# UNIT - 5 : NEWTON'S LAWS OF MOTION(WITHOUT FRICTION) [JEE – MAIN CRASH COURSE]

## Inertia

The inherent property of all the bodies by virtue of which they cannot change their state of rest or uniform motion along a straight line by their own is called inertia.

- Inertia is not a physical quantity, it is only a property of the body which depends on mass of the body.
- Inertia has no units and no dimensions.
- Two bodies of equal mass, one in motion and another is at rest, possess same inertia because it is a factor of mass only and does not depend upon the velocity.

## Linear Momentum

Linear momentum of a body is the quantity of motion contained in the body.

- It is measured in terms of the force required to stop the body in unit time.
- It is measured as the product of the mass of the body and its velocity, i.e., momentum = mass × velocity.
- If a body of mass $m$ is moving with velocity $\vec{v}$ then its linear momentum $\vec{p}$ is given by $\vec{p} = m\vec{v}$.
- Linear momentum is a vector quantity and its direction is the same as the direction of velocity of the body.
- If two objects of different masses have same momentum, the lighter body possesses greater velocity.

$$p = m_1 v_1 = m_2 v_2 = \text{constant}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{m_2}{m_1} \text{ i.e., } v \propto \frac{1}{m} \qquad [\text{As } p \text{ is constant}]$$

## Newton's First Law (Law of Inertia)

A body continues to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some external force to change the state.

- If no net force acts on a body, then the velocity of the body cannot change, i.e., the body cannot accelerate.
- Newton's first law defines inertia and is rightly called the law of inertia.

## Type of inertia

**Inertia of rest** It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest and cannot start moving by its own.

**Inertia of motion** It is the inability of a body to change itself its state of uniform motion, i.e., a body in uniform motion can neither accelerate nor retard by its own.

**Inertia of direction** It is the inability of a body to change by itself the direction of motion.

## Newton's Second Law

The rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force.

If a body of mass $m$, moves with velocity $\vec{v}$, then its linear momentum can be given by $\vec{p} = m\vec{v}$ and if force $\vec{F}$ is applied on a body, then

$$\vec{F} \propto \frac{d\vec{p}}{dt}$$

$$\Rightarrow \quad F = K\frac{d\vec{p}}{dt}$$

or $$\vec{F} = \frac{d\vec{p}}{dt} \quad (K = 1 \text{ in CGS and SI units})$$

$$= \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$$

(as $a = \frac{d\vec{v}}{dt} =$ acceleration produced in the body)

$$\therefore \quad \vec{F} = m\vec{a}$$

Force = mass × acceleration

## Force

Force is an external effect in the form of a push or pulls which produces or tries to produce motion in a body at rest, stops or tries to stop a moving body, and changes or tries to change the direction of motion of the body.

- $\vec{F} = m\vec{a}$ formula is valid only if force is changing the state of rest or motion and the mass of the body is constant and finite.
- If $m$ is not constant $\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$
- If force and acceleration have three components along $x$, $y$, and $z$ axis, then

  $$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \text{ and } \vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \quad (1)$$

  From (1) it is clear that

  $$F_x = ma_x, F_y = ma_y, F_z = ma_z$$

- No force is required to move a body uniformly along a straight line.

  $$\vec{F} = ma$$

  $$\therefore \quad \vec{F} = 0 \quad (\text{as } a = 0)$$

- Out of so many natural forces, for distance $10^{-15}$ m, nuclear force is the strongest while gravitational force is the weakest.

  $$F_{\text{nuclear}} > F_{\text{electromagnetic}} > F_{\text{gravitational}}$$

- Ratio of electric force and gravitational force between two electron, $F_e/F_g = 10^{43}$.

  $$\therefore \quad F_e \gg F_g$$

**Constant force** If the direction and magnitude of a force is constant. It is said to be a constant force.

**Central force** If a position-dependent force is always directed toward or away from a fixed point it is said to be central otherwise non-central.

**Conservative or non-conservative force** If under the action of a force the work done in a round trip is zero or the work is path independent, the force is said to be conservative otherwise non-conservative.

***Example*:** Conservative force: Gravitational force, electric force, elastic force.
Non-conservative force: Frictional force, viscous force.

