibration is given by $S_0 = \frac{P_0}{Bk}$ where k is wave number.

The pressure amplitude is $P_0 = \frac{1.0\times10^{-3}\ \text{Nm}^{-2}}{2} = 0.5\times1.0\times10^{-3}\ \text{Nm}^{-2}$

The displacement amplitude s_0 is given by $P_0 = Bks_0$

or $s_0 = \frac{P_0}{Bk} = \frac{P_0\lambda}{2\pi B} = \frac{0.5\times1.0\times10^{-3}\ \text{Nm}^{-2}\times\left(40\times10^{-2}\text{m}\right)}{2\times3.14\times1.4\times10^{5}\text{Nm}^{-2}} = 2.2\times10^{-10}\text{m}.$

Illustration 3: Assume that a wave is propagating on a long stretched string along its length taken as the positive x-axis. The wave equation is given as $y = y_0 \exp\left(\frac{t}{T} - \frac{x}{\lambda}\right)^2$ where y_0 = 4 mm, T =1 s, and λ = 4 cm. Now, (a) Find the velocity of the wave.

(b) Find the function $f(t)$ giving the displacement of the particle at x = 0.

(c) Find the function $g(x)$ giving the shape of the string at t = 0.

(d) Plot the shape $g(x)$ of the string at t = 0.

(e) Plot the shape of the string at t = 5 s. **(JEE MAIN)**

Sol: The wave moves having natural frequency of ν and wavelength λ has velocity $V = \nu\lambda$. As the frequency is $\nu = \frac{1}{T}$ the velocity of the wave is then $V = \frac{\lambda}{T}$.

(a) The wave equation may be written as $y = y_0 \exp\left[\frac{1}{T}\left(t - \frac{x}{\lambda / T}\right)^2\right]$

Comparing with the general equation, $y = f(t - x/v)$ we see that $\nu = \frac{\lambda}{T} = \frac{4\text{cm}}{1.0\text{s}}/\text{sec}$

(b) Substituting x = 0 in the given equation, we have $f(t) = y_0 e^{-(t/T)^2}$ ... (i)

(c) Substituting t = 0 in the given equation, we have $g(x) = y_0 e^{-(x/\lambda)^2}$ ... (ii)

Figure 12.6

4. VELOCITY OF SOUND WAVES

We know that the sound waves travel in air or in gaseous media as longitudinal waves. Further, when these waves travel longitudinally, then compression and rarefaction are produced in the layers of air in such a way that the particles in layers of the air move in a to and fro fashion about their mean position in the direction exactly as that of the direction of propagation of sound waves. Therefore, the speed v of longitudinal waves in an elastic medium of modulus of elastic E and density ρ is given by $v = \sqrt{\frac{E}{\rho}}$.

For both liquids and gases, E is the bulk modulus of elasticity. For a thin solid rod, E is Young's modulus. However, for large solids, E depends upon the bulk modulus and shear modulus.

Newton assumed that the changes produced due to propagation of sound in gases are isothermal; this implies that a compressed layer of air at higher temperature loses heat immediately to the surroundings, whereas a rarefied layer of at lower temperature gains heat from the surroundings so that temperature of air remains constant. As the modulus of elasticity for isothermal change is equal to the pressure P according to Newton's formula for change,

$v = \sqrt{\frac{P}{\rho}}$.

Laplace showed that the sound is propagated in air or gases under adiabatic change. This is because the compression and rarefactions produced due to the propagation of sound follow each other so rapidly that there is no time available for the compressed layer at a higher temperature and rarefied layer at a lower temperature to equalize their temperature with the surroundings. Thus, the velocity v of sound travelling under adiabatic conditions in a gas is given by Laplace's formula as:

$$v = \sqrt{\frac{E_{adiabatic}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} \text{ ; because } E_{adiabatic} = \gamma P \text{ and } \gamma = \frac{C_p}{C_v}$$

By substituting γ = 1.41 for air, density of air = 1.293 kg/m^3, atmospheric pressure = $1.013 \times 10^5 N/m^2$, the velocity of sound in air, v = 332 m/s. However, in general, the velocity of sound in solid is greater than the velocity of sound in liquids and the velocity of sound in liquids is greater than the velocity of sound in gases.

4.1 Sound Wave in Solids

Usually, sound waves travel in solids just like they travel in fluids. The speed of longitudinal sound waves in a solid rod can be shown to be $v = \sqrt{Y/\rho}$,

where Y is the Young's modulus of the solids and ρ its density.

However, for extended solids, the speed is a more complicated function of bulk modulus and shear modulus. The table provided hereunder gives the speed of sound in some common materials.

Medium	**Speed (m/s)**	**Medium**	**Speed (m/s)**
Air (dry 0^0C)	332	Copper	3810
Hydrogen	1330	Aluminum	5000
Water	1486	Steel	5200

4.2 Sound Wave in Fluids

A sound wave in air is a typical longitudinal wave. As a sound wave passes through air, its potential energy is usually associated with periodic compression and expansion of small volume element of the air. The unique property that determines the extent to which an element in the medium changes its volume as the pressure applied to it increases or decreases is the bulk modulus, B. $B = \frac{-\Delta P}{\Delta V / V}$

where $\frac{\Delta V}{V}$ is the fractional change in volume produced by a change in pressure ΔP.

Let us now suppose that air of density ρ is filled inside a tube of cross-sectional area A under a pressure P. Initially, the air is at rest.

At t = 0, the piston at the left end of the tube (as shown in the Fig. 12.7) is set to motion toward the right with a speed μ. After a time interval Δt, all portions of the air to the left of section 1 are moving with speed u, whereas all portions to the right of the section are at rest. Further, the boundary between the moving and the stationary portion travels to the right with v, the speed of the elastic wave (or sound wave). In the time interval Δt, the piston has moved $u\,\Delta t$ and the elastic disturbance has moved across a distance of $v\,\Delta t$.

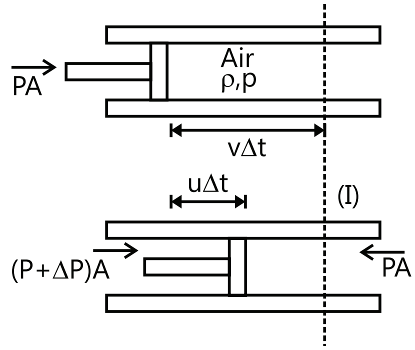


Figure 12.7

The mass of air that has attained a velocity u in time Δt is taken as $v = \sqrt{\frac{B}{\rho}}\;P(\Delta x)A$. Therefore, now the momentum imparted is $\left[Pv(\Delta t)A\right]u$ and the net impulse $= (\Delta PA).\Delta t$.

Thus, impulse = change in momentum $(\Delta PA).\Delta t = \left[Pv(\Delta t)A.\right]u$ or $\Delta P = Pvu$ ...(xv)

Since $B = \frac{\Delta P}{\Delta V / V}$ $\therefore$ $\Delta P = B\left(\frac{\Delta V}{V}\right)$ where $V = Av\Delta t$ and $\Delta V = Au\,\Delta t$

$\therefore \frac{\Delta V}{V} = \frac{Au\,\Delta t}{Av\Delta t} = \frac{u}{v}$ thus, $\Delta P = B\frac{u}{v}$ ...(xvi)

From (xv) and (xvi) $v = \sqrt{\frac{B}{P}}$.

4.3 Speed of Sound in a Gas: Newton's Formula and Laplace Correction

The speed of sound in a gas can be expressed in terms of its pressure and density. We now summarize these properties hereunder:

(a) For a given mass of an ideal gas, the pressure, volume and the temperature are related as $\frac{PV}{T}$ = constant.

However, if the temperature remains constant (called an isothermal process), then the pressure and the volume of a given mass of a gas satisfy PV = constant. Here, T is the absolute temperature of the gas. This is known as Boyle's law.

(b) If no heat is supplied to a given mass of a gas (called an adiabatic process), then its pressure and volume satisfy PV^{γ}= constant where γ is a constant for the given gas. It is, in fact, the ratio C_p / C_V of two specific heat capacities of the gas.

Newton assumed that when a sound wave is propagated through a gas, the temperature variation in the layer of compression and rarefaction is negligible. Hence, the condition here is isothermal and hence Boyle's law will be applicable.

$PV = \text{constant}$ or, $P\Delta V + V\Delta P = 0$ or, $B = -\frac{\Delta P}{\Delta V / V} = P$ ...... (i)

Using the above result, the speed of sound in the gas is given by $v = \sqrt{P/\rho}$.

Laplace, however, suggested that the compression or rarefaction takes place too rapidly and the gas element being compressed or rarefied is hardly left with enough time to exchange heat with the surroundings. It is hence an adiabatic process and therefore one should use the equation $PV^{\gamma} = \text{constant}$. Taking logarithms, in P+in V = constant.

Now, by taking differentials, $\frac{\Delta P}{P} + \gamma \frac{\Delta V}{V} = 0$ or $B = -\frac{\Delta P}{\Delta V / V} = \gamma P$

Thus, the speed of sound is $v = \sqrt{\frac{\gamma P}{\rho}}$.

5. EFFECT OF PRESSURE, TEMPERATURE AND HUMIDITY AND SPEED OF SOUND IN AIR

(a) Effect of temperature as PV = nRT and $\rho = \frac{m}{V}$ $\therefore$ $v = \sqrt{\frac{\gamma PV}{m}} = \sqrt{\frac{\gamma RT}{M_m}}$ where M_m is mass of one mole of gas.

Thus, the velocity of sound is directly proportional to the square root of the absolute temperature. If v_t and v_0 are velocities of sound at t^0C and 0 ^{0}C, respectively, then $\frac{V_t}{V_0} = \sqrt{\frac{T_t}{T_0}} = \sqrt{\frac{273+t}{273}}$

where T_t and T_0 are respective absolute temperatures...

(b) Effect of pressure. If the temperature of the gas remains constant, then the velocity of sound does not change with the change of pressure because $\frac{p}{\rho}$ is a constant quantity. PM = ρRT

(c) Effect of humidity. As the density of water vapor at STP, 0.8kg/m^3, is lower than the density of dry air, 1.29km/m^3, the speed of sound in air increases when the humidity increases in the moist air.

6. INTENSITY OF SOUND

Normally, the intensity of sound I at any point is the quantum of energy transmitted per second across a unit area normal to the direction of propagation of sound waves. The intensity follows the pattern of an inverse square law of distance, i.e., $I \propto 1/R^2$ and I is proportional to the square of amplitude. Further, the level of intensity of sound as perceived by humans is called loudness. Thus, the intensity level or loudness L is quantitatively measured as compared to a minimum intensity of sound audible to human ear. Hence, the intensity level or loudness, measured in unit of decibel, dB, is given as $L = 10 \log_{10}\left(\frac{I}{I_0}\right)$..

where I_0 is the minimum audible intensity which is equal to 10^{-12} watt/m^2. Thus, the intensity of sound increases by a factor of 10 when the intensity level or loudness increases by 10 decibels.

Now, let us consider again a sound wave travelling along the x-direction. Let the equation for the displacement of the particles and the excess pressure developed by the wave be given as

$$s = s_0 \sin\omega(t - x/v) \text{ and } P = P_0 \cos\omega(t - x/v) \quad \text{... (i)}$$

where $P_0 = \frac{B\omega s_0}{v}$

Now, consider a cross section of area 'A' perpendicular to x-direction. The power W, transmitted by the wave across the section considered is $W=(PA)\frac{\delta s}{\delta t}$; $W=AP_0\cos\omega(t-x/v)\omega s_0\cos\omega(t-x/v)$ $=\frac{A\omega^2 s_0^2 B}{n}\cos^2\omega(t-x/v)$

The intensity 'I' is thus $I=\frac{1}{2}\frac{\omega^2 s_0^2 B}{u}=\frac{2\pi^2 B}{u}s_0^2 V^2.$; $I=\frac{P_0^2 u}{2B}$

As $B=Pv^{-2}$, the intensity can also be written as $I=\frac{v}{2pv^2}P_0^2=\frac{P_0^2}{2pv}$.

Loudness: Our ear is sensitive for an extremely large range of intensity. Therefore, a logarithmic rather than a linear scale in this regard is convenient. Accordingly, the intensity level β of t = a sound wave is defined by the equation $\beta=10\log\left(\frac{I}{I_0}\right)$ Decibel, where $I_0=10^{-12}\,W/m^2$ is the reference or threshold intensity level to which any intensity I is compared.

PLANCESS CONCEPTS

Intensity is directly proportional to the square of the pressure amplitude.

Nivvedan (JEE 2009, AIR 113)

Illustration 4: Assume that the pressure amplitude in a sound wave from a radio receiver is $2.0\times10^{-2}Nm^{-2}$ and the intensity at a point is $5.0\times10^{-7}Wm^{-2}$. If by turning the "volume" knob the pressure amplitude is increased to $2.5\times10^{-2}Nm^{-2}$, then evaluate the intensity. **(JEE MAIN)**

Sol: The intensity of the wave is proportional to square of the pressure amplitude of wave. If we increase the pressure amplitude then the intensity of sound will accordingly.

The intensity is proportional to the square of the pressure amplitude.

Thus, $\frac{I'}{I}=\left(\frac{P'_0}{P_0}\right)^2$ or $I'=\left(\frac{P'_0}{P_0}\right)^2 I=\left(\frac{2.5}{2.0}\right)^2\times5.0\times10^{-7}Wm^{-2}=7.8\times10^{-7}Wm^{-2}$

7. PERCEPTION OF SOUND TO HUMAN EAR

There are three parameters which govern the perception of sound to human ear. They are listed hereunder.

(a) Pitch and frequency,

(b) Loudness, and

(c) Quality and waveform.

7.1 Pitch and Frequency

The frequency of a wave generally signifies how often the particles of the medium vibrate when a wave travels through the medium. It is measured as the number of complete back-and-forth vibrations of a particle of the medium per unit of time. Further, the sensation of frequency is commonly referred to as the pitch of a sound. Therefore, a high pitch sound generally corresponds to a frequency sound wave and a low pitch sound corresponds to a low frequency sound wave. Our ability to perceive pitch is associated with the frequency of the sound wave that impinges upon our ear. This is because sound waves travelling through air are longitudinal waves that produce high- and low-pressure disturbances of the particles of the air at a given frequency. Therefore, our ear has an ability to detect such frequencies and associate them with the pitch of the sound.

7.2 Loudness

Loudness is that characteristic of a sound that is primarily a psychological correlate of physical strength (amplitude). However, more formally it is defined as "that attribute of auditory sensation in terms of which sounds can be ordered on a scale extending from quiet to loud." Further, loudness is also affected by parameters other than sound pressure, including frequency, bandwidth and duration.

7.3 Quality of Waveform

The quality of sound is typically an assessment of the accuracy, enjoyability, or intelligibility of audio output from an electronic device. Therefore, quality of sound can be measured objectively, such as when tools are used to gauge the accuracy with which the device reproduces an original sound; or it can be measured subjectively, such as when we respond to the sound or gauge its perceived similarity to another sound. Thus, we differentiate between the sound from a table and that from a mridang on the basis of their quality alone.

Illustration 5: Suppose that a source emitting sound of frequency 180 Hz is placed in front of a wall at a distance of 2 m from it. Further, a detector is also placed in front of the wall at the same distance from it. Find the minimum distance between the source and the detector for which the detector detects a maximum loudness. Speed of sound in air = 360 m/s. **(JEE ADVANCED)**

Sol: As there is a wall at a distance of 2 m from the source, the wave will reflect from the wall and interfere with the wave directly from the source. If constructive interference takes place between the reflected wave and original wave then the maximum loudness is heard. The condition of constrictive interference is $\Delta = n\lambda$.

The situation is visualized in the Fig. 12.8. Now, suppose that the detector is placed at a distance of x meter from the sources. Then, the wave received from the source after reflection from the wall has travelled a distance of $2\left[(2)^2 + x^2/4\right]^{1/2}$ m. Therefore, the difference between the two waves is $\Delta = \left\{2\left[(2)^2 + \frac{x^2}{4}\right]^{1/2} - x\right\}$ m.

However, constructive interference will take place when $\Delta = \lambda, 2\lambda$. Thus, the minimum distance x for a maximum loudness corresponds to $\Delta = \lambda$ ... (i)

The wavelength is $\lambda = \frac{u}{\nu} = \frac{360\text{m}/\text{s}}{180}\text{s}^{-1} = 2\text{m}$

Thus, by (i), $2\left[(2)^2 + x^2/4\right]^{1/2} - x = 2$ or, $\left[4 + \frac{x^2}{4}\right]^{1/2} = 1 + \frac{x}{2}$

Or, $4 + \frac{x^2}{4} = 1 + \frac{x^2}{4} + x$ or $3 = x$.

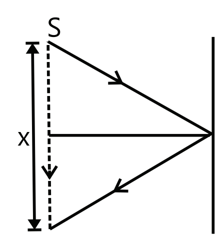


Figure 12.8

Thus, condition here is that the detector should be placed at a distance of 3 m from the source. Note, however, that there is no abrupt phase change.

8. INTERFERENCE OF SOUND WAVES

Superposition of waves: When two or more waves travelling in the same direction act on the particles simultaneously, then the intensity of the resultant wave is modified due to superposition of the wave according to the principle discussed hereunder.

Superposition principle: If two or more waves arrive at a point simultaneously, then displacement at any point is equal to the vector sum of the displacement due to individual waves:

$\therefore\ y = y_1 + y_2 + ... + y_n$

where y is the resultant displacement due to the superposition of displacement y_1, y_2 .... y_n

Superposition can, in turn, give rise to the following phenomena:

Interference: When two waves of the same frequency and of constant phase difference travelling in the same direction superpose, then they can effect modification in intensity in the form of alternate maximum and minimum intensities which is called the interference phenomenon.