## Equilibrium of concurrent force.

- If all the forces working on a body are acting on the same point, then they are said to be concurrent.
- A body, under the action of concurrent forces, is said to be in equilibrium when there is no change in the state of rest or of uniform motion along a straight line.
- The necessary condition for the equilibrium of a body under the action of concurrent forces is that the vector sum of all the forces acting on the body must be zero.
- Mathematically, for equilibrium, $\sum F_{\text{net}} = 0$

  or $\sum F_x = 0$ ; $\sum F_y = 0$ ; $\sum F_z = 0$

- Three concurrent forces will be in equilibrium if they can be represented completely by three sides of a triangle taken in order (Fig. 1).

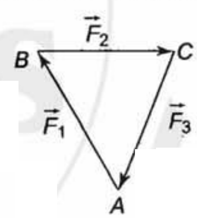


Fig. 1

- Lami's theorem: For concurrent forces,

  $$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

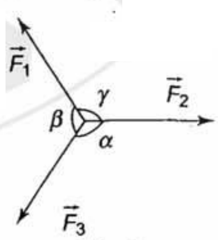


Fig. 2

## Newton's Third Law.

To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.

- When a body exerts a force on any other body, the second body also exerts an equal and opposite force on the first.
- Forces in nature always occur in pairs. A single isolated force is not possible.
- Any agent, applying a force also experiences a force of equal magnitude but in opposite direction. The force applied by the agent is called *Action* and the counter force experienced by it is called *reaction.*
- Action and reaction never act on the same body. If it were so, the total force on a body would have always been zero, i.e., the body will always remain in equilibrium.
- If $\vec{F}_{AB}$ = force exerted on body $A$ by body $B$ (Action) and $\vec{F}_{BA}$ = force exerted on body $B$ by body $A$ (Reaction), then according to Newton's third law of motion, $\vec{F}_{AB} = -\vec{F}_{BA}$

## Frame of Reference

A frame in which an observer is situated and makes his observations is known as his Frame of reference.

The reference frame is associated with a co-ordinate system and a clock to measure the position and time of events happening in space. We can describe all the physical quantities such as position, velocity, acceleration etc. of an object in this coordinate system.

Frame of reference are of two types: (1) Inertial frame of reference and (2) non-inertial frame of reference.

### Inertial frame of reference

- A frame of reference which is at rest or which is moving with a uniform velocity along a straight line is called an inertial frame of reference.
- In inertial frame of reference Newton's laws of motion hold good.
- Inertial frame of reference are also called unaccelerated frame of reference or Newtonian or Galilean frame of reference.
- Ideally no inertial frame exists in universe. For practical purpose, a frame of reference may be considered as inertial if its acceleration is negligible with respect to the acceleration of the object to be observed.

### Non-inertial frame of reference

- Accelerated frame of references are called non-inertial frame of reference.
- Newton's laws of motion are not applicable in non-inertial frame of reference.

## Impulse.

When a large force works on a body for very small time interval, it is called impulsive force.

An impulsive force does not remain constant, but changes first from zero to maximum and then from maximum to zero. In such case, we measure the total effect of force.

- Impulse of a force is a measure of total effect of force.
- $\vec{I} = \int_{t_1}^{t_2} \vec{F}\, dt$ .
- Impulse is a vector quantity and its direction is same as that of force.
- Force–time graph: Impulse is equal to the area under $F$–$t$ curve (Fig. 3).

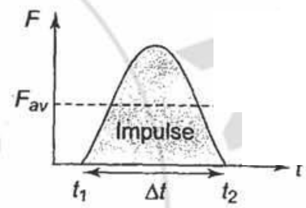


Fig. 3

If $F_{av}$ is the average magnitude of the force, then

$$I = \int_{t_1}^{t_2} F\, dt = F_{av}\int_{t_1}^{t_2} dt = F_{av}\Delta t$$

- From Newton's second law $\vec{F} = \frac{d\vec{p}}{dt}$

or $\int_{t_1}^{t_2} \vec{F}\, dt = \int_{p_1}^{p_2} d\vec{p} \Rightarrow \vec{I} = \vec{p}_2 - \vec{p}_1 = \overline{\Delta p}$

i.e., the impulse of a force is equal to the change in momentum. This statement is known as *impulse momentum theorem*.

## Law of Conservation of Linear Momentum

If no external force acts on a system (called isolated) of constant mass, the total momentum of the system remains constant with time.