If the waves $y_1 = a_1 \sin(\omega t - kx)$ and $y_2 = a_2 \sin(\omega t - kx + \phi)$ superimpose, then by applying the principle of superposition, $y = y_1 + y_2 = R\sin(\omega t - kx + \phi)$

where the resultant amplitude, R = $\sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$ and phase angle $\theta = \tan^{-1}\left[\frac{a_1\sin\phi}{a_1 + a_2\cos\phi}\right]$

When $\phi = 2\pi n$ where n = 0, 1, 2... it produces constructive interference which gives R = $R_{max} = a_1 + a_2$.

However, when $\phi = (2n+1)\pi$ where n = 0, 1, 2..., R = $R_{min} = a_1 - a_2$ or amplitude is minimum due to destructive interference.

As intensity is proportional to the square of amplitude, the ratio of maximum intensity, I_{max}, to minimum intensity , I_{min}, is given by $\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$.

8.1 Coherent and Incoherent Sources

Two sources are called coherent sources only when their phase difference remains constant in time. In case if the phase difference does not remain constant in time, then the sources are incoherent.

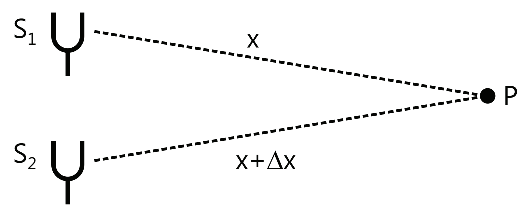


Figure 12.9

The Fig. 12.9 here shows two tuning forks s_1 and s_2, placed side by side, which vibrate with equal frequency and equal amplitude. The point p is situated at a distance x from s_1 and $x + \Delta x$ from S_2.

Now, suppose that the two forks are vibrating in phase so that $\delta_0 = 0$. Also, let p_{01} and p_{02} be the amplitudes of the wave from s_1 and s_2, respectively. Now, let us examine the resultant change in pressure at a point p. The pressure change at A due to the two waves are described by

$$P_1 = P_{01}\sin(kx - \omega t);\ P_2 = P_{02}\sin\left[k(x + \Delta x) - \omega t\right] = P_{02}\sin\left[(kx - \omega t) + \delta\right],$$

where $\delta = k\Delta x = \frac{2\pi \Delta x}{\lambda}$ ... (i)

is the phase difference between two waves reaching P.

The resultant wave is thus given by $p = p_0\sin\left[(kx - \omega t) + \delta\right]$ where $p_0^2 = p_{01}^2 + p_{02}^2 + 2p_{01}p_{02}\cos\delta$

And $\tan\varepsilon = \frac{p_{02}\sin\delta}{p_{01} + p_{02}\cos\delta}$

The resultant amplitude is maximum when $\delta = 2n\pi$ and minimum when $\delta = (2n+1)\pi$ where n is an integer. These are correspondingly the conditions for constructive and destructive interference:

$\delta = 2n\pi$ constructive interference

$\delta = (2n+1)\pi$ destructive interference ... (ii)

Using Eq. (i), i.e., $\delta = \frac{2\pi}{\lambda}\Delta x$, these conditions may be written in terms of the path difference as $\Delta x = n\lambda$ (constructive) or $\Delta x = (n + 1/2)\lambda$ (destructive) ... (iii)

At constructive interference, $p_0 = p_{01} + p_{02}$.

And at destructive interference, $p_0 = |p_{01} + p_{02}|$.

However, if the sources have an initial phase difference δ_0 between them, then the wave reaching 'p' at time t is represented by $p = p_{01}\sin[kx - \omega t]$ and $p = p_{02}\sin[k(x+\Delta x) - \omega t + \delta_0]$

The phase difference between these waves, therefore, is $\delta = \delta_0 + k\Delta x = \delta_0 + \frac{2\pi\Delta x}{\lambda}$.

Illustration 6: Two sound waves, originating from the same source, travel along different paths in air and then meet at a point. Now, if the source vibrates at frequency of 1.0 KHz and one path is 83 cm longer than the other, what will be the nature of interference? The speed of sound in air is 33 ms^{-1} **(JEE ADVANCED)**

Sol: The phase difference between the sound waves, is given by $\delta = \frac{2\pi}{\lambda}\Delta x$ where λ is the wavelength of the wave and Δx is the path difference between the waves

The wavelength of sound wave is $\lambda = \frac{u}{\nu}$; $= \frac{332\ \text{ms}^{-1}}{1.0\times10^3\ \text{Hz}} = 0.332\ \text{m}$

The phase difference between the waves arriving at the point of observation is

$$\delta = \frac{2\pi}{\lambda}\Delta x = 2\pi\times\frac{0.83\text{m}}{0.332\text{m}} = 2\pi\times2.5 = 5\pi$$

As this is an odd multiple of π, the waves interfere destructively.

9. REFLECTION OF SOUND

When there is discontinuity in the medium, sound waves obviously gets reflected. Therefore, when a sound wave gets reflected from a rigid boundary, then the particles at the boundary are unable to vibrate. Thus, a reflected wave is generated which interferes with the incoming wave to produce zero displacement at the boundary. At these points, however, the pressure variation is maximum. Thus, a reflected pressure wave has the same phase as the indicated wave.

Alternatively, a sound wave can also get reflected if it encounters a low pressure region. The reflected pressure wave interferes destructively with the incoming waves in this case. Thus, there is a phase change of $\prod$ in this case.

10. STANDING/LONGITUDINAL WAVES

When two progressive waves of the same frequency moving in the opposite direction superpose, then stationary waves are formed.

Let us now consider the superposition of two such waves along a stretched string having fixed ends where a harmonic wave travels toward right as $y_1 = a\sin\frac{2\pi}{\lambda}(vt - x)$. This wave is reflected from the second point and due to the reflection, its amplitude becomes $-a$ due to phase change of π. Further, the reflected wave y_2 = a sin $\frac{2\pi}{\lambda}(vt - x)$ travels toward left and these waves superpose due to a phase change of pi, and hence give the resultant displacement y.

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) - a\sin\frac{2\pi}{\lambda}(vt + x) = -2a\cos\left(\frac{2\pi vt}{\lambda}\right)\sin\frac{2\pi x}{\lambda} = -A\cos\left(\frac{2\pi vt}{\lambda}\right) \text{ where } A = 2a\sin\left(\frac{2\pi vt}{\lambda}\right)$$

The strings here apparently execute harmonic motion such that the particles of the string vibrate with the same frequency but with different amplitudes. Such a resultant wave is called a standing or stationary wave. The portion along the string where the amplitude is zero is called a node and where the amplitude is maximum is called an antinode.

For nodes: $A = 2a\sin\left(\frac{2\pi vt}{\lambda}\right) = 0; \Rightarrow \quad \frac{2\pi vt}{\lambda} = n\pi$ where n = 0, 1, 2, 3,

The relation $x = \frac{n\lambda}{2}$ gives the position of the nth node and the distance between successive nodes is $\frac{\lambda}{2}$.

For antinodes, $A = 2a\sin\left(\frac{2\pi vt}{\lambda}\right) = 2a, \frac{2\pi vt}{\lambda} = (2n-1)\frac{\pi}{2}$ where n = 1, 2, 3...

$x = (2n-1)\frac{\lambda}{4},$ i.e., $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots\ldots\ldots$

Such points are called antinodes with maximum amplitude of $2a$.

The distance between the successive nodes and antinodes is $\frac{\lambda}{4}$.

11. MODE OF VIBRATION IN AIR COLUMNS

Longitudinal/stationary waves can be generated in both open- and closed pipes like organ pipes having both open ends and one closed end, respectively. If a tuning fork produces a sound wave at the open end, then it is reflected from the second end such that the incident and reflected wave superpose to generate stationary waves. Further, the closed end is always a node and the open end is always an antinode.

11.1 Open pipe

The first three modes of vibration of an open pipe are given as follows:

For fundamental or first harmonic mode in the Fig. 12.10

(a) $l = \frac{\lambda_1}{2}; \lambda_1 = 2l; n_1 = \frac{v}{2l}$

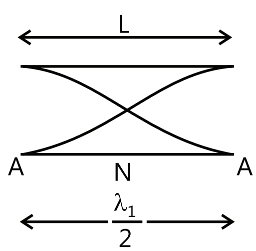


Figure 12.10

For the first overtone or the second harmonic in the Fig. 12.11

(b) $l = \lambda_2; n_2 = \frac{v}{2}$

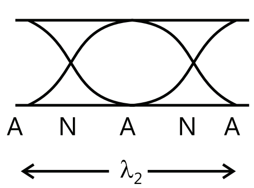


Figure 12.11

For the second overtone or the third harmonic mode in Fig. 12.12

$l = \frac{3\lambda_3}{2}; \lambda_3 = \frac{21}{3};$ or $\lambda_3 = \frac{3v}{2l};$ for path harmonic, $n_p = \frac{pv}{2l}$

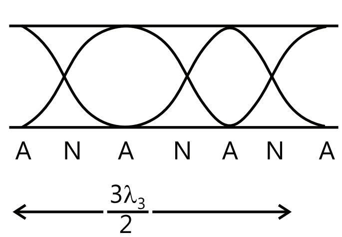


Figure 12.12

11.2 Closed pipe

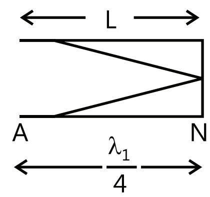


Figure 12.13

The first three modes of vibration of a closed pipe are given as follows:

For fundamental or first harmonic mode in Fig. 12.13.

$$l = \frac{\lambda_1}{4};\ \lambda_1 = 4l;\ n_1 = \frac{v}{4l}$$

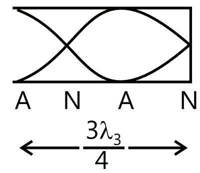


Figure 12.14

For the first overtone or third harmonic in Fig. 12.14

$$l = \frac{3\lambda_2}{4};\ \ \lambda_2 = \frac{4l}{3};\ \ n_2 = \frac{3v}{4l}$$

For the second overtone or the fifth harmonic,

$$l = \frac{5\lambda_2}{4};\ \lambda_3 = \frac{4l}{5};\ n_3 = \frac{5v}{4l}$$

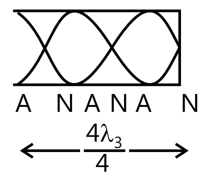


Figure 12.15

For the pth overtone or (2p+1)th harmonic, $n = \frac{(2p+1)v}{2l}$.

At the open end of the pipe, the antinode is formed at a small distance outside the open end. Thus, the correct length of the closed pipe is $l + e$ and that for an open pipe, it is $l + 2e$ and e is equal to 0.3D where D is the internal diameter of the pipe.

12. DETERMINATION OF SPEED OF SOUND IN AIR

12.1 Resonance Column Method

Generally, systems have one or more natural vibrating frequencies. Further, when a system is driven at a natural frequency, then there is a maximum energy transfer and the vibrating amplitude steadily increases till up to a maximum. However, when a system is driven at a natural frequency, we say that the system is in resonance (with the driven source) and refer to the particular frequency at which this occurs as a resonance frequency. Moreover, from the relationship between the frequency f, the wavelength λ, and the wave speed v, which is $\lambda f = v$, it is very obvious that if both the frequency and wavelength are known, then the wave speed can be easily determined. Further, if the wavelength and speed are known, then the frequency can be determined.

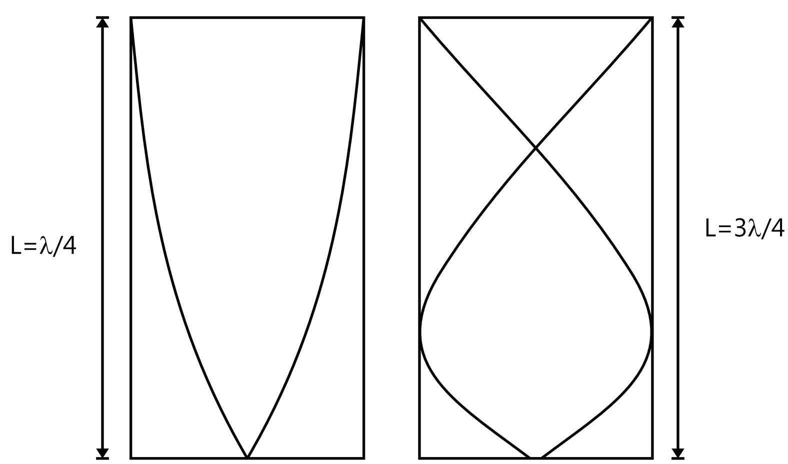


Figure 12.16

We know that air column in pipes or tubes of fixed length has particular resonant frequencies. Moreover, the interference of the waves travelling down the tube and the reflected waves traveling up the tube produces (longitudinal) standing wave which must have a node at the closed end of the tube and an antinode at the open end of the tube.

The resonance frequencies of a pipe or tube usually depend on its length L. As we observe from the Fig. 12.16, only a certain number of lengths or “loops” can be “fitted” into the tube length with the node–antinode requirements.

However, since each loop corresponds to one-half wavelength, resonance occurs when the tube is nearly equal to an odd number of quarter wavelength, i.e., $L = \lambda / 4, L = 3\lambda / 4, L = 5\lambda / 4$, etc

or in general, $L = (2n+1)\lambda / 4$; $\lambda = 4L / (2n+1)$; $f_n = (2n+1)v / 4L$

Hence, an air column (tube) of length L has particular resonance frequencies and therefore will be in resonance with the corresponding odd harmonic driving frequencies.

As we can observe from the above equation, the three experimental parameters involved in the resonance condition of an air column are f, V, and L. However, to study the resonance in this experiment, the length L of an air column will be varied for a given driven frequency. The length of the air column achieved by changing the position of the movable piston in the tube is as seen in the Fig. 12.17.

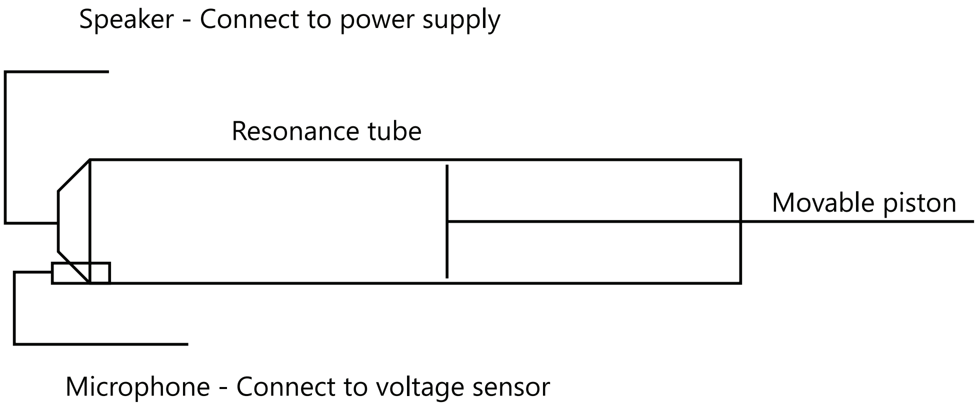


Figure 12.17

Further, as the piston is removed, increasing the length of the air column, more wavelength segments will fit into the tube, consistent with the node–antinode requirements at the ends. Thus, the difference in the tube lengths when successive antinodes are at the open end of the tube and resonance occurs is equal to a half wavelength; for example: $\Delta L = L_2 - L_1 = 3\lambda / 4 - \lambda / 4 = \lambda / 2$

Further, when an antinode is at the open end of the tube, a loud resonance tone is heard. Hence, the tube length for antinodes to be at the open end of the tube can be determined by moving the piston away from the open end of the tube and "listening" for resonances. However, no end correction is needed for the antinode occurring slightly above the end of the tube since in this case, difference in tube lengths for successive antinodes is equal to λ/2. Further, if we know the frequency of the driving source, then the wavelength is determined by measuring difference in tube length between successive antinodes, $\Delta L = \lambda / 2$ or $\lambda = 2\Delta L$, the speed of sound in air, $v_s = \lambda f$.

12.2 Kundt's Tube Method

In the Kundt's method, a gas is filled in a long cylindrical tube closed at both the ends, one by disk and the other by a movable piston. A metal rod is welded with the disk and is clamped exactly at the middle point. The length of the tube in this method can be varied by moving the movable piston. Some powder is sprinkled in the tube along its length.

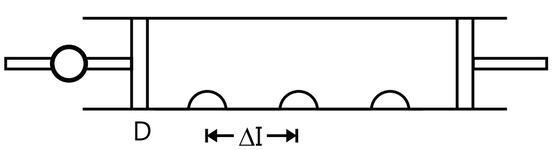


Figure 12.18

The rod in the setup is set into longitudinal vibrations either electronically or by rubbing it with some cloth or otherwise. Further, if the length of the gas column is such that one of its resonant frequency is equal to the frequency of the longitudinal vibration of the rod, then standing waves originate in the gas. Moreover, the powder particles at the displacement antinodes fly apart due to the inherent violent disturbance there, whereas the powder at the displacement nodes remain undisturbed because the particles here do not vibrate. Thus, the powder which was initially dispersed along the whole length of the tube gets collected in a heap at the displacement nodes. By measuring the seperation Δl between the successive heaps, we can find the wavelength of the sound in the enclosed gas. $\lambda = 2\Delta l$

However, it should be noted that the length of the column is adjusted by moving the piston such that the gas resonates and wavelength λ is obtained.

The speed of sound is given by $\nu = \lambda v = 2\Delta l * v$.

Further, if the frequency of the longitudinal vibration in the rod is not known, then the experiment is repeated with air filled in the tube. Now, the length between the heaps of the powder, $\Delta l'$ is measured. The speed of sound in air is then $\nu = 2\Delta l' v$ . ... (i)

Now, $\frac{\nu}{\nu} = \frac{\Delta l}{\Delta l'}$ or $\nu = \nu' \frac{\Delta l}{\Delta l'}$

By calculating the speed of ν' of sound in air, we can find the speed of sound in the gas.

13. BEATS

It two sources of slightly different frequencies produce sound waves in the same direction at the same point, these waves then superpose to produce alternate loud and feeble sounds. Such variations in loudness are called beats. The number of times such a fluctuation in loudness from maxima to minima takes place per second is called the beat frequency.

If two waves $y_1 = a\sin(2\pi n_1 t)$ and $y_2 = a\sin(2\pi n_2 t)$ of respective frequencies n_1 and n_2 superpose at the same place x = 0, then $y = y_1 + y_2 = a[\sin(2\pi n_1 t) + (\sin 2\pi n_2 t)]$

$$\therefore\ y = 2\,a\cos\left[\frac{2\pi(n_1 - n_2)t}{2}\right] \times \sin\left[\frac{2\pi(n_1 + n_2)t}{2}\right] = 2\,a\cos\left[2\pi(n_1 - n_2)t\right] \times \sin\left[2\pi(n_1 + n_2)t\right]$$

$$y = A\,\sin\left[\pi(n_1 + n_2)t\right]\ ;\ A = 2a\cos\left[\pi(n_1 - n_2)t\right]$$

The resultant wave is a harmonic wave with a frequency $\left(\frac{n_1 + n_2}{2}\right)$ but its amplitudes vary harmonically as a function of the difference in the frequency $n_1 - n_2$. The beat frequency n_B is $n_B = n_1 - n_2$.

If $n_1 - n_2$ is small, i.e., the number of times the intensity of sound fluctuates between maxima and minima per second is small, i.e., less than about 10 to 15, then the beats can be heard distinctly.

Illustration 7: Suppose that a string of length 25 cm and 2.5 g is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, then 8 beats per second are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m/s, then find the tension in the string **(JEE ADVANCED)**

Sol: The fundamental frequency of the string and the closed organ pipe are $\nu_s = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$ and $\nu_p = \frac{v}{4\ell}$. When two waves of equal amplitude and slightly different frequencies superimpose with each other, phenomenon called beats take place. Number of beats n = $\Delta\nu$ where $\Delta\nu$ is the difference in the frequencies of superimposed waves.

Fundamental of the string $\nu_s = \frac{1}{2\ell}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 0.25}\sqrt{\frac{T}{10^{-2}}} = 20\sqrt{T}$

The fundamental frequency of a closed pipe $\nu_p = \frac{v}{4\ell} = \frac{320}{4 \times 0.40} = 200$ Hz

The frequency of the first overtone of the string = $2\nu_s = 40\sqrt{T}$

Since there are 8 beat per second, $2\nu_s - \nu_p = 8$ or $40\sqrt{T} - 200 = 8$

Since on decreasing the tension, the beat frequency decreases, $2\nu_s$ is definitely greater than ν_p

$\therefore 40\sqrt{T} - 200 = 8$ or $T = 27.04N$

Illustration 8: A sonometer wire of 100 cm in length has a fundamental frequency of 330 Hz. Find

(a) The velocity of propagation of transverse waves along the wire and

(b) The wavelength of the resulting sound in air if velocity of sound in air is 330 ms^{-1}. **(JEE ADVANCED)**

Sol: As the wave travelling on the sonometer wire is the standing wave, the wavelength of the wire is $\lambda = 2L$. And the velocity of the wave is given by $v = f\lambda = 2fL$.

(a) In the case of transverse vibration of a string for fundamental mode: $L = (\lambda / 2) \Rightarrow \lambda = 2L = 2\times 1 == 2m$

i.e., the wavelength of transverse wave propagating on the string is 2 m. Now, as the frequency of the wire is given to be 330 Hz, so from $v = f\lambda$, the velocity of transverse wave along the wire will be $v = 330\times 2 = 660 m/s$

(b) Here, the vibrating wire will act as a source and produce sound, i.e., longitudinal waves in air: Now, as the frequency does not change with change in medium so $f = 330Hz$, and as velocity in air is given to be = 330 m/s

so from relation $v = f\lambda$ we get $\lambda_{air} = \frac{v_{air}}{f} = \frac{330}{330} = 1m$

i.e., for sound (longitudinal mechanical waves) in air produced by vibration of wire (body),

$f = 330\ s^{-1}, \lambda_{air} = 1m$ and $v = f\times\lambda = 330m/s$

14. DOPPLER EFFECT

We are familiar with the fact that when a source of sound or an observer or both are moving relative to each other, then there is an apparent change in the frequency of sound as heard by an observer and this is called Doppler Effect. Further, the apparent frequency increases if the source is moving toward the observer or the observer is moving toward the source. On the contrary, the apparent frequency decreases if either the source is moving away from the observer or the observer is moving away from the source. This apparent change in the frequency is principally due to the basic effect of motion of source to change the effective wavelength and the basic effect of motion of observer is the change in the number of waves received per second by the observer.

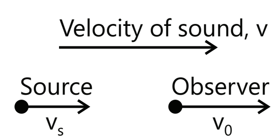


Figure 12.19

However, if both the source and the observer are moving in the positive direction of x-axis, then sound of frequency 'n' propagating in air with velocity in still air will result in an apparent frequency n' heard by observed as $n' = \left(\frac{v - v_0}{v - v_s}\right)n$

Moreover, if the direction of motion of source or observer is changed, then the signs of v_0 and v_s are accordingly changed from negative to positive. Thus, the frequency n', in still air for the different cases is obtained as follows:

(a) Both the source and observer are moving toward right when the source is approaching a receding observer toward right $n' = \left(\frac{v - v_0}{v - v_s}\right)n$

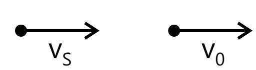


Figure 12.20

(b) Both the source and observer are receding from each other $n' = \left(\frac{v - v_0}{v + v_s}\right)n$

Figure 12.21

(c) Both the source and observer are moving toward each other $n' = \left(\frac{v + v_0}{v - v_s}\right)n$


Figure 12.22

(d) When the observer is approaching a receding source, $n' = \left(\dfrac{v+v_0}{v+v_s}\right)n$ If the wind is blowing with a velocity ω in the direction of sound, then ω is added to ν and if the wind is blowing with velocity ω opposite to direction of wind, then ω is subtracted from ν. The general formula for the apparent frequency n′ due to Doppler effect is, $n' = \left(\dfrac{\nu \pm \omega \mp \nu_0}{\nu \pm \omega \mp \nu_s}\right)n$

Figure 12.23

Illustration 9: Assume that a siren emitting a sound of frequency 2000 Hz moves away from you toward a cliff at a speed of 8 m/s.

(a) What is the frequency of the sound you hear coming directly from the siren?

(b) What is the frequency of sound you hear reflected off the cliff? Speed of sound in air is 330 m/s. **(JEE MAIN)**

Sol: As the siren being source is moving away from you the observer on cliff, the apparent frequency is given by $f' = f_0\left(\dfrac{v}{v+v_s}\right)$. Where f_0 is natural frequency of the sound wave. The intensity of the sound wave appears to be decreasing. When sound reflects from cliff it moves towards observer (cliff) and hence the frequency of the sound wave is $f' = f_o\left(\dfrac{v}{v-v_s}\right)$. When source moves towards the observer, the intensity of sound wave appears to be increasing.

(a) The frequency of sound heard directly is given by

$$f_1 = f_0\left(\frac{v}{v+v_s}\right);\ v_s = 8m/s;\ \therefore f_1 = \left(\frac{330}{330+8}\right)\times 2000$$

(b) The frequency of the reflected sound is given by

$$f_2 = f_1\left(\frac{v}{v-v_s}\right);\ \therefore f_2 = \left(\frac{330}{330-8}\right)\times 2000;\ \ f_2 = \frac{330}{322}\times 2000 = 2050\text{Hz}.$$

Illustration 10: Let us suppose that a sound detector is placed on a railway platform. A train, approaching the platform at a speed of 36 km h^{-1}, sounds its horn. The detector detects 12.0 kHz as the most dominant frequency in the horn. If the train stops at the platform and sounds the horn, what would be the most dominant frequency detected? The speed of sound in air is 340 ms^{-1}. **(JEE MAIN)**

Sol: In the first case, when train is moving towards the stationary observer on the platform, the intensity of the wave appears to be increasing. And the frequency is given by $f' = f_o\left(\dfrac{v}{v-v_s}\right)$. In the second case both the train and the observer are stationary so we hear the natural frequency f_0 of the sound wave.

Here, the observer (detector) is at rest with respect to the medium (air). Suppose that dominant frequency as emitted by the train is v_0. When the train is at rest at the platform, the detector will select the dominant frequency as v_0. When this same train was approaching the observer, then frequency detected would be

$$v' = \frac{\nu}{\nu-u_s}v_0\ ;\ \text{ or }\ v_0 = \frac{\nu-u_s}{\nu}v' = \left(1-\frac{u_s}{\nu}\right)v'$$

The speed of the source is $u_s = 36\text{kmh}^{-1} = \dfrac{36\times 10^3\text{m}}{3600\text{s}} = 10\text{ms}^{-1}$

Thus $v_0 = \left(1-\dfrac{10}{340}\right)\times 12.0\text{kHz} \quad = 11.6\text{kHz}$

14.1 Change in Wavelength

If a source moves with respect to the medium, then wavelength becomes different from the wavelength observed when there is no relative motion between the source and the medium. Thus, the formula for calculation of apparent wavelength may be derived immediately from the relation $\lambda = \nu / v$. It is given as $\lambda = \frac{\nu - u}{\nu}\lambda.$

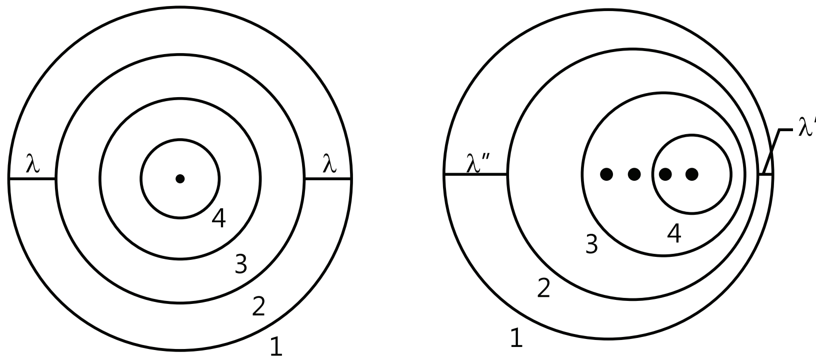


Figure 12.24

15. SONIC BOOM AND MACH NUMBER

A sonic boom is basically the sound associated with the shock waves created by an object travelling through air faster than the speed of sound. This boom generates a huge amount of energy, sounding much like an apparent explosion. The crack of a supersonic bullet passing overhead is an excellent example of a sonic boom in miniature.

Mach number is purely a dimensionless quantity representing the ratio of speed of an object moving through a fluid and the local speed of sound, $M = \frac{\nu}{\nu_{sound}}$ where M is the Mach number,

ν is the velocity of the source relative to the medium, and ν_{sound} is the speed of sound in the medium.

16. MUSICAL SCALE

A musical scale is a sequence of frequencies which has a particularly pleasing effect on our ear. A widely used musical scale, called diatonic scale, has eight frequencies covering an octave. We call each frequency as a note.

17. ACOUSTICS OF A BUILDING

Good concert halls: Good concert halls are so designed to eliminate unwanted reflection and echoes and to optimize the quality of the sound perceived by the audience. This is accomplished by suitably engineering the shape of the room and the walls, as well as to include sound-absorbing materials in areas that may cause echoes.

Lecture hall: Similar consideration such as the one made in the above must be made particularly in a college lecture hall, so that the professor can be heard by all of the students in the session. Although the sound quality need not be as good as in a concert hall where music is being played, it still must be good enough to prevent echoes and other things that will distort the audio quality of the speech delivered by the professor.

Work buildings: In an office building where there are cubicles with a divider in a large work area, there is often the problem of noise from conversation and activities. However, in this case the quality of the sound is not an issue as much as suppressing unwanted noise.

17.1 Echo

An echo (plural echoes) is a reflection of sound, arriving back at the listener particularly sometime after the direct sound.

17.2 Reverberation and Reverberation Time

Reverberation is the persistence of sound in a particular space after the original sound is produced. A reverberation, or reverb, is generated when sound is produced in an enclosed space causing a large number of echoes to build up and then slowly decay as the sound is absorbed by the wall and air.

Reverberation time: The interval between the initial direct arrival of a sound wave and the last reflected wave is called the reverberation time.

18. APPLICATION OF ULTRASONIC WAVES

Biomedical application: Ultrasound has very good therapeutic applications, which can be highly beneficial when used with appropriate dosage precautions. Relatively high power ultrasound can eliminate stony deposits or tissue, accelerate the effect of drugs in a targeted area, assist in the measurement of the elastic properties of tissue, and can also be used to sort cells or small particles for research.

Ultrasonic impact treatment: Ultrasonic impact treatment (UIT) is a technique wherein ultrasound is used to enhance the mechanical and physical properties of metals. Basically, it is a metallurgical processing technique in which ultrasonic energy is applied to a metal object.

Ultrasonic welding: In ultrasonic welding of plastics, high frequency (15 kHz to 40 kHz) low amplitude vibration is used to create heat by way of friction between the materials to be joined. The interface of the two parts is specially designed so as to concentrate the energy for the maximum weld strength.

Sonochemistry: Power ultrasound in the 20–100 kHz range alone is used in chemistry. The ultrasound does not interact directly with molecules to induce the chemical change, as its typical wavelength (in the millimeter range) is too long compared to the molecules. Instead, the energy causes cavitation, which generates extremes of temperature and pressure in the liquid where the reaction takes place.

19. SHOCK WAVES

A shock wave is one type of propagating disturbance. Similar to an ordinary wave, it carries energy and can propagate through a medium (solid, liquid, gas or plasma) or in some cases even in the absence of a material medium, through a field such as an electromagnetic field. Generally, shock waves are characterized by an abrupt, nearly discontinuous change in the characteristics of the medium.

PROBLEM-SOLVING TACTICS

1. Most of the questions are naturally related with the concepts of wave on a string. Therefore, one must be thorough with the concept of that particular topic. (E.g., standing waves formed in open pipe here are analogous to string tied at both ends. Further, many of the cases can be related in the same way.)

2. Questions dealing with physical experiments form another set of questions. Therefore, one must be familiar with usual as well as unusual (or specific) terminology of each experiment. Mostly, it happens that if we do not know the term, then we are usually stuck (E.g., end correction is one term used with the resonance column method, which is directly related with the radius of the tube.)

3. Path difference between two sources form another set of questions and this is the only place where some mathematical complexity can be involved. Hence, one must take care of them.

4. Questions related to Doppler effect and beats are generally formulae specific; therefore, one must carefully use the formulae. (It is, however, also advised that one must know about the derivation of these formulae.)

FORMULAE SHEET

S. No.	Term	Description
1.	Wave	It is a disturbance, which travels through the medium due to repeated periodic motion of particles of the medium about their equilibrium position. Examples include sound waves travelling through an intervening medium, water waves, light waves, etc.
2.	Mechanical waves	Waves requiring material medium for their propagation. These are basically governed by Newton's laws of motion. Sound waves are mechanical waves in the atmosphere between source and the listener and hence require medium for their propagation.
3.	Non-mechanical waves	These waves do not require material medium for their propagation. Examples include waves associated with light or light waves, radio waves, X-rays, micro waves, UV light, visible light and many more.
4.	Transverse waves	These are waves in which the displacements or oscillations are perpendicular to the direction of propagation of wave.
5.	Longitudinal waves	These are those waves in which displacement or oscillations in medium are parallel to the direction of propagation of wave, for example, sound waves.
6.	Equation of harmonic wave	At any time t, displacement y of the particle from its equilibrium position as a function of the coordinate x of the particle is y(x,y) = A $\sin(\omega t - kx)$ where A is the amplitude of the wave, k is the wave number, ω is the angular frequency of the wave, And $(\omega t - kx)$ is the phase.
7.	Wave number	Wave length λ and wave number k are related by the relation k = 2n/λ.
8.	Frequency	Wavelength λ and wave number k are related by the relation v = ω / k = λ / T = λf.
9.	Speed of a wave	Speed of a wave is given by $v = \omega / k = \lambda / T = \lambda f$.
10.	Speed of a transverse wave	Speed of a transverse wave on a stretched string depends only on tension and the linear mass density of the string but not on frequency of the wave, i.e., $v = \sqrt{T / \mu}$.
11.	Speed of a longitudinal wave	Speed of longitudinal waves in a medium is given by v = B = bulk modulus; ρ = density of medium; Speed of longitudinal waves in an ideal gas is $v = \sqrt{\gamma p / \rho}$ P = pressure of the gas , ρ = density of the gas and y = C_p/C_v.
12.	Principle of super position	When two or more waves traverse through the same medium, then the displacement of any particle of the medium is the sum of the displacements that the individual waves would give it, i.e., $y = \sum y_i(x,t)$.