- According to this law, for a system of particles, $\vec{F} = \frac{d\vec{p}}{dt}$

In the absence of external force, $\vec{F} = 0$, then $\vec{p} =$ constant

i.e., $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots =$ constant

or $m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots =$ constant (2)

Equation (2) shows that in absence of external force for a closed system the linear momentum of individual particles may change but their sum remains unchanged with time.

- Law of conservation of linear momentum is independent of the frame of reference though linear momentum depends on the frame of reference.
- Conservation of linear momentum is equivalent tc Newton's third law of motion.

For a system of two particles in absence of external force by law of conservation of linear momentum.

$$\vec{p}_1 + \vec{p}_2 = \text{constant.}$$

$$\therefore \quad m_1\vec{v}_1 + m_2\vec{v}_2 = \text{constant.}$$

Differentiating above with respect to time,

$$m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} = 0$$

$$\Rightarrow \quad m_1\vec{a}_1 + m_2\vec{a}_2 = 0$$

$$\Rightarrow \quad \vec{F}_1 + \vec{F}_2 = 0$$

$$\therefore \quad \vec{F}_2 = -\vec{F}_1$$

i.e., for every action there is equal and opposite reaction which is Newton's third law of motion.

## Apparent Weight of a Body in a Lift.

When a body of mass $m$ is placed on a weighing machine which is placed in a lift, then actual weight of the body is $mg$.

This acts on a weighing machine which offers a reaction $R$ given by the reading of weighing machine. This reaction exerted by the surface of contact on the body is the *apparent weight* of the body.

## Acceleration of Block on Smooth Inclined Plane.

**When an inclined plane is given a horizontal acceleration $b$** Since the body lies in an accelerating frame, an inertial force ($mb$) acts on it in the opposite direction (Fig. 4).

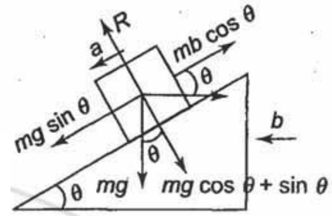


Fig. 4

Normal reaction $R = mg\cos\theta + mb\sin\theta$

and $ma = mg\sin\theta - mb\cos\theta$

$\therefore \quad a = g\sin\theta - b\cos\theta$

*Note:* The condition for the body to be at rest relative to the inclined plane:

$a = g\sin\theta - b\cos\theta = 0$

$\therefore \quad b = g\tan\theta$

## Wedge Constraint

Conditions for the application of wedge constraint are as follows:

- There is a regular contact between two objects.
- Objects are rigid.

The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact. In other words, components of velocity along the perpendicular direction to the contact plane of two objects is always equal if there are no deformations and they remain in contact (Fig. 5).

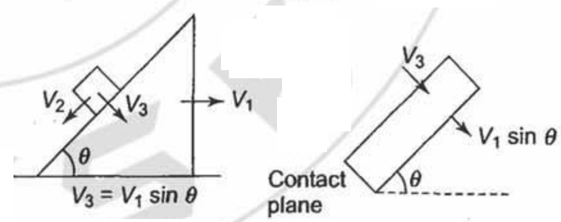


Fig. 5

*Note:* If a rod moves vertically downward on the surface of movable wedge of mass as shown in Fig. 6, the relation between the velocity of rod and that of the wedge at any instant can be found using the concept above.

## Pulley Constraint

In cases where pulley moves along with the blocks connected on both sides, we can say the displacement of the pulley is the average of the displacement on both sides of the pulley (Fig. 7).

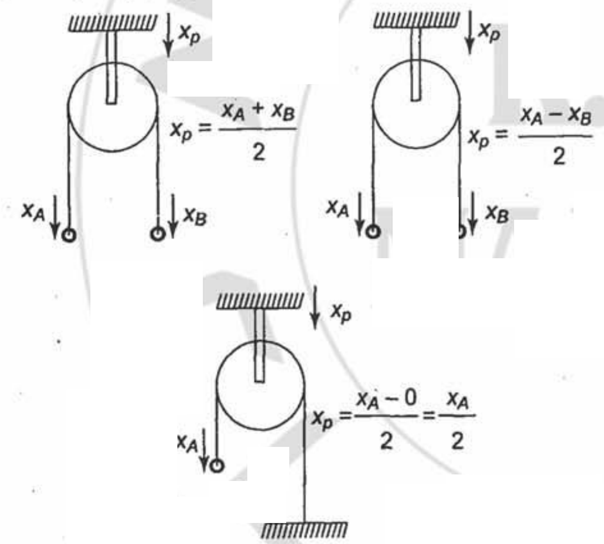


Fig. 7

If one end of the string is connected with fixed end, the displacement of that end can be considered as zero.