S. No.	Term	Description
13.	Interference of waves	If two sinusoidal waves of the same amplitude and wavelength travel in the same direction, then they interfere to produce a resultant sinusoidal wave travelling in the direction with resultant wave given by the relation $y'(x,t) = \left[2A_m \cos(u/2)\right]\sin(\omega t - kx + u/2)$ where u is the phase difference between the two waves. If u = 0, then interference would be fully constructive. If u = π , then waves would be out of phase and their interference would be destructive.
14.	Reflection of waves	When a pulse or travelling wave encounters any boundary, it gets reflected. However, if an incident wave is represented by $y_i(x,t) = A\ \sin(\omega t - kx),$ then the reflected wave at rigid boundary is $y_r(x,t) = A\ \sin(\omega t + kx + n)\ = -A\ \sin(\omega t + kx)$ and for reflection at open boundary, reflected waves is given by $y_r(x,t) = A\ \sin(\omega t + kx)$.
15.	Standing waves	The interference of two identical waves moving in opposite directions produces standing waves. The particle displacement in a standing wave is given by $y(x,t) = \left[2A\cos(kx)\right]\sin(\omega t)$. In standing waves, amplitude of waves is different at different points, i.e., at nodes, amplitude is zero and at antinodes, amplitude is maximum which is equal to sum of amplitudes of constituting waves.
16.	Normal modes of stretched string	Frequency of transverse motion of stretched string of length L fixed at both the ends is given by f = nv/2L where n = 1, 2, 3, 4. The set of frequencies given by the above relation is called normal modes of oscillation of the system. Mode n = 1 is called the fundamental mode with the frequency f_1 = v/2L. Second harmonic is the oscillation mode with n = 2 and so on. Thus, a string has infinite number of possible frequencies of vibration which are harmonics of fundamental frequency f_1 such that $f_n = nf_{1.}$
17.	Beats	Beats arise when two waves having slightly differing frequency V_1 and V_2 and comparable amplitudes are superposed.
18.	Doppler effect	Doppler effect is a change in the observed frequency of the wave when the source S and the observer O move relative to the medium. There are three different ways where we can analyze this change in frequency as listed hereunder. (1) when observer is stationary and source is approaching observer v = $v_0(1+V_s/V)$ where V_s = velocity of the source relative to the medium v = velocity of wave relative to the medium V = observed frequency of sound waves in terms of source frequency V_0 = source frequency Change in the frequency when source recedes from stationary observer is v = $V_0(1–V_S/V)$ Observer at rest measures higher frequency when source approaches it and it measures lower frequency when source recedes from the observer. (2) observer is moving with a velocity V_0 toward a source and the source is at rest is V = $V_0(1+V_0/V)$ (3) both the source and observer are moving, then frequency observed by observer is V = V_0 $(V+V_0)/(V+V_S)$ and all the symbols have respective meanings as discussed earlier

Solved Examples

JEE Main/Boards

Example1: A tube closed at one end has a vibrating diaphragm at the other end, which may be assumed to be displacement node. It is found that when the frequency of the diaphragm is 2000 Hz, then a stationary wave pattern is set up in which the distance between adjacent nodes is 8 cm. When the frequency is gradually reduced, then the stationary wave pattern disappears but another stationary wave pattern reappears at a frequency of 1600 Hz. Calculate

(i) The speed of sound in air,

(ii) The distance between adjacent nodes at a frequency of 1600 Hz,

(iii) The distance between the diaphragm and the closed end, and

(iv) The next lower frequencies at which stationary wave pattern will be obtained.

Sol: The standing waves generated inside the tube closed at one end, have the wavelength $n\,\lambda = 2L$ where L is length of the tube. The velocity of the wave in air is given by v = fλ, where n is the frequency of the sound wave.

Since the node-to-node distance is

or $\lambda/2 = 0.08$ or $\lambda = 0.16\text{m}$

(i) $v = f\lambda \quad \therefore v = 2000 \times 0.16 = 320\text{ m/s}$

(ii) $320 = 1600 \times \lambda$ or $\lambda = 0.2\text{ m}$

$\therefore$ Distance between nodes = 0.2/2 = 0.1 m = 10 cm

(iii) Since there are nodes at the ends, the distance between the closed end and the diaphragm must be an integral multiple of $\lambda/2$

$\therefore L = n\lambda/2 = n \times 0.2/2 = n' \times 0.16/2$

$\Rightarrow \frac{n}{n'} = \frac{4}{5}$ when $n' = 5, n = 4$

$$L = \frac{n' \times 0.16}{2} = 0.4\text{ m} = 40\text{ cm}$$

(iv) For the next lower frequency $n = 3,\ 2,1$

$\therefore \quad 0.4 = 3\lambda/2 \quad$ or $\quad \lambda = 0.8/3$

since $\quad v = f\lambda,\ f = \frac{320}{0.8/3} = 1200\text{Hz}$

$\therefore f = 320/0.4 = 800\text{Hz}$

again $\quad 0.4 = 1\lambda/2$ or $\lambda = 0.4\text{m}$

$\therefore \quad n = 320/0.4 = 800\text{Hz}$

Example 2: A tuning fork of frequency 256 Hz and an open orange pipe of slightly lower frequency are at 17^{0}C. When sounded together, they produce 4 beats per second. On altering the temperature of the air in the pipes, it is observed that the number of beats per second first diminishes to zero and then increases again to 4. By how much and in what direction has the temperature of the air in the pipe been altered?

Sol: In a open organ pipe the frequency of the wave is $n = \frac{V_t}{\lambda}$ where V_t is the velocity of wave at temperature t and λ=2L is the wavelength of the vibrating wave. If temperature of air inside the organ pipe changes, the velocity of wave also changes, since $V \propto \sqrt{T}$.

$n = \frac{V_{17}}{2l}$ where L = length of the pipe

$\therefore 256 - \frac{V_{17}}{2L} = 4$ or $\frac{V_{17}}{2L} = 252$

Since beats decrease first and then increase to 4, the frequency of the pipe increases. This can happen only if the temperature increases.

Let t be the final temperature, in Celsius,

$n = \frac{V_t}{2l} - 256 = 4$ or $\frac{V_t}{2l} = 260$

dividing $\frac{V_t}{V_{17}} = \frac{260}{252}$ or $\sqrt{\frac{273+t}{273+17}} = \frac{260}{252}$

$\left(\because V < \sqrt{T}\right)$ or $t = 308.7 - 273 = 35.7 - 17$

$= 18.7^0\text{C}.$

$\therefore$ Rise in temperature $= 35.7 - 17 = 18.7^0\text{C}.$

Example 3: Find the fundamental and the first overtone of a 15 cm pipe

(a) If the pipe is closed at one end,

(b) If the pipe is open at both ends,

(c) How many overtones may be heard by a person of normal hearing in each of the above cases? Velocity of sound in air = 330 ms^{-1}

Sol: For the organ pipe closed at one end, the fundamental frequency of the wave of wavelength λ is given by, $n_0 = \frac{v}{4L}$.The frequency of i^{th} over tone is given by $n_i = (i+1) \times n_0$ where i=1,2,3.... etc.

(a) $n_0 = \frac{v}{4L}$ where n_0 = frequency of the

fundamental $\Rightarrow n_0 = \frac{330}{4 \times 0.15} = 550\text{Hz}$

(b) The first four overtones are $2n_0$, $3n_0$, $4n_0$,and 5 n_0. So, the required frequencies are 1100, 2200, 3300, 4400, and 5500 Hz.

(c) the frequency of the nth overtone is (2n + 1)r

$$\therefore (2n+1)n_0 = 20000; \text{or} (2n+1)550 = 20000$$

$$\text{or } n = 17.68$$

Or n = 17.18 the acceptable value is 17.

Example 4: The wavelength of the note emitted by a tuning fork of frequency 512 Hz in air at 17^0 C is 66.6 cm. If the density of air at STP is 1.293 gram per liter, calculate γ for air.

Sol: The bulk modulus of gas γ is given by $\gamma = \frac{V^2 p}{P_o}$.

Here V is velocity of wave, and p is the pressure at a point. And P_o is the atmospheric pressure.

n = 512 Hz,= 66.6 cm ; $\nu = n\lambda$

$= 512 \times 66.6 = 340.48 \text{ m/s}$; $\gamma = \frac{\nu^2 p}{P_o}$

$P_o = 1.013 \times 10^5 \text{Nm}^{-2}$; $p = 1.293 \text{ kg/m}^3$.;

$$\therefore \gamma = \frac{(330)^2 \times 1.293}{1.013 \times 10^5} = 1.39.$$

Example 5: A source of sound is moving along orbit of radius 3 m with an angular velocity of 10 rad/s. A source detector located far away from the source is executing linear simple harmonic motion along the line BD as shown in the figure with an amplitude BC = CD = 6 m. The frequency of oscillation of the detector is $5/\pi$ per second. The source is at the point A when the detector is at the point B. If the source emits a continuous wave of frequency 340 Hz, then find the maximum and the minimum frequency recorded by the detector.

Sol: Here both source and detector are performing periodic motion. When source and detector are moving away from each other, the detector will record the minimum frequency and vice versa.

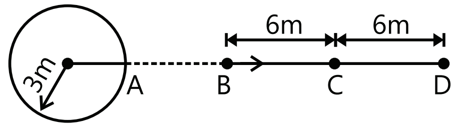
Speed of source, $V_s = r\omega = 3 \times 10 = 30$ m/s

Maximum velocity of detector $v_0 = A\omega'$

$v_0 = A \times 2\pi f' = 6 \times 2\pi \times (5/\pi) = 60 \text{ m/s}$

Actual frequency of source n = 340Hz

The frequency recorded by the detector is maximum when both the source and detector travel along the same direction.



$$n_{max} = \frac{v + v_0}{v + v_s} n = \frac{330 + 60}{330 - 30} \times 340 = 442\text{Hz}$$

The frequency recorded will be minimum when both the source and detector are travelling in opposite directions.

$$n_{max} = \frac{v + v_0}{v + v_s} n = \frac{330 - 60}{330 + 30} \times 340 = 255\text{Hz}$$

JEE Advanced/Boards

Example 1: Two sources S_1 and S_2 separated by 2.0 m, vibrate according to equation

$y_1 = 0.03 \sin(\pi t)$ and $y_2 = 0.02 \sin(\pi t)$

Where y_1, y_2 and t are in M.K.S. units. They send out waves of velocity 1.5m/s.

Calculate the amplitude of the resultant motion of the particle collinear with S_1 and S_2 and located at a point.

(a) To the right of s_2

(b) To the left of s_1 and

(c) In the middle of S_1 and S_2

Sol: The phase difference between the two waves is given by $\phi = \frac{2\pi x}{\lambda}$ where x= 2.0 m is the path difference between the two waves at points near to S_1 or S_2.

The resultant amplitude of the superimposed wave is $a = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi}$.

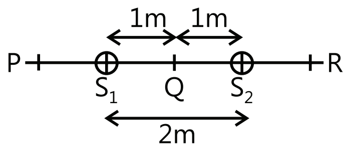
Let P and R be respective points to the left of S_1 and right of S_2, respectively.

The oscillations y_1 and y_2 have amplitude a_1 = 0.03 m and a_2 = 0.02 m, respectively. These have equal period T = 2 s and same frequency $n = \frac{1}{T} = \frac{1}{2} = 0.5 s^{-1}$.

The wavelength of each vibration

$$\lambda = \frac{v}{n} = \frac{1.5}{0.5} = 3.0\,m$$

(a) The path difference for point R to the



right of S_2 = Δ =($S_1R - S_2R$) = S_1S_2 = 2m

$$\therefore \text{Phase difference } \phi = \frac{2\pi}{\lambda}x \ = \frac{2\pi}{3} \times 2.0 = \frac{4\pi}{3}$$

The resultant amplitude for point R is given by

$$\sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} =$$

$$\sqrt{\left\{(0.03)^2 + (0.02)^2 + 2 \times 0.03 \times 0.02 \times \cos(4\pi/3)\right\}}$$

Solving, we obtain a = 0.02565 m.

(b) The path difference for all point p to the left of S_1 is $\Delta = S_2P - S_1P = 2.0$ m.

Hence, the resultant amplitude for all points to the left of S_1 is 0.0265 m.

(c) for a point Q, midway between $S_1$and S_2,

the path difference is zero i.e., $\phi = 0$

$$\text{Hence } a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2}$$

$$= \sqrt{\left\{(0.03)^2 + (0.02)^2 + 2(0.03)(0.02)\right\}}$$

$$= 0.03 + 0.02 = 0.05\,m$$

Example 2: A progressive and stationary simple harmonic wave each have the same frequency of 250 Hz, and the same velocity of 30 m/s. Calculate

(a) The phase difference between two vibrating points on the progressive waves which are 10 cm apart.

(b) The equation of motion of the progressive wave if its amplitude is 0.03 m.

(c) The distance between nodes in the stationary wave,

(d) The equation of motion of the stationary wave if its amplitude is 0.01 m.

Sol: The simple harmonic progressive waves, is represented by $y = a\sin\omega\left(\frac{t}{T} - \frac{x}{\lambda} + \phi\right)$ where ϕ is the phase constant of the wave. The phase difference is $\delta = \frac{2\pi}{\lambda}\Delta x$ where wavelength is λ and Δx is the path difference. The distance between two successive node or two successive antinode is $\lambda/2$.

Given, n= 250 Hz, v = 30 m/s

given, n = 250 Hz, v = 30 m / s

$$\therefore \quad \lambda = \frac{v}{n} = \frac{30}{250} = \frac{3}{25}m = 12cm$$

(a) $\therefore$ Phase difference for a distance of 10 cm

$$= \frac{2\pi}{\lambda} \times 10 = \frac{2\pi}{12} \times 10 = \frac{5}{3}\pi$$

(b) Now $a = 0.03m,\ \lambda = (3/25)m$

and $\frac{1}{T} = n = 250Hz$

The equation of a plane progressive

wave is given by $y = a\ \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda} + \phi\right)$

$\therefore$ where ϕ is initial phase;

$y = 0.03\ \sin 2\pi(250t - 25x/3 + \phi)$;

(c) The distance between nodes in stationary

wave $= \frac{\lambda}{2} = \frac{12}{6} = 6$ cm

(d) Equation of a stationary wave is given by

$$y = 2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$$

If there is antinode at $x = 0 = 2a\cos\frac{2\pi x}{\lambda}\sin$

As $a = 0.01m, \lambda = \frac{3}{25}m$ and $\frac{1}{T} = 250$ Hz

$y = 0.02\cos\left(\frac{50\pi x}{3}\right)\sin(500\pi t)m$ where x and

y are n meter and t in sec

Example 3: The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. Find the length of the pipes.

Sol: The difference in frequencies of the first overtones of the open organ pipe and closed organ pipe is 2.2 Hz. Write the frequencies in terms of length of the pipes and get the relation between the lengths of the pipes. The fundamental frequency of the closed organ pipe is given so its length can be easily found.

The beat are produced when the wave of same amplitude but different frequencies, resonate with each other.

Let the length of open and closed pipes be l_1 and l_2, respectively.

The frequency of first over tone of open organ pipe is

$$n_1 = \frac{2v}{2l_1} = \frac{v}{l_1}$$

The frequency of first over tone of closed organ pipe is

$$n_2 = \frac{3v}{4l_2}$$

Fundamental frequency of closed organ pipe

$$n = \frac{v}{4l_2};\ \therefore = \frac{v}{4l_2} = \frac{330}{4l_2} = 110$$

$$l_2 = \frac{330}{4 \times 110} = 0.75\text{m}$$

As beat frequency = 2.2 Hz

$$= \frac{v}{l_1} - \frac{3v}{4l_2} \Rightarrow \frac{330}{l_1} - \frac{3 \times 330}{4 \times 0.75} = 2.2$$

$$\therefore\ l_1 = \frac{330}{332.2} = 0.993\text{m};$$

$$\text{Beat frequency} = \frac{3v}{4l_2} - \frac{v}{l_1} = 2.2$$

$$\text{or}\ \frac{3 \times 330}{4 \times 0.75} - \frac{330}{l_1} = 2.2;\ \frac{330}{l_1} = 327.8$$

l_1 = 1.006 m.

Example 4: The speed of sound in hydrogen is 1270 m/s. Calculate the speed of sound in the mixture of oxygen and hydrogen in which they are mixed in 1:4 ratio.

Sol: The density of the mixture is given as $\rho = \frac{V_1\rho_1 + V_2\rho_2}{V_1 + V_2}$

here $V_1:V_2$ =1:4. The speed of sound in gas is $v \propto \sqrt{\frac{1}{\rho}}$.

Let V_1 and V_2 be respective volume of oxygen and hydrogen.

Let d_1, m_1 be density and mass of oxygen in the mixture and d_2 m_2 be density and mass of hydrogen in the mixture, respectively.

$$\therefore\ \rho = \frac{\text{Total mass}}{\text{total volume}} = \frac{V_1\rho_1 + V_2\rho_2}{V_1 + V_2}$$

$$= \frac{V_2\rho_2\left(V_1\rho_1 / V_2\rho_2 + 1\right)}{V_2\left(V_1 / V_2 + 1\right)} = \frac{d_2\left(V_1\rho_1 / V_2\rho_2 + 1\right)}{\left(V_1 / V_2 + 1\right)}$$

$$\because\ V_1 / V_2 = 1/4\ \text{and}\ \frac{\rho_1}{\rho_2} = \frac{32}{2} = 16$$

$$\therefore \rho = \frac{\rho_2\left(\frac{1}{4} \times 16 + 1\right)}{\left(\frac{1}{4} + 1\right)} = 4\ \rho_2\ \Rightarrow \frac{\rho}{\rho_2} = 4.$$

Let V_1 and V_2 be the speed of sound in the mixture and hydrogen, respectively.

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}}\ \text{and}\ v_2 = \sqrt{\frac{\gamma P}{\rho_2}};\ \therefore\ \frac{v_1}{v_2} = \sqrt{\frac{\rho_1}{\rho_2}}$$

$$= \sqrt{4} = 2\ \text{or}\ v_1 = \frac{v_2}{2} = \frac{1270}{2} = 635\ \text{m / sec}$$

Example 5: The difference between the apparent frequency of a source as perceived by an observer during its approaching and recession is 2% of the natural frequency of the source. Find the velocity of the source. Take the velocity of sound as 350 m/s.

Sol: By the Doppler's method use the formula for apparent frequency in terms of source velocity to express the difference in two frequencies of approach and recession of the source in terms of its velocity.

For the source approaching a stationary observer,

$$n' = n\left[\frac{v}{v - v_s}\right];\ \text{As}\ v >> v_s,$$

$$n' = n\left[\frac{1}{1 - \left(v_s / v\right)}\right] = n\left[1 - \frac{v_s}{v}\right]^{-1}$$

$$\therefore\ n' \simeq n\left[1 + \frac{V_s}{V}\right] \quad \text{...(i)}$$

When the source is receding, then

$$n'' \cong n\left[1 - \frac{V_s}{V}\right] \quad \text{...(ii)}$$

From Eqs. (i) and (ii)

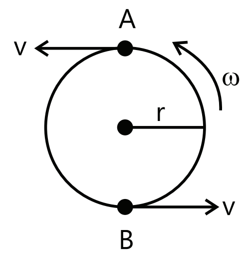
$$n' - n'' = \left[1 + \frac{V_s}{V}\right] - \left[1 - \frac{V_s}{V}\right] = \frac{2nv_s}{v}$$

$$\text{Or}\ \frac{n' - n''}{n} = \frac{2\nu_s}{\nu}$$

$$\text{Percentage change in frequency} = \left(\frac{2\nu_s}{\nu}\right) \times 100 = 2$$

$$\text{Or}\ \nu_s = 3.5\ \text{m / s};\ \frac{2n\nu_s}{\nu} \times 100 = 2$$

Example 6: A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5 m length and rotates with an angular velocity of 20 rad/s in the horizontal plane. Calculate the range of frequency heard by an observer stationed at a large distance from the whistle.



Sol: As the whistle is moved in the circle in horizontal plane, it sometimes moves away and sometimes towards the stationary observer. Thus the observer will hear the minimum frequency of $n_{min} = n\left(\frac{v}{v+v_s}\right)$ when whistle is moving away from him. The observer will hear maximum frequency of $n_{max} = n\left(\frac{v}{v-v_s}\right)$ when the whistle is moving towards him.

Velocity of source $= v_s = r\omega = 1.5 \times 20 = 30$ m/s

Frequency n= 440 Hz..

And speed of sound, v = 330 m/s, the maximum frequency n_{max} will correspond to a position when source is approaching the observer

$$n_{max} = n\left(\frac{v}{v-v_s}\right) = 440\left(\frac{330}{330-30}\right)$$

$$= \frac{440 \times 330}{300} = 484$$

The minimum frequency n_{max} will correspond to a position when source is receding the observer.

$$n_{min} = n\left(\frac{v}{v+v_s}\right) = 440\left(\frac{330}{330+30}\right)$$

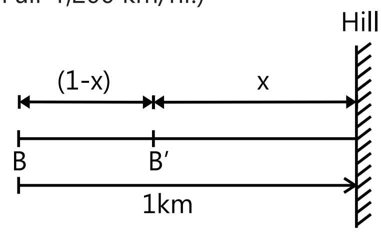
$$= \frac{440 \times 330}{360} = 403\text{Hz}$$

The range of frequency is from 403 Hz to 484 Hz.

Example 7: A train approaching a hill at a speed of 40 km/hr. sounds its horn of frequency 580 Hz when it is at a distance of 1 km from the hill. A wind with a speed of 40 km/hr is blowing in the direction of motion of the train. Find

(a) The frequency of the horn as heard by the observer on the hill,

(b) The distance from the hill at which the echo from the hill is heard by the driver and its frequency (velocity of sound in air 1,200 km/hr.)



Sol: As train is moving towards the stationary observer on the hill. And the wind is in direction of the motion of train, the frequency of the sound waves from horn heard to the observer on hill is given by $n' = n\left[\frac{v'}{v'-v_s}\right]$

where v′ = v+w (sum of velocities of train and train). When this sound wave reflects from the hill, and travels towards the moving train, the frequency heard by the driver is $n' = n\left[\frac{v-w+v_s}{(v-w)}\right]$.

(a) The apparent frequency is given by

$$n' = n\left[\frac{v+w}{(v+w)-v_s}\right]$$

V = 1200 km/hr., w = 40 km/hr., v_s = 40 km/hr. and n = 580 Hz.

$$\therefore \quad n' = 580\left[\frac{1200+40}{(1200+40)-40}\right] = 599.3\text{Hz}$$

(b) As shown in the figure, let the driver hear the echo when he is at a distance x km from the hill. Time taken by the train to reach the point B′

$$t = \frac{(1-x)}{\text{velocity of train}} = \frac{1-x}{40}\text{hr};$$

Time taken by the train to reach the point B′

$$t = \frac{x}{\text{velocity of sound} - \text{velocity of wind}} = \frac{x}{1200-40}\text{hr}$$

$$\frac{1-x}{40} = \frac{x}{1200-40};\ x = 0.966 \text{ km}$$

Frequency heard by driver.

$$n' = 580\left[\frac{1200-40+40}{(1200-40)}\right] = 600 \text{ Hz}$$

Example 8: A band playing music at frequency f is moving toward a wall with velocity $V_{s.}$ A motorist is following the band with a speed of V_{m}. If V is speed of sound, obtain an expression for the beat frequency heard by the motorist.

Sol: In this case, both the source and the observer moving with different speeds towards the wall so the frequency of sound heard by motorist is given as $f' = f_0\left[\frac{v+v_m}{v+v_s}\right]$.

While the sound reflected from the wall is moving towards the motorist. Hence the frequency heard by the motorist will be $f'' = f_w\left[\frac{v+v_m}{v}\right]$. These two waves superimpose with each other to create beats and number of beats heard is given by $n = f'' - f'$.

The frequency, f, of band heard by the motorist directly is given by

$$f' = f\left[\frac{v+v_m}{v+v_s}\right]$$

The frequency f_w reaching the wall is

$$f_w = f\left[\frac{v+0}{v+v_s}\right] = f\left[\frac{v}{v-v_b}\right]$$

The frequency f" reaching the motorist is given by

$$f'' = f_w\left[\frac{v+v_m}{v+0}\right] = f\left[\frac{v}{v-v_b}\right]\left(\frac{v+v_m}{v}\right)$$

$$= f\left[\frac{v+v_m}{v-v_b}\right] \therefore \text{ Beat frequency} = f'' - f' = \Delta f$$

$$\therefore \Delta f = f\left[\frac{v+v_m}{v-v_b}\right] - f\left[\frac{v+v_m}{v+v_b}\right];$$

$$= f\left[\frac{(v+v_m)(v+v_b)-(v+v_m)(v+v_b)}{(v^2-v_b^2)}\right]$$

$$= f\left[\frac{(v+v_m)(2v_b)}{(v^2-v_b^2)}\right]$$

JEE Main/Boards

Exercise 1

Q.1 The velocity of sound in air at NTP is 331 ms^{-1}. Find its velocity when the temperature rises to 91^{0}C and its pressure is doubled.

Q.2 A displacement wave is represented by $\xi = 0.25\times10^{-3}\sin(500t - 0.025x)$

Deduce (i) amplitude (ii) period (iii) angular frequency (iv) Wavelength (v) amplitude of particle velocity (vi) amplitude of particle acceleration. ξ, t and x are in cm, sec and meter respectively.

Q.3 Calculate the velocity of sound in gas, in which two wave lengths 2.04m and 2.08m produce 20 beats in 6 seconds.

Q.4 What type of mechanical wave do you expect to exist in (a) vacuum (b) air (c) inside the water (d) rock (e) on the surface of water?

Q.5 What will be the speed of sound in a perfectly rigid rod?

Q.6 A stone is dropped into a well in which water is 78.4m deep. After how long will the sound of splash be heard at the top? Take velocity of sound in air = 332ms^{-1}

Q.7 From a cloud at an angle of 30^0 to the horizontal, we hear the thunder clap 8 s after seeing the lightening flash. What is the height of the cloud above the ground if the velocity of sound in air is 330 m/s?

Q.8 A fork of frequency 250Hz held over tube and maximum sound is obtained when the column of air is 31cm or 97 cm. Determine (i) velocity of sound (ii) the end correction (iii) the radius of tube.

Q.9 In an experiment, it was found that a tuning fork and a sonometer gave 5 beats/sec, both when length of wire was 1m and 1.05m. Calculate the frequency of the fork.

Exercise 2

Single Correct Choice Type

Q.1 A firecracker exploding on the surface of lake is heard as two sounds at a time interval t apart by a man on a boat close to water surface. Sound travels with a speed u in water and a speed v in air. The distance from the exploding firecracker to the boat is

(A) $\frac{utv}{u+v}$ (B) $\frac{t(u+v)}{uv}$

(C) $\frac{t(u-v)}{uv}$ (D) $\frac{utv}{u-v}$

Q.2 A sonometer wire has a total length of 1 m between the fixed ends. Two wooden bridges are placed below the wire at a distance 1/7m from one end and 4/7m from the other end. The three segments of the wire have their fundamental frequencies in the ratio:

(A) 1: 2: 3 (B) 4: 2: 1

(C) 1: 1/2: 1/3 (D) 1: 1: 1

Q.3 A person can hear frequencies only up to 10 kHz. A steel piano wire 50 cm long of mass 5 g is stretched with a tension of 400 N. The number of the highest overtone of the sound produced by this piano wire that the person can hear is

(A) 4 (B) 50 (C) 49 (D) 51

Q.4 How many times intense is 90 dB sound than 40dB sound?

(A) 5 (B) 50 (C) 500 (D) 10^5

Q.5 At a prayer meeting, the disciples sing jai-ram jai-ram. The sound amplified by a loudspeaker comes back after reflection from a builder at a distance of 80m from the meeting. What maximum time interval can be kept between one jai-ram and the next jai-ram so that the echo does not disturb a listener sitting in the meeting? Speed of sound in air is 320 ms^{-1}.

(A) 20 Seconds (B) 0.3 Seconds

(C)40 Seconds (D) 0.5 Seconds

Q.6 A man stands before a large wall at a distance of 50.0 m and claps his hands at regular intervals. Initially, the interval is large. He gradually reduces the interval and fixes it at a value when the echo of a clap merges with the next clap 10 times during every 3 seconds. Find the velocity of sound in air.

(A) 420 m/s (B) 333 m/s

(C) 373 m/s (D) 555 m/s

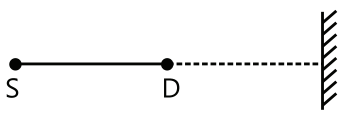
Q.7 Find the minimum and maximum wavelength of sound in water that is in the audible range (20-20000 Hz) for an average human ear. Speed of sound in water =1450 ms^{-1}.

(A) 72.5 m (B) 70.5 m (C) 71.5 m (D) 70.9 m

Q.8 The sound level at a point 5.0 m away from a point source is 40 dB. What will be the level at a point 50 m away from the source?

(A) 25 lb (B) 5 lb (C) 20 db (D) 40 lb

Q.9 A source of sound S and a detector D are placed at some distance from one another. A big cardboard is placed near the detector and perpendicular to the line SD as shown in figure. It is gradually moved away and it is shown that the intensity change from a maximum to a minimum as the board is moved through a distance of 20cm. What will be the frequency of the sound emitted. Velocity of sound in air is 336 ms^{-1}.



(A) 420 Hz (B) 422 Hz (C) 450 Hz (D) 410 Hz

Q.10 Two sources of sound, s_1 and s_2, emitting waves of equal wavelength 20.0 cm, are placed with a separation of 20.0 cm between them. A detector can be moved on a line parallel to s_1 s_2 and at a distance of 20.0cm from it. Initially, the detector is equidistant from the two sources. Assuming that the waves emitted by the sources are in phase, find the minimum distance through which the detector should be shifted to detect a minimum frequency of sound.

(A) 12 cm (B) 24 cm (C) 36 cm (D) 48 cm

Q.11 A cylindrical metal tube has a length of 50 cm and is open at both ends. Find the frequencies between 1000 Hz and 2000 Hz at which the air column in the tube can resonate. Speed of sound in air is 340 ms^{-1}.

(A) 1020 Hz, 1360 Hz, 1700 Hz

(B) 1200 Hz, 1400 Hz, 1700 Hz

(C) 1020 Hz, 1360 Hz, 2000 Hz

(D) 1000 Hz, 1360 Hz, 1800 Hz

Q.12 The first overtone frequency of a closed organ pipe p_1 is equal to the fundamental frequency of an open organ pipe p_2. If the length of the pipe p_1 is 30cm. What will be the length of p_2?

(A) 12 cm (B) 24 cm (C) 20 cm (D) 38 cm

Previous Years' Questions

Q.1 A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5kHz, while the train approaches the siren. During his return journey in a different train B, he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the train B to that of train A is ***(2002)***

(A) 242/252 (B) 2

(C) 5/6 (D) 11/6

Q.2 A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by mass M, the wire resonates with the same tuning fork forming three nodes and antinodes for the same position of the bridges. The value of M is ***(2002)***

(A) 25kg (B) 5kg

(C) 12.5kg (D) 1/25kg

Q.3 In the experiment for the determination of the speed of sound in air using the resonance column method, the length of air column that resonates in the fundamental mode, with a tuning fork is 0.1m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction. ***(2003)***

(A) 0.012m (B) 0.025m

(C) 0.05 m (D) 0.024

Q.4 A source of sound of frequency 600Hz is placed inside water. The speed of sound in water is 1500m/s and in air it is 300m/s. the frequency of sound recorded by an observer who is standing in air is ***(2004)***

(A) 200Hz (B) 3000Hz

(C) 120 Hz (D) 600 Hz

Q.5 A vibrating string of certain length l under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats/s when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased, the number of beats reduces to 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is ***(2008)***

(A) 344 (B) 336

(C) 117.3 (D) 109.3

Q.6 A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/h toward a tall building which reflects the sound waves. The speed of sound in air is 320m/s. The frequency of the siren by the car driver is ***(2011)***

(A) 8.50 kHz (B) 8.25 kHz

(C) 7.75 kHz (D) 7.50 kHz

Q7 Sound waves of frequency 660 Hz fall normally on a perfectly wall. The shortest distance from the wall at which the air particle have maximum amplitude of vibration is..........................m. speed of sound =330m/s. ***(1984)***

Q.8 In a sonometer wire, the tension is maintained by suspending a 50.7kg mass from the free end of the wire. The suspended mass has a volume of $0.0075m^3$. The fundamental frequency of vibration of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become......................Hz. ***(1987)***

Q.9 The ratio of the velocity of sound in hydrogen gas $\left(\gamma = \frac{7}{5}\right)$ to that in helium gas $\left(\gamma = \frac{5}{3}\right)$ at the same temperature is $\sqrt{21/5}$. State whether true or false

(1983)

Q.10 A plane wave of sound travelling in air is incident upon a plane water surface. The angle of incidence is 60^0. Assuming Snell's law to be valid for sound waves, it follows that the sound wave will be refracted into water away from the normal. State whether true or false ***(1984)***

Q.11 A source of sound wave with frequency 256 Hz is moving with a velocity ν towards a wall and an observer is stationary between the source and the wall.

When the observer is between the source and the wall, he will hear beats. State whether true or false ***(1985)***

Q.12. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then ***(2008)***

(A) 18 > x (B) x >54

(C) 54 > x > 36 (D) 36 > x > 18

Q.13 A motor cycle starts from rest and accelerates along a straight path at 2 m/s^2. At the starting point of the motor cycle there is a stationary electric sire. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (speed of sound = 330 ms^{-1}). ***(2009)***

(A) 49 m (B) 98 m

(C) 147 m (D) 196 m

Q14. Three sound waves of equal amplitudes have frequencies (v – 1), v, (v + 1). They superpose to give beats. The number of beats produced per second will be ***(2009)***

(A) 4 (B) 3 (C) 2 (D) 1

Q15. A cylindrical tube, open at both ends, has a fundamental frequency, f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now ***(2012)***

(A) f (B) $\frac{f}{2}$ (C) $\frac{3f}{4}$ (D) 2f

Q.16. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg) ***(2014)***

(A) 16 cm (B) 22 cm

(C) 38 cm (D) 6 cm

Q.17. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to : ***(2015)***

(A) 6% (B) 12% (C) 18% (D) 24%

Q.18 A pipe open at both ends has fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now : ***(2016)***

(A) $\frac{3f}{4}$ (B) 2f (C) f (D) $\frac{f}{2}$

JEE Advanced/Boards

Exercise 1

Q.1 Find the intensity of sound wave whose frequency is 250Hz. The displacement amplitude of particles of the medium at this position is 1×10^{-8} m. The density of the medium is kg/m^3, bulk modulus of elasticity of the medium is 400N/m^2.

Q.2 In a mixture of gases, the average number of degrees of freedom per molecule is 6. The rms speed of the molecules of the gas is c. find the velocity of sound in the gas.

Q.3 The loudness level at a distance R from a long linear source is found to be 40dB. At this point, the amplitude of oscillation of air molecules is 0.01cm. Then find the loudness level & amplitude at a point at distance '10R' from the source.

Q.4 Two identical sounds A and B reach a point in the same phase. The resultant sound is C. The loudness of C is n dB higher than the loudness of A. Find the value of n.

Q.5 Sound of wavelength λ passes through a Quincke's tube which is adjusted to give a maximum intensity I_0.

Find the distance the sliding tube should be moved to give an intensity $I_0/2$.

Q.6 The first overtone of a pipe closed at one end resonates with the third harmonic of a string fixed at its ends. The ratio of the speed of sound to the speed of transverse wave travelling on the string is 2: 1. Find the ratio of the length of pipe to the length of string.

Q.7 An open organ pipe filled with air has a fundamental frequency 500Hz. The first harmonic of another organ pipe closed at one end is filled with carbon dioxide has the same frequency as that the first harmonic of the open organ pipe. Calculate the length of each pipe. Assume that the velocity of sound in air and in carbon dioxide to be 330 and 164m/s respectively.

Q.8 A, B and C are three tuning forks. Frequency of A is 350 Hz. Beats produced by A and B are 5 per second by B and C are 4 per second. When a wax is put on A, beat frequency between A and B is 2Hz and between A and C is 6Hz. Then, find the frequency of B and C respectively.

Q.9 Tuning fork A when sounded with fork B of frequency 480Hz gives 5 beats per second. When the prongs of A are loaded with wax, it gives 3 beats per second. Find the original frequency of A.

Q.10 A car is moving towards a huge wall with a speed=c/10, where c=speed of sound in still air. A wind is also blowing parallel to the velocity of the car in the same direction and with the same speed. If the car sounds a horn of frequency f, then what is the frequency of the reflected sound of the horn headed by driver of the car?