## Applications of the pulley constraints

### Case I

- As pulley 1 is fixed, hence the displacement should be zero (Fig. 8). If the displacement of block $A$ is $x_A$ (down), then the displacement of other end should

$$x_{p_1} = 0 = \frac{x_A + x_D}{2} \Rightarrow x_D = -x_A.$$

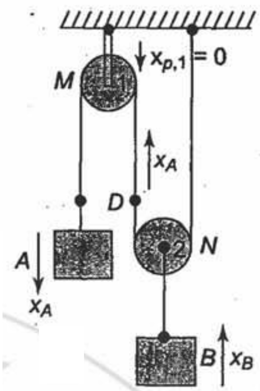


Fig. 8

- Displacement of block $B$ = Displacement of pulley 2

$$x_{p2} = \frac{x_A + 0}{2} \Rightarrow x_A = 2x_{p2}$$

$$\Rightarrow x_A = 2x_B$$

### Case II

- As pulley 1 is fixed, $|x_{p,2}| = |x_A|$

 f block $A$ moves up by $x_A$, pulley 2 should move $x_A$ in downward direction.

- 'or pulley 2

$$x_{p2} = x_A = \frac{x_B + 0}{2}$$

$$x_B = 2x_A$$

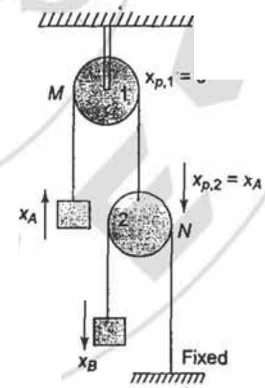


Fig. 9

## Some Useful Cases in Constraint Relation

In cases where distance between two points is always fixed, we can say the relative velocity of one point of an object with respect to any other point of the same object in the direction of the line joining them will always remain zero, as their separation always remains constant.

## Application of constraint relation

### Case I

Consider a rod of length $l$ resting on a wall and the floor (Fig. 10). Its lower end $A$ is pulled towards left with a constant velocity $u$. Result of this end $B$ starts moving down along the wall. Let us find the velocity of the other end $B$ downward when the rod makes an angle $\theta$ with the horizontal.

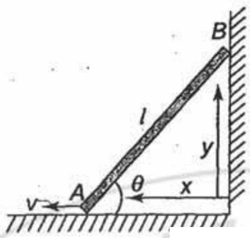

Fig. 10

Here the distance between the points $A$ and $B$ of the rod always remains constant, thus the two points must have same velocity components in the direction of their line joining i.e., along the length of the rod.

If point $B$ is moving down with velocity $v_B$, its component along the length of the rod is $v_B \sin\theta$. Similarly, the velocity component of point A along the length of rod is $v \cos\theta$. Thus, we have

$$v_B \sin\theta = u\cos\theta$$

or $$v_B = u\cot\theta$$

### Case II

Consider a ball of mass $m_1$ and a block of mass $m_2$ are joined together with an inextensible string (Fig. 11). The ball can slide on a smooth horizontal surface. If $v_1$ and $v_2$ are the respective speeds of the ball and the block, let us find the constraint relation between the two.

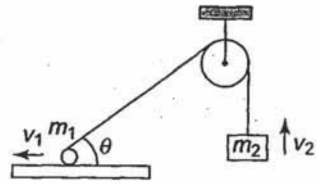

Fig. 11

As length of the string is constant, hence the velocity of end points along the string is same. Obviously, from Fig. 12, $v_1\cos\theta = v_2$.

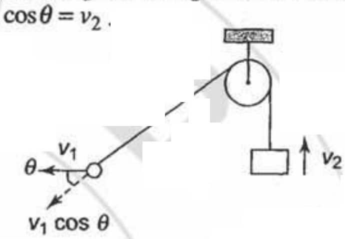

Fig. 12