Q.11 A fixed source of sound emitting a certain frequency appears as f_a when the observer is approaching the source with speed v and frequency f_r when the observer recedes from the source with same speed. Find the frequency of the source.

Q.12 Two stationary sources A and B are sounding notes of frequency 680Hz. An observer moves from A to B with a constant velocity u. If the sound is 340 ms^{-1}, what must be the value of u so that he hears 10 beats per second?

Exercise 2

Single Correct Choice Type

Q.1 Two successive resonance frequencies in an open organ pipe are 1944 Hz and 2592 Hz. What will be the length of the tube. The speed of sound in air is 324ms^{-1}.

(A) 20 cm (A) 25 cm (A) 33 cm (A) 16 cm

Q.2 A piston is fitted in a cylindrical tube of small cross section with the other end of the tube open. The tube resonates with a tuning fork of frequency 412 Hz. The piston is gradually pulled out of the tube and it is found that a second resonance occurs when the piston is pulled out through a distance of 320.0cm. What will be the speed of sound in the air of the tube.

(A) 328 m/s (B) 300 m/s

(C) 333 m/s (D) 316 m/s

Q.3 The fundamental frequency of a closed pipe is 293 Hz when the air in it is at a temperature of 20^0C. What will be its fundamental frequency when the temperature changes to 22^0C?

(A) 300 Hz (B) 283 Hz (C) 294 Hz (D) 262 Hz

Q.4 A tuning fork produces 4 beats per second with another tuning fork of frequency 256Hz. The first one is now loaded with a little wax and the beat frequency is found to increase to 6 per second. What was the original frequency of the tuning fork?

(A) 252 Hz (B) 220 Hz

(C) 250 Hz (D) 222 Hz

Q.5 What will be the frequency of beats produced in air when two sources of sound are activated, one emitting a wavelength of 32 cm and the other 32.2 cm. The speed of sound in air is 350ms^{-1}.

(A) 11 Hz (B) 13 Hz (C) 15 Hz (D) 7 Hz

Q6 A traffic policeman standing on a road sounds a whistle emitting a frequency of 2.00 kHz. What could be the apparent frequency heard by a scooter-driver approaching the policeman at a speed of 36.0 kmh^{-1}?

(A) 1181 Hz (B) 1183 Hz

(C) 1185 Hz (D) 1187 Hz

Assertion Reasoning Type

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I

(B) Statement-I is true, statement-II is true and statement-II is not the correct explanation for statement-I

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is correct.

Q.7 Statement-I: When a closed organ pipe vibrates, the pressure of the gas at the closed end remains constant.

Statement-II: In a stationary-wave system, displacement nodes are pressure antinodes, and displacement antinodes are pressure nodes.

Q.8 Statement-I: The pitch of wind instruments rises and that of string instruments falls as an orchestra warms up.

Statement-II: When temperature rises, speed of sound increases but speed of wave in a string fixed at both ends decreases.

Previous Years' Questions

Paragraph 1:

Two plane harmonic sound waves are expressed by the equations $y_1(x,t) = A\cos(\pi x - 100\pi t)$;

(All parameters are in MKS) ***(2006)***

Q.1 How many times does an observer hear maximum intensity in one second?

(A) 4 (B) 10 (C) 6 (D) 8

Q.2 What is the speed of the sound?

(A) 200 m/s (B) 180 m/s

(C) 192 m/s (D) 96m/s

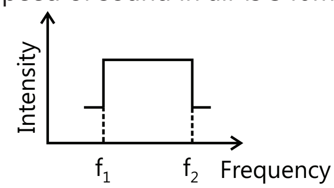
Q.3 At x=0, how many times is the amplitude of y_1+y_2 zero in one second?

(A) 192 (B) 48 (C) 100 (D) 96

Paragraph 2:

Two trains A and B are moving with a speed 20 m/s and 30 m/s respectively in the same direction on the same straight track, with B ahead of A, The engines are at the front ends, The engine of train A blows a long whistle.

Assume that the sound of the whistle is composed of components varying in frequency from f_1=800 Hz to f_2=1120 Hz, as shown in the figure. The spread in the frequency (highest frequency lowest frequency) is thus 320Hz. The speed of sound in air is 340m/s. ***(2007)***



Q.4 The speed of sound of the whistle is

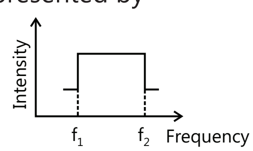
(A) 340m/s for passengers in A and 310 m/s for passengers in B

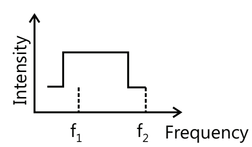
(B) 360m/s for passengers in A and 310 m/s for passengers in B

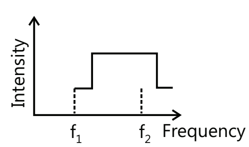
(C) 310 m/s for passengers in A and 360 m/s for passengers in B

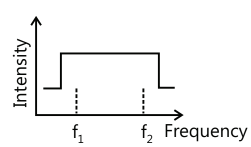
(D) 340 m/s for passenger in both the trains.

Q.5 The distribution of the sound intensity of the whistle as observed by the passenger in train A is best represented by

(A)


(B)


(C)


(D)


Q.6 The spread of frequency as observed by the passenger in train B is

(A) 310 Hz (B) 330 Hz (C) 350 Hz (D) 290 Hz

Q.7 Velocity of sound in air is 320m/s. A pipe closed at one end has a length of 1m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency ***(1989)***

(A) 80 Hz (B) 240 Hz (C) 320 Hz (D) 400 Hz

Q.8 A sound wave of frequency travels horizontally to the right and is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is c. ***(1995)***

(A) The number of waves striking the surface per second is $f\frac{(c+v)}{c}$

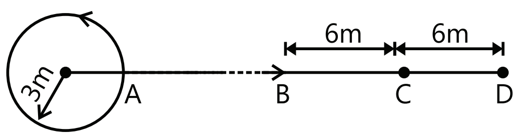
(B) The wavelength of reflected wave is $\frac{c(c-v)}{f(c+v)}$

(C) The frequency of the reflected wave is $f\frac{(c+v)}{(c-v)}$

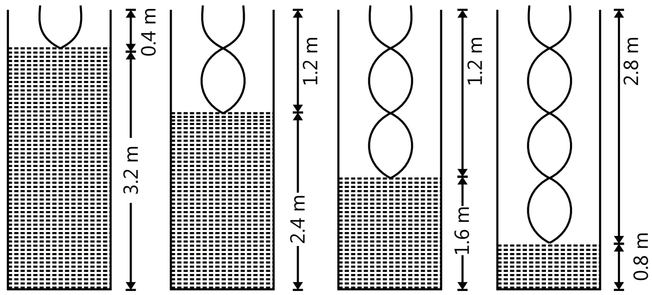
(D) The number of beats heard by a stationary listener to the left of the reflecting surface is $f.\frac{v}{c-v}$

Q.9 A source of sound of frequency 256 Hz is moving rapidly towards a wall with a velocity of 5m/s. How many beats per second will be heard by the observer on source itself if sound travels at a speed of 330 m/s? ***(1981)***

Q.10 A source of sound is moving along a circular path of radius 3m with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD (see figure) with an amplitude BCD=6m. The frequency of an oscillation of the detector is $5/\pi$ per second. The source is at the point A when the detector is at the point B. If the source emits a continuous sound wave of frequency 340 Hz, find the maximum and the minimum frequencies recorded by the detector. (Speed of sound=340 m/s) ***(1990)***



Q.11 A 3.6 m long pipe resonates with a frequency 212.5 Hz when water level is at a certain height in the pipe. Find the heights of water level (from the bottom of the pipe) at which resonances occur. Neglect end correction. Now the pipe is filled to a height H($\approx$ 3.6 m). A small hole is drilled very close to its bottom and water is allowed to leak. Obtain an expression for the rate of fall of water level in the pipe as a function of H. If the radii of the pipe and the hole are 2×10^{-2} m and 1×10^{-3} m respectively, calculate the time interval between the occurrence of first two resonances. Speed of sound in air is 340 m/s and $g=10 m/s^2$ ***(2000)***



Q.12 An observer standing on a railway crossing receives frequency of 2.2 kHz and 1.8 kHz when the train approaches and recedes from the observer. Find the velocity of the train. ***(2005)***

(The speed of the sound in air is 300 m/s)

Q.13 A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speed of the cars (in km per hour) to the nearest? The cars are moving at constant speeds much smaller than the speed of sound which is $330 ms^{-1}$. ***(2010)***

Q.14 A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is ***(2011)***

(A) 8.50 kHz (B) 8.25 kHz

(C) 7.75 kHz (D) 7.50 kHZ

Q.15 A person blows into open-end of a long pipe. As a result, a high pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe, ***(2012)***

(A) A high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

(B) A low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

(C) A low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

(D) A high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

Q.16 A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005)m, the gas in the tube is

(Useful information: $\sqrt{167RT}$ = 640 $J^{1/2}mole^{-1/2}$; $\sqrt{140RT}$ =590 $J^{1/2}mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.) ***(2014)***

(A) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

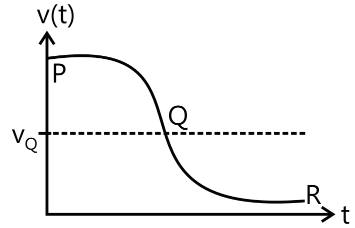
(B) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(C) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(D) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

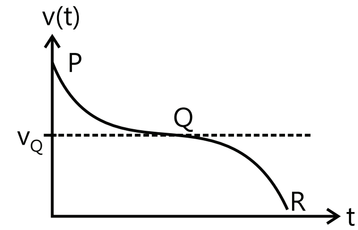
Q.17 Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let v(t) represent the beat frequency measured by a person sitting in the car at time t. Let vP, vQ and vR be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 m/s. Which of the following statement(s) is(are) true regarding the sound heard by the person? ***(2016)***

(A) The plot below represents schematically the variation of beat frequency with time



(B) $v_P + v_R = 2_{vQ}$

(C) The plot below represents schematically the variation of beat frequency with time



(D) The rate of change in beat frequency is maximum when the car passes through Q.

Q.18 A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air column is the second resonance. Then, ***(2009)***

(A) The intensity of the sound heard at the first resonance was more than that at the second resonance

(B) The prongs of the tuning fork were kept in a horizontal plane above the resonance tube

(C) The amplitude of vibration of the ends of the prongs is typically around 1 cm

(D) The length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air

Q.19. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is ***(2010)***

(A) 5 grams (B) 10 grams

(C) 20 grams (D) 40 grams

Q.20 A student is performing the experiment of Resonance Column. The diameter of the column tube is 4 cm. The distance frequency of the tuning for k is 512 Hz. The air temperature is 38°C in which the speed of sound is resonance occurs, the reading of the water level in the column is ***(2012)***

(A) 14.0 (B) 15.2 (C) 16.4 (D) 17.6

Q.21 Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are) ***(2013)***

(A) If the wind blows from the observer to the source, $f_2 > f_1$.

(B) If the wind blows from the source to the observer, $f_2 > f_1$.

(C) If the wind blows from observer to the source, $f_2 < f_1$.

(D) If the wind blows from the source to the observer $f_2 < f_1$.

Q.22 Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\pi/3$, $2\pi/3$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is ***(2015)***

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q. 6 Q.7 Q.8

Exercise 2

Q. 1 Q.2 Q.3

Q.12 Q.13 Q.14

Q.19 Q.21

JEE Advanced/Boards

Exercise 1

Q. 1 Q.6 Q.8

Q.12

Exercise 2

Q.1 Q.2 Q.3

Q.8 Q.14 Q.15

Q.16 Q.24 Q.26

Answer Key

JEE Main/Boards

Exercise 1

Q.1 382.2 ms^{-1} **Q.2** (i) 0.25×10^{-3} cm (ii) $\pi/250$ s (iii) 500 rad (iv) 80π m (v) 0.125 cm/s (iv) 62.5 cm/sec^2

Q.3 353.6 ms^{-1}

Q.4 (a) No wave (b) longitudinal waves (c) longitudinal (d) transverse or longitudinal or both (separately) (e) combined longitudinal and transverse (ripples)

Q.5 The speed of sound in a perfectly rigid rod will be infinite

Q.6 4.2 s **Q.7** 1.32 km **Q.8** 330 ms^{-1}, 0.02 m; 0.033 m **Q.9** 205 Hz

Exercise 2

Single correct choice type

Q.1 D **Q.2** B **Q.3** C **Q.4** D **Q.5** D **Q.6** B

Q.7 A **Q.8** C **Q.9** A **Q.10** A **Q.11** A **Q.12** C

Previous Years' Questions

Q.1 B **Q.2** A **Q.3** B **Q.4** D **Q.5** A **Q.6** A

Q.7 0.125 **Q.8** 240 **Q.9** False **Q.10** True **Q.11** False **Q.12** B

Q.13 B **Q.14** C **Q.15** A **Q.16** A **Q.17** B **Q.18** C

JEE Advanced/Boards

Exercise 1

Q.1 $\frac{\pi^2 \times 10^{-9}}{4} W/m^2$ **Q.2** 2C/3 **Q.3** 30 dB, $10\sqrt{10}$ μm

Q.4 6 **Q.5** $\lambda/8$ **Q.6** 1:1

Q.7 33 cm and 13.2 cm **Q.8** 345, 341 or 349 Hz **Q.9** 485 Hz

Q.10 11f/9 **Q.11** $\frac{f_r + f_a}{2}$ **Q.12** 2.5 ms^{-1}

Exercise 2

Single Correct Choice Type

Q.1 B **Q.2** A **Q.3** C **Q.4** A **Q.5** D **Q.6** A

Assertion Reasoning Type

Q.7 D **Q.8** A

Previous Years' Questions

Q.1 A **Q.2** A **Q.3** C **Q.4** B **Q.5** A **Q.6** A

Q.7 A, B, D **Q.8** A, B, C **Q.9** 7.87 Hz **Q.10** 438.7 Hz, 257.3 Hz

Q.11 3.2 m, 2.4 m, 1.6 m, 0.8 m, $-\frac{dH}{dt} = \left(1.11 \times 10^{-2}\right)\sqrt{H}, 43s$ **Q.12** v_T= 30 m/s

Q.13 7 **Q.14** A **Q.15** B, D **Q.16** D **Q.17** A, B, D **Q.18** A, D

Q.19 B **Q.20** B **Q.21** A, B **Q.22** 3

Solutions

JEE Main/Boards

Exercise 1

Sol 1: $V' = \sqrt{\frac{4}{3}}\,V = \sqrt{\frac{4}{3}} \times 331$

$= 382.2\ ms^{-1}$

Sol 2: (i) $A = 0.25 \times 10^{-3}$ cm

(ii) $T = \frac{2\pi}{500} = \frac{\pi}{250}$ s

(iii) ω : 500 rad/s

(iv) $\lambda = \frac{2\pi}{0.025}$ m $= 80\pi$m

(v) $V_{max} = 0.25 \times 10^{-3} \times 500\ cm\ s^{-1}$

$V_{max.} = 0.125\ cms^{-1}$

(vi) $a_{max.} = V_{max}\,w = 0.125 \times 500$

$a_{max.} = 62.5\ cms^{-2}$

Sol 3: $\frac{V}{2.04} - \frac{V}{2.08} = \frac{20}{6}$

$V\left(\frac{0.04}{2.04 \times 2.05}\right) = \frac{20}{6}$

$V = 353.6\ ms^{-1}$

Sol 4: (a) No wave possible as there is no particle.

(b) Longitudinal waves (direction of motion of particles parallel to direction of propagation of wave)

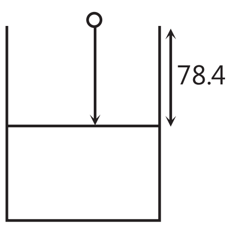
(c) Longitudinal

(d) Both are possible

(e) Combined longitudinal & transverse (ripples)

Sol 5: Infinite as young's modulus of a rigid body is infinite

Sol 6:

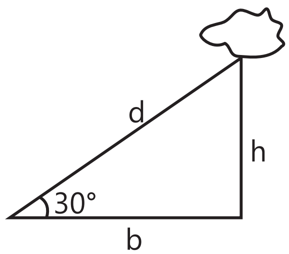


Time to reach water $= \sqrt{\frac{2 \times 78.4}{9.8}} = 4$ s

Time for sound to reach top $= \frac{78.4}{332} = 0.23$ s

Total time = 4.23 s

Sol 7:



$\Rightarrow d\left(\frac{1}{330} - \frac{1}{3 \times 10^8}\right) = 8$

$\Rightarrow d\,(3\times 10^8 - 330) = 8 \times 330 \times 3 \times 10^8$

$\Rightarrow d \cong 8 \times 330$

$\Rightarrow d = 264$ m

Height of cloud = 1320 m = 1.32 km

Sol 8: f = 250 Hz

$\Rightarrow (31 + h) = \frac{\lambda}{2}$

$\Rightarrow (97 + h) = \frac{3\lambda}{4}$

$\Rightarrow 66 = \frac{\lambda}{2}$

$\lambda = 132$ cm

$V = f\lambda = 250 \times 1.32\ ms^{-1}$

$V = 330\ ms^{-1}$

H = 132/4-31 = 2 cm = 0.02 m

Radius of tube $= \frac{\text{End Cross Section}}{0.6}$

$= \frac{0.02}{0.6} = \frac{0.2}{6} = \frac{0.1}{3} = 0.033$ m

Sol 9: $\frac{V}{2} - f = 5$

$F - \frac{V}{2.1} = 5$

$$\frac{V}{2}-\frac{V}{2.1} = 10$$

$V = 420\ ms^{-1}$

$$F = 5 + \frac{420}{2.1} = 205\text{ Hz}$$

Exercise 2

Single Correct Choice Type

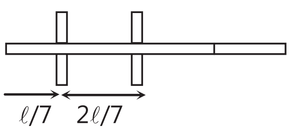
Sol 1: (D) $d = u\,t_0$

$\Rightarrow d = v(t_0 + t)$

$\Rightarrow (v-u)t_0 + vt = 0$

$$t_0 = \frac{vt}{u-v}$$

$$d = \frac{uvt}{u-v}$$

Sol 2: (B) 

λ 2l 4l

$$\ell = \frac{7\lambda}{2}$$

$$\lambda = \frac{2\ell}{7}$$

λ ratio : 1 : 2 : 4

ν ratio : 4 : 2 : 1

Sol 3: (C) $\mu = \dfrac{5\times10^{-3}}{0.5} = 0.01$

T = 400 N

$$\nu = \frac{(n+1)}{2\times0.5}\sqrt{\frac{400}{0.01}}$$

$\nu = 200(n + 1) < 10^4$

$\Rightarrow (n + 1) < 50$

$\Rightarrow n < 49$

Sol 4: (D) $10\log\left(\dfrac{I_2}{I_1}\right) = 50$

$I_2 = I_1 \times 10^5$

Sol 5: (D) Here given S=80m x 2=160m.

V=320m/s

So the maximum time interval will be

T=5/v=160/320=0.5seconds.

Sol 6: (B) He has to clap 10 times in 3 seconds.

So time interval between two clap =(3/10 second).

So the time taken go the wall

=(3/2 x 10)=3/20 seconds =333 m/s.

Sol 7: (A) For minimum wavelength n=20 KHZ

$$\Rightarrow v = n\lambda \Rightarrow \lambda = \left(\frac{1450}{20\times10^3}\right) = 7.25\text{cm.}$$

(b) For maximum wavelength n should be minimum

$\Rightarrow v = n\lambda \Rightarrow \lambda = v/n \Rightarrow 1450/20 = 72.5\text{m.}$

Sol 8: (C) We know that $\beta = 10\log_{10}\left(\dfrac{I}{I_0}\right)$

$$\beta_A = 10\log\frac{I_A}{I_0}, \beta_B = 10\log\frac{I_B}{I_0}$$

$$\Rightarrow I_A/I_0 = 10^{(\beta_A/10)} \Rightarrow I_B/I_0 = 10^{(\beta_B/10)}$$

$$\Rightarrow \frac{I_A}{I_B} = \frac{r_B^2}{r_A^2} = \left(\frac{50}{5}\right)^2 \Rightarrow 10^{(\beta_A\beta_B)=10^2}$$

$$\Rightarrow \frac{\beta_A-\beta_B}{10} = 2 \Rightarrow \beta_B = 40-20 = 20d\beta$$

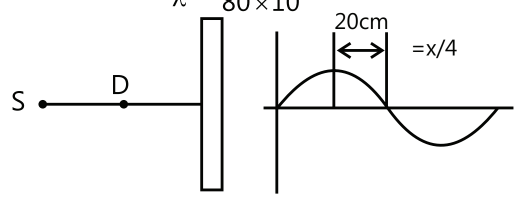
Sol 9 (A) According to the given data

V=336m/s,

$\lambda/4$ =distance between maximum and minimum intensity

$= (20\,\text{cm}) \Rightarrow \lambda = 80\,\text{cm}$

$$\Rightarrow n = \text{frequency} = \frac{V}{\lambda} = \frac{336}{80\times10^{-2}} = 420\text{Hz.}$$



Sol 10: (A) According to the data

$\lambda = 20\text{cm}, S_1S_2 = 20\text{cm}, BD = 20\text{cm}$

Let the detector is shifted to left for a distance x for hearing the minimum sound.

So path difference AI=BC-AB

$$=\sqrt{(20)^2+(10+x)^2}-\sqrt{(20)^2+(10-x)^2}$$

So the minimum distances hearing for minimum

$$=\frac{(2n+1)\lambda}{2}=\frac{\lambda}{2}=\frac{20}{2}=10\text{cm}$$

$$\Rightarrow\sqrt{(20)^2+(10+x)^2}-\sqrt{(20)^2+(10-x)^2}=10$$

Solving we get x=12. 0 cm.

Sol 11: (A) Here given that 1=50cm, v=340m/s

As it is an open organ pipe, the fundamental frequency f_1 =(v/21)

$$=\frac{340}{2\times50\times10^{-2}}=340\text{Hz}.$$

So, the harmonies are

f_3=3 x 340=1020 Hz

f_5=5 x 340=1700, f_6=6 x 340=2040 Hz

So, the possible frequencies are between 1000Hz and 2000Hz are 1020, 1360, 1700.

Sol 12: (C) According to the questions f_1 first overtone of a closed organ pipe

$$P_1=3v/4I=\frac{3\times V}{4\times30}$$

f_2 fundamental frequency of a open organ pipe $P_2=\frac{V}{2I_2}$

Here given $\frac{3V}{4\times30}=\frac{V}{2I_2}\Rightarrow I_2=20\text{cm}$

$\therefore$ Length of the pipe P_2 will be 20 cm.

Previous Years' Questions

Sol 1: (B) Using the formula f' = f $\left(\frac{v+v_0}{v}\right)$

we get, $5.5=5\left(\frac{v+v_A}{v}\right)$ ..... (i)

and $6.0=5\left(\frac{v+v_B}{v}\right)$ ..... (ii)

Hence, v = speed of sound

v_A = speed of train A

v_B = speed of train B

Solving Eqs. (i) and (ii), we get

$$\frac{v_B}{v_A}=2$$

Sol 2: (A) Let f_0 = frequency of tuning fork

Then, $f_0=\frac{5}{2\ell}\sqrt{\frac{9g}{\mu}}$ (μ = mass per unit length of wire)

$$=\frac{3}{2\ell}\sqrt{\frac{Mg}{\mu}}$$

Solving this, we get M = 25 kg

In the first case, frequency corresponds to fifth harmonic while in the second case it corresponds to third harmonic

Sol 3: (B) Let $\Delta\ell$ be the end correction.

Given that, fundamental tone for a length 0.1 m = first overtone for the length 0.35 cm.

$$\frac{v}{4(0.1+\Delta\ell)}=\frac{3v}{4(0.35+\Delta\ell)}$$

Solving this equation, we get $\Delta\ell$ = 0.025 m = 2.5 cm

Sol 4: (D) The frequency is a characteristic of source. It is independent of the medium.

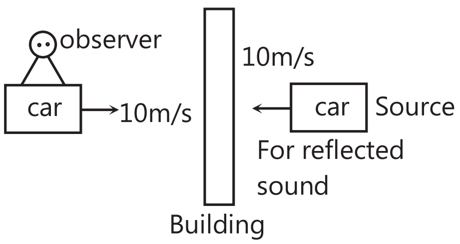
Sol 5: (A) With increase in tension, frequency of vibrating string will increase. Since number of beats are decreasing. Therefore, frequency of vibrating string or third harmonic frequency of closed pipe should be less than the frequency of tuning fork by 4.

$\therefore$ Frequency of tuning fork

= Third harmonic freq1uency of closed pipe + 4

$$=3\left(\frac{v}{4\ell}\right)+4=3\left(\frac{340}{4\times0.75}\right)+4=344\text{ Hz}$$

Sol 6: (A) 36 km/h = $36\times\frac{5}{18}$ = 10 m/s



Apparent frequency of sound heard by car driver (observer) reflected from the building will be

$$f' = f\left(\frac{v+v_0}{v-v_s}\right) = 8\left(\frac{320+10}{320-10}\right) = 8.5 \text{ kHz}$$

Sol 7: Wall will be a node (displacement). Therefore, shortest distance from the wall at which air particles have maximum amplitude of vibration (displacement antinode) should be $\lambda/4$

Here, $\lambda = \frac{v}{f} = \frac{330}{660} = 0.5$ m

$\therefore$ Desired distance is $\frac{0.5}{4} = 0.125$ m

Sol 8: Fundamental frequency $f = \frac{v}{2\ell} = \sqrt{\frac{T\ell\mu}{2\ell}}$

or $f \propto \sqrt{T}$

$$\frac{f'}{f} = \sqrt{\frac{w-F}{w}}$$

Here, w = weight of mass and

F = upthrust

$$f' = f\sqrt{\frac{w-F}{w}}$$

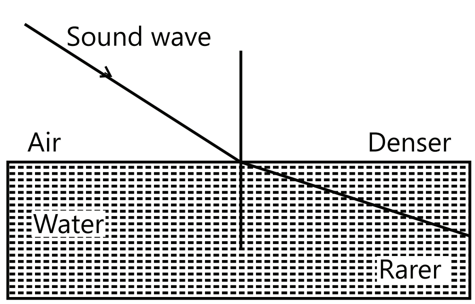
Substituting the values, we have

$$f' = 260\sqrt{\frac{(50.7)g-(0.0075)(10^3)g}{(50.7)g}} = 240 \text{ Hz}$$

Sol 9: $v_{sound} = \sqrt{\frac{\gamma RT}{M}}$

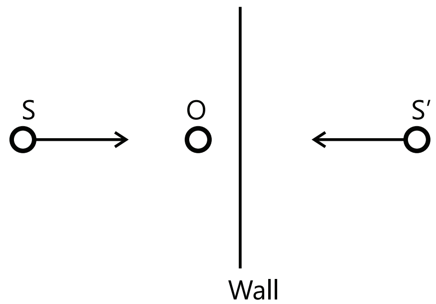
$$\frac{v_{H_2}}{v_{He}} = \frac{\sqrt{\gamma_{H_2}/M_{H_2}}}{\sqrt{\gamma_{He}/M_{He}}} = \frac{\sqrt{(7/5)/2}}{\sqrt{(5/3)/4}} = \frac{42}{25}$$

Sol 10:



For sound wave water is rarer medium because speed of sound wave in water is more. When a wave travels from a denser medium to rarer medium it refracts away from the normal

Sol 11: For reflected wave an image of source S′ can assumed as shown. Since, both S and S′ are approaching towards observer, no beats will be heard



Sol 12: (B)

$$n = \frac{1}{4x}\sqrt{\frac{\gamma RT}{M}} \Rightarrow xn = \frac{1}{4}\sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow x \propto \sqrt{T}$$

Sol 13: (B) Motor cycle, u = 0, a = 2 m/s^2

Observer is in motion and source is at rest.

$$\Rightarrow n' = n\frac{v-v_0}{v+v_s} \Rightarrow \frac{94}{100}n = n\frac{330-v_0}{330} \Rightarrow 330 - v_0$$

$$= \frac{330\times 94}{100}$$

$$\Rightarrow v_0 = 330 - \frac{94\times 33}{10} = \frac{33\times 6}{10} m/s$$

$$s = \frac{v^2-u^2}{2a} = \frac{9\times 33\times 33}{100} = \frac{9\times 1089}{100} \simeq 98m$$

Sol 14: (C) Maximum number of beats = $\nu + 1 - (\nu - 1)$ = 2

Sol 15: (A) $f_0 = \frac{v}{2\ell}$, $f_C = \frac{v}{2\ell}$

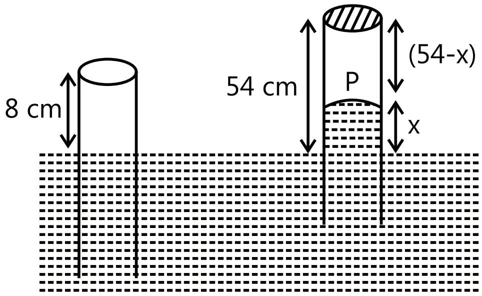
Sol 16: (A) P + x = P_0

P = (76 – x)

8 × A × 76 = (76 – x) × A × (54 – x)

x = 38

Length of air column = 54 – 38 = 16 cm.



Sol 17: (B)

$$f_{\text{before crossing}} = f_0\left(\frac{c}{c-v_s}\right) = 1000\left(\frac{320}{320-20}\right)$$

$$f_{\text{after crossing}} = f_0\left(\frac{c}{c+v_s}\right) = 1000\left(\frac{320}{320+20}\right)$$

$$\Delta f = f_0\left(\frac{2cv_s}{c^2-v_s^2}\right)$$

$$\frac{\Delta f}{f}\times 100\% = \frac{2\times 320\times 20}{300\times 340}\times 100 = 12.54\% \approx 12\%$$

Sol 18: (C) Open organ pipe

$$f = \frac{V}{2\ell} \qquad \text{...(i)}$$

For closed organ pipe

$$f' = \frac{V}{4\left(\frac{\ell}{2}\right)} = \frac{V}{2\ell} = f$$

JEE Advanced/Boards

Exercise 1

Sol 1: f = 250 Hz $V = \sqrt{\frac{B}{\rho}}$ = 20 ms^{-1}A = 10^{-8}m

ρ = 1 kg/m^3

B = 400 N/m^2

$$p_0 = \frac{B\omega S_0}{V} = \frac{400\times 2\pi\times 250\times 10^{-8}}{20}$$

p_0 = 3.14 × 10^{-4} N/m^2

$$I = \frac{p_0^2}{2\rho V} = 2.467\times 10^{-9}\ \text{W/m}^2$$

Intensity = 2.467 × 10^{-9} W/m^2

Sol 2: $V = \sqrt{\frac{\gamma RT}{M}}$

$$\sqrt{\frac{RT}{M}} = \sqrt{\frac{C}{\sqrt{3}}}$$

$$\gamma = 1 + \frac{2}{6} = \frac{4}{3}$$

$$\nu = \sqrt{\gamma}\times\frac{C}{\sqrt{3}} = \sqrt{\frac{4}{3}}\times\frac{C}{\sqrt{3}}$$

$$\nu = \frac{2}{3}C$$

Sol 3: For linear source, Intensity $\propto \frac{1}{R}$

$$A \propto \frac{1}{R^{1/2}}$$

$\therefore$ At 10R

$$\text{Loudness} = 10\log\frac{I/10}{I_0} = 40\ \text{dB} - 10\ \text{dB}$$

Loudness = 30 dB

$$\text{Amplitude} = \frac{0.01}{\sqrt{10}}\ \text{cm} = 10\sqrt{10}\ \mu\text{m}$$

Sol 4: I′ = 4I

$$\text{Loudness} = 10\log\frac{4I}{I_0} = 10(\log 4 + L_0)$$

$= 20\log 2 + L_0 = 6.010 + L_0$

$= 6.010 + L_0 = L_0 + 6.01$ dB

n = 6.01 dB

Sol 5: $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Here $I_1 = I_2$

$I = 2I_1(1 + \cos\phi)$

$I_0 = 4I_1$

$I_0/2 = 2I_1 = 2I_1(1 + \cos\phi)$

$\cos\phi = 0 \Rightarrow \phi = \frac{\pi}{2}$

$$\Rightarrow \phi = 2\pi\,\frac{(2\Delta x)}{\lambda}$$

$$\Rightarrow \Delta x = \frac{\frac{\pi}{2}\times\lambda}{2\pi\times 2} = \frac{\lambda}{8}$$

Sol 6: $\dfrac{V_p}{V_s} = 2$

$\dfrac{3}{4\ell_p} V_p = \dfrac{3}{2\ell_p} V_s$

$\Rightarrow \dfrac{\ell_p}{\ell_s} = \dfrac{V_p}{2V_s} = 1$

$\Rightarrow \lambda_p : \lambda_s = 1 : 1$

Sol 7: $500 = \dfrac{\nu_A}{\lambda_0}$

Closed pipe: $\lambda_0 = \dfrac{330}{500} = 2\lambda_1$

$\lambda_1 = \dfrac{330}{1000} \text{m} = 0.33\text{m}$

$\lambda_1 = 0.33$ m

Open pipe: $4\lambda_2 = \dfrac{264}{500}$

$\lambda_2 = 0.132$ m

Sol 8: $f_A = 350$ Hz

$|f_A - f_B| = 5$ Hz

$|f_B - f_C| = 4$ Hz

After waxing

$|f_A^1 - f_B| = 2$Hz

$|f_A^1 - f_c| = 6$ Hz

$f_A > f_B$ initially as on waxing f_A decreases.

$f_A - f_B$ 5Hz $\Rightarrow f_B = 345$ Hz

Case-I : $F_B > F_C$ $F_B - F_C = 4$ Hz $\Rightarrow f_C = 341$ Hz

$f_A^1 = 347$ Hz or 343 Hz

$f_C = 341$ Hz

$f_A^{'} = 347$ Hz

$f_B = 345$ Hz

$f_C = 341$ Hz

Case-II : $f_C > f_B$

$f_C = 349$ Hz

$f_A^{'} = 343$ Hz

$f_B = 345$ Hz

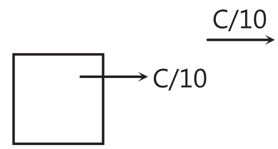
Sol 9: $f_B = 480$ Hz

$|f_B - f_A|$ decreases on waxing

$\therefore f_A > f_B$

$f_A = 485$ Hz

Sol 10:



$f_w = \dfrac{(C + C/10)}{(C + C/10) - C/10} \times f_0$

$f_w = \dfrac{11}{10} f_0$

$f_d = f_w \times \dfrac{\left(C - \dfrac{C}{10}\right) + \dfrac{C}{10}}{\left(C - \dfrac{C}{10}\right)}$

$= \dfrac{10}{9} f_w$

$= \dfrac{10}{9} \times \dfrac{11}{10} f_0$

$\Rightarrow f_d = \dfrac{11}{9} f_0$

Sol 11: $f_a = \dfrac{C + \nu}{C} f$

$f_r = \dfrac{C - \nu}{C} f$

$\Rightarrow f = \dfrac{f_a + f_r}{2}$

Sol 12: $f\left(\dfrac{C+u}{C} - \left(\dfrac{C-u}{C}\right)\right) = 10$

$\dfrac{2fu}{C} = 10$

$u = \dfrac{5C}{f} \Rightarrow u = \dfrac{5 \times 340}{680}$

$\Rightarrow u = 2.5 \text{ ms}^{-1}$

Exercise 2

Single Correct Choice Type

Sol 1: (B) Let the length of the resonating column will be=1

Here V=320 m/s

Then the two successive resonance frequencies are

$\frac{(n+1)v}{4l}$ and $\frac{nv}{4l}$

Here given $\frac{(n+1)v}{4l} = 2592; \lambda = \frac{nv}{4l} = 1944$

$\Rightarrow \frac{(n+1)v}{4l} - \frac{nv}{4l} = 2592 - 1944$

$= 548cm = 25cm.$

Sol 2: (A) Let, the piston resonates at length l_1 and l_2 Here, l=32cm; v=?,n=512 Hz

Now $\Rightarrow 512 = v/\lambda \Rightarrow v = 512 \times 0.64 = 328 m/s$

Sol 3: (C) We know that the frequency = f, T = temperatures

$f \propto \sqrt{T}$

So $\frac{f_1}{f_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$; $\Rightarrow \frac{293}{f_2} = \frac{\sqrt{293}}{\sqrt{295}}$

$\Rightarrow f_2 = \frac{293 \times \sqrt{295}}{\sqrt{293}} = 294$ Hz

Sol 4: (A) A tuning fork produces 4 beats with a known tuning fork whose frequency =256 Hz

So the frequency of unknown tuning fork=either 256-4=252 or 256+4=260 Hz

Now as the first one is load its mass/unit length increases. So, its frequency decreases.

As it produces 6 beats now original frequency must be 252 Hz.

260 Hz is not possible as on decreasing the frequency the beats decreases which is not allowed here.

Sol 5: (D)

Group I	Group II
Given V=350	V=350
$\lambda_1 = 32cm = 32 \times 10^{-2} m$	$\lambda_2 = 32.2cm = 32.2 \times 10^{-2} m$
So $\eta_2 = 350/32 \times 10^{-2}$ = 1093 Hz	$\eta_2 = 350/32.2 \times 10^{-2}$ = 1086 Hz

So beat frequency =1093-1086=7 Hz.

Sol 6: (A) Here given $f_s = 16 \times 10^3 Hz$

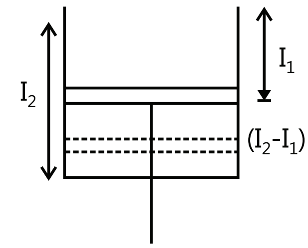
Apparent frequency $f' = 20 \times 10^3 Hz$ (greater than that value)

Let the velocity of the observer $=v_0$

Given v_s=0. So,

$$20 \times 10 = \left(\frac{330 + v_0}{330 + 0}\right) \times 16 \times 10^3$$

$$\Rightarrow v_0 = \frac{20 \times 330 - 16 \times 330}{4} = \frac{330}{4} m/s = 297 km/h$$



Assertion Reasoning Type

Sol 7: (D) Closed end is displacement node. So, it must be pressure antinode.

Sol 8: (A) Statement-II explains statement-I

Previous Years' Questions

Sol 1: (A) In one second number of maximas is called the beat frequency.

Hence, $f_b = f_1 - f_2 = \frac{100\pi}{2\pi} - \frac{92\pi}{2\pi} = 4$ Hz

Sol 2: (A) Speed of wave $v = \frac{\omega}{k}$

or $v = \frac{100\pi}{0.5\pi}$ or $\frac{92\pi}{0.46\pi} = 200$ m/s

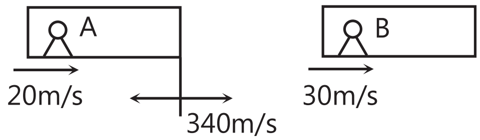
Sol 3: (C) At x = 0, y = $y_1 + y_2$ = 2A cos 96πt cos 4 πt

Frequency of cos (96 πt) function is 45 Hz and that of cos (4πt) function is 2Hz.

In one second, cos function becomes zero at 2f times, where f is the frequency. Therefore, first function will become zero at 96 times and the second at 4 times. But second will not overlap with first. Hence, net y will become zero 100 times in 1 s.

Sol 4: (B) $v_{SA} = 340 + 20 = 360$ m/s

$v_{SB} = 340 - 30 = 310$ m/s



Sol 5: (A) For the passengers in train A. There is no relative motion between source and observer, as both are moving with velocity 20 m/s. Therefore, there is no change in observed frequencies and correspondingly there is no change in their intensities.

Sol 6: (A) For the passengers in train B, observer is receding with velocity 30 m/s and source is approaching with velocity 20 m/s.

$$f_1' = 800\left(\frac{340-30}{340-20}\right) = 775 \text{ Hz}$$

and $$f_2' = 1120\left(\frac{340-30}{340-20}\right) = 1085 \text{ Hz}$$

$\therefore$ Spread of frequency = $f_2' - f_1' = 310$ Hz

Sol 7: (A, B, D) For closed pipe, $f = n\left(\frac{v}{4\ell}\right)$; n = 1, 3, 5 ....

For n = 1, $f_1 = \frac{v}{4\ell} = \frac{320}{4\times 1} = 80$ Hz

For n = 3, $f_3 = 3f_1 = 240$ Hz

For n = 5, $f_5 = 5f_1 = 400$ Hz

Sol 8: (A, B, C) Moving plane is like a moving observer. Therefore, number of waves encountered by moving plane.

$$f_1 = f\left(\frac{v+v_0}{v}\right) = f\left(\frac{c+v}{c}\right)$$

Frequency of reflected wave,

$$f_2 = f_1\left(\frac{v}{v-v_s}\right) = f\left(\frac{c+v}{c-v}\right)$$

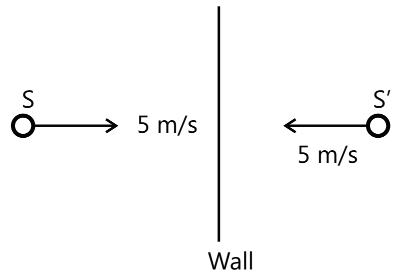
Wavelength of reflected wave

$$\lambda_2 = \frac{v}{f_2} = \frac{c}{f_2} = \frac{c}{f}\left(\frac{c-v}{c+v}\right)$$

Sol 9: Frequency heard by the observer due to S' (reflected wave)

$$f' = f\left(\frac{v+v_0}{v-v_s}\right)$$

$$= 256\left(\frac{330+5}{330-5}\right) = 263.87 \text{ Hz}$$

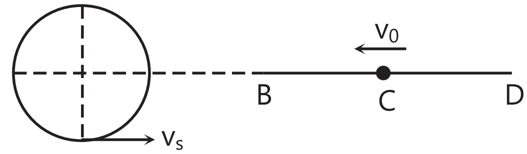


$\therefore$ Beat frequency $f_b = f' - f = 7.87$ Hz

Sol 10: Angular frequency of detector

$$\omega = 2\pi f = 2\pi\left(\frac{5}{\pi}\right) = 10 \text{ rad/s}$$

Since, angular frequency of source of sound and of detector are equal, their time periods will also be equal.



Maximum frequency will be heard in the position shown in figure. Since, the detector is far away from the source, we can use,

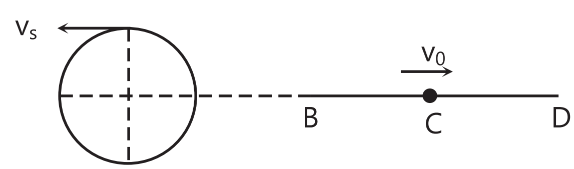
$$f_{max} = f\left(\frac{v+v_0}{v-v_s}\right)$$

Here, v = speed of sound = 340 m/s

(given) $v_s = R\omega = 30$ m/s

$v_o = \omega A = 60$ m/s

$$\therefore f_{max} = 340\frac{(340+60)}{(340-30)} = 438.7 \text{ Hz}$$



Minimum frequency will be heard in the condition shown in figure. The minimum frequency will be:

$$f_{min} = f\left[\frac{v-v_0}{v+v_s}\right] = 340\frac{(340-60)}{(340+30)} = 257.3 \text{ Hz}$$

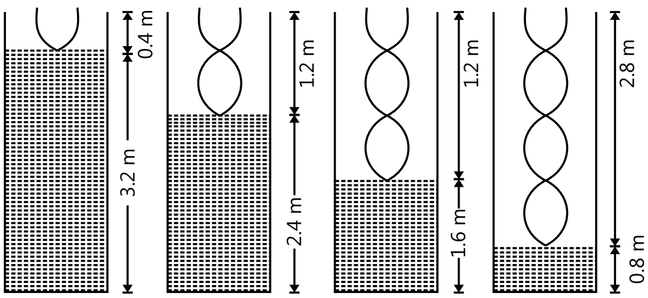
Sol 11: Speed of sound v = 340 m/s

Let λ_0 be the length of air column corresponding to the fundamental frequency. Then,

$$\frac{v}{4\ell_0} = 212.5$$

$$\text{or } \lambda_0 = \frac{v}{4(212.5)} = \frac{340}{4(212.5)} = 0.4 \text{ m}$$

In closed pipe only odd harmonics are obtained. Now let $\lambda_1, \lambda_2, \lambda_3, \ell_4$, etc., be the lengths corresponding to the 3rd harmonic, 4th harmonic, 7th harmonic etc. Then



$$3\left(\frac{v}{4\ell_1}\right) = 212.5 \Rightarrow \lambda_1 = 1.2 \text{ m}$$

$$5\left(\frac{v}{4\ell_2}\right) = 212.5 \Rightarrow \lambda_2 = 2.0 \text{ m}$$

$$\text{and } 7\left(\frac{v}{4\ell_3}\right) = 212.5 \Rightarrow \lambda_3 = 2.8 \text{ m}$$

$$9\left(\frac{v}{4\ell_4}\right) = 212.5 \Rightarrow \ell_4 = 3.6 \text{ m}$$

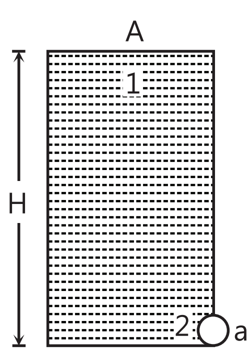
or heights of water level are (3.6 – 0.4) m, (3.6 – 1.2) m, (3.6 – 2.0)m and (3.6 – 2.8)m.

$\therefore$ Heights of water level are 3.2 m, 2.4 m, 1.6 m and 0.8 m

Let A and a be the area of cross-sections of the pipe and hole respectively. Then

$A = \pi(2 \times 10^{-2})^2 = 1.26 \times 10^{-3}\ \text{m}^2$

and $a = \pi(10^{-3})^2 = 3.14 \times 10^{-6}\ \text{m}^2$



Velocity of efflux, $v = \sqrt{2gH}$

Continuity equation at 1 and 2 gives

$$a\sqrt{2gH} = A\left(\frac{-dH}{dt}\right)$$

$\therefore$ Rate of fall of water level in the pipe,

$$\left(\frac{-dH}{dt}\right) = \frac{a}{A}\sqrt{2gH}$$

Substituting the values, we get

$$\frac{-dH}{dt} = \frac{3.14\times10^{-6}}{1.26\times10^{-3}}\sqrt{2\times10\times H}$$

$$\text{or } -\frac{dH}{dt} = (1.11 \times 10^{-2})\sqrt{H}$$

Between first two resonances, the water level falls from 3.2 m to 2.4 m.

$$\therefore \frac{dH}{\sqrt{H}} = -(1.11 \times 10^{-2})\, dt$$

$$\text{or } \int_{3.2}^{2.4}\frac{dH}{\sqrt{H}} = -(1.11 \times 10^{-2})\int_0^1 dt$$

$$\text{or } 2[\sqrt{2.4} - \sqrt{3.2}] = -(1.11 \times 10^{-2})t$$

or t = 43 s

Note: Rate of fall of level at a height h is

$$\left(\frac{-dh}{dt}\right) = \frac{a}{A}\sqrt{2gh} \propto \sqrt{h}$$

i.e., rate decreases as the height of water (or any other liquid) decreases in the tank. That is why, the time required to empty the first half of the tank is less than the time required to empty the rest half of the tank.

Sol 12: From the relation, $f' = f\left(\frac{v}{v \pm v_s}\right)$,

$$\text{we have } 2.2 = f\left[\frac{300}{300 - v_T}\right] \quad \text{.... (i)}$$

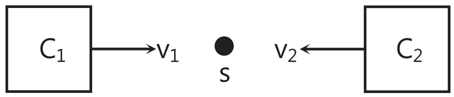
$$\text{and } 1.8 = f\left[\frac{300}{300 + v_T}\right] \quad \text{..... (ii)}$$

Here, $v_T = v_s$ = velocity of source/train

Solving Eqs. (i) and (ii), we get

$v_T = 30$ m/s

Sol 13: Firstly, car will be treated as an observer which is approaching the source. Then, it will be treated as a source, which is moving in the direction of sound.



Hence, $f_1 = f_0\left(\frac{v+v_1}{v-v_1}\right)$

$f_2 = f_0\left(\frac{v+v_2}{v-v_2}\right)$

$\therefore\ f_1 - f_2 = \left(\frac{1.2}{100}\right) f_0 = f_0\left[\frac{v+v_1}{v-v_1} - \frac{v+v_2}{v-v_2}\right]$

or $\left(\frac{1.2}{100}\right) f_0 = \frac{2v(v_1-v_2)}{(v-v_1)(v-v_2)} f_0$

As v_1 and v_2 are very very less than v.

We can write, $(v - v_1)$ or $(v - v_2) \approx v$

$\therefore\ \left(\frac{1.2}{100}\right) f_0 = \frac{2(v_1-v_2)}{v} f_0$

or $(v_1 - v_2) = \frac{v\times 1.2}{200} = \frac{330\times 1.2}{200} = 1.98\ ms^{-1}$

$= 7.128\ kmh^{-1}$

$\therefore$ The nearest integer is 7

Sol 14: (A) $f = \frac{320}{320-10}\times 8\times 10^3 \times \frac{320+10}{320} = 8.5\text{kHz}$

So 15: (B, D) At the open end, the phase of a pressure wave changes by π radian due to reflection. At the closed end, there is no change in the phase of a pressure wave due to reflection.

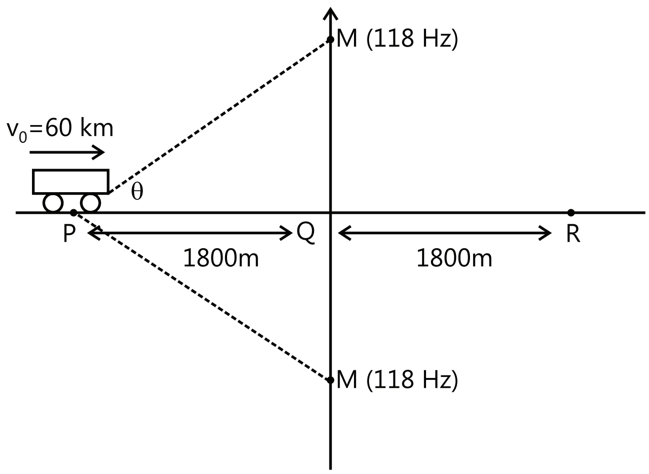
Sol 16: (D) $\ell = \frac{1}{4v}\sqrt{\frac{\gamma RT}{M}}$

Calculations for $\frac{1}{4v}\sqrt{\frac{\gamma RT}{M}}$ for gases mentioned in options A, B, C and D, work out to be 0.459 m, 0.363 m 0.340 m & 0.348 m respectively. As ℓ = (0.350 ± 0.005) m; Hence correct option is D.

Sol 17: (A, B, D) Frequency of M received by car

$f_1 = 118\left(\frac{V+V_0\cos\theta}{V}\right)$

$f_2 = 121\left(\frac{V+V_0\cos\theta}{V}\right)$



No. of beats $n = \Delta f = f_2 - f_1$

$n = 3\left(\frac{V+V_0\cos\theta}{V}\right)$

$n = 3\left(1+\frac{V_0}{V}\cos\theta\right)$

As θ ↑, cos θ ↓, n ↓

Rate of change of beat frequency $\frac{dn}{d\theta} = 3\left[\frac{V_0}{V}(-\sin\theta)\right]$

$\frac{dn}{d\theta}$ is maximum when sinθ = 1; θ=90°

i.e. car is at point Q.

$\nu_p = 3\left(1+\frac{V_0}{V}\cos\theta\right)$

$\nu_R = 3\left(1-\frac{V_0}{V}\cos\theta\right)$

At Q

No. of beats ν_Q =121-118 = 3

$\nu_Q = \frac{\nu_P + \nu_R}{2}$

Sol 18: (A, D) Larger the length of air column, feebler is the intensity.

Sol 19: (B)

$$\frac{v_S}{4L_P} = \frac{2\sqrt{\frac{T}{\mu}}}{2\ell_S}$$

$$\mu\ell_S = 10\,gm$$

Sol 20: (B)

$$\frac{V}{4(\ell + e)} = f$$

$$\Rightarrow \ell + e = \frac{V}{4f} \Rightarrow \ell = \frac{V}{4f} - e$$

Here $e = (0.6)r = (0.6)(2) = 1.2$ cm

So $\ell = \frac{336 \times 10^2}{4 \times 512} - 1.2 = 15.2\,cm$

Sol 21: (A, B) If wind blows from source to observer

$$f_2 = f_1\left(\frac{V + w + u}{V + w - u}\right)$$

When wind blows from observer towards source

$$f_2 = f_1\left(\frac{V - w + u}{V - w - u}\right)$$

In both cases, $f_2 > f_1$.

Sol 22: First and fourth wave interfere destructively. So from the interference of 2nd and 3rd wave only,

$$\Rightarrow I_{net} = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\cos\left(\frac{2\pi}{3} - \frac{\pi}{3}\right) = 3I_0$$

$$\Rightarrow n = 3$$